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vectors could be obtained. These are the values of the voltages at the junctions when the network is in resonance. Here only the ratios of the voltages are significant. With only a few changes in the wiring and with the use of a good grade of attenuator, values within a few percent for the characteristic vectors should be obtainable.

SOLUTION OF SIMULTANEOUS EQUATIONS

In principle it should also be possible to modify the network to solve simultaneous equations such as those of Eq. (3). To do this external currents I_1 of known amplitude and phase need to be introduced at the junctions and the junction voltages read.

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The Use of the Expansion of Gases in a Centrifugal Field as Cooling Process*

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The design of a vortex tube of good efficiency in which the expansion of a gas in a centrifugal field produces cold is described. The important variables in construction and operation are discussed and data for several tubes under various operating conditions are given. Low pressure gas, 2 to 11 atmospheres, enters the tube and two streams of air, one hot and the other cold, emerge at nearly atmospheric pressure. The cold stream may be as much as 68°C below inlet temperatures. Efficiencies and applications are discussed.

1. INTRODUCTION

The adiabatic expansion of a gas with external work, which may take place in a reciprocal engine or in a turbine, is the most effective cooling process. It requires, however, a considerable technical effort, especially in the region of low temperatures, where many technical difficulties arise. In the technique of gas liquification, therefore, the expansion with external work is usually replaced by the use of the much simpler throttle effect, which does not require any moving parts. This cooling process is much less efficient, and is only possible because real gases deviate from the perfect gas law.

Under these circumstances it seems important to investigate an arrangement which was described by Ranque in 1939.[1] In this arrangement compressed air is permitted to enter through a tangential nozzle into a cylindrical container, formed by a tube of 12 mm diameter. A turbulent flow of gas, in a screw-like motion, escapes through both ends. This rotating air stream produces a region of increased pressure near the wall inside the cylinder, and a region of decreased pressure near the axis. If one end of the cylinder is closed by a diaphragm which permits the escape of air only from the central region, while the other end is throttled, the air escaping through the central diaphragm has a reduced temperature, while the air escaping through the other end shows a temperature increase. The observed temperature difference between the two streams, using compressed air of 6-atmos. gauge pressure at 20°C, was 70°C, while the lowest temperature of the cold stream was −12°C.

The purpose of this investigation is to design an arrangement of this kind which can be used as a cooling process of maximum efficiency, and to determine the largest efficiency obtainable.

2. CONSTRUCTION AND FUNCTION OF A "VORTEX TUBE"[2]

The results of numerous preliminary experiments showed that for a favorable cooling effect

* Submitted by: R. M. Milton, Department of Chemistry, Johns Hopkins University; Translated by: I. Estermann, Physics Department, Carnegie Institute of Technology.

[1] M. G. Ranque, J. de. phys. et rad. [7], 4, 112 (1933).

[2] Literal translation of the German word "*Wirbelröhre.*"

the diaphragm should be installed as near as possible to the nozzle. The circular air flow near the nozzle and the diaphragm should be as nearly as possible of rotational symmetry. This is achieved by a peculiar method of introducing the compressed air, which is shown in Fig. 1b and 1c. Two pieces of tubing of internal diameter R are attached to the cylinder by means of flanges and overlapping nuts; one of the flanges acting as diaphragm B located near the nozzle.

For an explanation of the mechanism of the vortex tube we refer to Fig. 1a. If both ends of the attached tubes are open, the air entering tangentially through D escapes with a screw-like motion along the wall of the left tube, while the centrifugal force and the internal friction of the gas produce a lower pressure in the axial region. Hence, air will be sucked in through B from the right tube. This can be avoided if the flow through the left tube is throttled by a valve attached to the end of this tube. This valve should be sufficiently far away from the nozzle D (about 50 R) so that the gas reaching it will have lost most of its screw-like motion because of internal friction. By partial closing of the valve, it is possible to force a fraction, μ, of the air stream to escape to the right through the diaphragm B. This fraction increases with increasing internal pressure p_i which is measured in the left tube near the valve, and it originates in the region near the axis of the vortex in the left part of the tube. The air escaping through B has been expanded in the centrifugal field from a region of high pressure near the wall of the cylinder to a pressure p_i in the region near the axis; and it has transferred a considerable part of its kinetic energy by means of internal friction to the peripheral layers. These flow to the right and acquire a higher temperature. Hence, a fraction μ of the expanded air leaves the tube to the right with reduced temperature, and a fraction $(1-\mu)$ escapes to the left with increased temperature.

