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Dis 2A

2.6

4

$$\frac{\partial F(x, y)}{\partial x} = x(x^2 + y^2)^{-\frac{3}{2}} dx$$

$$\frac{\partial F(x, y)}{\partial x} = y(x^2 + y^2)^{-\frac{3}{2}} dy$$

$$F'(x, y) = x(x^2 + y^2)^{-\frac{3}{2}} dx + y(x^2 + y^2)^{-\frac{3}{2}} dy$$

22

$$\frac{1}{xy^2}(y^2 - xy)dx + \frac{1}{xy^2}x^2 dy = 0$$

$$\Rightarrow \left(\frac{1}{x} + \frac{1}{y}\right)dx + \frac{x}{y^2}dy = 0$$

$$\phi(x, y) = \int \left(\frac{1}{x} + \frac{1}{y}\right)dx + g(y)$$

$$\Rightarrow \phi(x, y) = -\frac{x}{y} + \ln|x| + g(y)$$

$$\frac{\partial \phi}{\partial y} = \frac{x}{y^2} + g'(y) = \frac{x}{y^2}$$

$$g(y) = C$$

$$\phi(x, y) = -\frac{x}{y} + \ln|x| + C$$

26

Let $M = y$ and $N = (x^2y - x)$

$$\frac{\partial M}{\partial y} = 1, \frac{\partial N}{\partial x} = 2xy - 1$$

$$\frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{2}{x}$$

$$\Rightarrow I = \int -\frac{2}{x} dx = \frac{1}{x^2}$$

$$\frac{y}{x^2} dx + \frac{xy - 1}{x} dy = 0$$

$$\phi(x, y) = \int \frac{y}{x^2} dx + g(y) = -\frac{y}{x} + g(y)$$

$$\frac{\partial \phi}{\partial x} = -\frac{1}{x} + g'(y) = \frac{xy - 1}{x}$$

$$\Rightarrow g'(y) = y, g(y) = \frac{y^2}{2}$$

$$\phi(x, y) = \frac{y}{x} + \frac{y^2}{2}$$

28

Let $M = 2y$ and $N = (x + y)$

$$\frac{\partial M}{\partial y} = 2, \frac{\partial N}{\partial x} = 1$$

$$-\frac{1}{M} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = \frac{1}{2y}$$

$$I = \int \frac{1}{2y} dy = \frac{1}{\sqrt{y}}$$

$$\frac{2y}{\sqrt{y}} dx + \frac{x + y}{\sqrt{y}} dy = 0$$

$$\phi(x, y) = \int \frac{2y}{\sqrt{y}} dx + g(y) = \frac{2xy}{\sqrt{y}} + g(y)$$

$$\frac{\partial \phi}{\partial y} = 3x\sqrt{y} + g'(y) = \frac{x + y}{\sqrt{y}}$$

$$\Rightarrow g(y) = 2x\sqrt{y} + \frac{2}{3}y^{3/2} - 2xy^{3/2}$$

$$\phi(x, y) = 2x\sqrt{y} + \frac{2}{3}y^{3/2}$$

30

$$\frac{\partial M}{\partial y} = 2, \frac{\partial N}{\partial x} = 3$$

$$\mu(x, y) = xy^2$$

$$2xy^3 dx + 3x^2 y^2 dy = 0$$

$$\phi(x, y) = \int 2xy^3 dx + g(y) = x^2 y^3 + g(y)$$

$$\frac{\partial \phi}{\partial y} = 3x^2 y^2 + g'(y) 3x^2 y^2$$

$$g'(y) = 0, g(y) = C$$

$$\phi(x, y) = x^2 y^3 + C$$

42

$$\frac{dy}{dx} = -\frac{2x}{y}$$

$$\phi(x, y) = 2x^2 + y^2 = C$$

34

$$f_1(x, y) = \ln x - \ln y, f_1(tx, ty) = \ln tx - \ln ty$$

So, f_1 is homogeneous to degree 0;

$$f_2(x, y) = 1, f_2(tx, ty) = 1$$

So $\ln x - \ln y$ and 1 are homogeneous to degree 0;

40

Let $y = x$ and $dy = vdx + xdv$

$$(xv + 2xe^{-v})dx - x(vdx + xdv) = 0$$

After canceling,

$$2e^{-v} - xdv = 0$$

The integrating factor is $\frac{1}{x(2e^{-v})}$.

$$\int \frac{dx}{x} - \int \frac{dv}{2e^{-v}} = \int 0 dx$$

$$\Rightarrow \ln|x| - \frac{1}{2}e^v = C$$

$$\Rightarrow \ln|x| - \frac{1}{2}e^{\frac{y}{x}} = C$$