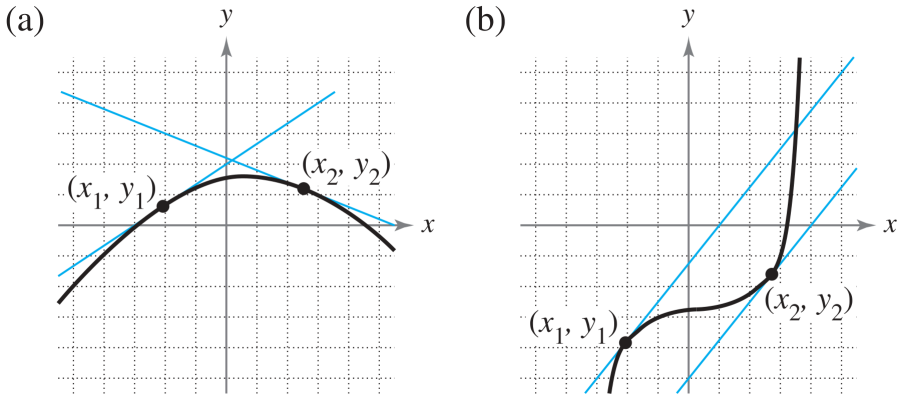
2.1 The Derivative and the Tangent Line Problem

Pages to Read: 96 - 103
Problem's Page: 103
Assigned Problems: 2 - 10 Evens; 16, 20, 24, 26, 30, 32, 34, 38, 39 - 42; 46 - 52; 54, 56, 58, 60, 62, 64, 74, 78, 80, 83 - 88; 94, 96, 98; 101 - 104

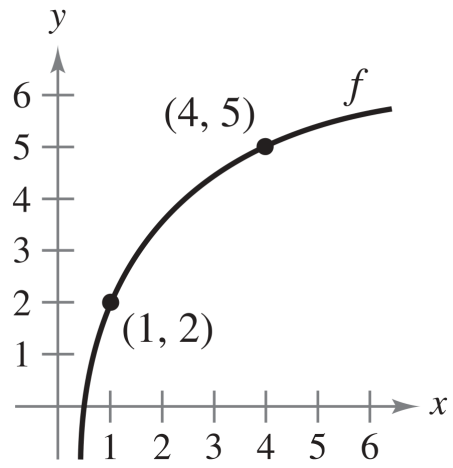
2.1.1 Question 2

Estimate the slope of the graph at points (x_1, y_1) ***and*** (x_2, y_2)



2.1.2 Question 4

Insert the proper inequality symbol $(< \text{ or } >)$ ***between the given quantities:***



(a) $\dfrac{f(4)-f(1)}{4-1}$ ($<$ or $>$) $\dfrac{f(4)-f(3)}{4-3}$

(b) $\dfrac{f(4)-f(1)}{4-1}$ ($<$ or $>$) $f'(1)$

2.1.3 Question 6

Find the slope of the tangent line to the graph of the function at the given point.

$$g(x) = \frac{3}{2}x + 1, (-2, -2)$$

2.1.4 Question 8

Find the slope of the tangent line to the graph of the function at the given point.

$$g(x) = 6 - x^2, (1, 5)$$

2.1.5 Question 10

Find the slope of the tangent line to the graph of the function at the given point.

$$h(t) = t^2 + 3, (-2, 7)$$

2.1.6 Question 16

Find the derivative by the limit process.

$$f(x) = 8 - \frac{1}{5}x$$

2.1.7 Question 20

Find the derivative by the limit process.

$$f(x) = x^3 + x^2$$

2.1.8 Question 24

Find the derivative by the limit process.

$$f(x) = \frac{4}{\sqrt{x}}$$

2.1.9 Question 26

(a) Find an equation of the tangent line to the graph of f ***at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the*** derivative ***feature of a graphing utility to confirm your results.***

$$f(x) = x^2 + 3x + 4, \quad (-2, 2)$$

2.1.10 Question 30

(a) Find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the **derivative** ***feature of a graphing utility to confirm your results.***

$$f(x) = \sqrt{x-1}, \quad (5, 2)$$

2.1.11 Question 34

(a) Find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the **derivative** ***feature of a graphing utility to confirm your results.***

$$f(x) = \frac{1}{x+1}, \quad (0,1)$$

2.1.12 Question 38

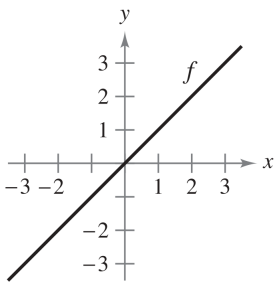
Find an equation of the line that is tangent of the graph of f **and** ***parallel to the given line.***

$$\text{Function: } f(x) = \frac{1}{\sqrt{x-1}}, \text{ Line: } x + 2y + 7 = 0;$$

2.1.13 Question 39, 40, 41, & 42

The graph of f is given. Select the graph of f'

39.



