

Finite difference kernel:

$$\frac{\partial^2 u}{\partial p^2} \approx \frac{u_{p-1} - 2u_p + u_{p+1}}{\Delta p^2}$$

1D Wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$\frac{u_{x,t-1} - 2u_t + u_{x,t+1}}{\Delta t^2} = c^2 \frac{u_{x-1,t} - 2u_x + u_{x+1,t}}{\Delta x^2}$$

$$0 = \frac{u_{x,t-1} - 2u_t + u_{x,t+1}}{\Delta t^2} - c^2 \frac{u_{x-1,t} - 2u_x + u_{x+1,t}}{\Delta x^2}$$

$$0 = \Delta x^2 (u_{x,t-1} - 2u_t + u_{x,t+1}) - \Delta t^2 c^2 (u_{x-1,t} - 2u_x + u_{x+1,t})$$

$$0 = \Delta x^2 (u_{x,t-1} + u_{x,t+1}) - \Delta t^2 c^2 (u_{x-1,t} + u_{x+1,t}) + (-2\Delta x^2 + 2c^2 \Delta t^2) u_{x,t}$$

$$0 = \Delta x^2 (u_{x,t-1} + u_{x,t+1}) - \Delta t^2 c^2 (u_{x-1,t} + u_{x+1,t}) + 2(c^2 \Delta t^2 - \Delta x^2) u_{x,t}$$