In the absence of internal friction, and with a sufficient pressure gradient, the velocity of the air would increase to supersonic speed during the expansion between the circumference and the axis of the vortex tube. The internal friction, however, is particularly effective in this region. It causes a flow of energy from the axis to the circumference by trying to establish a constant angular velocity throughout the cross section of the tube. There will be, therefore, a decrease in the heat content of the axial part of the air stream flowing to the right and an increase in the heat content of the stream flowing to the left, if heat exchange with the surroundings through the wall of the tube is prevented.

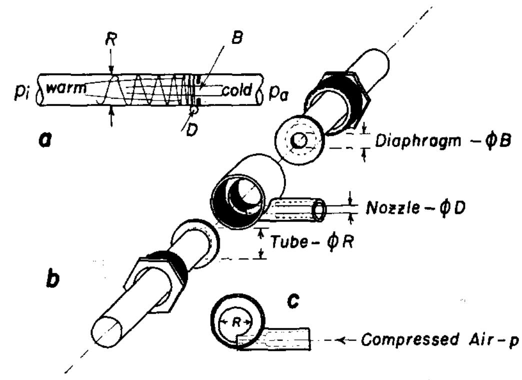

FIG. 1. Vortex tube; (a) schematic sketch; (b) tube taken apart (thread surfaces black); (c) central piece with nozzle.

The demonstration of such a vortex tube is simply astonishing. With proper choice of the fraction μ and $1-\mu$, compressed air of a few atmospheres pressure and 20°C will easily produce a temperature of +200°C in the left tube and −50°C in the right tube.

3. DETERMINATION OF THE MOST FAVORABLE CONDITIONS FOR THE OPERATION OF A VORTEX TUBE

In the construction of a vortex tube according to Fig. 1 the results of several preliminary experiments have been utilized. It appears that the efficiency is not affected by any particular shape of the cylindrical container, or of the "warm" tube. We used, therefore, a smooth cylindrical tube of sufficient length. The tube carrying the diaphragm B may have any shape which does not interfere with the flow of the cold air. Attempts to increase the efficiency by using diaphragms of special shapes had no apparent results. The use of a thin-walled German silver tube and the arrangement of screws and flanges for the assembly as described earlier reduces the heat transfer between the warm and the cold part to a small value.

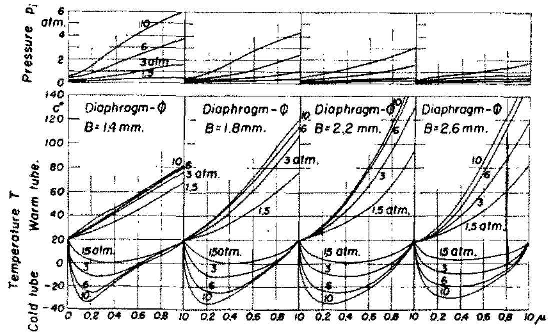

FIG. 2. Temperature of cold and hot gas stream and internal pressure in tube No. 1 as function of external pressure, size of diaphragm, and μ. μ=mass of cold/mass of total air. Constants: $R=4.6$ mm, $D=1.1$ mm, $T_0=20°C$, $p_a=1$ atmos. Parameter: pressure p at the nozzle.

While the general design is fixed by the simple principles listed above, there are a number of variables to consider which affect the operation. Since it seems rather difficult to derive these influences theoretically from an analysis of the behavior of the compressible flow in a quantitative form, they were subjected to an experimental investigation. The temperatures of the gas in the two flow directions depend on the following variables:

(1) Temperature, T_c, pressure p; and rate of flow G of the expanding compressed air.
(2) External pressure p_a outside the vortex tube.
(3) Diameters of the tube (R), the diaphragm (B), and the nozzle (D); these quantities determine G.
(4) Ratio of distribution of air between the flow in both directions, μ-mass of cold air/mass of total air.

The measurement of the temperature of the streaming gas streams presents a separate problem. Thermocouples arranged in the gas stream would give incorrect results, since it is certain that the expanded cold air has not completely given off its kinetic energy and still contains a rotating flow. For this reason, the temperatures T were simply measured at the thermally insulated walls of the tubes. These measurements yield temperature differences which are certainly not too favorable, but will probably be too small.

