Contents

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Discrete Mathematics-2021

Section -A

1.(a) Define contradiction, tautology, and contingency with example.

- Tautology

A tautology is a statement that is always true, no matter what. For example, the statement "All bachelors are unmarried" is a tautology. This is because the definition of "bachelor" is "an unmarried man." Therefore, it is impossible for a bachelor to be married.

- Contradiction

A contradiction is a statement that is always false, no matter what. For example, the statement "This statement is false" is a contradiction. This is because if the statement is true, then it is false, and if it is false, then it is true. This is a logical impossibility.

- Contingency

A contingency is a statement that can be either true or false, depending on the situation. For example, the statement "It will rain tomorrow" is a contingency. It is possible that it will rain tomorrow, but it is also possible that it will not rain.

Here are some more examples of tautologies, contradictions, and contingencies:

- Tautologies:
  - All squares are rectangles.
  - Every even number is divisible by 2.
  - The sum of all interior angles in a triangle is 180 degrees.
- Contradictions:
  - This statement is false.
  - A square is a circle.
  - 2 + 2 = 5.
- Contingencies:
  - It will snow tomorrow.
  - I will pass my next math test.
  - I will win the lottery.

(b) Show that that ¬ (pV(¬pAq)) and ¬p A ¬q are logically equivalent.

To show that ¬ (p V (¬p ∧ q)) and ¬p ∧ ¬q are logically equivalent, we can use logical equivalences and truth tables.

Let's start by using De Morgan's laws to rewrite ¬ (p V (¬p ∧ q)):

¬ (p V (¬p ∧ q))

= ¬p ∧ ¬(¬p ∧ q) (De Morgan's law for negation of a disjunction)

= ¬p ∧ (¬¬p ∨ ¬q) (De Morgan's law for negation of a conjunction)

= ¬p ∧ (p ∨ ¬q) (Double negation)

= (¬p ∧ p) ∨ (¬p ∧ ¬q) (Distributive law)

= F ∨ (¬p ∧ ¬q) (Negation law: p ∧ ¬p is always false)

= ¬p ∧ ¬q (Identity law: F ∨ p is always p)

The expression simplifies to $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$.

p	q	¬p^¬pV¬q	¬ (pV(¬pAq))
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

(c) Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives. Let the domain consist of all people.

(i) Someone in your class can speak Hindi.
(ii) Everyone in your class is friendly.
(iii) There is a person in your class who was not born in Rajshahi.
(iv) No student in your class has taken a course in logic programming.

Let's translate each statement into logical expressions using predicates, quantifiers, and logical connectives.

(i) Someone in your class can speak Hindi.

Let's define the predicate:

$S(x)$: "x is in your class."

$H(x)$: "x can speak Hindi."

The statement can be translated as:

$\exists x (S(x) \wedge H(x))$

(ii) Everyone in your class is friendly.

Let's define the predicate:

$S(x)$: "x is in your class."

$F(x)$: "x is friendly."

The statement can be translated as:

$\forall x (S(x) \rightarrow F(x))$

(iii) There is a person in your class who was not born in Rajshahi.

Let's define the predicate:

S(x): "x is in your class."

B(x): "x was born in Rajshahi."

The statement can be translated as:

$\exists x\,(S(x) \land \neg B(x))$

(iv) No student in your class has taken a course in logic programming.

Let's define the predicate:

S(x): "x is in your class."

C(x): "x has taken a course in logic programming."

The statement can be translated as:

$\neg\exists x\,(S(x) \land C(x))$

Note: The translations above assume that the domain consists of all people in your class. Adjustments to the predicates and quantifiers may be necessary if the domain is different

2.(a) Define the following with example

(i) Existential quantifier

(ii) Universal quantifier

- **Existential quantifier** is a symbol that is used to express the idea that there exists at least one object that satisfies a given property. The symbol for existential quantifier is ∃. For example, the statement $\exists x P(x)$ means that there exists an object x such that $P(x)$ is true.

  For example, consider the statement:

  ∃x: x is an even number.

- **Universal quantifier** is a symbol that is used to express the idea that all objects satisfy a given property. The symbol for universal quantifier is ∀. For example, the statement $\forall x P(x)$ means that for all objects x, $P(x)$ is true.
- For example, consider the statement:
- ∀x: x is a positive number.

(b) Show that Vx (P(x) → Q(x)) and ∃x (P(x) A¬Q(x)) are logically equivalent.

Proof:

Assume that Vx (P(x) → Q(x)) is true.

Then, for all x in the domain, if P(x) is true, then Q(x) is true.

Hence, there must exist at least one x such that P(x) is true and Q(x) is false.

Therefore, ∃x (P(x) A¬Q(x)) is true.

Conversely, assume that ∃x (P(x) A¬Q(x)) is true.

2. Then, there must exist at least one x such that P(x) is true and Q(x) is false.

3. But, if P(x) is true and Q(x) is false, then the implication P(x) → Q(x) is false.

4. Therefore, Vx (P(x) → Q(x)) is false.

Therefore, Vx (P(x) → Q(x)) and ∃x (P(x) A¬Q(x)) are logically equivalent.

(c) Translate the statement "The sum of two positive integers is always positive" into a logical expression.

Let P(x, y) be the statement "x and y are positive integers."

Let Q(x, y) be the statement "x + y is positive."

The logical expression representing the given statement would be:

$\forall x \forall y(x > 0 \land y > 0 \rightarrow x + y > 0)$

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(d) Translate the following statement into English, where C(x) is "x has a computer," F(x, y) is "x and y are friends," and the domain for both x and y consists of all students in your school.
$\forall x(C(x) \lor \exists y(C(y) \land F(x, y)))$

Every student in the school either has a computer or has a friend who has a computer.

The statement uses the following predicates:

C(x): "x has a computer"

F(x, y): "x and y are friends"

The domain of the statement is the set of all students in the school. The quantifiers in the statement mean the following:

∀x: For every student x in the school

∃y: There exists a student y in the school

The statement can be broken down into two parts:

∀x(C(x) ∨ ∃y(C(y) ∧ F(x, y))): For every student x in the school, either x has a computer (C(x)) or there exists a student y in the school such that y has a computer (C(y)) and x and y are friends (F(x, y)).

In other words, every student in the school either has a computer themselves or knows someone who does.

3.(a) 'Fallacy of denying the hypothesis' is a type of wrong reasoning. Explain with example.
(b) Show that the premises "A student in this class has not read the book," and "Everyone in 3 this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book."

(c) Prove the theorem "Prove that if n is an integer and n^2 is odd, then n is odd." by using appropriate method.

Given:

n is an integer

n^2 is odd

To prove:

n is odd

Proof by contrapositive:

Assume that n is even.

Then n can be written as 2k for some integer k.

So $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$.

Since $2k^2$ is an integer, n^2 is even.

This contradicts our assumption that n^2 is odd.

Therefore, our assumption must be false, which means that n must be odd.

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4.(a) The bit strings for the sets {1,2,3,4,5) and (1,3,5,7,9) are 1111100000 and 1010101010, respectively. Use bit strings to find the union and intersection of these sets.

The union of the sets {1, 2, 3, 4, 5} and {1, 3, 5, 7, 9} is {1, 2, 3, 4, 5, 7, 9},

and The intersection is {1, 3, 5}.

(b) Proof the following statements using set laws:

i) A-(B U C) = (A-B) n (A-C)

ii) (A-B')U (A-C') = A n (B U C).

Statement i:

A-(B U C) = (A-B) n (A-C)

Use code with caution. Learn more

Proof:

We know that A-(B U C) = {x in A | x not in B and x not in C}.

We also know that (A-B) n (A-C) = {x in A | x in A-B and x in A-C}.

Therefore, A-(B U C) = (A-B) n (A-C) because the two sets have the same elements.

Proof by set difference:

A-(B U C) = A - (B U C)

= A - B - C

= (A - B) - C

= (A-B) n (A-C)

Proof by set intersection:

A-(B U C) = A - (B U C)

= A - (B n C)

= (A - B) n (A - C)

Use code with caution. Learn more

Statement ii:

(A-B')U (A-C') = A n (B U C).

Proof:

We know that A-B' = {x in A | x not in B} and A-C' = {x in A | x not in C}.

We also know that B U C = {x in U | x in B or x in C}.

Therefore, (A-B')U (A-C') = A n (B U C) because the two sets have the same elements.

Proof by set difference:

(A-B')U (A-C') = A - B' - C'

= A - (B' n C')

= A n (B U C)

Proof by set intersection:

(A-B')U (A-C') = (A-B') n (A-C')

= (A - B) n (A - C)

= A n (B U C)

(c) Write down the properties of binary relations.

Properties of Binary Relations:

1. Reflexivity: A relation R on a set A is reflexive if every element of A is related to itself. In other words, for all a ∈ A, (a, a) ∈ R.
2. Irreflexivity: A relation R on a set A is irreflexive if no element of A is related to itself. In other words, for all a ∈ A, (a, a) ∉ R.
3. Symmetry: A relation R on a set A is symmetric if for every pair (a, b) ∈ R, the pair (b, a) is also in R. In other words, if (a, b) ∈ R, then (b, a) ∈ R.

4. Antisymmetry: A relation R on a set A is antisymmetric if for any distinct elements a and b in A, if (a, b) ∈ R and (b, a) ∈ R, then a = b. In other words, if (a, b) ∈ R and (b, a) ∈ R, then a = b.
5. Transitivity: A relation R on a set A is transitive if for any three elements a, b, and c in A, if (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R. In other words, if (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R.
6. Asymmetry: A relation R on a set A is asymmetric if for every pair (a, b) ∈ R, the pair (b, a) is not in R. In other words, if (a, b) ∈ R, then (b, a) ∉ R.
7. Antireflexivity: A relation R on a set A is antireflexive if no element of A is related to itself. In other words, for all a ∈ A, (a, a) ∉ R.
8. Connectedness: A relation R on a set A is connected if for every pair of distinct elements (a, b) ∈ R or (b, a) ∈ R. In other words, for any two distinct elements a and b in A, either (a, b) ∈ R or (b, a) ∈ R

Section-B

5.(a)Use Warshall's Algorithm to find the transitive closures of the relation

R = {(2, 1), (2,3), (3,1), (3,4), (4,1), (4,3)} on {1,2,3,4)

(b) Define 'poset' and 'lattice'.

Poset' stands for partially ordered set. It is a mathematical structure that consists of a set of elements along with a binary relation that defines a partial order among those elements

'Lattice' is a special type of poset. A lattice is a poset in which every pair of elements has both a unique greatest lower bound (also called meet or infimum) and a unique least upper bound (also called join or supremum

(**c) Determine whether the posets represented by each of the Hasse diagrams (E and F) in the following are lattices**

6.(a) How many edges are there in a graph with 10 vertices each of degree six?

In a graph, the sum of degrees of all vertices is equal to twice the number of edges. Therefore, for a graph with 10 vertices each of degree six, the sum of degrees would be 10 * 6 = 60.

Let's denote the number of edges in the graph as "E". According to the handshaking lemma, the sum of degrees is 2E. So we have:

2E = 60

Dividing both sides by 2, we get:

E = 60 / 2 = 30

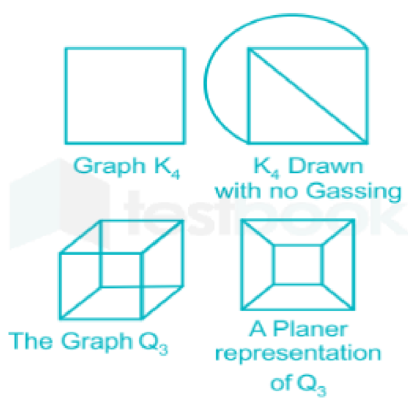
Therefore, in a graph with 10 vertices each of degree six, there would be 30 edges.

(b) Define n-Cube graph with example. Draw the Q3 graph with example.

The **n-Cube graph**, also known as the **hypercube graph or the n-dimensional cube graph**, is a graph that represents the corners and edges of an n-dimensional hypercube. It is a generalization of the cube graph (Q3) to higher dimensions.

The n-Cube graph has **2^n vertices**, each representing a corner of the hypercube, and an edge exists between two vertices if and only if the corresponding corners in the hypercube are adjacent.

