উত্তর (Operator method)

2015

6.

a) Solve $y'' + 4y' + 4y = 2x + 6$

Here,

A.E. $m^2 + 4m + 4 = 0$

$\cancel{m^2 + 2.2.m + 2^2 = 0}$

$\cancel{(m+2)^2 = 0}$

$\cancel{m+2 = 0}$

$\cancel{m =}$

$m^2 + 2m + 2m + 4 = 0$

বা, $m(m+2) + 2(m+2) = 0$

$\Rightarrow (m+2)(m+2) = 0$

$\therefore m = -2, -2$

$\therefore y_c = c_1 e^{-2x} + x c_2 e^{-2x}$

$\because$ This is because the roots are real and equal. Hence we multiply an extra x.

Rewrite equation is

$(D^2 + 4D + 4)y = 2x + 6$

$$y_p = \frac{1}{D^2+4D+4}(2x+6)$$

$$= \frac{1}{4\left(1 + D + \frac{D^2}{4}\right)}(2x+6)$$

$$= \frac{1}{4\left\{1 + \left(D + \frac{D^2}{4}\right)\right\}}(2x+6)$$

put, $\lambda = D + \frac{D^2}{4}$

$$= \frac{1}{4(1+\lambda)}(2x+6)$$

$$= \frac{1}{4}(1+\lambda)^{-1}(2x+6)$$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots\ldots$

$$= \frac{1}{4}\left(1 - \lambda + \lambda^2 + \cdots\right)(2x+6)$$

$$= \frac{1}{4}(1-\lambda)(2x+6)$$

$$4y_p = (1-\lambda)(2x+6)$$

$$= (2x+6) - \lambda(2x+6)$$

$$= (2x+6) - 2$$

$$= 2x+4$$

$$y_p = \frac{2(x+2)}{4}$$

$$= \frac{x+2}{2}$$

here;

$$\lambda(2x+6) = \left(D + \frac{D^2}{4}\right)(2x+6)$$

$$= D(2x+6) + \frac{D^2}{4}(2x+6)$$

$$= 2 + 0 + 0$$

$$= 2$$

$$\therefore y = y_c + y_p = c_1 e^{-2x} + x c_2 e^{-2x} + \frac{x+2}{2}$$

(Ans)

b) $y'' - y = x^2 e^x + 5$

A.E, $m^2 - 1 = 0$

$$m^2 = 1$$

$$m = \pm 1$$

$$y_c = c_1 e^x + c_2 e^{-x}$$

Rewrite the equation as

$$(D^2-1)y = x^2e^x+5$$

$$y_p = \frac{1}{D^2-1}(x^2e^x+5)$$

$$= \frac{1}{D^2-1}x^2e^x + \frac{1}{D^2-1}5$$

$$= e^x \frac{1}{D^2-1}x^2 + \frac{1}{-(1-D^2)}5$$

Now, $-\frac{1}{(1-D^2)}5 = K$

$$\Rightarrow -(1-D^2)^{-1}5 = K$$

$$\Rightarrow -(1+D^2)5 = K$$

$$\Rightarrow -5-D^2 5 = K$$

$$\Rightarrow K = -5$$

Again, $e^x \cdot \frac{1}{D^2-1}x^2$

$$= e^x\frac{1}{(D+1)^2-1}x^2 \quad [D=D+1]$$

$$= e^x\frac{1}{D^2+2D+1-1}x^2$$

$$= e^x\frac{1}{D^2+2D}x^2$$

$$= e^x\frac{1}{2D(1+\frac{D}{2})}x^2$$

$$= e^x\frac{1}{2D}\left(1-\frac{D}{2}+\frac{D^2}{4}\right)x^2$$

$$= e^x\frac{1}{2D}\left(x^2-x+\frac{1}{2}\right)$$

$$= e^x\frac{1}{2}\left(\frac{x^3}{3}-\frac{x^2}{2}+\frac{1}{2}x\right)$$

$$= e^x\left(\frac{x^3}{6}-\frac{x^2}{4}+\frac{x}{4}\right)$$

$$\therefore y_p = e^x\left(\frac{x^3}{6}-\frac{x^2}{4}+\frac{x}{4}\right)+(-5)$$

$$= e^x\left(\frac{x^3}{6}-\frac{x^2}{4}+\frac{x}{4}\right)-5$$

$$\therefore y = y_c + y_p$$

$$= c_1e^x+c_2e^{-x}+e^x\left(\frac{x^3}{6}-\frac{x^2}{4}+\frac{x}{4}\right)-5$$

(Ans)

