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The Heat Pump in a Vortex Tube

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Abstract

The temperature splitting phenomenon of a Ranque Hilsch vortex tube in which a stream of gas divides itself into a hot and a cold flow, has been identified as a natural heat pump mechanism, which is enabled by a secondary circulation. We have developed an analytical model that quantitatively explains the measurements.

The Vortex Tube

A vortex tube, Figure 1a, is a cylindrical pipe of radius R (typically 1 cm) and length L (typically $L/2R \approx 30$). The gas first enters the plenum chamber at the mass flow rate i_0, the pressure p_{pl}, and temperature T_{pl}. It then streams through one or more inlet nozzles located near the periphery. The fraction $y = j_c/j_0$ emerges in the state p_c, T_c out of the cold port, an outlet hole (radius $R_c \approx 0.30\ R$) on the axis at the inlet plane, and the remainder leaves the hot port at the far end with the pressure and temperature p_h, T_h. By selecting p_{pl}, T_{pl}, p_c, and the cold mass fraction y, the temperature spread $T_h - T_c$ can be varied over a considerable range.

This temperature separation has baffled researchers ever since it was discovered by Ranque [1] and studied by Hilsch [2]. Many different qualitative explanations have been offered, see for instance [3], however none of these explanations so far has lead to a quantitative model for the Ranque Hilsch effect [4].

In a recent analysis of the velocity field [5] we found a secondary circulation with a mass flow rate j_s of the same magnitude as j_h and j_c. The velocity field of the secondary circulation u_s is aligned at right angle to the velocity u_φ of the primary vortex. j_s convects material between the axial region where the pressure is low and the peripheral region where the pressure is high. This circulation has all the features of a working fluid in a heat pump, and thereby provides a new mechanism to explain the Ranque Hilsch effect.

Heat pumps employ three processes: (i) the working fluid moves heat or enthalpy continuously between a high pressure and a low pressure region, (ii) the compressed working fluid is hotter than the surrounding medium to give off heat, (iii) the expanded

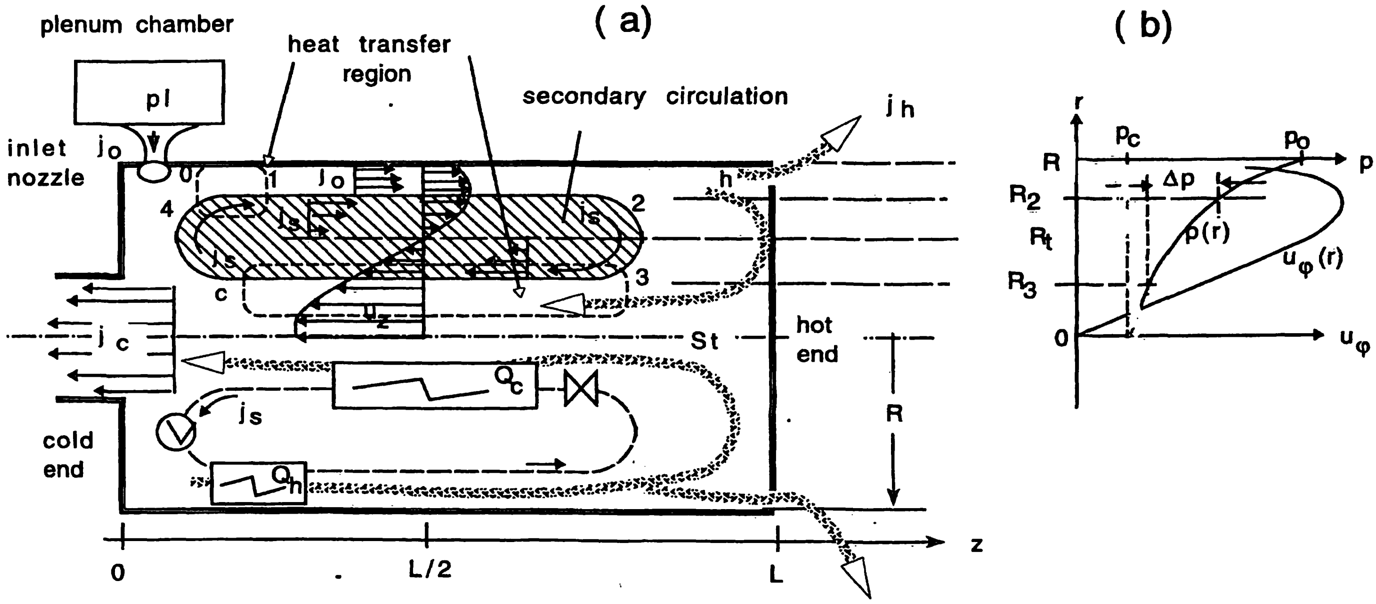

Fig. 1: (a) Vortex tube with region of secondary flow. $R_3 \leq r \leq R_2$, and velocity profile $u_z(r)$ at $z = L/2$, lower part of figure shows a schematic of the heat pump. (b) Pressure $p(r)$ and azimuthal velocity $U_\varphi(r)$ near the entrance plane.

