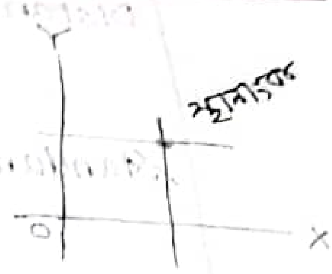
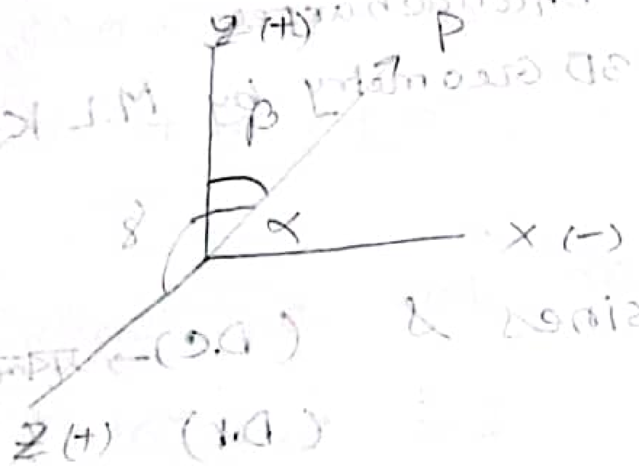
Analytical Geometry

Altab Sir 18.07.23

Geometry
- 2 D — সমতলীয় বিন্দু
- 3 D — all existing point



৪ টি ভাগে বিভক্ত



তিনটি plane XY, XZ, YZ

৮ টি octen তৈরী হয়/ অক্ষগুলো ৮-টি ভাগে বিভক্ত করে।

* Right handed screw rule, vector formula মেনে চলে।

inline অক্ষ হইলে oblique ~~co~~-ordinates system.

তিনঅক্ষ 90° হইলে Rectangular ordinat system.

Distance

Standard books: (I) Askwith, Ana. Geo
(II) 3D. Geo. by J. T. Bell

Solution type → (I) Geometry, by Rahman & Bhattacharjee.
(II) 3D Geometry by M.L Khanna

Direction ~~coff~~ cosines & (D.C) → সরলরেখায় হয়
Direction cosine ratios (D.r)

Direction cosines of a strait line:

$\cos\alpha$, $\cos\beta$, $\cos\gamma$ → Direction cosines

Direction cosines ratio হলো ঐ তিন d.c এর সাথে কত গুন আকারে থাকে।

if $\cos\alpha \to l, m, n$ are the d.c.s of the line AB than a, b, c are called d.r.s of the line AB if,

$$\boxed{\frac{l}{a} = \frac{m}{b} = \frac{n}{c}}$$

$l = ak$
$m = bk$
$n = ck$

$\cos\alpha + \cos\beta + \cos\gamma = 1$

$l^2 + m^2 + n^2 = 1$

of a strait line

Q: Define d.c and d.r and prove that

প্রমাণ, $l^2 + m^2 + n^2 = 1$

Attab sir: 20-07-23

Define d.c.s. and d.r.s of a st. line & if l, m, n are the d.c.s of a line $l^2+m^2+n^2=1$

Example দিতে হবে x অক্ষ x অক্ষের সাথে $0°$

so, $\cos 0°$ ... 90

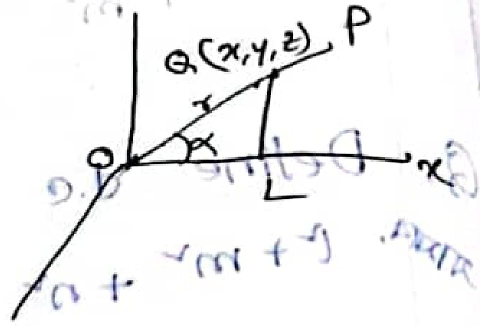
so, $\cos 90°$

সমান্তরাল রেখা d.c same.

পৃথিবীর সকল " , vector এর মান same.

