MASTER-AI

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Learn the AI/ML Project on:

Auto-Mpg Estimation

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★ AI / ML Project - Auto MPG Prediction ★

Description:

The data is technical spec of cars. The dataset is downloaded from UCI Machine Learning Repository

"The data concerns city-cycle fuel consumption in miles per gallon, to be predicted in terms of 3 multivalued discrete and 5 continuous attributes." (Quinlan, 1993) Number of Instances: 398 Number of Attributes: 9 including the class attribute

Acknowledgements

Dataset: UCI Machine Learning Repository
Data link : [https://archive.ics.uci.edu/ml/datasets/auto+mpg (https://archive.ics.uci.edu/ml/datasets/auto+mpg)](https://archive.ics.uci.edu/ml/datasets/auto+mpg)

Objective:

- Understand the Dataset & cleanup (if required).
- Build Regression models to predict the sales w.r.t a single & multiple feature.
- Also evaluate the models & compare thier respective scores like R2, RMSE, etc.

1. Data Exploration

In [2]:

```
#Importing the basic librarires

import numpy as np
import pandas as pd
import seaborn as sns
from IPython.display import display

import matplotlib.pyplot as plt
plt.rcParams['figure.figsize'] = [10,6]

import warnings
warnings.filterwarnings('ignore')
```

In [3]:

```
#Importing the dataset

df = pd.read_csv('auto-mpg.csv', names=['mpg', 'cylinders', 'displacement', 'horsepower', '
       'acceleration', 'model_year', 'origin', 'car_name'], skiprows=1)
df.reset_index(drop=True, inplace=True)
original_df = df.copy(deep=True)
display(df.head())

print('\n\033[1mInference:\033[0m The Datset consists of {} features & {} samples.'.format(
```

	mpg	cylinders	displacement	horsepower	weight	acceleration	model_year	origin	car_nai
0	18.0	8	307.0	130	3504	12.0	70	1	chevrc cheve mal
1	15.0	8	350.0	165	3693	11.5	70	1	bu skyl: 3
2	18.0	8	318.0	150	3436	11.0	70	1	plymo satel
3	16.0	8	304.0	150	3433	12.0	70	1	amc re
4	17.0	8	302.0	140	3449	10.5	70	1	ford tor

Inference: The Datset consists of 9 features & 398 samples.

In [4]:

```
#Checking the dtypes of all the columns

df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 398 entries, 0 to 397
Data columns (total 9 columns):
 #   Column        Non-Null Count  Dtype  
---  ------        --------------  -----  
 0   mpg           398 non-null    float64
 1   cylinders     398 non-null    int64  
 2   displacement  398 non-null    float64
 3   horsepower    398 non-null    object 
 4   weight        398 non-null    int64  
 5   acceleration  398 non-null    float64
 6   model_year    398 non-null    int64  
 7   origin        398 non-null    int64  
 8   car_name      398 non-null    object 
dtypes: float64(3), int64(4), object(2)
memory usage: 28.1+ KB
```

In [5]:

```
#Checking number of unique rows in each feature

df.nunique()
```

Out[5]:

```
mpg             129
cylinders         5
displacement     82
horsepower       94
weight          351
acceleration     95
model_year       13
origin            3
car_name        305
dtype: int64
```

In [6]:

```
#Checking the stats of all the columns

display(df.describe())
```

	mpg	cylinders	displacement	weight	acceleration	model_year	origi
count	398.000000	398.000000	398.000000	398.000000	398.000000	398.000000	398.00000
mean	23.514573	5.454774	193.425879	2970.424623	15.568090	76.010050	1.57286
std	7.815984	1.701004	104.269838	846.841774	2.757689	3.697627	0.80205
min	9.000000	3.000000	68.000000	1613.000000	8.000000	70.000000	1.00000
25%	17.500000	4.000000	104.250000	2223.750000	13.825000	73.000000	1.00000
50%	23.000000	4.000000	148.500000	2803.500000	15.500000	76.000000	1.00000
75%	29.000000	8.000000	262.000000	3608.000000	17.175000	79.000000	2.00000
max	46.600000	8.000000	455.000000	5140.000000	24.800000	82.000000	3.00000

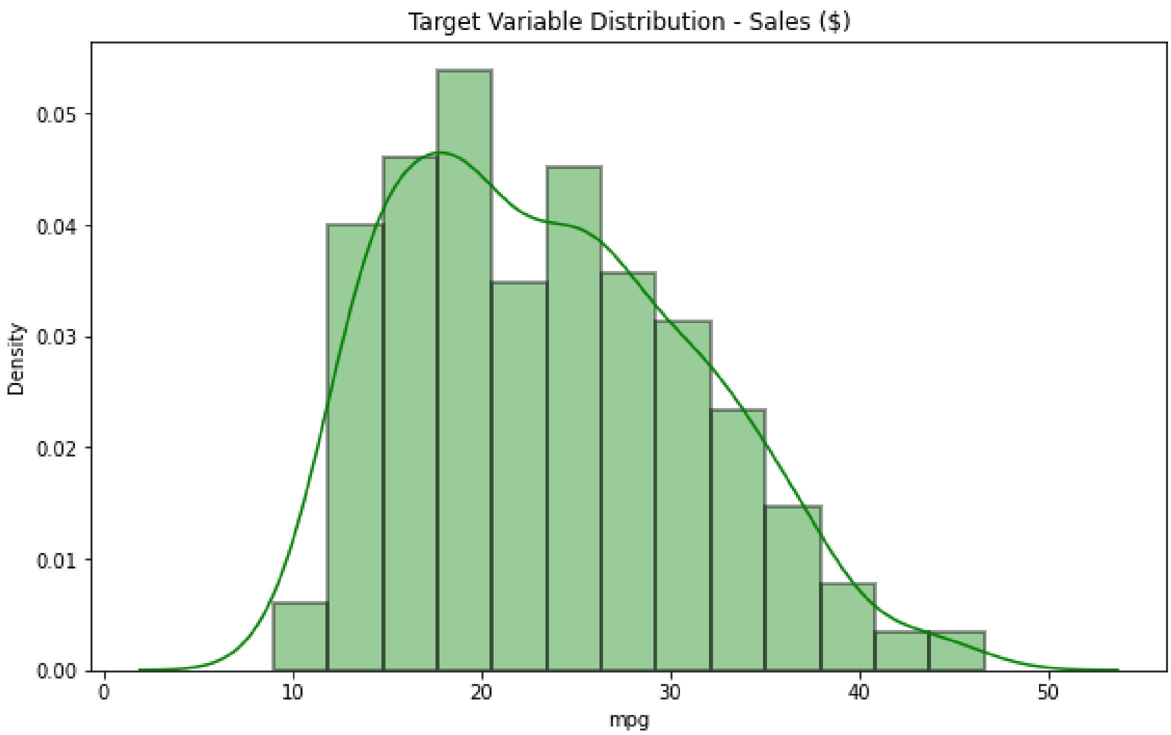
Inference: The stats seem to be fine, let us do further analysis on the Dataset

---

2. Exploratory Data Analysis (EDA)

In [7]:

```
#Let us first analyze the distribution of the target variable

c = df.columns
sns.distplot(df[c[0]], color='g',hist_kws=dict(edgecolor="black", linewidth=2))
plt.title('Target Variable Distribution - Sales ($)')
plt.show()
```

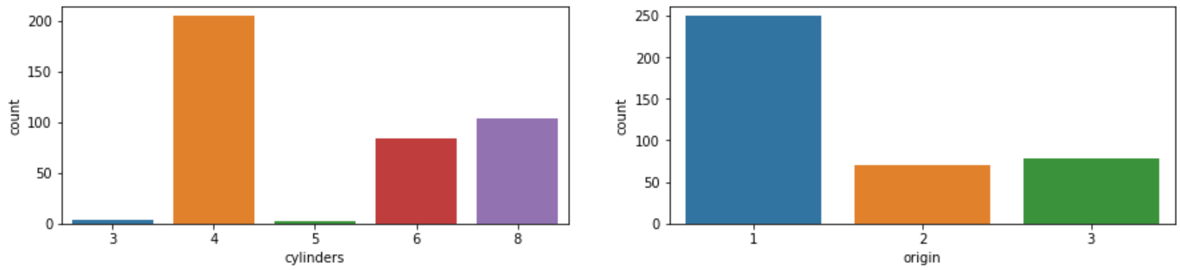


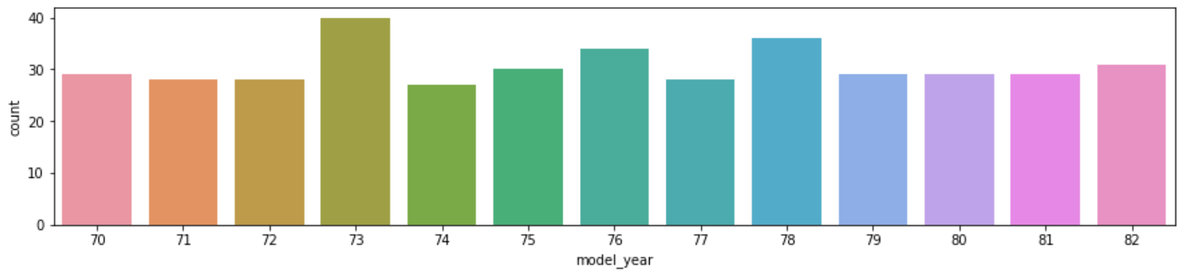
Inference: The Target Variable seems to be be normally distributed, averaging around 12$(units)

In [8]:

```
#Visualising the categorical features

cf = ['cylinders','origin','model_year']

print('\033[1mVisualising Categorical Features:'.center(120))

plt.figure(figsize=[15,3])
plt.subplot(1,2,1)
sns.countplot(df[cf[0]])
plt.subplot(1,2,2)
sns.countplot(df[cf[1]])
plt.show()

plt.figure(figsize=[15,3])
plt.subplot(1,1,1)
sns.countplot(df[cf[2]])
plt.show()
```

Visualising Categorical Features:



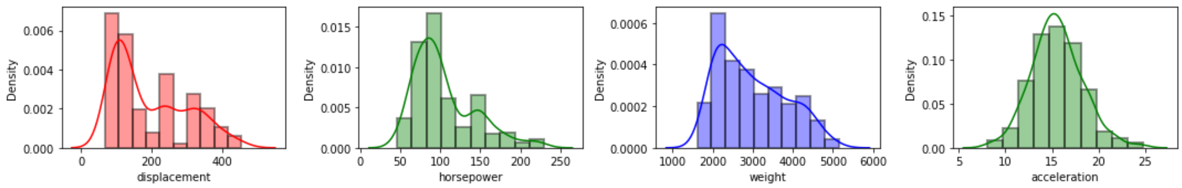


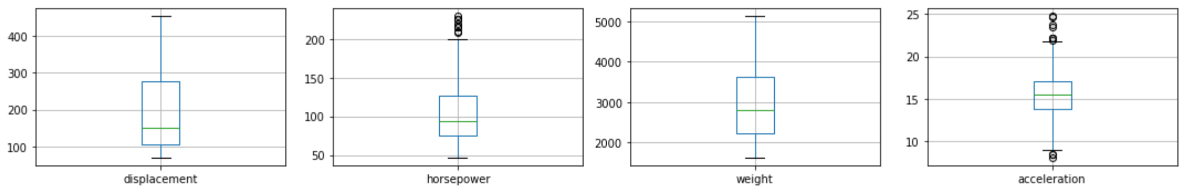
Inference: 4 cylinders & 1 origin seem to be domiant classes, while the frequency of classes in model_year are relatively consistent