working fluid is colder than its surrounding, to absorb heat. All three conditions are met in the vortex tube:

Fluid starting in the plenum chamber at the conditions T_{pl}, p_{pl} passes adiabatically through the inlet nozzle(s) thereby cooling down and speeding up. The entrance temperature T_0 and velocity u_0 in the vortex tube depend on the pressure drop $p_{pl} - p_0$. From the momentum equation applied in the entrance nozzle

$$M_0^2 = u_0^2/a_0^2 = mu_0^2/\gamma R_g T_0 \approx (p_{pl} - p_0)/\gamma p_{pl} \tag{1}$$

one can find the entrance Mach number M_0. In (1) a_0 is the speed of sound, m the molecular weight, R_g the gas constant, and γ is the adiabatic exponent. Due to the tangential orientation of the inlet nozzle system the entrance flow (components $u_\varphi >> u_z, u_r$) sets up the primary azimuthal vortex with velocity and pressure profiles as indicated in Figure 1b. The lowest pressure in the entrance plane is found on the axis. It is approximately equal to the exit pressure p_c of the cold stream. p_c adjusts itself so that the radial pressure balance is consistent with the entrance Mach number. The radial integration of the momentum equation yields:

$$M_0^2 = u_0^2/a_0^2 = (2/\gamma)(p_0 - p_c)/p_0 =: (2/\gamma)x, \tag{2}$$

An important parameter of a vortex tube is the normalized pressure drop $x = (p_0 - p_c)/p_0$. Unfortunately the entrance pressure p_0 is generally not known. However, it can be eliminated between equations (1) and (2). Therefore x can be expressed as functions of the easily measured pressures p_{pl} and p_c.

$$x = (p_0 - p_c)/p_0 \approx (p_{pl} - p_c)/(p_{pl} + 2p_c) \tag{3}$$

Experiments by Ahlborn and Groves [5], Lay [6], Sibulkin [7], Bruun [8], and Takahama and Yokosawa [9] show a convoluted flow field inside the vortex tube in which a secondary circulation with the mass flow rate j_s (and the velocity components u_r, u_z) is embedded in the primary vortex. Our measurements [5] yield $j_s \approx 0.5 j_0$ for y in the range $0.2 \leq y \leq 0.8$. Secondary flow cells are well known from every day experiences. An example is the meridional motion of tea leaves in a tea cup, which is seen when the tea is stirred with a spoon: The tea leaves ascend along the axis and drop down near the periphery. This happens because the rotating fluid is slowed down by wall friction so that the centrifugal acceleration is larger at the top than at the bottom of the cup. Similarly the swirl velocity is larger in the entrance plane of the vortex tube than near the hot end.

A typical radial profile of the axial velocity u_z at $z = L/2$ is shown in Figure 1a. In the outer regions, $r > R_t$, the gas flows towards the hot end at velocities of typical $u_z \approx 30$ m/sec. At radial positions $r < R_t$ the gas flows towards the cold end. In this "back flow core" the velocity reaches $u_c \approx -100$ m/s. Note that there is no axial motion at the radius R_t. Fluid volumes located in this region are trapped and circulate for a long time before they may escape the trap due to random fluctuations. Another place where fluid may be held for a long time is the stagnation region St on the tube axis near the hot end. For radii $r < R_t$ the secondary circulation, j_s, is part of the back flow core, and is in close contact with the flow of gas that eventually exits as the cold stream. For $r > R_t$ the secondary circulations flows next to the inlet stream j_0. On its cyclical path the secondary circulation j_s convects mass elements between the inner regions where the pressure is low and the outer regions where the pressure is high. The compression and expansion of j_s is likely powered by its deceleration and acceleration, and it is an adiabatic process, since the radial motion of j_s occurs well away from the ends of the vortex tube. It is not clear from the data if the secondary circulation is open or closed. A closed circulation would form a flattened vortex ring of separated fluid that never mixes with the rest. In an open circulation fluid continually mixes by diffusion or turbulent interaction so that some fluid enters the secondary circulation while an equal amount of fluid leaves. However, in either case the secondary circulation has all the feature (i) of the working fluid in a heat pump.