The **Q3 graph**, also known as the **cube graph** or the 3-dimensional hypercube, represents the corners and edges of a cube. It consists **of 8 vertices and 12 edges.(half of vertices+vertices)**

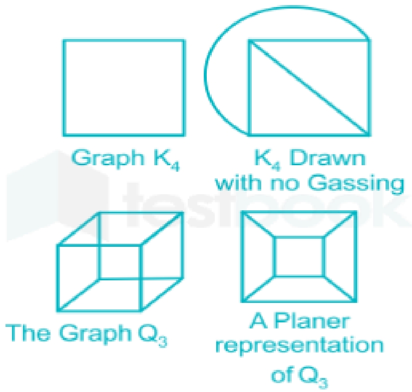


(c) How is adjacency matrix used to represent a graph? Discuss with example. Draw a graph with the following adjacency matrix

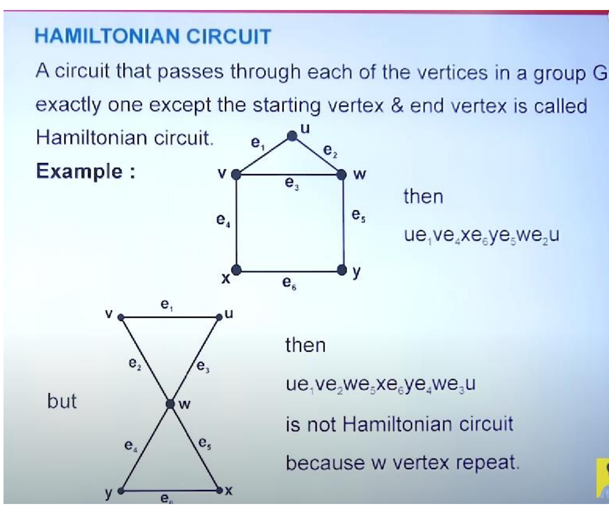
d) Give the reasons whether the following graphs G and H are isomorphic or not

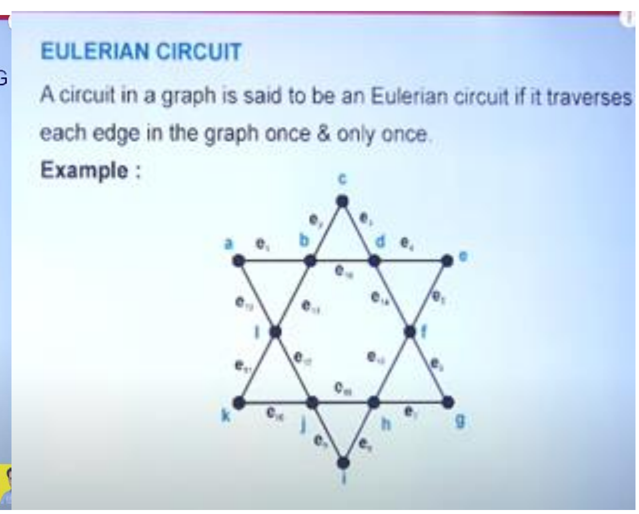
7. (a) What is planar graph? Is Q3 planar? Justify your answer with figure.

A planar graph is one that can be drawn in a plane without any edges crossing. For example, the complete graph K4 is planar,



(b) Define Hamilton circuit and Euler circuit. How do they differ from each other?





(c) How many paths of length four are there from a to d in the following simple graph?

(c) adjancy matrix of G (ordering the vertices as a,b,c,d) is

$$A = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 1 & 0 & 0 & 1 \\ c & 1 & 0 & 0 & 1 \\ d & 0 & 1 & 1 & 0 \end{array}$$

Hence the number of paths of length four from a to d is the (1,4) the entry of A^4. Because

$$A^4 = \begin{bmatrix} 8 & 0 & 0 & 8 \\ 0 & 8 & 8 & 0 \\ 0 & 8 & 8 & 0 \\ 8 & 0 & 0 & 8 \end{bmatrix}$$

there are exactly eight paths of length our from a to d By inspection of the graph, we see that a,b, a,b,d, a,b, a,c,d, a,b,d,b,d, a,b,d,c,d, a,c,a,b,d, a,c,d, a,c,d,b, d, and a,c,d,c,b are eight paths of length

8.(a) What is the prefix form for ((x + y) ↑ 2) + ((x-4)/3)?

\+ ↑ + x y 2 / - x 4 3

What is the value of the postfix expression 7 2 3 * -4 ↑ 9 3 / + ?

1. 7 2 3 * -4 ↑ 9 3 / +
2. 7 6 * -4 ↑ 9 3 / +
3. 42 -4 ↑ 9 3 / +
4. 38 9 / +
5. 38 3 +
6. 41 +

(b) What are the differences between graph and tree? Prove that a tree with n vertices has (n-1) edges.

(c) **Draw a binary tree to represent the following mathematical expression: (a - b)/(c+ (d-)).**

