Building a fínite-difference mesh and selecting a turbulence model for numerical simulations of a vortex tube in OpenFOAM software

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Building a finite-difference mesh and selecting a turbulence model for numerical simulations of a vortex tube in OpenFOAM software

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Abstract. In the article, the usability of standard two-parameters models of turbulence for numerical simulation of a vortex tube is discussed. Both $k-\varepsilon$ and $k-\omega$ turbulence models were checked for subsonic air flows into the vortex tube channel. The main attention was paid to the development of the mesh, the simplicity of the implementation of simulation, and the physical concordance of the model results. Simulations performed by rhoPimpleFoam and sonicFoam solvers using k-ε and k-ω turbulence models gave similar results. The choice was made in favor of the $k-\varepsilon$ model.

1. Introduction

The vortex tube is a special device that separates an inlet stream of compressed gas (air) into two outlet streams so that one of it is colder and the other one is hotter. This device is notable simple, has no moving parts, and easy to manufacture. A vortex tube also known as Ranque-Hilsch tube is a well-known device used in a wide variety of technical fields. It should be noted that the described situation was not formed right away. Thus, the first article by Georges-Joseph Ranque [1], published in 1933, was negatively perceived in the scientific community that is explained by the seeming violation of energy conservation law.

Interest in the described device appeared only after the publication of Rudolf Hills [2], made in 1947. From this point, there has been an explosion of vortex tube researches and, accordingly, publications on this topic. So, by 1957, there were more than a hundred publications dealt with the study of vortex tubes [3]. Interest in this topic continues to this day, as evidenced by the continuous increase in the number of publications [4–6].

The main reason for a large number of vortex tube studies is the fact that despite the active use of the device in various fields [7–9], the physical mechanism of the phenomenon remains largely unexplained, which complicates the development of really effective vortex tubes [4–6].

There are two main ways to study a vortex tube. The first is an experimental study of some physical parameters that depend on the mechanical changes in the tube. This approach is limited both by the cost of the experimental device and the time it takes to obtain each experimental value. The second approach is based on theoretical models of gas-dynamic processes in a vortex tube channel. The most popular theoretical research direction is a numerical simulation. This way supposedly allows exploring quickly and cheaply many different physical configurations and inlet-outlet parameters.

In this paper, some difficulties in the numerical simulation are described. Certain techniques are shown to make a computer experiment really cheap and fast.

2. The model and methods

2.1. The simulated area

A vortex tube considered in the work is a cylindrical device that has four tangential inlet channels and two outlet diaphragms. The length of the simulated tube is 120 mm, and the diameter is equal to 16 mm. The cold diaphragm diameter is 5 mm. The hot diaphragm has circled shape and its width is 0.5 mm.

The initial conditions correspond to standard ones. Inlet parameters are constant pressure (600 kPa) and constant temperature (293 K). An inlet velocity is determined by the pressure gradient. Outlet parameters correspond to the normal pressure (100 kPa) and flow boundary conditions for velocity and temperature.

Boundary conditions for physical parameters on the wall are slip for the k-w turbulence model case and no-slip for the k-e model. Boundary conditions for turbulence parameters on the wall are described by corresponding wall functions.

The simulated area is shown in figure 1. It is one-quarter of the cylindrical device. This case makes two special boundaries with cyclic boundary conditions. Additional outlet zones that offset the outlet from the vortex tube diaphragms are shown.