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c) $y'' - y' - 12y = e^{4x}$

A.E,

$$m^2 - m - 12 = 0$$

$$m^2 - 4m + 3m - 12 = 0$$

$$m(m-4) + 3(m-4) = 0$$

$$(m-4)(m+3) = 0$$

$$m = -3, 4$$

$y_c = c_1 e^{-3x} + c_2 e^{4x}$

$\therefore (D^2 - D - 12) y = e^{4x}$

$y_p = \frac{1}{D^2 - D - 12} e^{4x}$

$= x \frac{e^{4x}}{2D - 1}$

$= x \frac{e^{4x}}{2 \cdot 4 - 1}$

$= x \frac{e^{4x}}{8 - 1}$

$= \frac{x e^{4x}}{7}$

$\therefore y = y_c + y_p$

$= c_1 e^{-3x} + c_2 e^{4x} + \frac{x e^{4x}}{7}$

(Ans)

#

2018

5(c) variation of parameters: $y'' + y = \cos^2 x$ — ①

A.E,

$m^2 + 1 = 0$

$m^2 = -1$

$m = \pm i$

$y_c = e^{0x}(c_1 \sin x + c_2 \cos x)$

$= c_1 \sin x + c_2 \cos x$

$\therefore y_p = u \sin x + v \cos x$ — (ii)

$= u(x) \sin x + v(x) \cos x$

$y_p' = u \cos x - v \sin x + u' \sin x + v' \cos x$

$= u \cos x - v \sin x$ $[\because \underline{u' \sin x + v' \cos x = 0}]$ (iii)

$y_p'' = -u \sin x - v \cos x + u' \cos x - v' \sin x$

$= -u \sin x - v \cos x + u' \cos x - v' \sin x$ — (iv)

eq (ii), (iii), (iv) we get,

$-u \sin x - v \cos x + u' \cos x - v' \sin x + u \sin x + v \cos x = \cos^2 x$

$\therefore u' \cos x - v' \sin x = \cos^2 x$ — ①

$u' \sin x + v' \cos x = 0$ — ②

① × cos x ④ ② × sin x we get

$u'\cos^2 x - v'\sin x\cos x = \cos 3x$

$u'\sin^2 x + v'\sin x\cos x = 0$

$u'(\cos^2 x + \sin^2 x) = \cos 3x$

$u' . 1 = \cos^3 x$

$u' = \cos^3 x$

$\therefore u = \int u'\, dx = \int \cos^3 dx$

$= \int \cos x(1-\sin^2 x)\, dx$

$= \int (\cos x - \cos x \sin^2 x)\, dx$

$= \int \cos x\, dx - \int \cos x \sin^2 x\, dx$

$= \sin x - [\cos x \int \sin^2 dx - \int (\frac{d}{dx} \cos x \int \sin^2 dx)$

$= \sin x - \cos x$

$\therefore u = \int u'\, dx = \int \cos^3 dx = \int \cos^2 x \cos x\, dx$

$= \int \cos x(1-\sin^2 x)\, dx$

$= \int (1-z^2)\, dz$

$= z - \frac{z^3}{3} + c$

$= \sin x - \frac{\sin^3}{3} + c$

Let,
$\sin x = z$
$\cos x\, dx = dz$

eq ① × $\sin x$ − ② × $\cos x$ we get

$$u' \sin x \cos x - v' \sin^2 x = \sin x \cos^2 x$$
$$\underline{u' \sin x \cos x + v' \cos^2 x = 0}$$
$$-v'(\sin^2 x + \cos^2 x) = \sin x \cos^2 x$$
$$-v' = \sin x \cos^2 x$$
$$v' = -\sin x \cos^2 x$$

$$\therefore v = \int v' dx = -\int \sin x \cos^2 x \, dx$$
$$= \int \cos^2 x (-\sin x) \, dx$$