4. RESULTS

Figure 2 contains results obtained with a vortex tube of a diameter $R=4.6$ mm. The effect of the nozzle diameter D cannot be ascertained from this figure, but it was found by experiments that $D=1.1$ mm was a favorable value. Smaller values of D would certainly be less favorable. If they were chosen, the rotating flow would decrease too fast on account of the small rate of flow and the more important contribution of the friction between gas and walls. Larger values of D would result in such large rates of flow that it could not be carried by the tube without increasing the pressure p_i to too large values. This would make the expansion ratio p/p_i unfavorable. The pressure difference p_i-p_a, moreover, causes losses at the throttle.

Figure 2 shows the temperatures of the cold and warm tubes as functions of μ for four different values of the initial gas pressure p. The ratio u can be changed by means of a valve attached to the free end of the warm tube, which was about 30 cm long. At such a distance from the nozzle, the rotating component of the gas motion had decreased sufficiently. The value of μ was measured with two gas meters arranged in the cold and warm stream, respectively. The total rate of flow is proportional to the absolute pressure p of the entering gas, but is practically independent of the setting of the valve and, therefore, of μ. For $D=1.1$ mm and air, the rate of flow was, for instance, $G=7.0$ m^3/hr. for $p=10$-atmos. gauge pressure. The internal pressure p_i was measured near the throttle valve and is also plotted in Fig. 2. The influence of the diameter of the diaphragm B is shown for four different values.

There is always a certain value of μ producing a minimum value of the temperature of the cold air. The larger the value of μ, the higher the temperature of the warm end. A small diaphragm ($B=1.4$ mm) is rather unfavorable, especially at high values of μ, since then the internal pressure p_i and the throttle losses become too large. On the other hand, the largest diaphragm ($B=2.7$ mm) gives a smaller cooling effect in the minimum of the curve because it permits the entrance of air from regions where the rotating flow is still considerable. For large values of μ (e.g., 0.8), the most favorable cooling effect is obtained with the largest diaphragm. The choice of the diaphragm diameter B depends, consequently, on whether it is desired to reach very low temperatures or to produce large quantities of cold air. The value $B=2.2$ mm may be considered as a most favorable compromise solution for these two factors.

Many more details may be seen in the curves. It should be pointed out, in particular, that the cooling of the cold air and the heating of the warm air with respect to room temperature are equal for $\mu=0.5$. It is easy to see, moreover, that the relation

$$\mu \text{ (cooling)} = (1-\mu) \text{ (heating)}$$

must be always valid since the amount of heat removed from the cooled air must be equal to the amount of heat given to the heated air. It should also be remarked that the setting of the throttle valve giving minimum temperature can be found easily without special measurements; a boiling sound is audible if the valve is set for the correct value μ (e.g., $\mu=0.3$ for 10 atmos. and $B=2.2$ mm, $T=-35°\text{C}$). It should be realized that the temperature of the cold air is measured too high if μ is very small, and the temperature of the warm air is measured too low if μ is near 1, because in both cases the rate of flow through the tube in question is very small.

It would be necessary to repeat all the measurements for different values of the tube diameter R, in order to investigate the influence of this quantity. It is, however, sufficient to report the measurements in the cold air stream with the most favorable value of the diameter of the diaphragm B for a wider tube (Fig. 3). The diameter of the tube R has been increased to more than twice its former value, and the nozzle diameter D in proportion. In this case, the temperatures of the cold air are lower than those in Fig. 2 for all values of p. Additional data for the lowest temperatures obtained with a still larger tube as function of p are given in Fig. 4. The total rate of flow G for all tubes may be represented by the relation

$$G=\text{const.}\cdot F\cdot p,$$

where F is the area of the nozzle in mm², p the initial air pressure in atmos., G the rate of air flow in kg/hr., and const. $=0.80$ kg/(h mm² atmos.).

Finally, we show in Fig. 3 the product $\mu(T_0-T)$ as function of μ for tube No. 2. This product is proportional to the total cooling effect. It is apparent that the maximum heat removal does not occur at the value of μ for which the lowest temperature is obtained.

