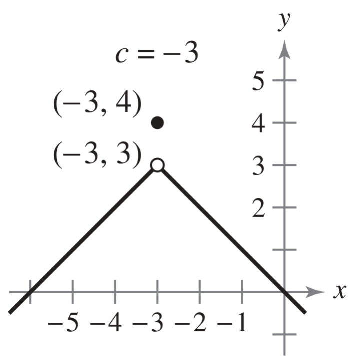
1.4 Continuity and One-Sided Limits

Pages to Read: 70 - 78
Problem's Page: 78
Assigned Problems: 4, 6, 12, 18, 24, 28, 36, 44, 50, 52, 54, 64 - 72 Evens, 78, 80, 82, 84, 90, 94, 96, 98, 100, 102, 114

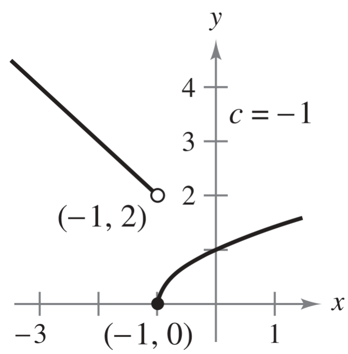
1.4.1 Question 4

Use the graph to determine the limit and discuss the continuity of the function:



1.4.2 Question 6

Use the graph to determine the limit and discuss the continuity of the function:



1.4.3 Question 12

Find the limit if it exists. If it does not exist, explain why.

$$\lim_{x \to 9^-} \frac{\sqrt{x} - 3}{x - 9}$$

1.4.4 Question 18

Find the limit if it exists. If it does not exist, explain why.

$$\lim_{x \to 2} f(x), \text{where} f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \geq 2 \end{cases}$$

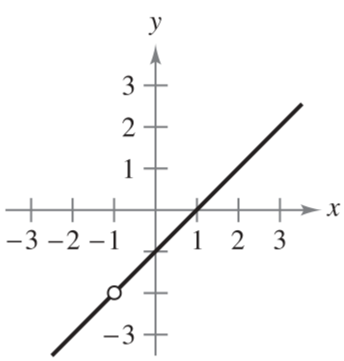
1.4.5 Question 24

Find the limit if it exists. If it does not exist, explain why.

$$\lim_{x \to 2^+} (2x - [\![x]\!])$$

1.4.6 Question 28

Discuss the continuity of each function.: $f(x) = \dfrac{x^2 - 1}{x + 1}$



1.4.7 Question 36

Find the x-values (if any) at which f is not continuous. Which of the discontinuitieis are removable?

$$f(x) = \frac{3}{x-2}$$

1.4.8 Question 44

Find the x-values (if any) at which f is not continuous. Which of the discontinuitieis are removable?

$$f(x) = \frac{x}{x^2-1}$$

1.4.9 Question 50

Find the x-values (if any) at which f is not continuous. Which of the discontinuitieis are removable?

$$f(x) = \frac{|x-8|}{x-8}$$

1.4.10 Question 52

Find the x-values (if any) at which f is not continuous. Which of the discontinuitieis are removable?

$$f(x) = \begin{cases} -2x+3, & x < 1 \\ x^2, & x \geq 1 \end{cases}$$

1.4.11 Question 54

Find the x-values (if any) at which f is not continuous. Which of the discontinuitieis are removable?

$$f(x) = \begin{cases} -2x, & x \leq 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$$

1.4.12 Question 64

Find the constant(s) a and/or b such that the function is continuous on the entire real line

$$f(x) = \begin{cases} 3x^3, & x \leq 1 \\ ax + 5, & x > 1 \end{cases}$$

1.4.13 Question 66

Find the constant(s) a and/or b such that the function is continuous on the entire real line

$$g(x) = \begin{cases} \dfrac{4\sin x}{x}, & x < 0 \\ a - 2x, & x \geq 0 \end{cases}$$

1.4.14 Question 68

Find the constant(s) a and/or b such that the function is continuous on the entire real line

$$g(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & x \neq a \\ 8, & x = a \end{cases}$$

