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Dis 2A

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$y(t) = Ae^{2t}$
$y'(t) = 2Ae^{2t}$
$y''(t) = 4Ae^{2t}$
plug into $y'' + 3y' - 18y = 18e^{2t}$
$4Ae^{2t} + 6Ae^{2t} - 18Ae^{2t} = 18e^{2t}$
$-8A = 18 \Rightarrow A = -\frac{9}{4}$
$y(t) = -\frac{9}{4}e^{2t}$

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$y(t) = acos2t + bsin2t$
$y'(t) = -2asin2t + 2bcos2t$
$y''(t) = -4acos2t - 4bsin2t$
plug into $y'' + 9y = sin2t$
$-4acos2t - 4bsin2t + 9acost2t + 9bsin2t = sin2t$

$$\begin{cases} 5b = 1 \\ 5a = 0 \end{cases} \Rightarrow \begin{cases} a = 0 \\ b = \frac{1}{5} \end{cases}$$

$y(t) = \frac{1}{5}sin2t$

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$y(t) = At + B \Rightarrow y'(t) = A \Rightarrow y''(t) = 0$
plug into $y'' + 5y' + 4y = 2 + 3t$
$4At + 5A + 4B = 3t + 2$

$$\begin{cases} 4A = 3 \\ 5A + 4B = 2 \end{cases} \Rightarrow \begin{cases} A = \frac{3}{4} \\ B = -\frac{7}{16} \end{cases}$$

$y(t) = \frac{3}{4}t - \frac{7}{16}$

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$\lambda^2 + 3\lambda + 2 = 0 \Rightarrow (\lambda + 1)(\lambda + 2) = 0$
$\lambda_1 = -1, \lambda_2 = -2$
$y_g(t) = C_1e^{-t} + C_2e^{-2t}$
$y_p(t) = Ae^{-4t}$
$y'_p(t) = -4Ae^{-4t}$
$y''_p(t) = 16Ae^{-4t}$
plug into $y'' + 3y' + 2y = 3e^{-4t}$
$16Ae^{-4t} - 12Ae^{-4t} + 2Ae^{-4t} = 3e^{-4t}$
$6A = 3 \Rightarrow A = \frac{1}{2}$
$y(t) = C_1e^{-t} + C_2e^{-2t} + \frac{1}{2}e^{-4t}$
$y'(t) = -C_1e^{-t} - 2C_2e^{-2t} - 2e^{-4t}$
Because $y(0) = 1$ and $y'(0) = 0$,
$C_1 = 3, C_2 = -\frac{5}{2}$
$y(t) = 3e^{-t} - \frac{5}{2}e^{-2t} + \frac{1}{2}e^{-4t}$

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$\lambda^2 + 4\lambda + 4 = 0 \Rightarrow (\lambda + 2)^2 = 0$
$\lambda = -2$
$y_g(t) = C_1e^{-2t} + C_2te^{-2t}$
$y_p(t) = At + B \Rightarrow y'_p(t) = A \Rightarrow y''_p(t) = 0$
plug into $y'' + 4y' + 4y = 4 - t$
$4A + 4(At + B) = 4 - t$

$$\begin{cases} 4A = -1 \\ 4B = 5 \end{cases} \Rightarrow \begin{cases} A = -\frac{1}{4} \\ B = \frac{5}{4} \end{cases}$$

$y(t) = C_1e^{-2t} + C_2te^{-2t} - \frac{1}{4}t + \frac{5}{4}$
$y'(t) = -2C_1e^{-2t} + C_2e^{-2t} - 2C_2te^{-2t} - \frac{1}{4}$
Because $y(0) = -1$ and $y'(0) = 0$
$C_1 = -\frac{9}{4}, C_2 = \frac{17}{4}$
$y(t) = -\frac{9}{4}e^{-2t} + \frac{17}{4}te^{-2t} - \frac{1}{4}t + \frac{5}{4}$

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$y(t) = R\left\{4e^{2it}\right\} = R\left\{4(cos2t + isin2t)\right\}$
$y_p(t) = Ate^{2it}$

