University of Rajshahi
Dept. of Computer Science and Engineering
B.Sc. Engg. Part-I (Even Semester) Examination - 2021
Course: MATH-1221 (Co-ordinate Geometry, Vector Analysis and Complex Variable)
Full Marks: 52.50
Time: 03 hours

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(Answer any THREE questions from each section. Use separate answer script for each section)

Section A

1.a) Define Direction Cosines and Direction Ratios. Find the direction cosines of the line passing through two points (-2, 4, -5) and (1, 2, 3). 2.00

b) Define Plane. Find the equation of the plane which passes through the points (1, 0, -1) and (2, 1, 1) and parallel to the line joining the points (-2, 1, 3) and (5, 2, 0). 4.00

c) Find the equation of the straight line passing through the point (2, -1, 1) and parallel to the line joining the points (1, 2, 3) and (-1, 1, 2). 2.75

2.a) Define Dot and Cross product of two vectors. Find an equation for the plane perpendicular to the vector $A = 2i + 3j + 6k$ and passing through the terminal point of the vector $B = i + 5j + 3k$. 3.00

b) Show that $A.(B \times C)$ is in absolute value equal to the volume of a parallelepiped with sides A, B and C. 3.00

c) A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $i - 3j + 2k$. 2.75

3.a) If $F = (2x + y)i + (3y - x)j$ then evaluate $\int_c F.dr$ where C is the curve in xy-plane consisting of the straight line from (0, 0) to (2, 0) and then to (3, 2). 2.75

b) Evaluate $\iint_s A.nds$, where $A = 18zi - 12j + 3yk$ and S is that part of the plane $2x + 3y + 6z = 12$ which is in the first octant. 4.00

c) Evaluate $\int_v Fdv$ where $F = 2xzi - xj + y^2k$ and V is the region bounded by the surface $x = 0, y = 0, z = 0, x = 1, y = 1$, and $z = 1$. 2.00

4.a) Define Gradient of a scalar function $\phi(x, y, z)$. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2). 3.00

b) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2i - j - 2k$. 2.75

c) Define divergence of a vector. If $\phi = 2x^3y^2z^4$, then find div grad ϕ. 3.00

Section B

5.a) Define complex conjugate of a complex number. Discuss the graphical representation of a complex number and prove that $|z_1 + z_2| \geq |z_1| - |z_2|$. 3.00

b) Show that if $(z) = \frac{z}{\bar{z}}$, the limit $\lim\limits_{z \to 0} f(z)$ does not exist. 3.00

c) Show that $f(z) = \bar{z}$ is non-analytic anywhere. 2.75

6.a) Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$ along the straight lines from (0, 3) to (2, 3) and then (2, 3) to (2, 4). 3.75

b) Evaluate $\int_C \frac{z^2 - z + 1}{z - 1} dz$ where $C: |z| = \frac{1}{2}$ is taken in anti-clockwise direction. 3.00

c) Define Simply and Multiply connected regions. 2.00

7.a) State Cauchy's integral formula and explain why Cauchy's integral formula are quite remarkable. 3.00

b) Evaluate $\int_C \frac{dz}{z-a}$ where C is any simple closed curve C and $z = a$ is (i) Outside C, (ii) Inside C. 2.75

c) Expand $f(z) = \sin(z)$ in Taylor series about $z = \frac{\pi}{4}$. 3.0

8. a) Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$. 3.

b) State residue theorem. 1.

c) Evaluate $\oint_C \frac{z}{(z-1)(z+1)^2} dz$ around the circle C defined by $|z| = 2$ using residue theorem. 4.

University of Rajshahi
Department of Computer Science and Engineering
B.Sc. Engg. Part-I, Even Semester, Examination 2020
Course Code: MATH-1221
Course Title: Co-ordinate Geometry, Vector Analysis and Complex Variable

Time: 3 Hours **Full Marks: 52.5**

[N.B. Answer SIX questions taking at least THREE from each Section.]

Section-A

1. (a) Construct the rotation matrix when the point is rotated through an angle θ. 3.75

(b) If the point is rotated through 45°, find the new coordinates of the point whose coordinates are (1,1) using the rotation matrix. 3

(c) Prove that rotation is distance invariant. 2

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2. (a) Rotate axes to eliminate the xy-term from the equation $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0$ and draw the figure. 3

(b) What surfaces in $\mathbb{R}^3$ are represented by the following equations? 1.75
$(i)\ x = 5 \quad (ii)\ z = 4.$

(c) A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube; show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$. 4

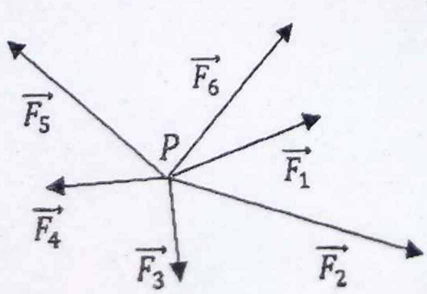
3. (a) Prove that the general equation of first degree in x, y, z *i.e.*, $Ax + By + Cz + D = 0$ represents a plane. 3

(b) Find the equation of the straight lines through $(2, -1, 4)$, which are 2.75
(i) parallel to y-axis (ii) perpendicular to y-axis.