Proof: Let OP be the line passing through the ~~line~~ origin and parallel to given line where d.s.c are l, m, n

Then, any point Q (x, y, z) on OP can be given by



[$l = \cos\alpha$] $x = OQ \cdot l = lr$ where

$OQ = r$

Similarly,

$$y = OQ \cdot m$$
$$= mr$$

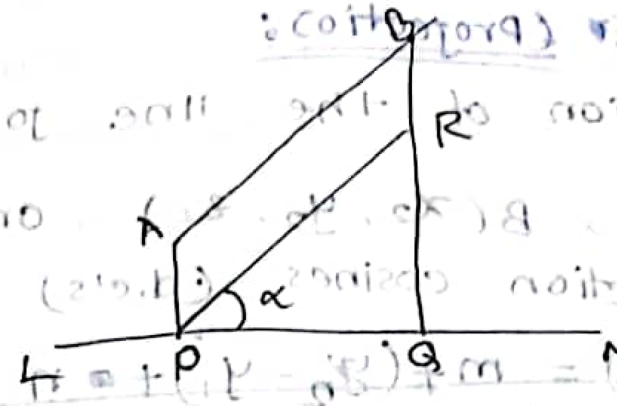
$$z = OQ \cdot n$$
$$= nr$$

now, $x^2 + y^2 + z^2 = r^2(l^2 + m^2 + n^2)$

$$\Rightarrow r^2 = r^2(l^2 + m^2 + n^2)$$

$$[\because r = OQ = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$$

Projection $\longrightarrow$ Perpendicular image



Projection of the segment AB on another line LM.

$AB\cos\alpha$

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = K$$

$l = aK,\ m = bK,\ n = cK$

$$\Rightarrow l^2 + m^2 + n^2 = K^2(a^2 + b^2 + c^2)$$

$$\Rightarrow \boxed{K = \frac{1}{\sqrt{a^2+b^2+c^2}}}$$

$\boxed{K = d.r.}$ formula

direction cosine ratios

(x_1, y_1, z_1) ———— (x_2, y_2, z_2)

$*(x,y,z)$ $(0,0,0)$ d.c $=(x,y_2)$

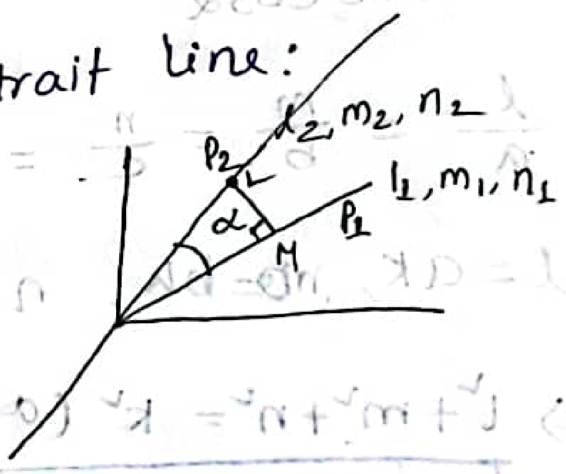
formula (Projection):

Projection of the line joining $A(x_1,y_1,z_1)$, $B(x_2,y_2,z_2)$ on the line whose direction cosines (d.c's) are l,m,n

~~is $(x_2-x_1)=m+(y_2-y_1)+n$, (z~~

is $\boxed{(x_2-x_1)l+(y_2-y_1)m+(z_2-z_1)n}$

Angle between two strait line:



$OL\cos\alpha = OL\,l_2 l_1 + OL\,m_2 m_1 + OL\,n_2 n_1$

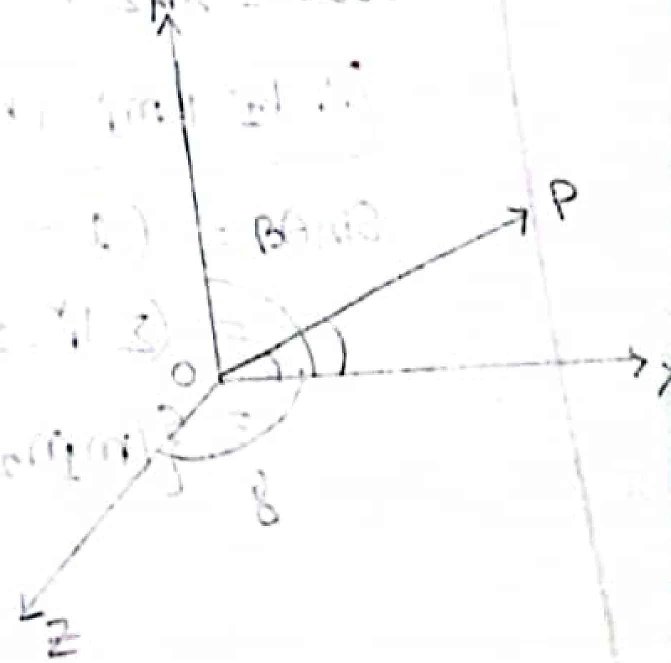
$L \equiv (OL\,l_2, OL\,m_2, OL\,n_2)$

Phn 2 [illegible]

Direction cosines: In analytical geometry the directional cosines are also known as direction cosines of a vector. Is defined as the cosines of the angles between the three coordinate ~~system~~ axes and the vector.

if α, β, γ are the directional angles of a directed line L, then $\cos\alpha$, $\cos\beta$, and $\cos\gamma$ are called the directional cosines of directed Line OP

Direction cosines are denoted by l, m, n respectively.



Thus, $l = \cos\alpha$
$m = \cos\beta$
$n = \cos\gamma$

Direction cosine ratios :- A set of three numbers a, b, c which are proportional to the direction cosines l, m, n respectively are called Direction ratios (d.r's) of a line.

Example: $a = kl$
$b = km$
$c = kn$, where k is constant.

25.07.23 – Angle between two lines

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2 =$$

$$\boxed{\therefore l_1^2 + m_1^2 + n_1^2 = 1 = l^2 + m^2 + n^2}$$

$$\sin\theta = (1 - \cos^2\theta)^{\frac{1}{2}}$$

$$= (\Sigma l_1^2 . \Sigma l_2^2 - (l_1 l_2 + m_1 m_2 + n_1 n_2)$$

$$= \{(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - l_1 n_2)^2 + (l_1 m_2 - l_2 m_1)\}^{\frac{1}{2}}$$

$$= (\Sigma m_1 m_2 - m_2 n_1)^2$$

$$\tan\theta = \frac{\sqrt{m_1 n_2 - m_2 n_1}}{l_1 l_2 + m_1 m_2 + n_1 n_2}$$

i) if the two straight line are traingular then,

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$$

ii) if the two strait line are parallel then

$$\frac{l_1}{l_2} = \frac{m_1}{m_2} \neq \frac{n_1}{n_2} = \frac{\sqrt{l^2}}{\sqrt{l_2^2}} = \frac{1}{1} = 1$$

$$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2} = \frac{\sum l^2}{\sqrt{l_2^2}} = 1$$

As well, $l_1 = l_2$; $m_1 = m_2$; $n_1 = n_2$

Direction cosine : $l_1^2 + m_1^2 + n_1^2 = l_2^2 + m_2^2 + n_2^2$

$$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

Direction cosine $\cos\alpha$, $\cos$

Exercise Math:

Example : 2, 5, 6, 7, 8, 10, 11, 12, 13, 15, 7.

Rahman & Bhattacharje

Ex. 6 Solid geometry P.-11 Rahman S BHATTA...

A line makes angles α, β, γ with the four diagonals of a cube. Prove that, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$

Ans.