Let,
$\cos x = z$
$-\sin x \, dx = dz$

$$= \int z^2 dz$$
$$= \frac{z^3}{3} + c$$
$$= \frac{\cos^3 x}{3} + c$$

$$\therefore y_p = \left[\sin x - \frac{\sin^3 x}{3} + c\right] \sin x + \left[\frac{\cos^3 x}{3} + c\right] \cos x$$
$$= \sin^2 x - \frac{\sin^4 x}{3} + \frac{\cos^4 x}{3} + c$$

$$\therefore y = y_c + y_p$$
$$= c_1 \sin x + c_2 \cos x + \sin^2 x - \frac{1}{3}\sin^4 x + \frac{1}{3}\cos^4 x$$
$\cancel{+c}$

(Ans)

2016

6.

a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = xe^{-x}$

A.E, $m^2+2m+2=0$

$\cancel{m^2+2m+m+2=0}$

$\cancel{m(m+2)+1(m+2)=0}$

$\cancel{(m+2)(m+1)=0}$

$\cancel{m=-1,-2}$

A.E, $m^2+2m+2=0$

$m^2+2m+2=0$

$m = \frac{-2 \pm \sqrt{4-4\cdot1\cdot2}}{2\cdot1}$

$= \frac{-2 \pm \sqrt{-4}}{2}$

$= \frac{-2 \pm 2i}{2}$

$= -1 \pm i$

$y_c = e^{-x}(c_1 \sin x + c_2 \cos x)$

$y_p = \frac{1}{D^2+2D+2} xe^{-x}$

$= e^{-x} \frac{1}{D^2+2D+2} x$ $\quad [D = D-1]$

$= e^{-x} \frac{1}{(D-1)^2+2(D-1)+2} x$

$$= e^{-x} \frac{1}{D^2 - \cancel{2D} + 1 + \cancel{2D} - \cancel{2} + 2} \cdot x$$

$$= e^{-x} \frac{1}{1+D^2} x$$

$$= e^{-x} (1+D^2)^{-1} x$$

$$= e^{-x} (1-D^2) x$$

$$= e^{-x} \cdot x$$

$$y_p = x e^{-x}$$

$$\therefore y = y_c + y_p = e^{-x}(c_1 \sin x + c_2 \cos x) + x e^{-x}$$

(Ans)

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b) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$

A.E. $m^2 - 2m + 4 = 0$

$$m = \frac{2 \pm \sqrt{4 - 4.1.4}}{2.1}$$

$$= \frac{2 \pm \sqrt{4-16}}{2}$$

$$= \frac{2 \pm 2\sqrt{3}i}{2}$$

$$= 1 \pm \sqrt{3}i$$

$$y_c = e^x (c_1 \sin(\sqrt{3}x) + c_2 \cos(\sqrt{3}x))$$

$$= e^{-x}\frac{1}{D^2-\cancel{2D}+1+\cancel{2D}-\cancel{2}+\cancel{2}}\cdot x$$

$$= e^{-x}\frac{1}{1+D^2}x$$

$$= e^{-x}(1+D^2)^{-1}x$$

$$= e^{-x}(1-D^2)x$$

$$= e^{-x}\cdot x$$

$$y_p = xe^{-x}$$

$$\therefore y = y_c + y_p = e^{-x}(c_1\sin x + c_2\cos x) + xe^{-x}$$

(Ans)

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b) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x\cos x$

A.E. $m^2-2m+4=0$

$$m = \frac{2\pm\sqrt{4-4.1.4}}{2.1}$$

$$= \frac{2\pm\sqrt{4-16}}{2}$$

$$= \frac{2\pm 2\sqrt{3}i}{2}$$

$$= 1\pm\sqrt{3}i$$

$$y_c = e^x(c_1\sin(\sqrt{3}x) + c_2\cos(\sqrt{3}x))$$

$$y_p = \frac{1}{(D^2-2D+4)} e^x \cos x$$

$$= e^x \frac{1}{D^2-2D+4} \cos x$$

$$= e^x \frac{1}{(D+1)^2-2(D+1)+4} \cos x \quad [D = D+1]$$

$$= e^x \frac{1}{D^2+\cancel{2D}+1-\cancel{2D}-2+4} \cos x$$

$$= e^x \frac{1}{D^2+3} \cos x \qquad f(D^2) \to f(-a^2)$$

$$D^2 = -a^2$$

$$= e^x \frac{1}{-1^2+3} \cos x \qquad \cos ax = \cos x$$

$$= e^x \frac{1}{2} \cos x$$

$$= \frac{e^x \cos x}{2}$$

$$\therefore y = y_c + y_p$$

$$= e^x \left[c_1 \sin(\sqrt{3}x) + c_2 \cos(\sqrt{3}x)\right] + \frac{e^x \cos x}{2}$$

