Quine-McCluskey (Tabular) Minimization

- Two step process utilizing tabular listings to:
  - Identify prime implicants (implicant tables)
  - Identify minimal PI set (cover tables)
- All work is done in tabular form
  - Number of variables is not a limitation
  - Basis for many computer implementations
  - Don’t cares are easily handled
- Proper organization and term identification are key factors for correct results

Quine-McCluskey Minimization (cont.)

- Terms are initially listed one per line in groups
  - Each group contains terms with the same number of true and complemented variables
  - Terms are listed in numerical order within group
- Terms and implicants are identified using one of three common notations
  - full variable form
  - cellular form
  - 1,0,- form

Notation Forms

- Full variable form - variables and complements in algebraic form
  - hard to identify when adjacency applies
  - very easy to make mistakes
- Cellular form - terms are identified by their decimal index value
  - Easy to tell when adjacency applies; indexes must differ by power of two (one bit)
  - Implicants identified by term nos. separated by comma; differing bit pos. in () following terms

Notation Forms (cont.)

- 1,0,- form - terms are identified by their binary index value
  - Easier to translate to/from full variable form
  - Easy to identify when adjacency applies, one bit is different
  - - shows variable(s) dropped when adjacency is used
- Different forms may be mixed during the minimization

Example of Different Notations

$$F(A, B, C, D) = \Sigma\, m(4,5,6,8,10,13)$$

	Full variable	Cellular	1,0,-
1	$\bar{A}B\bar{C}\bar{D}$	4	0100
	$A\bar{B}\bar{C}\bar{D}$	8	1000
2	$\bar{A}B\bar{C}D$	5	0101
	$\bar{A}BC\bar{D}$	6	0110
	$A\bar{B}C\bar{D}$	10	1010
3	$AB\bar{C}D$	13	1101

Implication Table (1,0,-)

- Quine-McCluskey Method
  - Tabular method to systematically find all prime implicants
  - $f(A,B,C,D) = \Sigma\, m(4,5,6,8,9,10,13) + \Sigma\, d(0,7,15)$
  - Part 1: Find all prime implicants
  - Step 1: Fill Column 1 with active-set and DC-set minterm indices. Group by number of true variables (# of 1's).

NOTE: DCs <u>are</u> included in this step!

Implication Table		
Column I		
0000		
0100		
1000		
0101		
0110		
1001		
1010		
0111		
1101		
1111		

Implication Table (cellular)

- Quine-McCluskey Method
  - Tabular method to systematically find all prime implicants
  - $f(A,B,C,D) = \Sigma\, m(4,5,6,8,9,10,13) + \Sigma\, d(0,7,15)$
  - Part 1: Find all prime implicants
  - Step 1: Fill Column 1 with active-set and DC-set minterm indices. Group by number of true variables (# of 1's).

NOTE: DCs are included in this step!

Implication Table		
Column I		
0		
4		
8		
5		
6		
9		
10		
7		
13		
15		

Minimization - First Pass (1,0,-)

- Quine-McCluskey Method
  - Tabular method to systematically find all prime implicants
  - $f(A,B,C,D) = \Sigma\ m(4,5,6,8,9,10,13) + \Sigma\ d(0,7,15)$
  - Part 1: Find all prime implicants
  - Step 2: Apply Adjacency - Compare elements of group with N 1's against those with N+1 1's. One bit difference implies adjacent. Eliminate variable and place in next column.

    E.g., 0000 vs. 0100 yields 0-00

    0000 vs. 1000 yields -000

    When used in a combination, mark with a check. If cannot be combined, mark with a star. These are the prime implicants.

    Repeat until nothing left.

Implication Table

Column I	Column II	
0000 ✓	0-00 -000	
0100 ✓ 1000 ✓	010- 01-0 100- 10-0	
0101 ✓ 0110 ✓ 1001 ✓ 1010 ✓	01-1 -101 011- 1-01	
0111 ✓ 1101 ✓	-111 11-1	
1111 ✓		

Minimization - First Pass (cellular)

- Quine-McCluskey Method
  - Tabular method to systematically find all prime implicants
  - $f(A,B,C,D) = \Sigma\, m(4,5,6,8,9,10,13) + \Sigma\, d(0,7,15)$
  - Part 1: Find all prime implicants
  - Step 2: Apply Adjacency - Compare elements of group with N 1's against those with N+1 1's. 2^n difference implies adjacent. Next col is numbers with diff in parentheses.

