Engineering Economics Semester Final

Made by Md. Mehedi Hasan Rafy
[updated version]

Topics:

- ***Demand and Supply:*** Definition of Demand and Supply, Law of Demand, Law of Supply, Demand Schedule, Supply Schedule, Demand and Supply Curves, Market Equilibrium, Determination of Equilibrium, Market price and quantity, Price elasticity of demand.

- ***National Income:*** GDP, GNP, Economic Growth.

- ***Inflation:*** Inflation, Measure of Inflation,

- ***Future Value:*** Present value and Future Value, One time capital, Series of Uniform Cashflow.

- ***Interest Rates:*** Simple and Compound Interest rates.

- ***Assignment:*** PV and FV calculation for:
  - One time investment
  - Uniform series cashflow
  - Non-uniform series cashflow
  - Inflation adjustment
  - Internal rate of return (IRR)

- ***Tax:*** Tax, Direct Tax and Indirect Tax, Tax adjustment in investment, Gross income and Taxable income, Net Operating Income After Tax, Cashflow After Tax, Cashflow Before Tax, Depreciation.

- **Extended:** Production, Factors of Production, Cost Benefit Analysis, Theory of Cost, Growth rate

Demand and Supply

*Definition of Demand and Supply***

- ***Demand:*** The quantity of a good or service that consumers are willing and able to buy at various prices, holding other factors constant.

- ***Supply:*** The quantity of a good or service that producers are willing and able to offer for sale at various prices holding other factors constant.

*Law of Demand**

The law of demand states that, *"There's an inverse relationship between price and quantity demanded by consumers."* Mathematically, $P \propto \frac{1}{D}$.

As the price of a good decreases, the quantity demanded increases, all else being equal, and vice versa.

*Law of Supply**

The law of supply states that, *"Price of a product is directly proportionate to its quantity supplied by producers"*. Mathematically, $P \propto S$

As the price of a good increases, the quantity supplied increases, all else being equal.

*Demand Schedule***

A table showing the quantity of a good or service that consumers are willing to buy at various prices, while other factors are constant.

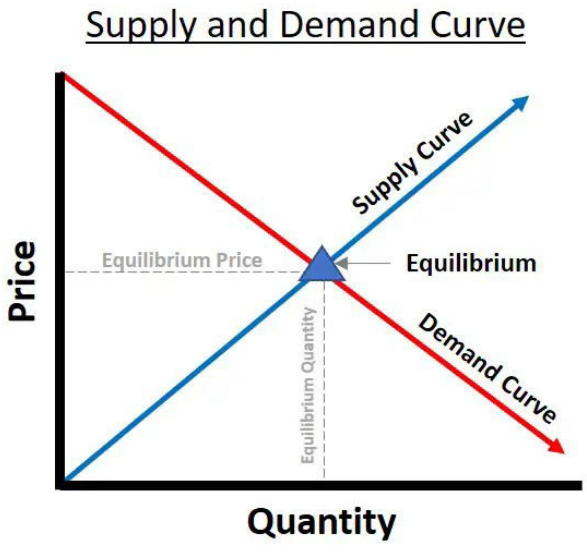
*Supply Schedule**

A table illustrating the quantity of a good or service that producers are willing to offer for sale at different prices, while other factors remain constant.

*Demand and Supply Curves*****

- ***Demand Curve:*** A graphical representation of the relationship between the price of a good and the quantity demanded by consumers.

- ***Supply Curve:*** A graphical representation of the relationship between the price of a good and the quantity supplied by producers.



Demand Curve*

$Q_d = a - bP$

Where,

Q_d = Quantity Demanded

a = Intercept of the demand curve (quantity demanded when price is 0)

b = Slope of the demand curve (change in quantity demanded per unit change in price)

P = Price

Supply Curve

$Q_S = c + dP$

Where,

Q_S = Quantity Supplied

c = Intercept of the supply curve (quantity supplied when price is 0)

d = Slope of the supply curve (change in quantity supplied per unit change in price)

P = Price

Market and Equilibrium

Market is any place where consumers and producers meet to exchange Goods and services willingly. At equilibrium,

$$Q_D = Q_S$$

Examples-01: Draw a demand curve using the demand equation $Q_d = 50 - 5p$

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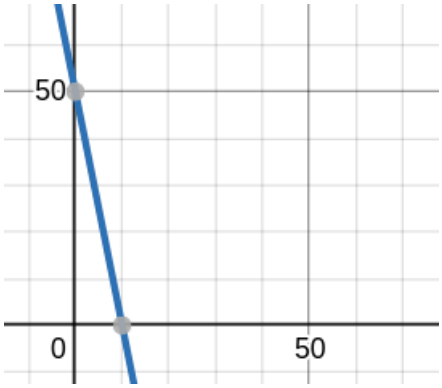
Demand schedule for different values of price(p):

Price	Quantity
0	50-5*0=50
2	50-5*2=40
5	50-5*5=25
10	50-5*10=0
12	50-5*12=-10

Putting Price(p) on the x-axis and Quantity demanded(Q_d) on the y-axis we get the following graph:



Shift in Demand and Supply Curves*

A shift in the demand and supply curves in economics refers to a change in the entire relationship between the price of a good and the quantity demanded or supplied at every price level. When a curve shifts, it means that for any given price, buyers or sellers are willing to purchase or supply a different quantity than before the shift.

One thing to remember, if demand curve shifts to the left or supply curve shifts to the right price decreases holding other factors constant.

Factors that causes shift in Demand*

- *Income*
- *Consumer preferences*
- *Population*
- *Price of goods*
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Factors that causes shift in Supply**

- *Input prices*
- *Government regulations and taxes*
- *Number of suppliers*
- *Producers choice*

Market Equilibrium**

The point at which the quantity demanded equals the quantity supplied, $Q_d = Q_s$ resulting in no excess demand or excess supply. At this point, price is called Equilibrium Price P_E and quantity is called Equilibrium Quantity Q_E .

Equilibrium in a market can be disrupted by various factors, such as changes in consumer preferences, shifts in supply, or government interventions. When equilibrium is disturbed, market forces work to restore it:

- Price Adjustment
- Market Forces
- Government Intervention
- Market Signals

Example-01: Consider a commodity $Q_d = 17 - 3p$ and $Q_s = 3 + 4p$, where Q_d , Q_s and p denotes quantity demanded, quantity supply and price respectively. Find the equilibrium price and quantity for that commodity.

At, Market equilibrium,

$$Q_d = Q_s$$
$$\rightarrow 17 - 3p = 3 + 4p$$
$$\rightarrow 7p = 14$$
$$\rightarrow p = 2.$$

So, Equilibrium price P_E is 2. Equilibrium quantity, $Q_e = Q_d = 17 - 3 \times 2 = 11.$

Example-02: Using the following demand and supply equations, find the equilibrium quantity and market price: $Q_d = 20 - 2p, Q_s = 10 + 3p.$

Solution:

At, Market equilibrium,

$$Q_d = Q_s$$
$$\rightarrow 20 - 2p = 10 + 3p$$
$$\rightarrow 5p = 10$$
$$\rightarrow p = 2.$$

So, Equilibrium price is 2.

Equilibrium quantity, $Q_e = Q_d = 20 - 2 * 2 = 16.$

Determination of Equilibrium

Equilibrium is established when the demand and supply curves intersect. At this point, the market clears and there's no pressure for prices to change.

Market Price and Quantity

The price and quantity at which the demand and supply curves intersect, establishing equilibrium in the market.

Surplus and Deficit in the Market*

- ***Surplus:*** A surplus occurs when the quantity of a good or service supplied exceeds the quantity demanded by consumers or buyers at a given price. In other words, there is an excess supply of the product in the market. Mathematically, when $Q_S > Q_E$ or $Q_D < Q_E$.
  This typically results in a buildup of unsold inventory and can lead to price reductions as sellers attempt to clear the surplus.

- ***Deficit:*** A deficit occurs when the quantity of a good or service demanded exceeds the quantity supplied at a given price. Mathematically, $Q_S < Q_E$ or $Q_D > Q_E$.
  In this case, there is not enough supply to meet consumer demand. Deficits can lead to shortages, price increases, and unmet consumer needs.

Question-01: How Surplus and Deficit Are Tackled in the Market?

Solution:

Market forces and mechanisms naturally respond to surplus and deficit situations to restore equilibrium. Here's how each situation is typically addressed:

Tackling Surplus

- ***Price Adjustment:*** In response to a surplus, sellers may lower the price of the product to encourage more consumers to purchase it. Lower prices can stimulate demand and help reduce the surplus.

- ***Promotions and Discounts:*** Businesses may run promotions, discounts, or sales events to incentivize consumers to buy more of the surplus product. These strategies aim to clear excess inventory.

- ***Exporting:*** In some cases, surplus goods may be exported to international markets where demand is higher. This can help reduce the surplus domestically.

- ***Production Reduction:*** To prevent future surpluses, producers may reduce production levels or adjust their supply to align with consumer demand.

Tackling Deficit

- ***Price Increase:*** When a deficit occurs, sellers may raise prices to reflect the increased demand. Higher prices can help balance supply and demand by reducing the number of buyers.

- ***Imports:*** To address a deficit, businesses may import the needed goods from other regions or countries where supply is more abundant. This can help meet consumer demand.

- ***Production Increase:*** Producers may respond to a deficit by increasing production to match consumer demand. This often requires investing in additional resources, labor, or production capacity.

- ***Substitute Goods:*** In some cases, consumers may turn to substitute goods or services that are readily available to satisfy their needs during a deficit.

- ***Government Intervention:*** In extreme cases or when critical goods are in deficit, governments may intervene through policies or regulations to ensure the availability of essential items to the public.

Elasticity of Demand

*Elasticity of demand**

Elasticity of Demand is a concept in economics that measures how responsive the quantity demanded of a good is to changes in its price.