Let the d.c.s of the line be, l, m, n. The eight vertics can taken as $O(0,0,0)$, $P(a,a,a)$, $A(a,0,0)$, $B(0,a,0)$, $C(0,0,a)$, $L(0,a,a)$. $M(a,0,a)$, $N(a,a,0)$

AL, BM, CN and OP are the diagonals of the cube. fig.1. the d.c.'s of the four diagonals are

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{3}\right)$$

~~cos~~

The direction cosines of the line which is incline at angles $\alpha, \beta, \gamma, \delta$ respectively to the four diagonals are

$$\cos\alpha = \frac{l}{\sqrt{3}} + \frac{m}{\sqrt{3}} + \frac{n}{\sqrt{3}} = \frac{l+m+n}{\sqrt{3}}$$

Similarly $\cos\beta = \dfrac{-l+m+n}{\sqrt{3}}$, $\cos\gamma = \dfrac{l-m+n}{\sqrt{3}}$

$$\cos\delta = \frac{l+m-n}{\sqrt{3}}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{1}{3}\cdot 4(l^2+m^2+n^2)$$

$$= \frac{4}{3} \quad \text{[Proved]}$$

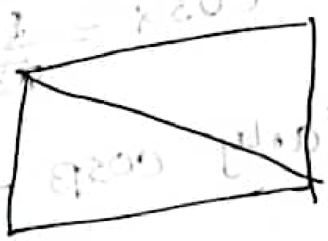
27.07.23 Aftab sir

Parallelopiped

Rectantugalar parallelopiped

Cube

Principal diagonal



6 নং Math sir বলে দিয়েছে।

বই এ আছে Rahman & Vattacharge

Plane

Area পাওয়া গেলে surface

সমতল → যার উপর যেকোন দুটি বিন্দু নিলে ঐ রেখার উপর সমস্ত বিন্দু ঐরেখার উপর থাকবে-

সরলরেখার আদর্শ সমীকরন : $ax+by+c$

ঢাল এর " : $y=mx+c$

কোন point (x_1, y_1) সমীকরনকে সিদ্ধ করলে

Strait line or plane কিনা ~~Taste~~ test করা যায়

* একঘাতের তিন variable এর সরলরেখা সমীকরণ:

$$\boxed{ax + by + cz + d = 0} \rightarrow \text{এর normal abc}$$

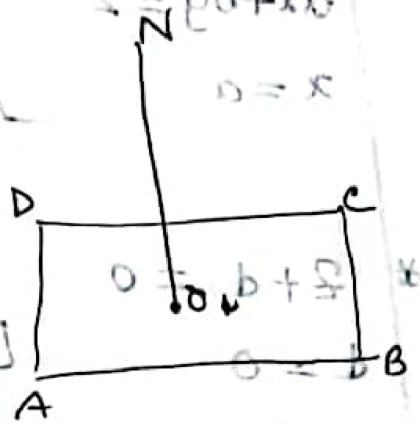
↓

সমতলের সমীকরণ

সকল একঘাতী সমীকরণ all indicate → plane

→ xy, plane এর সমীকরণ $z = 0$

01 August, 2023



Plane:

equation of plane:

$$\boxed{ax + by + cz + d = 0}$$

General equation

$S = t$

a, b, c are the d.c's or d.r's of the normal to the plane.

normal form

$$\boxed{ax + by + cz = p} \rightarrow \text{origin থেকে ⊥ দূরত্ব}$$

where a, b, c are the d.c's of the normal. Then p will the distance of the plane from the origin.

Distance of the plane $ax+by+cz+d=0$ from (x_1, y_1, z_1) is

$$\left| \frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}} \right|$$

$ax+by=2$
$x=a$

এগুলো 3D তে Plane এর সমীকরণ।

* $z+d=0$
$d=0$

[z মূলবিন্দু দিয়ে যায়]
[x, y plane এ]

* $x+d=0$
$x=-d$

[yz plane এ]

$$\frac{a}{\sqrt{a^2+b^2+c^2}}x+\frac{b}{\sqrt{a^2+b^2+c^2}}y+\frac{c}{\sqrt{a^2+b^2+c^2}}z+\frac{d}{\sqrt{a^2+b^2+c^2}}=0$$

$$=\frac{d}{\sqrt{a^2+b^2+c^2}}$$

(normal form এ আনার জন্য)

CamScanner

Example 12:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Find the the equ.