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      /
    /   \
   -     +
  / \   /  \
 a   b c    -
           / \
```

d e

Discrete Mathematics-2020

Section-A

1.(a) **Define Proposition with example. Consider the following sentences and determine which one is proposition or not.**

(i) **Colombo is the capital of Pakistan.**

(ii) **2+3=5**

iii) **x + 2 = 11**

(iv) **x+y=y+x, for every pair of real numbers x and y.**

(a) In logic, a **proposition** is a statement that can be either true or false. It is a declarative sentence that makes a claim about a subject. For example, "The Earth is round" is a proposition because it makes a claim about the shape of the Earth that can be either true or false.

Now let's consider the sentences given in the question:

(i) "Colombo is the capital of Pakistan." is a proposition because it is a false statement. Islamabad is the capital of Pakistan, not Colombo.

(ii) "2+3=5" is a proposition because it makes a claim that is true.

(iii) "x+2=11" is not a proposition **because it is an equation**, not a statement that can be either true or false.

(iv) "x+y=y+x, for every pair of real numbers x and y." is a proposition because it makes a claim that is true for all pairs of real numbers x and y.

(b**) What are the "contrapositive", "converse" and "inverse" of the conditional statement: "The home team wins whenever it is raining"?**

The conditional statement is "The home team wins whenever it is raining". Let's break down the terms:

- Hypothesis: "It is raining"
- Conclusion: "The home team wins"

The contrapositive, converse, and inverse of this conditional statement can be determined as follows:

1. **Contrapositive:** "If the home team does not win, then it is not raining." The contrapositive of a conditional statement flips the hypothesis and conclusion and negates both.
2. **Converse**: "Whenever the home team wins, it is raining." The converse of a conditional statement flips the hypothesis and conclusion.
3. **Inverse:** "If it is not raining, then the home team does not win." The inverse of a conditional statement negates both the hypothesis and the conclusion.

It's important to note that the truth value of the original statement, contrapositive, converse, and inverse can be different. In this case, the contrapositive and inverse statements are logically equivalent to the original statement and will have the same truth value, but the converse statement may not be true in all cases.

(c) Show that (p -q) (pvq) is a tautology.

p	q	¬p	(p v q)	[¬p ∧ (p v q)]	[¬p ∧ (p v q)] → q
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

(d) Translate the following sentence into logical expression: "You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

Let's break down the given English sentence and translate it into a logical expression step by step:

Let: P = "You can ride the roller coaster."

Q = "You are under 4 feet tall."

R = "You are older than 16 years old."

The sentence can be translated as follows:

"You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

Translated logical expressions:

$(Q \wedge \neg R) \rightarrow \neg P$

$P \rightarrow (\neg Q \vee R)$

Explanation: $(Q \wedge \neg R) \rightarrow \neg P$: "If you are under 4 feet tall and not older than 16 years old, then you cannot ride the roller coaster." $P \rightarrow (\neg Q \vee R)$: "If you can ride the roller coaster, then either you are not under 4 feet tall or you are older than 16 years old."

Therefore, $(Q \wedge \neg R) \rightarrow \neg P$ and $P \rightarrow (\neg Q \vee R)$ represent the corrected logical expressions for the given sentence.

2.(a) Express the statement "Every student in this class has studied calculus" using predicates and quantifiers.

$\forall x(S(x) \rightarrow C(x))$

Here is the breakdown of the expression:

- The first quantifier, $\forall x$, means "for all x."
- The predicate S(x) means "x is a student in this class."
- The predicate C(x) means "x has studied calculus."
- The arrow ($\rightarrow$) means "implies."

(b) Let Q(x, y) denote the statement "x+y=0." What are the truth values of the quantification $\exists y \forall x$ Q(x,y) and $\forall x \exists y$ Q(x,y) where the universe of discourse for the variables are the set of real number? Explain.

To evaluate the truth values of the quantifications $\exists y \forall x\, Q(x, y)$ and $\forall x \exists y\, Q(x, y)$, let's analyze them one by one:

$\exists y \forall x\, Q(x, y)$: This quantification states "There exists a y such that for all x, Q(x, y) is true."

To determine the truth value, we need to find a y that satisfies Q(x, y) for all x. In this case, Q(x, y) is true if $x + y = 0$.

Since the universe of discourse for the variables is the set of real numbers, we need to find a y that, when added to any real number x, yields zero. However, this is not possible since there is no single value of y that can satisfy this condition for all real numbers x.

Therefore, the statement ∃y∀x Q(x, y) is false.

∀x∃y Q(x, y): This quantification states "For all x, there exists a y such that Q(x, y) is true."

To determine the truth value, we need to check if there exists a y for each x such that Q(x, y) is true. Again, Q(x, y) is true if x + y = 0.

In this case, for any real number x, we can find a corresponding y such that x + y = 0. Specifically, if we choose y = -x, then x + y = x + (-x) = 0, which satisfies Q(x, y).

Since we can find a suitable y for each x, the statement ∀x∃y Q(x, y) is true.

To summarize:

The quantification ∃y∀x Q(x, y) is false.

The quantification ∀x∃y Q(x, y) is true.

(c) Let M(x, y) be "x has sent y an e-mail message" and T(x, y) be "x has telephoned y" where the universe of discourse is the set of all students in your class. Use quantifiers to express each of the following statements. (Assume all e-mail messages that were sent are received.)

(i) Every student in your class has sent e-mail message to Kamal.

(ii) No one in your class has telephoned Nondini.

(iii) Everyone in class has either telephoned Arup or sent him an e-mail message.

(ii) There is someone in your class who has either sent an e-mail message or telephoned everyone else in your class.

Sure, here are the statements expressed in quantifiers:

(i) Every student in your class has sent e-mail message to Kamal.

∀x(M(x, Kamal))

(ii) No one in your class has telephoned Nondini.

∀x(¬T(x, Nondini))

(iii) Everyone in class has either telephoned Arup or sent him an e-mail message.

$\forall x(T(x, Arup) \lor M(x, Arup))$

(iv) There is someone in your class who has either sent an e-mail message or telephoned everyone else in your class.

$\exists x(\forall y(M(x, y) \lor T(x, y)))$.

3.(**a) What do you mean by rules of inference? What rules of inference are used in the following famous argument? "All men are mortal. Socrates is a man. Therefore, Socrates is mortal."**

The argument you provided, "All men are mortal. Socrates is a man. Therefore, Socrates is mortal," can be understood as an application of the following rules of inference:

Universal Instantiation (UI): This rule allows us to instantiate a universally quantified statement by replacing the variable with a specific instance. In this argument, it is used to conclude that Socrates is mortal based on the premise "All men are mortal." By instantiating the universal statement with the specific instance of Socrates, we derive the conclusion that Socrates is mortal.

Modus Ponens (MP): Modus Ponens is a valid form of deductive reasoning that allows us to infer a conclusion from a conditional statement (if-then statement) and its antecedent (the "if" part) being true. In this argument, it is used to infer the conclusion "Socrates is mortal" from the premise "Socrates is a man" and the conditional statement "All men are mortal."

So, to summarize:

Universal Instantiation (UI) is used to instantiate the universal statement "All men are mortal" with the specific instance of Socrates.

Modus Ponens (MP) is used to derive the conclusion "Socrates is mortal" from the premise "Socrates is a man" and the conditional statement "All men are mortal."

(b) Show that the hypotheses H1, H2, and H3 lead to the conclusion C, where

H1: "If you send me an e-mail message, then I will finish writing the program."

H2: "If you do not send me e-mail message, then I will go to sleep early."

H3: "If1 go to sleep early, then I will wake up feeling refreshed."

C: "If I do not finish writing the program, then I will wake up feeling refreshed."

1. H1: "If you send me an e-mail message, then I will finish writing the program." This can be represented as: (E → F)
2. H2: "If you do not send me an e-mail message, then I will go to sleep early." This can be represented as: (¬E → S)
3. H3: "If I go to sleep early, then I will wake up feeling refreshed." This can be represented as: (S → R)
4. To prove C: "If I do not finish writing the program, then I will wake up feeling refreshed." This can be represented as: (¬F → R)

- 1.E → F (PREMISE)
- 2.¬E → S (PREMISE)
- 3.S → R (PREMISE)
- 4. ¬F→ ¬E (Contrapositive -1)
- 5.¬E→ R (2,3 hypothetical sylligsm)
- 6.¬E → R (4,5 hypothetical sylligsm)

(c) Suppose the domain of the propositional function P(x, y) consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.

i) 3x P(x, 3), ii) vy P(1, y), iii) By-P2, y) and iv) VxP(x, 2).

To write the given propositions using disjunctions and conjunctions, let's break them down one by one:

i) 3x P(x, 3): This proposition asserts that for all values of x (1, 2, and 3), P(x, 3) is true. Using disjunction (∨), we can express this proposition as: P(1, 3) ∨ P(2, 3) ∨ P(3, 3)

This disjunction represents the statement "P is true for x = 1 and y = 3 OR P is true for x = 2 and y = 3 OR P is true for x = 3 and y = 3."

ii) vy P(1, y): This proposition states that there exists a value of y for which P(1, y) is true. Using conjunction (∧), we can express this proposition as: P(1, 1) ∧ P(1, 2) ∧ P(1, 3)

This conjunction represents the statement "P is true for x = 1 and y = 1 AND P is true for x = 1 and y = 2 AND P is true for x = 1 and y = 3."

iii) Bx P(x, 2): It seems that there is a typo in this proposition. Assuming it was meant to be "∀x P(x, 2)" (for all x, P(x, 2)), we can represent it as a conjunction using: P(1, 2) ∧ P(2, 2) ∧ P(3, 2)

This conjunction represents the statement "P is true for x = 1 and y = 2 AND P is true for x = 2 and y = 2 AND P is true for x = 3 and y = 2."

iv) Vx P(x, 2): This proposition states that there exists a value of x for which P(x, 2) is true. Using disjunction (V), we can express this proposition as: P(1, 2) ∨ P(2, 2) ∨ P(3, 2)

This disjunction represents the statement "P is true for x = 1 and y = 2 OR P is true for x = 2 and y = 2 OR P is true for x = 3 and y = 2."

To summarize: i) 3x P(x, 3) is expressed as P(1, 3) ∨ P(2, 3) ∨ P(3, 3). ii) vy P(1, y) is expressed as P(1, 1) ∧ P(1, 2) ∧ P(1, 3). iii) Bx P(x, 2) is expressed as P(1, 2) ∧ P(2, 2) ∧ P(3, 2). iv) Vx P(x, 2) is expressed as P(1, 2) ∨ P(2, 2) ∨ P(3, 2).

4.(a) **Define composite function. Find fog and gof, where f(x) = x^2+1 and g(x) = x + 2 are functions from R to R.**

A composite function is a function that is obtained by applying one function to the result of another function. If we have two functions f and g, then the composite function fog (read as "f composed with g") is defined by (fog)(x) = f(g(x)), and the composite function gof (read as "g composed with f") is defined by (gof)(x) = g(f(x)).

In this case, we have f(x) = $x^2 + 1$ and g(x) = x + 2. Therefore:

(fg)(x) = f(g(x)) = f(x + 2) = $(x + 2)^2 + 1 = x^2 + 4x + 5$

(gof)(x) = g(f(x)) = g(x^2 + 1) = $x^2 + 1 + 2 = x^2 + 3$

So, (fg)(x) = $x^2 + 4x + 5$ and (gof)(x) = $x^2 + 3$.

(b) Suppose that the universal set is U= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10).

(i) Express the bit strings of the sets A = {2,4,7) and B = (1,3,6,9).

(ii) Find the value of A-B and AOB using bit strings.

(i) To express the sets A = {2, 4, 7} and B = {1, 3, 6, 9} as bit strings, we can use a 10-bit string where each bit corresponds to a number in the universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. If a bit is 1 in the bit string, it indicates that the corresponding number is in the set, and if it is 0, it indicates that the corresponding number is not in the set.

The bit string for set A is 0010010100, where the first bit corresponds to 1, the second bit corresponds to 2, the third bit corresponds to 3, and so on. The bits are 1 for the numbers 2, 4, and 7, and 0 for the other numbers in U.

The bit string for set B is 1001010001, where the first bit corresponds to 1, the second bit corresponds to 2, and so on. The bits are 1 for the numbers 1, 3, 6, and 9, and 0 for the other numbers in U.

(ii) To find the value of A - B using bit strings, we can use the following formula: A - B = A AND NOT(B)

Here, NOT(B) is the complement of the bit string for set B, which is obtained by flipping all the bits in the bit string.

NOT(B) = 0110101110

Now we can perform the AND operation between the bit string for A and the complement of B to get the bit string for A - B:

A AND NOT(B) = 0010010100 AND 0110101110 = 0010000100

Therefore, the set A - B = {4}.

To find the value of A U B using bit strings, we can use the following formula: A U B = A OR B

We can perform the OR operation between the bit strings for A and B to get the bit string for A U B:

A OR B = 0010010100 OR 1001010001 = 1011010101

Therefore, the set A U B = {1, 2, 3, 4, 6, 7, 9}.

(c) What is the bit string corresponding to the symmetric difference of two sets? Suppose that the universal set is U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10). Express each of these sets with bit strings where the ith bit in the string is 1 if i is in the set and 0 otherwise : i) (3, 4, 5) and ii) (1, 3, 6, 10).

the symmetric difference of two sets A and B is the set of elements that are in either A or B but not in their intersection. In other words, the symmetric difference of A and B is the union of A and B minus their intersection.

We can express the symmetric difference of two sets A and B using bit strings as follows:

1. Compute the bit string of A minus B: A - B
2. Compute the bit string of B minus A: B - A
3. Compute the bit string of the symmetric difference of A and B: (A - B) OR (B - A)

To express the sets {3, 4, 5} and {1, 3, 6, 10} with bit strings where the ith bit in the string is 1 if i is in the set and 0 otherwise, we can use the following 10-bit string where each bit position corresponds to a number in the universal set U. If a bit is 1 in the bit string, it indicates that the corresponding number is in the set, and if it is 0, it indicates that the corresponding number is not in the set.

The bit string for set A = {3, 4, 5} is 0011100000, where the first bit corresponds to 1, the second bit corresponds to 2, the third bit corresponds to 3, and so on. The bits are 1 for the numbers 3, 4, and 5, and 0 for the other numbers in U.

