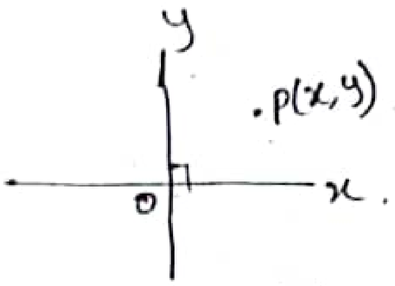
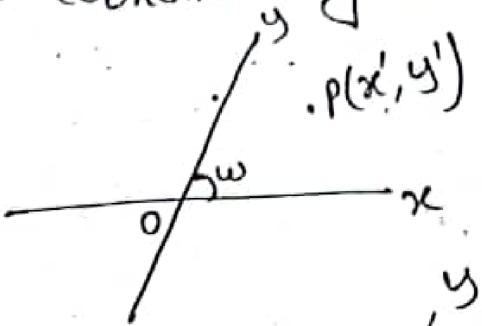
7.

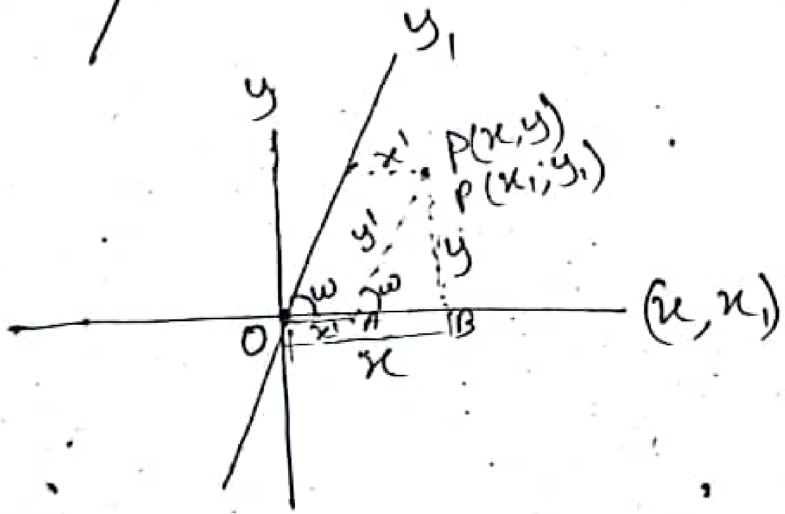
a)

Rectangular co-ordinate system:



Oblique coordinate system:





Now,

$$\sin\omega = \frac{y}{y_1}$$

$\Rightarrow y_1 = y\,\mathrm{cosec}\,\omega$ or $y = y_1 \sin\omega$

again,

$x = x_1' + AB$

$= x_1' + y_1 \cos\omega$

$\cos\omega = \frac{AB}{y'}$

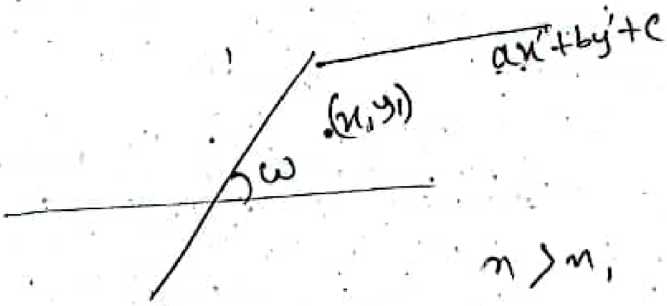
$\Rightarrow AB = y_1 \cos\omega$

The relation between rectangular and ~~oo~~ oblique cordinate System is,

$$x = x_1 + y_1 \cos\omega$$
$$y = y_1 \sin\omega$$

where, (x, y) is the point in rectangular system on the other hand (x_1, y_1) point is the same point in the oblique co-ordinate system representation.

\# given,
a point (x_1, y_1) and a line $ax+by+c=0$ and the angle between the axes is ω.



$n > n_1$

$n_1 + \varsigma = n$

we know,

$$x = x_1 + y_1 \cos\omega$$
$$y = y_1 \sin\omega$$ [from previous]

$$x_1 = \underline{x - y\cot\omega}$$
$$y_1 = \underline{y\,\mathrm{cosec}\,\omega}$$

The equation of the line in rectangular co-ordinate System is,

$$a(x - y\cot w) + b(y\operatorname{cosec} w) + c = 0$$

$$\Rightarrow ax - ay\cot w + by\operatorname{cosec} w + c = 0$$

$$\Rightarrow ax + y(b\operatorname{cosec} w - a\cot w) + c = 0$$

Now, the point (x_1, y_1) in rectangular representation is (x, y) or $(x_1 + y_1\cos w,\ y_1\sin w)$