$$Ax + By + Cz \pm d = 0$$

$$A = -\frac{d}{a}$$
$$B = -\frac{d}{b}$$
$$C = -\frac{d}{c}$$

$$A \equiv (a, 0, 0)$$
$$B \equiv (0, b, 0)$$
$$C \equiv (0, 0, c)$$

Equ. of the plane,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \text{——} \ (1)$$

if (1) is at constant distance p from the origin then we have,

$$p = \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

or $a^{-2}+b^{-2}+c^{-2}=p^{-2}$

Now planes through A, B, & C and parallel to the co-ordinate planes, are,

$x=a,\ y=b,\ z=c$ respectively

if (x_1, y_1, z_1) is the pt. of intersection of three plane $x_1=a,\quad y_1=b,\quad z_1=c$

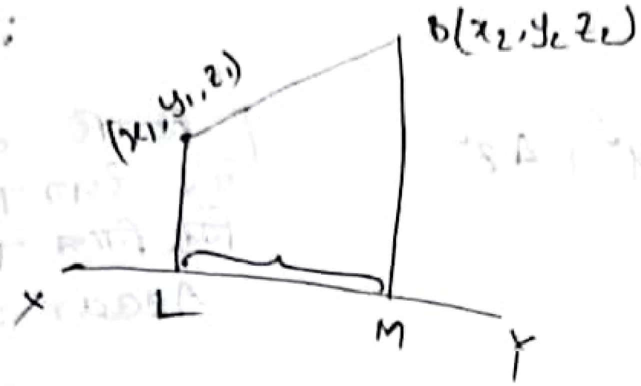
Altab Sir, ~~10.08.23~~ 10.08.23

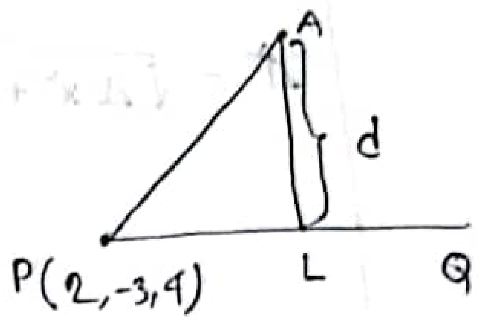
Find the distance of the point $A(1,2,3)$ from the line PQ through $P(2,-3,4)$ which makes equal angle with the axes.

2. Define projection of a line segment on a line.

Find the distance of the point $A(1,2,3)$ from the line PQ through $(2,-3,4)$ which makes equal angles with the axes.

Solve:





Form $=(x_2-x_1)l-(y_2-y_1)m-(z_2-z_1)n$

Since the given line PQ makes equal angles with the axes, the d.c.s of the line PQ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Now, consider the figure drawn drawn in right and we see that,

PL = projection of the line segment PA on the line PQ.

$$= (2-1)\frac{1}{\sqrt{3}} + (-3+2)\frac{1}{\sqrt{3}} + (4-3)\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

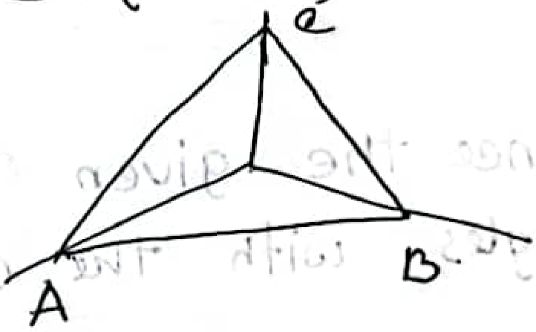
Therefore, required distance $= AL$

$$= \sqrt{PA^2 - PL^2}$$

$$\Delta^2 = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

[তিনটি axes এর উপর তিনটি বিন্দু নিলে ত্রিভুজটির Area সূত্র]

$A \equiv (a,0,0)$ $B \equiv (0,b,0)$ $C \equiv (0,0,c)$



Then for ΔABC

$$\Delta x = \frac{1}{2}bc, \quad \Delta y = \frac{1}{2}ca$$

$$\Delta z = \frac{1}{2}ab$$

$$\boxed{lx + my + nz = p}$$

$A \equiv (\frac{r}{l}, 0, 0)$ $B \equiv (0, \frac{r}{m}, 0)$ $C \equiv (0, 0, \frac{r}{n})$