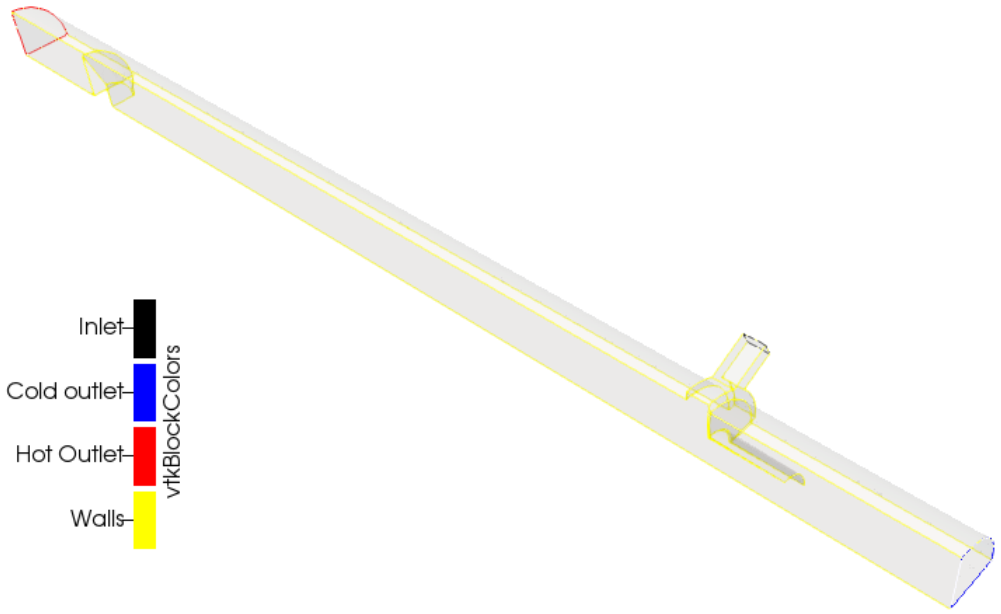


Figure 1. The simulated area (cyclic sides are not highlighted).

2.2. The mathematical model

The hydrodynamics processes in a vortex tube channel can be described by the standard system of continuous media equations:

- the continuity equation
  $\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0$;
- Navier–Stokes equation (the equation of impulses)
  $\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = -\frac{\partial \rho}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$;
- the energy conservation equation
  $\frac{\partial (\rho E)}{\partial t} + \frac{\partial (\rho u_j E)}{\partial x_j} = -\frac{\partial (p u_j)}{\partial x_j} + \frac{\partial (\tau_{ik} u_k)}{\partial x_j}$;
- a closure equation of state
  $e = \frac{p}{(\gamma - 1)\rho}$.

The ideal gas state equation is used in this work. The following notations were used: γ is the adiabatic exponent; u_i are velocity vector components; ρ is the gas density; $E = e + \frac{\sum u_i^2}{2}$ is the specific total energy of the gas, e is the specific internal energy; and p is the pressure. The viscous stress tensor τ_{ij} is calculated as:

$$\tau_{ij} = (\mu + \mu_t)\left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right]$$

Here μ is a dynamic viscosity; μ_t is a dynamic eddy viscosity calculated as $\mu_t = \nu_t \rho$, where ν_t is the kinematic eddy viscosity; δ_{ij} is the Kronecker symbol.

The goal of this work was to compare the applicability of two turbulence models.

The $k - \omega$ turbulence model is considered one of the best for the case where there is a solid wall in the calculation domain. The vortex tube is just this case. The model is expressed by the following conservation equations:

- the turbulence energy equation
  $\frac{\partial k}{\partial t} + \frac{\partial (u_j k)}{\partial x_j} = \tau_{ij}\frac{\partial u_i}{\partial x_j} - \beta^* \omega k + \frac{\partial}{\partial x_j}\left[(\nu + \sigma^* \nu_t)\frac{\partial k}{\partial x_j}\right]$;
- and the specific dissipation rate equation
  $\frac{\partial \omega}{\partial t} + \frac{\partial (u_j \omega)}{\partial x_j} = \frac{\alpha \omega}{k}\tau_{ij}\frac{\partial u_i}{\partial x_j} - \beta \omega^2 + \frac{\partial}{\partial x_j}\left[(\nu + \sigma \nu_t)\frac{\partial \omega}{\partial x_j}\right]$.

