operator method: 2021-3.?

(5) $(D^2-4D+4)y = 3x^2 e^{2x} \sin 2x$

A.E $\quad m^2-4m+4=0$

$m^2-2m-2m+4=0$

$m(m-2)-2(m-2)=0$

$(m-2)(m-2)=0$

$m=2,2$

$\therefore y_c = c_1 e^{2x} + x c_2 e^{2x}$

$\cancel{y_p = \frac{1}{D^2-4D+4} 3x^2 e^{2x} \sin 2x}$

$\cancel{= 3e^{2x} \frac{1}{D+2}}$ $\quad D^2-4D+4=(D-2)^2$

$\cancel{= 3e^{2x} \frac{1}{D+2-2}}$

$\therefore y_p = \frac{1}{(D-2)^2} 3x^2 e^{2x} \sin 2x$

$= 3e^{2x} \frac{1}{(D+2-2)^2} x^2 \sin 2x$

$= 3e^{2x} \frac{1}{D^2} [x^2 \sin 2x]$

$= 3e^{2x} \frac{1}{D} \int x^2 \sin 2x\, dx$

$= 3e^{2x} \frac{1}{D} \left[x^2\left(-\frac{\cos 2x}{2}\right) - \int 2x \left(\frac{-\cos 2x}{2}\right) dx \right]$

$= 3e^{2x} \frac{1}{D} \left[-\frac{x^2 \cos 2x}{2} + \left(\frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} dx \right) \right]$

$= 3e^{2x} \frac{1}{D} \left[-\frac{x^2 \cos 2x}{2} + \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} \right]$

$= -\frac{3}{2} e^{2x} \frac{1}{D} \left[+ x^2 \cos 2x - x \sin 2x - \frac{\cos 2x}{2} \right]$

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$$= -\frac{3}{2}e^{2x}\left[\int x^2\cos 2x\,dx - \int x\sin 2x\,dx - \int \frac{\cos 2x}{2}dx\right]$$

Hence $\int x^2 \cos 2x\,dx$

$$= x^2\frac{\sin 2x}{2} - \int 2x\frac{\sin 2x}{2}dx$$

$$= x^2\frac{\sin 2x}{2} - \left[x\left(-\frac{\cos 2x}{2}\right) - \int -\frac{\cos 2x}{2}dx\right]$$

$$= \frac{x^2\sin 2x}{2} + \frac{x\cos 2x}{2} + \frac{\sin 2x}{4}$$

Again, $\int x\sin 2x\,dx$

$$= -\frac{x\cos 2x}{2} + \frac{\sin 2x}{4}$$

Again, $\int \frac{\cos 2x}{2}dx$

$$= \frac{\sin 2x}{4}$$

$$\therefore y_p = -\frac{3}{2}e^{2x}\left[\frac{x^2\sin 2x}{2} + \frac{x\cos 2x}{2} + \frac{\sin 2x}{4} + \frac{x\cos 2x}{2} - \frac{\sin 2x}{4} - \frac{\sin 2x}{4}\right]$$

$$= -\frac{3}{4}e^{2x}\left[x^2\sin 2x + x\cos 2x + x\cos 2x - \frac{\sin 2x}{2}\right]$$

$$= -\frac{3}{4}e^{2x}\left[x^2\sin 2x + 2x\cos 2x - \frac{\sin 2x}{2}\right]$$

$$= -\frac{3}{4}e^{2x}\left[\left(x^2 - \frac{1}{2}\right)\sin 2x + 2x\cos 2x\right]$$

$$\therefore y = (c_1 + xc_2)e^{2x} - \frac{3}{4}e^{2x}\left[\left(x^2-\frac{1}{2}\right)\sin 2x + 2x\cos 2x\right]$$

(Ans)