We Know the perpendicular distance between a point (x_1, y_1) and a line $ax + by + c = 0$ is,

$$\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$$

$$\therefore \text{the distance} = \frac{a(x_1 + y_1\cos w) + y_1\sin w(b\operatorname{cosec} w - a\cot w) + c}{\sqrt{a^2 + (b\operatorname{cosec} w - a\cot w)^2}}$$

b) Let, two equations are,

$lx + my + n = 0$ — (i)

$lx + my + n_1 = 0$ — (ii)

$l^2x^2 + lmxy + ln_1x + lmxy + m^2y^2 + mn_1y + lnx + mny + nn_1 = 0$

$\therefore$ $a = l^2$

$h = lm$

$b = m^2$

$g = \frac{ln_1 + ln}{2}$

$f = \frac{mn + mn_1}{2}$

$nn_1 = 0$

Let (x_1, y_1) be the point in equation (1)

$\therefore$ the distance of (x_1, y_1) and the line (2) is

$$\frac{lx_1 + my_1 + n_1}{\sqrt{l^2 + m^2}}$$

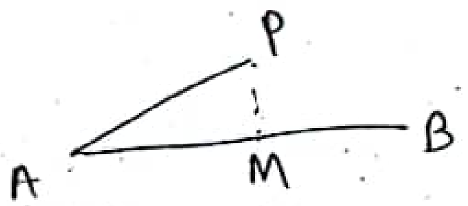
$$= \frac{n_1 - n}{\sqrt{l^2 + m^2}}$$

$$= \frac{\sqrt{(n_1 + n)^2 - 4n_1n_2}}{\sqrt{l^2 + m^2}} = \pm \frac{\sqrt{\left(\frac{2g}{l}\right)^2 - 4 \times 0}}{\sqrt{a+b}} = \pm \frac{\sqrt{\frac{4g^2}{a}}}{\sqrt{a+b}} = \pm 2g\sqrt{\frac{1/a}{a+b}}$$

$$= \frac{\pm 2g}{\sqrt{a^2 + ab}}$$

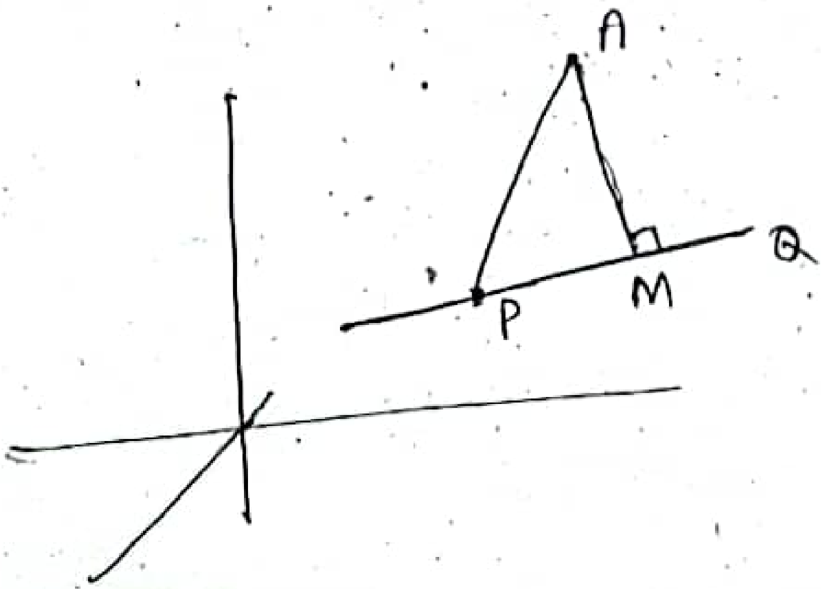
2.

a) The projection of a line segment on another line is like the shadow you see when light falls on an object. It helps ~~are~~ us understand where the segment would hit the other line if it were extended in a straight line.



here, AM is the projection of AP on AB line.

#



Since PQ line makes equal angle with the axes so the direction cosines of PQ line are,

$$l = \frac{1}{\sqrt{3}},\ m = \frac{1}{\sqrt{3}},\ n = \frac{1}{\sqrt{3}}$$

and, PM = The projection of the line AP on PQ line.

$$= \left|(x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n\right|$$

$$= \left|\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\right|$$

$$= \frac{2}{\sqrt{3}}$$

We know,

$$AM = \sqrt{AP^2 - PM^2}$$

$$AP = \sqrt{1+1+4}$$

$$= \sqrt{6}$$

$$\therefore AM = \sqrt{6 - \frac{4}{3}}$$

$$= \sqrt{\frac{14}{3}}$$

b)