The bit string for set B = {1, 3, 6, 10} is 1001001001, where the first bit corresponds to 1, the second bit corresponds to 2, and so on. The bits are 1 for the numbers 1, 3, 6, and 10, and 0 for the other numbers in U.

Now, we can compute the bit string of the symmetric difference of A and B:

A - B = 0011000000 B - A = 1000101001

(A - B) OR (B - A) = 1011101001

The bit string 1011101001 represents the set {1, 4, 5, 6, 10}, which is the symmetric difference of the sets A and B.

Section-B

5.(a) Define binary relation with example. Explain the properties of relation.

(a) a binary relation is a collection of pairs of elements from A.

Example: Let's consider a set A = {1, 2, 3}. A binary relation R on A could be R = {(1, 1), (1, 2), (2, 1)}. This relation R indicates that 1 is related to itself (reflexive property), 1 is related to 2, and 2 is related to 1 (symmetry property).

The properties of a relation are as follows:

Reflexivity: A relation R is reflexive if every element of A is related to itself. In other words, for all a ∈ A, (a, a) ∈ R.

Irreflexivity: A relation R is irreflexive if no element of A is related to itself. In other words, for all a ∈ A, (a, a) ∉ R.

Symmetry: A relation R is symmetric if for every pair (a, b) ∈ R, the pair (b, a) is also in R. In other words, if (a, b) ∈ R, then (b, a) ∈ R.

Antisymmetry: A relation R is antisymmetric if for any distinct elements a and b in A, if (a, b) ∈ R and (b, a) ∈ R, then a = b. In other words, if (a, b) ∈ R and (b, a) ∈ R, then a = b.

Transitivity: A relation R is transitive if for any three elements a, b, and c in A, if (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R. In other words, if (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R.

Asymmetry: A relation R is asymmetric if for every pair (a, b) ∈ R, the pair (b, a) is not in R. In other words, if (a, b) ∈ R, then (b, a) ∉ R.

Antireflexivity: A relation R is antireflexive if no element of A is related to itself. In other words, for all a ∈ A, (a, a) ∉ R.

Connectedness: A relation R is connected if for every pair of distinct elements (a, b) ∈ R or (b, a) ∈ R. In other words, for any two distinct elements a and b in A, either (a, b) ∈ R or (b, a) ∈ R.

(b) Let A = (1,2,3,4) and R be the relation on A where (a, b) E R if and only if a> b. Represent the relation in arrow diagram, zero-one matrix and graph.

(b) Let A = {1, 2, 3, 4} and R be the relation on A where (a, b) ∈ R if and only if a > b.

Arrow diagram representation:

1 → 2

1 → 3

1 → 4

2 → 3

2 → 4

3 → 4

Zero-one matrix representation:

	1	2	3	4
1	0	0	0	0
2	1	0	0	0
3	1	1	0	0
4	1	1	1	0

Graph representation:

The graph would have vertices representing elements of set A and directed edges pointing from the smaller element to the larger element based on the relation.

  1

 ↙ ↘

2   3

↓ ↘ ↓

  4 →

(c) How many reflexive relations are there on a set with n elements? Explain.

(c) Reflexive relations on a set with n elements:

For a set with n elements, each element can either be related to itself or not. Thus, for each element, there are two choices (either include the pair (a, a) or exclude it). Since there are n elements in the set, there are 2^n ways to choose reflexive pairs for each element.

Therefore, the total number of reflexive relations on a set with n elements is 2^n.

6.(a) What is lattice? Show that every totally ordered set is a lattice.

A lattice is a partially ordered set (L, ≤) in which every subset {a, b} consisting of two elements has a least upper bound and a greatest lower bound.

SOLUTION

To proof: Every totally ordered set is a lattice.

PROOF

Let (S, R) be a totally ordered set.

Poset

Since (S, R) is a totally ordered set, (S, R) is a partially ordered set (poset).

Lattice

Let a and b be two elements of S.

Since (S, R) is a totally ordered set, a and b are comparable.

Since a and b are comparable: $a < b$ or $a > b$ or $a = b$.

If $a < b$, then the least upper bound is b and the greatest lower bound is a.
If $a > b$, then the least upper bound is a and the greatest lower bound is b.
If $a = b$, then the least upper bound is a and the greatest upper bound b.

Thus we note that there exists a least upper bound and a greatest lower bound in each case, which implies that (S, R) is a lattice.
□

LCM({1, 2, 4, 5, 10}) = 20 (since 20 is the smallest number divisible by all five numbers)

The least upper bound (lub) of the sets {3, 9, 12} and {1, 2, 4, 5, 10} in the poset (Z+, |) is the least common multiple of the sets, which is 20.

In summary, the greatest lower bound (glb) of the sets {3, 9, 12} and {1, 2, 4, 5, 10} in the poset (Z+, |) is 1, and the least upper bound (lub) is 20.

(c) Use Warshall's Algorithm to find the transitive closures of the relation R=((1,2), (1,3), (1,4), (2,3), (2,4), (3,4)) on (1,2,3,4)

R = { (1,2), (1,3), (1,4), (2,3), (2,4), (3,4) }

Matrix representation:

| 1 2 3 4

----

1 | 0 1 1 1

2 | 0 0 1 1

3 | 0 0 0 1

4 | 0 0 0 0 Now, we will apply Warshall's Algorithm to update the matrix to find the transitive closure:Warshall's Algorithm:

Step 1 (k = 1):

Matrix after k = 1:

| 1 2 3 4

----

1 | 0 1 1 1

2 | 0 0 1 1

3 | 0 0 0 1

4 | 0 0 0 0

Step 2 (k = 2):

Matrix after k = 2:

| 1 2 3 4

----

1 | 0 1 1 1

2 | 0 0 1 1

3 | 0 0 0 1

4 | 0 0 0 0

Step 3 (k = 3):

Matrix after k = 3:

| 1 2 3 4

----

1 | 0 1 1 1

2 | 0 0 1 1

3 | 0 0 0 1

4 | 0 0 0 0

Step 4 (k = 4):

Matrix after k = 4:

| 1 2 3 4

----

1 | 0 1 1 1

2 | 0 0 1 1

3 | 0 0 0 1

4 | 0 0 0 0

The final matrix represents the transitive closure of the relation R. The entry (i, j) in the matrix is 1 if there exists a directed path from i to j in the relation R, and 0 otherwise.

In this case, the transitive closure matrix is the same as the original matrix because the relation R is already transitive. All pairs that can reach each other directly are the only ones that can reach each other through any number of intermediate steps as well.

7.(0) Define 'Pseudograph' and 'Multigraph' with example. Write the differences between them.

Differences between Pseudograph and Multigraph:

1. Parallel Edges:
    - Pseudograph: Allows multiple edges (parallel edges) between the same pair of vertices.
    - Multigraph: Also allows multiple edges (parallel edges) between the same pair of vertices.
2. Loops:
    - Pseudograph: Permits loops, which are edges that connect a vertex to itself.
    - Multigraph: Does not allow loops; there are no edges that start and end at the same vertex.

(Find the adjacency matrix of the given graphs:

(c) Define incidence matrix with example. Represent the following graph with an incidence matrix.

9. **(a) Explain Euler path with example. Discuss the necessary and sufficient condition to have a Euler path in a graph.**

An Euler path is a path in a graph that visits each edge exactly once. To have an Euler path in a graph, there are two necessary and sufficient conditions:

Connected Graph: The graph must be connected, meaning there is a path between every pair of vertices. If the graph is not connected, there will be disconnected components, and it won't be possible to traverse all edges with a single path.

Eulerian or Semi-Eulerian Graph: The graph must be either Eulerian or semi-Eulerian.

a. Eulerian Graph: An Eulerian graph is a graph where every vertex has an even degree (the number of edges incident to the vertex). In other words, all vertices in the graph have an even number of edges connected to them.

b. Semi-Eulerian Graph: A semi-Eulerian graph is a graph where exactly two vertices have an odd degree, and all other vertices have an even degree.

(b) What is full m-ary tree?

A full m-ary tree is a type of m-ary tree in which every internal (non-leaf) node has exactly m children. In other words, all internal nodes in a full m-ary tree have a fixed number of children, and all leaves have zero children

When a rooted m-ary tree is called balanced?

Discrete Mathematics-2019

Section A

1.(a) Define proposition and predicate calculus with example.

- **Proposition**: A proposition is a statement that can be either true or false, but not both. It is a declarative sentence that is capable of being assigned a truth value. Propositions are denoted by letters, such as P, Q, R, etc. Examples of propositions include:
- "The sun is shining."
- "2 + 2 = 4."
- "Water boils at 100 degrees Celsius."
- **A predicate** is a statement that takes one or more variables as input and returns a truth value. For example, the predicate "x is taller than y" can be used to represent the statement "The person x is taller than the person y." In predicate calculus, we can use quantifiers to state that a predicate is true for all or some members of a set. For example, the statement "All dogs have four legs" can be expressed in predicate calculus as follows:

∀x(Dog(x) -> HasFourLegs(x))

(b) Give the converse, inverse and contrapositive of the implication" If it is raining then I get wet".

The implication "If it is raining then I get wet" can be represented as:

P: It is raining. Q: I get wet.

Converse: If I get wet, then it is raining. Inverse: If it is not raining, then I do not get wet. Contrapositive: If I do not get wet, then it is not raining.

(c) Check whether ((PQ)→R) V-Pis a tautology.

(d) Show that (¬P ∧ (¬QAR)) V (QAR) V(PAR) = R. [Use only Laws]

2. (a) Find the disjunctive and conjunctive normal form for P^(P-Q)

The propositional statement is P ∧ (P → ¬Q).

The disjunctive normal form (DNF) represents the statement as a disjunction (logical OR) of conjunctions (logical AND) of literals (variables or their negations).

To convert the statement to DNF:

1. Expand the implication using the material implication rule: P → Q is equivalent to ¬P ∨ Q.
2. Distribute ∧ over ∨ using the distributive property: (A ∧ B) ∨ C is equivalent to (A ∨ C) ∧ (B ∨ C).

Applying these steps, we have: P ∧ (P → ¬Q) = P ∧ (¬P ∨ ¬Q) = (P ∧ ¬P) ∨ (P ∧ ¬Q) = ⊥ ∨ (P ∧ ¬Q) = P ∧ ¬Q

Therefore, the disjunctive normal form (DNF) for the statement P ∧ (P → ¬Q) is P ∧ ¬Q.

The conjunctive normal form (CNF) represents the statement as a conjunction of disjunctions of literals.

To convert the statement to CNF:

1. Expand the implication using the material implication rule: P → Q is equivalent to ¬P ∨ Q.
2. Distribute ∨ over ∧ using the distributive property: A ∨ (B ∧ C) is equivalent to (A ∨ B) ∧ (A ∨ C).

Applying these steps, we have: P ∧ (P → ¬Q) = P ∧ (¬P ∨ ¬Q) = (P ∧ ¬P) ∨ (P ∧ ¬Q) = ⊥ ∨ (P ∧ ¬Q) = (P ∨ ⊥) ∧ (¬Q ∨ ⊥) = P ∧ ¬Q

Therefore, the conjunctive normal form (CNF) for the statement P ∧ (P → ¬Q) is P ∧ ¬Q.

(b) Give reasons for each step needed to show that the following argument is valid, using

rules of inference: [P ∧ (P→Q)A(S∧ R) ∧ R¬Q] → (SVT).

(c) Let p(x, y), q(x, y) and r(x,y) represent open statements, with replacements for the variables x, y chosen from some prescribed universe. Write the negation mentioning the rules used: Vxy [(p(x, y) Ag(x, y))→r(x, y)].

3.(a) Prove the theorem "If 3n+2 is odd, then n is odd" by using appropriate proof method. (b) Define composite function. Find f°g 8°f and where

(c) The bit strings for the sets {1,2,3,4,5} and {1,3,5,7,9} are 1111100000 and 3

1010101010respectively. Use bit strings to find the union and intersection of these sets.

f(x)=2x+3

g(x)=3x+2

are functions from R to R.

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4.

(a) Define complement of a set and power set.

The complement of a given set consists of all the elements in the universal set that are not present in the given set.

In other words, it is the set of all elements that do not belong to the given set.

Formally, if A is a set and U is the universal set, the complement of A is denoted as A' or A^c and is defined as:

A' = {x ∈ U | x ∉ A}

The power set of a set is the set of all possible subsets of that set, including the empty set and the set itself. In other words,

it is the collection of all possible combinations of elements from the original set.

To illustrate with an example, let's consider a set A = {1, 2}. The power set of A, denoted as P(A), would be:

P(A) = { {}, {1}, {2}, {1, 2} }.

(b). Give a proof using set laws: (i) A - (B ∪ C) = (A - B) ∪ (A - C).) (ii) (A-B') U(A-

1.

To prove the equality (i) A - (B ∪ C) = (A - B) ∪ (A - C), we can use set laws and logical reasoning. Let's break it down step by step:

Start with the left-hand side of the equation: A - (B ∪ C).

Apply the definition of set difference: A ∩ (B ∪ C)'.

Use De Morgan's law: A ∩ (B' ∩ C').

Distribute the intersection operator (∩) over the union operator (∪): (A ∩ B') ∩ (A ∩ C').

Apply the definition of set difference: (A - B) ∩ (A - C).

Use the distributive law of sets: (A - B) ∪ (A - C).

Now, we have arrived at the right-hand side of the equation: (A - B) ∪ (A - C).

By following these steps, we have shown that the left-hand side of the equation is equal to the right-hand side, and hence, we have proven the equality (i):

A - (B ∪ C) = (A - B) ∪ (A - C).

3

3

C) AN (BNC)

(c) If A = {1, 2, 3, 4} give an example of a relation R on set A which is i) Reflexive and 2.75 symmetric, but not transitive ii) symmetric and transitive, but not reflexive.

Section B

5.(a) Explain inverse of a relation, with suitable example.

(a) The inverse of a relation R on a set A is a new relation that reverses the direction of the ordered pairs in R. In other words, if (a, b) is in R, then (b, a) will be in the inverse relation.

Example: Let's consider a relation R on the set A = {1, 2, 3} defined as R = {(1, 2), (2, 3)}. The inverse relation of R, denoted as R^-1, will have the reversed pairs:

R^-1 = {(2, 1), (3, 2)}

Here, (1, 2) in R is reversed to (2, 1) in R^-1, and (2, 3) in R is reversed to (3, 2) in R^-1.

(b) Given R = {(x,y): X+ 3y = 12};

i) Write R as a set of ordered pairs

ii) Find domain, 4 range, inverse of R

iii) Find R.R.

Given R = {(x, y): x + 3y = 12}: i) Writing R as a set of ordered pairs involves finding all pairs (x, y) that satisfy the equation x + 3y = 12.

By rearranging the equation, we have: x = 12 - 3y

Therefore, R can be written as: R = {(12 - 3y, y): y is a real number}

ii) Finding the domain and range of R: Domain of R: The set of all x-values that appear in the ordered pairs of R. In this case, the domain is the set of all possible values of (12 - 3y). As y can take any real number, the domain is (-∞, +∞).

Range of R: The set of all y-values that appear in the ordered pairs of R. Since y can take any real number, the range is also (-∞, +∞).

Inverse of R: To find the inverse of R, we need to reverse the ordered pairs. In this case, the inverse relation will be:

R^-1 = {(y, 12 - 3y): y is a real number}

iii) Finding R.R (composition of R with itself): To find R.R, we need to perform the composition of R with itself. This involves taking each element from the first set of ordered pairs and finding the corresponding elements in the second set.

R.R = {(x, z): (x, y) ∈ R and (y, z) ∈ R}

In this case, we substitute the values of x and z into the equation x + 3y = 12, and solve for y.