2017/7 b) $(D^2+4)y = \cos x$

A.E, $m^2+4=0$

$m^2 = -4$

$m = \pm 2i$

$y_c = c_1 \sin 2x + c_2 \cos 2x$

$y_p = \frac{1}{D^2+4} \cos x$

$= \frac{1}{-1^2+4} \cos x$

$= \frac{1}{3} \cos x$

$\left\{ \begin{array}{l} f(D) \to f(-a^2) \\ D^2 \to -a^2 \\ \cos ax = \cos x \end{array} \right.$

$\therefore y = y_c + y_p = c_1 \sin 2x + c_2 \cos 2x + \frac{1}{3} \cos x$

—*—

2017 variation of parameters

7.b) $4y'' - 4y' - 8y = 8e^{-x}$

A.E, $4m^2 - 4m - 8 = 0$

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2) = 0$

$(m-2)(m+1) = 0$

$m = -1, 2$

$\therefore y_c = c_1 e^{-x} + c_2 e^{2x}$

$\cancel{y_p = \frac{1}{4D^2 - 4D - 8} 8e}$

$y_p = ue^{-x} + ve^{2x}$ ——— (i)

$y_p' = -ue^{-x} + 2ve^{2x} + u'e^{-x} + v'e^{2x}$

$= -ue^{-x} + 2ve^{2x}$ ——— (ii) $[u'e^{-x} + v'e^{2x} = 0]$

$y_p'' = ue^{-x} + 4ve^{2x} - u'e^{-x} + 2v'e^{2x}$ ——— (iii)

Form (i), (ii) and (iii) we get,

$4ue^{-x} + 16ve^{2x} - 4u'e^{-x} + 8v'e^{2x} - 4ue^{-x} + 8ve^{2x}$

$- 8ue^{-x} + 8ve^{2x} = 8e^{-x}$

$\therefore -4u'e^{-x} + 8v'e^{2x} = 8e^{-x}$ ——— (iv)

$u'e^{-x} + v'e^{2x} = 0$ ——— (v)

(v) $\times 4$ + iv, we get

$$-4u'e^{-x}+8v'e^{2x}=8e^{-x}$$
$$\cancel{4}u'e^{-x}+4v'e^{2x}=0$$

$$12v'e^{2x}=8e^{-x}$$
$$v'=\frac{\cancel{8}^{2}e^{-x}}{\cancel{12}_{3}e^{2x}}$$
$$=\frac{2}{3}e^{-3x}$$
$$\therefore v=\int v'dx=\frac{2}{3}\int e^{-3x}dx$$
$$=\frac{2}{3}\cdot\frac{e^{-3x}}{-3}$$
$$=-\frac{2}{9}e^{-3x}$$

Again (IV) − (V) × 8, we get,

$$\Rightarrow -4u'e^{-x}+\cancel{8v'e^{2x}}=8e^{-x}$$
$$\underset{-}{8}u'e^{-x}\underset{-}{+}\cancel{8}v'e^{2x}=\underset{-}{0}$$

$$-12u'e^{-x}=8e^{-x}$$
$$u'=-\frac{8e^{-x}}{12e^{-x}}$$
$$=-\frac{2}{3}$$
$$\therefore u=\int u'dx=-\int\frac{2}{3}dx$$
$$=-\frac{2}{3}x$$
$$\therefore y_p=\left[-\frac{2}{3}x\right]e^{-x}+\left[-\frac{2}{9}e^{-3x}\right]e^{+2x}$$
$$=-\frac{2}{3}xe^{-x}-\frac{2}{9}e^{-x}$$
$$\therefore y=y_c+y_p=c_1e^{-x}+c_2e^{2x}-\frac{2}{3}xe^{-x}-\frac{2}{9}e^{-x}$$ —*— (Solved