    E.g., 0 vs. 4 yields 0,4(4)

    5 vs. 7 yields 5,7(2)

    When used in a combination, mark with a check. If cannot be combined, mark with a star. These are the prime implicants.

    Repeat until nothing left.

Implication Table

Column I	Column II	
0 ✓	0,4(4) 0,8(8)	
4 ✓ 8 ✓	4,5(1) 4,6(2) 8,9(1) 8,10(2)	
5 ✓ 6 ✓ 9 ✓ 10 ✓	5,7(2) 5,13(8) 6,7(1) 9,13(4)	
7 ✓ 13 ✓	7,15(8) 13,15(2)	
15 ✓		

Minimization - Second Pass (1,0,-)

- Quine-McCluskey Method
  - Step 2 cont.: Apply Adjacency - Compare elements of group with N 1's against those with N+1 1's. One bit difference implies adjacent. Eliminate variable and place in next column.

    E.g., 0000 vs. 0100 yields 0-00

    0000 vs. 1000 yields -000

    When used in a combination, mark with a check. If cannot be combined, mark with a star. These are the prime implicants.

    Repeat until nothing left.

Implication Table

Column I	Column II	Column III
0000 ✓	0-00 *	01-- *
	-000 *	
0100 ✓		-1-1 *
1000 ✓	010- ✓	
	01-0 ✓	
0101 ✓	100- *	
0110 ✓	10-0 *	
1001 ✓		
1010 ✓	01-1 ✓	
	-101 ✓	
0111 ✓	011- ✓	
1101 ✓	1-01 *	
1111 ✓	-111 ✓	
	11-1 ✓	

Minimization - Second Pass (cellular)

- Quine-McCluskey Method
  - Step 2 cont.: Apply Adjacency - Compare elements of group with N 1's against those with N+1 1's. 2^n difference implies adjacent. Next col is numbers with differences in parentheses.

    E.g., 4,5(1) and 6,7(1) yields 4,5,6,7(3)

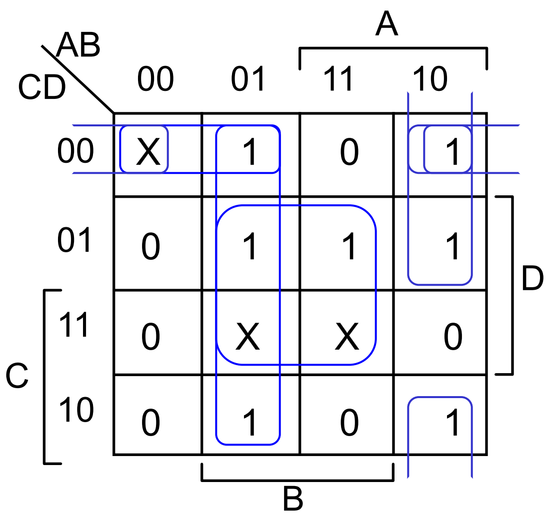
    When used in a combination, mark with a check. If cannot be combined, mark with a star. These are the prime implicants.

    Repeat until nothing left.

Implication Table

Column I	Column II	Column III
0 ✓	0,4(4) *	4,5,6,7(3) *
	0,8(8) *	
4 ✓		5,7,13,15 (10) *
8 ✓	4,5(1) ✓	
	4,6(2) ✓	
5 ✓	8,9(1) *	
6 ✓	8,10(2) *	
9 ✓		
10 ✓	5,7(2) ✓	
	5,13(8) ✓	
7 ✓	6,7(1) ✓	
13 ✓	9,13(4) *	
15 ✓	7,15(8) ✓	
	13,15(2) ✓	

Prime Implicants



Prime Implicants:

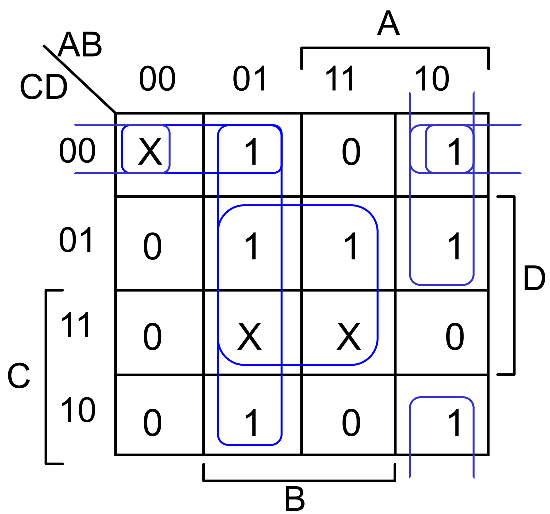
$0\text{-}00 = \bar{A}\bar{C}\bar{D}$ $\quad$ $\text{-}000 = \bar{B}\bar{C}\bar{D}$

$100\text{-} = A\bar{B}\bar{C}$ $\quad$ $10\text{-}0 = A\bar{B}\bar{D}$

$1\text{-}01 = A\bar{C}D$ $\quad$ $\text{-}1\text{-}1 = BD$

$01\text{-}\text{-} = \bar{A}B$

Prime Implicants (cont.)



Prime Implicants:

$0\text{-}00 = \bar{A}\bar{C}\bar{D}$ $\quad$ $\text{-}000 = \bar{B}\bar{C}\bar{D}$

$100\text{-} = A\bar{B}\bar{C}$ $\quad$ $10\text{-}0 = A\bar{B}\bar{D}$

$1\text{-}01 = A\bar{C}D$ $\quad$ $\text{-}1\text{-}1 = BD$

$01\text{-}\text{-} = \bar{A}B$

Stage 2: find smallest set of prime implicants that cover the active-set

recall that essential prime implicants must be in final expression

Coverage Table

Coverage Chart

	4	5	6	8	9	10	13
0,4(0-00)	X						
0,8(-000)				X			
8,9(100-)				X	X		
8,10(10-0)				X		X	
9,13(1-01)					X		X
4,5,6,7(01--)	X	X	X				
5,7,13,15(-1-1)		X					X

rows = prime implicants
columns = ON-set elements
place an "X" if ON-set element is
covered by the prime implicant

Note: <u>Don't</u> include DCs in coverage table; they don't have covered by the final logic expression!

Coverage Table (cont.)

Coverage Chart

	4	5	6	8	9	10	13
0,4(0-00)	X						
0,8(-000)				X			
8,9(100-)				X	X		
8,10(10-0)				X		X	
9,13(1-01)					X		X
4,5,6,7(01--)	X	X	X				
5,7,13,15(-1-1)		X					X

	4	5	6	8	9	10	13
0,4(0-00)	X						
0,8(-000)				X			
8,9(100-)				X	X		
8,10(10-0)				X		X	
9,13(1-01)					X		X
4,5,6,7(01--)	X	X	X				
5,7,13,15(-1-1)		X					X

rows = prime implicants
columns = ON-set elements
place an "X" if ON-set element is
covered by the prime implicant

If column has a single X, than the implicant associated with the row is essential. It must appear in minimum cover

Coverage Table (cont.)

	4	5	6	8	9	10	13
0,4(0-00)	X						
0,8(-000)				X			
8,9(100-)				X	X		
8,10(10-0)				X		X	
9,13(1-01)					X		X
4,5,6,7(01--)	X	X	X				
5,7,13,15(-1-1)		X					X

Eliminate all columns covered by essential primes

Coverage Table (cont.)

	4	5	6	8	9	10	13
0,4(0-00)	X						
0,8(\000)				X			
8,9(100-)				X	X		
8,10(10-0)				X		X	
9,13(1-01)					X		X
4,5,6,7(01--)	X	X	X				
5,7,13,15(-1-1)		X					X

	4	5	6	8	9	10	13
0,4(0-00)	X						
0,8(\000)				X			
8,9(100-)				X	X		
8,10(10-0)				X		X	
9,13(1-01)					X		X
4,5,6,7(01--)	X	X	X				
5,7,13,15(-1-1)		X					X

Eliminate all columns covered by essential primes

Find minimum set of rows that cover the remaining columns

$$F = A\overline{B}\,\overline{D} + A\overline{C}D + \overline{A}B$$