(c) A= (a, b, c), R= {(a, a), (a, b), (b, c), (c, c)}, find reflexive, symmetric and transitive closure.

Given A = {a, b, c} and R = {(a, a), (a, b), (b, c), (c, c)}:

Reflexive Closure: The reflexive closure of R is the smallest reflexive relation that contains R. To make R reflexive, we need to add (b, b) and (c, c) to R.

Reflexive closure of R: {(a, a), (a, b), (b, c), (c, c), (b, b)}

Symmetric Closure: The symmetric closure of R is the smallest symmetric relation that contains R. We need to add pairs (b, a) and (c, b) to R to make it symmetric.

Symmetric closure of R: {(a, a), (a, b), (b, c), (c, c), (b, b), (b, a), (c, b)}

Transitive Closure: The transitive closure of R is the smallest transitive relation that contains R. Since R is already transitive, no additional pairs need to be added.

Transitive closure of R: {(a, a), (a, b), (b, c), (c, c), (b, b), (b, a), (c, b)}

Therefore, the reflexive closure of R is {(a, a), (a, b), (b, c), (c, c), (b, b)}, the symmetric closure is {(a, a), (a, b), (b, c), (c, c), (b, b), (b, a), (c, b)}, and the transitive closure is {(a, a), (a, b), (b, c), (c, c), (b, b), (b, a), (c, b)}.

6.(a) What is Hasse diagram?

A Hasse diagram is a *graphical representation* of the relation of elements of a partially ordered set (poset) with an implied *upward orientation*.

Draw the Hasse diagram for the poset ({1, 2, 3, 4, 6, 8, 12), 1).

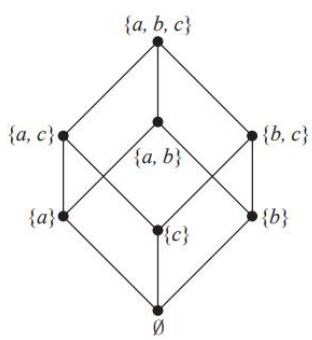


FIGURE 4 **The Hasse Diagram of $(P(\{a, b, c\}), \subseteq)$.**

(b) Define lattice. Determine whether the posets ({1,2,3,4, 5},1) and ({1,2,4,8,16},1) are lattices.

(c) Answer the following questions concerning the poset ({3, 5, 9, 15, 24, 45}, |) Find:

i) the maximal element

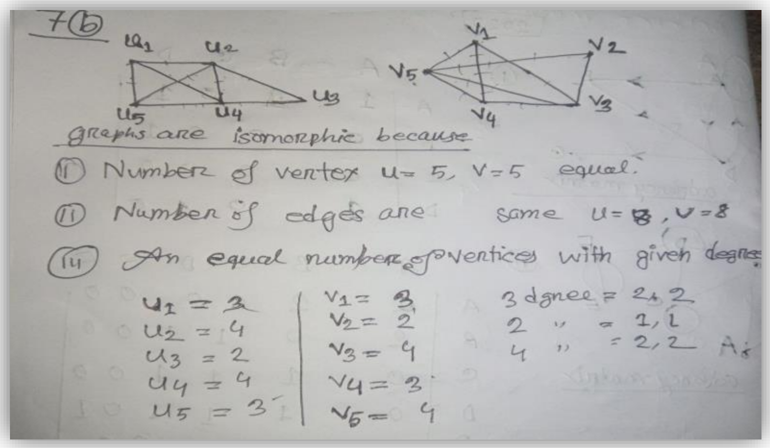
ii) the minimal element

iii) the least upper bound of (3, 5), if it exists.

iV) The greatest lower bound of {15, 45}, if it exist.

7.(a) Define Bipartite Graph and complete bipartite graph with example.

b) Examine whether the following pair of graphs are isomorphic. If not isomorphic, give the reasons.



(c) Define Eulerian graph and Hamiltonian graph. Give an example of a graph which is Eulerian but not Hamiltonian and vice versa.

(d) Show that the complete bipartite graph K,, has the Hamiltonian cycle.

8.(a) What is the prefix form for ((x + y) ↑ 2) + ((x-4)/3)? what is the value of the postfix expression 7 2 3*-4193/+?

(b) What is the difference between graph and tree?

Graph

- A graph is a non-linear data structure that consists of a set of vertices and a set of edges that connect pairs of vertices.
- A graph can have any number of vertices and edges, and there can be multiple paths between any two vertices.
- Graphs can be directed or undirected. In a directed graph, the edges have a direction, which indicates the order in which the vertices are connected. In an undirected graph, the edges do not have a direction.

Prove that a tree with n vertices has (n-1) edges.

-

Tree

- A tree is a special type of graph that is connected and acyclic.
- A connected graph is a graph in which there is a path between any two vertices.
- An acyclic graph is a graph that does not contain any cycles.
- A tree with n vertices has n-1 edges.

(c) Find a spanning tree for the graph shown by removing edges in simple circuits.

Discrete Mathematics-2018

Section A

1.(a) What are contradiction and tautology? Explain with example.

- Tautology

A tautology is a statement that is always true, no matter what. For example, the statement "All bachelors are unmarried" is a tautology. This is because the definition of "bachelor" is "an unmarried man." Therefore, it is impossible for a bachelor to be married.

- Contradiction

A contradiction is a statement that is always false, no matter what. For example, the statement "This statement is false" is a contradiction. This is because if the statement is true, then it is false, and if it is false, then it is true. This is a logical impossibility.

- Tautologies:
  - All squares are rectangles.
  - Every even number is divisible by 2.
  - The sum of all interior angles in a triangle is 180 degrees.
- Contradictions:
  - This statement is false.
  - A square is a circle.
  - 2 + 2 = 5.

(b) Show that (p →r) V (q →r) = (p A q) → r using logical equivalence.

Proof using Logical Equivalence

$(p \to r) \lor (q \to r)$	
$\equiv (\neg p \lor r) \lor (\neg q \lor r)$	Definition of implication
$\equiv \neg p \lor r \lor \neg q \lor r$	Associative
$\equiv \neg p \lor \neg q \lor r \lor r$	Commutative
$\equiv (\neg p \lor \neg q) \lor (r \lor r)$	Associative
$\equiv \neg (p \land q) \lor r$	De Morgan, Idempotent
$\equiv (p \land q) \to r$	Definition of implication

Therefore, we have shown that $(p \to r) \lor (q \to r)$ is logically equivalent to $(p \land q) \to r$.

(c) Express the following statements using the propositions as below:

p: The message is scanned for viruses

q: The message was sent from an unknown system together with logical connectives (including negations):

i) The message is scanned for viruses whenever the message was sent

from an unknown system.

ii) The message was sent from an unknown system but it was not

scanned for viruses.

iii) It is necessary to scan the message for viruses whenever it was sent

from an unknown system.

iv) When a message is not sent from an unknown system it is not

scanned for viruses.

i) The message is scanned for viruses whenever the message was sent from an unknown system.

This statement can be represented as the implication: $q \to p$

Explanation:

- q: The message was sent from an unknown system.
- p: The message is scanned for viruses.

- q → p: Represents the implication that if the message was sent from an unknown system, then it is scanned for viruses.

ii) The message was sent from an unknown system but it was not scanned for viruses.

This statement can be represented as the conjunction with negation: q ∧ ¬p

Explanation:

- q: The message was sent from an unknown system.
- p: The message is scanned for viruses.
- q ∧ ¬p: Represents the conjunction of "The message was sent from an unknown system" and "It was not scanned for viruses."

iii) It is necessary to scan the message for viruses whenever it was sent from an unknown system.

This statement can be represented as the implication: q → p

Explanation:

- q: The message was sent from an unknown system.
- p: The message is scanned for viruses.
- q → p: Represents the implication that if the message was sent from an unknown system, then it is necessary to scan it for viruses.

iv) When a message is not sent from an unknown system, it is not scanned for viruses.

This statement can be represented as the implication with negation: ¬q → ¬p

Explanation:

- ¬q: The message is not sent from an unknown system.
- ¬p: The message is not scanned for viruses.
- ¬q → ¬p: Represents the implication that if the message is not sent from an unknown system, then it is not scanned for viruses.

So, the corrected logical expressions for the given statements are:

i) q → p ii) q ∧ ¬p iii) q → p iv) ¬q → ¬p

2. **(a) State the rules of inferences for propositional logic**

The rules of inference for propositional logic are as follows:

1. Modus Ponens (MP): If we have a conditional statement (P → Q) and we also have P, we can infer Q. Symbolically: (P → Q), P ⊢ Q.
2. Modus Tollens (MT): If we have a conditional statement (P → Q) and we also have ¬Q, we can infer ¬P. Symbolically: (P → Q), ¬Q ⊢ ¬P.
3. Disjunctive Syllogism (DS): If we have a disjunction (P ∨ Q) and we know ¬P, we can infer Q. Symbolically: (P ∨ Q), ¬P ⊢ Q.
4. Addition (ADD): If we have a proposition P, we can infer the disjunction (P ∨ Q) for any proposition Q. Symbolically: P ⊢ (P ∨ Q).
5. Simplification (SIMP): If we have a conjunction (P ∧ Q), we can infer either P or Q individually. Symbolically: (P ∧ Q) ⊢ P or (P ∧ Q) ⊢ Q.
6. Conjunction (CONJ): If we have two propositions P and Q, we can infer their conjunction (P ∧ Q). Symbolically: P, Q ⊢ (P ∧ Q).
7. Constructive Dilemma (CD): If we have two conditional statements (P → Q) and (R → S), and a disjunction (P ∨ R), we can infer the disjunction (Q ∨ S). Symbolically: (P → Q), (R → S), (P ∨ R) ⊢ (Q ∨ S).

(b) Show that R∧ (P V Q) is a valid conclusion from the premises P V Q, Q→R, P→M and ¬M.

1. P ∨ Q (Premise)
2. Q → R (Premise)
3. P → M (Premise)
4. ¬M (Premise)
5. ¬(R ∧ (P ∨ Q)) (Assumption for contradiction)
6. ¬R ∨ ¬(P ∨ Q) (De Morgan's law, applied to 5)
7. ¬R ∨ (¬P ∧ ¬Q) (De Morgan's law, applied to 6)
8. (¬R ∨ ¬P) ∧ (¬R ∨ ¬Q) (Distribution, applied to 7)
9. (P ∨ Q) → (¬R ∨ ¬P) (Contrapositive, applied to 8)
10. (P ∨ Q) → (P → ¬R) (Implication, applied to 9)

11. (P ∨ Q) → (¬R) (Hypothetical syllogism, applied to 10 and 3)
12. ¬(¬R) ∨ (P ∨ Q) (Implication, applied to 11)
13. R ∨ (P ∨ Q) (Double negation, applied to 12)
14. (R ∨ P) ∨ Q (Associativity, applied to 13)
15. P ∨ (R ∨ Q) (Associativity, applied to 14)
16. R ∨ Q (Disjunctive syllogism, applied to 15 and 1)
17. Q ∨ R (Commutation, applied to 16)
18. R ∧ (Q ∨ R) (Conjunction, applied to 16 and 17)
19. R ∧ (P ∨ Q) (Associativity, applied to 18)

(c) Show that the hypothesis,

"It is not sunny this afternoon and it is colder than 3 yesterday",

"We will go swimming only if it is sunny",

"If we do not go swimming, then we will take a canoe trip" and

"If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset".

1. Hypothesis: It is not sunny this afternoon and it is colder than 3 yesterday. This can be represented as: ¬S ∧ C
2. Hypothesis: We will go swimming only if it is sunny. This can be represented as: S → G
3. Hypothesis: If we do not go swimming, then we will take a canoe trip. This can be represented as: ¬G → T
4. Hypothesis: If we take a canoe trip, then we will be home by sunset. This can be represented as: T → H

We want to prove the conclusion: We will be home by sunset (H).

1. ¬S ∧ C (premise)
2. S → G (premise)
3. ¬G → T (premise)
4. T → H (premise)
5. ¬S (conjunction elimination of step 1)

6. ¬G (modus ponens of step 2 and 5)

7. T (modus ponens of step 3 and 6)

8. H (modus ponens of step 8 and 4)

EXAMPLE 6 Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."

Extra Examples

Solution: Let p be the proposition "It is sunny this afternoon," q the proposition "It is colder than yesterday," r the proposition "We will go swimming," s the proposition "We will take a canoe trip," and t the proposition "We will be home by sunset." Then the premises become $\neg p \wedge q$, $r \to p$, $\neg r \to s$, and $s \to t$. The conclusion is simply t. We need to give a valid argument with premises $\neg p \wedge q$, $r \to p$, $\neg r \to s$, and $s \to t$ and conclusion t.

We construct an argument to show that our premises lead to the desired conclusion as follows.

Step	Reason
1. $\neg p \wedge q$	Premise
2. $\neg p$	Simplification using (1)
3. $r \to p$	Premise
4. $\neg r$	Modus tollens using (2) and (3)
5. $\neg r \to s$	Premise
6. s	Modus ponens using (4) and (5)
7. $s \to t$	Premise
8. t	Modus ponens using (6) and (7)

3.**(a) Define Disjunctive Normal Form and Conjunctive Normal Form. '**

Disjunctive Normal Form (DNF) and Conjunctive Normal Form (CNF) are two specific forms used to represent logical expressions in propositional logic.

Disjunctive Normal Form (DNF): DNF is a way of representing a logical expression as a disjunction (OR) of one or more conjunctions (AND) of literals. A literal is either a variable or its negation. In DNF, the whole expression is true if at least one of the conjunctions is true. Here's an example of a logical expression in DNF

(b) Show that the premises " A student in this class has not read the book", and

"Everyone in this class passed the first exam" imply the conclusion

"Someone who passed the first exam has not read the book"

(c) Write the negation of the statements i) (∃x)(∀y) p(x, y) ii) ∀ y(x^2>x) and ∃x (x^2=2).

he negation of the statements would be as follows:

i) (∃x)(∀y) p(x, y)

Negation: (∀x)(∃y) ¬p(x, y)

The negation of $(\exists x)(\forall y)\ p(x, y)$ is $(\forall x)(\exists y)\ \neg p(x, y)$, which asserts that for every x, there exists a y for which p(x, y) is false.

ii) $(\forall y)(x^2 > x) \land (\exists x)(x^2 = 2)$

Negation: $(\exists y)(x^2 \leq x) \lor (\forall x)(x^2 \neq 2)$

(d) Let P(x,y) denote the statement x = y+3. What are the truth values of the Proposition P(1,2) and P(3,0).

The statement P(x, y) denotes the statement "x = y + 3." To determine the truth values of the propositions P(1, 2) and P(3, 0), we substitute the values of x and y into the statement and evaluate it:

P(1, 2): Substitute x = 1 and y = 2 into the statement: 1 = 2 + 3

This statement is not true since 1 does not equal 2 + 3. Therefore, the truth value of P(1, 2) is False.

P(3, 0): Substitute x = 3 and y = 0 into the statement: 3 = 0 + 3

This statement is true since 3 does equal 0 + 3. Therefore, the truth value of P(3, 0) is True.

To summarize:

P(1, 2) is False.

P(3, 0) is True.

4.(a) Prove DeMorgan's law for set intersection, (AB)'=A' UB!

(b) Suppose A is the set of distinct letters in the word elephant, B is the set of distinct letters in the word sycophant, C is the set of distinct letters in the word fantastic, and D is the set of distinct letters in the word student. The universe U is the set of 26 lowercase letters of the English alphabet. Find:

i) AUB

ii) Anc An (CUD)

(c) A={1, 2, 3, 4, 5}; B= (6, 7, 8, 9, 10); C-(a, b, c, d, e} f::A→B,

D= {7, 8, 9, 10};

f={(1, 7), (2, 6), (3, 9), (4, 7), (5, 10)} g={(6, b), (7, a), (6, d), (8, c), (10, b)}

(i) Is f a function? Why or why not? (ii) Is f injective (that is, one-to-one)? Why or why not? (iii) Is f surjective (that is, onto)? Why or why not? (iv) Is g a function? Why or why not?

Section B

5.(a) Define the properties of relation with suitable example. (b) Let R be the relation represented by the matrix:

2 3

[0 1 1]

MR = 1 1 0 L1 0 1 :

Find the matrix representing

i) R1, ii) R and iii) R2 (c) Let R, and R_2 be relations on a set A represented by the matrices

[0 1 0]

[0 1 01

MR11 1 1A MR2 =0 1 1

L1 0 0

Find the matrices that represent (1) R, R2 (ii) R2 R

6.(a) Draw the Hasse diagram representing the partial ordering ((a, b) | a divides b} set {1, 2, 3, 4, 6, 8, 12).

on 3

(b) Use Warshall's Algorithm to find the transitive closures of the relation R= {(1, (1, 4), (2, 1), (2, 3), (3, 1), (3, 2), (3, 4), (4, 2)} on set {1, 2, 3, 4}.

1), 4

(c) Let R be the relation on the set of real numbers such that xRy if and only if x and y are real numbers that differ by less than 1, that is x - y < 1. Show that R is not an equivalence relation.

7.(a) Define regular graph and a complete graph. What is meant by isomorphism of graphs?

(b) Draw the complete graph Ks with vertices A, B, C, D, E. Draw all complete sub graph of K, with 4 vertices. Are the sub graphs bipartite, explain why or why not?

(c) Explain Euler paths and circuits in a graph with example.

8.(a) Draw a binary tree to represent the following mathematical expression: (a - b)/(c*(d-e))