2018 operator method

6.a) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = \sin 2x$

A.E, $m^2 + 4m + 1 = 0$

$$m = \frac{-4 \pm \sqrt{16 - 4.1.1}}{2.1}$$

$$= \frac{-4 \pm 2\sqrt{3}}{2}$$

$$= -2 \pm \sqrt{3}$$

$$y_c = e^{-2x}\left(c_1 \sin\sqrt{3}x + c_2 \cos\sqrt{3}\,x\right)$$

$f(D) = f(-a^2)$

$D^2 \Rightarrow -a^2$

$$y_p = \frac{1}{D^2 + 4D + 1}\sin 2x$$

$$= \frac{1}{-2^2 + 4D + 1}\sin 2x$$

$$= \frac{1}{4D - 3}\sin 2x$$

$$= \frac{4D + 3}{16D^2 - 9}\sin 2x$$

$$= \frac{4D + 3}{16(-2^2) - 9}\sin 2x$$

$$= \frac{4D + 3}{-64 - 9}\sin 2x$$

$$= \frac{(4D + 3)\sin 2x}{-\cancel{107}\,73}$$

$$= \frac{4.2\cos 2x + 3\sin 2x}{-73}$$

$$= -\frac{8}{73}\cos 2x - \frac{3}{73}\sin 2x$$

$$\therefore y = y_c + y_p = e^{-2x}\left(c_1 \sin\sqrt{3}x + c_2 \cos\sqrt{3}\,x\right) - \frac{8}{73}\cos 2x - \frac{3}{73}\sin x$$

solved

2020
~~2019~~

Operator method

7. b) $(D^2+4)y = x^2 e^{2x}$

A.E,

$$m^2+4=0$$
$$m^2=-4$$
$$m=\pm 2i$$

$$y_c = c_1 \sin 2x + c_2 \cos 2x$$

$$y_p = \frac{1}{D^2+4} x^2 e^{2x}$$
$$= e^{2x} \frac{1}{D^2+4} x^2$$
$$= e^{2x} \frac{1}{(D+2)^2+4} x^2$$
$$= e^{2x} \frac{1}{D^2+4D+4+4} x^2$$
$$= e^{2x} \frac{1}{D^2+4D+8} x^2$$
$$= e^{2x} \frac{1}{8\left(1+\frac{D}{2}+\frac{D^2}{8}\right)} x^2$$
$$= e^{2x} \frac{1}{8(1+\lambda)} x^2 \qquad \text{Let, } \lambda = \frac{D}{2}+\frac{D^2}{8}$$
$$= e^{2x} \frac{1}{8} (1+\lambda)^{-1} x^2$$
$$= e^{2x} \frac{1}{8} (1-\lambda+\lambda^2) x^2$$
$$= e^{2x} \frac{1}{8} \left[x^2 - \lambda x^2 + \lambda^2 x^2\right]$$

Now, $\lambda x^2 = \left(\frac{D}{2}+\frac{D^2}{8}\right) x^2 = \frac{D}{2}x^2 + \frac{D^2}{8}x^2$
$$= \left(x + \frac{1}{4}\right)$$

Again, $\lambda^2 x^2 = \left(\frac{D}{2}+\frac{D^2}{8}\right)^2 x^2$

$= \left(\frac{D^2}{4} + \cancel{2}\cdot\frac{D}{\cancel{2}}\cdot\frac{D^2}{8} + \frac{D^4}{64}\right) x^2$

$= \left(\frac{D^2}{4} + \frac{D^3}{8} + \frac{D^4}{64}\right) x^2$

$= \frac{D^2}{4}x^2 + \frac{D^3}{8}x^2 + \frac{D^4}{64}x^2$

$= \frac{1}{2} + 0 + 0$

Now, $y_p = e^{2x}\frac{1}{8}\left(x^2 - x - \frac{1}{4} + \frac{1}{2}\right)$

$= e^{2x}\frac{1}{8}\left(x^2 - x + \frac{1}{4}\right)$

$-\frac{1}{4}+\frac{1}{2} = \frac{-1+2}{4} = \frac{1}{4}$

$= e^{2x}\left(\frac{x^2}{8} - \frac{x}{8} + \frac{1}{32}\right)$

$\therefore y = y_c + y_p = c_1 \sin 2x + c_2 \cos 2x + e^{2x}\left(\frac{x^2}{8} - \frac{x}{8} + \frac{1}{32}\right)$

