1. Define electric field, electric dipole and dipole moment.
2. State and prove Gauss's Law in electrostatics.
3. Why do we apply Gauss's Law?
4. How do you calculate the electric field density due to an electric dipole?
5. Write down the applications of Gauss Law.
6. Define electric Flux.
7. What is dipole moment? Obtain an expression for the potential due to an electric dipole.
8. Find the expression for the electric field due to a dipole. Compare the expression with that for a point charge.

- Calculate the maximum torque on the dipole.
- Find the field E at points inside and outside the cylinder.
- Calculate the electric field intensity due to a uniformly charged sphere.
- Calculate the magnitude of the torque on the dipole.

Electric Field: Electric field is the environment created by an electric charge in the space around it, such that if any other electric charge is present in the space, it will come to know of its presence and exert a force on it.

$E = \frac{F}{q} = NC^{-1}$

Coulombs Law: Force of Interaction between two stationary point charges is directly proportional to the product of the charges, inversely proportional to the square of the distance between them, and acts along the straight line joining the two charges.

Coulomb's Formula

$$F = \frac{1}{4\pi\varepsilon_o\varepsilon_r} \cdot \frac{q_1 q_2}{d^2} = \frac{1}{4\pi\varepsilon_o K} \cdot \frac{q_1 q_2}{d^2}$$

$$F = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{d^2}$$

$\varepsilon_0 = 8.854 \times 10^{-12}$ Coul^2/Nm^2

Electric Dipole: An electric dipole is defined as a couple of opposite charges "q" and "–q" separated by a distance "d". By default, the direction of electric dipoles in space is always from negative charge "-q" to positive charge "q". The midpoint "q" and "–q" is called the centre of the dipole.

Electric Field, $E = \frac{F}{q_0}$

Electric Flux, $d\phi = \vec{E} \cdot d\vec{A}$

$= E\,dA\cos\theta$

Electric dipole moment, ~~P=q~~

$P = q \times d$

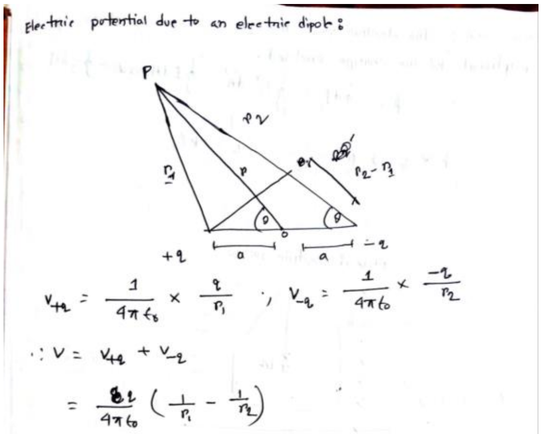
$\Rightarrow \vec{P} = q\vec{d}$

$\Rightarrow P = qd\cos\theta$

Electric Dipole Moment: The electric dipole moment is a measure of the separation of positive and negative electrical charges within a system, that is, a measure of the system's overall polarity. The SI unit for electric dipole moment is the coulomb-meter (C·m).

Electric Flux: Electric flux is the measure of flow of the electric field through a given area.

- **Obtain an expression for the potential due to an electric dipole.**

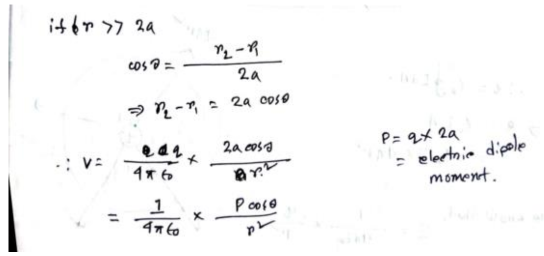


Electric potential due to an electric dipole:

$$V_{+q} = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r_1} \quad ; \quad V_{-q} = \frac{1}{4\pi\epsilon_0} \times \frac{-q}{r_2}$$