Due to wall friction the velocity drops gradually as the gas in the peripheral region moves from region 1 towards the hot end, h. Therefore the gas heats up. At $z \approx L$ the velocity has become quite small compared to u_0. The associated temperature increase is found with the energy equation $T_h = T_1 + (\gamma - 1) M_0^2 T_1/2$, where M_0 can be replaced by x using equation (2).

$$T_h = T_1 + (\gamma - 1) x T_1/\gamma, \tag{4}$$

Feature (ii) of a heat pump requires that the adiabatically compressed and heated fluid comes into contact with a cooler medium for a sufficiently long time to exchange heat. This condition is indeed satisfied in the heat transfer region near the entrance nozzle.

Point 4 on the path of the secondary circulation is close to the injection point 0 of the fluid which has been adiabatically cooled as it passed through the injection nozzles. This is the coldest place in the vortex tube. Since the azimuthal velocity u_φ in this region is much larger than the radial and axial velocities volume elements from the secondary circulation j_s and from the injected flow j_0 swirl around side by side for a fairly long time and exchange heat predominantly by turbulent mixing and possibly by some heat conduction. In this convoluted flow field the injected flow j_0 and the secondary circulation which we will now call the "working fluid", stay in contact over a much longer path than the trace of j_s in the r-z plane would suggest. Hence condition (ii) of a heat pump is also satisfied.

Feature (iii) of a heat pump requires that the expanded and cooled working fluid comes into contact with a warmer stream. When the working fluid emerges out of the high pressure heat exchange region, it travels towards the hot end and then turns inwards at to some point 2 to join the back flow core at some point 3. Provided there is still a sizable angular velocity, the pressure at point 2 must be higher than the pressure at point 3, so that j_s must undergo an adiabatic expansion and cooling. From point 3 on j_s comes into contact with the inner core of the flow which eventually becomes the cold stream j_c. Since this axial stream started with a temperature $T \approx T_h$ at the hot end it must be hotter than the adiabatically cooled working fluid, so that a heat flux Q_c is transferred back into j_s. Enthalpy transfer in this region is enhanced by the fact that the contact time is relatively long since fluid moves very slowly near the stagnation point St. Hence this low pressure heat exchange region satisfies condition (iii) of a heat pump.

Over a full cycle the working fluid maintains constant internal energy, like any heat transfer medium in an ordinary heat pump: Therefore the absorbed heat Q_c and the rejected quantity Q_h must be nearly equal assuming the net work of compression added to the mass unit of j_s to be small compared to either Q_c or Q_h.

A quantitative model of the heating process must consider four effects: (a) The adiabatic compression of the secondary circulation between points c and 4, which raises the temperature in j_s from T_c to T_4, (b) the mixing and exchange of enthalpy between points 4 and 1, which transfers the heat flux Q_h out of the working fluid lowering the temperature of j_s from T_4 to T_1 and raising the temperature of the entrance flow from T_0 to T_1, (c) the adiabatic deceleration of the fluid near the wall between the points 1 and h, which increases the temperature from T_1 to T_h, and (d) the mixing and exchange of enthalpy between points 3 and c, which transfers the heat flux Q_c back into the working fluid. We assume that the secondary flow is an open circulation system.

The processes (a) and (b), although conceptually quite distinct, cannot be separated easily because one does not know exactly at which pressure the adiabatic compression stops and where the mixing starts. However assuming that the working fluid j_s does not heat up, the heat flow Q_c in the low pressure region must equal the heat release Q_h in the high pressure zone.

In the process (b) two fluid elements of the streams j_c and j_0 have the total energy $E_4 = C(j_0 T_0 + j_s T_4)\Delta t$ before the heat exchange and $E_1 = C(j_0 + j_s) T_1 \Delta t$ thereafter. Conservation of energy requires $E_4 = E_1$ or $j_0(T_1 - T_0) = j_s(T_4 - T_1)$. Similarly in the process (d) some energy E_3 is exchanged, and one finds $j_s(T_c - T_3) = j_c(T_h - T_c)$. Since the working fluid j_s cannot heat up in steady state, the heat exchanged in process (b) must be equal to the energy exchanged in process (d) and one has $j_0(T_1 - T_0) = j_c(T_h - T_c)$ or

