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Dis 2A

2.5

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Let $y(t)$ represents the amount of salt in solution, and r represent the rate that water enter.

$$\frac{dy}{dt} = -\frac{r}{500}x$$

Let $c(t)$ represent the consentration of the solution.

$$c'(t) = -\frac{r}{500}c$$

$$c = 0.05e^{-r/500}t$$

plug in $c(60) = 0.01$

$$r = \frac{25}{3}In5$$

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$$\frac{dS}{dt} = -\frac{3S}{100-t}$$

$$S(t) = 5\times 10^{-6}\times(100-t)^3$$

when volume is $V(t) = 100-t = 50$

$$S(t) = 0.625lb$$

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$$\frac{\partial F(x,y)}{\partial x} = x(x^2+y^2)^{-\frac{3}{2}}dx$$

$$\frac{\partial F(x,y)}{\partial x} = y(x^2+y^2)^{-\frac{3}{2}}dy$$

$$F'(x,y) = x(x^2+y^2)^{-\frac{3}{2}}dx + y(x^2+y^2)^{-\frac{3}{2}}dy$$

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$$\frac{1}{xy^2}(y^2-xy)dx + \frac{1}{xy^2}x^2dy = 0$$

$$\Rightarrow (\frac{1}{x}+\frac{1}{y})dx + \frac{x}{y^2}dy = 0$$

$$\text{ø}(x,y) = \int(\frac{1}{x}+\frac{1}{y})dx + g(y)$$

$$\Rightarrow \text{ø}(x,y) = -\frac{x}{y} + In|x| + g(y)$$

$$\frac{\partial \text{ø}}{\partial y} = \frac{x}{y^2} + g'(y) = \frac{x}{y^2}$$

$$g(y) = C$$

$$\text{ø}(x,y) = -\frac{x}{y} + In|x| + C$$

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Let $M = y$ and $N = (x^2y-x)$

$$\frac{\partial M}{\partial y} = 1, \frac{\partial N}{\partial x} = 2xy-1$$

$$\frac{1}{N}(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}) = -\frac{2}{x}$$

$$\Rightarrow I = \int -\frac{2}{x}dx = \frac{1}{x^2}$$

$$\frac{y}{x^2}dx + \frac{xy-1}{x}dy = 0$$

$$\text{ø}(x,y) = \int \frac{y}{x^2}dx + g(y) = -\frac{y}{x} + g(y)$$

$$\frac{\partial \text{ø}}{\partial x} = -\frac{1}{x} + g'(y) = \frac{xy-1}{x}$$

$$\Rightarrow g'(y) = y, g(y) = \frac{y^2}{2}$$

$$\text{ø}(x,y) = \frac{y}{x} + \frac{y^2}{2}$$

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Let $M = 2y$ and $N = (x+y)$

$$\frac{\partial M}{\partial y} = 2, \frac{\partial N}{\partial x} = 1$$

$$-\frac{1}{M}(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}) = \frac{1}{2y}$$

$$I = \int \frac{1}{2y} dy = \frac{1}{\sqrt{y}}$$

$$\frac{2y}{\sqrt{y}} dx + \frac{x+y}{\sqrt{y}} dy = 0$$

$$ø(x,y) = \int \frac{2y}{\sqrt{y}} dx + g(y) = \frac{2xy}{\sqrt{y}} + g(y)$$

$$\frac{\partial ø}{\partial y} = 3x\sqrt{y} + g'(y) = \frac{x+y}{\sqrt{y}}$$

$$\Rightarrow g(y) = 2x\sqrt{y} + \frac{2}{3} y^{3/2} - 2xy^{3/2}$$

$$ø(x,y) = 2x\sqrt{y} + \frac{2}{3} y^{3/2}$$

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$$\frac{\partial M}{\partial y} = 2, \frac{\partial N}{\partial x} = 3$$

$$\mu(x,y) = xy^2$$

$$2xy^3 dx + 3x^2y^2 dy = 0$$

$$ø(x,y) = \int 2xy^3 dx + g(y) = x^2y^3 + g(y)$$

$$\frac{\partial ø}{\partial y} = 3x^2y^2 + g'(y) 3x^2y^2$$

$$g'(y) = 0, g(y) = C$$

$$ø(x,y) = x^2y^3 + C$$

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$$f_1(x,y) = Inx - Iny, f_1(tx,ty) = Intx - Inty$$

So, f_1 is homgeneous to degree 0;

$$f_2(x,y) = 1, f_2(tx,ty) = 1$$

So $Inx - Iny$ and 1 are homgeneous to degree 0;

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Let $y = x$ and $dy = vdx + xdv$

$$(xv + 2xe^{-v})dx - x(vdx + xdv) = 0$$

After canceling,

$$2e^{-v} - xdv = 0$$

The integrating factor is $\frac{1}{x(2e^{-v})}$.

$$\int \frac{dx}{x} - \int \frac{dv}{2e^{-v}} = \int 0 dx$$

$$\Rightarrow In|x| - \frac{1}{2} e^{v} = C$$

$$\Rightarrow In|x| - \frac{1}{2} e^{\frac{y}{x}} = C$$

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$$\frac{dy}{dx} = -\frac{2x}{y}$$

$$ø(x,y) = 2x^2 + y^2 = C$$