```
  ÷
 / \
-   *
/ \ / \
```

a b c -

/ \

d e

(b) Write down the vertex sequence for the pre-order and also the post-order traversal of the tree (to be created) in (a)

(c) Use Kruskal's algorithm to find a minimal spanning tree for the following graph, where the numbers represent the weight of the corresponding edges. What is the total weight of the minimal spanning tree? Also draw the minimal spanning tree.

Discrete Mathematics-2017

Part-A

1.(a) Construct the truth table of the compound proposition (p V ¬q) → (p∧ q).

p	q	¬q	p V ¬q	p ∧ q	(p V ¬q) → (p ∧ q)
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

(b) **How can this English sentence be translated into a logical expression? "You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."**

Let's break down the given English sentence and translate it into a logical expression step by step:

Let: P = "You can ride the roller coaster."

Q = "You are under 4 feet tall."

R = "You are older than 16 years old."

The sentence can be translated as follows:

"You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

Translated logical expressions: $(Q \land \neg R) \rightarrow \neg P$

$$P \rightarrow (\neg Q \lor R)$$

Explanation: $(Q \land \neg R) \rightarrow \neg P$: "If you are under 4 feet tall and not older than 16 years old, then you cannot ride the roller coaster." $P \rightarrow (\neg Q \lor R)$: "If you can ride the roller coaster, then either you are not under 4 feet tall or you are older than 16 years old."

Therefore, $(Q \land \neg R) \rightarrow \neg P$ and $P \rightarrow (\neg Q \lor R)$ represent the corrected logical expressions for the given sentence.

~~(c)Determine the output for the combinatorial circuit in the following Figure. If the value of p, q~~

~~and rare T, T and F respectively.~~

~~A combinatorial circuit~~

(d) Build a digital circuit that produces the output (p V ¬r) A (¬p V (q V¬r)) when given input bits p, q, and r.

2.(a) Define Universal quantification and Existential quantification with example.

- Universal quantification is a logical operator that states that a predicate is true for all values of a variable. It is denoted by the symbol ∀. For example, the statement $\forall x(x > 0)$ means that for all x, x is greater than 0.
- Existential quantification is a logical operator that states that there exists at least one value of a variable for which a predicate is true. It is denoted by the symbol ∃. For example, the statement $\exists x(x > 0)$ means that there exists at least one x such that x is greater than 0.

Here are some examples of universal and existential quantification:

- Universal quantification:
  - ∀x(x is a human) means that all x are human.
  - ∀x(x is a mammal) means that all x are mammals.
  - ∀x(x is a square) means that all x are squares.
- Existential quantification:
  - ∃x(x is a human) means that there exists at least one x that is human.
  - ∃x(x is a mammal) means that there exists at least one x that is a mammal.

- ∃x(x is a square) means that there exists at least one x that is a square.

(b) Show that [¬p∧ (p v q)] →q is a tautology.

p	q	¬p	(p v q)	[¬p ∧ (p v q)]	[¬p ∧ (p v q)] → q
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

(c) Express the statements " If somebody is female and is a parent, then this person is someone's mother" as a logical expression.

F: Somebody is female.

P: Somebody is a parent.

M: This person is someone's mother.

The correct logical expression for the statement is:

(F ∧ P) → M

Explanation:

- (F ∧ P): Represents the conjunction (and) of "Somebody is female" and "Somebody is a parent."
- M: Represents the statement "This person is someone's mother."
- (F ∧ P) → M: Represents the implication (→) that if somebody is female and a parent, then this person is someone's mother.

So, the correct logical expression for the given statement is (F ∧ P) → M.

(d) Express the following statement using logical connectives:

i) The automated reply cannot be sent when the file system is full.

ii) you cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.

iii) you can access the internet from campus only if you are a computer science major or you are not a freshman.

Sure, here are the expressions of the statements using logical connectives:

i) The automated reply cannot be sent when the file system is full.

(File system is full) → (Automated reply cannot be sent)

- A: The automated reply can be sent.
- F: The file system is full.

The correct logical expression for the statement is: $\neg(F \to A)$

ii) you cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.

(Under 4 feet tall) ∧ ¬(Over 16 years old) → (Cannot ride the roller coaster)

- R: You can ride the roller coaster.
- H: You are under 4 feet tall.
- O: You are older than 16 years old.

The correct logical expression for the statement is: $(H \land \neg O) \to \neg R$

iii) you can access the internet from campus only if you are a computer science major or you are not a freshman.

(Computer science major) ∨ ¬(Freshman) → (Can access the internet from campus)

- A: You can access the internet from campus.
- C: You are a computer science major.
- F: You are a freshman.

The correct logical expression for the statement is: $(C \lor \neg F) \to A$

3.(a) What do you mean by rules of inference? Define Modus ponens and Modus tollens rules of inference with example?

Modus ponens is a rule of inference that states that if p implies q, and p is true, then q must be true. For example, if we know that "If it is raining, then the ground is wet" and that "It is raining", then we can conclude that "The ground is wet".

Modus tollens is a rule of inference that states that if p implies q, and not q is true, then not p must be true. For example, if we know that "If it is raining, then the ground is wet" and that "The ground is not wet", then we can conclude that "It is not raining".

Here are some examples of how modus ponens and modus tollens can be used in arguments:

- Modus ponens:

Premise 1: If I study hard, I will pass the test.

Premise 2: I studied hard.

Conclusion: I will pass the test.

- Modus tollens:

Premise 1: If I am a dog, I have four legs.

Premise 2: I do not have four legs.

Conclusion: I am not a dog.

(b) Show that the hypotheses H1, H2, and H3 lead to the conclusion C. where H1: "If you send me an e-mail message, then I will finish writing the program.", H2: "If you do not send me an e-mail message, then I will go to sleep early.", H3: "If I go to sleep early, then I will wake up feeling refreshed.", C: "If I do not finish writing the program, then I will wake up feeling refreshed."

5. H1: "If you send me an e-mail message, then I will finish writing the program." This can be represented as: $(E \rightarrow F)$
6. H2: "If you do not send me an e-mail message, then I will go to sleep early." This can be represented as: $(\neg E \rightarrow S)$
7. H3: "If I go to sleep early, then I will wake up feeling refreshed." This can be represented as: $(S \rightarrow R)$
8. To prove C: "If I do not finish writing the program, then I will wake up feeling refreshed." This can be represented as: $(\neg F \rightarrow R)$

To derive the conclusion C, we will assume the antecedent $(\neg F)$ and try to derive the consequent (R):

5. Assume $\neg F$ (the negation of F).

6. **Applying Modus Tollens to H1 (E → F) and the assumption (¬F): E → F ¬F**

Therefore, ¬E (by Modus Tollens)

7. **Applying Modus Ponens to H2 (¬E → S) and the result of step 6 (¬E): ¬E → S ¬E**

Therefore, S (by Modus Ponens)

8. **Applying Modus Ponens to H3 (S → R) and the result of step 7 (S): S → R S**

Therefore, R (by Modus Ponens)

9. Thus, we have derived R from the assumption (¬F).
10. Therefore, we can conclude that (¬F → R), which is equivalent to the statement C.

Hence, the hypotheses H1, H2, and H3 lead to the conclusion C.

c)What do you mean by proof by contradiction?

Prove the theorem "If 3n+2 is odd, then n is odd" by using proof by contradiction.

Proof by contradiction is a method of proving a statement by showing that the opposite of the statement leads to a contradiction. In other words, we assume the opposite of the statement to be true and then show that this leads to a logical impossibility. If we can do this, then we can conclude that the original statement must be true.

To prove the theorem "If 3n+2 is odd, then n is odd" by using proof by contradiction, we will assume the opposite of the statement to be true. In other words, we will assume that n is even.

If n is even, then we can write n = 2k for some integer k. Substituting this into the statement, we get the following:

3(2k) + 2 = 6k + 2

This expression is even, since the sum of two even numbers is even. However, we were told that 3n+2 is odd. This is a contradiction, since we cannot have an even number be odd.

4.(a) Define binary relation with example. How many different relations can we define on a set A with n elements?

Let A={1,2,3,4) and R be the relation on A where (a,b) e R if and only if a>b. Represent the

relation in arrow diagram, zero-one matrix and graph.

(b)

(c) Define when a relation R on a set A is called symmetric, antisymmetric and asymmetric. Are the following relations on (1,2,3,4) symmetric, antisymmetric and asymmetric: iv) R= ((4, 4), (3, 3), (1, 4))

i) R={(1, 1), (1, 2), (2, 1), (3, 3), (4, 4)} ii) {(1, 1)}

(i) {(1, 3), (3,2), (2, 1)}

Part-B

5.(a) What is the closure of a relation? Find the reflexive, symmetric and transitive closure of the relation R= {(1, 3), (1, 4), (2, 1), (3, 2)) on the set A= {1, 2, 3, 4)

The closure of a relation is a process where we add additional pairs to the original relation to make it satisfy certain properties. The most common types of relation closures are the reflexive closure, symmetric closure, and transitive closure.

1. **Reflexive Closure**: The reflexive closure of a relation R on a set A adds all the missing reflexive pairs (a, a) where a is an element of A. In other words, it ensures that every element in A is related to itself.
2. **Symmetric Closure:** The symmetric closure of a relation R on a set A adds pairs (b, a) whenever (a, b) is in R. In other words, it ensures that if a is related to b, then b is also related to a, making the relation symmetric.
3. **Transitive Closure:** The transitive closure of a relation R on a set A adds pairs (a, c) whenever there exist pairs (a, b) and (b, c) in R. In other words, it ensures that if a is related to b and b is related to c, then a is also related to c, making the relation transitive.

Now, let's find the reflexive, symmetric, and transitive closures of the given relation R on the set A = {1, 2, 3, 4}.

Given relation R = {(1, 3), (1, 4), (2, 1), (3, 2)}

1. **Reflexive Closure**: To make the relation reflexive, we need to add the reflexive pairs (a, a) for every a in A. Reflexive pairs: (1, 1), (2, 2), (3, 3), (4, 4)

Reflexive Closure R_r: R_r = {(1, 3), (1, 4), (2, 1), (3, 2), (1, 1), (2, 2), (3, 3), (4, 4)}

2. **Symmetric Closure**: To make the relation symmetric, we need to add pairs (b, a) whenever (a, b) is in R. Symmetric pairs: (3, 1), (4, 1), (1, 2), (2, 3)

Symmetric Closure R_s: R_s = {(1, 3), (1, 4), (2, 1), (3, 2), (3, 1), (4, 1), (1, 2), (2, 3)}

3. **Transitive Closure**: To make the relation transitive, we need to add pairs (a, c) whenever there exist pairs (a, b) and (b, c) in R. Transitive pairs: (1, 2), (3, 3), (1, 2)

Transitive Closure R_t: R_t = {(1, 3), (1, 4), (2, 1), (3, 2), (1, 1), (2, 2), (3, 3), (4, 4), (1, 2)}

(b) Define totally ordered set with example. Is (Z^+, |) a totally ordered poset? Why?

Here '|' means the 'divides by' relation.

(c) Define lattice. Is the poset (Z, D) a lattice? Explain it.

A partially ordered set in which every pair of elements has both a least upper bound and a greatest lower bound is called a lattice. Lattices have many special properties.

In the poset (Z+, **|), Z+ represents the set of positive integers** (excluding zero), and **| represents the divisibility relation.**

Let a and b be two positive integers. The least upper bound and greatest lower bound of these two integers are the least common multiple and the greatest common divisor of these integers, respectively, as the reader should verify. It follows that this poset is a lattice.

6.(a) Define n-Cube graph with example. Draw the Q3 graph.

The **n-Cube graph**, also known as the **hypercube graph or the n-dimensional cube graph**, is a graph that represents the corners and edges of an n-dimensional hypercube. It is a generalization of the cube graph (Q3) to higher dimensions.

The n-Cube graph has **2^n vertices**, each representing a corner of the hypercube, and an edge exists between two vertices if and only if the corresponding corners in the hypercube are adjacent.

The **Q3 graph**, also known as the **cube graph** or the 3-dimensional hypercube, represents the corners and edges of a cube. It consists **of 8 vertices and 12 edges.(half of vertices)**

(b) Define bipartite graph with example. Is C6 graph bipartite? Justify your answer.

A bipartite graph is a graph a graph in which every edge connects **a vertex from one set to a vertex in the other set.**

The C6 graph refers to a cycle graph with 6 vertices. It is a cyclic arrangement of 6 vertices, where each vertex is connected to its two adjacent vertices, forming a hexagon shape.

1----2

/ \

6 3

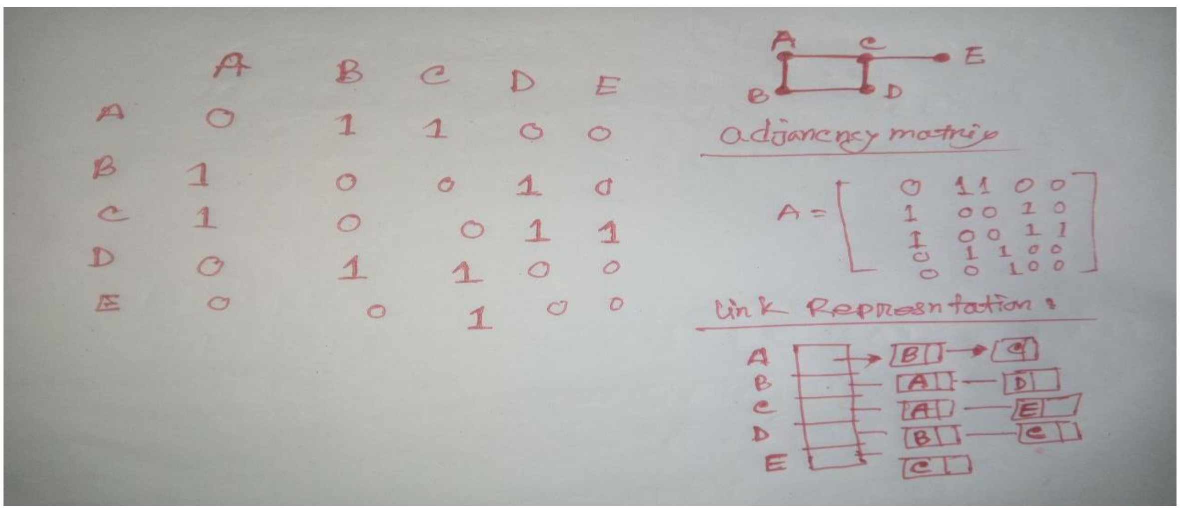
| |

5--------4

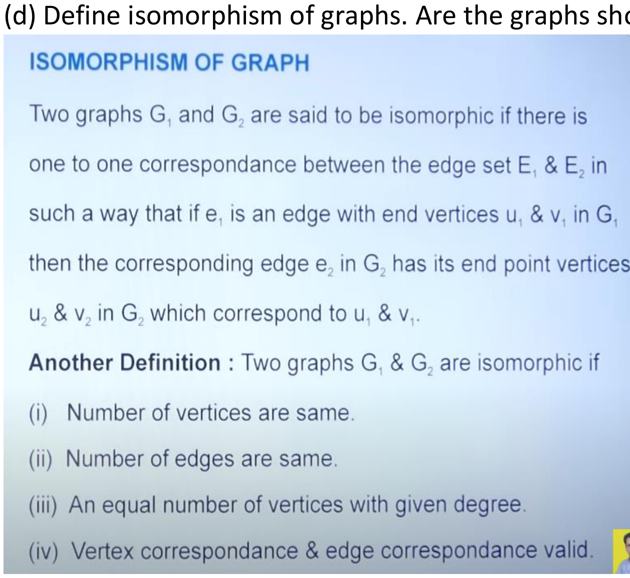
To determine if the C6 graph is bipartite, we can try to divide its vertex set into two sets. Let's label them Set A and Set B. **Set A: {1, 3, 5} Set B: {2, 4, 6}**

By observing the graph, we can see that every edge connects a vertex from Set A to a vertex from Set B. There are no edges within the same set. Therefore, the C6 graph can be divided into two disjoint sets, satisfying the definition of a bipartite graph.

(c) Represent the graph shown below by adjacency matrix and linked representation.



(d) Define isomorphism of graphs. Are the graphs shown below are isomorphic? Explain.



ISOMORPHISM OF GRAPH

Two graphs G_1 and G_2 are said to be isomorphic if there is one to one correspondance between the edge set E_1 & E_2 in such a way that if e_1 is an edge with end vertices u_1 & v_1 in G_1 then the corresponding edge e_2 in G_2 has its end point vertices u_2 & v_2 in G_2 which correspond to u_1 & v_1.

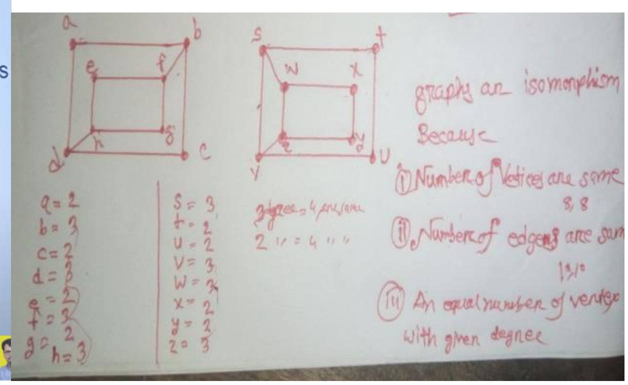
Another Definition : Two graphs G_1 & G_2 are isomorphic if

(i) Number of vertices are same.

(ii) Number of edges are same.

(iii) An equal number of vertices with given degree.

(iv) Vertex correspondance & edge correspondance valid.



7.(a) What is planar graph? Is Q3 planar? Justify your answer with figure.

A planar graph is one that can be drawn in a plane without any edges crossing. For example, the complete graph K_4 is planar,

(b) Define Hamilton circuit and Euler circuit. How do they differ from each other?

Circuit manei closed korte hobe ,kintu hamiltonian == no repeated vertices and eulerian == no repeated edges

(c) What do you know about chromatic number of a graph? What is the chromatic number of the graph K3,4. Explain with diagram.

8.**(a) Find the level of each vertex in the rooted tree shown in the following Figure. What is the height of this tree?**

(b)Form a binary search tree for the words mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry (using alphabetical order).

(c) Use Huffman coding to encode the following symbols with the frequencies listed: A: 0.08, B: 0.10, C: 0.12, D: 0.15, E: 0.20, F: 0.35. What is the average number of bits used to encode a character?

(d)What is the ordered rooted tree that represents the expression ((x + y) ^ 12) + ((x-4)/3)?

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Discrete Mathematics-2016

Part-A

1.

(a) What do you mean by propositional logic?

Propositional logic is a branch of logic that deals with propositions, which are statements that can be either true or false. Propositional logic does not consider the meaning of propositions, only their truth value.

Propositional logic has a set of logical connectives, which are used to combine propositions into more complex propositions. The most common logical connectives are:

- And (∧)

- Or (∨)
- Not (¬)
- If...then (→)
- If and only if (↔)