$$T_1 = T_0 + y(T_h - T_c) = T_c - \varepsilon_0 T_c + y(T_h - T_1 + \varepsilon_1 T_c), \tag{5}$$

where we have defined the positive quantities ε_1 and ε_2 by the relations $T_1 = T_c + \varepsilon_1 T_c$, and $T_0 = T_c - \varepsilon_0 T_c$. Equation (5) describes the heat exchange processes (b) and (d) in the high pressure and the low pressure regions. It implicitly also includes the effects of the process (a) since heat could not be transferred to the inlet stream unless the temperature in the working fluid had been raised by adiabatic compression. Process (c) is described by equation (4). Combination of equations (4) and (5) leads to the temperature ratio

$$Z = \frac{T_h}{T_c} = 1 + (1 + \varepsilon_1)\frac{(\gamma - 1)}{\gamma}x(1 + y) + \varepsilon_1\left(y - \frac{\varepsilon_0}{\varepsilon_1}\right). \tag{6}$$

The first two terms on the right hand side represent the principal effect of the pressure function x and the mass flow ratio y. For one particular setting in our vortex tube the exit temperatures were measured as $T_h = 310$ K and $T_c = 271$ K, the entrance velocity was determined as $u_0 = 227$ m/s, $T_0 \approx 263$ K was calculated from nozzle theory, and $T_1 \approx 280$ was obtained with equation (4). Such numbers suggest the approximations $\varepsilon_1 << 1$, and $\varepsilon_0 << 1$. When evaluating the temperature ratio in first approximation it is safe to neglect ε_1 compared to 1 so that the first two terms on the right hand side of equation (6) can be simplified as $1 + x(1 + y)(\gamma - 1)/\gamma =: Z_0$. The third term on the right hand side $\Delta Z = \varepsilon_1(y - \varepsilon_0/\varepsilon_1)$ could in principle be of order ε_1, because y and $\varepsilon_0/\varepsilon_1$ are both of order 1. Fortunately y and $\varepsilon_0/\varepsilon_1$ have a different sign, and if they are of similar magnitude ΔZ could well be small compared to 1. To check this possibility we evaluated $\Delta Z = Z_{\exp} - Z_0$ for a large number of data points with $0 \leq y \leq 1$ at the plenum pressure $p_{pl} = 3$ bar. $Z_{\exp} = (T_h/T_c)_{\exp}$ is the measured temperature ratio. y was directly measured, and x was determined according to equation (3). These calculations yield ΔZ for 240 data points in the range $-0.02 < \Delta Z < -0.01$, with an average value $\Delta Z_{av} = -0.013$. The data did not show any dependence of ΔZ on y. Therefore

$$Z = \frac{T_h}{T_c} \approx Z_0 = 1 + \frac{(\gamma - 1)}{\gamma}x(y + 1) \tag{7}$$

Figure 2 shows the measured ratio $(T_h/T_c)_{\exp}$ as function of Z_0 as calculated from the measured values of x and y. The linear, dashed line in Figure 2, runs below $Z_{\exp} = Z_0$ by the amount ≈ 0.01. This difference is the error made by neglecting ΔZ.

For many applications it is more important to know the increase and decrease of the temperature in the hot and the cold stream. These can be obtained as follows: For energetic reasons the temperature changes in the hot stream $\Delta T_h = T_h - T_{pl}$ and in the cold component $\Delta T_c = T_{pl} - T_c$ must be equal for the mass fraction $y = 0.5$. This condition can be combined with (7) to find the temperature spread at $y = 0.5$,

$$\Delta T_h(0.5) \approx T_{pl}(B \cdot x)/(1 + B \cdot x), \tag{8}$$

where $B = 3(\gamma - 1)/4\gamma$. According to measurements of all investigators ΔT_h rises about linearly with y in the range $0 < y < 0.8$. The slope of this curve is obtained from