In [9]:

```
#Visualising the numeric features

print('\033[1mNumeric Features Distribution'.center(130))

nf = ['displacement','horsepower','weight','acceleration']

df = df[~df['horsepower'].isin(['?'])]
df.reset_index(drop=True, inplace=True)
df['horsepower']= df['horsepower'].astype(int)

clr=['r','g','b','g','b','r']

plt.figure(figsize=[15,2.5])
for i in range(4):
    plt.subplot(1,4,i+1)
    sns.distplot(df[nf[i]],hist_kws=dict(edgecolor="black", linewidth=2), bins=10, color=cl
plt.tight_layout()
plt.show()

plt.figure(figsize=[15,2.5])
for i in range(4):
    plt.subplot(1,4,i+1)
    df.boxplot(nf[i])
plt.tight_layout()
plt.show()
```

Numeric Features Distributio
n

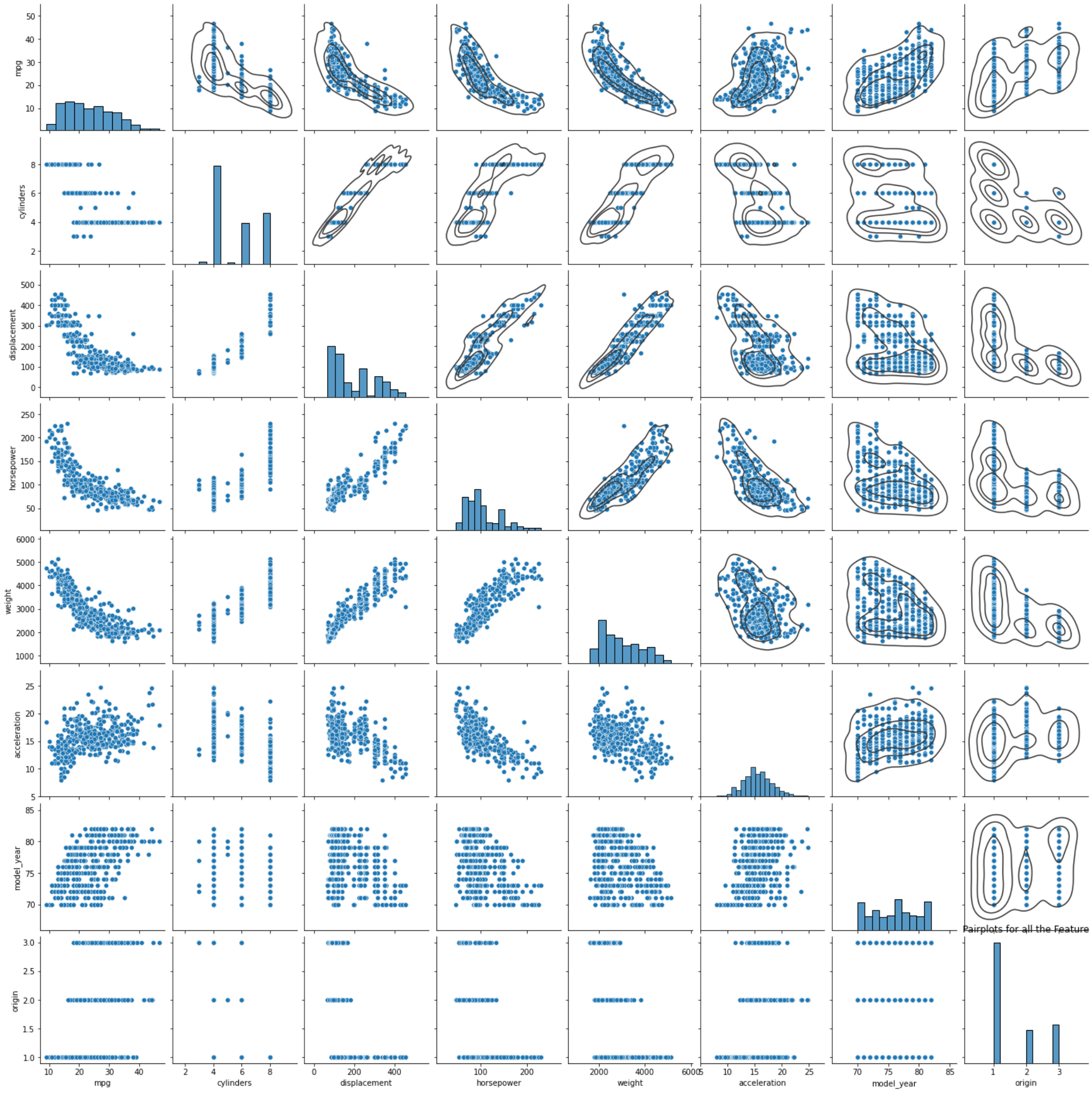




Inference: There seem to be some outliers in the horsepower & acceleration features

In [11]:

```
#Understanding the relationship between all the features

g = sns.pairplot(df)
plt.title('Pairplots for all the Feature')
g.map_upper(sns.kdeplot, levels=4, color=".2")
plt.show()
```



Inference: We can notice that few have linear relationship, like that of displacement & horspower, while others features are not more randomly distributed

3. Data Preprocessing

In [12]:

```
#Check for empty elements

print(df.isnull().sum())
print('\n\033[1mInference:\033[0m The dataset had {} inconsistant/null elements which were
```

```
mpg             0
cylinders       0
displacement    0
horsepower      0
weight          0
acceleration    0
model_year      0
origin          0
car_name        0
dtype: int64
```

Inference: The dataset had 6 inconsistant/null elements which were dropped.

In [13]:

```
#Removal of any Duplicate rows (if any)

counter = 0
rs,cs = df.shape

df.drop_duplicates(inplace=True)

if df.shape==(rs,cs):
    print('\n\033[1mInference:\033[0m The dataset doesn\'t have any duplicates')
else:
    print(f'\n\033[1mInference:\033[0m Number of duplicates dropped/fixed ---> {r-df.shape[
```

Inference: The dataset doesn't have any duplicates

In [13]:

```
#Removal of outlier:

df = df.drop(['car_name'], axis=1)

for i in df.columns:
    Q1 = df[i].quantile(0.25)
    Q3 = df[i].quantile(0.75)
    IQR = Q3 - Q1
    df = df[df[i] <= (Q3+(1.5*IQR))]
    df = df[df[i] >= (Q1-(1.5*IQR))]
    df = df.reset_index(drop=True)
display(df)
print('\n\033[1mInference:\033[0m After removal of outliers, The dataset now has {} feature
```

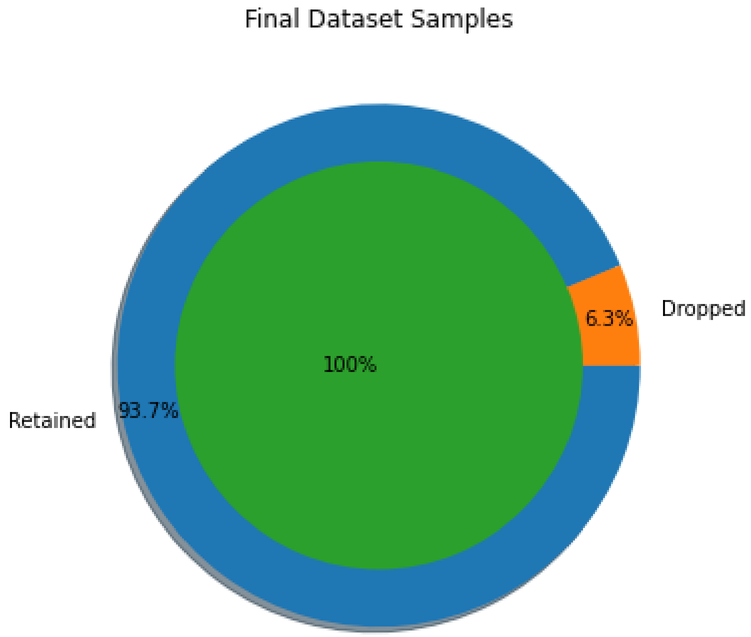
	mpg	cylinders	displacement	horsepower	weight	acceleration	model_year	origin
0	18.0	8	307.0	130	3504	12.0	70	1
1	15.0	8	350.0	165	3693	11.5	70	1
2	18.0	8	318.0	150	3436	11.0	70	1
3	16.0	8	304.0	150	3433	12.0	70	1
4	17.0	8	302.0	140	3449	10.5	70	1
...	...	...	...	...	...	...	...	...
368	27.0	4	151.0	90	2950	17.3	82	1
369	27.0	4	140.0	86	2790	15.6	82	1
370	32.0	4	135.0	84	2295	11.6	82	1
371	28.0	4	120.0	79	2625	18.6	82	1
372	31.0	4	119.0	82	2720	19.4	82	1

373 rows × 8 columns

Inference: After removal of outliers, The dataset now has 8 features & 373 s
amples.

In [17]:

```
#Final Dataset size after performing Preprocessing

plt.title('Final Dataset Samples')
plt.pie([df.shape[0], original_df.shape[0]-df.shape[0]], radius = 1, labels=['Retained','Dr
        autopct='%1.1f%%', pctdistance=0.9, explode=[0,0], shadow=True)
plt.pie([df.shape[0]], labels=['100%'], labeldistance=-0, radius=0.78)
plt.show()

print(f'\n\033[1mInference:\033[0m After the cleanup process, {original_df.shape[0]-df.shap
while retaining {df.shape[0]/(original_df.shape[0]-df.shape[0])}% of the data.')
```



Inference: After the cleanup process, 25 samples were dropped, while retaini
ng 14.92% of the data.

