Trapping Models

No Trapping

$$\dot{A} = -\frac{1}{\tau_A} A \qquad \dot{B} = -\frac{1}{\tau_B} B \quad \Rightarrow \quad \boxed{\begin{aligned} A &= A_0\, e^{-\frac{1}{\tau_A}t} \\ B &= B_0\, e^{-\frac{1}{\tau_B}t} \end{aligned}}$$

$$\Rightarrow -\dot{N} = -\dot{A} - \dot{B} = \boxed{A_0 \frac{1}{\tau_A} e^{-\frac{1}{\tau_A}t} + B_0 \frac{1}{\tau_B} e^{-\frac{1}{\tau_B}t}}$$

Population	app. lifetime	app. intensity
A	τ_A	$\frac{A_0}{N_0}$
B	τ_B	$\frac{B_0}{N_0}$

Trapping with one Defect Type

$$\dot{A} = -\frac{1}{\tau_A} A - \lambda A \qquad \dot{B} = -\frac{1}{\tau_B} B + \lambda A$$

$$\Rightarrow \boxed{A = N_0\, e^{-\left(\frac{1}{\tau_A}+\lambda\right)t}} = N_0\, e^{-\frac{1}{\tau_r}t}$$

$$\Rightarrow \dot{B} = -\frac{1}{\tau_B} B + \lambda N_0\, e^{-\frac{1}{\tau_r}t}$$

Ansatz: $B = c(t)\, e^{-\frac{1}{\tau_B}t}$

$$\Rightarrow \dot{B} = -c \frac{1}{\tau_B} e^{-\frac{1}{\tau_B}t} + \dot{c}\, e^{-\frac{1}{\tau_B}t}$$

$$\Rightarrow -c \frac{1}{\tau_B} e^{-\frac{1}{\tau_B}t} + \dot{c}\, e^{-\frac{1}{\tau_B}t} = -c \frac{1}{\tau_B} e^{-\frac{1}{\tau_B}t} + \lambda N_0\, e^{-\frac{1}{\tau_r}t}$$

$$\dot{c} = \lambda N_0\, e^{-\left(\frac{1}{\tau_r}-\frac{1}{\tau_B}\right)t} \quad \Rightarrow \quad c = -\frac{\lambda N_0}{\frac{1}{\tau_r}-\frac{1}{\tau_B}} e^{-\left(\frac{1}{\tau_r}-\frac{1}{\tau_B}\right)t} + c_0$$

$$\Rightarrow B = -\frac{\lambda N_0}{\frac{1}{\tau_r}-\frac{1}{\tau_B}} e^{-\frac{1}{\tau_r}t} + c_0\, e^{-\frac{1}{\tau_B}t}$$

$$B(0) = 0 \implies \boxed{B = \frac{\lambda N_0}{\frac{1}{\tau_r} - \frac{1}{\tau_B}} \left(e^{-\frac{1}{\tau_B} t} - e^{-\frac{1}{\tau_r} t} \right)}$$

$$\implies -\dot{N} = N_0 \frac{1}{\tau_r} e^{-\frac{1}{\tau_r} t} + \frac{\lambda N_0}{\frac{1}{\tau_r} - \frac{1}{\tau_B}} \left(\frac{1}{\tau_B} e^{-\frac{1}{\tau_B} t} - \frac{1}{\tau_r} e^{-\frac{1}{\tau_r} t} \right)$$

$$\boxed{= N_0 \frac{1}{\tau_r} \left(1 - \frac{\lambda}{\frac{1}{\tau_r} - \frac{1}{\tau_B}} \right) e^{-\frac{1}{\tau_r} t} + N_0 \frac{1}{\tau_B} \frac{\lambda}{\frac{1}{\tau_r} - \frac{1}{\tau_B}} e^{-\frac{1}{\tau_B} t}}$$

Population	app. lifetime	app. intensity
A	$\tau_r = \left(\frac{1}{\tau_A} + \lambda \right)^{-1}$	$1 - \frac{\lambda}{\frac{1}{\tau_r} - \frac{1}{\tau_B}}$
B	τ_B	$\frac{\lambda}{\frac{1}{\tau_r} - \frac{1}{\tau_B}}$

$$\implies \boxed{\lambda = I_B \left(\frac{1}{\tau_r} - \frac{1}{\tau_B} \right)} \implies \boxed{\frac{1}{\tau_A} = \frac{1}{\tau_r} - I_B \left(\frac{1}{\tau_r} - \frac{1}{\tau_B} \right) = \frac{I_A}{\tau_r} + \frac{I_B}{\tau_B}}$$

Saturation Trapping with one Defect Type

$\lambda \gg \frac{1}{\tau_A} > \frac{1}{\tau_B}$ ($\tau_B > \tau_A$ is always assumed)

$$\implies \frac{1}{\tau_r} \to \lambda, \quad \frac{\lambda}{\frac{1}{\tau_r} - \frac{1}{\tau_B}} \to 1$$

$$\implies -\dot{N} = N_0 \frac{1}{\tau_B} e^{-\frac{1}{\tau_B} t}$$ $\implies$ λ cannot be calculated

