Date:

2018

5.(a) Let centre is $(h,0)$ and radius r

Equation

$$(x-h)^2+y^2=r^2$$

Diff w.r.t x

$$2(x-h)+2y\frac{dy}{dx}=0$$

Again Diff w.r.t x

$$2(1-0)+2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2=0$$

or, $y\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2+1=0$

which is the required DE.

—x—

(b) Given that,

$$\frac{dy}{dx}=x^3y^3-xy$$

or, $\dfrac{dy}{dx}+xy=x^3y^3$ —— (i)

Equation (i) is in the form

$$\frac{dy}{dx}+P(x)y=Q(x)y^n$$

this is a Barnoulli DE.

$\therefore$ $\dfrac{dy}{dx}y^{-3}+xy^{-2}=x^3$ —— (ii)

D-Star

put, $v = y^{-2}$

or, $\frac{dv}{dx} = -2y^{-3}\frac{dy}{dx}$

or, $y^{-3}\frac{dy}{dx} = -\frac{1}{2}\frac{dv}{dx}$

Replace this at eq. (ii)

$-\frac{1}{2}\frac{dv}{dx} + xv = x^3$

or, $\frac{dv}{dx} - 2vx = -2x^3$ —— (iii)

The form of eq. (iii) is

$\frac{dv}{dx} + p(x)v = Q(x)$

so, this is a linear DE.

where, $p(x) = -2x$

I.F is -

$e^{\int -2x dx} = e^{-2x\frac{x^2}{2}} = e^{-x^2}$

Multiply eq. (iii) by e^{-x^2} and integrating

~~$e^{-x^2}\frac{dv}{dx} - 2.vx e^{-x^2} = -2x^3 e^{-x^2}$~~

~~or, $\frac{d}{dx}(e^{-x^2}.v)$~~

$v.e^{-x^2} = \int e^{-x^2}(-2x^3)\,dx$

or, $v.e^{-x^2} = -\int e^{-x^2}.x^2.2x\,dx$

or, $v.e^{-t} = -\int e^{-t}.t.dt$

Let,
$x^2 = t$
or, $2x\,dx = dt$

or, $v.e^{-t} = -\left[t\int e^{-t}dt - \int\left(\frac{d}{dt}(t)\int e^{-t}d(t)\right)dt\right]$

or, $v.e^{-t} = te^{-t} + \int -e^{-t}dt$

or, $v.e^{-t} = te^{-t} + e^{-t} + c$

or, $v = t + 1 + ce^{t}$

or, $\frac{1}{y^2} = 1 + x^2 + ce^{x^2}$ (solved)

—x—

(c) $(\sin x\cos y + e^{2x})dx + (\cos x\sin y + \tan y)dy = 0$

Here, $M = \sin x.\cos y + e^{2x}$, $N = \cos x.\sin y + \tan y$

$\frac{\partial M}{\partial y} = \sin x.(-\sin y) + 0$

$= -\sin x.\sin y$

$\frac{\partial N}{\partial x} = \cancel{\cos}\ \sin y(-\sin x) +$

$= -\sin x.\sin y$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

It is an exact DE.

This general solution is

$\int M dx$ (y as constant) $+ \int N dy$ (terms free from x) $= c$

or, $\int (\sin x . \cos y + e^{2x}) dx + \int \tan y \, dy = c$

or, $\cos y . (-\cos x) + \frac{e^{2x}}{2} + \log(\sec y) = c$

$\therefore -\cos x . \cos y + \frac{e^{2x}}{2} + \log(\sec y) = c$

where c is an arbitary constant.

(Ans)

—x—

Date:

6.(a) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = \sin 2x$

Auxiliary equation,

$$m^2+4m+1=0$$

$$\text{or, } m = \frac{-4 \pm \sqrt{16-4}}{2}$$

$$\text{or, } m = \frac{-4 \pm 2\sqrt{3}}{2}$$

$$\therefore m = -2 \pm \sqrt{3}$$

$$\therefore y_c = c_1 e^{(-2+\sqrt{3})x} + c_2 e^{(-2-\sqrt{3})x}$$

For particular solution, we have

$$y_p = \frac{1}{D^2+4D+1} \sin 2x$$

$$\text{or, } y_p = \frac{1}{-4+4D+1} \sin 2x$$

$$\text{or, } y_p = \frac{1}{4D-3} \sin 2x$$

$$\text{or, } y_p = \frac{4D+3}{16D^2-9} \sin 2x$$

$$\text{or, } y_p = \frac{4D+3}{16\times(-4)-9} \sin 2x$$

$$\text{or, } y_p = -\frac{1}{73}(4D+3) \sin 2x$$

$$\text{or, } y_p = -\frac{1}{73}(8\cos 2x + 3\sin 2x)$$

$\therefore$ General solution, $y = y_c + y_p$

$$\therefore y = c_1 e^{(-2+\sqrt{3})x} + c_2 e^{(-2-\sqrt{3})x} - \frac{1}{73}(8\cos 2x + 3\sin 2x)$$