CT-1 $(D^3-8D^2+21D-18)y = 6e^{2x}+5e^{3x}$

A.E, $m^3-8m^2+21m-18=0$

$\Rightarrow m^3-2m^2-6m^2+12m+9m-18=0$

$\Rightarrow m^2(m-2)-6m(m-2)+9(m-2)=0$

$\Rightarrow (m-2)(m^2-6m+9)=0$

$\Rightarrow m=2 \qquad \Rightarrow m^2-6m+9=0$

$\Rightarrow m^2-3m-3m+9=0$

$\Rightarrow m(m-3)-3(m-3)=0$

$\Rightarrow (m-3)(m-3)=0$

$\Rightarrow m=3,3$

$\therefore \alpha=3,\ \beta=3,\ \gamma=2$

$\therefore y_c = e^{3x}(c_1+xc_2)+c_3e^{2x}$

$y_p = \frac{1}{D^3-8D^2+21D-18}(6e^{2x}+5e^{3x})$

$= \frac{1}{D^3-8D^2+21D-18}6e^{2x}+\frac{1}{D^3-8D^2+21D-18}5e^{3x}$

$= 6x\frac{1}{3D^2-16D+21}e^{2x}+5x\frac{1}{3D^2-16D+21}e^{3x}$

$= 6x\frac{1}{12-32+21}e^{2x}+5x^2\frac{1}{6D-16}e^{3x}$

$= 6x.e^{2x}+5x^2\frac{1}{18-16}e^{3x}$

$= 6xe^{2x}+\frac{5}{2}x^2e^{3x}$

$\therefore y = y_c+y_p$

$$\therefore y = e^{3x}(c_1 + c_2 x) + c_3 e^{2x} + 6xe^{2x} + \frac{5}{2}x^2 e$$

(Ans)

CT-2 $(D^4+2D^2+1)y = x^2\sin x$

A.E, $m^4+2m^2+1=0$

$(m^2)^2+2m^2.1+1=0$

$(m^2+1)^2=0$

$(m^2+1)(m^2+1)=0$

$m=\pm i \quad , \quad m=\pm i$

$y_c = e^{0x}[(c_1+c_2x)\cos x+(c_3+c_4x)\sin x]$

$=(c_1+c_2x)\cos x+(c_3+c_4x)\sin x$

$y_p = \frac{1}{D^4+2D^2+1}(x^2\sin x)$

$e^{ix}=\cos x+i\sin x$

Real part of $e^{ix}=\cos x$

Img. part of $e^{ix}= i\sin x$

Real part of this $= \frac{1}{(D^2+1)^2} x^2 e^{ix}$

$= e^{ix}\frac{1}{[(D+i)^2+1]^2}x^2$

$= e^{ix}\frac{1}{(D^2+i^2+2iD+1)^2}x^2$

$= e^{ix}\frac{1}{(D^2+2iD)^2}x^2$

$= e^{ix}\frac{1}{D^4+4i^2D^2+4.iD^3}x^2$

$= e^{ix}\frac{1}{D^4-4D^2+4iD^3}x^2$

$= e^{ix}\frac{1}{-4D^2}\frac{1}{[1+(-\frac{D^2}{4}-iD)]}x^2$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$

$= -\frac{e^{ix}}{4}\frac{1}{D^2}\left[1+\left(-\frac{D^2}{4} - iD\right)\right]^{-1} x^2$

$= -\frac{e^{ix}}{4}\frac{1}{D^2}\left[1+\left(\frac{D^2}{4}+iD\right)+\left(\frac{D^2}{4}+iD\right)^2 + \cdots\right] x^2$

$= -\frac{e^{ix}}{4}\frac{1}{D^2}\left[1+\frac{D^2}{4}+iD+\frac{D^4}{16}+i^2D^2+\frac{D^3 i}{2}+\cdots\right] x^2$

$= -\frac{e^{ix}}{4}\frac{1}{D^2}\left[x^2+\frac{1}{2}+2xi-2\right]$

$= -\frac{e^{ix}}{4}\frac{1}{D}\int\left(x^2+\frac{1}{2}+2xi-2\right)dx$

$= -\frac{e^{ix}}{4}\frac{1}{D}\left[\frac{x^3}{3}+\frac{x}{2}+\frac{x^2}{\cancel{2}}i-2x\right]$

$= -\frac{e^{ix}}{4}\int\left(\frac{x^3}{3}+\frac{x}{2}+\frac{x^2}{\cancel{2}}i-2x\right)dx$

$= -\frac{e^{ix}}{4}\left(\frac{x^4}{12}+\frac{x^2}{4}+\frac{x^3}{3}i-x^2\right)$