It follows from Fig. 4 that increasingly larger temperature reductions and cooling effects may

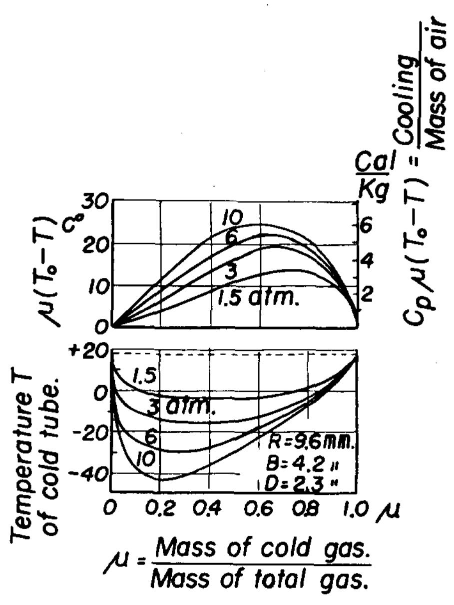

FIG. 3. Temperature and heat content of the cold air stream of tube No. 2 with most favorable diaphragm.

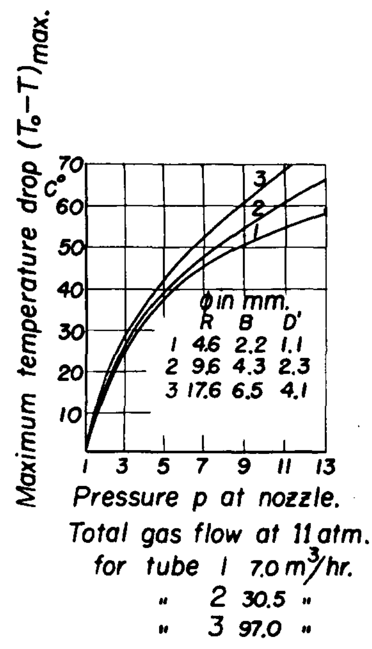

FIG. 4. Maximum temperature drop for tubes Nos. 1, 2, and 3.

be obtained with larger vortex tubes at the same initial pressure. The rates of flow, however, become considerable, and the capacity of the compressor which was available did not permit the use of larger tubes. It is easy to see that the efficiency increases for larger tube diameters, because for larger tube diameters the heat losses due to heat conduction from the axis to the circumference of the vortex tube become less important. The larger the tube diameter R, the smaller is the ratio B/R, where B is the most favorable diameter of the diaphragm. It should be remembered, however, that a relatively small diaphragm favors transmission of cold air which has been well expended and is practically free from revolving motion.

5. THERMODYNAMIC EFFICIENCY OF A VORTEX TUBE

The calculation of the cooling efficiency of a vortex tube will be carried out by comparing it with the cooling efficiency of the adiabatic expansion of a perfect gas in an expansion engine delivering external work. Such an engine will, theoretically, deliver a continuous flow of cold air of the temperature T_1, produced by expanding a gas of the temperature T_0 from the pressure p to the pressure p_a. For this process,

$$T_0/T_1 = (p/p_a)^{(\gamma-1)/\gamma}. \tag{1}$$

The work E delivered by the expansion engine per unit mass is $E = c_p(T_0 - T_1)$ and is equivalent to the amount of heat Q removed from the gas. The minimum amount of work required for

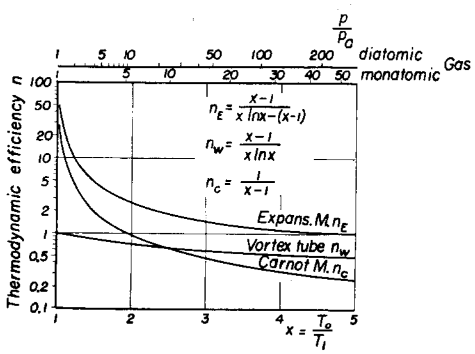

FIG. 5. Efficiency of 3 refrigerating machines for comparison.

(isothermal) the compression is

$$A = RT_0 \ln \frac{p}{p_a}. \tag{2}$$

Substituting for p/p_a from Eq. (1), one obtains

$$A = c_p T_0 \ln \frac{T_0}{T_1}. \tag{3}$$

The net work required is $A - E$, and the result obtained for the efficiency of the expansion engine is

$$\eta_E = \frac{Q}{A-E} = \frac{T_0 - T_1}{T_0 \ln(T_0/T_1) - (T_0 - T_1)} = \frac{x-1}{x \ln x - (x-1)}, \tag{4}$$

where $x = T_0/T_1$.

The efficiency of the vortex tube is less than that of the expansion engine, since only a fraction μ of the gas is expanded into a cold stream and heat is removed by another fraction of the gas $(1-\mu)$. Moreover, no expansion work is gained.

