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Dis 2A

4.1

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$$\begin{bmatrix} t^2 & t|t| \\ 2t & 2|t| \end{bmatrix} = 2|t|t^2 - 2tt|t| = 0$$

So, it is linear dependent.

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$$\begin{bmatrix} e^t & e^{-3t} \\ e^t & -3e^{-3t} \end{bmatrix} = -4e^{-2t}$$

$$y(0) = 1 = C_1e^0 + C_2e^{-3\times 0} = C_1 + C_2 = 1$$
$$y'(0) = -2 = C_1e^0 - C_2 3e^{-3\times 0} = C_1 - (-3)C_2$$
$$C_1 = \frac{1}{4}, C_2 = \frac{3}{4}$$

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$$y_2(t) = y_1(t)x = xt$$
$$(xt)' = x't + x, (x't + x)' = x''t + 2x'$$
$$t^2(x''t + 2x') - 2t(x't + x) + 2xt = 0$$
$$\Rightarrow t^3x'' = 0 \Rightarrow x'' = 0 \Rightarrow x = t \Rightarrow y_2(t) = t^2$$
$$t^2(2) - 2t(2t) + 2t^2 = 0$$
$$y = C_1t + C_2t^2$$

4.3

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$$6\lambda^2 + \lambda - 1 = 0 = (3\lambda - 1)(2\lambda + 1)$$
$$\lambda_1 = \frac{1}{3}, \lambda_2 = -\frac{1}{2}$$
$$y(t) = C_1e^{\frac{1}{3}} + C_2e^{-\frac{1}{2}}$$

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$$6\lambda + 5\lambda - 6 = 0 = (2\lambda + 3)(3\lambda - 2)$$
$$\lambda_1 = \frac{2}{3}, \lambda_2 = -\frac{3}{2}$$
$$y(t) = C_1e^{\frac{2}{3}} + C_2e^{-\frac{3}{2}}$$

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$$\lambda^2 + 2\lambda + 3 = 0$$
$$\lambda = -1 \pm \sqrt{2}i$$
$$y(t) = e^{-t}(C_1cos\sqrt{2}t + C_2sin\sqrt{2}t)$$

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$$\lambda^2 + 2\lambda + 2 = 0$$
$$\lambda = -1 \pm i$$
$$y(t) = e^{-t}(C_1cost + C_2sint)$$

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$$4\lambda^2 + 12\lambda + 0 = 9 = (2\lambda + 3)(2\lambda + 3)$$
$$\lambda = -\frac{3}{2}$$
$$y(t) = C_1e^{-\frac{3}{2}t} - C_2te^{-\frac{3}{2}t}$$

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$$\lambda^2 + 8\lambda + 16 = (\lambda + 4)(\lambda + 4) = 0$$
$$\lambda = -4$$
$$y(t) = C_1e^{-4t} + C_2te^{-4t}$$

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$$\lambda^2 - 2\lambda - 3 = (\lambda - 3)(\lambda + 1) = 0$$
$$\lambda_1 = 3, \lambda_2 = -1$$
$$y(t) = C_1e^{3t} + C_2e^{-t}$$

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Because $\lambda = \lambda_1$,

$$\lambda^2 + p\lambda + q = (\lambda - \lambda_1)(\lambda - \lambda_1) = \lambda^2 - 2\lambda_1\lambda + \lambda_1^2$$

$$\Rightarrow p = -2\lambda_1, q = \lambda_1^2$$

$$\frac{dy}{dt} = e^{\lambda_1 t} + \lambda_1 t e^{\lambda_1 t}$$

$$\frac{d^2y}{dx^2} = \lambda_1 e^{\lambda_1 t}(2 + \lambda_1 t)$$

plug in $\frac{dy}{dt}$ and $\frac{d^2y}{dx^2}$ into $y'' + py' + qy$

$$\Rightarrow \lambda_1 e^{\lambda_1 t}(2+\lambda_1 t) + p(e^{\lambda_1 t} + \lambda_1 t e^{\lambda_1 t}) - q(te^{\lambda_1 t})$$

plug in $p = -2\lambda_1$ and $q = \lambda_1^2$

$$\Rightarrow 2\lambda_1 e^{\lambda_1 t} + \lambda_1^2 t e^{\lambda_1 t} - 2\lambda_1 e^{\lambda_1 t} - 2\lambda_1^2 t e^{\lambda_1 t} + \lambda^2 t e^{\lambda_1 t} = 0$$