

Name: Chen Chen

UID: 004710308

Dis 2A

4

$$y(t) = Ae^{2t}$$

$$y'(t) = 2Ae^{2t}$$

$$y''(t) = 4Ae^{2t}$$

$$\text{plug into } y'' + 3y' - 18y = 18e^{2t}$$

$$4Ae^{2t} + 6Ae^{2t} - 18Ae^{2t} = 18e^{2t}$$

$$-8A = 18 \Rightarrow A = -\frac{9}{4}$$

$$y(t) = -\frac{9}{4}e^{2t}$$

6

$$y(t) = a\cos 2t + b\sin 2t$$

$$y'(t) = -2a\sin 2t + 2b\cos 2t$$

$$y''(t) = -4a\cos 2t - 4b\sin 2t$$

$$\text{plug into } y'' + 9y = \sin 2t$$

$$-4a\cos 2t - 4b\sin 2t + 9a\cos 2t + 9b\sin 2t = \sin 2t$$

$$\begin{cases} 5b = 1 \\ 5a = 0 \end{cases} \Rightarrow \begin{cases} a = 0 \\ b = \frac{1}{5} \end{cases}$$

$$y(t) = \frac{1}{5}\sin 2t$$

14

$$y(t) = At + B \Rightarrow y'(t) = A \Rightarrow y''(t) = 0$$

$$\text{plug into } y'' + 5y' + 4y = 2 + 3t$$

$$4At + 5A + 4B = 3t + 2$$

$$\begin{cases} 4A = 3 \\ 5A + 4B = 2 \end{cases} \Rightarrow \begin{cases} A = \frac{3}{4} \\ B = -\frac{7}{16} \end{cases}$$

$$y(t) = \frac{3}{4}t - \frac{7}{16}$$

18

$$\lambda^2 + 3\lambda + 2 = 0 \Rightarrow (\lambda + 1)(\lambda + 2) = 0$$

$$\lambda_1 = -1, \lambda_2 = -2$$

$$y_g(t) = C_1e^{-t} + C_2e^{-2t}$$

$$y_p(t) = Ae^{-4t}$$

$$y'_p(t) = -4Ae^{-4t}$$

$$y''_p(t) = 16Ae^{-4t}$$

$$\text{plug into } y'' + 3y' + 2y = 3e^{-4t}$$

$$16Ae^{-4t} - 12Ae^{-4t} + 2Ae^{-4t} = 3e^{-4t}$$

$$6A = 3 \Rightarrow A = \frac{1}{2}$$

$$y(t) = C_1e^{-t} + C_2e^{-2t} + \frac{1}{2}e^{-4t}$$

$$y'(t) = -C_1e^{-t} - 2C_2e^{-2t} - 2e^{-4t}$$

$$\text{Because } y(0) = 1 \text{ and } y'(0) = 0,$$

$$C_1 = 3, C_2 = -\frac{5}{2}$$

$$y(t) = 3e^{-t} - \frac{5}{2}e^{-2t} + \frac{1}{2}e^{-4t}$$

22

$$\lambda^2 + 4\lambda + 4 = 0 \Rightarrow (\lambda + 2)^2 = 0$$

$$\lambda = -2$$

$$y_g(t) = C_1e^{-2t} + C_2te^{-2t}$$

$$y_p(t) = At + B \Rightarrow y'_p(t) = A \Rightarrow y''_p(t) = 0$$

$$\text{plug into } y'' + 4y' + 4y = 4 - t$$

$$4A + 4(At + B) = 4 - t$$

$$\begin{cases} 4A = -1 \\ 4B = 5 \end{cases} \Rightarrow \begin{cases} A = -\frac{1}{4} \\ B = \frac{5}{4} \end{cases}$$

$$y(t) = C_1e^{-2t} + C_2te^{-2t} - \frac{1}{4}t + \frac{5}{4}$$

$$y'(t) = -2C_1e^{-2t} + C_2e^{-2t} - 2C_2te^{-2t} - \frac{1}{4}$$

$$\text{Because } y(0) = -1 \text{ and } y'(0) = 0$$

$$C_1 = -\frac{9}{4}, C_2 = \frac{17}{4}$$

$$y(t) = -\frac{9}{4}e^{-2t} + \frac{17}{4}te^{-2t} - \frac{1}{4}t + \frac{5}{4}$$