According to the explanation of the mechanism given in Section 2, the gas is expanded along a spiral path and withdrawn through a diaphragm. The internal friction tends to produce a uniform angular velocity along the path of expansion in the centrifugal field. The same effect could be obtained by installing a turbine wheel in front of the nozzle. This would transform the vortex tube into an expansion turbine, where the compressed gas enters near the circumference and leaves at low pressure near the axis while mechanical work is delivered by the turbine rotor. In this theoretical case all the gas could be removed as cold gas through the diaphragm; and the efficiency would be η_E as derived in Eq. (4). The work developed at the turbine rotor could be changed into heat by a friction clutch on the shaft. This heat could be removed by branching off a very small fraction of the expanded gas towards the warm side of the vortex tube which would escape with any desired high temperature.

The efficiency of such a hypothetical vortex tube with a hidden turbine would be larger than

TABLE I. Efficiency of tube No. 3. $R=17.6$ mm, $T_0=18°C$, $p_a=1$ atmos.

1. Pressure ratio used (p/p_a)	2.5	4	7	11
2. Maximum temperature drop $(T_0-T)\text{max}$	24°C	36°C	52°C	68°C
3. Theoretical temperature drop (T_0-T_1)	67°C	95°C	124°C	144°C
4. Temperature efficiency $\alpha=(T_0-T)\text{max}/(T_0-T_1)$	0.36	0.38	0.42	0.47
5. $[\mu(T_0-T)]\text{max}$	15.5°C	21°C	24°C	27.5°C
6. Cooling efficiency $B=[\mu(T_0-T)]\text{max}/(T_0-T_1)$	0.23	0.22	0.19	0.19
7. Theoretical efficiency η_W	0.88	0.81	0.77	0.72
8. Practical efficiency $B\eta_W$	0.20	0.17	0.15	0.14

that of a real vortex tube, since almost all the gas would escape as a cold stream through the diaphragm. No expansion work would be developed in this case; its heat equivalent would be carried off by the minute amount of gas escaping through the warm tube. The efficiency of such an ideal vortex tube shall be compared with that of the actual vortex tube; it is obtained by omitting the expansion work E in the denominator of Eq. (4). The result is

$$\eta_W=(x-1)/x \ln x. \tag{5}$$

Figure 5 shows the dependence of η_E and η_W on x, also, and for comparison only, the efficiency of a Carnot engine η_c. While such an engine is not very practical for the production of a continuous stream of cold air of the temperature T_1, it is frequently used in practice for such a purpose. The upper scales in Fig. 5 show the pressure ratio p/p_a for which the temperature ratio T_0/T_1 is reached for mono- and diatomic gases. For $x=1$, η_c and η_E approach ∞, while all values of η approach zero for large values of x.

The theoretical efficiency of the vortex tube is rather favorable, expecially for large pressure ratios where it approaches the value η_E.

The results of these theoretical considerations may now be compared with data for actual vortex tubes found by experiment (Table I). The data given in this table refer to the largest tube No. 3 with a diameter $R=17.6$ mm. Data given in line 2 are taken from the curves in Fig. 4. The values for the quantity $[\mu(T_0-T)]_{\max}$ in line 5, which is proportional to the amount of cold Q, are taken from the curves in Fig. 3 and have been multiplied with a factor 1.1 (see Fig. 4), in order to obtain corresponding values for tube No. 3. It is apparent that the obtained temperature drop α reaches almost 50 percent of the theoretical value. The "cooling efficiency" B is defined as the ratio between the actually obtained efficiency and the theoretically calculated efficiency, it amounts to about 20 percent. The practical thermodynamic efficiency in line 8 is defined as B_{NW}; this definition would be valid if an ideal air compressor would be used.

It is probable that the real efficiencies are somewhat better than the data in Table I indicate. This is caused by, firstly the method of temperature measurement which gives unfavorable values, and secondly, to the presence of moisture in the compressed air, which appears in the cold stream as a fog of ice crystals.

6. APPLICATIONS OF THE VORTEX TUBE

Since results for the efficiency of a well-designed vortex tube operated with compressed air are now available, it may be in order to discuss possible applications.

There is little probability that vortex tubes will replace the customary refrigerating machines, since their efficiencies are much better in the region of small pressure ratios. There may, however, be special cases where a vortex tube would be more desirable because of its simple construction (e.g., air cooling in mine shafts).

The situation is different at low initial temperatures and high pressure ratios. It may be expected that the vortex tube will be superior to the throttle expansion (Joule-Thompson effect) for gas liquefaction. For these applications to lowest temperatures additional measurements are planned and will be reported later.