$y'_p(t) = At^{2it} + 2itAe^{2it}$
$y''_p(t) = 4iAe^{2it} - 4tAe^{2it}$
$4iAe^{2it} - 4tAe^{2it} + 4Ate^{2it} = 4e^{2it}$
$A = -i$
$z(t) = -it(cos2t + isin2t) = -itcos2t + sin2t$
$R\{z(t)\} = y_p(t) = tsin2t$

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$\alpha y''_f + \alpha p y'_f + \alpha q y_f = \alpha f(t)$
$\beta y''_g + \beta y'_g + \beta y_g = \beta g(t)$
$(\alpha y_f + \beta y_g)'' + p(\alpha p y_f + \beta y_g)' + q(\alpha p y_f + \beta y_g) = \alpha f(t) + \beta g(t)$
$\Rightarrow z(t) = \alpha f(t) + \beta g(t)$

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$y''_1 + 2y'_1 + y_1 = 3$
$y''_2 + 2y'_2 + y_2 = -e^{-t}$
$y_{1p} = A, y'_{1p} = 0, y'_{1p} = 0$
$A = 3 \Rightarrow y_{1p} = 3$
$y_{2p} = Ate^{-t}$
$y'_{2p} = (A - At)e^{-t}$
$y''_{2p} = (At - A)e^{-t}$
$(At - A)e^{-t} + (2A - 2At)e^{-t} + Ate^{-t} = -e^{-t}$
$A = -1 \Rightarrow y_{2p} = -e^{-t}$
$y(t) = 3 - e^{-t}$

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$y''_1 + 2y'_1 + 2y_1 = 3cost$
$y''_2 + 2y'_2 + 2y_2 = -sint$
$z(t) = Ae^{it}, z'(t) = Aie^{it}, z''(t) = i^2Ae^{it}$
plug into $z'' + 2z' + 2z = e^{it}$
$A(i^2 + 2i + 2)e^{it} = e^{it}$
$A = \frac{1-2i}{5} \Rightarrow z(t) = \frac{1-2i}{5}(cost + isint)$
$\frac{y_{1p}}{3} = R\{z(t)\}\, \frac{1}{5}cost + \frac{2}{5}sint$
$y_{2p} = im\{z(t)\} = \frac{1}{5}sint - \frac{2}{5}cost$
$y(t) = cost + sint$

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$y_p(t) = (At^2 + Bt + C)e^{-2t}$
$y'_p(t) = (-2At^2 + 2At - 2Bt + B - 2C)e^{-2t}$
$y''_p(t) =$
$(4At^2 - 8At + 4Bt + 2A - 4B - 4C)e^{-2t}$
plug into $y'' + 2y' + y = t^2e^{-2t}$
$\{(4At^2 - 8At + 4Bt + 2A - 4B - 4C)+$
$(-2At^2 + 2At - 2Bt + B - 2C)+$
$(At^2 + Bt + C)\}\, e^{-2t} = t^2e^{-2t}$
$At^2 - 4At + Bt + 2A - 2B + C = t^2$
$$\begin{cases} A = 1 \\ -4A + B = 0 \\ 2A - 2B + C = 0 \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = 4 \\ C = 6 \end{cases}$$
$y(t) = (t^2 + 4t + 6)e^{-2t}$

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$y_p(t) = (bsint + acost)e^{-2t}$
$y'_p(t) = \{(-a - 2b)sint + (-2a + b)cost\}\, e^{-2t}$
$y''_p(t) = \{(4a + 3b)sint + (3a - 4b)cost\}\, e^{-2t}$
plug into $y'' + 2y' + 2y = e^{-2t}sint$
$\{(4a + 3b)sint + (3a - 4b)cost +$
$((-2a - 4b)sint + (-4a + 2b)cost)+$
$(2bsint + 2acost)\}\, e^{-2t} = e^{-2t}sint$
$\Rightarrow (2a + b)sint + (a - 2b)cost = sint$
$$\begin{cases} 2a + b = 1 \\ a - 2b = 0 \end{cases} \Rightarrow \begin{cases} a = \frac{2}{5} \\ b = \frac{1}{5} \end{cases}$$
$y(t) = \frac{2}{5}cost + \frac{1}{5}sint$