(Ans)

— x —

5(b). $(D^3 - 7D - 6)y = e^{2x} x^2$

Auxilary equation

$$m^3 - 7m - 6 = 0$$

~~or,~~ $\cancel{m^2(m+1) - m(m+1) - 6(m+1) = 0}$

or, $m^3 + m^2 - m^2 - m - 6m - 6 = 0$

or, $m^2(m+1) - m(m+1) - 6(m+1) = 0$

or, $(m+1)(m^2 - m - 6) = 0$

or, $(m+1)(m^2 - 3m + 2m - 6) = 0$

or, $(m+1)\{m(m-3) + 2(m-3)\} = 0$

or, $(m+1)(m-3)(m+2) = 0$

$\therefore m = -1, -2, 3$

$\therefore y_c = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{3x}$

For particular solution, we have

Date:

$$y_p = \frac{1}{D^3-7D-6} e^{2x} x^2$$

or, $$y_p = e^{2x} \times \frac{1}{(D+2)^3-7(D+2)-6} x^2$$

or, $$y_p = e^{2x} \times \frac{1}{D^3+6D^2+5D-12} x^2$$

or, $$y_p = e^{2x} \times \frac{1}{-12} \left\{ \frac{1}{1-\left(\frac{5D}{12}+\frac{D^2}{2}+\frac{D3}{12}\right)} \right\} x^2$$

or, $$y_p = -\frac{1}{12} e^{2x} \left(1+\frac{5D}{12}+\frac{D^2}{2}+\frac{D^3}{12}+\frac{25D^2}{144}+\cdots\right) x^2$$

or, $$y_p = -\frac{1}{12} e^{2x} \left(x^2+\frac{5}{12}\times 2x+\frac{2}{2}+0+\frac{25\times 2}{144}\right)$$

or, $$y_p = -\frac{1}{12} e^{2x} \left(x^2+\frac{5x}{6}+1+\frac{25}{72}\right)$$

$\therefore$ $$y_p = -\frac{1}{12} x^2 e^{2x} - \frac{5xe^{2x}}{72} - \frac{97}{864} e^{2x}$$

$\therefore$ General solution, $y = y_c + y_p$

$\therefore$ $$y = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{3x} - \frac{1}{12} x^2 e^{2x} - \frac{5xe^{2x}}{72} - \frac{97}{864} e^{2x}$$

(Ans)

D-Star

6(c) $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$

2017

5(a) **Order:** The order of the differential equation is the order of the highest order derivative present in the equation.

Example: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$. The order is 2.

Degree: The degree of the differential equation is represented by the power of the highest order derivative in the given differential equation.

Example: $\frac{d^4y}{dx^4} + \left(\frac{d^2y}{dx^2}\right)^2 - 3\frac{dy}{dx} + y = 9$. Here, degree = 1

$$2\frac{d^3y}{dx^3} + 3\left(\frac{d^2y}{dx^2}\right)^4 + \frac{dy}{dx} + y = \sin 4x$$

Order = 3
Degree = 1 (Ans)

—x—

5(b) **Homogeneous ODE:**

A first order differential equation is homogeneous if it takes the form: $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$
where, $F\left(\frac{y}{x}\right)$ is a homogeneous function.

$$(2xy+3y^2)dx-(2xy+x^2)dy=0 \quad \text{--- (i)}$$

or, $(2xy+3y^2)\,dx=(2xy+x^2)\,dy$

or, $$\frac{dy}{dx}=\frac{2xy+3y^2}{2xy+x^2}$$

or, $$\frac{dy}{dx}=\frac{2\times\frac{y}{x}+3\left(\frac{y}{x}\right)^2}{2\times\frac{y}{x}+1} \quad \text{[}\because\text{ multiplying by } x^2\text{]}$$

--- (ii)

put,

$y=vx$

or, $$\frac{dy}{dx}=v+x\cdot\frac{dv}{dx}$$

or, $$\frac{2\times\frac{y}{x}+3\left(\frac{y}{x}\right)^2}{2\times\frac{y}{x}+1}=v+x\cdot\frac{dv}{dx}$$

or, $$v+x\cdot\frac{dv}{dx}=\frac{2v+3v^2}{2v+1} \quad \left[\because v=\frac{y}{x}\right]$$

or, $$x\cdot\frac{dv}{dx}=\frac{2v+3v^2}{2v+1}-v$$

$$\frac{2v+3v^2-2v^2-v}{2v+1}=\frac{v+v^2}{2v+1}$$

or, $$x\cdot\frac{dv}{dx}=\frac{v+v^2}{2v+1}$$

or, $$\frac{dx}{x}=\left(\frac{2v+1}{v+v^2}\right)dv$$

or, $\log x+c=\log(v+v^2)$

or, $\log x+\log c=\log(v+v^2)$

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or, $xc = v + v^2$

or, $xc = \frac{y}{x} + \frac{y^2}{x^2}$

$\therefore y^2 + xy = x^3 c$

where c is an arbitrary constant.