Arc Elasticity:

- Arc elasticity calculates elasticity over a range or interval of prices and quantities.
- It considers the initial and final points on the demand curve to calculate the percentage change in price and quantity.
- Arc elasticity is useful when you have discrete data points or when you want to measure elasticity over a range.
- The formula for arc elasticity is: $$Arc\ Elasticity = \frac{\frac{Q_2 - Q_1}{Q_2 + Q_1}}{\frac{P_2 - P_1}{P_2 + P_1}}$$

Example of Arc Elasticity: Suppose the quantity demanded for a good decreases from 10 units to 8 units when the price increases from $5 to $7. Using arc elasticity, we would use both sets of initial and final points to calculate the elasticity over the entire range.

$$Arc\ Elasticity = \frac{\frac{Q_2 - Q_1}{Q_2 + Q_1}}{\frac{P_2 - P_1}{P_2 + P_1}} = \frac{\frac{10 - 8}{10 + 8}}{\frac{7 - 5}{7 + 5}} \approx 0.6667$$

Point Elasticity:

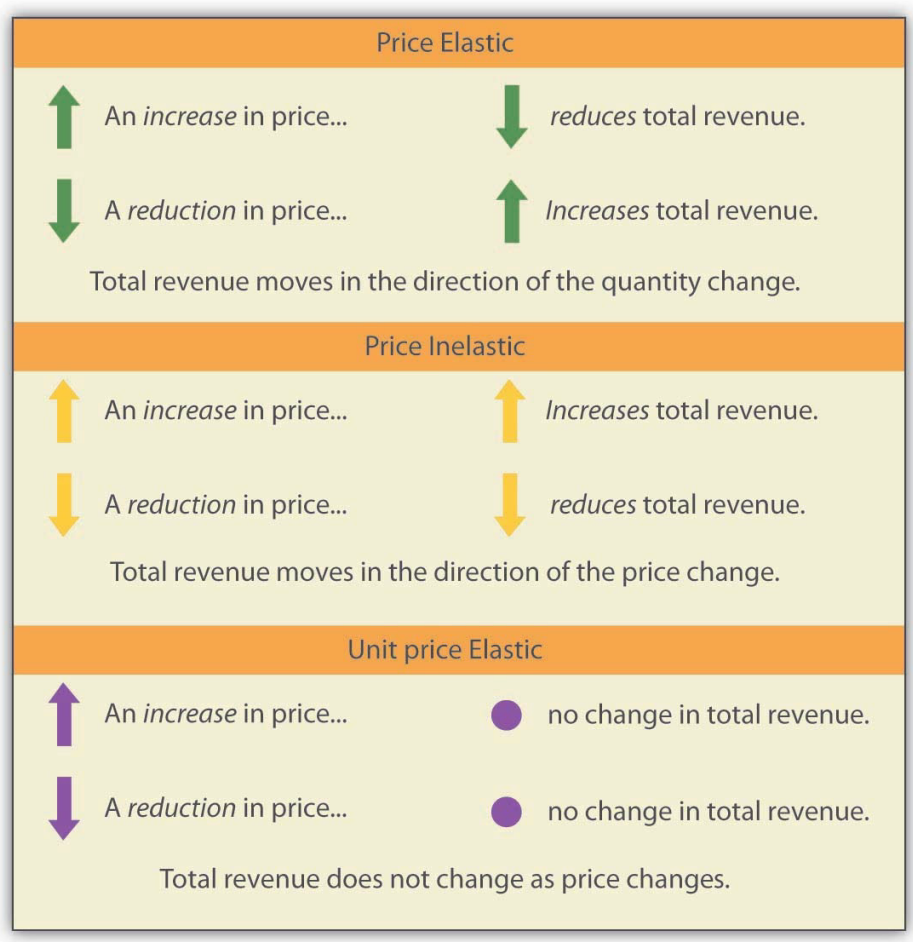
- Point elasticity calculates elasticity at a specific point on the demand curve.
- It considers the elasticity at a single point without considering a range.
- Point elasticity is used when you need to know the elasticity at a specific price-quantity combination.
- The formula for point elasticity is: $$Point\ Elasticity = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}}$$

Example of Point Elasticity: Suppose A product is sold at a price of 10 and quantity of 20 units. You want to know the elasticity of demand at a price of $5 and a quantity of 10 units. You would use point elasticity to calculate the elasticity at this specific point.

$$Point\ elasticity = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\frac{5-10}{10}}{\frac{10-20}{20}} = 1.$$

Price Elasticity of Demand (PED)**

A measure of how much the quantity demanded of a good responds to a change in its price.



Formula: $PED = \frac{\%\ change\ of\ quantity\ demanded}{\%change\ of\ price} = \frac{\frac{Q_2 - Q_1}{Q_1}}{\frac{P_2 - P_1}{P_1}}$

- If $PED < 0$, demand is **inelastic**. An increase in price leads to a fall in quantity demanded or the demand curve is downward sloping.
- If $PED = 0$ (or very small), demand is **perfectly inelastic**. Quantity demanded does not change in response to price changes.

- If $0<PED<1$, demand is **inelastic**. A percentage change in price leads to a smaller percentage change in quantity demanded.
- If $PED=1$, demand is **unit elastic**. A percentage change in price leads to an equal percentage change in quantity demanded.
- If $PED>1$, demand is **elastic.** A percentage change in price leads to a larger percentage change in quantity demanded.
- If $PED=\infty$ (or very large), demand is **perfectly elastic**. Quantity demanded is highly responsive to price changes.

Example: If the price of a product increases by 10%, and as a result, the quantity demanded decreases by 20%, the price elasticity of demand would be:
Now, $PED=\frac{\%change\in quantity}{\%change\in price}=\frac{20}{10}=2$
This indicates that the good has a elastic demand since the value of $PED>1$ indicating increase in price reduces total revenues holding other factor constant.

Cross-Price Elasticity of Demand (XED)

Cross-price elasticity of demand (XED) measures how sensitive the quantity demanded of one good is to changes in the price of another related good. It helps us understand whether two goods are substitutes or complements.

Formula: $XED=\frac{\%\,Change\ of\ Quantity\ Demanded\ of\ Good\ A}{\%\,Change\in Price\ of\ Good\ B}=\frac{\frac{A_2-A_1}{A_1}}{\frac{B_2-B_1}{B_1}}$

- If $XED>0$, the goods are **substitutes.** An increase in the price of one good leads to an increase in the quantity demanded of the other (and vice versa).
- If $XED=0$, the goods are **unrelated.** Changes in the price of one good have no effect on the quantity demanded of the other.
- If $XED<0$, the goods are **complements.** An increase in the price of one good leads to a decrease in the quantity demanded of the other (and vice versa).

Income Elasticity of Demand (YED)**

Income elasticity of demand (YED) measures how sensitive the quantity demanded of a good is to changes in consumer income. It helps us understand whether a good is a normal or inferior good and how it relates to changes in consumer incomes.

Formula: $YED = \frac{\%\ Change\ of\ Quantity\ Demanded}{\%\ Change\ of\ Income} = \frac{\frac{Q_2 - Q_1}{Q_1}}{\frac{I_2 - I_1}{I_1}}$

- If $YED > 1$ the good is a **luxury.** An increase in income leads to a proportionally larger increase in the quantity demanded.
- If $YED < 1$ but still positive, the good is a **necessity.** An increase in income leads to an increase in the quantity demanded, but it is proportionally smaller.
- If $YED < 0$ (negative), the good is an **inferior good.** An increase in income leads to a decrease in the quantity demanded.

Example-01: Discuss what are meant by price elasticity of demand, cross price elasticity of demand and income elasticity of demand.

Price elasticity of demand: A measure of how much the quantity demanded of a good responds to a change in its price.

Cross-Price Elasticity of Demand (XED): A measure of how sensitive the quantity demanded of one good is to changes in the price of another related good. It helps us understand whether two goods are substitutes or complements.

Income Elasticity of Demand (YED): A measure of how sensitive the quantity demanded of a good is to changes in consumer income. It helps us understand whether a good is a normal or inferior good and how it relates to changes in consumer incomes.

Example-02: If the demand for the commodity Y increases from 5 units to 8 units as a result of increase in income of a consumer from Tk. 50,000.00 to Tk. 65,000.00, find the income elasticity of demand and comment on the nature of the commodity.

Solution:

Income elasticity of demand(YED):

$$YED = \frac{\%\ Change\ of\ Quantity\ Demanded}{\%\ Change\ of\ Income} = \frac{\frac{Q_2 - Q_1}{Q_1}}{\frac{I_2 - I_1}{I_1}} = \frac{\frac{8-5}{5}}{\frac{65{,}000 - 50{,}000}{50{,}000}} = 60/30 = 2$$

Interpretation:

Here, YED>1, which suggests that the commodity Y is a luxury good. In this case, a 1% increase in income leads to a 2% increase in the quantity demanded of the commodity Y. Consumers consider it a non-essential or luxury item.

Example-03: Show that the price elasticity of demand ranges from zero to infinity.
Price Elasticity of Demand (PED): A measure of how much the quantity demanded of a good responds to a change in its price.

Formula: $PED = \frac{\%\ change\ of\ quantity\ demanded}{\%change\ of\ price} = \frac{\frac{Q_2 - Q_1}{Q_1}}{\frac{P_2 - P_1}{P_1}}$

- If PED = 0 (or very small), demand is perfectly inelastic. Quantity demanded does not change in response to price changes.
- If 0<PED< 1, demand is inelastic.
  A percentage change in price leads to a smaller percentage change in quantity demanded.
- If PED = 1, demand is unit elastic.
  A percentage change in price leads to an equal percentage change in quantity demanded.
- If PED > 1, demand is elastic.
  A percentage change in price leads to a larger percentage change in quantity demanded.
- If PED = ∞ (or very large), demand is perfectly elastic.
  Quantity demanded is highly responsive to price changes.

So, it can be showed that Price Elasticity of Demand ranges from zero to infinity.

National Income

National Income**

National income is a measure of the total factor incomes generated within a country's borders over a specific period, usually a year.
Factor income is incomes of factors of production.
Factors of production:

Production	Income
Land	Rent
Labor	Wages
Capital	Interest
Entrepreneur	Profit

It represents the sum of all incomes earned by residents and entities within a nation's economy during that time frame.
National income is a key indicator used in economics to assess the overall economic performance and standard of living in a country.

The three primary methods of measuring national income are:

1. Production or Value-Added Approach: This method calculates national income by summing the value added at each stage of production within an economy. It considers the value of goods and services produced and subtracts intermediate consumption (the value of goods and services used up in the production process).

Formula: $National\ Income = Gross\ Value\ of\ Output - Value\ of\ Intermediate\ Consumption$
Advantages: This approach provides a detailed view of the production process and how value is added at each stage.
It is well-suited for analyzing the contributions of different industries to the economy.
Disadvantages: It can be challenging to obtain accurate data on intermediate consumption, and double-counting can occur if not properly accounted for.

2. Income Approach: The income approach calculates national income by summing all forms of income earned by households and businesses within a country. It includes wages, salaries, profits, rents, interest, taxes, and subsidies.

Formula: National Income = Compensation of Employees + Gross Operating Surplus + Gross Mixed Income + Taxes on Production and Imports - Subsidies on Production and Imports
Advantages: This method provides a comprehensive view of income distribution and the sources of income in an economy.
Disadvantages: Data collection can be challenging, and it may not capture the underground or informal economy.

3. Expenditure Approach: The expenditure approach calculates national income by summing all expenditures within an economy. It includes consumption, investment, government spending, and net exports (exports minus imports).

Formula:

$$National\ Income = Consumption + Investment + Government\ Spending + Exports - Imports$$

Advantages: This approach provides insights into how economic activity is distributed across different sectors, such as consumer spending, business investment, and government contributions.

