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Dis 2A

SECTION 2.1

1

$$\o(t,y,y') = t^2y' + (1+t)y = 0$$

$$y' = -\frac{(1+t)y}{t^2}$$

8

(a)

$$(t^2+y^2)' = (C^2)'$$

$$\Rightarrow 2t + 2yy' = 0$$

$$\Rightarrow t + yy' = 0$$

(b)

$$y = \pm\sqrt{C^2 - t^2}$$

$$\Rightarrow y' = \mp\frac{t}{\sqrt{C^2-t^2}}$$

plug into $t + yy' = 0$:

$$t + \frac{\pm\sqrt{C^2-t^2}t}{\mp\sqrt{C^2-t^2}} = 0$$

$$\Rightarrow t - t = 0$$

(c)

$$Because, \sqrt{C^2-t^2} \geq 0$$

$$\Rightarrow C^2 \geq t^2$$

$$\Rightarrow -C \leq t \leq C$$

(d)

15

20

SECTION 2.2

9

$$\frac{dy}{dx} = \frac{yIny}{x+1}$$

$$\int \frac{dy}{yIny} = \int \frac{dx}{x+1}$$
$$In|In(y)| = In|x+1|$$
$$In(y) = x+1$$
$$y = e^{x+1}$$

15

$$\int ydy = \int sinxdx$$
$$\frac{1}{2}y^2 + C = -cosx + C$$
$$y(x) = \pm\sqrt{-2cosx+C}$$

plug in x=$\frac{\pi}{2}$

$$y(x) = \sqrt{-2cosx+1}$$

Interval of existence:

$$-2cosx+1 \geq 0$$
$$\Rightarrow \frac{\pi}{3} < x < \frac{5\pi}{3}$$

22

$$\int \frac{y}{y^2+1}dy = \int dx$$
$$In|y^2+1| + C = x + C$$
$$y^2+1 = e^{2x+2C}$$

Because e^{2C} is a constant, we can substitute it with u.

$$y(x) = \pm\sqrt{ue^{2x}-1}$$

plug in $x = 1$

$$4 = ue^2 - 1$$
$$u = 5e^{-2}$$

So, solution is:

$$y(x) = \sqrt{5e^{2x-2}-1}$$

Interval of existence:

$$5e^{2x-2} - 1 > 0$$
$$x > 1 - \frac{In(5)}{2}$$

24
a

$$\lambda = 1.551 * 10^{-8}$$

b

$$N_0 = 1000andN(t) = 100$$
$$t = In\frac{10}{\lambda} = 1.485 \times 10^8 years$$

SECTION 2.4

8

$$u(x) = e^{-In(1+x^3)} = \frac{1}{1+x^3}$$
$$\frac{1}{1+x^3}y' - \frac{3x^2}{(1+x^3)^2}y = \frac{x^2}{1+x^3}$$
$$y(x) = \frac{1}{3(1+x^3)In(1+x^3)} + C(1+x^3)$$

13
a

$$u(x) = e^{-\int a(x)dx} = e^{sinx}$$
$$e^{sinx}(y' + ycosx) = e^{sinx}cosx$$
$$e^{sinx}y = e^{sinx} + C$$
$$y(x) = \frac{e^{sinx}+C}{e^{sinx}}$$

b

$$\frac{dy}{1-y} = cosxdx$$
$$-In|1-y| = sinx + C$$
$$|1-y| = e^{-sinx-C} \Rightarrow 1-y = \pm e^{-sinx-C}$$

let u=$\pm e^{-C}$

$$y(x) = 1 - ue^{-sinx}$$

19

$$u(x) = e^{\int -\frac{1}{2x+3}dx} = |2x+3|^{-\frac{1}{2}}$$

$$(2x+3)^{-\frac{1}{2}}y' - (2x+3)^{-\frac{3}{2}} = (2x+3)^{-1}$$

$$y(x) = \frac{1}{2}(2x+3)^{\frac{1}{2}}In(2x+3) + C(2x+3)^{\frac{1}{2}}$$

Because $y(-1) = 0 = C$,

$$y(x) = \frac{1}{2}(2x+3)^{\frac{1}{2}}In(2x+3).$$

interval of existence: $(-\frac{3}{2}, +\infty)$

24

let $z = y^{-1}$

$$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx} = y^{-2}(y^2 - y) = -1 + z$$

$$e^{-x}(\frac{dz}{dx} - z) = -e^{-x}$$

$$\Rightarrow e^{-x}z = e^{-x} + C$$

$$\Rightarrow z(x) = 1 + Ce^{x}$$

plug in $z = y^{-1}$

$$y(x) = \frac{1}{1 + Ce^{x}}$$