Calculus Formulas

Power Rules: $\frac{d}{dx}x^n = nx^{n-1}$ and $\int x^n dx = \frac{x^{n+1}}{n+1} + c$		***Product Rule:*** $\frac{d}{dx}\left[f(x)\cdot g(x)\right] = f(x)\cdot g'(x) + f'(x)\cdot g(x)$
Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2}$		***Reciprocal Rule:*** $\frac{d}{dx}\left[\frac{1}{g(x)}\right] = \frac{-g'(x)}{[g(x)]^2}$
Chain Rule: $\frac{d}{dx}(f\circ g)(x) = f'[g(x)]\cdot g'(x)$		***Integration-by-Parts:*** $\int u\,dv = uv - \int v\,du$

Trigonometric Functions

Derivative	***Integral***
$\frac{d}{dx}\sin x = \cos x$	$\int \sin x\,dx = -\cos x + c$
$\frac{d}{dx}\cos x = -\sin x$	$\int \cos x\,dx = \sin x + c$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \tan x\,dx = \ln\lvert\sec x\rvert + c$ $\int \sec^2 x\,dx = \tan x + c$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + c$ $\int \csc^2 x\,dx = -\cot x + c$
$\frac{d}{dx}\sec x = \sec x\cdot\tan x$	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + c$ $\int \sec x\cdot\tan x\,dx = \sec x + c$
$\frac{d}{dx}\csc x = -\csc x\cdot\cot x$	$\int \csc x\,dx = \ln\lvert\csc x - \cot x\rvert + c$ $\int \csc x\cdot\cot x\,dx = -\csc x + c$

Inverse Trigonometric Functions

Derivative	***Integral***
$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-u^2}}\,dx = \sin^{-1}\frac{u}{a} + c$
$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$	
$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$	$\int \frac{1}{a^2+u^2}\,dx = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$
$\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$	
$\frac{d}{dx}\sec^{-1} x = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\int \frac{1}{u\sqrt{u^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\frac{u}{a} + c$
$\frac{d}{dx}\csc^{-1} x = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	

Identities:

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $\sin 2x = 2\sin x\cos x$ $\qquad$ $\cos^2 x = \frac{1+\cos 2x}{2}$

$1 + \cot^2 x = \csc^2 x$ $\qquad$ $\cos 2x = \cos^2 x - \sin^2 x$ $\qquad$ $\sin^2 x = \frac{1-\cos 2x}{2}$

$\tan^2 x + 1 = \sec^2 x$ $\qquad$ $\cos(x+y) = \cos x\cos y - \sin x\sin y$ $\qquad$ $\sin(x+y) = \sin x\cos y + \cos x\sin y$

Exponential Functions

Derivative	***Integral***
$\frac{d}{dx}\left(e^x\right) = e^x$	$\int e^x\,dx = e^x + c$
$\frac{d}{dx}\left(b^x\right) = (\ln b)b^x$	$\int b^x\,dx = \frac{b^x}{\ln b} + c$

Logarithmic Functions

Derivative	***Integral***
$\frac{d}{dx}\left(\ln\lvert x\rvert\right) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}\left(\log_b\lvert x\rvert\right) = \frac{1}{(\ln b)x}$	

Definition of Log base b: $\log_b N = x \Leftrightarrow b^x = N$

Change of Base Formula: $\log_b x = \frac{\ln x}{\ln b} = \frac{\log x}{\log b}$

Identities:

$\ln\left(e^x\right) = x$ $\qquad$ $e^{\ln x} = x$ $\qquad$ $\ln e = \log 10 = \log_b b = 1$

$\log_b\left(b^x\right) = x$ $\qquad$ $b^{\log_b x} = x$ $\qquad$ $\ln 1 = \log 1 = \log_b 1 = 0$

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Calculus Summary Formulas

Differentiation Formulas

1. $\frac{d}{dx}(x^n) = nx^{n-1}$
2. $\frac{d}{dx}(fg) = fg' + gf'$
3. $\frac{d}{dx}(\frac{f}{g}) = \frac{gf' - fg'}{g^2}$
4. $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$
5. $\frac{d}{dx}(\sin x) = \cos x$
6. $\frac{d}{dx}(\cos x) = -\sin x$
7. $\frac{d}{dx}(\tan x) = \sec^2 x$
8. $\frac{d}{dx}(\cot x) = -\csc^2 x$
9. $\frac{d}{dx}(\sec x) = \sec x \tan x$
10. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
11. $\frac{d}{dx}(e^x) = e^x$
12. $\frac{d}{dx}(a^x) = a^x \ln a$
13. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
14. $\frac{d}{dx}(Arc\sin x) = \frac{1}{\sqrt{1-x^2}}$
15. $\frac{d}{dx}(Arc\tan x) = \frac{1}{1+x^2}$
16. $\frac{d}{dx}(Arc\sec x) = \frac{1}{|x|\sqrt{x^2-1}}$
17. $\frac{dy}{dx} = \frac{dy}{dx} \times \frac{du}{dx}$ *Chain Rule*

Trigonometric Formulas

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \csc^2\theta$
4. $\sin(-\theta) = -\sin\theta$
5. $\cos(-\theta) = \cos\theta$
6. $\tan(-\theta) = -\tan\theta$
7. $\sin(A + B) = \sin A\cos B + \sin B\cos A$
8. $\sin(A - B) = \sin A\cos B - \sin B\cos A$
9. $\cos(A + B) = \cos A\cos B - \sin A\sin B$
10. $\cos(A - B) = \cos A\cos B + \sin A\sin B$
11. $\sin 2\theta = 2\sin\theta\cos\theta$
12. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
13. $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\cot\theta}$
14. $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$
15. $\sec\theta = \dfrac{1}{\cos\theta}$
16. $\csc\theta = \dfrac{1}{\sin\theta}$
17. $\cos(\dfrac{\pi}{2} - \theta) = \sin\theta$
18. $\sin(\dfrac{\pi}{2} - \theta) = \cos\theta$

Integration Formulas

Definition of a Improper Integral

$\int_a^b f(x)\,dx$ is an improper integral if

1. f becomes infinite at one or more points of the interval of integration, or
2. one or both of the limits of integration is infinite, or
3. both (1) and (2) hold.

1. $\int a\,dx = ax + C$
2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{1}{x} dx = \ln|x| + C$
4. $\int e^x\,dx = e^x + C$
5. $\int a^x dx = \frac{a^x}{\ln a} + C$
6. $\int \ln x\,dx = x\ln x - x + C$
7. $\int \sin x\,dx = -\cos x + C$
8. $\int \cos x\,dx = \sin x + C$
9. $\int \tan x\,dx = \ln|\sec x| + C$ or $-\ln|\cos x| + C$
10. $\int \cot x\,dx = \ln|\sin x| + C$
11. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
12. $\int \csc x\,dx = \ln|\csc x - \cot x| + C$
13. $\int \sec^2 x\,dx = \tan x + C$
14. $\int \sec x \tan x\,dx = \sec x + C$
15. $\int \csc^2 x\,dx = -\cot x + C$
16. $\int \csc x \cot x\,dx = -\csc x + C$
17. $\int \tan^2 x\,dx = \tan x - x + C$
18. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} Arc\tan\left(\frac{x}{a}\right) + C$
19. $\int \frac{dx}{\sqrt{a^2 - x^2}} = Arc\sin\left(\frac{x}{a}\right) + C$
20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} Arc\sec\frac{|x|}{a} + C = \frac{1}{a} Arc\cos\left|\frac{a}{x}\right| + C$