The kinematic eddy viscosity is calculated as

$$\nu_t = \frac{k}{\omega}.$$

Closure coefficients are empirical values

$$\alpha = \frac{13}{25},\ \beta = \frac{9}{125},\ \beta^* = \frac{9}{100},\ \sigma = \sigma^* = \frac{1}{2}.$$

However, the $k - \varepsilon$ turbulence model is more simple in its numerical implementation. The model equations are:

- the turbulence energy equation
  $\frac{\partial k}{\partial t} + \frac{\partial (u_j k)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\nu_t\right)\frac{\partial k}{\partial x_j}\right] - \frac{2}{3}k\frac{\partial u_j}{\partial x_j} - \rho\varepsilon$;
  The turbulence dissipation rate equation
  $\frac{\partial \varepsilon}{\partial t} + \frac{\partial (u_j \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] - \frac{2}{3}C_1 \varepsilon \frac{\partial u_j}{\partial x_j} - C_2 \frac{\varepsilon^2}{k}$.

The kinematic eddy viscosity in the model is calculated as $\nu_t = C_\mu \frac{k^2}{\varepsilon}$.

The model empirical closure coefficients are:

$$C_1 = 1.44,\ C_2 = 1.92,\ \sigma_k = 1,\ \sigma_\varepsilon = 1.3.$$

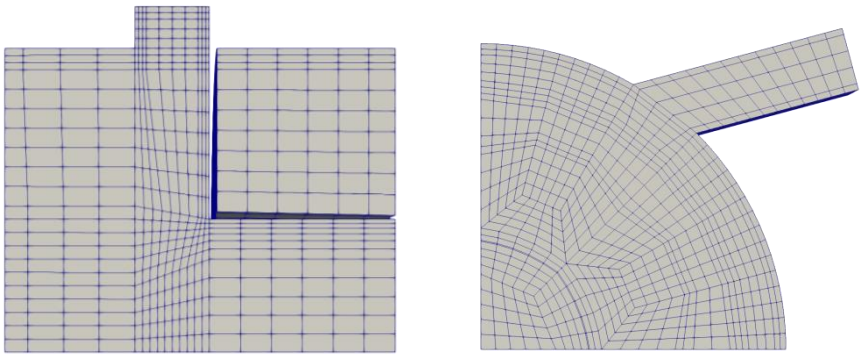
3. Results and discussion

Numerical simulations were conducted using OpenFOAM software. When preparing the simulation, considerable attention was paid to the construction of a finite-volume mesh [10]. Several actions were taken to preserve the relative uniformity of the mesh and the orthogonality of the boundaries of the finite volumes.

It should be noted that the mesh for the $k-\omega$ model is significantly different from that for the $k-\varepsilon$ model. This is because the $k-\omega$ model requires consideration of a thin near-wall layer. At the same time, several cells should be located perpendicular to the wall in the near-wall region.

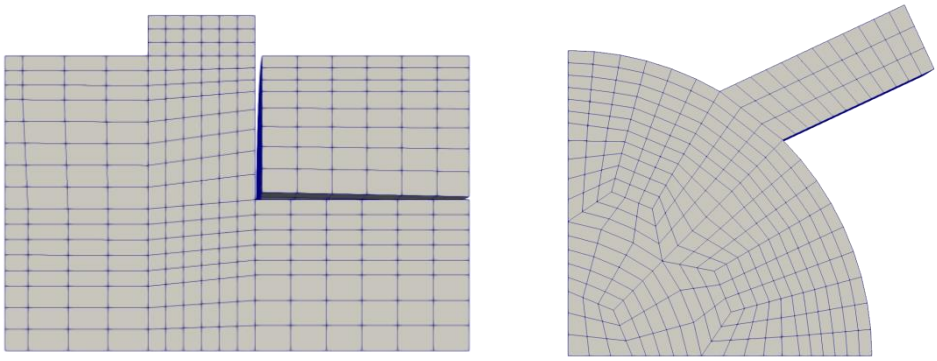
This fact leads to a special type of mesh as shown in figure 2.