Disadvantages: Accurate data on consumption, investment, and government spending can be challenging to obtain. In open economies, net exports can significantly affect the calculation.

Gross Domestic Product (GDP)**

GDP is the total value of all goods and services produced within a country's borders in a given time period (usually a year). It's a comprehensive measure of a country's economic activity and is often used to gauge the overall health and size of an economy.

GDP can be calculated using ***three*** approaches: **production, income,** and **expenditure.** These should yield the same value when calculated correctly.

For example, if a country produces $500 billion worth of goods and services in a year, then its GDP for that year is $500 billion.

Formula for Real GDP Growth Rate:

$$GR = \frac{(RG_C - RG_P)}{RG_P} \times 100$$

Where,

GR = Real GDP Growth Rate

RG_C = Real GDP in Current Year

RG_P = Real GDP in Previous Year

$$Per\ Capita\ GDP = \frac{GDP}{Total\ Population}$$

Suppose Country A's GDP was $500 billion in 2020 and increased to $550 billion in 2021. The population in both years was 100 million.

- $GR = \frac{(550 - 500)}{500} \times 100 = 10\,\%$

This growth indicates a positive economic trend in Country A, with an increase in both GDP and per capita GDP.

Gross National Product (GNP)**

GNP is the total value of all goods and services produced by a country's residents, both domestically and internationally, within a specific time frame. It takes into account the income earned by a country's residents from abroad and subtracts the income earned by foreigners within the country.

GNP takes into account the earnings of a country's residents, whether they are within the country or abroad.

For example, Suppose Country B has a GNP of $1.2 trillion. This means that the total value of all goods and services produced by the citizens and businesses of Country B, whether they are located within the country or abroad, is $1.2

trillion. It also includes the income earned by Country B's citizens and businesses from their investments and operations abroad.

Key Distinction: The key distinction between GDP and GNP is in their geographic scope. GDP measures the production within a country's geographical boundaries, while GNP measures the production by a country's residents and businesses, regardless of their location.
Let's say Country C has a GDP of $800 billion and a GNP of $850 billion in a given year. This would mean that the total economic activity within Country C's borders (GDP) is $800 billion, but when you consider the income earned by Country C's citizens and businesses from their activities abroad, it amounts to $850 billion (GNP).

Economic Growth**

Economic growth refers to the increase in an economy's output of goods and services over time. It's typically measured by the percentage change in GDP from one period to another.
Economic growth signifies an increase in an economy's capacity to produce goods and services, leading to a higher standard of living for its citizens.
For example, If a country's GDP grows from $1 trillion in one year to $1.1 trillion the following year, its economic growth rate is 10% ($100 billion increase / $1 trillion initial GDP).

Importance of economic growth*

1. Higher Living Standards: Economic growth leads to an increase in the production of goods and services, resulting in higher income levels for individuals and households. This, in turn, improves living standards by providing access to better housing, healthcare, education, and a wider range of consumer goods and services.

2. Reducing Poverty: Economic growth plays a vital role in poverty reduction. By creating jobs and income opportunities, it helps lift people out of poverty. A growing economy generates employment, raises wages, and provides a pathway for individuals and families to improve their economic well-being.

3. Job Creation: Economic growth leads to the creation of new jobs across various sectors of the economy. This not only reduces unemployment but also provides individuals with the means to support themselves and their families. Job creation is a critical driver of economic stability and social well-being.

4. Investment in Human Capital: Growing economies often invest in education, skills development, and healthcare. As incomes rise, governments and businesses allocate more resources to develop human capital, leading to a more skilled and productive workforce. This, in turn, contributes to sustained economic growth.

5. Innovation and Technological Advancements: Economic growth fosters innovation and technological progress. It encourages businesses to invest in research and development, leading to advancements in various industries. Technological improvements enhance productivity, drive economic competitiveness, and improve overall quality of life.

*Economic Development**

Economic development is a broader and more multidimensional concept. It encompasses not only the quantitative aspect of economic growth but also qualitative improvements in living standards, human well-being, and the quality of life for a population.

Economic development places a strong emphasis on improving the living conditions of a country's citizens, reducing poverty, inequality, and unemployment, and achieving sustainable economic progress. It considers factors beyond GDP growth, such as education, healthcare, access to clean water, housing, and social services.

Inflation

*Inflation**

Inflation refers to the sustained increase in the general price level of goods and services in an economy over a period of time. It erodes the purchasing power of money, as each unit of currency buys fewer goods and services.

*Inflation Rate**

The inflation rate is the percentage increase in the general price level of goods and services over a specific period. It reflects the erosion of purchasing power due to rising prices. Inflation reduces the value of money, meaning that a fixed amount of money can buy fewer goods or services over time.

Inflation Rate: $IR = \frac{(C_i - P_i)}{P_i} \times 100$

Where,

IR = Inflation Rate

C_i = Price index in Current Period

P_i = Price index in Previous Period

Example-01:

Let's say the CPI for a certain year is 180 and for the previous year, it was 170. To calculate the inflation rate:

$$Inflation\ Rate = \frac{(180-170)}{170} \times 100 = 5.88\%$$

This indicates a 5.88% increase in the average price level over the specified time period.

Measures of Inflation*

- ***Consumer Price Index (CPI):***
  The CPI measures changes in the average prices paid by urban consumers for a fixed basket of goods and services. It's widely used to gauge inflation's impact on households.

- ***Producer Price Index (PPI):***
  The PPI measures average changes in the selling prices received by domestic producers for their output. It reflects price changes at the production level before goods reach consumers.

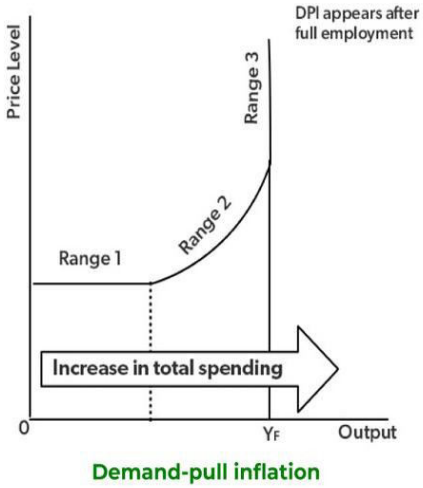
- ***GDP Deflator:***
  The GDP deflator measures changes in the overall price level for all goods and services included in the GDP. It's a broader indicator than the CPI, covering both consumption and investment goods.

- ***Personal Consumption Expenditures (PCE) Price Index:***
  Similar to the CPI, the PCE Price Index measures the average change in prices paid by consumers for goods and services. It's often considered the Federal Reserve's preferred inflation gauge.

Types of Inflation/Causes of Inflation*

- *Demand-pull Inflation*
- *Cost-push Inflation*
- *Monetary Inflation*

Demand-Pull Inflation



Demand-pull inflation

Demand-pull inflation occurs when the overall demand for goods and services in an economy exceeds their supply, leading to an increase in prices. It is driven by strong consumer and business demand, which "pulls up" prices. This type of inflation is often associated with periods of economic growth.

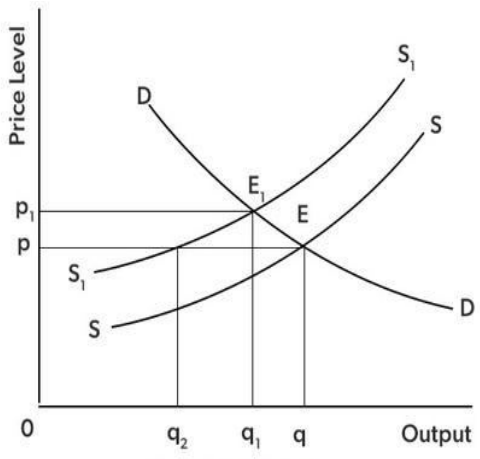
Example of Demand-Pull Inflation: Imagine an economy where consumer and business confidence is high:

- Consumer Spending: Consumers have more disposable income and are confident about the future, leading to increased spending on goods and services.
- Business Investment: Businesses are expanding their operations, investing in new equipment, and hiring more workers to meet rising demand.
- Government Spending: The government may implement infrastructure projects or increase public spending, injecting more money into the economy.

As a result of these factors, the overall demand for goods and services in the economy rises significantly. When demand outpaces supply, businesses may respond by raising prices to take advantage of the increased demand. This leads to an increase in the general price level, resulting in demand-pull inflation.

Cost-Push Inflation

Cost-push inflation occurs when the prices of production inputs (such as labor, raw materials, and energy) increase, causing businesses to raise their prices to maintain their profit margins. In this case, rising production costs "push up" prices, leading to inflation.



Cost-Push Inflation

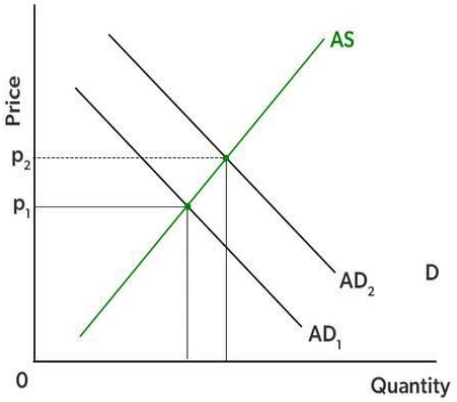
Example of Cost-Push Inflation: Consider an economy heavily dependent on imported oil:

- Oil Price Increase: The global price of oil rises significantly due to geopolitical tensions or supply disruptions in oil-producing regions.
- Production Costs: Industries and businesses that rely on oil as a key input experience higher production costs. This includes transportation, manufacturing, and agriculture.
- Passing on Costs: To maintain their profit margins, businesses pass on these higher production costs to consumers by increasing the prices of goods and services.

As a result of the oil price increase and the subsequent rise in production costs across various sectors, the general price level in the economy starts to climb. This is an example of cost-push inflation, where the increase in the cost of a critical input leads to higher prices for a wide range of goods and services.

Monetary Inflation

Monetary inflation, often simply referred to as inflation, is a sustained increase in the general price level of goods and services in an economy over time. It can be caused by factors other than just demand-pull or cost-push influences. One significant factor is the expansion of the money supply by a central bank. When the money supply grows faster than the growth of goods and services in the economy, it can lead to an increase in prices.



Example of Monetary Inflation: Suppose a country's central bank decides to increase the money supply by printing more money:

- Increased Money Supply: The central bank injects a substantial amount of new money into the economy through various mechanisms.
- Excess Money Chasing Goods: With more money in circulation, consumers and businesses have extra cash on hand. This can lead to increased demand for goods and services.

- Rising Prices: As demand increases and exceeds supply, businesses may respond by raising prices. Consumers, having more money, may be willing to pay these higher prices.