Propositional logic can be used to represent and reason about arguments. An argument is a set of propositions, one of which is the conclusion, and the others of which are the premises. An argument is valid if the conclusion is true whenever all of the premises are true.

Propositional logic is a powerful tool for reasoning about the world. It can be used to analyze arguments, to make predictions, and to solve problems.

(b) How can this English sentence be translated into a logical expression? "You can access the Internet from campus only if you are a Computer Science major or you are not a freshman."

P: "You can access the Internet from campus."

Q: "You are a Computer Science major."

R: "You are a freshman."

Translated logical expression:

$P \rightarrow (Q \lor \neg R)$

Explanation:

$P \rightarrow (Q \lor \neg R)$ represents the logical expression for the given sentence. It states that if you can access the Internet from campus (P), then either you are a Computer Science major (Q) or you are not a freshman (¬R).

In this expression, the → symbol represents the implication operator, ∧ represents the conjunction operator (AND), ∨ represents the disjunction operator (OR), and ¬ represents the negation operator (NOT).

(c) Show that the following two sentences are logically equivalent.

(i) "It is not the case that roses are red and violets are blue".

(ii) "Roses are not red, or violets are not blue".

Let: P = "Roses are red."

Q = "Violets are blue."

The sentence can be represented as: ¬(P ∧ Q)

Sentence 2: "Roses are not red, or violets are not blue." This can be represented as: ¬P ∨ ¬Q

To show logical equivalence, we need to show that ¬(P ∧ Q) and ¬P ∨ ¬Q have the same truth values for all possible combinations of truth values for P and Q.

Truth table:

P	**Q**	**P ∧ Q**	**¬(P ∧ Q)**	**¬P**	**¬Q**	**¬P ∨ ¬Q**
True	True	True	False	False	False	False
True	False	False	True	False	True	True
False	True	False	True	True	False	True
False	False	False	True	True	True	True

As we can see from the truth table, ¬(P ∧ Q) and ¬P ∨ ¬Q have the same truth values for all possible combinations of truth values for P and Q. Therefore, we can conclude that the two sentences are logically equivalent.

2

a)Define 'Propositional Function'.

A propositional function is a statement that contains one or more variables, and can be made into a proposition by replacing the variables with specific values. For example, the statement "x is a dog" is a propositional function. It can be made into a proposition by replacing the variable x with a specific value, such as "Fido". The resulting proposition is "Fido is a dog".

Propositional functions are used in predicate logic, which is a more powerful form of logic than propositional logic. Predicate logic can be used to represent more complex statements, such as "All dogs are mammals".

Here are some examples of propositional functions:

- x is a dog
- y is taller than z

- The sum of x and y is equal to z

These propositional functions can be made into propositions by replacing the variables with specific values. For example, the propositional function "x is a dog" can be made into the proposition "Fido is a dog" by replacing the variable x with the value "Fido".

Propositional functions are a powerful tool that can be used to represent and reason about complex statements. They are used in many different fields, including mathematics, computer science, and philosophy.

b) Let P(x) denote the statement "x> 3". What are the truth values of the quantifications ∃xP(x) and VxP(x), where the domain consists of all real numbers?

Let's first define the following:

- P(x) = x > 3
- ∃xP(x) = There exists an x such that P(x)
- ∀xP(x) = For all x, P(x)

The domain of discourse is all real numbers.

- ∃xP(x) is true because there are real numbers that are greater than 3, such as 4, 5, and 6.
- ∀xP(x) is false because there are real numbers that are not greater than 3, such as 0, 1, and 2.

Therefore, the truth value of ∃xP(x) is true and the truth value of ∀xP(x) is false

(c) Negate each of the following statements:

(i) ∃xVyP(x, y); (ii) Vx∃yP(x, y); (iii) ∃y∃xVzP (x, y, z);

To negate each of the following statements, we can apply the rules of negation to the quantifiers and the predicate:

(i) ∃xVyP(x, y) To negate this statement, we change the existential quantifier (∃) to a universal quantifier (∀) and negate the predicate P(x, y):

¬(∃xVyP(x, y))

Negated statement: ∀x¬VyP(x, y)

(ii) ∀x∃yP(x, y) To negate this statement, we change the universal quantifier (∀) to an existential quantifier (∃) and negate the predicate P(x, y):

¬(∀x∃yP(x, y))

Negated statement: ∃x¬∃yP(x, y)

(iii) ∃y∃x∀zP(x, y, z) To negate this statement, we change the existential quantifiers (∃) to universal quantifiers (∀) and negate the predicate P(x, y, z):

¬(∃y∃x∀zP(x, y, z))

Negated statement: ∀y∀x¬∀zP(x, y, z)

3.

(a)

There are 6 people in a room; each of them shakes hands with other. If no one shakes hands with any other person more than once, how many handshakes take place? A group of 30 people have been trained as astronauts to go on the first mission to Mars. How

(b)

many ways are there to select a crew of six people to go on this mission (assuming that all crew members have the same job)? (c) Find the minimum number of students in a class to be sure that three of them are born in the

same month.

4.

(a) Write down the properties of binary relations. (b) What do you mean by closure of the relation? Let the relation R on a set {1,2,3} is represented by the following matrix. Find R*.

0 1 0 MR = 0 1 01

(c) Define 'Equivalence Relation' with example.

Part-B

5.

(a) **Define 'Poset' and 'Lattic'.**

Poset' stands for partially ordered set. It is a mathematical structure that consists of a set of elements along with a binary relation that defines a partial order among those elements

'Lattice' is a special type of poset. A lattice is a poset in which every pair of elements has both a unique greatest lower bound (also called meet or infimum) and a unique least upper bound (also called join or supremum

(b) Give a direct proof of the theorem, "If n is an odd integer, then n^2 is odd".

(c) The following defines a grammar G:

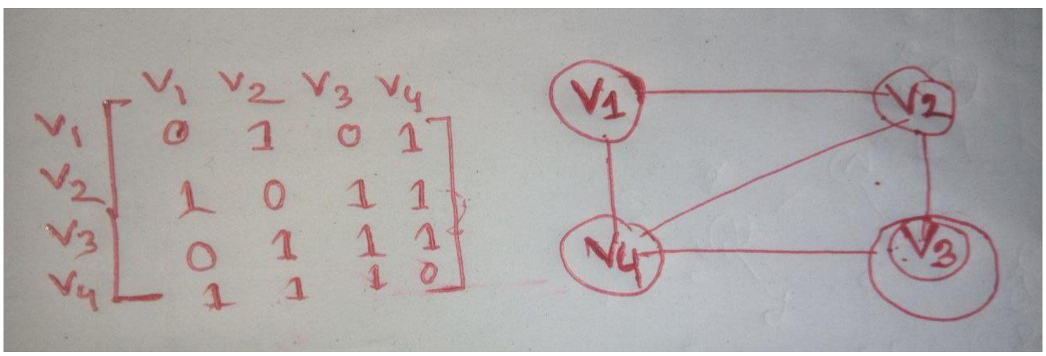
V = {A, B, S, a, b}, T = {a, b},

P = {S →AB, A → Aa, B→ Bb, A→ a, B→ b}.

Write the production in abbreviated form. Using production rules obtain w = a^2 b^4.

6.(a) Define 'Psedograph' and 'Multigraph' with example. What is the difference between them?

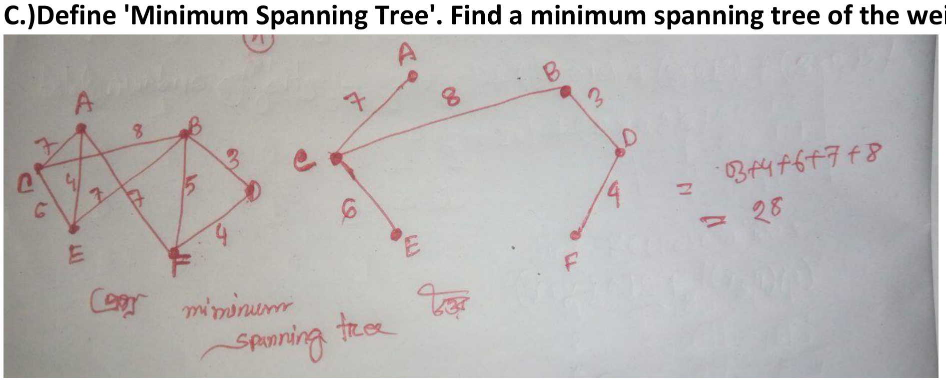
(b) State the "Handshaking Theorem' with example. How adjacency matrix is used to represent a graph? Discuss with example. Draw a graph



7. **A)What do you mean by 'Isomorphic' and 'Homomorphic' Graphs? Give examples**

B) Define 'Distance', 'Diameter', 'Cutpoints' and 'Bridge' with proper diagram.

C.)Define 'Minimum Spanning Tree'. Find a minimum spanning tree of the weighted graph G in Fig-7c.



8.

(a) Define 'Planar Graph'. What do you mean by 'Map' and 'Dual Map"?

(b) Define 'Spanning Tree' with example. Use BFS to find a spanning tree for the graph shown in Fig-8b choosing the root vertex as 'e'. Write the step and draw the spanning tree.

Discrete Mathematics-2015

Part-A

1. (a) Define 'Proposition'. Consider the following sentences and determine which one is proposition or not.

(i) What time is it?

(ii) Read this carefully.

(iii) x+1 = 2

(iv) x + y = z.

(a) In logic, a proposition is a declarative statement that is either true or false. It is a meaningful expression that can be assigned a truth value.

Now let's consider the sentences provided:

(i) "What time is it?" - This is not a proposition because it is a question rather than a declarative statement.

(ii) "Read this carefully." - This is not a proposition either because it is an imperative sentence, giving a command rather than expressing a statement that can be evaluated as true or false.

(iii) "x+1 = 2" - This is a proposition. It is a mathematical equation that can be evaluated for truth or falsity.

(iv) (iv) "x + y = z" - This is also a proposition. It is a mathematical equation that can be evaluated for truth or falsity, depending on the values assigned to the variables.

(b) How can this English sentence be translated into a logical expression? "You cannot ride the roller coaster if you are less than 4 feet tall unless you are older than 16 years old".

Let's break down the given English sentence and translate it into a logical expression step by step:

Let: P = "You can ride the roller coaster."

Q = "You are under 4 feet tall."

R = "You are older than 16 years old."

The sentence can be translated as follows:

"You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

Translated logical expressions: $(Q \land \neg R) \rightarrow \neg P$

$$P \rightarrow (\neg Q \lor R)$$

Explanation: $(Q \land \neg R) \rightarrow \neg P$: "If you are under 4 feet tall and not older than 16 years old, then you cannot ride the roller coaster." $P \rightarrow (\neg Q \lor R)$: "If you can ride the roller coaster, then either you are not under 4 feet tall or you are older than 16 years old."

Therefore, $(Q \land \neg R) \rightarrow \neg P$ and $P \rightarrow (\neg Q \lor R)$ represent the corrected logical expressions for the given sentence.

(c) Show that the following two sentences are logically equivalence. "It is not the case that roses are red and violets are blue". "Roses are not red, or violets are not blue".

Let: P = "Roses are red."

Q = "Violets are blue."

The sentence can be represented as: $\neg(P \land Q)$

Sentence 2: "Roses are not red, or violets are not blue." This can be represented as: $\neg P \lor \neg Q$

To show logical equivalence, we need to show that $\neg(P \land Q)$ and $\neg P \lor \neg Q$ have the same truth values for all possible combinations of truth values for P and Q.

Truth table:

P	**Q**	**P ∧ Q**	**¬(P ∧ Q)**	**¬P**	**¬Q**	**¬P V ¬Q**
True	True	True	False	False	False	False
True	False	False	True	False	True	True
False	True	False	True	True	False	True
False	False	False	True	True	True	True

As we can see from the truth table, ¬(P ∧ Q) and ¬P ∨ ¬Q have the same truth values for all possible combinations of truth values for P and Q. Therefore, we can conclude that the two sentences are logically equivalent.

2.

(a) Define 'Tautology' and 'Contradiction'.

- Tautology

A tautology is a statement that is always true, no matter what. For example, the statement "All bachelors are unmarried" is a tautology. This is because the definition of "bachelor" is "an unmarried man." Therefore, it is impossible for a bachelor to be married.

- Contradiction

A contradiction is a statement that is always false, no matter what. For example, the statement "This statement is false" is a contradiction. This is because if the statement is true, then it is false, and if it is false, then it is true. This is a logical impossibility.

- A tautology is always true.
- A contradiction is always false.

(b) Let Q(x) denote the statement "x=x+1". What is the truth value of the quantification ∃x(Q(x)) where the domain consists of all real numbers?

The statement Q(x) = "x = x + 1" is not true for any real number x because no real number is equal to itself plus one.

When evaluating the quantification ∃x(Q(x)) where the domain consists of all real numbers, we are checking whether there exists a real number for which the statement Q(x) is true.

Since Q(x) is never true for any real number x, the quantification ∃x(Q(x)) is false. There does not exist a real number x such that x is equal to x + 1.

(c) What are rules of inference?

Show that the following argument is valid. If today is Tuesday, I have a test in Mathematics or Economics. If my Economics professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics professor is sick. Therefore, I have a test in Mathematics.

Rules of inference are logical rules or principles that allow us to derive valid conclusions from given premises in formal logic. These rules ensure that the logical reasoning process is sound and consistent.

Common rules of inference include modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism, and more.

Now let's examine the argument provided:

Premises:

1. If today is Tuesday, I have a test in Mathematics or Economics.
2. If my Economics professor is sick, I will not have a test in Economics.
3. Today is Tuesday and my Economics professor is sick.

Conclusion: 4. Therefore, I have a test in Mathematics.

To demonstrate that the argument is valid, we can use the rules of inference.

1. From premise 3 (Today is Tuesday and my Economics professor is sick) and premise 1 (If today is Tuesday, I have a test in Mathematics or Economics), we can use the disjunctive syllogism rule:

Today is Tuesday and my Economics professor is sick If today is Tuesday, I have a test in Mathematics or Economics Therefore, I have a test in Mathematics

Thus, using the disjunctive syllogism rule, we can derive the conclusion (I have a test in Mathematics) from the given premises. Therefore, the argument is valid.

3.

(a) Define 'Set' and 'Symmetric difference of sets'.

(b) In a class of 80 students, 50 students know English, 55 students know French and 46 students know German language, 37 students know English and French, 28 students know French and German, 25 students know English and German, 7 students know none of the languages. How many students know all the 3 languages?

(c) How many seven-letter words can be formed using the letters of the word

"BENZENE"?

4.

(a) What do you mean by 'Relation'? Explain the properties of relation.

(b) Suppose R, the relation {(1, 2), (2, 3), (3, 3), (2, 4), (3, 1)) from (1, 2, 3) to {1, 2, 3, 4} and R_2 the relation {(1, 2), (2, 3), (3, 1), (3, 3), (4, 2)} from {1, 2, 3, 4} to {1, 2, 3). Determine the adjacency matrix of R_1 and R_2.

(c) Suppose R = {(1, 2), (2, 2), (2, 3), (5, 4)) is a relation on S = {1, 2, 3, 4, 5}. What is the reflexive and symmetric closure of R?

(d) What is on-to function?

Part-B

5.(a) Define 'Partial order' and 'Partially ordered set'

(b) Let A = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24) be ordered by the relation "x divides y". Draw aHosse diagram of A. Find out the maximal, first and last element(s) of A.

(c) Define 'Lattice' and 'Bounded Lattice'.

6.

(a) Count the number of vertices (V), the number of edges (E), and the number of regions (R) of the following map and verify Euler's formula. Also find out the degree of the outside region.

3

(b) What are the differences between tree and graph?

(c) Let A, B, C, D, E, F, G, H be eight data items with the following assigned weights Data item A B C D E F G H Weight 5 11 19 2 11 25 5 Construct a 2 tree with a minimum weighted path length P using the above data as external nodes

(b) The main differences between trees and graphs are as follows:

Trees:

- Trees are a type of graph that consists of nodes connected by edges.

- Trees have a hierarchical structure where each node has a unique parent node (except for the root) and can have zero or more child nodes.

- Trees do not contain cycles or loops, meaning there is only one path between any two nodes.

- Trees have a specific root node from which all other nodes are reachable.

- Trees can be categorized into different types based on their properties, such as binary trees, AVL trees, B-trees, etc.

Graphs:

- Graphs are a collection of nodes (vertices) connected by edges (links).

- Graphs can have different types of connections between nodes, such as directed edges (arcs) or undirected edges.

- Graphs can have cycles or loops, meaning there can be multiple paths between nodes.

- Graphs do not have a specific root node and can have disconnected components.

- Graphs can be categorized based on various properties, such as directed graphs, weighted graphs, bipartite graphs, etc.

In summary, trees are a specific type of graph with a hierarchical structure, while graphs are a more general concept that can have various structures and connections between nodes.

(c) To construct a binary tree with minimum weighted path length using the given data, we can follow these steps:

1. Start with the data items and their assigned weights:

Data item: A B C D E F G H

Weight: 5 11 19 2 11 25 5

2. Sort the data items in ascending order based on their weights:

Data item: D A H G B E C F

Weight: 2 5 5 11 11 11 19 25

3. Build the binary tree by repeatedly choosing the two data items with the smallest weights and creating a new node with their combined weight:

```
          (45)
          /  \
        (16)  (29)
        / \    / \
      (7) (9) (11) (18)
      / \ / \ / \ / \
    (3) (4) (2) (5) (6) (5) (11) (8)
    / \
  (2) (2)
```

In this tree, the external nodes represent the data items, and the internal nodes represent the combined weights. The minimum weighted path length, P, is 45.

7. (a) Define bipartite graph with example. Is the following graph bipartite? Explain why or why not.

(b) Define incidence matrix with example. Represent the following the graph using an incidence matrix.

(c) Explain Hamilton path and Hamilton circuit in a graph with example.

8. (a) Define operation on a nonempty set.

(b) Consider the set of positive integers, N. Determine whether addition, multiplication, subtraction and division are operations on N.

(c) Define 'Semigroup', 'Group' and 'Ring'.

(d) Define 'Grammar' and 'Language'.

(a) An operation on a nonempty set is a rule or function that combines two elements from the set to produce a unique result in the same set. In other words, it is a binary operation that takes two elements from the set and returns a single element of the set.

(b) In the set of positive integers, N:

- Addition: Addition is an operation on N. When you add two positive integers, the result is always another positive integer. For example, adding 2 and 3 gives 5, which is also a positive integer.

- Multiplication: Multiplication is an operation on N. When you multiply two positive integers, the result is always another positive integer. For example, multiplying 2 and 3 gives 6, which is also a positive integer.

- Subtraction: Subtraction is not an operation on N. While subtracting two positive integers may result in a positive integer, it is not always the case. For example, subtracting 3 from 2 gives -1, which is not a positive integer.

- Division: Division is not an operation on N. Dividing two positive integers may result in a non-integer or a fractional number, which is not a positive integer. For example, dividing 6 by 2 gives 3, which is a positive integer, but dividing 3 by 2 gives 1.5, which is not a positive integer.

(c) Definitions of mathematical structures:

- Semigroup: A semigroup is a set together with an associative binary operation. The operation combines any two elements of the set and produces another element in the set. The operation does not need to have an identity element or inverse elements. For example, the set of positive integers with the operation of addition forms a semigroup.

- Group: A group is a set together with an associative binary operation, an identity element, and inverse elements for every element in the set. The operation combines any two elements of the set and produces another element in the set. For example, the set of integers with the operation of addition forms a group.

- Ring: A ring is a set together with two binary operations, usually addition and multiplication, that satisfy certain properties. The set forms an abelian group under addition and is closed under multiplication. The operations also follow distributive laws. For example, the set of integers with the operations of addition and multiplication forms a ring.

(d) Definitions of language-related terms:

- Grammar: A grammar is a set of rules that specify the structure of a language. It defines the syntax and structure of sentences and phrases in a language. Grammars are commonly used in linguistics, computer science, and other fields to describe the rules of a language.

- Language: In the context of formal language theory, a language is a set of strings or sequences of symbols drawn from an alphabet. It represents a set of valid expressions or sentences in a particular system or formalism. Languages can be natural languages like English or programming languages like Python.