---

4. Data Manipulation

In [70]:

```
#Splitting the data intro training & testing sets

from sklearn.model_selection import train_test_split

X = df.drop(['mpg'],axis=1)
Y = df.mpg
Train_X, Test_X, Train_Y, Test_Y = train_test_split(X, Y, train_size=0.8, test_size=0.2, ra
Train_X.reset_index(drop=True,inplace=True)
Train_X.reset_index(drop=True,inplace=True)
Train_X.reset_index(drop=True,inplace=True)
Train_X.reset_index(drop=True,inplace=True)

print('Original set  ---> ',X.shape,Y.shape,'\nTraining set  ---> ',Train_X.shape,Train_Y.s
```

```
Original set  --->  (373, 7) (373,)
Training set  --->  (298, 7) (298,)
Testing set   --->  (75, 7)  (75,)
```

In [71]:

```
#Feature Scaling (Standardization)

from sklearn.preprocessing import StandardScaler

std = StandardScaler()

print('\033[1mStandardardization on Training set'.center(80))
Train_X_std = std.fit_transform(Train_X)
Train_X_std = pd.DataFrame(Train_X_std, columns=X.columns)
display(Train_X_std.describe())

print('\n','\033[1mStandardardization on Testing set'.center(80))
Test_X_std = std.transform(Test_X)
Test_X_std = pd.DataFrame(Test_X_std, columns=X.columns)
display(Test_X_std.describe())
```

Standardardization on Training set

	cylinders	displacement	horsepower	weight	acceleration	model_yea
count	2.980000e+02	2.980000e+02	2.980000e+02	2.980000e+02	2.980000e+02	2.980000e+0:
mean	-1.132577e-16	-6.258975e-17	1.639255e-16	2.354567e-16	6.139757e-16	-7.600184e-1(
std	1.001682e+00	1.001682e+00	1.001682e+00	1.001682e+00	1.001682e+00	1.001682e+0(
min	-1.434665e+00	-1.193437e+00	-1.661251e+00	-1.579042e+00	-2.122077e+00	-1.666395e+0(
25%	-8.367204e-01	-8.415279e-01	-7.759633e-01	-8.610395e-01	-6.720466e-01	-8.341285e-0
50%	-8.367204e-01	-4.845917e-01	-2.152813e-01	-2.629963e-01	-5.060508e-02	-1.861894e-0:
75%	3.591678e-01	6.968168e-01	5.224582e-01	7.010703e-01	6.019085e-01	8.304047e-0
max	1.555056e+00	2.416143e+00	2.883225e+00	2.548743e+00	2.600879e+00	1.662671e+0(

Standardardization on Testing set

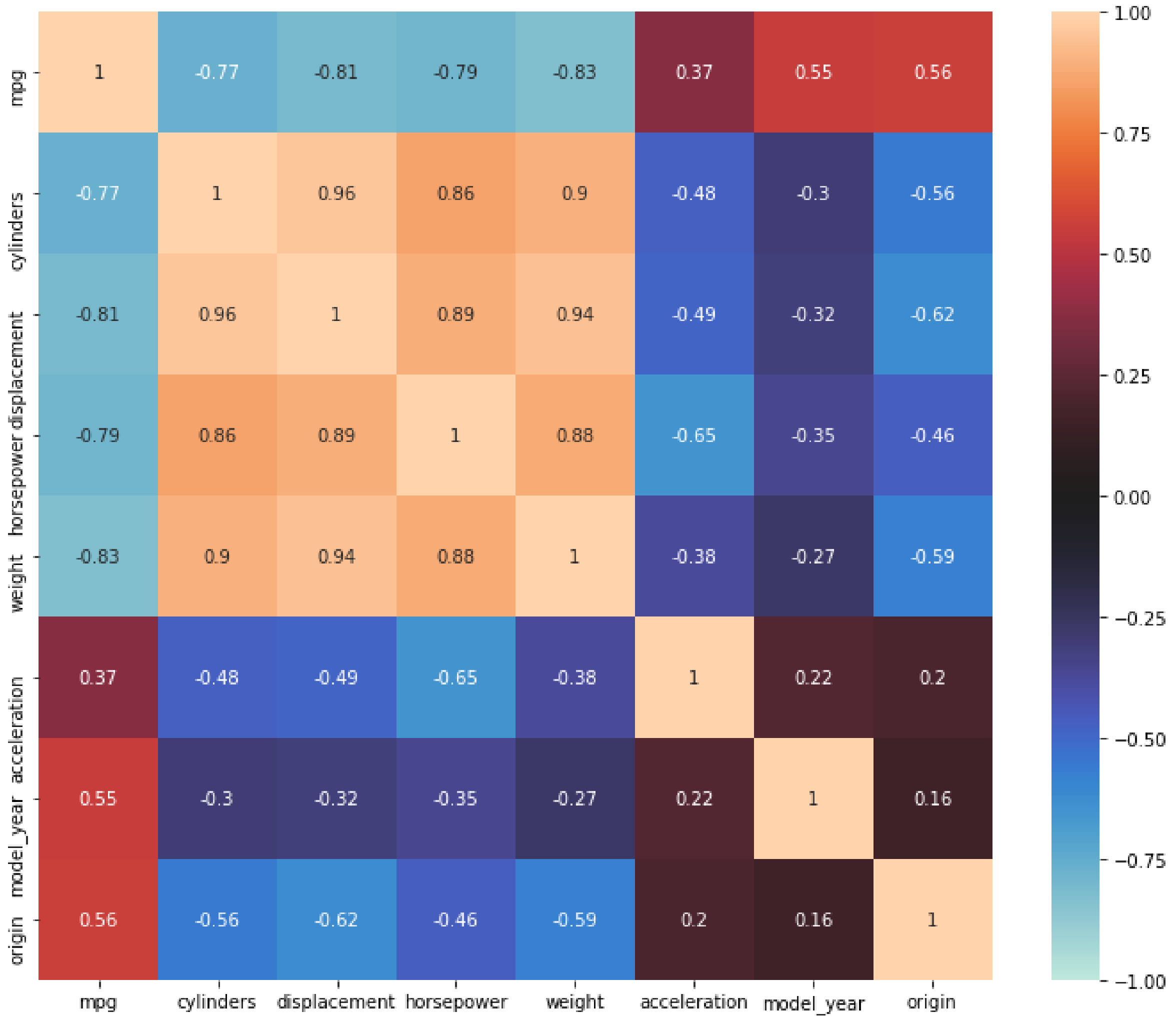
	cylinders	displacement	horsepower	weight	acceleration	model_year	origin
count	75.000000	75.000000	75.000000	75.000000	75.000000	75.000000	75.000000
mean	0.024319	0.002919	-0.066553	-0.100686	-0.121311	0.142398	-0.115254
std	0.998812	0.958443	0.958450	0.830193	1.010223	1.018150	0.993879
min	-1.434665	-1.213546	-1.572722	-1.366041	-2.536371	-1.666395	-0.752195
25%	-0.836720	-0.841528	-0.790718	-0.909023	-0.713476	-0.695417	-0.752195
50%	-0.836720	-0.379019	-0.362829	-0.148888	-0.050605	0.275560	-0.752195
75%	0.359168	0.656599	0.345401	0.597203	0.529407	0.830405	0.472691
max	1.555056	2.416143	2.824206	1.613642	1.855149	1.662671	1.697577

5. Feature Selection/Extraction

In [72]:

```
#Checking the correlation

print('\033[1mFinal Dataset Samples'.center(80))
plt.figure(figsize=[12,10])
sns.heatmap(df.corr(), annot=True, center=0, vmin=-1, vmax=1)
plt.show()
```

Final Dataset Samples



Inference: There seems to be strong multi-correlation between the features. Let us try to fix those in the Modelling section

In [73]:

```python
#Testing a Linear Regression model with statsmodels

from statsmodels.formula import api

Train_xy = pd.concat([Train_X_std,Train_Y.reset_index(drop=True)],axis=1)
a = Train_xy.columns.values

API = api.ols(formula=f'{a[-1]} ~ {a[0]} + {a[1]} + {a[2]} + {a[3]} + {a[4]} + {a[5]} + {a[
#print(API.conf_int())
#print(API.pvalues)
API.summary()
```

Out[73]:

OLS Regression Results

Dep. Variable:	mpg	**R-squared:**	0.818
Model:	OLS	**Adj. R-squared:**	0.814
Method:	Least Squares	**F-statistic:**	186.1
Date:	Fri, 12 Nov 2021	**Prob (F-statistic):**	2.55e-103
Time:	14:40:45	**Log-Likelihood:**	-777.87
No. Observations:	298	**AIC:**	1572.
Df Residuals:	290	**BIC:**	1601.
Df Model:	7		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	23.5379	0.193	121.775	0.000	23.157	23.918
cylinders	-0.3307	0.679	-0.487	0.627	-1.667	1.006
displacement	1.1648	0.957	1.218	0.224	-0.718	3.048
horsepower	-1.1618	0.630	-1.844	0.066	-2.402	0.078
weight	-4.9058	0.752	-6.522	0.000	-6.386	-3.425
acceleration	-0.1691	0.315	-0.538	0.591	-0.788	0.450
model_year	2.6255	0.209	12.546	0.000	2.214	3.037
origin	1.1068	0.259	4.268	0.000	0.596	1.617

Omnibus:	28.397	**Durbin-Watson:**	2.072
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	46.017
Skew:	0.590	**Prob(JB):**	1.02e-10
Kurtosis:	4.521	**Cond. No.**	13.0

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Inference: We can fix these multicollinearity with two techniques:

1. Manual Method - Variance Inflation Factor (VIF)
2. Automatic Method - Recursive Feature Elimination (RFE)

5a. Manual Method - VIF

In [74]:

```
# Calculate the VIFs to remove multicollinearity
from statsmodels.stats.outliers_influence import variance_inflation_factor

vif = pd.DataFrame()
X = Train_xy.drop(['mpg'],axis=1)
vif['Features'] = X.columns
vif['VIF'] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

Out[74]:

	Features	VIF
1	displacement	24.50
3	weight	15.14
0	cylinders	12.34
2	horsepower	10.62
4	acceleration	2.65
6	origin	1.80
5	model_year	1.17

In [75]:

```
#Iter 1

Train_xy_1 = Train_xy.drop(['displacement'], axis=1)
a = Train_xy_1.columns.values

API = api.ols(formula=f'{a[-1]} ~ {a[0]} + {a[1]} + {a[2]} + {a[3]} + {a[4]} + {a[5]}', dat
#print(API.conf_int())
#print(API.pvalues)
API.summary()
```

Out[75]:

OLS Regression Results

Dep. Variable:	mpg	**R-squared:**	0.817
Model:	OLS	**Adj. R-squared:**	0.813
Method:	Least Squares	**F-statistic:**	216.6
Date:	Fri, 12 Nov 2021	**Prob (F-statistic):**	3.47e-104
Time:	14:41:15	**Log-Likelihood:**	-778.63
No. Observations:	298	**AIC:**	1571.
Df Residuals:	291	**BIC:**	1597.
Df Model:	6		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	23.5379	0.193	121.675	0.000	23.157	23.919
cylinders	0.2535	0.481	0.527	0.599	-0.693	1.200
horsepower	-1.1748	0.630	-1.864	0.063	-2.416	0.066
weight	-4.4173	0.637	-6.937	0.000	-5.670	-3.164
acceleration	-0.2545	0.307	-0.829	0.408	-0.858	0.349
model_year	2.6029	0.209	12.476	0.000	2.192	3.013
origin	1.0086	0.247	4.089	0.000	0.523	1.494

Omnibus:	29.814	**Durbin-Watson:**	2.074
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	49.873
Skew:	0.603	**Prob(JB):**	1.48e-11
Kurtosis:	4.600	**Cond. No.**	8.47

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [76]:

```
# Calculate the VIFs to remove multicollinearity
from statsmodels.stats.outliers_influence import variance_inflation_factor

vif = pd.DataFrame()
X = Train_xy_1.drop(['mpg'],axis=1)
vif['Features'] = X.columns
vif['VIF'] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

Out[76]:

	Features	VIF
2	weight	10.83
1	horsepower	10.62
0	cylinders	6.18
3	acceleration	2.52
5	origin	1.63
4	model_year	1.16

In [77]:

```
#Iter 2

Train_xy_2 = Train_xy.drop(['displacement','weight'],axis=1)
a = Train_xy_2.columns.values

API = api.ols(formula=f'{a[-1]} ~ {a[0]} + {a[1]} + {a[2]} + {a[3]} + {a[4]}', data=Train_x
#print(API.conf_int())
#print(API.pvalues)
API.summary()
```

Out[77]:

OLS Regression Results

Dep. Variable:	mpg	**R-squared:**	0.787
Model:	OLS	**Adj. R-squared:**	0.783
Method:	Least Squares	**F-statistic:**	215.5
Date:	Fri, 12 Nov 2021	**Prob (F-statistic):**	9.77e-96
Time:	14:41:16	**Log-Likelihood:**	-801.44
No. Observations:	298	**AIC:**	1615.
Df Residuals:	292	**BIC:**	1637.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	23.5379	0.208	112.905	0.000	23.128	23.948
cylinders	-1.4507	0.446	-3.256	0.001	-2.328	-0.574
horsepower	-4.2234	0.487	-8.671	0.000	-5.182	-3.265
acceleration	-1.3975	0.279	-5.009	0.000	-1.947	-0.848
model_year	2.3736	0.222	10.692	0.000	1.937	2.811
origin	1.4420	0.257	5.607	0.000	0.936	1.948

Omnibus:	27.561	**Durbin-Watson:**	2.116
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	42.713
Skew:	0.592	**Prob(JB):**	5.31e-10
Kurtosis:	4.428	**Cond. No.**	5.09

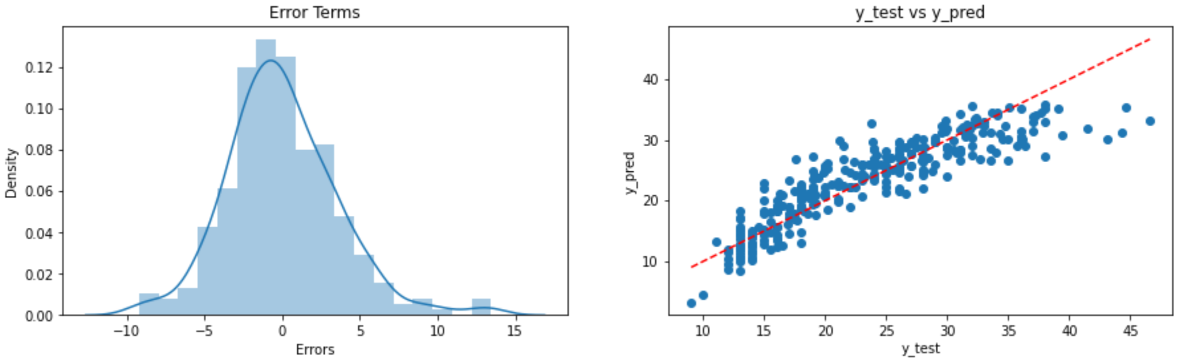
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [78]:

```
#Evaluating the Multiple Linear Regression Model

from sklearn.linear_model import LinearRegression
from sklearn.metrics import r2_score, mean_absolute_error, mean_squared_error

print('{}{}\033[1mEvaluating Simple Linear Regression Model\033[0m{}{}\n'.format('<'*3,'-'*

MLR = LinearRegression().fit(Train_X_std.drop(['displacement','weight'],axis=1),Train_Y)

print('The Coeffecient of the Linear Regresion Model was found to be ',MLR.coef_)
print('The Intercept of the Linear Regresion Model was found to be ',MLR.intercept_)

#Plotting predicted predicteds alongside the actual datapoints

pred = MLR.predict(Train_X_std.drop(['displacement','weight'],axis=1))

print('\n\n{}Training Set Metrics{}'.format('-'*20, '-'*20))
pred1 = MLR.predict(Train_X_std.drop(['displacement','weight'],axis=1))#Test_X_sm)
print('\nR2-Score on Training set --->',round(r2_score(Train_Y, pred1),2))
print('Residual Sum of Squares (RSS) on Training set  --->',round(np.sum(np.square(Train_Y-
print('Mean Squared Error (MSE) on Training set       --->',round(mean_squared_error(Train_
print('Root Mean Squared Error (RMSE) on Training set --->',round(np.sqrt(mean_squared_erro

print('\n{}Test Set Metrics{}'.format('-'*20, '-'*20))
pred2 = MLR.predict(Test_X_std.drop(['displacement','weight'],axis=1))#Test_X_sm)
print('\nR2-Score on Testing set --->',round(r2_score(Test_Y, pred2),2))
print('Residual Sum of Squares (RSS) on Training set  --->',round(np.sum(np.square(Test_Y-p
print('Mean Squared Error (MSE) on Training set       --->',round(mean_squared_error(Test_Y
print('Root Mean Squared Error (RMSE) on Training set --->',round(np.sqrt(mean_squared_erro
print('\n{}Residual Plots{}'.format('-'*20, '-'*20))

# Plotting y_test and y_pred to understand the spread.

plt.figure(figsize=[15,4])

plt.subplot(1,2,1)
sns.distplot((Train_Y - pred))
plt.title('Error Terms')                    # Plot heading
plt.xlabel('Errors')

plt.subplot(1,2,2)
plt.scatter(Train_Y,pred)
plt.plot([Train_Y.min(),Train_Y.max()],[Train_Y.min(),Train_Y.max()], 'r--')
plt.title('y_test vs y_pred')                # Plot heading
plt.xlabel('y_test')                           # X-label
plt.ylabel('y_pred')                           # Y-label
plt.show()
```

```
<<<-----------------------------------Evaluating Simple Linear Regression
 Model----------------------------------->>>

The Coeffecient of the Linear Regresion Model was found to be  [-1.4507119
7 -4.2233568  -1.39745292  2.37359801  1.4419944 ]
The Intercept of the Linear Regresion Model was found to be  23.5379194630
87256


--------------------Training Set Metrics--------------------
```

```
R2-Score on Training set ---> 0.79
Residual Sum of Squares (RSS) on Training set  ---> 3781.91
Mean Squared Error (MSE) on Training set       ---> 12.69
Root Mean Squared Error (RMSE) on Training set ---> 3.56

--------------------Test Set Metrics--------------------

R2-Score on Testing set ---> 0.8
Residual Sum of Squares (RSS) on Training set  ---> 758.53
Mean Squared Error (MSE) on Training set       ---> 10.11
Root Mean Squared Error (RMSE) on Training set ---> 3.18

--------------------Residual Plots--------------------
```



5b. Automatic Method - RFE

In [79]:

```python
# Applyin
from sklearn.feature_selection import RFE
from sklearn.linear_model import LinearRegression

# Running RFE with the output number of the variable equal to 10
lm = LinearRegression()
lm.fit(Train_X_std, Train_Y)

rfe = RFE(lm,n_features_to_select=5)             # running RFE
rfe = rfe.fit(Train_X, Train_Y)

list(zip(Train_X.columns,rfe.support_,rfe.ranking_))
```

Out[79]:

```
[('cylinders', False, 2),
 ('displacement', True, 1),
 ('horsepower', True, 1),
 ('weight', False, 3),
 ('acceleration', True, 1),
 ('model_year', True, 1),
 ('origin', True, 1)]
```

In [80]:

```
#Testing a Linear Regression model with statsmodels

from statsmodels.formula import api

Train_xy = pd.concat([Train_X_std[Train_X.columns[rfe.support_]],Train_Y.reset_index(drop=T
a = Train_xy.columns.values

API = api.ols(formula=f'{a[-1]} ~ {a[0]} + {a[1]} + {a[2]} + {a[3]} + {a[4]}', data=Train_x
#print(API.conf_int())
#print(API.pvalues)
API.summary()
```

Out[80]:

OLS Regression Results

Dep. Variable:	mpg	**R-squared:**	0.791
Model:	OLS	**Adj. R-squared:**	0.788
Method:	Least Squares	**F-statistic:**	221.4
Date:	Fri, 12 Nov 2021	**Prob (F-statistic):**	4.41e-97
Time:	15:29:41	**Log-Likelihood:**	-798.27
No. Observations:	298	**AIC:**	1609.
Df Residuals:	292	**BIC:**	1631.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	23.5379	0.206	114.112	0.000	23.132	23.944
displacement	-2.1784	0.527	-4.136	0.000	-3.215	-1.142
horsepower	-3.6221	0.537	-6.739	0.000	-4.680	-2.564
acceleration	-1.3214	0.278	-4.760	0.000	-1.868	-0.775
model_year	2.3786	0.220	10.829	0.000	1.946	2.811
origin	1.1700	0.275	4.254	0.000	0.629	1.711

Omnibus:	29.947	**Durbin-Watson:**	2.104
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	50.605
Skew:	0.602	**Prob(JB):**	1.03e-11
Kurtosis:	4.621	**Cond. No.**	6.00

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [81]:

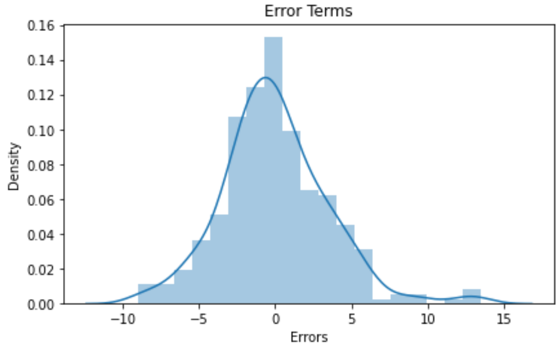
```
from sklearn.linear_model import LinearRegression
from sklearn.metrics import r2_score, mean_absolute_error, mean_squared_error

MLR = LinearRegression().fit(Train_X_std[Train_X.columns[rfe.support_]],Train_Y)

print('The Coeffecient of the Linear Regresion Model was found to be ',MLR.coef_)
print('The Intercept of the Linear Regresion Model was found to be ',MLR.intercept_)

#Plotting predicted predicteds alongside the actual datapoints

pred = MLR.predict(Train_X_std[Train_X.columns[rfe.support_]])
```