(c) Find the shortest distance between the lines 3

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}; \qquad \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$

4. (a) Forces $\vec{F_1}, \vec{F_2}, \vec{F_3}, \vec{F_4}, \vec{F_5}, \vec{F_6}$ act as shown on object P. What force is needed to prevent P from moving? 1.75



		3
(b)	Prove that $\vec{A}.(\vec{B}\times\vec{C}) = \vec{B}.(\vec{C}\times\vec{A}) = \vec{C}.(\vec{A}\times\vec{B})$.	4
(c)	Prove that $[\vec{A}\times\vec{B}\ \vec{B}\times\vec{C}\ \vec{C}\times\vec{A}] = [\vec{A}\ \vec{B}\ \vec{C}]^2$.	

Section-B

5. (a) A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\hat{\imath} + \sin\omega t\hat{\jmath}$, where ω is a constant. Show that — 6

    (i) the velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$,

    (ii) the acceleration $\vec{a}$ is directed towards the origin and has magnitude proportional to the distance from origin, and

    (iii) $\vec{r}\times\vec{v} = a$ constant vector.

   (b) If $\vec{A}$ has constant magnitude, then show that $\vec{A}$ and $\frac{d\vec{A}}{dt}$ are perpendicular if $\left|\frac{d\vec{A}}{dt}\right| \neq 0$. — 2.75

6. (a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the direction $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$. — 2.75

   (b) Define divergence of a vector. If $\phi = 2x^3y^2z^4$ then find div grad ϕ. — 3

   (c) Define curl of a vector. Prove that div curl $\vec{A} = 0$ for any vector $\vec{A}$. — 3

7. (a) If z_1 and z_2 are two complex numbers, then prove that $|z_1 + z_2| \leq |z_1| + |z_2|$. — 3

   (b) Find the branch points and branch lines of the function $f(z) = z^{\frac{1}{2}}$. — 3

   (c) State Cauchy's integral formula and explain why Cauchy's integral formula is quite remarkable. — 2.75

8. (a) Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$. — 3

   (b) State residue theorem. — 1.75

   (c) Evaluate $\oint_C \frac{z}{(z-1)(z+1)^2} dz$ around the circle C defined by $|z| = 2$ using residue theorem. — 4

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University of Rajshahi
Department of Computer Science and Engineering
B.Sc. Engineering Part I, **Even** Semester Examination **2018**
Course Code: MATH-**1211** Course Title: **Differential and Integral Calculus**
Time: **03** Hours Full Marks: **52.5**

Section **A**
Answer any **THREE** questions.

1. (a) Define function. Find domain and range of $f(x) = {|x|}/{x}$ and draw the graph of f. **03**

   (b) Define continuity of a function at a point. Let $f(x) = \begin{cases} x : 0 \le x < \frac{1}{2} \\ 1 - x : \frac{1}{2} \le x < 1 \end{cases}$ **3.25**

   Is this function continuous at x=1/2 ? Is it differentiable at x=1/2?

   (c) Evaluate $\mathrm{lt}_{x\to 0}\left(\frac{Sinx}{x}\right)^{1/x^2}$ **2.5**

2. (a) If $f(x) = \left(\frac{a+x}{b+x}\right)^{a+b+2x}$, show that $f'(0)=\left\{2Log\frac{a}{b} + \frac{b^2-a^2}{ab}\right\}\left(\frac{a}{b}\right)^{a+b}$ **03**

   (b) If $Siny = xSin(a+y)$, prove that $\frac{dy}{dx} = \frac{Sin^2(a+y)}{Sina}$ **2.75**

   (c) If $y = x^{n-1}logx$, then prove that $y_n = \frac{(n-1)!}{x}$ **03**

3. (a) State and prove Mean value theorem **03**

   (b) Show that the largest rectangle with a given perimeter is a square. **03**

   (c) Expand $2x^3 + 7x^2 + x - 1$ in power of (x-2) **2.75**

4. (a) Show that $(3x^2y - 2y^2)dx + (x^3 - 4xy + 6y^2)dy$ can be written as an exact differential of a function $\varphi(x, y)$ and find this function. **03**

   (b) Define homogeneous function. If $z = \sin^{-1}\frac{x^2+y^2}{x+y}$, show that $x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} =$ $tanz$ **2.75**

   (c) Show that in the curve $by^2 = (x+a)^3$, the square of the subtangent varies as the subnormal. **03**

Section **B**
Answer any **THREE** questions.