$= -\frac{\cos x + i\sin x}{4}\left[\frac{x^4}{12}-\frac{3x^2}{4}+\frac{x^3 i}{3}\right]$

Real part of $= -\frac{\cos x}{4}\left[\frac{x^4}{12}-\frac{3x^2}{4}\right] + \frac{\sin x}{4}\cdot\frac{x^3}{3}$

$\therefore y = y_c + y_p$

$= (c_1+c_2x)\cos x + (c_3+c_4x)\sin x - \frac{1}{4}\left[\frac{x^4}{12}-\frac{3x^2}{4}\right]\cos x + \frac{\sin x}{4}\cdot\frac{x^3}{3}$

(Ans)

Undetermind Co-efficient:

① $(D^2-4D+4)y = x^3e^{2x}+2e^{3x}$

A.E, $m^2-4m+4=0$

$(m-2)^2=0$

$m=2,2$

$\therefore y_c=(c_1+c_2x)e^{2x}$

~~y_p~~ Derivd set, $S=\{x^3e^{2x},\ e^{3x},\ x^2e^{2x},\ xe^{2x},\ e^{2x}\}$

multiply by x $=\{x^4e^{2x},\ x^3e^{2x},\ x^2e^{2x},\ xe^{2x},\ e^{3x}\}$

Again " " $x=\{x^5e^{2x},\ x^4e^{2x},\ x^3e^{2x},\ x^2e^{2x},\ e^{3x}\}$

$y_p = c_3x^5e^{2x}+c_4x^4e^{2x}+c_5x^3e^{2x}+c_6x^2e^{2x}+c_7e^{3x}$

$y_p' = 5c_3x^4e^{2x}+2c_3x^5e^{2x}+4c_4x^3e^{2x}+2c_4x^4e^{2x}$
$+3c_5x^2e^{2x}+2c_5x^3e^{2x}+2c_6xe^{2x}+2c_6x^2e^{2x}$
$+3c_7e^{3x}$

$y_p'' = 20c_3x^3e^{2x}+10c_3x^4e^{2x}+10c_3x^4e^{2x}+4c_3x^5e^{2x}+$
$12c_4x^2e^{2x}+8c_4x^3e^{2x}+8c_4x^3e^{2x}+4c_4x^4e^{2x}+$
$6c_5xe^{2x}+6c_5x^2e^{2x}+6c_5x^2e^{2x}+4c_5x^3e^{2x}+$
$2c_6e^{2x}+4c_6xe^{2x}+4c_6xe^{2x}+4c_6x^2e^{2x}+$
$9c_7e^{3x}$

$= x^3e^{2x}(20c_3+8c_4+8c_4+4c_5)+x^4e^{2x}(10c_3+10c_3+$
$4c_4)+x^5e^{2x}(4c_3)+x^2e^{2x}(12c_4+6c_5+6c_5+4c_6)$
$+xe^{2x}(6c_5+4c_6+4c_6)+2c_6e^{2x}+9c_7e^{3x}$

$(D^2-4D+4)y = x^3e^{2x} + 2e^{3x}$

$\therefore (20c_3+16c_4+4c_5)x^3e^{2x} + (20c_3+4c_4)x^4e^{2x} + 4c_3x^5e^{2x}$

$+(12c_4+12c_5+4c_6)x^2e^{2x} + (6c_5+8c_6)xe^{2x} +$

$2c_6e^{2x} + 9c_7e^{3x} - x^4e^{2x}(20c_3+8c_4) - 8c_3x^5e^{2x}$

$-x^3e^{2x}(16c_4+8c_5) - x^2e^{2x}(12c_5+8c_6) - 8c_6xe^{2x}$

$-12c_7e^{3x} + 4c_3x^5e^{2x} + 4c_4x^4e^{2x} + 4c_5x^3e^{2x} +$

$4c_6x^2e^{2x} + 4c_7e^{3x}$

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$20c_3+16c_4+4c_5-16c_4-8c_5+4c_5 = 1$ —— (i)

$9c_7-12c_7+4c_7 = 2$ —— (ii)

$20c_3+4c_4-20c_3-8c_4+4c_4 = 0$ —— (iii)

