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**Dis 2A**

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$$\frac{\partial F(x, y)}{\partial x} = x(x^2 + y^2)^{-\frac{3}{2}} dx$$

$$\frac{\partial F(x, y)}{\partial x} = y(x^2 + y^2)^{-\frac{3}{2}} dy$$

$$F'(x, y) = x(x^2 + y^2)^{-\frac{3}{2}} dx + y(x^2 + y^2)^{-\frac{3}{2}} dy$$

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$$\frac{1}{xy^2}(y^2 - xy)dx + \frac{1}{xy^2}x^2 dy = 0$$

$$\Rightarrow \left(\frac{1}{x} + \frac{1}{y}\right)dx + \frac{x}{y^2}dy = 0$$

$$\phi(x, y) = \int \left(\frac{1}{x} + \frac{1}{y}\right)dx + g(y)$$

$$\Rightarrow \phi(x, y) = -\frac{x}{y} + \ln|x| + g(y)$$

$$\frac{\partial \phi}{\partial y} = \frac{x}{y^2} + g'(y) = \frac{x}{y^2}$$

$$g(y) = C$$

$$\phi(x, y) = -\frac{x}{y} + \ln|x| + C$$

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Let  $M = y$  and  $N = (x^2y - x)$

$$\frac{\partial M}{\partial y} = 1, \frac{\partial N}{\partial x} = 2xy - 1$$

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = -\frac{2}{x}$$

$$\Rightarrow I = \int -\frac{2}{x} dx = \frac{1}{x^2}$$

$$\frac{y}{x^2} dx + \frac{xy - 1}{x} dy = 0$$

$$\phi(x, y) = \int \frac{y}{x^2} dx + g(y) = -\frac{y}{x} + g(y)$$

$$\frac{\partial \phi}{\partial x} = -\frac{1}{x} + g'(y) = \frac{xy - 1}{x}$$

$$\Rightarrow g'(y) = y, g(y) = \frac{y^2}{2}$$

$$\phi(x, y) = \frac{y}{x} + \frac{y^2}{2}$$

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Let  $M = 2y$  and  $N = (x + y)$

$$\frac{\partial M}{\partial y} = 2, \frac{\partial N}{\partial x} = 1$$

$$-\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{2y}$$

$$I = \int \frac{1}{2y} dy = \frac{1}{\sqrt{y}}$$

$$\frac{2y}{\sqrt{y}} dx + \frac{x + y}{\sqrt{y}} dy = 0$$

$$\phi(x, y) = \int \frac{2y}{\sqrt{y}} dx + g(y) = \frac{2xy}{\sqrt{y}} + g(y)$$

$$\frac{\partial \phi}{\partial y} = 3x\sqrt{y} + g'(y) = \frac{x + y}{\sqrt{y}}$$

$$\Rightarrow g(y) = 2x\sqrt{y} + \frac{2}{3}y^{3/2} - 2xy^{3/2}$$

$$\phi(x, y) = 2x\sqrt{y} + \frac{2}{3}y^{3/2}$$