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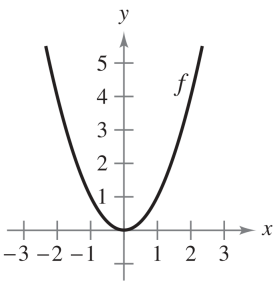

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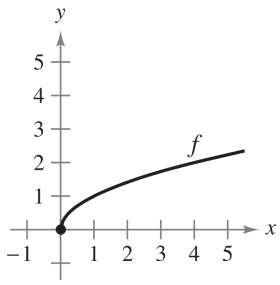

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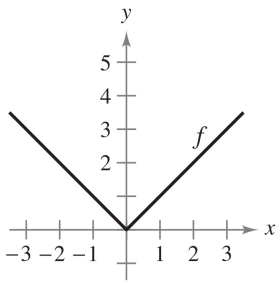
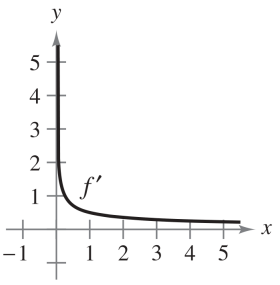
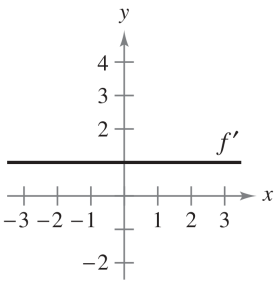


(a)



(b)



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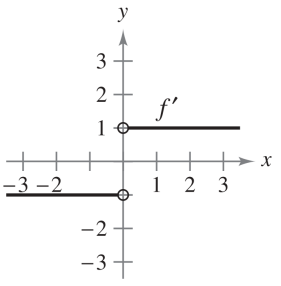
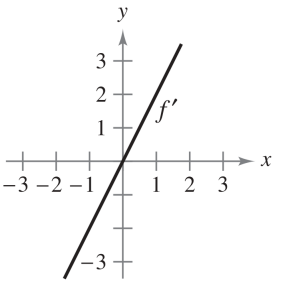
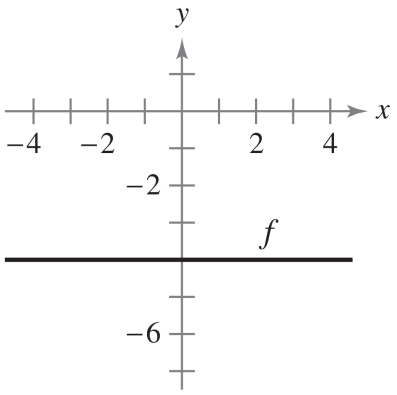


(d)



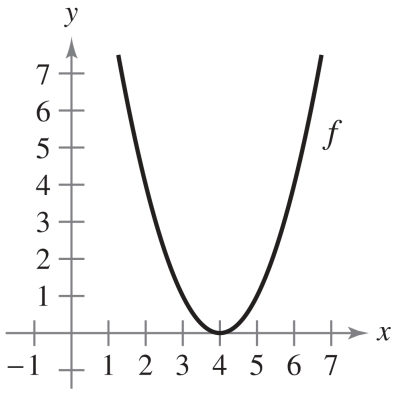
2.1.14 Question 46

Sketch the graph of f'. Explain how you found your answer



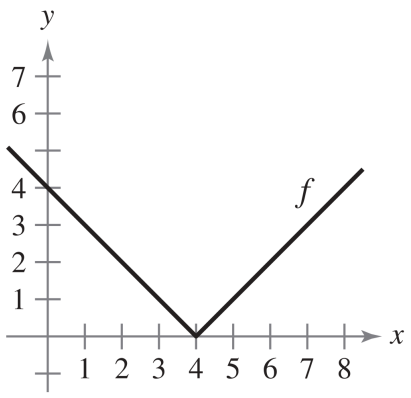
2.1.15 Question 47

Sketch the graph of f'. Explain how you found your answer



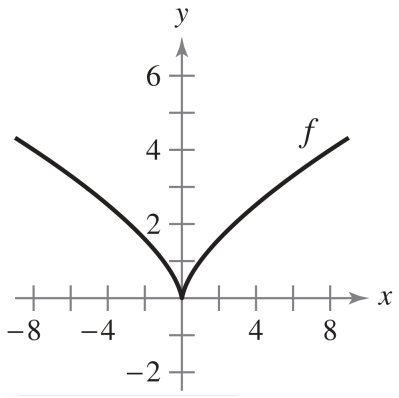
2.1.16 Question 48

Sketch the graph of f'. Explain how you found your answer



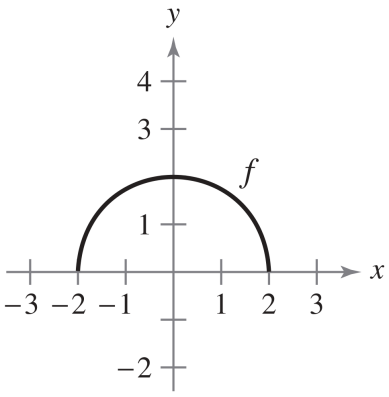
2.1.17 Question 49

Sketch the graph of f'. Explain how you found your answer



2.1.18 Question 50

Sketch the graph of f'. Explain how you found your answer



2.1.19 Question 51

Sketch a graph of a function whose derivative is always negative. Explain how you found your answer.