Here $OP = \sqrt{h^2+k^2+l^2} = p$ let, $x' = h$, $y' = k$, $z' = l$

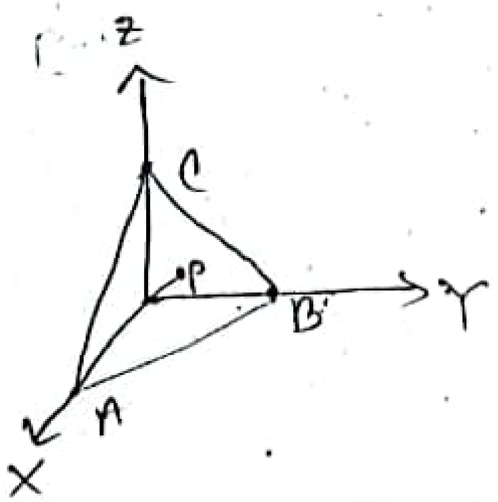
$\therefore$ DRS of OP are

$$\frac{h}{\sqrt{h^2+k^2+l^2}}, \frac{k}{\sqrt{h^2+k^2+l^2}}, \frac{l}{\sqrt{h^2+k^2+l^2}} \text{ or } \frac{h}{p}, \frac{k}{p}, \frac{l}{p}$$

Since OP is normal to the point plane, therefore equation of plane is

$$\frac{h}{p}x + \frac{h}{p}y + \frac{l}{p}z = p \quad \text{or} \quad hx+ky+lz = p^2$$

$$\therefore A \equiv \left(\frac{p^2}{h}, 0, 0\right), B \equiv \left(0, \frac{p^2}{k}, 0\right), C \equiv \left(0, 0, \frac{p^2}{l}\right)$$



Now, $\Delta ABC = \Delta = \sqrt{A_{xy}^2 + A_{yz}^2 + A_{zx}^2}$

where, A_{xy}^2 is area of projection of ΔABC on xy plane = area of ΔAOB

Now $A_{xy} = \frac{1}{2}\begin{vmatrix} p^2/h & 0 & 1 \\ 0 & p^2/k & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{p^4}{2|hk|}$

Similarly $A_{yz} = \frac{p^4}{2|kl|}$ and $A_{zx} = \frac{p^4}{2|lh|}$

$\therefore \Delta = \sqrt{A_{xy}^2 + A_{yz}^2 + A_{zx}^2} = \frac{p^5}{2hkl}$

Now replace $p = r$ and h, k, l with x_1, y_1, z_1 respectively

$\Delta = \frac{r^5}{2x_1y_1z_1}$ (showed)

3.

a) Let the equation of the variable plane be $lx + my + nz = p$

Here l, m, n vary but p is constant as the plane is always at constant distance from the origin

l, m, n are the actual direction cosines of the plane.

It cuts the axis of x at $A(\frac{p}{l}, 0, 0)$ axis of y at $B(0, p/m, 0)$ and axis of z at $C(0, 0, p/n)$

Now the planes through A, B, C are parallel to the co-ordinate planes, then the co-ordinates of point of intersection $(p/l, p/m; p/n)$,

$\therefore x = p/l, y = p/m, z = p/n$

Therefore, $\frac{p^2}{x^2} + \frac{p^2}{y^2} + \frac{p^2}{z^2} = l^2 + m^2 + n^2 = 1$

or $x^{-2} + y^{-2} + z^{-2} = p^{-2}$

b) Let l, m, n be the direction cosines of the line LM of shortest distance. As it is perpendicular two lines we have

$3l - m + 3n = 0$ and $-3l + 2m + 4n = 0$

$$\frac{l}{-9-6} = \frac{m}{-9-12} = \frac{n}{6-3}$$

$$\frac{l}{-10} = \frac{m}{-21} = \frac{n}{3} = \frac{1}{\sqrt{253}}$$

$$\therefore l = -\frac{10}{\sqrt{253}}, \quad m = -\frac{21}{\sqrt{253}}, \quad n = \frac{3}{\sqrt{253}}$$

The magnitude of the shortage distance in the projection of the joint of the pts $(3, 8, 3)$ and $(-3, -7, 6)$ on

ML and is $= (-3-3)\cdot\frac{-10}{\sqrt{253}} + (-7-8)\frac{-21}{\sqrt{253}} + (6-3)\frac{3}{\sqrt{253}}$

$= 24.1418 7316$

Again the equation of the plane containing the first of the two given lines and the line of the shortage distance is

$$\begin{vmatrix} x-3 & y-8 & z-3 \\ 3 & -1 & 3 \\ -10 & -21 & 3 \end{vmatrix} = 0$$

Also equation of the plane containing the second line and the S.D is

$$\begin{vmatrix} x+3 & y+7 & z-6 \\ -3 & 2 & 4 \\ -10 & -21 & 3 \end{vmatrix} = 0$$