1.4.15 Question 70

Discuss the continuity of the composite function $h(x) = f(g(x))$***.***

$$f(x) = \frac{1}{\sqrt{x}}$$
$$g(x) = x - 1$$

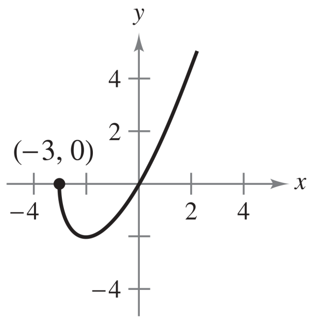
1.4.16 Question 72

Discuss the continuity of the composite function $h(x) = f(g(x))$***.***

$$f(x) = \sin x$$
$$g(x) = x^2$$

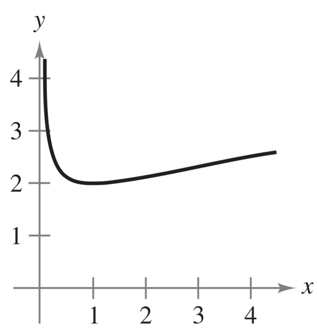
1.4.17 Question 78

Describe the interval(s) on which the function is continuous. $f(x) = x\sqrt{x+3}$



1.4.18 Question 80

Describe the interval(s) on which the function is continuous. $f(x) = \dfrac{x+1}{\sqrt{x}}$



1.4.19 Question 82

Use a graphing utility to graph the function on the interval $[-4, 4]$. Does the graph of the function appear to be continuous on the interval? Is the runction continuous on $[-4, 4]$? Write a short paragraph about the importance of examining a function analytically as well as graphically.

$$f(x) = \frac{x^3 - 8}{x - 2}$$

1.4.20 Question 84

Explain why the function has a zero in the given interval.

$$f(x) = x^3 + 5x - 3, \text{Interval } [0, 1]$$

1.4.21 Question 90

Use the Intermediate Value Theorem and a graphing utility to approximate the zer of the function in the interval $[0, 1]$. Repeatedly "zoom in" on the graph of the function to approximate the zero accurate to two decimal places. use the zero or root reature of the graphing utlity to approximate the zero accurate to four decimal places.

$$h(\theta) = 1 + \theta - 3\tan\theta$$

1.4.22 Question 94

Verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem

$$f(x) = \frac{x^2 + x}{x - 1}, \left[\frac{5}{2}, 4\right], f(x) = 6$$

1.4.23 Question 96

Sketch the graph of any function f ***such that*** $\lim_{x \to 3^+} f(x) = 1$ ***and*** $\lim_{x \to 3^-} f(x) = 0$***. Is the function continuous at*** $x = 3$***? Explain.***

1.4.24 Question 98

Describe the difference between a discontinuity that is removable and one that is nonremovable. In your explanation, give examples of the following descriptions.

(a) A function with a nonremovable discontinuity at $x = 4$

(b) A function with a removable discontinuity at $x = -4$

(c) A function that has both of the characteristics described in parts (a) and (b)

1.4.25 Question 100

True or False? **Determine whether the statment is true or false. If false, explain why or give an example that shows it is false.**

If $\lim_{x \to c} f(x) = L$ and $f(c) = L$, then f is continuous at c.

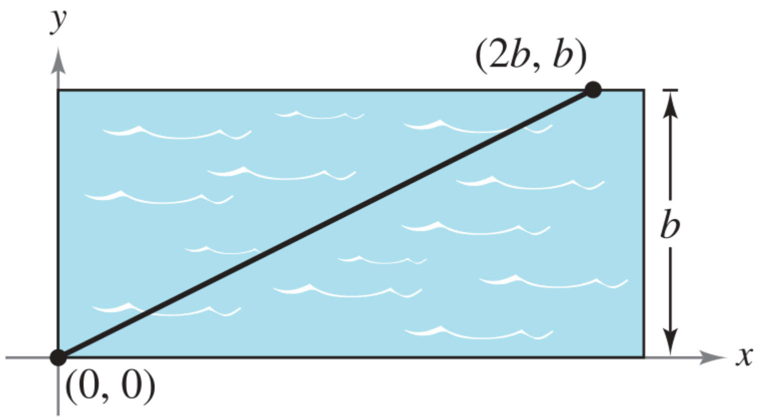
1.4.26 Question 102

True or False? **Determine whether the statment is true or false. If false, explain why or give an example that shows it is false.**

If $f(x) = g(x)$ for $x \neq c$ and $f(c) \neq g(c)$, then either f or g is not continuous at c.

1.4.27 Question 114

A swimmer crosses a pool of width b by swimming in a stright line from $(0, 0)$ to $(2b, b)$.



(a) Let f be a function defined as the y-coordinate of the point on the long side of the pool that is nearest the swimmer at any given time during the swimmer's crossing of the pool. Determine the function f and sketch its graph. Is f continuous? Explain.

(b) Let g by the minimum distance between the swimmer and the long sides of the pool. Determine the function g and sketch its graph. Is g continuous? Explain.