Δ Now $\Delta ABC = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$ where Δx is the projection of ΔABC on yz-plane & so on.

$$= \left[\left(\frac{1}{2}\frac{r^2}{mn}\right)^2 + \left(\frac{1}{2}\frac{r^2}{nl}\right)^2 + \left(\frac{1}{2}\frac{r^2}{lm}\right)^2\right]^{\frac{1}{2}}$$

$$= \frac{r^2}{2} \cdot \frac{1}{lmn}$$

Exm

$$= \frac{r^2}{2} \cdot \frac{1}{\frac{x'}{r} \cdot \frac{y'}{r} \cdot \frac{z'}{r}} = \frac{r^5}{2x'y'z'}$$

if ~~the plane~~

\# Find equ. of the plane restively then (1, 2, 3) from the line PQ through p (2, -3, 4) passing through a point (1, 2, 4) and parallel to x-axis where Δx is the projection of ΔABC on yz plane & so, o

\# Find the equation to the plane through two point and parall to an axis.

Altab Sir – 17.08.23

Distance between two parallel strait line:

$ax + by + c_1 = 0$
$ax + by + c_2 = 0$

(x_1, y_1) বিন্দু থেকে

R.D (Required distance) $= \dfrac{ax_1 + by_1 + c_2}{\sqrt{a^2+b^2}}$

$= \dfrac{-c_1 + c_2}{\sqrt{a^2+b^2}}$ $\left[ax_1 + by = -c_1\right.$ মান বসিয়ে $\left.\right]$

(x_1, y_1) – ① নং Equation

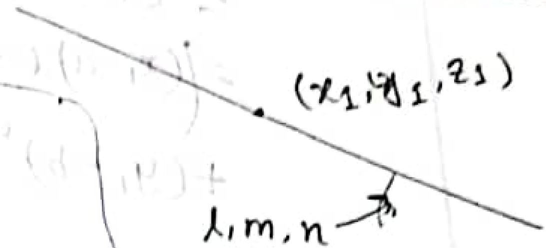
Shortest distance line (S.D line)

Shortest distance (S.D)

Formula

$\boxed{\text{Distance of the point } (x_1, y_1, z_1) \text{ from line whose equation is } \dfrac{x-a}{l} = \dfrac{y-b}{m} = \dfrac{z-c}{n}, \ l, m, n \text{ being the d.c.s of the}}$

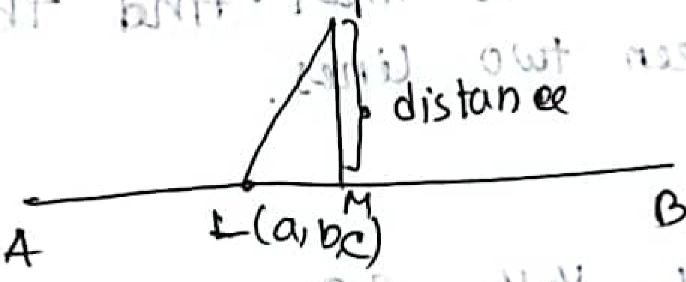
equation $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n} = r$



হলে, then r represent the actual distance any point x, y, z from the fix point (x_1, y_1, z_1).

প্রমাণ Exm - Distance of the point (x_1, y_1, z_1) from the line whose equ. is $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$, ~~l, m, n~~

$$= \left\{ \begin{vmatrix} \frac{x_1-a}{l} & \frac{y-b}{m} \end{vmatrix}^2 + \begin{vmatrix} \frac{y_1-b}{m} & \frac{z_1-c}{n} \end{vmatrix}^2 + \begin{vmatrix} \frac{z_1-c}{n} & \frac{x_1-a}{l} \end{vmatrix}^2 \right\}^{\frac{1}{2}}$$



L·M projection of PL on AB

$(x_1-a)l + (y_1-b)m + (z_1-c)n$

∴ Required distance = PM

$= (PL^2 - LM^2)^{\frac{1}{2}}$

$= \{(x_1-a)l + (y_1-b)m + (z_1-c)n\}^2 + (x_1-a)^2 + (y_1-b)^2 + (z_1-c)^2$

$= \{(x_1-a)^2 + (y_1-b)^2 + (z_1-c)^2\}(l^2+m^2+n^2) -$

[

$= \{(x_1-a)m - (y_1-b)l\}^2 + \cdots$

xm গুরুত্বপূর্ণ What do you mean by shortest distance between two lines. find the shortest between two lines.

