Arithmetic Geometry

1 Algebraic number theory foundations

1.1 Algebraic number fields 1.2 Rings of integers 1.3 Ideal class groups 1.4 Units and Dirichlet's unit theorem 1.5 Dedekind domains 1.6 Ramification theory 1.7 Local fields

2 Elliptic curves

2.1 Weierstrass equations 2.2 Group law on elliptic curves 2.3 Torsion points 2.4 Mordell-Weil theorem 2.5 Elliptic curve isogenies 2.6 Complex multiplication 2.7 Reduction modulo p

3 Modular forms

3.1 Modular groups 3.2 Cusp forms 3.3 Eisenstein series 3.4 Hecke operators 3.5 Newforms 3.6 Modularity theorem 3.7 Modular curves

4 Diophantine equations

4.1 Linear Diophantine equations 4.2 Quadratic Diophantine equations 4.3 Fermat's Last Theorem 4.4 Thue equations 4.5 Rational points on curves 4.6 Height functions 4.7 Diophantine approximation

5 Galois representations

5.1 Galois groups 5.2 l-adic representations 5.3 Artin representations 5.4 Serre's modularity conjecture 5.5 Fontaine-Mazur conjecture 5.6 Langlands program 5.7 Motives

6 L-functions and zeta functions

6.1 Riemann zeta function 6.2 Dirichlet L-functions 6.3 Dedekind zeta functions 6.4 Artin L-functions 6.5 Hasse-Weil zeta functions 6.6 Functional equations 6.7 Analytic continuation

7 Abelian varieties

7.1 Complex tori 7.2 Polarizations 7.3 Isogenies of abelian varieties 7.4 Endomorphism algebras 7.5 Jacobian varieties 7.6 Néron models 7.7 Tate modules

8 p-adic analysis and geometry

8.1 p-adic numbers 8.2 p-adic manifolds 8.3 Rigid analytic spaces 8.4 Berkovich spaces 8.5 p-adic modular forms 8.6 Coleman integration 8.7 p-adic Hodge theory

9 Arithmetic of algebraic varieties

9.1 Rational points 9.2 Local-global principle 9.3 Brauer-Manin obstruction 9.4 Hasse principle 9.5 Weak approximation 9.6 Arithmetic surfaces 9.7 Arithmetic threefolds

10 Class field theory

10.1 Hilbert class fields 10.2 Ray class fields 10.3 Artin reciprocity law 10.4 Idele class groups 10.5 Global class field theory 10.6 Local class field theory 10.7 Explicit reciprocity laws

11 Étale cohomology

11.1 Étale morphisms 11.2 Grothendieck topologies 11.3 Cohomology of sheaves 11.4 l-adic cohomology 11.5 Comparison theorems 11.6 Cycle class maps 11.7 Weil conjectures

12 Arithmetic dynamics

12.1 Dynamical systems on projective spaces 12.2 Height functions in dynamics 12.3 Periodic points 12.4 Preperiodic points 12.5 Dynamical Mordell-Lang conjecture 12.6 Arithmetic equidistribution 12.7 Dynamical Manin-Mumford conjecture

Model Theory

1 Model Theory: First-Order Logic Intro

1.1 Historical development and motivation for model theory 1.2 Basic concepts and terminology of first-order logic 1.3 Syntax and semantics of first-order languages 1.4 Applications of model theory in mathematics and computer science

2 Structures and Signatures

2.1 Definition and examples of mathematical structures 2.2 Signatures: function symbols, relation symbols, and constants 2.3 Homomorphisms and isomorphisms between structures

3 Terms, Formulas, and Satisfaction

3.1 Construction of terms and formulas in first-order logic 3.2 Free and bound variables 3.3 Truth and satisfaction in structures 3.4 Interpretations and models

4 Theories and Models

4.1 Axioms, theories, and models 4.2 Consistency and completeness of theories 4.3 Model-theoretic consequences and logical implications 4.4 Examples of theories and their models

5 Elementary Equivalence and Isomorphism

5.1 Definition and properties of elementary equivalence 5.2 Partial isomorphisms and back-and-forth constructions 5.3 Ehrenfeucht-Fraïssé games

6 Compactness and Löwenheim–Skolem Theorems

6.1 Compactness theorem and its applications 6.2 Downward Löwenheim-Skolem theorem 6.3 Upward Löwenheim-Skolem theorem 6.4 Consequences and limitations of these theorems

7 Types and Saturation

7.1 Types and type spaces 7.2 Realizations and omissions of types 7.3 Saturated and homogeneous models 7.4 Construction of saturated models

8 Ultraproducts and Ultrapowers

8.1 Ultrafilters and their properties 8.2 Construction of ultraproducts and ultrapowers 8.3 Łoś's theorem and its applications

9 Quantifier Elimination ŭ0026 Model Completeness

9.1 Quantifier elimination: definition and techniques 9.2 Model completeness and its relationship to quantifier elimination 9.3 Applications to specific theories (e.g., dense linear orders, real closed fields)

10 Omitting Types Theorem and Prime Models

10.1 Omitting types theorem and its proof 10.2 Applications of omitting types 10.3 Prime and atomic models

11 Categoricity and Completeness

11.1 Categoricity in power and its implications 11.2 Morley's categoricity theorem 11.3 Complete theories and their properties

12 Interpretations and Definability

12.1 Interpretations between theories 12.2 Definable sets and functions 12.3 Elimination of imaginaries

13 Algebraic Fields: Closed ŭ0026 Applications

13.1 Model theory of fields 13.2 Algebraically closed fields and their properties 13.3 Applications to algebraic geometry

14 Introduction to Stability Theory

14.1 Stable theories and their properties 14.2 Forking independence 14.3 Classification theory and dividing lines