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Dis 2A

SECTION 2.5

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a

Let $y(t)$ is the amount of sugar in the tank. $\frac{dy}{dt}$ is the rate of change of the sugar in the tank

$$\frac{dy}{dt} = 0.6 - 3\frac{y}{100}$$

$$\Rightarrow y(t) = 20(1 - e^{\frac{3t}{100}})$$

$$y(20) = 10(1 - e^{-0.6})(lb)$$

b

$$20(1 - e^{\frac{3t}{100}}) = 15$$

$$t = \frac{100In4}{3}(mm)$$

c

$$t \to \infty, y(t) \to 20$$

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Let $y(t)$ is the amount of salt in the tank. $\frac{dy}{dt}$ is the rate of change of the salt in the tank

$$\frac{dy}{dt} = 2 - \frac{x}{10+t}$$

$$y(t) = 10 + t - \frac{100}{(10+t)}$$

$$y(15) = 21(lb)$$

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a

Let $y(t)$ is the amount of pollutant in the tank.
$\frac{dy}{dt}$ is the rate of change of the pollutant in the tank.

$$\frac{dy}{dt} = 3 - \frac{4x}{50+t}$$

$$y(t) = \frac{3}{5}t + 30 - \frac{187500000}{(50+t)^4}$$

b

$$\frac{dy}{dt} = -\frac{2x}{30-t}$$

$$y(t) = \frac{215342}{9000000}(30-t)^2$$

$$t = 8.7868(min)$$

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Let $y(t)$ is the amount of salt in the tank I
$\frac{dy}{dt}$ is the rate of change of the salt in the tank I.

$$\frac{dy}{dt} = ab - \frac{b}{V}y$$

$$y(t) = aV + Ke^{-\frac{b}{V}t}$$

Because $y(0) = 0$

$$K = -aV$$

Let $z(t)$ is the amount of salt in the tank $\mathbb{I}$
$\frac{dz}{dt}$ is the rate of change of the salt in the tank $\mathbb{I}$.

$$\frac{dz}{dt} = \frac{b}{V}x - \frac{b}{V}y$$

$$y = aV - abte^{-\frac{b}{V}t} - aVe^{-\frac{b}{V}t}$$

SECTION 2.6

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$$\frac{\partial(In(xy) + x^2y^3)}{\partial x}$$

$$\Rightarrow (\frac{1}{x} + 2xy^3)dx$$

$$\frac{\partial(In(xy) + x^2y^3)}{\partial y}$$

$$\Rightarrow (\frac{1}{y} + 3x^2y^2)dy$$

$$dF = (\frac{1}{x} + 2xy^3)dx + (\frac{1}{y} + 3x^2y^2)dy$$

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$$\frac{\partial(tan^{-1}(\frac{x}{y})+y^4)}{\partial x}$$
$$\Rightarrow (\frac{y}{x^2+y^2})dx$$
$$\frac{\partial(tan^{-1}(\frac{x}{y})+y^4)}{\partial y}$$
$$\Rightarrow (-\frac{x}{x^2+y^2}+4y^3)dy$$
$$dF = (\frac{y}{x^2+y^2})dx + (-\frac{x}{x^2+y^2}+4y^3)dy$$

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$$ø(u,v) = \int \frac{2u}{u^2+v^2}du + g(v)$$
$$= In|v^2+u^2| + g(v)$$
$$\frac{\partial ø}{\partial v} = \frac{2v}{v^2+u^2} + g'(v)$$
$$\Rightarrow g'(v) = 0 \Rightarrow g(v) = C$$
$$ø(u,v) = In|v^2+u^2| + C$$

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$$\frac{(3y+y)(y+1)}{x^4}dx + \frac{-2(y+1)}{x^3}dy$$
$$ø(x,y) = \int \frac{(3y+y)(y+1)}{x^4}dx + g(y)$$
$$= -\frac{4y(1+y)}{3x^3} + g(y)$$
$$\frac{\partial ø}{\partial y} = \frac{4+8y}{3x^3} + g'(y) = \frac{-2(y+1)}{x^3}$$
$$g'(y) = \int(-\frac{2(y+1)}{x^3} - \frac{4+8y}{3x^3})dy$$
$$g(y) = -\frac{2(\frac{7y^2}{2}+5y)}{3x^3}$$
$$ø(x,y) = -\frac{2(\frac{7y^2}{2}+5y)}{3x^3} - \frac{4y(1+y)}{3x^3}$$

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$$M = y^2+2xy, N = -x^2$$
$$\frac{\partial M}{\partial y} = 2y+2x, \frac{\partial N}{\partial x} = -2x$$
$$\mu = e^{-\int \frac{2(2x+y)}{y(2x+y)}dx}$$
$$= e^{-2Iny} = \frac{1}{y^2}$$
$$(1+\frac{2x}{y})dx - \frac{x^2}{y^2}dy = 0$$
$$F(x,y) = \frac{xy+x^2}{y} = C$$
$$\mu(y) = \frac{1}{y^2}$$

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$$y = vx, \frac{dy}{dx} = v + \frac{dv}{dx}$$
$$\Rightarrow v + x\frac{dv}{dx} = \frac{v+v^3}{v^2-2}$$
$$x\frac{dv}{dx} = \frac{3v}{v^2-2}$$
$$\int(\frac{v^2-2}{3v})dv = \int \frac{dx}{x}$$
$$\Rightarrow \frac{v^2}{6} - \frac{2}{3}In(v) = In(x) + In(C)$$
$$y^{\frac{2}{3}} = Cx^{-\frac{1}{3}}e^{\frac{y^2}{6x^2}}$$