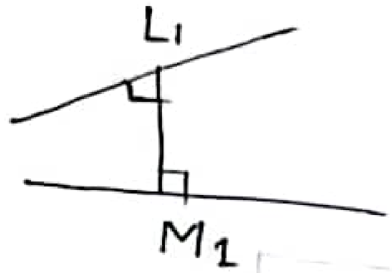
$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} = r_1$

$\frac{x-x_2}{l_2}$

১ম সরলরেখার উপর যেকোন সমান বিন্দু

$(x_1 + l_1 r_1, \; y_1 + m_1 r_1, \; z_1 + n_1 r_1)$

$(x_2 + l_2 r_2, \; y_2 + m_2 r_2, \; z_2 + n_2 r_2)$



দুটি সরলরেখা দেয়া থাকবে। Shortest distance বের করতে হবে। এ জাতীয় একটি প্রশ্ন থাকবে।

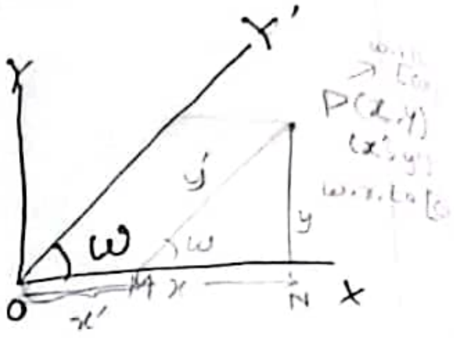
Find the conditions that the lines (~~no~~ at least two) are co-planer and then find the equation of the plane.

Exm
জানবে
(Important)

Allah Sir 22.08.23

new chapter

The changes of axes.



$$y = y' \sin\omega$$
$$y' = y \operatorname{cosec}\omega$$

→ Relation between rectangular and oblique axes Q.#

$$x = x' + y' \cos\omega$$
$$x' = x - y \cot\omega$$

1. Q. What are the relation between rectangular and oblique axes?

2. Find the length of the perpendicular from (x_1, y_1) on the line $ax + by + c = 0$, the axes being inclined at an angle ω

Solution: let (X, Y) & (x_1', y_1') be the co-ordinates of (x, y) w.r.to rectangular axes

Then $x = X - Y\cot\omega$ & $y = Y\operatorname{cosec}\omega$

$x_1' = x_1 + y_1\cos\omega$ & $y_1' = y_1\sin\omega$

Q.# Find the condition

Rotation of rectangular axes with origin unchanged

$\cancel{x' = OQ + QN}$

$$\boxed{\begin{aligned} x &= OM - LM = x'\cos\theta - y'\sin\theta \\ y &= OP + OL = x'\sin\theta + y'\cos\theta \end{aligned}}$$

Exam
গুরুত্ব

Q. $ax^2 + 2hxy + by^2 \rightarrow$ old axes

$$\downarrow$$

$a'x'^2 + 2h'x'y' + b'y'^2 \rightarrow$ new axis

Prove that $a+b = a'+b'$

$h^2 - ab = h'^2 - a'b'$

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$$\frac{a+b-2h\cos\omega}{\sin^2\omega} = \frac{a'+b'-2h'\cos\omega'}{\sin^2\omega'}$$

& $$\frac{ab-h^2}{\sin^2\omega} = \frac{a'b'-h'^2}{\sin^2\omega'}$$

$$\boxed{\theta = \frac{1}{2}\tan^{-1}\frac{2h}{a-b}}$$

আসবে
Exm এ

* $a_1x + b_1y + c_1 = 0$ —— (i)

