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Dis 2A

4

$y(t) = Ae^{2t}$
$y'(t) = 2Ae^{2t}$
$y''(t) = 4Ae^{2t}$
plug into $y'' + 3y' - 18y = 18e^{2t}$
$4Ae^{2t} + 6Ae^{2t} - 18Ae^{2t} = 18e^{2t}$
$-8A = 18 \Rightarrow A = -\frac{9}{4}$
$y(t) = -\frac{9}{4}e^{2t}$

6

$y(t) = acos2t + bsin2t$
$y'(t) = -2asin2t + 2bcos2t$
$y''(t) = -4acos2t - 4bsin2t$
plug into $y'' + 9y = sin2t$
$-4acos2t - 4bsin2t + 9acost2t + 9bsin2t = sin2t$
$\begin{cases} 5b = 1 \\ 5a = 0 \end{cases} \Rightarrow \begin{cases} a = 0 \\ b = \frac{1}{5} \end{cases}$
$y(t) = \frac{1}{5}sin2t$

14

$y(t) = At + B \Rightarrow y'(t) = A \Rightarrow y''(t) = 0$
plug into $y'' + 5y' + 4y = 2 + 3t$
$4At + 5A + 4B = 3t + 2$
$\begin{cases} 4A = 3 \\ 5A + 4B = 2 \end{cases} \Rightarrow \begin{cases} A = \frac{3}{4} \\ B = -\frac{7}{16} \end{cases}$
$y(t) = \frac{3}{4}t - \frac{7}{16}$

18

$\lambda^2 + 3\lambda + 2 = 0 \Rightarrow (\lambda + 1)(\lambda + 2) = 0$
$\lambda_1 = -1, \lambda_2 = -2$
$y_g(t) = C_1e^{-t} + C_2e^{-2t}$
$y_p(t) = Ae^{-4t}$
$y_p'(t) = -4Ae^{-4t}$
$y_p''(t) = 16Ae^{-4t}$
plug into $y'' + 3y' + 2y = 3e^{-4t}$
$16Ae^{-4t} - 12Ae^{-4t} + 2Ae^{-4t} = 3e^{-4t}$
$6A = 3 \Rightarrow A = \frac{1}{2}$
$y(t) = C_1e^{-t} + C_2e^{-2t} + \frac{1}{2}e^{-4t}$
$y'(t) = -C_1e^{-t} - 2C_2e^{-2t} - 2e^{-4t}$
Because $y(0) = 1$ and $y'(0) = 0$,
$C_1 = 3, C_2 = -\frac{5}{2}$
$y(t) = 3e^{-t} - \frac{5}{2}e^{-2t} + \frac{1}{2}e^{-4t}$

22

$\lambda^2 + 4\lambda + 4 = 0 \Rightarrow (\lambda + 2)^2 = 0$
$\lambda = -2$
$y_g(t) = C_1e^{-2t} + C_2te^{-2t}$
$y_p(t) = At + B \Rightarrow y_p'(t) = A \Rightarrow y_p''(t) = 0$
plug into $y'' + 4y' + 4y = 4 - t$
$4A + 4(At + B) = 4 - t$
$\begin{cases} 4A = -1 \\ 4B = 5 \end{cases} \Rightarrow \begin{cases} A = -\frac{1}{4} \\ B = \frac{5}{4} \end{cases}$
$y(t) = C_1e^{-2t} + C_2te^{-2t} - \frac{1}{4}t + \frac{5}{4}$
$y'(t) = -2C_1e^{-2t} + C_2e^{-2t} - 2C_2te^{-2t} - \frac{1}{4}$
Because $y(0) = -1$ and $y'(0) = 0$
$C_1 = -\frac{9}{4}, C_2 = \frac{17}{4}$
$y(t) = -\frac{9}{4}e^{-2t} + \frac{17}{4}te^{-2t} - \frac{1}{4}t + \frac{5}{4}$

26

$y(t) = R\left\{4e^{2it}\right\} = R\{4(cos2t + isin2t)\}$
$y_p(t) = Ate^{2it}$
$y_p'(t) = At^{2it} + 2itAe^{2it}$
$y_p''(t) = 4iAe^{2it} - 4tAe^{2it}$
$4iAe^{2it} - 4tAe^{2it} + 4Ate^{2it} = 4e^{2it}$
$A = -i$
$z(t) = -it(cos2t + isin2t) = -itcos2t + sin2t$
$R\{z(t)\} = y_p(t) = tsin2t$