$$\therefore V = V_{+q} + V_{-q}$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$



if $r >> 2a$

$$\cos\theta = \frac{r_2 - r_1}{2a}$$

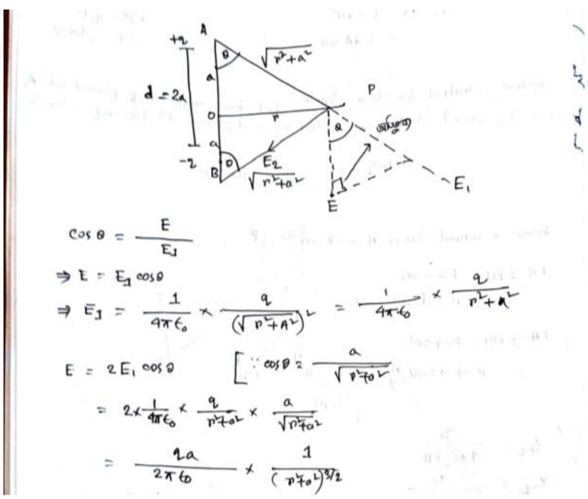
$$\Rightarrow r_2 - r_1 = 2a\cos\theta$$

$$\therefore V = \frac{q}{4\pi\epsilon_0} \times \frac{2a\cos\theta}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \times \frac{p\cos\theta}{r^2}$$

$p = q \times 2a$ = electric dipole moment.

- **Obtain the expression for the electric field of an electric dipole.**



$$\cos\theta = \frac{E}{E_1}$$

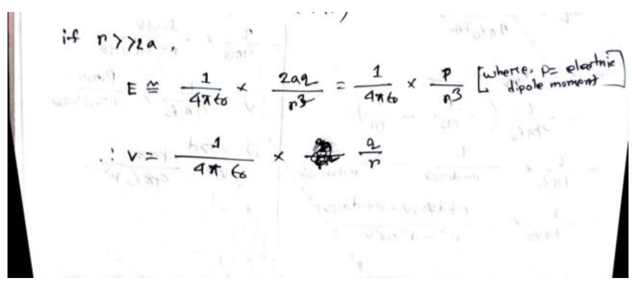
$$\Rightarrow E = E_1\cos\theta$$

$$\Rightarrow E_1 = \frac{1}{4\pi\epsilon_0} \times \frac{q}{(\sqrt{r^2+a^2})^2} = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2+a^2}$$

$$E = 2E_1\cos\theta \quad \left[\because \cos\theta = \frac{a}{\sqrt{r^2+a^2}}\right]$$

$$= 2 \times \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2+a^2} \times \frac{a}{\sqrt{r^2+a^2}}$$