(Ans)

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Undetermind co-efficient

2019

6.c) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x$

A.E,
$m^2 - 2m - 3 = 0$

$m^2 - 3m + m - 3 = 0$

$m(m-3) + 1(m-3) = 0$

$(m-3)(m+1) = 0 \quad \therefore m = -1, 3$

$\therefore y_c = c_1 e^{-x} + c_2 e^{3x}$

y_p: The solution set of $S = \{e^{-x}, e^{3x}\}$ Now, e^x, $\sin x$ are not part of S.

$y_p = c_3 e^x + c_4 \sin x + c_5 \cos x$

$y_p' = c_3 e^x + c_4 \cos x - c_5 \sin x$

$y_p'' = c_3 e^x - c_4 \sin x - c_5 \cos x$

$\therefore c_3 e^x - c_4 \sin x - c_5 \cos x - 2c_3 e^x - c_4 \cos x + c_5 \sin x - 3c_3 e^x - 3c_4 \sin x - c_5 \cos x = 2e^x - 10\sin x$

$\Rightarrow e^x(c_3 - 2c_3 - 3c_3) + \sin x(-c_4 + c_5 - 3c_4) + \cos x(-c_5 \cancel{\cos x} - c_4 - c_5) = 2e^x - 10\sin x$

$\Rightarrow -4c_3 e^x + (-4c_4 + c_5 \cancel{-3c_4}) \sin x + (-2c_5 - c_4)\cos x = 2e^x - 10\sin x$

Compearing.

$-4c_3 = 2$

$c_3 = \frac{2}{-4} = -\frac{1}{2}$

Again, $-4c_4 + c_5 = -10$ —— (I)

And $-2c_5 - c_4 = 0$ —— (II)

(I) $\times 2$ + (II)

$$-8c_4 + \cancel{2c_5} = -20$$
$$-c_4 - \cancel{2c_5} = 0$$

$$-9c_4 = -20$$
$$c_4 = 20/9$$
$$\therefore c_5 = 4c_4 - 10$$
$$= 4 \times \frac{20}{9} - 10$$
$$= \frac{80-90}{9}$$
$$= -10/9$$

$$\therefore y_p = -\frac{1}{2}e^{x} + \frac{20}{9}\sin x - \frac{10}{9}\cos x$$

$$\therefore y = y_c + y_p = c_1 e^{-x} + c_2 e^{3x} - \frac{1}{2}e^{x} + \frac{20}{9}\sin x - \frac{10}{9}\cos x$$

(Ans)

2021
~~2020~~

3. a) $\frac{d^2y}{dx^2} \cancel{+} - 2\frac{dy}{dx} - 3y = 2e^{x} - 10\cos x$

A.E, $m^2 - 2m - 3 = 0$

$m = -1, 3$

$y_c = c_1 e^{-x} + c_2 e^{3x}$

The solution set of $S = \{e^{-x}, e^{3x}\}$ $e^{x}, \cos x$ are not part of S.

$\therefore$ y_p, y_p', y_p'' are same previous question.

$\therefore -4C_3 e^x + (-4C_4 + C_5)\sin x + (-2C_5 - C_4)\cos x = 2e^x - 10\cos x$

Comparing

$\therefore -4C_3 \cancel{e^x} = 2$

$\therefore C_3 = -\frac{1}{2}$

$\therefore -4C_4 + C_5 = 0$ —— (i)

$-2C_5 - C_4 = -10$ —— (ii)

(i) $\times 2$ + (ii), we get,

$-8C_4 + \cancel{2C_5} = 0$

$-C_4 - \cancel{2C_5} = -10$

$-9C_4 = -10$

$C_4 = 10/9$

$\therefore C_5 = +4C_4$

$= 4 \cdot \frac{10}{9}$

$= \frac{40}{9}$

$\therefore y_p = -\frac{1}{2}e^x + \frac{10}{9}\sin x + \frac{40}{9}\cos x$

$\therefore y = y_c + y_p = C_1 e^{-x} + C_2 e^{3x} - \frac{1}{2}e^x + \frac{10}{9}\sin x + \frac{40}{9}\cos x$