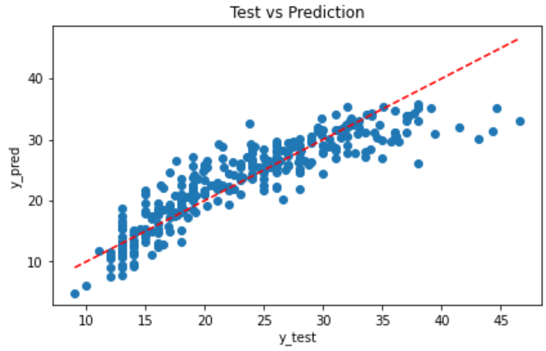
```
The Coeffecient of the Linear Regresion Model was found to be  [-2.17838243
-3.62206739 -1.32135996  2.37863175  1.17002319]
The Intercept of the Linear Regresion Model was found to be  23.537919463087
256
```

In [82]:

```
#Evaluating the Multiple Linear Regression Model

print('{}{}\033[1mEvaluating Simple Linear Regression Model\033[0m{}{}\n'.format('<'*3,'-'*

print('\n\n{}Training Set Metrics{}'.format('-'*20, '-'*20))
pred1 = MLR.predict(Train_X_std[Train_X.columns[rfe.support_]])#Test_X_sm)
print('\nR2-Score on Training set --->',round(r2_score(Train_Y, pred1),2))
print('Residual Sum of Squares (RSS) on Training set  --->',round(np.sum(np.square(Train_Y-
print('Mean Squared Error (MSE) on Training set       --->',round(mean_squared_error(Train_
print('Root Mean Squared Error (RMSE) on Training set --->',round(np.sqrt(mean_squared_erro

print('\n{}Test Set Metrics{}'.format('-'*20, '-'*20))
pred2 = MLR.predict(Test_X_std[Train_X.columns[rfe.support_]])#Test_X_sm)
print('\nR2-Score on Testing set --->',round(r2_score(Test_Y, pred2),2))
print('Residual Sum of Squares (RSS) on Training set  --->',round(np.sum(np.square(Test_Y-p
print('Mean Squared Error (MSE) on Training set       --->',round(mean_squared_error(Test_Y
print('Root Mean Squared Error (RMSE) on Training set --->',round(np.sqrt(mean_squared_erro
print('\n{}Residual Plots{}'.format('-'*20, '-'*20))

# Plotting y_test and y_pred to understand the spread.
plt.figure(figsize=[15,4])

plt.subplot(1,2,1)
sns.distplot((Train_Y - pred))
plt.title('Error Terms')                   # Plot heading
plt.xlabel('Errors')

plt.subplot(1,2,2)
plt.scatter(Train_Y,pred)
plt.plot([Train_Y.min(),Train_Y.max()],[Train_Y.min(),Train_Y.max()], 'r--')
plt.title('Test vs Prediction')                # Plot heading
plt.xlabel('y_test')                            # X-label
plt.ylabel('y_pred')                            # Y-label
plt.show()
```

```
<<<-----------------------------------Evaluating Simple Linear Regression Mo
del----------------------------------->>>



--------------------Training Set Metrics--------------------

R2-Score on Training set ---> 0.79
Residual Sum of Squares (RSS) on Training set  ---> 3702.32
Mean Squared Error (MSE) on Training set       ---> 12.42
Root Mean Squared Error (RMSE) on Training set ---> 3.52

--------------------Test Set Metrics--------------------

R2-Score on Testing set ---> 0.8
Residual Sum of Squares (RSS) on Training set  ---> 783.84
Mean Squared Error (MSE) on Training set       ---> 10.45
Root Mean Squared Error (RMSE) on Training set ---> 3.23

--------------------Residual Plots--------------------
```





Conclusion:

It is clear that both manual & automatic methods intend to drop two varibles, but comparing both models, we can see that following dropping the columns recommended by VIF Technique gave better generalizability & better metrics for test set. Hence it is better to drop the features 'weight' and 'displacement' in order to prevent issue of multicollinearity.

6. Predictive Modelling

In [83]:

```
#Let us first define a function to evaluate our models

Model_Evaluation_Comparison_Matrix = pd.DataFrame(np.zeros([5,8]), columns=['Train-R2','Tes
                                                                            'Train-MSE','Te


def Evaluate(n, pred1,pred2):
    #Plotting predicted predicteds alongside the actual datapoints
    plt.figure(figsize=[15,6])
    for e,i in enumerate(Train_X_std):
        plt.subplot(2,3,e+1)
        plt.scatter(y=Train_Y, x=Train_X_std[i], label='Actual')
        plt.scatter(y=pred, x=Train_X_std[i], label='Prediction')
        plt.legend()
    plt.show()

    #Evaluating the Multiple Linear Regression Model

    print('\n\n{}Training Set Metrics{}'.format('-'*20, '-'*20))
    print('\nR2-Score on Training set --->',round(r2_score(Train_Y, pred1),2))
    print('Residual Sum of Squares (RSS) on Training set  --->',round(np.sum(np.square(Trai
    print('Mean Squared Error (MSE) on Training set       --->',round(mean_squared_error(Tr
    print('Root Mean Squared Error (RMSE) on Training set --->',round(np.sqrt(mean_squared_

    print('\n{}Testing Set Metrics{}'.format('-'*20, '-'*20))
    print('\nR2-Score on Testing set --->',round(r2_score(Test_Y, pred2),2))
    print('Residual Sum of Squares (RSS) on Training set  --->',round(np.sum(np.square(Test
    print('Mean Squared Error (MSE) on Training set       --->',round(mean_squared_error(Te
    print('Root Mean Squared Error (RMSE) on Training set --->',round(np.sqrt(mean_squared_
    print('\n{}Residual Plots{}'.format('-'*20, '-'*20))

    Model_Evaluation_Comparison_Matrix.loc[n,'Train-R2']  = round(r2_score(Train_Y, pred1),
    Model_Evaluation_Comparison_Matrix.loc[n,'Test-R2']   = round(r2_score(Test_Y, pred2),2
    Model_Evaluation_Comparison_Matrix.loc[n,'Train-RSS'] = round(np.sum(np.square(Train_Y-
    Model_Evaluation_Comparison_Matrix.loc[n,'Test-RSS']  = round(np.sum(np.square(Test_Y-p
    Model_Evaluation_Comparison_Matrix.loc[n,'Train-MSE'] = round(mean_squared_error(Train_
    Model_Evaluation_Comparison_Matrix.loc[n,'Test-MSE']  = round(mean_squared_error(Test_Y
    Model_Evaluation_Comparison_Matrix.loc[n,'Train-RMSE']= round(np.sqrt(mean_squared_erro
    Model_Evaluation_Comparison_Matrix.loc[n,'Test-RMSE'] = round(np.sqrt(mean_squared_erro

    # Plotting y_test and y_pred to understand the spread.
    plt.figure(figsize=[15,4])

    plt.subplot(1,2,1)
    sns.distplot((Train_Y - pred))
    plt.title('Error Terms')
    plt.xlabel('Errors')

    plt.subplot(1,2,2)
    plt.scatter(Train_Y,pred)
    plt.plot([Train_Y.min(),Train_Y.max()],[Train_Y.min(),Train_Y.max()], 'r--')
    plt.title('Test vs Prediction')
    plt.xlabel('y_test')
    plt.ylabel('y_pred')
    plt.show()
```

Objective: Let us now try building multiple regression models & compare their evaluation metrics to choose the best fit model both training and testing sets...

6a. Multiple Linear Regression(MLR)

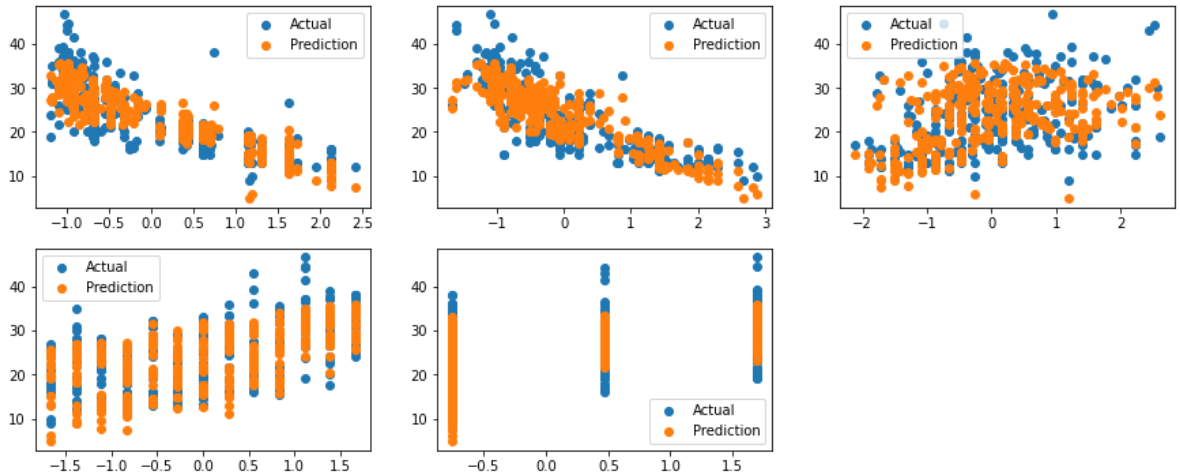
$$\boxed{Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ \dots + \beta_i X_i}$$

Y : Dependent variable
β_0 : Intercept
β_i : Slope for X_i
X = Independent variable

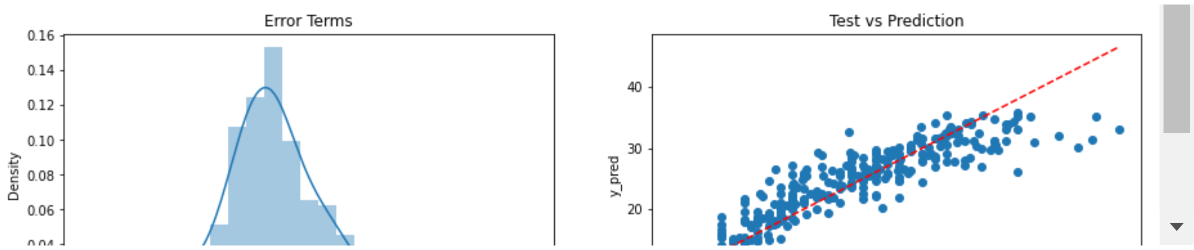
In [84]:

```
from sklearn.linear_model import LinearRegression
from sklearn.metrics import r2_score, mean_absolute_error, mean_squared_error

Train_X_std = Train_X_std[Train_X.columns[rfe.support_]]
Test_X_std = Test_X_std[Test_X.columns[rfe.support_]]

MLR = LinearRegression().fit(Train_X_std,Train_Y)
pred1 = MLR.predict(Train_X_std)
pred2 = MLR.predict(Test_X_std)

print('{}{}\033[1m Evaluating Multiple Linear Regression Model \033[0m{}{}\n'.format('<'*3,
print('The Coeffecient of the Regresion Model was found to be ',MLR.coef_)
print('The Intercept of the Regresion Model was found to be ',MLR.intercept_)

Evaluate(0, pred1, pred2)
```

```
<<<----------------------------------- Evaluating Multiple Linear Regression
Model ----------------------------------->>>

The Coeffecient of the Regresion Model was found to be  [-2.17838243 -3.6220
6739 -1.32135996  2.37863175  1.17002319]
The Intercept of the Regresion Model was found to be  23.537919463087256
```




```
--------------------Training Set Metrics--------------------

R2-Score on Training set ---> 0.79
Residual Sum of Squares (RSS) on Training set  ---> 3702.32
Mean Squared Error (MSE) on Training set       ---> 12.42
Root Mean Squared Error (RMSE) on Training set ---> 3.52

--------------------Testing Set Metrics--------------------

R2-Score on Testing set ---> 0.8
Residual Sum of Squares (RSS) on Training set  ---> 783.84
Mean Squared Error (MSE) on Training set       ---> 10.45
Root Mean Squared Error (RMSE) on Training set ---> 3.23

--------------------Residual Plots--------------------
```