$$= \frac{qa}{2\pi\epsilon_0} \times \frac{1}{(r^2+a^2)^{3/2}}$$

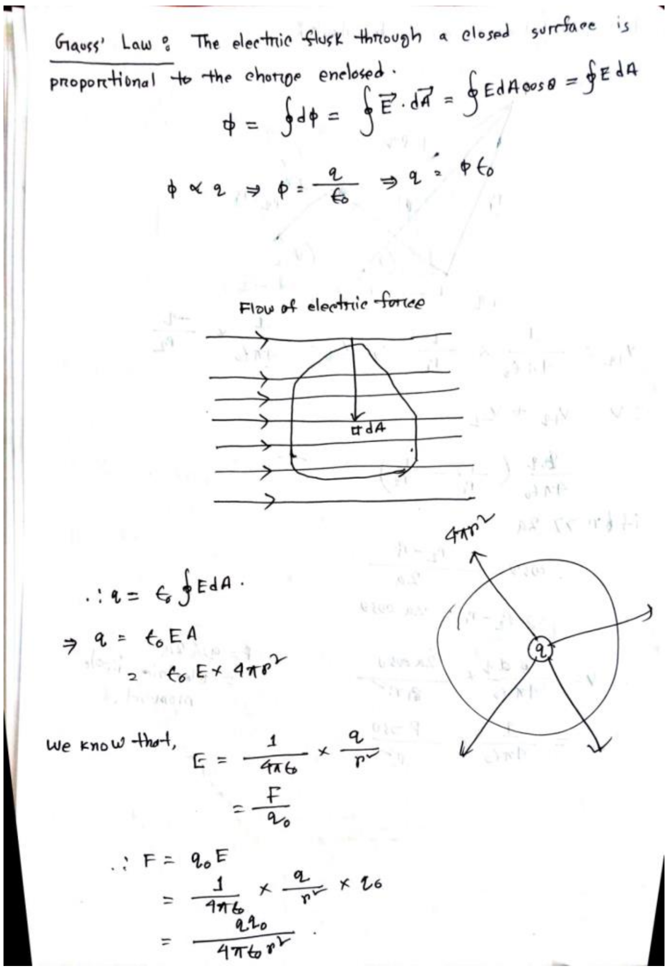


if $r >> 2a$,

$$E \cong \frac{1}{4\pi\epsilon_0} \times \frac{2aq}{r^3} = \frac{1}{4\pi\epsilon_0} \times \frac{p}{r^3} \quad [\text{where, } p = \text{electric dipole moment}]$$

$$\therefore V = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r}$$

- **State and prove Gauss's Law in electrostatics.**



Gauss' Law: The electric flux through a closed surface is proportional to the charge enclosed.

$$\phi = \oint d\phi = \oint \vec{E} \cdot d\vec{A} = \oint EdA\cos\theta = \oint EdA$$

$$\phi \propto q \Rightarrow \phi = \frac{q}{\epsilon_0} \Rightarrow q = \phi\epsilon_0$$

$$\therefore q = \epsilon_0 \oint EdA.$$

$$\Rightarrow q = \epsilon_0 EA$$

$$= \epsilon_0 E \times 4\pi r^2$$

We know that, $E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2}$

$$= \frac{F}{q_0}$$

$$\therefore F = q_0 E$$

$$= \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2} \times q_0$$

$$= \frac{qq_0}{4\pi\epsilon_0 r^2}.$$

- Why do we apply Gauss's Law?

Gauss's Law is a general law applying to any closed surface. It is an important tool since it permits the assessment of the amount of enclosed charge by mapping the field on a surface outside the charge distribution. For geometries of sufficient symmetry, it simplifies the calculation of the electric field.

- **Write down the applications of Gauss Law.**

Here are some of the key applications of Gauss's Law:

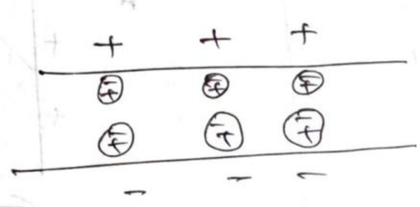
1. Finding Electric field due to infinite straight wire.
2. Finding Electric field due to infinite plate sheet.
3. Finding Electric field due to thin spherical shell.
4. Determining electric flux.
5. Solving Symmetric Problems.
6. Understanding Faraday Cages.
7. Analyzing dipoles.

Permittivity (ϵ): Permittivity (ϵ) is a measure of the polarizability of a di-electric.

$E = \frac{V}{d}$

ϵ_r = relative permittivity

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$



Electric Displacement:

$D = \epsilon_0 \epsilon_r E$ (in medium)

$D = \epsilon_0 E$

Electric Displacement / Flux Density

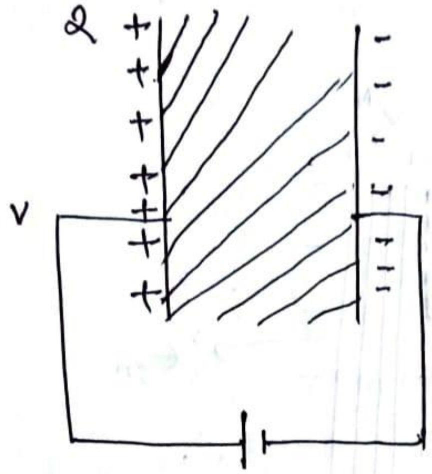
$$D = \frac{\Psi}{A}$$

A = area.