Ans

2021

3(c) variation of parameters

$(D^2+4)y = \cos^2 x$

$\therefore$ A.E., $m^2+4=0$

$m^2=-4$

$m=\pm 2i$

$y_c = c_1 \sin 2x + c_2 \cos 2x$

$y_p = u \sin 2x + v \cos 2x$ —— (i)

$y_p' = 2u \cos 2x - 2v \sin 2x + u' \sin 2x + v' \cos 2x$

$= 2u \cos 2x - 2v \sin 2x$ —— (ii) $[u' \sin 2x + v' \cos 2x = 0]$

$y_p'' = -4u \sin 2x - 4v \cos 2x + 2u' \cos 2x - 2v' \sin 2x$ —— (iii)

$\therefore -4u \sin 2x - 4v \cos 2x + 2u' \cos 2x - 2v' \sin 2x + 4u \sin 2x + 4v \cos 2x = \cos^2 x$

$\therefore 2u' \cos 2x - 2v' \sin 2x = \cos^2 x$ —— (iv)

$u' \sin 2x + v' \cos 2x = 0$ —— (v)

(iv) $\times \cos 2x$ + (v) $\times \sin 2x \times 2$, we get

$2u' \cos^2 2x - 2v' \sin 2x \cos 2x = \cos^2 x \cos 2x$

$2u' \sin^2 2x + 2v' \sin 2x \cos 2x = 0$

$2u' (\cos^2 2x + \sin^2 2x) = \cos^2 x \cos 2x$

$\Rightarrow 2u' = \cos^2 x \cos 2x$

$\therefore u' = \frac{1}{2} \cos^2 x \cos 2x$

Again, (iv) $\times \sin 2x -$ (v) $\times 2 \times \cos 2x$, we get

$$2u' \sin 2x \cos 2x - 2v' \sin^2 2x = \cos^2 x \sin 2x$$
$$2u' \sin 2x \cos 2x + 2v' \cos^2 2x = 0$$
$$- \quad - \quad -$$

$$-2v'(\sin^2 2x + \cos^2 2x) = \cos^2 x \sin 2x$$

$$v' = -\frac{1}{2} \cos^2 x \sin 2x$$

$$\therefore \quad u = \int u' dx = \frac{1}{2}\int \cos^2 x \cos 2x \, dx$$
$$= \frac{1}{2}\int \cos^2 x (\cos^2 x - \sin^2 x) dx$$
$$= \frac{1}{2}\int (\cos^4 x - \sin^2 x \cos^2 x) dx$$
$$= \frac{1}{2}\int (\cos^4 x - \sin^2 x + \sin^4 x) dx$$
$$= \frac{1}{2}\int [(\cos^2)^2 x + (\sin^2)^2 x - \sin^2 x] dx$$
$$= \frac{1}{2}\int (1 - \sin^2 x) dx$$
$$= \frac{1}{2} x + \frac{1}{2} \frac{\cos^3 x}{3}$$
$$= \frac{1}{4}\int (1 + \cos 2x) \cos 2x \, dx$$
$$= \frac{1}{4}\int (\cos 2x + \cos^2 2x) dx$$
$$= \frac{1}{4} \frac{\sin 2x}{2} + \frac{1}{4 \times 2}\int (1 + \cos 4x) dx$$
$$= \frac{1}{8} \sin 2x + \frac{1}{8} x + \frac{1}{8} \frac{\sin 4x}{4}$$

$v = \int v' \, dx = -\frac{1}{2}\int \cos^2 x \cdot \sin 2x \cdot dx$

$= -\frac{1}{2}\int \cos^2 x \cdot 2\sin x \cdot \cos x \, dx$

$= -\int \cos^3 x \cdot \sin x \, dx$

$= -\int z^3 \, dz$

$= -\frac{z^4}{4} + c$

$= -\frac{\cos^4 x}{4}$

Let ~~Sin~~ $\cos x = z$

$-\sin x \, dx = dz$

$\therefore y_p = \left(\frac{1}{8}\sin 2x + \frac{1}{8}x + \frac{1}{8}\frac{\sin 4x}{4}\right)\sin 2x + -\left(\frac{\cos^4 x}{4}\right) \cos 2x$

$y = y_c + y_p$

(Ans)