5. (a) Evaluate any three of the following **8.75**

   (i) $\int \frac{dx}{\sqrt{x(1+x)^5}}$

   (ii) $\int \frac{dx}{\sqrt{(x^2-7x+12)}}$

   (iii) $\int \frac{dx}{(2x-3)\sqrt{(2x^2-3x+4)}}$

   (iv) $\int (3x-2)\sqrt{(x^2+x+1)}dx$

6. (a) Evaluate $\int \frac{x^4+2x+6}{x^3+x^2-2x}dx$ 03

(b) Evaluate $\int_0^{\pi/2} \frac{Sinx}{Sinx+Cosx}dx$ 2.75

(c) If $I_n = \int_0^{\pi/4} tan^n x dx$ show that $I_n + I_{n-2} = \frac{1}{n-1}$ and deduce the value of I_5 03

7. (a) Evaluate $\int_0^a \frac{x^4}{\sqrt{a^2-x^2}}dx$ 03

(b) If $I_n = \int_0^{\pi/4} tan^n\theta d\theta$, show that $I_n = \frac{1}{n-1} - I_{n-2}$ 2.75

(c) Prove that $\int_0^1 \frac{\log(1+x)}{1+x^2}dx = \frac{\pi}{8}log2$ 03

8. (a) Find the length of the cardioide $r = a(1 + cos\theta)$ and show that arc of the upper half is bisected by $\theta = \pi/3$ 03

(b) Find the area of the segment cutoff from the parabola $y^2 = 2x$ by the straight line y=4x-1 03

(c) Evaluate $\int_{z=1}^{2}\int_{y=0}^{1}\int_{x=-1}^{1}(x^2 + y^2 + z^2)dxdydz$ 2.75

Section -B

5. Evaluate any three of the following 8.75

(i) $\int \frac{dx}{x(a+b\log x)}$ (ii) $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}} (\beta > \alpha)$ (iii) $\int \frac{dx}{5+4\cos x}$ (iv) $\int \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right) dx$

6.(a) Show that $\int_0^a \frac{a^2-x^2}{(a^2+x^2)^2} dx = \frac{1}{2a}$. 2.75

(b) Evaluate $\lim_{n\to\infty} \left[\frac{1}{n+m} + \frac{1}{n+2m} + \dots\dots\dots + \frac{1}{n+nm}\right]$ 3

(c) Show that $\int_0^{\frac{\pi}{2}} \log \sin x \, dx = \int_0^{\frac{\pi}{2}} \log \cos x \, dx = \frac{\pi}{2} \log \frac{1}{2}$ 3

7.(a) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta \, d\theta$, show that $I_n = \frac{1}{n-1} - I_{n-2}$. 2.75

(b) Prove that $u_n = \int_0^1 x^n \tan^{-1} x \, dx$ then $(n+1)u_n + (n-1)u_{n-2} = \frac{\pi}{2} - \frac{1}{n}$. 3

(c) Show that $\int_0^1 x^{n-1}(1-x)^{m-1} dx = \int_0^1 x^{m-1}(1-x)^{n-1} dx = \frac{1.2.3 \dots\dots (m-1)}{n(n+1) \dots\dots (n+m-1)}$. 3

8.(a) Find the area of quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ between the major and minor axes. 4

(b) Find the length of the arc of the parabola $y^2 = 4ax$ measured from the vertex to one extremity of the latus rectum. 4.75

University of Rajshahi
Department of Computer Science and Engineering
B.Sc.(Engg.), Part-1, Even Semester Examination-2017
Course: CSE1211 (Differential and Integral Calculus)
Total marks: 52.50 **Time: 3 hours**

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[Answer **three** questions from each section]

Section-A

1.(a) Define even function. Show that $f(x) = 2\cos x + \sin^2 x - \frac{3}{x^2} + x^4$ is an even function and $f(x) = x\sin^2 x - x^3$ is an odd function. 3

(b) If $f(x) = 3 + 2x \quad for \ -\frac{3}{2} < x \leq 0$
$= 3 - 2x \quad for \ 0 < x < \frac{3}{2}$ 3
Show that f(x) is continuous at $x = 0$ but $f'(0)$ does not exist.