Ψ =

- Define capacrtance and a capacitor. On what factor does the capacitance depend?
- Find an expression for the capacitance of a parallel plate capacitor.
- A parallel plate capacitor consists of two square metal plates with 5.0 cm of side separated by 1 0 2¾ cm. A Sulphur slab of 6 0 mm thick and with k = 4 is placed on the lower plate, calculate the capacitance.

Capacitor: Consists of two conducting surface seperated by a dielectric.



Capacitance: Capacitance is expressed as the ratio of the electric charge on each conductor to the potential difference between them.

$$C = \frac{Q}{V}$$
$$= F$$

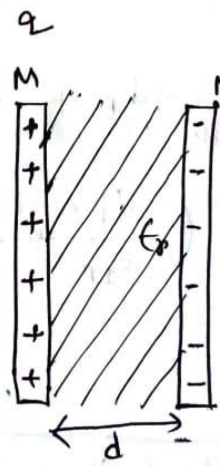
where,
Q = charge
V = voltage
C = capacitance

· Expression for capacitance of a spherical conductor:

$$V = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r} \quad \therefore C = \frac{q}{\frac{q}{4\pi\epsilon_0 r}} = 4\pi\epsilon_0 r$$

$$\boxed{\therefore C = 4\pi\epsilon_0 r}$$

· Parallel Plate capacitor:

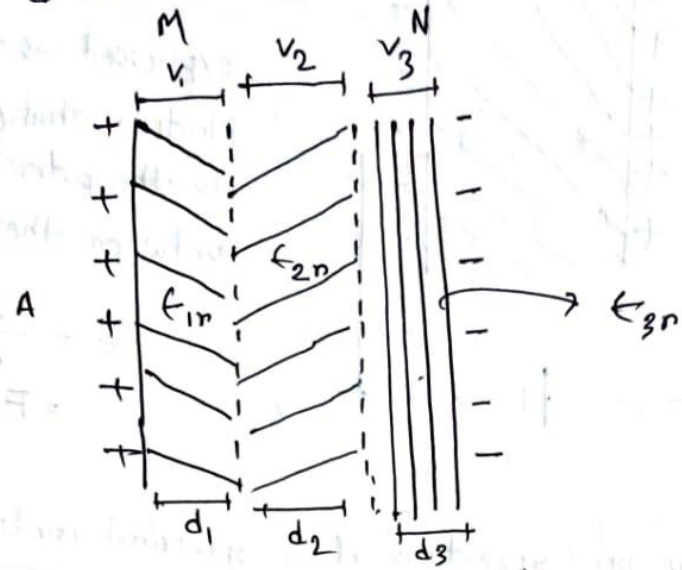


Electric displacement, $D = \frac{\psi}{A}$
$= \frac{q}{A}$

$$\therefore E = \frac{V}{d}$$
$$\Rightarrow D = \epsilon E$$
$$\Rightarrow \frac{q}{A} = \epsilon \times \frac{V}{d}$$
$$\Rightarrow \frac{q}{V} = \frac{A\epsilon}{d}$$

$\therefore C = \epsilon_0 \times \frac{A}{d}$ [in air]

$\therefore C = \epsilon_0 \epsilon_r \times \frac{A}{d}$ [in medium]



$V = V_1 + V_2 + V_3$

$= E_1 d_1 + E_2 d_2 + E_3 d_3$

$= \frac{D}{\epsilon_0 \epsilon_{1r}} \times d_1 + \frac{D}{\epsilon_0 \epsilon_{2r}} \times d_2 + \frac{D}{\epsilon_0 \epsilon_{3r}} \times d_3$