Trapping with multiple Defect Types

$$\dot{A} = -\frac{1}{\tau_A} A - \sum_i \lambda_i A$$

$$\dot{B}_i = -\frac{1}{\tau_i} B_i + \lambda_i A$$

$$\implies \boxed{A = N_0 e^{-\left(\frac{1}{\tau_A} + \sum_i \lambda_i \right) t}} = N_0 e^{-\frac{1}{\tau_r} t}$$

$$\implies \dot{B}_i = -\frac{1}{\tau_i} B_i + \lambda_i N_0 e^{-\frac{1}{\tau_r} t}$$

Ansatz: $B_i = c(t)\, e^{-\frac{1}{\tau_i} t}$

$$\implies \dot{B} = -c \frac{1}{\tau_i} e^{-\frac{1}{\tau_i} t} + \dot{c}\, e^{-\frac{1}{\tau_i} t}$$

$$\Rightarrow -c\frac{1}{\tau_i}e^{-\frac{1}{\tau_i}t} + \dot{c}\,e^{-\frac{1}{\tau_i}t} = -c\frac{1}{\tau_i}e^{-\frac{1}{\tau_i}t} + \lambda_i N_0 e^{-\left(\frac{1}{\tau_A}+\sum_i\lambda_i\right)t}$$

$$\dot{c} = \lambda_i N_0 e^{-\left(\frac{1}{\tau_r}-\frac{1}{\tau_i}\right)t}$$

$$\Rightarrow c = -\frac{\lambda_i N_0}{\frac{1}{\tau_r}-\frac{1}{\tau_i}} e^{-\left(\frac{1}{\tau_r}-\frac{1}{\tau_i}\right)t} + c$$

$$\Rightarrow B_i = -\frac{\lambda_i N_0}{\frac{1}{\tau_r}-\frac{1}{\tau_i}} e^{-\frac{1}{\tau_r}t} + c\,e^{-\frac{1}{\tau_i}t}$$

$$B_i(0) = 0 \Rightarrow \boxed{B_i = \frac{\lambda_i N_0}{\frac{1}{\tau_r}-\frac{1}{\tau_i}}\left(e^{-\frac{1}{\tau_i}t} - e^{-\frac{1}{\tau_r}t}\right)}$$

$$\Rightarrow -\dot{N} = N_0\frac{1}{\tau_r}e^{-\frac{1}{\tau_r}t} + \sum_i \frac{\lambda_i N_0}{\frac{1}{\tau_r}-\frac{1}{\tau_i}}\left(\frac{1}{\tau_i}e^{-\frac{1}{\tau_i}t} - \frac{1}{\tau_r}e^{-\frac{1}{\tau_r}t}\right)$$

$$\boxed{= N_0\frac{1}{\tau_r}\left(1-\sum_i\frac{\lambda_i}{\frac{1}{\tau_r}-\frac{1}{\tau_i}}\right)e^{-\frac{1}{\tau_r}t} + N_0\sum_i\frac{1}{\tau_i}\frac{\lambda_i}{\frac{1}{\tau_r}-\frac{1}{\tau_i}}e^{-\frac{1}{\tau_i}t}}$$

Population	app. lifetime	app. intensity
A	$\tau_r = \left(\frac{1}{\tau_A}+\sum_i\lambda_i\right)^{-1}$	$1-\sum_i\frac{\lambda_i}{\frac{1}{\tau_r}-\frac{1}{\tau_i}}$
B_i	τ_i	$\frac{\lambda_i}{\frac{1}{\tau_r}-\frac{1}{\tau_i}}$

$$\Rightarrow \boxed{\lambda_i = I_i\left(\frac{1}{\tau_r}-\frac{1}{\tau_i}\right)}$$

$$\Rightarrow \boxed{\frac{1}{\tau_A} = \frac{1}{\tau_r} - \sum_i\lambda_i = \frac{1}{\tau_r} - \sum_i I_i\left(\frac{1}{\tau_r}-\frac{1}{\tau_i}\right) = \frac{I_A}{\tau_r} + \sum_i\frac{I_i}{\tau_i}}$$

Saturation Trapping with multiple Defect Types

$I_A \to 0 \Rightarrow \tau_r$ unknown

$$\lambda_i = I_i \left(\frac{1}{\tau_r} - \frac{1}{\tau_i} \right) = I_i \left(\frac{1}{\tau_A} - \frac{1}{\tau_i} + \sum_j \lambda_j \right)$$

$$\Rightarrow \lambda_i = \frac{I_i}{1 - \sum_j I_j} \left(\frac{1}{\tau_A} - \sum_j \frac{I_j}{\tau_j} \right)$$ (breaks because $1 - \sum_j I_j = 0$)

=> λ_i cannot be calculated but ratios can:

$$\boxed{\frac{\lambda_i}{\lambda_j} = \frac{I_i}{I_j}}$$

=> Assume $\lambda_i = \alpha_i c_i$

α_i ... trapping coefficient of defect type i $\left[\frac{m^3}{s}\right]$

c_i ... concentration of defect type i $\left[\frac{1}{m^3}\right]$

also assume that defects are infinitely deep traps

=> α_i is a measure of how much volume an e^+ traverses per time which is the same for all defect types

=> $\alpha_i = \alpha$

$$\Rightarrow \boxed{\frac{I_i}{I_j} = \frac{c_i}{c_j}}$$