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**Dis 2A**

4.4

8

16

a:

$$k = 2kg/s^2$$

b:

$$y'' + 3y' + 2y = 0 \Rightarrow w_0 = \sqrt{2}, c = 1.5$$

$$\lambda^2 + 3\lambda + 2 = 0 \Rightarrow \lambda_1 = -1, \lambda_2 = -2$$

$$y(t) = C_1 e^{-t} + C_2 e^{-2t} \Rightarrow y'(t) = -C_1 e^{-t} - 2C_2 e^{-2t}$$

$$y(t) = -3e^{-t} + 2e^{-2t}$$

$$\tan \phi = -\frac{1}{\sqrt{3}} \Rightarrow \phi = -\frac{\pi}{6}$$

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$$0.2y'' + 5y = 0 \text{ or } y'' + 25y = 0$$

$$\lambda^2 + 25 = 0 \Rightarrow \lambda = \pm 5$$

$$y(t) = C_1 \cos 5t + C_2 \sin 5t$$

$$\Rightarrow y'(t) = -5C_1 \sin 5t + 5C_2 \cos 5t$$

$$y(0) = 0.5 \Rightarrow C_1 = 0.5, y'(0) = 0 \Rightarrow C_2 = 0$$

$$y(t) = 0.5 \cos 5t$$

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$$k = 1.5kg/s$$

$$0.1y'' + 1.5y' + 9.8y = 0 \text{ or } y'' + 15y' + 98y = 0$$

$$\lambda^2 + 15\lambda + 98 = 0 \Rightarrow \lambda = \frac{-15 \pm \sqrt{167}i}{2}$$

$$y(t) = e^{-\frac{15t}{2}} \left( C_1 \cos \frac{167}{2}t + C_2 \sin \frac{167}{2}t \right)$$

$$C_1 = \frac{1}{10}C_2 \approx 0.12$$

$$A = \sqrt{C_1^2 + C_2^2} \approx 0.15, \phi \approx 0.86$$

I. 4.5

A. 2

$$y(t) = ae^{-t} \Rightarrow y'(t) = -ae^{-t} \Rightarrow y''(t) = ae^{-t}$$

$$(ae^{-t}) + 6(-ae^{-t}) + 8(ae^{-t}) = -3e^{-t}$$

$$a = 3 \Rightarrow y_p = 3e^{-t}$$

B. 8

$$y(t) = a\cos 3t + b\sin 3t$$

$$y'(t) = -3a\sin 3t + 3b\cos 3t$$

$$y''(t) = -9a\cos 3t - 9b\sin 3t$$

$$-9a\cos 3t - 9b\sin 3t + 7(-3a\sin 3t + 3b\cos 3t) +$$

$$6(y(t) = a\cos 3t + b\sin 3t) = -4\sin 3t$$

$$\Rightarrow a + 21b = 0 \text{ and } -21a + b = -4$$

$$a = \frac{42}{221}, b = -\frac{2}{221}$$

$$y_p = \frac{42}{221}\cos 3t - \frac{2}{221}\sin 3t$$

C. 12

$$z = Ae^{i2t}$$

$$((2i)^2 + 7(2i) + 6)A = 3 \Rightarrow A = \frac{3}{100} - \frac{21i}{100}$$

$$z = (\frac{3}{100} - \frac{21i}{100})(\cos 2t + i\sin 2t)$$

$$y(t) = -\frac{21}{100}\cos 2t + \frac{3}{100}\sin 2t$$

D. 16

$$y(t) = at^2 + bt + c$$

$$y'(t) = 2at + b$$

$$y''(t) = 2a$$

$$2a + 5(2at + b) + 6(at^2 + bt + c) = -t^2 + 4$$

$$a = -\frac{1}{6}, b = \frac{5}{18}, c = \frac{53}{108}$$

$$y(t) = -\frac{1}{6}t^2 + \frac{5}{18}t + \frac{53}{108}$$

E. 20

$$y_h = e^{-t}(C_1 \cos t + C_2 \sin t)$$

$$z = Ae^{i2t} \Rightarrow z' = (2i)Ai2t \Rightarrow z'' = 4i^2 Ae^{i2t}$$

$$A = -\frac{1}{5} - \frac{2}{5}i$$

$$y_p = \frac{1}{5}\cos 2t + \frac{2}{5}\sin 2t$$

$$y(t) = e^{-t}(C_1 \cos t + C_2 \sin t) + \frac{1}{5}\cos 2t + \frac{2}{5}\sin 2t$$

$$C_1 = -\frac{9}{5}, C_2 = -\frac{13}{5}$$

$$y(t) = e^{-t}(-\frac{9}{5}\cos t + -\frac{13}{5}\sin t) + \frac{1}{5}\cos 2t + \frac{2}{5}\sin 2t$$

F. 28

$$y_h = (C_1 + C_2t)e^{-2t}$$

$$y_p(t) = Ate^{-2t}$$

$$y'_p(t) = 2Ae^{-2t}(t - t^2)$$

$$y + p''(t) = 2Ae^{-2t}(2t^2 - 4t + 1)$$

$$A = 1$$

$$y_p = t^2e^{-2t}$$

G. 33

$$y = at + b$$

$$25at + 25b = 2 + 3t$$

$$y = \frac{2}{25}t + \frac{3}{25}$$

$$\lambda^2 + 25 = 0 \Rightarrow \lambda = \pm 5i$$

$$z = Ate^{5it}$$

$$A = -\frac{1}{10i}$$

$$z = -\frac{1}{10}it(\cos 5t + i\sin 5t)$$

$$y = \frac{2}{25}t + \frac{3}{25} + \frac{1}{10}t\sin 5t$$

H. 40

$$y = (at + b)e^{-3t}$$

$$y' = e^{-3t}((a - 3b) - 3at)$$

$$y'' = 3e^{-3t}((-2a + 3b) + 3at)$$

$$a = \frac{1}{20}, b = \frac{9}{400}$$

$$y = \left(\frac{1}{20}t + \frac{9}{400}\right)e^{-3t}$$