$= \frac{q}{A \epsilon_0 \epsilon_{1r}} \times d_1 + \frac{q}{A \epsilon_0 \epsilon_{2r}} \times d_2 + \frac{q}{A \epsilon_0 \epsilon_{3r}} \times d_3$

$= \frac{q}{A \epsilon_0} \left(\frac{d_1}{\epsilon_{1r}} + \frac{d_2}{\epsilon_{2r}} + \frac{d_3}{\epsilon_{3r}} \right)$

$$\therefore C = \frac{q}{\frac{q}{A \epsilon_0} \left(\frac{d_1}{\epsilon_{1r}} + \frac{d_2}{\epsilon_{2r}} + \frac{d_3}{\epsilon_{3r}} \right)}$$

$$= \frac{A\epsilon_0}{\frac{d_1}{\epsilon_{1r}} + \frac{d_2}{\epsilon_{2r}} + \frac{d_3}{\epsilon_{3r}}}$$

$$\therefore C = \frac{A\epsilon_0}{\frac{d_1}{\epsilon_{1r}} + \frac{d_2}{\epsilon_{2r}} + \frac{d_3}{\epsilon_{3r}}}$$

$$\therefore dw = v\,dq$$

$$= v\,dcv$$

$$= cv\,dv$$

$$\therefore w = \int_0^v cv\,dv$$

$$= c\left[\frac{v^2}{2}\right]_0^v$$

$$= \frac{1}{2}cv^2 \text{ Joule}$$

$$= \frac{1}{2}QV \text{ Joule}$$

$$= \frac{1}{2}\frac{Q^2}{C} \text{ Joule.}$$

* A parallel plate capacitor having waxes paper as the insulator has a capacitance of 3800 pF, breakdown voltage 1500 V. The waxed paper has relative permittivity of 4.3 and breakdown voltage of $15 \times 10^6\ Vm^{-1}$. Find the spacing between two plates of the capacitor and the plate area.

$E = 15 \times 10^{-6} Vm^{-1}$
$V = 1500V$

$\Rightarrow E = \frac{V}{d}$

$\Rightarrow d = \frac{V}{E} = \frac{1500}{15 \times 10^{-6}} = 10^{-4} m = 0.1 mm$

$\Rightarrow C = \frac{\epsilon_0 \epsilon_r A}{d}$

$\Rightarrow A = \frac{Cd}{\epsilon_0 \epsilon_r} = \frac{3800 \times 10^{-9} \times 10^{-4}}{8.854 \times 10^{-12} \times 4.3}$

$= 0.01 m^2.$

* A parallel-plate capacitor has plates 0.15 mm apart and dielectric with relative permittivity of 3. Find the electric field ~~density~~ intensity and the voltage between plates if the surface charge is $5 \times 10^{-4} \mu C/cm^2$.

Soln

$\epsilon_r = 3$
$\epsilon_0 = 8.854 \times 10^{-12} coul^2/N$
$D = 5 \times 10^{-4} \mu C/cm^2$
$= 5 \times 10^{-6} C/m^2$

Electric intensity,

$E = \frac{D}{\epsilon_0 \epsilon_r}$

$\Rightarrow = \frac{5 \times 10^{-6}}{8.854 \times 10^{-12} \times 3}$

$= 188000 V/m.$

Calculation: From Eq. 25-9 we have

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{F/m})(115 \times 10^{-4}\ \text{m}^2)}{1.24 \times 10^{-2}\ \text{m}}$$
$$= 8.21 \times 10^{-12}\ \text{F} = 8.21\ \text{pF}. \qquad \text{(Answer)}$$

(b) What free charge appears on the plates?

Calculation: From Eq. 25-1,

$$q = C_0 V_0 = (8.21 \times 10^{-12}\ \text{F})(85.5\ \text{V})$$
$$= 7.02 \times 10^{-10}\ \text{C} = 702\ \text{pC}. \qquad \text{(Answer)}$$

Because the battery was disconnected before the slab was inserted, the free charge is unchanged.

(c) What is the electric field E_0 in the gaps between the plates and the dielectric slab?