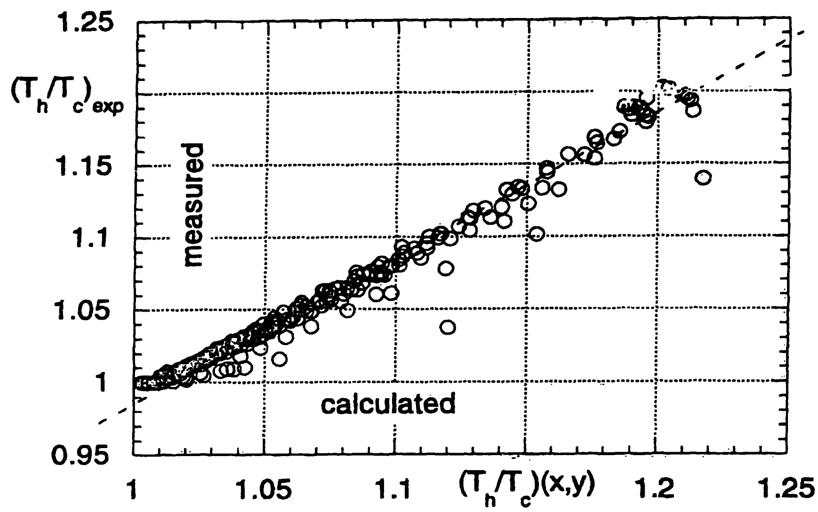

Fig. 2: Ratios of measured exit temperatures as functions of predicted values according to equation (7) for $p_{pl} = 3$ bar.

equation (8) and one gets the hot exit temperature

$$T_h(x, y) \approx T_{pl}\left\{1 + \frac{2B\cdot x\cdot y}{1 + B\cdot x}\right\} \tag{9}$$

The temperature at the cold exit is then found with equation (7) as

$$T_c \approx T_h/Z_0 \approx T_{pl}\left\{1 + \frac{2B\cdot x\cdot y}{1 + B\cdot x}\right\} \Big/ \left\{1 + \frac{\gamma - 1}{\gamma}(1 + \gamma)x\right\} \tag{10}$$

Figure 3 shows the experimental check of equations (8) and (9) for experimental points at $p_{pl} = 3$ atm and 4 atm and mass flow rates in the range $0.05 \le y \le 0.9$. The data for the two different plenum pressures fall onto the same line, indicating that the approximation $\Delta Z < < 1$, which was inferred from data with $p_{pl} = 3$ bar holds for the higher

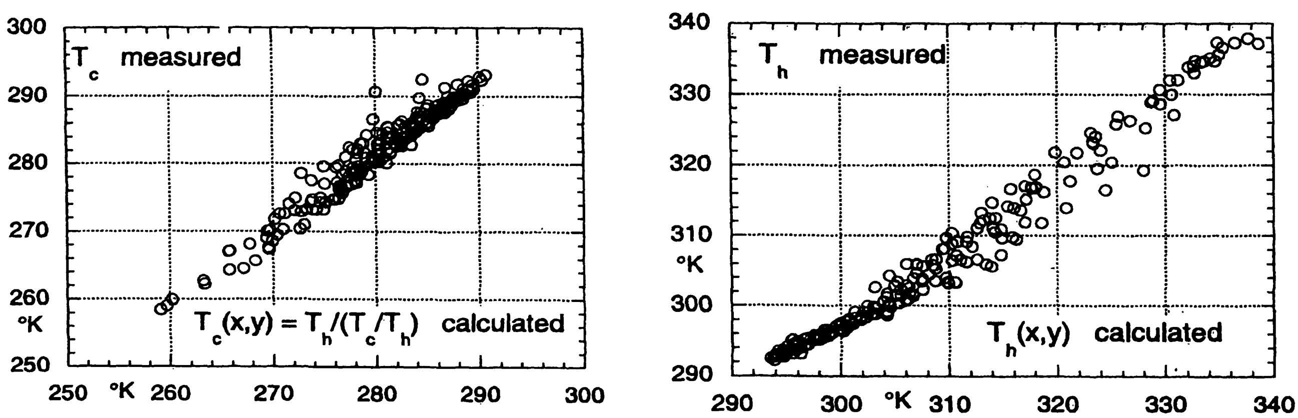

Fig. 3: Measured exit temperatures as functions of predicted values according to equations (8) and (9) for $p_{pl} = 3$ bar and 4 bar.

pressure as well. Predictions and measurements are in excellent agreement indicating that our model is a good quantitative representation of the processes in a vortex tube. Since the energy equation of the model was based on a mixing heat exchange process we conclude that the secondary circulation behaves like an open flow so that the "working fluid" in the vortex tube heat pump is not permanently trapped.

In summary the primary vortex in the vortex tube sets up a secondary circulation j_s that acts like the working fluid in a heat pump. The secondary stream absorbs heat near the axis at low pressure. Then an adiabatic compression and heating takes place as the gas is moved towards the periphery where energy is transferred to very cold fluid elements which are just emerging out of the injection nozzles. This process together with the conversion of kinetic energy into heat in the hot stream accounts fully for the observed Ranque. Hilsch effect, and thereby removes the mystery of an phenomenon that had been unexplained for over 60 years.

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