$4c_3-8c_3+4c_3 = 0$ —— (iv)

$2c_6 = 0$ —— (v)

$12c_4+12c_5+4c_6-12c_5-8c_6+4c_6 = 0$ —— (vi)

$6c_5+8c_6-8c_6 = 0$ —— (vii)

from, (v) eq, we get,

$c_6 = 0$

from (vii) eq, we get

$c_5 = 0$

from (ii) eq, we get,

$13c_7-12c_7 = 2$

$c_7 = 2$

from (vi) eq, we get

$12c_4 = 0$

$c_4 = 0$

from (i) eq, we get,

$20c_3 = 1$

$c_3 = \frac{1}{20}$

$\therefore y_P = \frac{1}{20}x^5e^{2x} + 0 + 0 + 0 + 2e^{3x}$

$\therefore y = (c_1+c_2x)e^{2x} + \frac{1}{20}x^5e^{2x} + 2e^{3x}$

(Ans)

C.T (11)

(iii) $(D^2+1)y = -2\sin x + 4x\cos x$

A.E, $m^2+1=0$

$m^2=-1$

$m=\pm i$

$y_c = e^{0.x}(c_1 \sin x + c_2 \cos x)$

$= c_1 \sin x + c_2 \cos x$

derived set, $\{\sin x, x\cos x, \cos x, x, x\sin x\}$

x mul from, $\{x^2\sin x, x^2\cos x, x\cos x, x^2, x\sin x\}$

$y_p = c_3 x^2\cos x + c_4 x^2 \sin x + c_5 x\cos x + c_6 x\sin x + c_7 x^2$

$y_p' = 2c_3 x\cos x - c_3 x^2\sin x + 2c_4 x\sin x + c_4 x^2\cos x + c_5\cos x - c_5 x\sin x + c_6 \sin x + c_6 x\cos x + 2c_7 x$

$y_p'' = +2c_3\cos x - 2c_3 x\sin x - 2c_3 x\sin x - c_3 x^2\cos x + 2c_4\sin x + 2c_4 x\cos x + 2c_4 x\cos x - c_4 x^2\sin x - c_5\sin x - c_5\sin x - c_5 x\cos x + c_6\cos x + c_6\cos x - c_6 x\sin x + 2c_7$

$= \cos x(2c_3+2c_6) + \sin x(2c_4-2c_5) - x\sin x(4c_3+c_6) + x\cos x(4c_4-c_5) - c_3x^2\cos x - c_4x^2\sin x + 2c_7$

$\therefore (D^2+1)y = \cos x(2c_3+2c_6) + \sin x(2c_4-2c_5) - x\sin x(4c_3+c_6) + x\cos x(4c_4-c_5) - c_3x^2\cos x - c_4x^2\sin x + 2c_7 + c_3x^2\cos x + c_4 x^2\sin x + c_5 x\cos x + c_6 x\sin x + c_7 x^2$

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$2c_4 - 2c_5 = -2$ —— (i) $\quad 2c_3 + 2c_6 = 0$ —— (ii)

$4c_4 - \cancel{c_5} + \cancel{c_5} = 4$ —— (iii) $\quad -4c_3 - c_6 = 0$ —— (iv)

$2c_7 = 0$ —— (v)

from (iii) eq, we get

$4c_4 = 4$

$c_4 = 1$

from (i) eq, we get,

$2 - 2c_5 = -2 \quad [c_4 = 1]$

$-2c_5 = -4$

$c_5 = 2$

from (v) eq, we get

$c_7 = 0$

from (ii) + 2× (iv) eq, we get

$$\begin{aligned} 2c_3 + 2c_6 &= 0 \\ -8c_3 - 2c_6 &= 0 \\ \hline -6c_3 &= 0 \end{aligned}$$

$c_3 = 0$

from (ii) eq, we get,

$2c_6 = 0 \quad [c_3 = 0]$

$c_6 = 0$

$\therefore y_p = 0 + x^2 \sin x + 2x \cos x + 0 + 0$

$\because y = y_c + y_p$

$= c_1 \sin x + c_2 \cos x + x^2 \sin x + 2x \cos x$

(Ans)