KEY IDEA

We need to apply Gauss' law, in the form of Eq. 25-36, to Gaussian surface I in Fig. 25-17.

Calculations: That surface passes through the gap, and so it encloses *only* the free charge on the upper capacitor plate. Electric field pierces only the bottom of the Gaussian surface. Because there the area vector $d\vec{A}$ and the field vector $\vec{E}_0$ are both directed downward, the dot product in Eq. 25-36 becomes

$$\vec{E}_0 \cdot d\vec{A} = E_0\, dA \cos 0° = E_0\, dA.$$

Equation 25-36 then becomes

$$\varepsilon_0 \kappa E_0 \oint dA = q.$$

The integration now simply gives the surface area A of the plate. Thus, we obtain

$$\varepsilon_0 \kappa E_0 A = q,$$

or

$$E_0 = \frac{q}{\varepsilon_0 \kappa A}.$$

We must put $\kappa = 1$ here because Gaussian surface I does not pass through the dielectric. Thus, we have

$$E_0 = \frac{q}{\varepsilon_0 \kappa A} = \frac{7.02 \times 10^{-10}\ \text{C}}{(8.85 \times 10^{-12}\ \text{F/m})(1)(115 \times 10^{-4}\ \text{m}^2)}$$
$$= 6900\ \text{V/m} = 6.90\ \text{kV/m}. \qquad \text{(Answer)}$$

Note that the value of E_0 does not change when the slab is introduced because the amount of charge enclosed by Gaussian surface I in Fig. 25-17 does not change.

(d) What is the electric field E_1 in the dielectric slab?

KEY IDEA

Now we apply Gauss' law in the form of Eq. 25-36 to Gaussian surface II in Fig. 25-17.

Calculations: Only the free charge $-q$ is in Eq. 25-36, so

$$\varepsilon_0 \oint \kappa \vec{E}_1 \cdot d\vec{A} = -\varepsilon_0 \kappa E_1 A = -q. \qquad (25\text{-}37)$$

The first minus sign in this equation comes from the dot product $\vec{E}_1 \cdot d\vec{A}$ along the top of the Gaussian surface because now the field vector $\vec{E}_1$ is directed downward and the area vector $d\vec{A}$ (which, as always, points outward from the interior of a closed Gaussian surface) is directed upward. With 180° between the vectors, the dot product is negative. Now $\kappa = 2.61$. Thus, Eq. 25-37 gives us

$$E_1 = \frac{q}{\varepsilon_0 \kappa A} = \frac{E_0}{\kappa} = \frac{6.90\ \text{kV/m}}{2.61}$$
$$= 2.64\ \text{kV/m}. \qquad \text{(Answer)}$$

(e) What is the potential difference V between the plates after the slab has been introduced?

KEY IDEA

We find V by integrating along a straight line directly from the bottom plate to the top plate.

Calculation: Within the dielectric, the path length is b and the electric field is E_1. Within the two gaps above and below the dielectric, the total path length is $d - b$ and the electric field is E_0. Equation 25-6 then yields

$$V = \int_-^+ E\, ds = E_0(d - b) + E_1 b$$
$$= (6900\ \text{V/m})(0.0124\ \text{m} - 0.00780\ \text{m})$$
$$+ (2640\ \text{V/m})(0.00780\ \text{m})$$
$$= 52.3\ \text{V}. \qquad \text{(Answer)}$$

This is less than the original potential difference of 85.5 V.

(f) What is the capacitance with the slab in place?

KEY IDEA

The capacitance C is related to q and V via Eq. 25-1.

Calculation: Taking q from (b) and V from (e), we have

$$C = \frac{q}{V} = \frac{7.02 \times 10^{-10}\ \text{C}}{52.3\ \text{V}}$$
$$= 1.34 \times 10^{-11}\ \text{F} = 13.4\ \text{pF}. \qquad \text{(Answer)}$$

This is greater than the original capacitance of 8.21 pF.