6b. Ridge Regression Model

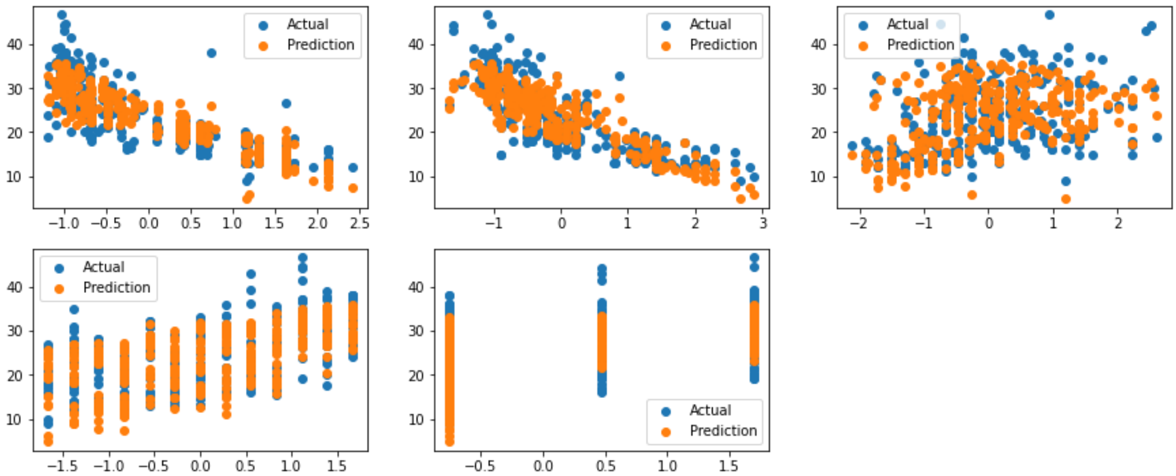
Ridge Formula: Sum of Error + Sum of the squares of coefficients

$$L = \underbrace{\sum(\hat{Y}i - Yi)^2} + \underbrace{\lambda \sum \beta^2}$$

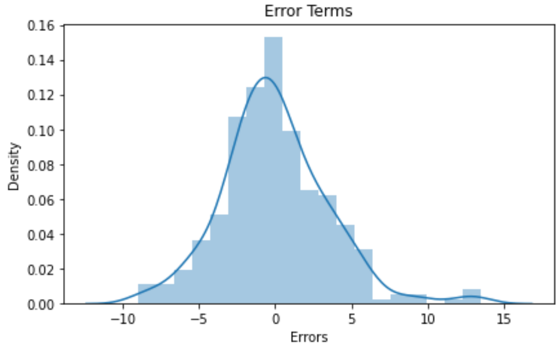
In [85]:

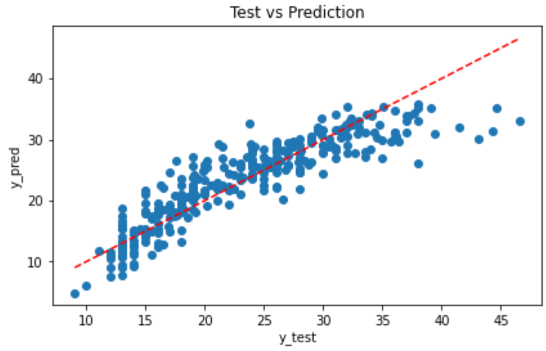
```python
#Creating a Ridge Regression model

from sklearn.linear_model import Ridge

Train_X_std = Train_X_std[Train_X.columns[rfe.support_]]
Test_X_std = Test_X_std[Test_X.columns[rfe.support_]]

RLR = Ridge().fit(Train_X_std,Train_Y)
pred1 = RLR.predict(Train_X_std)
pred2 = RLR.predict(Test_X_std)

print('{}{}\033[1m Evaluating Ridge Regression Model \033[0m{}{}\n'.format('<'*3,'-'*35 ,'-
print('The Coeffecient of the Regresion Model was found to be ',MLR.coef_)
print('The Intercept of the Regresion Model was found to be ',MLR.intercept_)

Evaluate(1, pred1, pred2)
```

```
<<<----------------------------------- Evaluating Ridge Regression Model ---
--------------------------------->>>

The Coeffecient of the Regresion Model was found to be  [-2.17838243 -3.6220
6739 -1.32135996  2.37863175  1.17002319]
The Intercept of the Regresion Model was found to be  23.537919463087256
```




```
--------------------Training Set Metrics--------------------

R2-Score on Training set ---> 0.79
Residual Sum of Squares (RSS) on Training set  ---> 3702.48
Mean Squared Error (MSE) on Training set       ---> 12.42
Root Mean Squared Error (RMSE) on Training set ---> 3.52

--------------------Testing Set Metrics--------------------

R2-Score on Testing set ---> 0.8
Residual Sum of Squares (RSS) on Training set  ---> 784.56
Mean Squared Error (MSE) on Training set       ---> 10.46
Root Mean Squared Error (RMSE) on Training set ---> 3.23

--------------------Residual Plots--------------------
```





6c. Lasso Regression Model

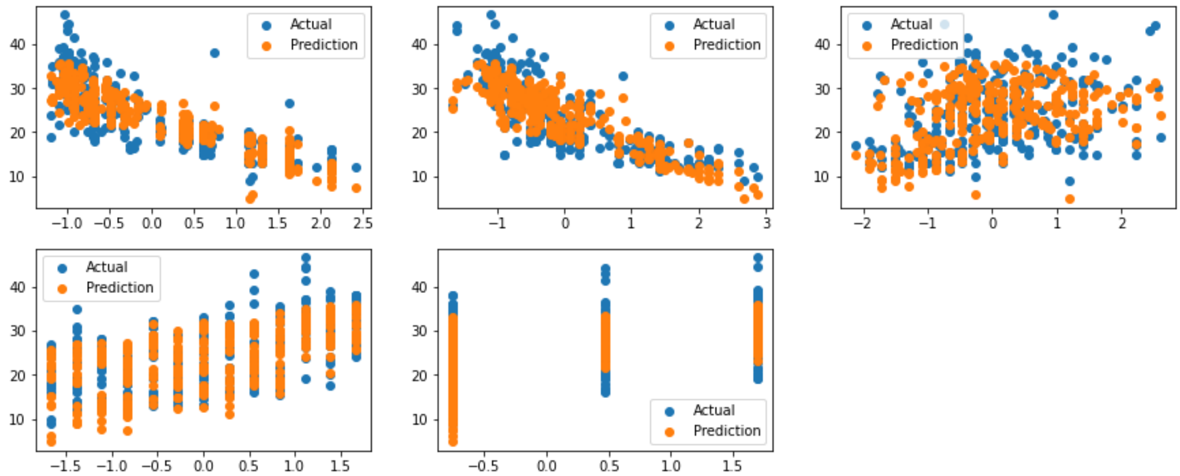
Lasso = *Sum of Error* + *Sum of the absolute value of coefficients*

$$L = \sum(\hat{Y}i - Yi)^2 + \lambda\sum|\beta|$$

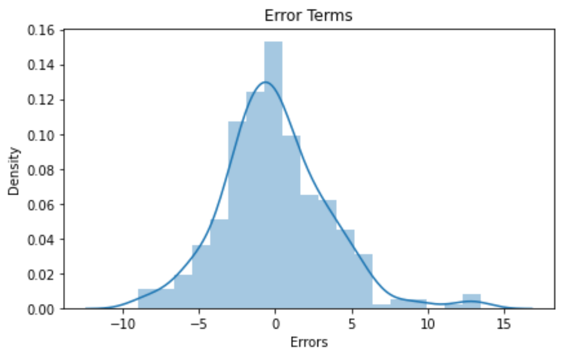
In [86]:

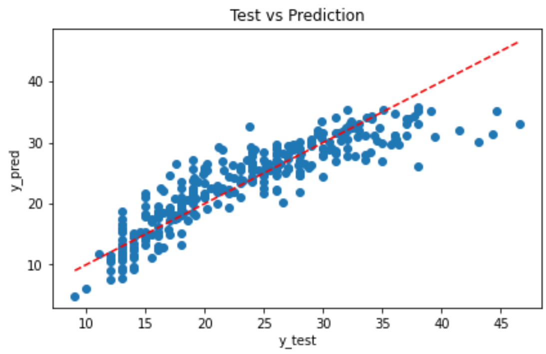
```
#Creating a Ridge Regression model

from sklearn.linear_model import Lasso

Train_X_std = Train_X_std[Train_X.columns[rfe.support_]]
Test_X_std = Test_X_std[Test_X.columns[rfe.support_]]

LLR = Lasso().fit(Train_X_std,Train_Y)
pred1 = LLR.predict(Train_X_std)
pred2 = LLR.predict(Test_X_std)

print('{}{}\033[1m Evaluating Lasso Regression Model \033[0m{}{}\n'.format('<'*3,'-'*35 ,'-
print('The Coeffecient of the Regresion Model was found to be ',MLR.coef_)
print('The Intercept of the Regresion Model was found to be ',MLR.intercept_)

Evaluate(2, pred1, pred2)
```

```
<<<----------------------------------- Evaluating Lasso Regression Model ---
-------------------------------->>>

The Coeffecient of the Regresion Model was found to be  [-2.17838243 -3.6220
6739 -1.32135996  2.37863175  1.17002319]
The Intercept of the Regresion Model was found to be  23.537919463087256
```




```
--------------------Training Set Metrics--------------------

R2-Score on Training set ---> 0.74
Residual Sum of Squares (RSS) on Training set  ---> 4540.38
Mean Squared Error (MSE) on Training set       ---> 15.24
Root Mean Squared Error (RMSE) on Training set ---> 3.9

--------------------Testing Set Metrics--------------------

R2-Score on Testing set ---> 0.75
Residual Sum of Squares (RSS) on Training set  ---> 967.67
Mean Squared Error (MSE) on Training set       ---> 12.9
Root Mean Squared Error (RMSE) on Training set ---> 3.59

--------------------Residual Plots--------------------
```