Over time, this ongoing increase in the money supply and demand for goods and services can lead to a sustained rise in the general price level, resulting in monetary inflation.

Key Differences:

- Cause: Demand-pull inflation is driven by strong demand, cost-push inflation by rising production costs, and monetary inflation by an expansion of the money supply.
- Effect on Output: Demand-pull inflation can result in increased production and economic growth, while cost-push inflation may lead to reduced output. Monetary inflation can have various effects on output, depending on how it is managed by policymakers.
- Policy Response: Central banks and policymakers may use different tools and strategies to address each type of inflation. For example, they may raise interest rates to combat demand-pull inflation, implement supply-side policies to address cost-push inflation, and use monetary policy to control monetary inflation.

Impact of inflation*

Inflation can have various effects on an economy:

1. **Purchasing Power Erosion:** Inflation reduces the value of money over time, diminishing consumers' purchasing power. This can lead to a decrease in real income and a lower standard of living, especially for fixed-income individuals.
2. **Uncertainty:** High and unpredictable inflation rates make it challenging for businesses and individuals to plan for the future. Long-term contracts become uncertain, and individuals may hesitate to save or invest.
3. **Savers and Lenders' Loss:** Inflation erodes the real value of savings and investments. Lenders receive back less real value than they lent, while borrowers benefit as they repay loans with less valuable currency.

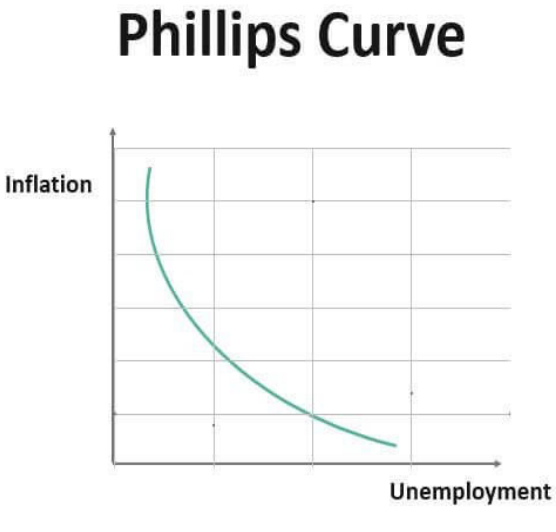
4. **Interest Rates:** Central banks often raise interest rates to combat high inflation. Higher interest rates can deter borrowing and spending, potentially slowing down economic growth.
5. **International Competitiveness:** High inflation can reduce a country's international competitiveness by raising domestic costs and prices, making exports less attractive.

Advantages of Inflation

Inflation, when moderate and well-managed, can have certain advantages for an economy. These advantages are often associated with low to moderate levels of inflation, typically within a range of 1% to 3% per year. Here are some of the potential advantages of inflation:

1. **Encouragement of Spending and Investment:** Mild inflation can encourage consumers and businesses to spend and invest rather than hoarding cash. When people expect that the value of money will decline slightly over time, they are more likely to put their money to work in the economy, supporting economic activity.
2. **Debt Relief:** Inflation can reduce the real burden of debt. If you have a fixed-rate loan, the amount you owe in nominal terms remains the same, but its real value (adjusted for inflation) decreases. This can benefit borrowers by making it easier to repay loans.
3. **Expectations of Future Growth:** Moderate inflation can signal expectations of economic growth and expansion.
4. **Revenue Generation:** For governments, inflation can lead to higher nominal tax revenues, especially if tax brackets are not fully indexed to inflation. This can provide additional funds for public spending and investment.
5. **Asset Price Appreciation:** Inflation can lead to the appreciation of asset prices, such as real estate and stocks. This can benefit investors and those who hold assets, although it can also contribute to wealth inequality.

Inflation and Unemployment



The relationship between inflation and unemployment in the short run is often explained using the Phillips Curve, which is a graphical representation of the inverse relationship between these two economic variables. The Phillips Curve suggests that, in the short run, there is a trade-off between inflation and unemployment, meaning that when one goes up, the other tends to go down, and vice versa.

Rate of Return

The rate of return is the gain or loss of an investment over a certain period of time. The rate of return is also known by the acronym RoR.

Formula: $ROR = \frac{Current\ value - Initial\ Value}{Initial\ Value} \times 100\%$

- $ROR > 0$ means Gain on investment.
- $ROR < 0$ means Loss

Annualized Rate of return: Annualized rate of return is the rate of return specifically for each year.

Formula: $Annualized\ ROR = \left(\frac{Ending\ value\ of\ Investment}{Beginning\ value\ of\ investment}\right)^{\frac{1}{Year}} - 1$

Nominal Rate of return: The rate of return given by investments before factoring in inflation, taxes, inflation fees etc.

Real Rate of return: The rate of return given by investments after factoring in inflation, taxes, inflation fees etc.

Formula: $Real\ rate\ of\ return = \frac{Nominal\ rate\ of\ return - inflation\ rate}{1 + inflation}$

Example-01: Money in a medium-risk investment makes a guaranteed 8% per year. Inflation rate has averaged 5.5% per year. What is the real rate of return on the investment?

$$Real\ Rate\ of\ Return = \frac{Nominal\ Rate\ of\ Return - Inflation\ Rate}{1 + Inflation\ Rate}$$

In this case:

- Nominal Rate of Return = 8%

- Inflation Rate = 5.5%

Plug these values into the formula:

$$Real\ Rate\ of\ Return = \frac{0.08 - 0.055}{1 + 0.055} \times 100\% = \frac{0.025}{1.055} \times 100\% \approx \frac{0.025}{1.055} \times 100\% \approx 0.0236 \times 100\% \approx 2.36\%$$

This means that your investment is expected to grow by about 2.36% in real terms, accounting for the impact of inflation.

Time Value of Money

The time value of money, which accounts for the fact that money available today is worth more than the same amount in the future due to earning potential (opportunity cost) and inflation.

Measures of Time value of Money:

- Present Value
- Future Value

Present Value (PV) and Future Value (FV)

- ***Present Value (PV)***

  PV is the value of a sum of money today, considering its worth at a future point in time, discounted at a specified rate. It's used to determine what a future cash flow is worth in terms of today's dollars.

  Formula: $PV = \frac{FV}{(1+r)^n}$

  Where,

  - FV = future value
  - PV = present value
  - r = interest rate per period
  - n = number of periods.

- ***Future Value (FV)***

  FV is the value that a current sum of money will have at a future date, taking into account the compounding effect over time.

  Formula: $FV = PV(1+r)^n$

One-Time Capital/Investment

When dealing with a single lump-sum cash flow, like an investment or loan, you can calculate its future value or present value based on a given interest rate and time period.

Series of Uniform Cash Flows

When you have a series of uniform cash flows (like an annuity), where the same amount is received or paid at regular intervals, you can calculate their future value or present value as well.

Present Value of Annuity or Uniform series of cashflow Formula:

$$PV = Pmt \times \frac{1-(1+r)^{-n}}{r}$$

Where,

Pmt = periodic payment

r = interest rate per period

n = number of periods.

Future Value of Annuity or Uniform series of cashflow Formula:

$$FV = Pmt \times \frac{(1+r)^{n}-1}{r}$$

Example: Imagine you invest $1,000 at an annual interest rate of 5% compounded annually. How much will it be worth in 5 years?

$$FV = 1000 \times (1+0.05)^{5} = \$1{,}276.28$$

Conversely, if you want to determine how much you need to invest today to have $2,000 in 3 years: $PV = \frac{2000}{(1+0.05)^{3}} \approx 1727.675$

If you have a series of annual payments of $500 for 10 years at an interest rate of 6%, the future value of this annuity would be:

$$FV = Pmt \times \frac{(1+r)^{n}-1}{r} = 500 \times \frac{(1+0.06)^{10}-1}{0.06} \approx 6590.3974$$

Assignment

1. Present Value (PV) and Future Value (FV) calculation for One time investment:

One-Time Investment

A one-time investment, also known as a lump-sum investment or single-payment investment, refers to the act of investing a fixed amount of money at a single point in time.

This is typically a one-time contribution made to an investment, savings account, or any other financial instrument.

Present Value (PV) for One time investment

Definition: Present value (PV) is a financial concept that calculates the current worth of a future sum of money, considering a specified interest rate or discount rate. It tells you how much a future cash flow is worth today.

Formula: The formula for calculating the present value (PV) of a future sum of money is:

$$PV = \frac{FV}{\left(1+\frac{r}{n}\right)^{nt}}$$

Where,

- PV = Present value
- FV = Future value
- r = Rate of interest (percentage ÷ 100)
- n = Number of times the amount is compounding per year
- t = Time in years

Examples Using Present Value Formula:

Example 1: Jonathan borrowed some amount from a bank at a rate of 7% compounded annually. If he finished paying his loan by paying $6,500 at the end of 4 years, then what is the amount of loan that he had taken? Round your answer to the nearest thousands.

Solution: The future value is, $FV = \$6500.$

The time is $t = 4$ years.

$n = 1$ (as the amount is compounded annually).

The rate of interest is, $r = 7\% = 0.07$.

Substitute all these values in the the present value formula:

$$PV = \frac{FV}{1+\left(\frac{r}{n}\right)^{nt}} = \frac{6500}{1+\left(\frac{0.07}{1}\right)^{1*4}} = \frac{6500}{1.07^4} \approx 5{,}000$$

Example-02: Mia invested some amount in a bank where her amount gets compounded daily at 5% annual interest. What is the amount invested by Mia if the amount she got after 10 years is $1,650? Round your answer to the nearest thousands.

Solution: The future value is, $FV = \$1650.$

The time, $t = 10$ years.

$n=365$ (as the amount is compounded daily).

The rate of interest is, $r=5\%=0.05$.

Substitute all these values in the present value formula:

$$PV=\frac{FV}{\left(1+\frac{r}{n}\right)^{(n\times t)}}=\frac{1650}{\left(1+\frac{0.05}{365}\right)^{(365\times 10)}}\approx 1000.8095$$

Future Value (FV) for One time investment:

Definition: Future value (FV) of a one-time investment calculates how much a present sum of money will be worth at a specified point in the future, taking into account a certain interest rate or rate of return.

Formula: The formula for calculating the future value (FV) of a one-time investment is:

$$FV=PV\left(1+\frac{r}{n}\right)^{nt}$$

Where,

- PV = Present value
- FV = Future value
- r = Rate of interest ($\frac{percentage}{100}$)
- n = Number of times the amount is compounding per year
- t = Time in years

Examples Using Future Value Formula (Compound Interest):

Example-01: David borrowed $5000 from a bank at a rate of 7% compounded annually. How much he has to pay back at the end of 4 years?

Solution:

To find: The future value of the borrowed amount after 4 years.