(c) Evaluate $\lim_{x\to\frac{\pi}{2}} (sinx)^{logx}$. 2.75

2.(a) Find the differential coefficient of $(sinx)^{cosx} + (cosx)^{sinx}$. 2.75

(b) Differentiating $\cos^{-1}\frac{1-x^2}{1+x^2}$ w.r.to $tan^{-1}\frac{2x}{1-x^2}$. 3

(c) If $y = \tan^{-1} x$ then 3
(i) $(1 + x^2)y_1 = 1$

and (ii) $(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$.

3.(a) Define partial derivatives. If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. 3

(b) If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. 2.75

(c) Given $\frac{x}{2} + \frac{y}{3} = 1$, find the maximum value of xy and minimum value of $x^2 + y^2$. 3

(a) Show that the tangent at (a, b) to the curve $(\frac{x}{a})^3 + (\frac{y}{b})^3 = 2$ is $\frac{x}{a} + \frac{y}{b} = 2$. 3

) If $lx + my = 1$ touches the curve $(ax)^n + (by)^n = 1$ show that $(\frac{l}{a})^{\frac{n}{n-1}} + (\frac{m}{b})^{\frac{n}{n-1}} = 1$. 3

Find the asymptotes of the curves $x^3 + 3x^2y - xy^2 - 3y^3 + x^2 - 2xy + 3y^2 + 4x + 5 = 0$. 2.7

7. a) Obtain a reduction formula for $\int sin^n x\, dx$ and hence deduce Walli's formula. 6

b) Evaluate $\int_0^{\frac{\pi}{2}} \sin x\, dx$ from the definition of definite integral as limit of a sum. 2.75

8. a) Find the volume generated by the revolution about x-axis of the area bounded by the loop of the curve $y^2 = x^2(2 - x)$. 4

b) Find the area bounded by the curve $y^2 = x^3$ and the line $y = 2x$. 2

c) Find the length of the perimeter of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. 2.

University of Rajshahi

Department of Computer Science and Engineering

B.Sc.In Engineering 1st Year 2nd Semester Examination-2019

Course: **MATH 1211** [Differential and Integral Calculus]

Marks: 52.50 Time: 3 Hours

[Answer any six (06) questions taking three (03) from each section.]

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Section-A

1.a) Define function and hence define domain codomain with an example. 3

b) Find the domain and range of $f(x) = |x+1| + |x|$ and also sketch the graph of $f(x)$. 3

c) Prove that every differential function is continuous but the converse is not true. 2.75

2.a) Define derivative of a function $f(x)$ at $x = c$. Show that the function

$$f(x) = \begin{cases} 3 + 2x \; for -\frac{3}{2} < x \le 0 \\ 3 - 2x \; for \; 0 < x \le \frac{3}{2} \end{cases}$$

is continuous at $x = 0$ but not differentiable at $x = 0$. 4

b) If $y = (\sin^{-1} x)^2$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$. 3

c) i) Evaluate $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$ ii) Differentiate $x^{\sin^{-1} x}$ with respect to $\sin^{-1} x$. 1.75

3.a) For a given curved surface of a right circular cone when the volume is maximum, show that the semi-vertical angle is $\sin^{-1}\frac{1}{\sqrt{3}}$. 4

b) Give the geometrical interpretation of Mean value theorem. 3

c) Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 8$ in [1, 4]. 1.75

4.a) If $u = f(x^2 + 2xyz, y^2 + 2zx)$, prove that $(y^2 - zx)\frac{\partial u}{\partial x} + (z^2 - xy)\frac{\partial u}{\partial y} + (x^2 - yz)\frac{\partial u}{\partial z} = 0$. 4

b) Define pedal equation of a curve. Prove that the curve $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{a'} + \frac{y^2}{b'} = 1$ will cut orthogonally if $a - b = a' - b'$. 3

c) Find the asymptotes of the curve $x^2y^2 - a^2(x^2 + y^2) = 0$. 1.75

Section-B

5. Answer any three of the following: 8.75

i) $\int \frac{x+1}{3+2x-x^2} dx$ ii) $\int \frac{dx}{\sqrt{(2x^2+3x+4)}}$

iii) $\int \frac{dx}{5+4\cos x}$ iv) $\int \frac{dx}{\sqrt{(x^2-2x+3)}\,(x^2-2x+1)}$

6.a) Evaluate $\int_0^1 \frac{\ln(1+x)}{1+x^2} dx$. 4

b) Evaluate $\int_2^4 \frac{x-3}{\sqrt{(5-x)(x-1)}} dx$. 3

c) Write down the five general properties of definite integral. 1.75