6d. Elastic-Net Regression

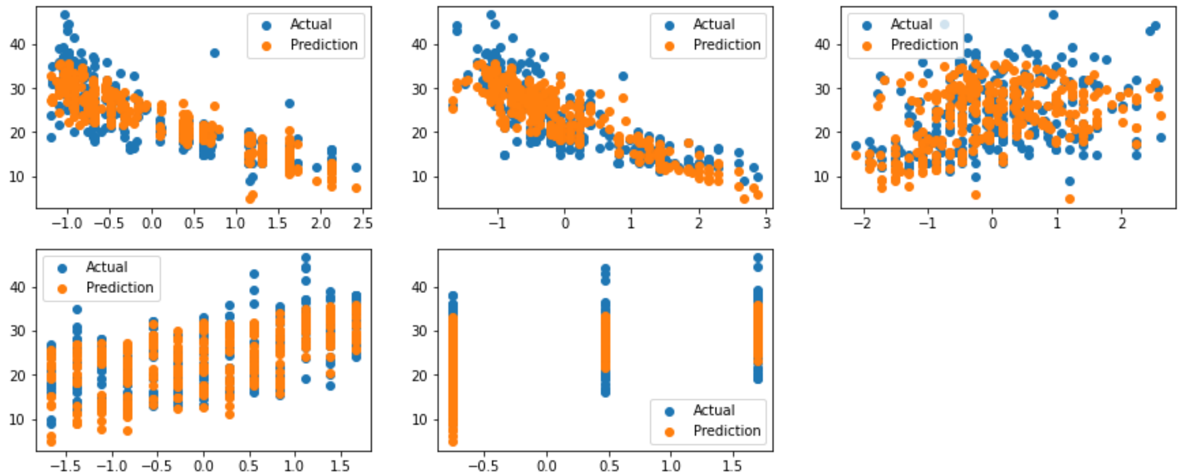
Elastic Net Formula: Ridge + Lasso

$$L = \sum(\hat{Y}i - Yi)^2 + \lambda\sum\beta^2 + \lambda\sum|\beta|$$

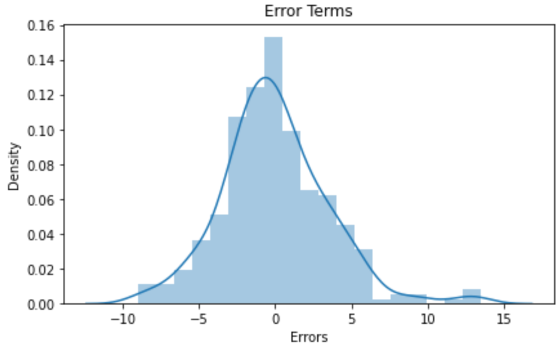
In [87]:

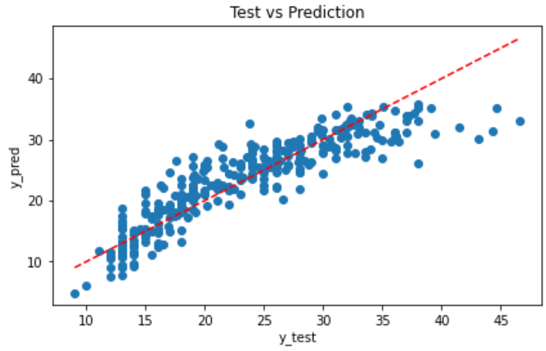
```
#Creating a ElasticNet Regression model

from sklearn.linear_model import ElasticNet

Train_X_std = Train_X_std[Train_X.columns[rfe.support_]]
Test_X_std = Test_X_std[Test_X.columns[rfe.support_]]

ENR = ElasticNet().fit(Train_X_std,Train_Y)
pred1 = ENR.predict(Train_X_std)
pred2 = ENR.predict(Test_X_std)

print('{}{}\033[1m Evaluating Elastic-Net Regression Model \033[0m{}{}\n'.format('<'*3,'-'*
print('The Coeffecient of the Regresion Model was found to be ',MLR.coef_)
print('The Intercept of the Regresion Model was found to be ',MLR.intercept_)

Evaluate(3, pred1, pred2)
```

```
<<<----------------------------------- Evaluating Elastic-Net Regression Mod
el ----------------------------------->>>

The Coeffecient of the Regresion Model was found to be  [-2.17838243 -3.6220
6739 -1.32135996  2.37863175  1.17002319]
The Intercept of the Regresion Model was found to be  23.537919463087256
```




```
--------------------Training Set Metrics--------------------

R2-Score on Training set ---> 0.72
Residual Sum of Squares (RSS) on Training set  ---> 4878.58
Mean Squared Error (MSE) on Training set       ---> 16.37
Root Mean Squared Error (RMSE) on Training set ---> 4.05

--------------------Testing Set Metrics--------------------

R2-Score on Testing set ---> 0.74
Residual Sum of Squares (RSS) on Training set  ---> 1013.88
Mean Squared Error (MSE) on Training set       ---> 13.52
Root Mean Squared Error (RMSE) on Training set ---> 3.68

--------------------Residual Plots--------------------
```





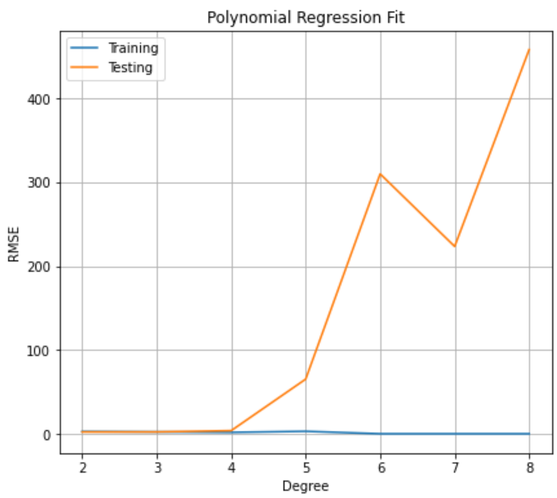
6e. Polynomial Regression Model

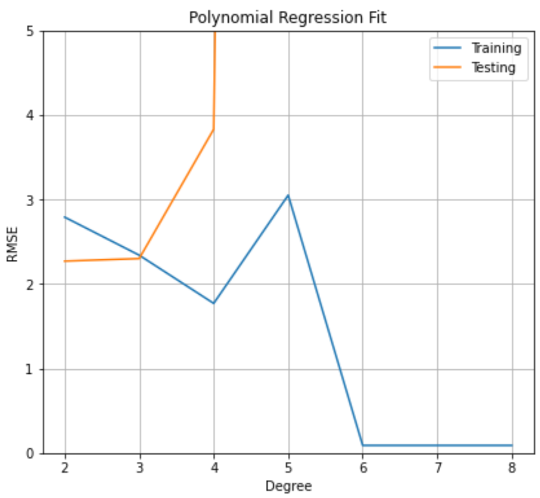
Polynomial Regression : Order-n

$$y = b_0 + b_1 x_1 + b_2 x_1^2 + \dots + b_n x_1^n$$

In [88]:

```python
#Checking polynomial regression performance on various degrees

from sklearn.preprocessing import PolynomialFeatures
Trr=[]; Tss=[]

for i in range(2,9):
    #print(f'{i} Degree')
    poly_reg = PolynomialFeatures(degree=i)
    X_poly = poly_reg.fit_transform(Train_X_std)
    X_poly1 = poly_reg.fit_transform(Test_X_std)
    LR = LinearRegression()
    LR.fit(X_poly, Train_Y)

    pred1 = LR.predict(X_poly)
    Trr.append(round(np.sqrt(mean_squared_error(Train_Y, pred1)),2))

    pred2 = LR.predict(X_poly1)
    Tss.append(round(np.sqrt(mean_squared_error(Test_Y, pred2)),2))

plt.figure(figsize=[15,6])
plt.subplot(1,2,1)
plt.plot(range(2,9),Trr, label='Training')
plt.plot(range(2,9),Tss, label='Testing')
#plt.plot([1,4],[1,4],'b--')
plt.title('Polynomial Regression Fit')
#plt.ylim([0,5])
plt.xlabel('Degree')
plt.ylabel('RMSE')
plt.grid()
plt.legend()
#plt.xticks()

plt.subplot(1,2,2)
plt.plot(range(2,9),Trr, label='Training')
plt.plot(range(2,9),Tss, label='Testing')
plt.title('Polynomial Regression Fit')
plt.ylim([0,5])
plt.xlabel('Degree')
plt.ylabel('RMSE')
plt.grid()
plt.legend()
#plt.xticks()
plt.show()
```



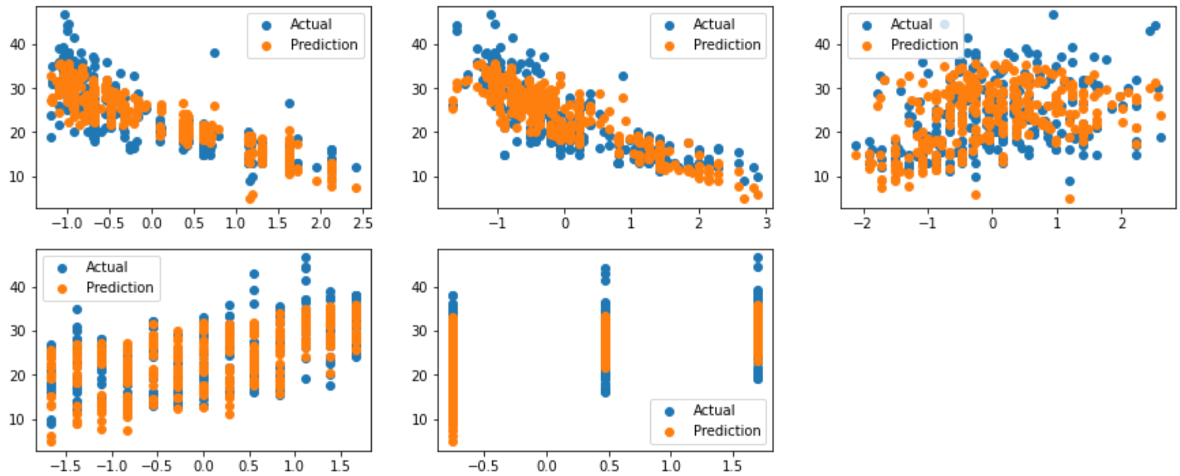


Inference: We can choose 3rd order polynomial regression as it gives the optimal training & testing scores...