The present value (investment) $, PV=\$5000.$

The rate of interest, $r=7\%=7/100=0.07$.

The time in years, $t=4.$

Since the amount is compounded annually, $n=1.$

Using the future value formula of compound interest:

$$FV=PV\left(1+\frac{r}{n}\right)^{(n\times t)}=5000\left(1+\frac{0.07}{1}\right)^{(1\times 4)}\approx 6578.5152$$

Example-02: You have invested $1000 in a bank where your amount gets compounded daily at 5% annual interest. Then what is the future value of the amount you have invested for 10 years?

Solution:

To find: Future value for an investment after 10 years.

The present value (investment), $PV = \$1000.$

The rate of interest, $r = 5\% = \frac{5}{100} = 0.05.$

The time in years, $t = 10.$

Since the amount is compounded daily, $n = 365.$

Using the future value formula of compound interest:

$$FV = PV\left(1+\frac{r}{n}\right)^{n*t} = 1000\left(1+\frac{0.05}{365}\right)^{365*10} = 1000\left(1+0.05365\right)^{365*10} = \$1648.66$$

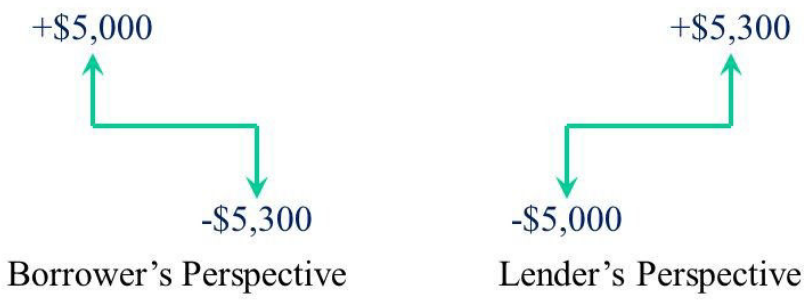
The future value = $1648.66.

2. Present Value (PV) and Future Value (FV) calculation for Uniform Series of Cash Flow:

Cashflow

Cash flow refers to the movement of money into and out of a business or individual's accounts over a specific period. It represents the inflows and outflows of cash, including revenues, expenses, investments, and financing activities.

- Consider a borrower that takes out a loan for $5,000 at 6% interest.
  From the borrower's perspective, the amount borrowed is an inflow.
  From the lender's perspective, it is an outflow.



Uniform Series of Cash Flow:

A uniform series of cash flows, often referred to as an annuity, represents a series of equal cash flows or payments occurring at regular intervals over a specified period.
These cash flows can be incoming for example income, investments or outgoing for example loan payments, lease payments.

Present Value (PV) of a Uniform Series of Cash Flow:

Definition: The present value (PV) of a uniform series of cash flow calculates the current worth of a series of future cash flows, assuming a specified discount rate. In other words, it determines the value of a stream of payments at the present time.
Formula: The formula for calculating the present value (PV) of a uniform series of cash flow is:

$$PV = Pmt \times \frac{1-(1+r)^{-n}}{r}$$

Where:

- PV = Present Value
- PMT = The uniform payment amount (cash flow)
- r = Discount rate or interest rate per period (expressed as a decimal)
- n = Number of periods

Example-01: Assume a person has the opportunity to receive an ordinary annuity that pays $50,000 per year for the next 25 years, with a 6% discount rate, or take a $650,000 lump-sum payment. Which is the better option? Using the above formula, the present value of the annuity is:

$$PV = \$50{,}000 \times \frac{1-\frac{1}{(1+0.06)^{25}}}{0.06} = \$639{,}168$$

Future Value (FV) of a Uniform Series of Cash Flow:

Definition: The future value (FV) of a uniform series of cash flow calculates the total value of a series of future cash flows at a specified point in the future, considering a certain interest rate.
Formula: The formula for calculating the future value (FV) of a uniform series of cash flow is:

$$FV = PMT \times \frac{(1+r)^{n}-1}{r}$$

Where:

- FV = Future Value
- PMT = The uniform payment amount (cash flow)
- r = Annual interest rate (expressed as a decimal)

- n = Number of periods

Example-02: Assume someone decides to invest $125,000 per year for the next five years in an annuity they expect to compound at 8% per year. In this example, the series of payments is a regular annuity in which the payments are made at the end of each period. The expected future value of this payment stream using the above formula is as follows:

$$FV = 125000 \times \frac{(1+0.08)^5 - 1}{0.08} \approx 733325.12$$

3. Present Value (PV) and Future Value (FV) calculation for Non-uniform Series of Cash Flow:

Non-Uniform Series of Cash Flow:

A non-uniform series of cash flows refers to a series of cash flows or payments that are not equal in amount and may vary at different time periods.

These cash flows can be either incoming for example variable income or outgoing for example irregular expenses.

Present Value (PV) of a Non-Uniform Series of Cash Flow:

Definition: The present value (PV) of a non-uniform series of cash flow calculates the current worth of a series of future cash flows, taking into account their varying amounts and a specified discount rate. This determines the value of a series of payments at the present time.

Formula: The formula for calculating the present value (PV) of a non-uniform series of cash flow is the summation of the present values of each individual cash flow:

$$PV = \sum_{1}^{n} \frac{CF_i}{(1+r)^{t_i}}$$

Where:

- PV = Present Value
- CF_t = Cash flow at time period t
- r = Discount rate or interest rate per period (expressed as a decimal)
- n = Total number of periods
- t = compounding per year

Future Value (FV) of a Non-Uniform Series of Cash Flow:

Definition: The future value (FV) of a non-uniform series of cash flow calculates the total value of a series of future cash flows at a specified point in the future, considering varying amounts and a certain interest rate.

Formula: The formula for calculating the future value (FV) of a non-uniform series of cash flow is the summation of the future values of each individual cash flow:

$$FV = \sum_{1}^{n} CF_i (1+r)^{t_i}$$

Where:

- FV = Future Value
- CF_t = Cash flow at time period t
- r = Annual interest rate (expressed as a decimal)
- n = Total number of periods
- t = compounding per year

Example-01: Let's consider an investment project with non-uniform cash flows over three years. The cash flows are $2,000, $3,000, and $4,000 for years 1, 2, and 3, respectively. We want to find both the present value (PV) and future value (FV) of these cash flows, assuming an annual interest rate of 8%.

Calculate the Present Value (PV):

$$PV = \frac{2000}{(1+0.08)^1} + \frac{3000}{(1+0.08)^2} + \frac{4000}{(1+0.08)^3} \approx 30747.3454$$

Calculate the Future Value (FV):

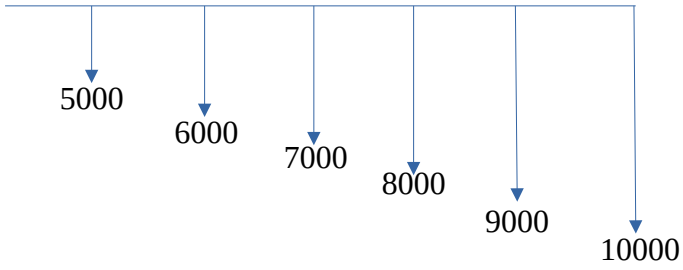
$$FV = 2000 \times (1+0.08)^3 + 3000 \times (1+0.08)^2 + 4000 \times (1+0.08)^1 \approx 10388.824$$

Example-02: A construction firm pays $5,000.00 for the first year of a maintenance service contract that increases by $1,000.00 in each subsequent year until year six. If the interest rate is 6%, calculate the present worth of the uniform annual series.

Solution:

Presenting the information on a cashflow diagram:

Year: 1 2 3 4 5 6



The present worth of a non-uniform series of cashflow can be calculated using this formula: $PV = \sum_{1}^{n} \frac{CF_i}{(1+r)^{t_i}}$

$$\rightarrow PV = \frac{5000}{(1+0.06)^1} + \frac{6000}{(1+0.06)^2} + \frac{7000}{(1+0.06)^3} + \frac{8000}{(1+0.06)^4} + \frac{9000}{(1+0.06)^5} + \frac{10000}{(1+0.06)^6} \approx 36045.9730$$

Where:

- PV = Present Value of the uniform annual series
- PMT = Payment amount in the first year (Year 1), which is $5,000
- r = Interest rate per year (6% or 0.06 as a decimal)
- n = Number of years (6 years)

4. Present Worth and Future Worth calculation for inflation adjustment:

Inflation

Inflation is the rate at which the general level of prices for goods and services rises, leading to a decrease in the purchasing power of a currency. Inflation erodes the real value of money over time, meaning that a given amount of money will have less purchasing power in the future than it does today.

Present Worth Calculation with Inflation Adjustment:

When calculating the present worth of cash flows while adjusting for inflation, we are essentially determining the current value of future cash flows, taking into account the expected rate of inflation. This allows us to account for the fact that money will have reduced purchasing power in the future due to inflation.

The formula for calculating the present worth (PW) with inflation adjustment is as follows:

$$PW = \frac{FV}{(1+r)^n}$$

Where:

- PW = Present Worth

- FV = Future Value (the amount of money in the future)
- r = Interest rate
- n = Number of periods

However, when adjusting for inflation, we need to use the real rate of return, which is the nominal rate (the rate without considering inflation) minus the inflation rate. So, the formula becomes: $PW = \frac{FV}{(1+r_{real})^n}$

where: $r_{real} = \frac{nominal\,rate - inflation\,rate}{1 + inflation\,rate}$

Example-01*:* Let's say you expect an annual inflation rate of 3% and you have $10,000 that you expect to receive in 5 years. You want to find the present worth of this amount, adjusting for inflation, using a nominal interest rate of 6%.

First, calculate the real discount rate:

$$r_{real} = 0.06 - 0.03 = 0.03$$

Now, use the present worth formula with the real discount rate:

$$PW = \frac{1000}{(1+0.03)^5} \approx 8626.0878$$

Future Worth calculation with inflation adjustment:

When calculating the future worth, we are determining the value of future cash flows at a specified point in the future, taking into account the expected rate of inflation.

The formula for calculating the future worth (FW) with inflation adjustment is as follows:

$$FW = PV(1+r_{real})^n$$

where:

- FW = Future Worth
- PV = Present Value (the current value of cash flows)
- r_{real} = Real discount rate = nominal rate - inflation rate
- n = Number of periods into the future

In this context, we are adjusting the present value (PV) for inflation by using the real discount rate r_{real} to find the value of the cash flows in terms of their purchasing power at a future date.

Example-01: Let's use the same scenario as in the previous example. You have $10,000 today, and you expect an annual inflation rate of 3%. You want to find the future worth of this amount in 5 years, adjusting for inflation, using a nominal interest rate of 6%.