26

$$y(t) = R\{4e^{2it}\} = R\{4(\cos 2t + i\sin 2t)\}$$

$$y_p(t) = Ate^{2it}$$

$$\begin{aligned}
y_p'(t) &= At^{2it} + 2itAe^{2it} \\
y_p''(t) &= 4iAe^{2it} - 4tAe^{2it} \\
4iAe^{2it} - 4tAe^{2it} + 4At e^{2it} &= 4e^{2it}
\end{aligned}$$

$$A = -i$$

$$z(t) = -it(\cos 2t + i \sin 2t) = -it \cos 2t + \sin 2t$$

$$R\{z(t)\} = y_p(t) = t \sin 2t$$

30

$$\begin{aligned}
\alpha y_f'' + \alpha p y_f' + \alpha q y_f &= \alpha f(t) \\
\beta y_g'' + \beta y_g' + \beta y_g &= \beta g(t) \\
(\alpha y_f + \beta y_g)'' + p(\alpha y_f + \beta y_g)' + q(\alpha y_f + \beta y_g) &= \alpha f(t) + \beta g(t) \\
\Rightarrow z(t) &= \alpha f(t) + \beta g(t)
\end{aligned}$$

34

$$\begin{aligned}
y_1'' + 2y_1' + y_1 &= 3 \\
y_2'' + 2y_2' + y_2 &= -e^{-t} \\
y_{1p} &= A, y_{1p}' = 0, y_{1p}'' = 0 \\
A = 3 &\Rightarrow y_{1p} = 3 \\
y_{2p} &= Ate^{-t} \\
y_{2p}' &= (A - At)e^{-t} \\
y_{2p}'' &= (At - A)e^{-t} \\
(At - A)e^{-t} + (2A - 2At)e^{-t} + Ate^{-t} &= -e^{-t} \\
A = -1 &\Rightarrow y_{2p} = -e^{-t} \\
y(t) &= 3 - e^{-t}
\end{aligned}$$

36

$$\begin{aligned}
y_1'' + 2y_1' + 2y_1 &= 3 \cos t \\
y_2'' + 2y_2' + 2y_2 &= -\sin t \\
z(t) &= Ae^{it}, z'(t) = Aie^{it}, z''(t) = i^2 Ae^{it} \\
\text{plug into } z'' + 2z' + 2z &= e^{it} \\
A(i^2 + 2i + 2)e^{it} &= e^{it} \\
A = \frac{1-2i}{5} &\Rightarrow z(t) = \frac{1-2i}{5}(\cos t + i \sin t) \\
\frac{y_{1p}}{3} = R\{z(t)\} &= \frac{1}{5} \cos t + \frac{2}{5} \sin t \\
y_{2p} = \text{im}\{z(t)\} &= \frac{1}{5} \sin t - \frac{2}{5} \cos t \\
y(t) &= \cos t + \sin t
\end{aligned}$$

41

$$\begin{aligned}
y_p(t) &= (At^2 + Bt + C)e^{-2t} \\
y_p'(t) &= (-2At^2 + 2At - 2Bt + B - 2C)e^{-2t} \\
y_p''(t) &= (4At^2 - 8At + 4Bt + 2A - 4B - 4C)e^{-2t} \\
\text{plug into } y'' + 2y' + y &= t^2 e^{-2t} \\
\{(4At^2 - 8At + 4Bt + 2A - 4B - 4C) + (-2At^2 + 2At - 2Bt + B - 2C) + (At^2 + Bt + C)\}e^{-2t} &= t^2 e^{-2t} \\
At^2 - 4At + Bt + 2A - 2B + C &= t^2 \\
\begin{cases} A = 1 \\ -4A + B = 0 \\ 2A - 2B + C = 0 \end{cases} &\Rightarrow \begin{cases} A = 1 \\ B = 4 \\ C = 6 \end{cases} \\
y(t) &= (t^2 + 4t + 6)e^{-2t}
\end{aligned}$$

44

$$\begin{aligned}
y_p(t) &= (b \sin t + a \cos t)e^{-2t} \\
y_p'(t) &= \{(-a - 2b) \sin t + (-2a + b) \cos t\}e^{-2t} \\
y_p''(t) &= \{(4a + 3b) \sin t + (3a - 4b) \cos t\}e^{-2t} \\
\text{plug into } y'' + 2y' + 2y &= e^{-2t} \sin t \\
\{(4a + 3b) \sin t + (3a - 4b) \cos t + ((-2a - 4b) \sin t + (-4a + 2b) \cos t) + (2b \sin t + 2a \cos t)\}e^{-2t} &= e^{-2t} \sin t \\
\Rightarrow (2a + b) \sin t + (a - 2b) \cos t &= \sin t \\
\begin{cases} 2a + b = 1 \\ a - 2b = 0 \end{cases} &\Rightarrow \begin{cases} a = \frac{2}{5} \\ b = \frac{1}{5} \end{cases} \\
y(t) &= \frac{2}{5} \cos t + \frac{1}{5} \sin t
\end{aligned}$$