A mesh for the case of $k-\varepsilon$ turbulence model is shown in figure 3. It is clear that this mesh is more simple and consist of less finite volumes.



a **b**

Figure 2. Mesh for calculations using $k-\omega$ turbulence model. The area near the inlet is shown along the channel (a) and in the radial section (b).



a **b**

Figure 3. Mesh for calculations using $k-\varepsilon$ turbulence model. The area near the inlet is shown along the channel (a) and in the radial section (b).

This approach to mesh building leads to an essentially different quantity of finite volumes in $k-\omega$ vs. $k-\varepsilon$ meshes. The following are listings of the checkMesh command output showing the results of the mesh analysis for the $k-\omega$ model (on the right) and that for $k-\varepsilon$ (on the left).

```
Mesh stats                          Mesh stats
    points:           89723             points:           36771
    faces:            251992            faces:            100338
    internal faces:   235628            internal faces:   91038
    cells:            81270             cells:            31896
```

It should be noted what the number of faces differs approximately 2.5 times. It leads to enlarging calculation time. Also, the simulation time increases, for the reason that the time step for the $k-\omega$ model is less. As a result, the full time of numerical simulation by the $k-\omega$ turbulence model is ten times longer then simulation time necessary for the $k-\varepsilon$ model.

Described meshes were used for numerical simulations of the vortex tube using rhoPimpleFoam and sonicFoam solvers from OpenFOAM software [11,12]. Both solvers were tested using $k-\omega$ and $k-\varepsilon$ turbulence models. As it was already noted, simulation time for $k-\varepsilon$ model is less.

Physical parameters calculated by each of the two models are slightly different as shown in figure 4. The figure shows temperature values near the cold diaphragm, averaged by section. Two differences in the data follow from the figure. First, the second model significantly smooths out the observed turbulent or numerical pulsations. The same smoothing effect is observed for other physical parameters. Secondly, as can be clearly seen, the obtained values do not coincide. Such differences exist for all considered physical quantities. However, the dependence of physical values on the model geometry is the same for each of the turbulence models.

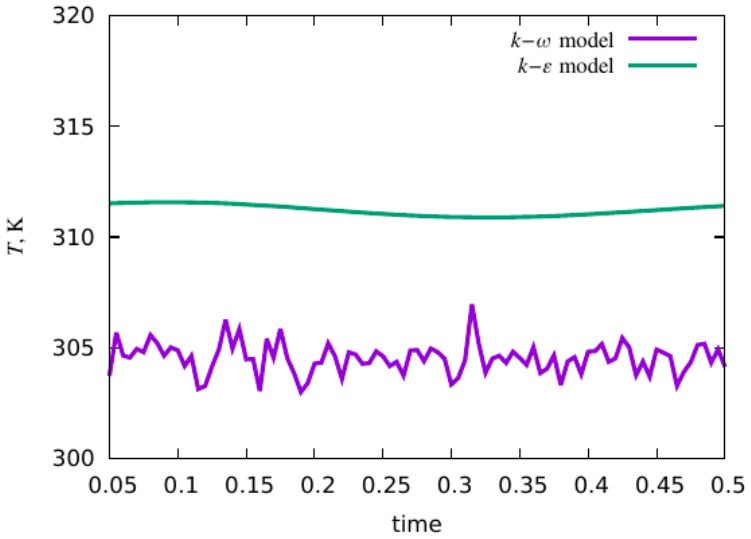


Figure 4. Comparison of the temperature behavior for both turbulence models.

Thus, for those problems for which a turbulent model was used, the most significant difference was revealed with regard to the simulation time. Proceeding from this, the $k-\varepsilon$ model of turbulence looks more preferable and it is this model that is used in further studies.

Conclusions

The paper compares standard two-parameter turbulence models. It is shown that for some tasks, the most significant difference between these models is the number of mesh nodes and, accordingly, the simulation time. From the above, a conclusion is drawn about the preference of the $k-\varepsilon$ turbulence model. However, for the final conclusion, it is necessary to compare the numerical results with the corresponding experimental data. To date, no such comparison has been made.

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