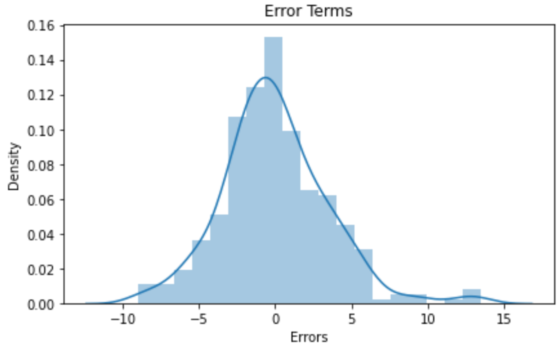
In [89]:

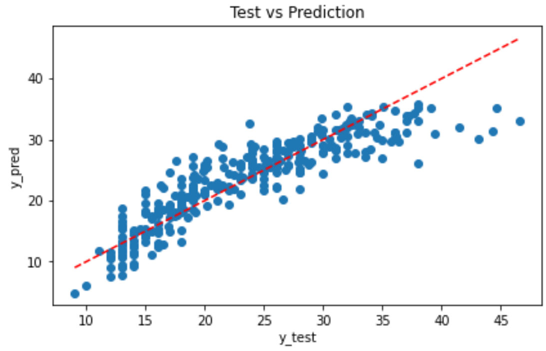
```
#Using the 3rd Order Polynomial Regression model (degree=3)

from sklearn.preprocessing import PolynomialFeatures
poly_reg = PolynomialFeatures(degree=3)
X_poly = poly_reg.fit_transform(Train_X_std)
X_poly1 = poly_reg.fit_transform(Test_X_std)
PR = LinearRegression()
PR.fit(X_poly, Train_Y)

pred1 = PR.predict(X_poly)
pred2 = PR.predict(X_poly1)

print('{}{}\033[1m Evaluating Polynomial Regression Model \033[0m{}{}\n'.format('<'*3,'-'*3
print('The Coeffecient of the Regresion Model was found to be ',MLR.coef_)
print('The Intercept of the Regresion Model was found to be ',MLR.intercept_)

Evaluate(4, pred1, pred2)
```

```
<<<----------------------------------- Evaluating Polynomial Regression Mode
l ----------------------------------->>>

The Coeffecient of the Regresion Model was found to be  [-2.17838243 -3.6220
6739 -1.32135996  2.37863175  1.17002319]
The Intercept of the Regresion Model was found to be  23.537919463087256
```




```
--------------------Training Set Metrics--------------------

R2-Score on Training set ---> 0.91
Residual Sum of Squares (RSS) on Training set  ---> 1626.65
Mean Squared Error (MSE) on Training set       ---> 5.46
Root Mean Squared Error (RMSE) on Training set ---> 2.34

--------------------Testing Set Metrics--------------------

R2-Score on Testing set ---> 0.9
Residual Sum of Squares (RSS) on Training set  ---> 395.27
Mean Squared Error (MSE) on Training set       ---> 5.27
Root Mean Squared Error (RMSE) on Training set ---> 2.3

--------------------Residual Plots--------------------
```





6f. Comparing the Evaluation Metics of the Models

In [48]:

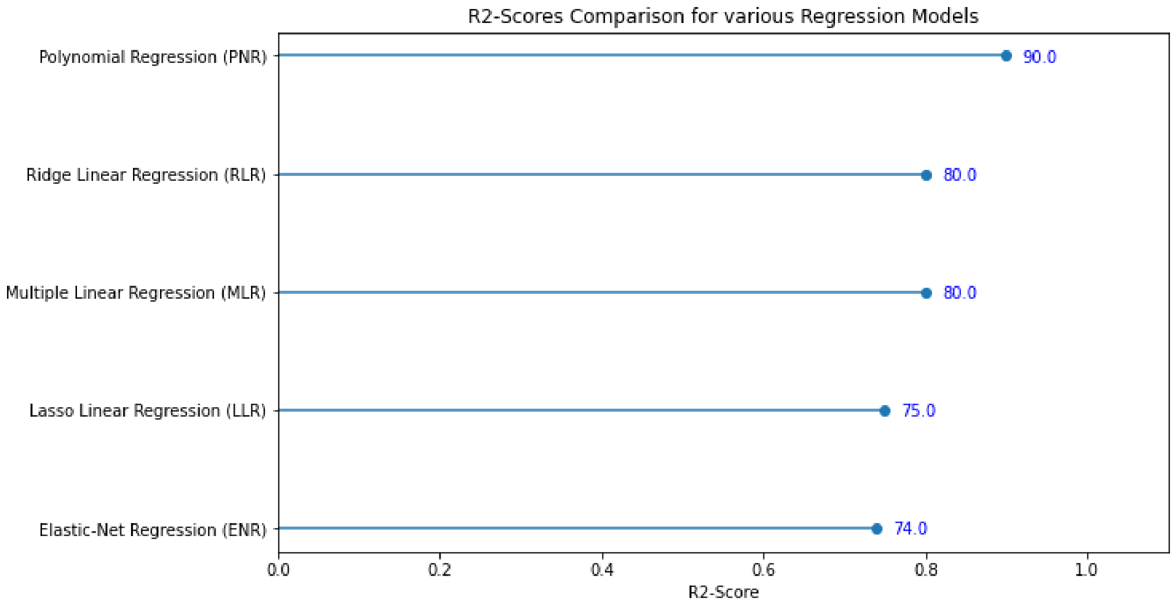
```
# Regression Models Results Evaluation

EMC = Model_Evaluation_Comparison_Matrix.copy()
EMC.index = ['Multiple Linear Regression (MLR)','Ridge Linear Regression (RLR)','Lasso Line
EMC
```

Out[48]:

	Train-R2	Test-R2	Train-RSS	Test-RSS	Train-MSE	Test-MSE	Train-RMSE	Test-RMSE
Multiple Linear Regression (MLR)	0.79	0.80	3702.32	783.84	12.42	10.45	3.52	3.23
Ridge Linear Regression (RLR)	0.79	0.80	3702.48	784.56	12.42	10.46	3.52	3.23
Lasso Linear Regression (LLR)	0.74	0.75	4540.38	967.67	15.24	12.90	3.90	3.59
Elastic-Net Regression (ENR)	0.72	0.74	4878.58	1013.88	16.37	13.52	4.05	3.68
Polynomial Regression (PNR)	0.91	0.90	1626.65	395.27	5.46	5.27	2.34	2.30

In [49]:

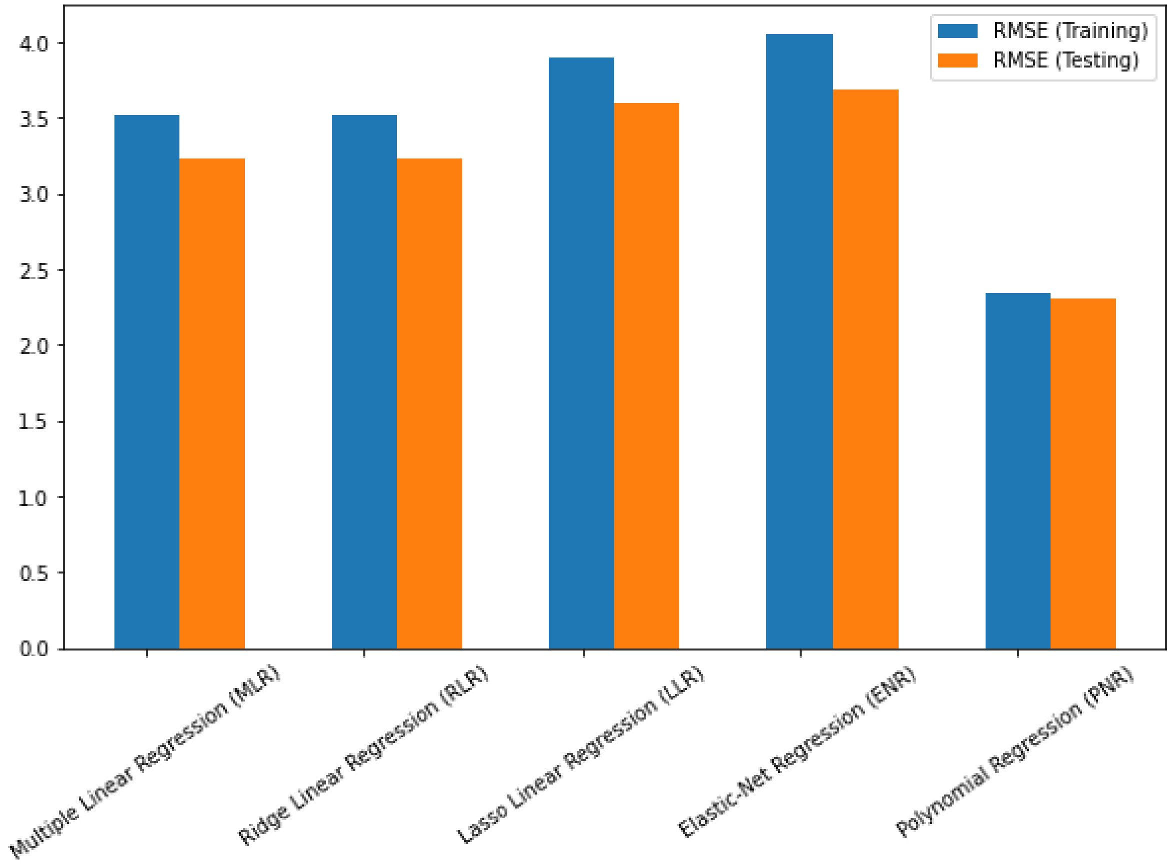
```
# R2-Scores Comparison for different Regression Models

R2 = EMC['Test-R2'].sort_values(ascending=True)
plt.hlines(y=R2.index, xmin=0, xmax=R2.values)
plt.plot(R2.values, R2.index,'o')
plt.title('R2-Scores Comparison for various Regression Models')
plt.xlabel('R2-Score')
#plt.ylabel('Regression Models')
for i, v in enumerate(R2):
    plt.text(v+0.02, i-0.05, str(v*100), color='blue')
plt.xlim([0,1.1])
plt.show()
```



Inference: From the above plot, it is clear that the polynomial regresion models have the highest explainability power to understand the dataset.

In [90]:

```
# Root Mean SquaredError Comparison for different Regression Models

cc = Model_Evaluation_Comparison_Matrix.columns.values
s=5

plt.bar(np.arange(s), Model_Evaluation_Comparison_Matrix[cc[-2]].values, width=0.3, label='
plt.bar(np.arange(s)+0.3, Model_Evaluation_Comparison_Matrix[cc[-1]].values, width=0.3, lab
plt.xticks(np.arange(s),EMC.index, rotation =35)
plt.legend()
plt.show()
```



Inference: Lesser the RMSE, better the model! Also, provided the model should have close proximity with the training & testing scores.
For this problem, it is can be said that polynomial regressions are the best choice to go with...

---

10. Project Outcomes & Conclusions

Here are some of the key outcomes of the project:

- The Dataset was quiet small totally just 398 samples & after preprocessing 6.3% of the datasamples were dropped.
- Visualising the distribution of data & their relationships, helped us to get some insights on the feature-set.
- The features had high multicollinearity, hence in Feature Extraction step, we used VIF & RFE Techniques to drop highly correlated features.
- Testing multiple algorithms with default hyperparamters gave us some understanding for various models performance on this specific dataset.
- While, Polynomial Regression (Order-3) gave the best overall scores for the current dataset, yet it wise to also consider simpler models like MLR & ENR as they are more generalisable.

In [ ]:

```
<<<----------------------------------------------THE END-----------------------------------
```