$a_2x + b_2y + c_2 = 0$ —— (ii)

(i) × (ii) = 0 হবে দুটি সরলরেখার একত্রিত সমী:
(Pair of straight line)

$y = m_1 x$ —— (i)

$y = m_2 x$ —— (ii)

সরলরেখা দুটির একত্রিত সমীকরণ

$(y - m_1 x)(y - m_2 x) = 0$

$x^2 - 2x + 1 = 0$

$\boxed{x^2 + y^2 = 0}$

$(x-1)^2 = 0$

বিন্দুবৃত্ত

$(x-1)(x-1)$

$x = 1$ ও $x = 1$

প্রমান কর যে —— সমীকরণটি তিনটি সরলরেখার সমীকরণ।

(i) উৎপাদক বের করতে হবে

$a(x+y)(bx+c)(x+dy)$
$\underset{0}{\overset{"}{}}$ $\underset{0}{\overset{"}{}}$ $\underset{0}{\overset{"}{}}$ তিনটি সরলরেখার

Exm ॥ $ax^2+2hxy+by^2=0$ represents two
9 ১। st. lines passing through the origin.

$ax^2+2hxy+by^2$

$\Rightarrow y^2+\frac{2hy}{b}+\frac{a}{b}x^2 = 0$

$\Rightarrow y^2+2\cdot y\cdot\frac{hx}{b}+\frac{h^2x^2}{b^2}-\frac{h^2x^2}{b^2}+\frac{a}{b}x^2=0$

$\Rightarrow \left(y+\frac{hx}{b}\right)^2-\frac{(h^2-ab)x^2}{b^2}=0$

$\Rightarrow y+\frac{hx}{b} = \sqrt{\frac{(h^2-ab)x^2}{b^2}}$

$\Rightarrow y+\frac{hx}{b} = \pm\frac{(\sqrt{h^2-ab})x}{b}$

$\Rightarrow y+\frac{hx}{b}+\frac{\sqrt{h^2-ab}}{b}x=0$ &

$\Rightarrow y+\frac{hx}{b}-\frac{\sqrt{h^2-ab}}{b}x=0$

Slope of the first line $-\left(\frac{h}{b}+\frac{\sqrt{h^2-ab}}{b}\right)\times$

$$\left(\frac{h}{b}+\frac{\sqrt{h^2-ab}}{b}\right)$$

$$=\frac{h^2}{b^2}-\frac{h^2-ab}{b^2}=-1$$

$$\Rightarrow h^2-h^2+ab=-b^2$$

$$\Rightarrow ab+b^2=0$$

$$\Rightarrow b(a+b)=0$$

$$\Rightarrow a+b=0$$

অথবা,

$$ax^2+2hxy+by^2=0$$

$$\Rightarrow abx^2+2hbxy+b^2y^2=0$$

$$\Rightarrow abx^2 = h^2x^2+(hx)^2+2\cdot hx\cdot by+(by)^2=0$$

$$\Rightarrow (hx+by)^2=(\sqrt{h^2-ab}\cdot x)^2$$

$\boxed{ax^3+3ax^2y+by^3=0} \rightarrow$ মূলবিন্দুগামী তিনটি সরলরেখার সমীকরণ।

Homogenous equation of 2nd degree

$y = m_1 x$ ও $y = m_2 x$

$$(y - m_1 x)(y - m_2 x) \equiv ax^2 + 2hxy + by^2$$

সহগ সমীকৃত করে,

$$b m_1 m_2 = a$$

$$m_1 m_2 = a/b$$ ও

$$m_1 + m_2 = \frac{2h}{b}$$

আসবে ③. $2x^2 + 8xy + y^2 = 0$

$x + y + 1 = 0$ এই তিনটি

সরলরেখা দ্বারা গঠিত ত্রিভুজের ক্ষেত্রফল কত?

Find the ~~equation~~ area of the triangle formed by lines given by $ax^2 + 2hxy + by^2$ ও $lx + my + n = 0$