First, calculate the real discount rate:

$$r_{real} = 0.06 - 0.03 = 0.03$$

Now, use the future worth formula with the real discount rate:

$$FW = 10000(1 + 0.03)^5 \approx 12762.8156$$

5. Calculation for Internal Rate of Return:

Internal Rate of Return(IRR)

The Internal Rate of Return (IRR) is a financial metric used to evaluate the potential profitability of an investment or project. It represents the discount rate at which the net present value (NPV) of the future cash flows from the investment becomes zero. In other words, the IRR is the rate of return that an investment is expected to generate over its lifetime.

Here are some key points about IRR:

- It helps in making investment decisions by comparing the expected rate of return from an investment to a required rate of return (often the cost of capital).
- A project or investment is considered economically viable if its IRR is greater than the required rate of return.
- When comparing multiple investment opportunities, the one with the higher IRR is generally considered more attractive.

Present Value (PV) Calculation for IRR:

Calculating the present value (PV) in the context of IRR is essentially finding the initial investment or the present worth of a series of future cash flows, discounted back to the present at the IRR. Here's the formula for PV when considering IRR:

$$PV = \frac{CF_0}{(1+IRR)^0} + \frac{CF_1}{(1+IRR)^1} + \ldots \frac{CF_n}{(1+IRR)^n} \ldots\ldots\ldots\ldots (1)$$

Where:

- PV = Present Value
- CF_0 , CF_1 …, CF_n = Cash flows for periods 0, 1, 2, ..., n
- IRR = The internal rate of return

This formula calculates the present value of all the cash flows associated with the investment, discounted at the IRR. If the calculated PV is positive, it indicates that the investment is expected to generate a return greater than the initial investment (a potentially good investment). If PV is negative, it suggests that the investment may not be financially viable.

Future Value (FV) Calculation for IRR:

Calculating the future value (FV) in the context of IRR helps us understand how much an initial investment or series of cash flows will be worth at a specified future point in time, assuming the IRR. The formula for FV with IRR is: $FV = CF_0(1+IRR)^n + CF_1(1+IRR)^{n-1} + \ldots CF_n(1+IRR)^0 \ldots\ldots\ldots (2)$

Where:

- FV = Future Value
- CF_0 , CF_1 ,…, Cf_n = Cash flows for periods 0, 1, 2, ..., n
- IRR = The internal rate of return
- n = Number of periods into the future

This formula calculates the future value of the cash flows at a specified point in the future, assuming the IRR as the discount rate. It provides an estimate of how the investment is expected to grow over time. Now, If all the Cash flows for the all the periods are same or its an one time investment, after merging equation (1) and equation (2) and further expanding we get,

$$IRR = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

Example-01: For an initial outlay of $10000 results in a single cash inflow of $13680 in 3 years. Determine the Internal Rate of Return(IRR).

Solution:

In this case,

PV = $10000

FV = $13680

n = 3 years

We can use the formula of internal rate of return,

$$IRR=(\frac{FV}{PV})^{\frac{1}{n}}-1=(\frac{13680}{10000})^{\frac{1}{3}}-1\approx 11\%$$

Interest Rates

Simple Interest

Simple interest is calculated only on the principal amount (the initial sum of money), and it remains constant over time. It doesn't consider the interest that accumulates on previously earned interest.

Simple Interest Formula: $SI=P\times r\times t$

Where,

SI =Simple Interest Rate

P = Principal

r = Interest rate

t = Time

Example-01: Let's say you invest $1,000 at a simple interest rate of 8% per year for 3 years. The simple interest earned would be:

SI = 1000 × 0.08 × 3 = $240

Compound Interest

Compound interest takes into account not only the principal amount but also the interest that has been earned on previous periods. This results in interest on interest, leading to exponential growth over time.

Compound Interest Formula: $FV=PV\left(1+\frac{r}{n}\right)^{nt}$

Where,

FV = Future Value

PV = Principal

r = Annual interest rate (expressed as a decimal)

n = Number of compounding periods per year

t = Number of years

Example-01: Suppose you invest $1,000 at an annual compound interest rate of 6%, compounded annually for 5 years.

$FV=1000\times 1+0.06^{5}=\$1{,}338.23$

Comparing Simple and Compound Interest

- Compound interest typically yields higher returns over time compared to simple interest, especially for longer investment horizons.
- Compound interest allows your money to grow at a faster rate due to the effect of compounding.

Nominal Interest Rate

The nominal interest rate, also known as the stated or advertised interest rate, is the rate quoted by a financial institution or lender before accounting for inflation or compounding. It is the rate used to calculate the interest payments on a loan or the interest earned on an investment. The nominal interest rate does not account for the effects of inflation or the frequency of compounding.

Effective Interest Rate (Annual Percentage Yield - APY or Annual Equivalent Rate - AER)

The effective interest rate, also known as the real interest rate, is the actual rate that takes into account the effects of compounding and, if applicable, inflation. It reflects the true cost of borrowing or the true return on an investment. The effective interest rate considers both the nominal rate and how often interest is compounded.

Example-01: Three different bank loan rates for electric generation equipment are listed below. Determine the effective rate on the basis of the compounding period for each rate.

i. 9% per year, compounded quarterly.
ii. 9% per year, compounded monthly.
iii. 4.5% per 6 months, compounded weekly.

Solution:

The graphic in Figure 4–1 indicates the effective rate per CP and how the interest rate is distributed over time.

Nominal r% per t	Compounding Period (CP)	Compounding Frequency (m)	Effective Rate per CP $\left(\frac{r}{m}\right)$	Distribution over Time Period t
(a) 9% per year	Quarter	4	2.25%	2.25% | 2.25% | 2.25% | 2.25% — 1 2 3 4 Quarter
(b) 9% per year	Month	12	0.75%	.75% .75% .75% .75% .75% .75% .75% .75% .75% .75% .75% .75% — 1 2 3 4 5 6 7 8 9 10 11 12 Month
(c) 4.5% per 6 months	Week	26	0.173%	0.173% — 1 12 14 16 26 Week

Relationship between Market, Real and Inflation Rate

- ***Market Interest Rate:*** This is the nominal interest rate that banks and financial institutions advertise.
- ***Real Interest Rate:*** This is the effective interest rate adjusted for inflation. It represents the purchasing power of your money after considering inflation. It is calculated as:

  $$\text{Real Interest Rate} = \text{Nominal Interest Rate} - \text{Inflation Rate}.$$
- ***Inflation Rate:*** This represents the rate at which the general price level of goods and services in an economy is rising. When inflation is considered, the real interest rate gives a more accurate picture of the return on an investment or the cost of borrowing because it accounts for the reduction in purchasing power due to rising prices.

The ***relationship*** can be summarized as follows:

$$\text{Market Interest Rate} = \text{Real Interest Rate} + \text{Inflation Rate}.$$

If the real interest rate is positive, it means that the investment is growing in purchasing power after accounting for inflation. If negative, it means the investment is not keeping pace with inflation.

TAX

Definition of Tax

Tax is a financial charge or levy imposed by a government on individuals, businesses, or other entities to fund public expenditures and government activities. Taxes are mandatory payments that citizens and organizations are legally obligated to pay to the government.

Taxes are used to support various public services and functions, such as healthcare, education, defense, infrastructure development, and social welfare programs.

Here are some key components and types of taxes:

1. Taxpayer: The individual or entity responsible for paying taxes is known as the taxpayer.

2. Taxing Authority: The government agency or authority responsible for collecting taxes and administering tax laws is known as the taxing authority.

3. Taxable Income: The portion of an individual's or entity's income or profits that is subject to taxation is referred to as taxable income. Deductions, exemptions, and tax credits can influence the calculation of taxable income.

4. Tax Rate: The tax rate is the percentage or fixed amount applied to taxable income to determine the tax liability. Tax rates can vary depending on the type of tax and the jurisdiction.

5. Tax Deductions: Tax deductions are expenses or items that can be subtracted from taxable income to reduce the overall tax liability. Common deductions include mortgage interest, charitable contributions, and business expenses.

6. Tax Credits: Tax credits are direct reductions in the amount of taxes owed. They are typically designed to incentivize certain behaviors or activities, such as energy-efficient home improvements or education expenses.

There are two main kind of tax citizens of a country pays. They are:

- ***Indirect Tax***
- ***Direct Tax***

Indirect Tax*

An indirect tax is a tax that is imposed on goods and services rather than on individuals or entities directly.
It is called "indirect" because the tax is typically collected by intermediaries, such as businesses, at the point of sale or production, and then passed on to the government.
Indirect taxes are usually included in the price of the goods or services and are ultimately paid by consumers as part of the purchase price.

Examples of Indirect Taxes:

1. Sales Tax: A sales tax is a common example of an indirect tax. It is imposed on the purchase of goods and services. It is typically collected by businesses from consumers at the point of sale and then remitted to the government. Sales Tax for consumers:

Formula: $\textit{Total Sales Tax} = \textit{Purchase Price} \times \textit{Sales Tax Rate}$

2. Value Added Tax (VAT): VAT is a consumption tax that is applied at each stage of production and distribution of goods and services. Businesses collect VAT on their sales, deduct the VAT they paid on purchases, and remit the difference to the government.

3. Excise Tax: Excise taxes are imposed on specific goods, such as tobacco, alcohol, gasoline, and luxury items. These taxes are typically included in the price of the product, and businesses collect and remit them.

Direct Tax*

A direct tax is a tax that is levied directly on individuals or entities based on their income, profits, or assets. Unlike indirect taxes, which are passed on to consumers, direct taxes are paid directly by the taxpayer to the government.

Examples of Direct Taxes:

1. Income Tax: This tax is imposed on an individual's or business's income, including wages, salaries, profits, and capital gains. The tax rate often varies based on income levels. Income Tax for individuals:

Formula: $Tax\ Liability = Taxable\ Income \times Tax\ Rate$

2. Property Tax: Property tax is assessed on the value of real estate properties, including homes, land, and commercial buildings. Property owners are responsible for paying property taxes. Property Tax Calculation for property owners:

Formula: $Property\ Tax = Property\ Value \times Property\ Tax\ Rate$

3. Wealth Tax: Some countries impose a wealth tax on individuals or entities based on their net wealth or assets. This tax is paid directly by the wealthy individuals or entities.

Differences Between Indirect and Direct Taxes

Components	*Indirect Tax*	*Direct Tax*
1. Taxpayer	In indirect taxes, the taxpayer is an intermediary business or retailer, which collects and remits the tax to the government.	In direct taxes, the taxpayer is the individual or entity that directly pays the tax to the government.
2. Tax Collection	Indirect taxes are collected at the point of sale or production of goods and services.	Direct taxes are collected based on income, assets, or other financial factors.
3. Visibility	Indirect taxes are often less visible to consumers because they are included in the price of goods and services.	Direct taxes, on the other hand, are more transparent as taxpayers are aware of the tax they owe.
4. Progressiveness	Indirect taxes are typically regressive, as they apply uniformly to all consumers regardless of their income.	Direct taxes can be structured to be progressive. Individuals with higher incomes pay a higher percentage of their income in taxes.

