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Dis 2A

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$y_1(t) = cos2t$ and $y_2(t) = sin2t$
$y = v_1 cos2t + v_2 sin2t$
$y' = v_1' cos2t + v_2' sin2t - 2v_1 cos2t + 2v_2 sin2t$
Because $v_1' cos2t + v_2 sin2t = 0$,
$y' = -2v_1 cos2t + 2v_2 sin2t$
$y'' =$
$-2v_1' cos2t + 2v_2' sin2t - 4(v_1 cos2t + v_2 sin2t)$
$y'' + 4y = -2v_1' cos2t + 2v_2' sin2t = sec2t$
Because $v_1' cos2t + v_2 sin2t = 0$ and
$-2v_1' cos2t + 2v_2' sin2t = sec2t$
$v_1' = -\frac{1}{2} tan2t$ and $v_2' = \frac{1}{2}$
$y = \frac{1}{4} cos2t \cdot In(cos2t) + \frac{t}{2} sin2t$

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Because $W(cost, sint) = 1$,
$x_p = v_1 x_1 + v_2 x_2$
$v_1' = -sect tant + sint$
$v_1 = -sect - cost$
$v_2' = sect - cost$
$v_2 = In|sect + tant| - sint$
$x_p = -2 + sint \cdot In|sect + tant|$

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$y'' + \frac{3}{t} y' + \frac{1}{t^2} y = \frac{1}{t^3}$
plug in $y_1(t) = t^{-1}$
$2t^{-3} - \frac{3}{t} t^{-2} + \frac{1}{t^2} t^{-1} = 0$
$y_2(t)' = t^{-2}(1 - Int)$ and
$y_2(t)'' = t^{-3}(-3 + 2Int)$
plug in,
$t^{-3}(-3 + 2Int) + 3t^{-3}(1 - Int) + t^{-3} Int = 0$
$W(t^{-1}, t^{-1} Int) = t^{-3}$
$v_1' = t^{-1} Int$, So $v_1 = -\frac{1}{2}(Int)^2$
$v_2' = t^{-1}$, So $v_2 = Int$
$y_p = \frac{1}{2} t^{-1} (Int)^2$
$y(t) = C_1 t^{-1} + C_2 t^{-1} Int + \frac{1}{2} t^{-1} (Int)^2$
$t > 0$

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$A = \begin{pmatrix} -4 & 1 \\ -2 & 1 \end{pmatrix}$
$p(\lambda) = det(A - \lambda I) = \lambda^2 + 3\lambda - 2$
$\lambda_1 = \frac{(-3-\sqrt{17})}{2}$, $\lambda_2 = \frac{-3-\sqrt{17}}{2}$

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$A = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$
$p(\lambda) = det(A - \lambda I) = (\lambda + 3)^2$
$\lambda = -3$

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$p(\lambda) = -(1 - \lambda) \begin{vmatrix} 3 - \lambda & 2 \\ -4 & -3\lambda \end{vmatrix} =$
$(\lambda - 1)^2 (\lambda + 1)$
$\lambda_1 = -1$ and $\lambda_2 = 1$

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$p(\lambda) = det \begin{pmatrix} -3 - \lambda & -4 \\ 2 & 3 - \lambda \end{pmatrix} =$
$(\lambda + 1)(\lambda - 1)$
$\lambda_1 = -1$ and $\lambda_2 = 1$
$A + I = \begin{pmatrix} -2 & -4 \\ 2 & 4 \end{pmatrix}$
$y_1(t) = e^{-t} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$
$A - I = \begin{pmatrix} -4 & -4 \\ 2 & 2 \end{pmatrix}$
$y_2(t) = e^{t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

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$p(\lambda) = (3 + \lambda) \begin{vmatrix} -5 - \lambda & -6 \\ 4 & 5 - \lambda \end{vmatrix} =$
$(\lambda + 3)(\lambda + 1)(\lambda - 1)$
$\lambda_1 = -3$ and $\lambda_2 = -1$ and $\lambda_3 = 1$
$A + 3I = \begin{pmatrix} -2 & 0 & -6 \\ 26 & 0 & 38 \\ 4 & 0 & 8 \end{pmatrix}$
$y_1(t) = e^{-3t} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$
$A + I = \begin{pmatrix} -4 & 0 & -6 \\ 26 & -2 & 38 \\ 4 & 0 & 6 \end{pmatrix}$

$$y_2(t) = e^{-t}\begin{pmatrix}-3\\-1\\2\end{pmatrix}$$

$$A - I = \begin{pmatrix}-6 & 0 & -6\\26 & -4 & 38\\4 & 0 & 4\end{pmatrix}$$

$$y_3(t) = e^{t}\begin{pmatrix}-3\\3\\1\end{pmatrix}$$

$$det[y_1(0), y_2(0), y_3(0)] = 1$$