2.1.20 Question 52

Sketch a graph of a function whose derivative is always positive. Explain how you found your answer.

2.1.21 Question 54

The limit represents $f'(c)$ ***for a function*** f ***and a number*** c***. Find*** f ***and*** c***.***

$$\lim_{\Delta x \to 0} \frac{(-2 + \Delta x)^3 + 8}{\Delta x}$$

2.1.22 Question 56

The limit represents $f'(c)$ ***for a function*** f ***and a number*** c***. Find*** f ***and*** c***.***

$$\lim_{x \to 9} \frac{2\sqrt{x} - 6}{x - 9}$$

2.1.23 Question 58

Identify a function f that has the given characteristics. Then sketch the function.

$$f(0) = 4; f'(0) = 0;$$
$$f'(x) < 0 \text{ for } x < 0;$$
$$f'(x) > 0 \text{ for } x > 0;$$

2.1.24 Question 60

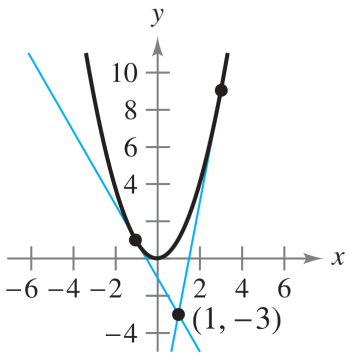
Identify a function f that has the given characteristics. Then sketch the function.

Assume that $f'(c) = 3$. Find $f'(-c)$ if (a) f is an odd function and if (b) f is an even function.

2.1.25 Question 62

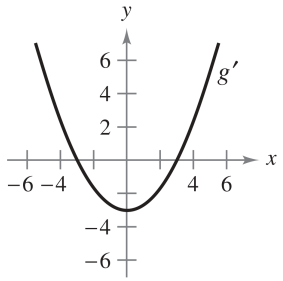
Find the equations of the two tangent lines to the graph of f that pass through the indicated point.

$$f(x) = x^2$$



2.1.26 Question 64

The figure shows the graph of g'



(a) $g'(0) =$

(b) $g'(3) =$

(c) What can you conclude about the graph of g knowing that $g'(1) = -\frac{8}{3}$?

(d) What can you conclude about the graph of g knowing that $g'(-4) = \frac{7}{3}$?

(e) Is $(6) - g(4)$ positive or negative? Explain.

(f) Is it possible to find $g(2)$ from the graph? Explain.

2.1.27 Question 74

Use the alternative form of the derivative to find the derivative at $x = c$ ***(if it exists)***

$$g(x) = x(x-1), \quad c = 1$$

2.1.28 Question 80

Use the alternative form of the derivative to find the derivative at $x = c$ ***(if it exists)***

$$g(x) = (x+3)^{1/3}, \quad c = -3$$

2.1.29 Question 83

Describe the x-values at which f is differentiable (Optional Graphs are in the Textbook).

$$f(x) = \frac{2}{x-3}$$

2.1.30 Question 84

Describe the x-values at which f is differentiable (Optional Graphs are in the Textbook).

$$f(x) = \left|x^2 - 9\right|$$

2.1.31 Question 85

Describe the x-values at which f is differentiable (Optional Graphs are in the Textbook).

$$f(x) = (x+4)^{2/3}$$

2.1.32 Question 86

Describe the x-values at which f is differentiable (Optional Graphs are in the Textbook).

$$f(x) = \frac{x^2}{x^2-4}$$

2.1.33 Question 87

Describe the x-values at which f is differentiable (Optional Graphs are in the Textbook).

$$f(x) = \sqrt{x-1}$$

2.1.34 Question 88

Describe the x-values at which f is differentiable (Optional Graphs are in the Textbook).

$$f(x) = \begin{cases} x^2 - 4, & x \leq 0 \\ 4 - x^2, & x > 0 \end{cases}$$

2.1.35 Question 94

Find the derivative from the left and from the right at $x = 1$ ***(if they exist. Is the function differentiable at*** $x = 1$***?)***

$$f(x) = \sqrt{1 - x^2}$$

2.1.36 Question 96

Find the derivative from the left and from the right at $x = 1$ ***(if they exist. Is the function differentiable at*** $x = 1$***?)***

$$f(x) = \begin{cases} x, & x \leq 1 \\ x^2, & x > 1 \end{cases}$$

2.1.37 Question 98

Determine whether the function is differentiable at $x = 2$

$$f(x) = \begin{cases} \frac{1}{2}x + 1, & x < 2 \\ \sqrt{2x}, & x \geq 2 \end{cases}$$

2.1.38 Question 101, 102, 103, 104

True or False? **In 101 - 104, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

102. The slope of the tangent line to the differentiable function f at the point $(2, f(2))$ is $\dfrac{f(2+\Delta x)-f(2)}{\Delta x}$.

103. If a function is continuous at a point, then it is differentiable at that point.

104. if a function has derivatives from both the right and the left at a point, then it is differentiable at that point.

105. if a function is differentiable at a point, then it is continuous at that point.