Tax Adjustment in Investment

Tax adjustment in investment refers to the consideration and calculation of the tax implications associated with an investment decision. It involves evaluating how taxes will impact the return on investment and making adjustments to financial calculations accordingly. The goal is to assess the after-tax profitability of an investment and make informed decisions that optimize returns while complying with tax regulations.

Formulas for Tax Adjustment: To incorporate tax adjustments into investment analysis, you can use the following formulas:

1. $Taxable\ Income = Total\ Income - Tax\ Deductions$
2. $Tax\ Liability = Taxable\ Income \times Tax\ Rate$
3. $After-Tax\ Cash\ Flow = Cash\ Flow - Tax\ Liability$
4. $After\ Tax\ Return\ of\ Investment = \frac{After\ Tax\ Cash\ Flow}{Initial\ Investment} \times 100\,\%$

Example-01: Suppose you are considering investing in a rental property. Here are the details:

- Initial Investment (Purchase Price): $200,000
- Annual Rental Income: $15,000
- Annual Operating Expenses (excluding taxes): $5,000
- Mortgage Interest Deduction: $7,000
- Property Depreciation Deduction: $3,000
- Marginal Tax Rate: 25%

Solution:

Step 1: Calculate Taxable Income:

$$Taxable\ Income = Total\ Income - Tax\ Deductions = \$15{,}000 - \$7{,}000 - \$3{,}000 = \$5{,}000$$

Step 2: Calculate Tax Liability:

$$Tax\ Liability = Taxable\ Income \times Tax\ Rate = \$5{,}000 \times 0.25 = \$1{,}250$$

Step 3: Calculate After-Tax Cash Flow:

$$After-Tax\ Cash\ Flow = Cash\ Flow - Tax\ Liability = \$15{,}000 - \$5{,}000 - \$1{,}250 = \$8{,}750$$

Step 4: Calculate After-Tax ROI:

$$After-Tax\ ROI = \frac{After-Tax\ Cash\ Flow}{Initial\ Investment} \times 100\,\% = \frac{\$8{,}750}{\$200{,}000} \times 100\,\% = 4.375\,\%$$

Gross Income

Gross income refers to the total income earned or received by an individual or entity before any deductions or taxes are applied. It includes all sources of income, both earned and unearned, and serves as the starting point for calculating various taxes, such as income tax.

Formula: $Gross\ Income = Income\ ¿\ Employment + Income\ ¿\ Investments + Other\ Income$

Example of Gross Income: Let's consider an individual's sources of income:

- Income from Employment: $50,000
- Income from Investments (Interest and Dividends): $5,000

- Rental Income: $7,000
- Business Income: $10,000
Using the formula for gross income:

$$Gross\ Income = \$50{,}000 + \$5{,}000 + \$7{,}000 + \$10{,}000 = \$72{,}000$$

In this example, the individual's gross income is $72,000, representing the total income earned or received from all sources.

Taxable Income

Taxable income, on the other hand, is the portion of an individual's or entity's gross income that is subject to taxation after applying various deductions, exemptions, and credits allowed by tax laws. It is the income on which taxes are calculated and paid.

Formulas: $Taxable\ Income = Gross\ Income - Deductions + Exemptions - Tax\ Credits$

Example of Taxable Income: Let's continue with the example of the individual's gross income:
- Gross Income: $72,000
- Deductions: $10,000
- Exemptions (e.g., personal and dependent exemptions): $8,000
- Tax Credits (e.g., education tax credits): $2,000
Using the formula for taxable income:

$$Taxable\ Income = \$72{,}000 - \$10{,}000 + \$8{,}000 - \$2{,}000 = \$68{,}000$$

In this example, the individual's taxable income is $68,000, which represents the income subject to taxation after accounting for deductions, exemptions, and tax credits.

Net Operating Income After Tax (NOIAT)

NOIAT is a measure of a company's or property's operating income after accounting for taxes. It reflects the income generated by operations that is available to both debt and equity investors after tax obligations have been met.

Formula: $NOIAT = NOI - Taxes$

Where:
- NOI (Net Operating Income) is the income generated from the core operations of a business or property before taxes.
- Taxes represent the total tax liability, which includes income taxes, property taxes, and other applicable taxes.

Example of NOIAT: Suppose a real estate property generates $100,000 in annual rental income and incurs $40,000 in operating expenses. The property also has a property tax liability of $5,000 and an income tax liability of $15,000.

$$NOI = \$100{,}000 - \$40{,}000 = \$60{,}000$$
$$Taxes = \$5{,}000\,(Property\ Tax) + \$15{,}000\,(Income\ Tax) = \$20{,}000$$
$$NOIAT = \$60{,}000 - \$20{,}000 = \$40{,}000$$

In this example, the Net Operating Income After Tax (NOIAT) for the property is $40,000.

Cash Flow After Tax (CFAT)

CFAT represents the actual cash flow generated by an investment or business after all taxes (including income taxes) have been accounted for. It provides a more accurate picture of the cash available for distribution to investors and for reinvestment in the business.

Formula: $CFAT = Net\ Income + Non - Cash\ Expenses - Taxes$

Where:

- Net Income is the profit or income generated by the investment or business before taxes.
- Non-Cash Expenses typically include items like depreciation, which are expenses that reduce net income but do not involve an actual cash outflow.
- Taxes represent the total tax liability.

Main elements of after tax cash flows*

The main elements of after-tax cash flows refer to the components that are considered when calculating the cash flows of a project or investment after accounting for taxes. These elements include:

- ***Revenues:*** After-tax cash flows consider the revenue generated by the project or investment. This includes sales revenue, rental income, service fees, or any other income generated from the project.
- ***Operating Expenses:*** Operating expenses are the costs incurred in running the project or investment. These expenses are deducted from the revenues to determine the operating profit or income before taxes. Examples of operating expenses include raw materials, labor costs, rent, utilities, maintenance expenses, marketing expenses, and administrative costs.
- ***Depreciation and Amortization:*** Depreciation and amortization expenses represent the allocation of the cost of assets over their useful life. These expenses are tax-deductible and are subtracted from the operating profit to calculate taxable income. Depreciation refers to the gradual reduction in value of tangible assets (e.g., machinery, buildings), while amortization refers to the gradual reduction in value of intangible assets (e.g., patents, copyrights).
- ***Interest Expenses:*** Interest expenses represent the costs of borrowing funds for the project or investment. These expenses are deductible for tax purposes and are subtracted from the taxable income. Interest expenses may include loan interest, bond interest, or any other interest payments related to the financing of the project.

- ***Taxation:*** After-tax cash flows take into account the taxes payable on the taxable income of the project. The tax rate and tax laws applicable to the project determine the amount of taxes owed. Taxes are typically calculated based on the taxable income after deducting allowable expenses and applying applicable tax rates.
- ***Salvage Value:*** The salvage value represents the estimated residual value of the project or investment at the end of its useful life. It is the amount that can be realized from the sale or disposal of the project's assets. The salvage value is considered in the calculation of after-tax cash flows as it affects the net cash flow at the end of the project's life.

Example of CFAT: Suppose a small business has a net income of $80,000, incurs $10,000 in depreciation expenses, and has a total tax liability (including income tax) of $20,000.

$$CFAT = \$80{,}000\,(Net\ Income) + \$10{,}000\,(Depreciation) - \$20{,}000\,(Income\ Tax) = \$70{,}000$$

In this example, the Cash Flow After Tax (CFAT) for the business is $70,000.

Cash Flow Before Tax (CFBT)

CFBT is the cash flow generated by an investment or business before accounting for taxes. It represents the pre-tax cash flow and is often used to evaluate the financial performance of an investment without considering the tax implications.

Formula: $CFBT = Net\ Income + Non-Cash\ Expenses$

Where:

- Net Income is the profit or income generated by the investment or business before taxes.
- Non-Cash Expenses typically include items like depreciation, which are expenses that reduce net income but do not involve an actual cash outflow.

Example of CFBT: Using the same small business as in the previous example, with a net income of $80,000 and depreciation expenses of $10,000:

$$CFBT = \$80{,}000\,(Net\ Income) + \$10{,}000\,(Depreciation) = \$90{,}000$$

In this example, the Cash Flow Before Tax (CFBT) for the business is $90,000.

Depreciation

Depreciation is an accounting method used to allocate the cost of a tangible asset over its useful life. It represents the reduction in the value of an asset due to wear and tear, obsolescence, or other factors. Depreciation is an important concept in accounting and financial reporting because it allows businesses to match the cost of an asset with the revenue it generates over time, providing a more accurate picture of financial performance.

Key Points:

- Depreciation applies to tangible assets such as buildings, machinery, vehicles, and equipment, but not to intangible assets like patents or trademarks.
- Depreciation is a non-cash expense, meaning it represents a reduction in the asset's book value without any actual cash outflow.
- It is used for both financial reporting (income statement) and tax purposes (tax deduction).

Example-01: A medium-sized profitable corporation may buy a $15,000 used pickup truck for use by the shipping and receiving department. During the truck's 5-year useful life, it is estimated the firms will save $4000 per year after all the costs of owning and operating the truck have been paid. Truck salvage value is estimated at $4500.
i. What is the before-tax rate of return?
ii. What is the after-tax rate of return on this capital expenditure? Assume straight-line depreciation.

I. $$Depreciation = \frac{Initial\,cost - Salvage\,value}{Useful\,life} = \frac{\$15{,}000 - \$4{,}500}{5} = \$2{,}100\ per\ year$$

where,

- Initial cost = $15,000
- Salvage value = $4,500
- Useful life = 5 years

$$The\ before-tax\ rate\ of\ return = \frac{Annual\ savings - Depreciation}{Initial\,cost} \times 100\,\%$$

$$\rightarrow The\ before-tax\ rate\ of\ return = \frac{4000 - 2100}{15000} \times 100\,\% = 12.67\,\%$$

Where:

- Annual savings = $4,000
- Depriciation = $2100

$$II.\ The\ after-tax\ rate\ of\ return = \frac{(Annual\ savings - Depreciation)(1 - Tax\ rate)}{Initial\,cost} \times 100\,\%$$

$$\rightarrow The\ after-tax\ rate\ of\ return = \frac{(4000 - 2100)(1 - 0.35)}{1500} \times 100\,\% = 8.23\,\%$$

Where:

- Annual savings = $4,000
- Tax rate = 35%

Production

Production refers to the process of transforming inputs, or factors of production, into outputs, which are goods and services that satisfy human wants and needs.

Factors of Production

There are typically ***four primary factors*** of production:

a. ***Land***: This includes all natural resources used in production, such as land itself, minerals, water, and forests.

b. ***Labor***: The human effort and skill used in the production process.

c. ***Capital***: Refers to the physical and human-made resources used in production, such as machinery, tools, and infrastructure.

d. ***Entrepreneurship***: The innovation, organization, and risk-taking abilities of individuals who combine the other factors of production to create goods and services.

Production Function

The production function represents the relationship between inputs and outputs. It shows how different combinations of inputs result in different levels of output. Mathematically, a production function can be represented as follows:

Q = f(L, K, M, T)

Where:

- Q = Quantity of output
- L = Quantity of labor
- K = Quantity of capital
- M = Quantity of land
- T = Technological level or efficiency

Law of Diminishing Marginal Returns

This law states that as more units of a variable input (e.g., labor) are added to a fixed quantity of other inputs (e.g., capital and land), the marginal product of the variable input will eventually decrease. In other words, there's a point where adding more of a particular input leads to smaller increases in output.

Total, Marginal, and Average Product

- ***Total Product (TP)***: The total quantity of output produced.

- ***Marginal Product (MP)***: The additional output produced by using one more unit of a particular input.
- ***Average Product (AP)***: The output per unit of a particular input.

Mathematically, these can be expressed as:

$TP = f(L, K, M, T)$

$MP = \frac{\Delta Q}{\Delta L}$ (change in output / change in labor)

$AP = \frac{TP}{L}$

Short Run vs. Long Run

In the short run, at least one factor of production is fixed (usually capital), and only variable inputs (like labor) can be adjusted. In the long run, all factors of production are variable and can be adjusted.

Cost-Benefit Analysis (CBA)

Cost-Benefit Analysis is a systematic approach used to evaluate the costs and benefits of a proposed project, policy, or decision.

It aims to determine whether the benefits of a particular action outweigh the costs, thereby assisting decision-makers in making informed choices.

Components of CBA

1. ***Costs***:
   - ***Explicit Costs***: Direct monetary expenses involved in the project, such as labor, materials, and equipment.
   - ***Implicit Costs***: Indirect costs, often opportunity costs, related to the resources used in the project.
2. ***Benefits***:
   - ***Direct Benefits***: Measurable gains resulting directly from the project, such as increased revenue or saved costs.
   - ***Indirect Benefits***: Non-monetary gains that might not be directly measurable, like improved quality of life or environmental benefits.

Methodology

1. ***Identify Project Scope***: Clearly define the project, including its goals, objectives, and expected outcomes.
2. ***Determine Costs and Benefits***: List all costs and benefits associated with the project over its lifecycle.

3. ***Monetize***: Convert non-monetary benefits into monetary terms for comparability.
4. ***Discounting***: Apply a discount rate to future costs and benefits to account for time preference and opportunity cost.
5. ***Calculate Net Present Value (NPV)***:

- $NPV = \Sigma \frac{Benefits - Costs}{(1+r)^t}$
- $NPV = \frac{R_t}{(1+t)^t}$

Where:

- R_t =net cashflow over time.
- r = Discount rate
- t = Time period

6. ***Sensitivity Analysis***: Assess the impact of changing variables (like discount rate or project lifespan) on the results.

Example:
Suppose you're considering a project to build a new public park. The project will cost $1 million upfront and generate annual benefits of $300,000 (including both direct and indirect benefits) for 10 years. The discount rate is 5%.

Theory of Cost

The theory of cost is a fundamental concept that examines how firms produce goods and services, the factors that influence production costs, and the relationship between costs and output levels.

Cost Categories

1. ***Fixed Costs (FC)***:
Fixed costs are expenses that do not change with changes in the level of production or output. They remain constant regardless of how much a firm produces. Examples include rent, salaries of permanent employees, and certain administrative costs.

2. ***Variable Costs (VC)***:
Variable costs change in direct proportion to changes in production levels. They rise as production increases and decrease as production decreases. Examples

include raw materials, wages of hourly workers, and electricity for manufacturing processes.

3. ***Total Costs (TC)***:

Total costs are the sum of fixed and variable costs. Mathematically,

$$TC = FC + VC$$

4. ***Average Costs***:

- ***Average Fixed Costs (AFC)***: AFC is calculated by dividing total fixed costs by the level of output. As output increases, AFC decreases because fixed costs are spread over a larger quantity of goods.
- ***Average Variable Costs (AVC)***: AVC is calculated by dividing total variable costs by the level of output. It represents the variable cost per unit of output.
- ***Average Total Costs (ATC)***: ATC is calculated by dividing total costs by the level of output. Mathematically: $ATC = AFC + AVC$

5. ***Marginal Cost (MC)***:

Marginal cost is the additional cost incurred by producing one more unit of output. It is calculated by the change in total cost divided by the change in quantity. Mathematically: $MC = \frac{\Delta TC}{\Delta Q}$

Short-Run and Long-Run Costs

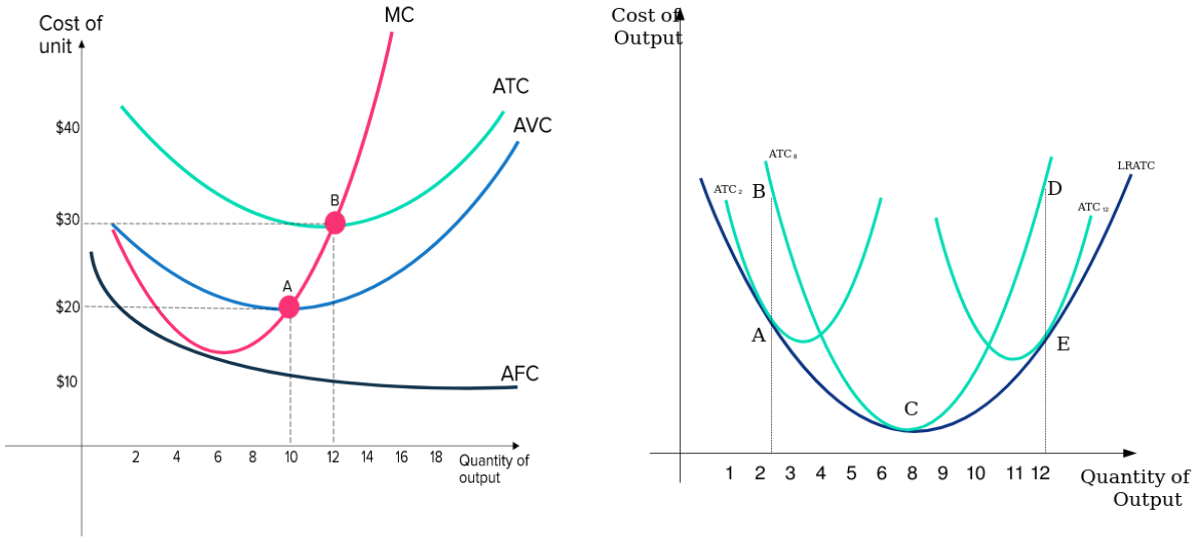
In the short run, firms may have fixed factors of production (like capital) that cannot be easily adjusted. Therefore, they can change output by varying variable factors (like labor).

In the long run, all factors of production are variable, and firms can adjust both fixed and variable inputs.

Relationships and Graphical Representation

- AFC decreases as output increases since the fixed costs are spread over more units.
- AVC usually decreases at lower levels of output due to increasing specialization and better utilization of resources, then it starts to increase as diminishing marginal returns set in.
- ATC often has a U-shaped curve, reflecting the combined effect of AFC and AVC.

Here's a simple graphical representation of these cost curves:



Example:

Let's consider a toy manufacturer with the following cost information:

- Fixed Costs (FC): $10,000
- Variable Costs (VC): $5 per unit
- Quantity (Q): 100 units

1. Calculate Total Costs (TC):

$$TC = FC + (VC\ per\ unit \times Q) = \$10{,}000 + (\$5 \times 100) = \$10{,}500$$

2. Calculate Average Variable Costs (AVC):

$$AVC = \frac{VC}{Q} = \frac{\$5}{100} = \$0.05\ per\ unit$$

3. Calculate Average Total Costs (ATC):

$$ATC = \frac{TC}{Q} = \frac{\$10{,}500}{100} = \$105\ per\ unit$$

4. Calculate Marginal Cost (MC): If producing 101 units costs $10,520, then:

$$MC = \frac{\Delta TC}{\Delta Q} = \frac{\$10{,}520 - \$10{,}500}{101 - 100} = \$20$$

Growth Rate

Growth rates are a fundamental concept used to measure the change in a variable over time, often in terms of percentage change. They are commonly

used to analyze economic indicators, such as GDP, population, and various economic variables, to understand how they change from one period to another.

Formula for Calculating Growth Rate

$$Growth\,Rate = \frac{Final\,Value - Initial\,Value}{Initial\,Value} \times 100\,\%$$

Where:

- ***Final Value***: The value of the variable at the end of the time period.
- ***Initial Value***: The value of the variable at the beginning of the time period.

The growth rate is usually expressed as a percentage. A positive growth rate indicates an increase in the variable, while a negative growth rate indicates a decrease.

Example:

Let's say you want to calculate the annual growth rate of a country's GDP over a five-year period. The GDP values for each year are as follows:

- Year 1 GDP: $1,000 billion
- Year 2 GDP: $1,200 billion
- Year 3 GDP: $1,350 billion
- Year 4 GDP: $1,500 billion
- Year 5 GDP: $1,800 billion

To calculate the growth rate between Year 1 and Year 2:

$$Growth\,Rate = \frac{\$1{,}200\,billion - \$1{,}000\,billion}{\$1{,}000\,billion} \times 100\,\% = (0.2) \times 100 = 20\,\%$$

Interpreting Growth Rates

Interpreting growth rates depends on the context of the variable being measured. Positive growth rates typically indicate economic expansion, while negative growth rates suggest contraction. High growth rates are often associated with robust economic performance, while low or negative growth rates can indicate economic challenges.

Compound Annual Growth Rate (CAGR)

For variables that change continuously over multiple periods, the compound annual growth rate (CAGR) is often used. CAGR calculates the average annual

growth rate that, if applied over the entire time period, would result in the same final value. $CAGR = \left(\frac{Ending\ Value}{Beginning\ Value}\right)^{\frac{1}{Number\ of\ Years}} - 1$

CAGR is useful for analyzing long-term trends and investments.

Importance of Growth Rates

Growth rates are critical for assessing economic performance, making investment decisions, and formulating economic policies. They provide insights into the pace of economic development, changes in living standards, and potential future trends.

Economists and policymakers analyze growth rates to understand the health of an economy and identify areas for improvement. Businesses use growth rates to assess market trends and make strategic decisions. Investors use growth rates to evaluate potential returns on investments.