(Ans)

—x—

5(c)

Exact differential equation:

Consider the differential equation

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

where M and N have continuous first partial derivatives at all points (x,y) in a rectangular domain D. If

$$\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x}$$

for all $(x,y) \in D$, then the DE is exact in D.

Given that,

$$3x(xy-2)\,dx + (x^3+2y)\,dy = 0$$

Here, $M = 3x(xy-2)$, $N = x^3 + 2y$

$= 3x^2y - 6x$

$\frac{\partial M}{\partial y} = 3x^2$, $\frac{\partial N}{\partial x} = 3x^2$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, the DE is exact.

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The general solution is -

$\int M dx$ (y as constant) + $\int N dy$ (terms free from x) = c

or, $\int (3x^2y - 6x)\,dx + \int (2y)\,dy = c$

or, $3 \times \frac{x^3}{3} \times y - 6 \times \frac{x^2}{2} + 2 \times \frac{y^2}{2} = c$

or, $x^3y - 3x^2 + y^2 = c$

where c is an arbitary constant.

(Ans)

—x—

6(a) <u>Bernoulli's equation:</u>

In mathematics, an ordinary differential equation is called a Bernoulli's differential equation if it is of the form

$$y' + P(x)\,y = Q(x)\,y^n,$$

where n is a real number.

Given that,

$$\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$$

or, $\frac{1}{y(\log y)^2}\frac{dy}{dx} + \frac{y}{x} \times \frac{1}{y(\log y)^2} \times \log y = \frac{1}{x^2}$

or, $\frac{1}{y(\log y)^2}\frac{dy}{dx} + \frac{1}{x(\log y)} = \frac{1}{x^2}$ ——— (i)

put,

$v = \frac{1}{\log y}$

or, $\frac{dv}{dy} = -\frac{1}{y(\log y)^2}\cdot\frac{dy}{dx}$

$\therefore \frac{1}{y(\log y)^2}\cdot\frac{dy}{dx} = -\frac{dv}{dy}$

Replace this at equation (i)

$-\frac{dv}{dy} + \frac{v}{x\cancel{y}} = \frac{1}{x^2}$

$\therefore \frac{dv}{dy} - \frac{v}{x} = -\frac{1}{x^2}$ ——— (ii)

I.F $\Rightarrow e^{\int -\frac{1}{x}dx} = e^{-\log x} = e^{\log x^{-1}} = \frac{1}{x}$

For general solution,

$v\cdot\frac{1}{x} = \int \frac{1}{x}\left(-\frac{1}{x^2}\right)dx + c$

$\frac{x^{-3+1}}{-3+1} = \frac{1}{-2x^2}$

or, $v\cdot\frac{1}{x} = -\int\frac{1}{x^3}dx + c$

or, $v\cdot\frac{1}{x} = \frac{1}{2x^2} + c$

$\therefore \frac{1}{x(\log y)} = \frac{1}{2x^2} + c$ (where c is an arbitary constant) (Ans)

6(b)

(i) $y + px = p^2x^4$ —— (i)

or, $y = -px + p^2x^4$

or, $\frac{dy}{dx} = -p - x \cdot \frac{dp}{dx} + 4p^2x^3 + 2px^4 \cdot \frac{dp}{dx}$

or, $p + p - 4p^2x^3 + x\frac{dp}{dx} - 2px^4\frac{dp}{dx} = 0$

or, $2p - 4p^2x^3 + x\frac{dp}{dx}(1 - 2px^3) = 0$

or, $2p(1-2px^3) + x\frac{dp}{dx}(1-2px^3) = 0$

or, $(1-2px^3)\left(2p + x\frac{dp}{dx}\right) = 0$

$\therefore 1 - 2px^3 = 0$

$\therefore p = \frac{1}{2x^3}$

which doesn't any arbitary constant.

or, $2p + x\frac{dp}{dx} = 0$

or, $x\frac{dp}{dx} = -2p$

or, $\frac{1}{2}\frac{dp}{p} = -\frac{dx}{x}$

$\therefore \cancel{\frac{1}{2}}\log p = -2\log x + \log c$

or, $\log p = \log x^{-2} + \log c$

or, $\log p = \log\frac{c}{x^2}$

$\therefore p = \frac{c}{x^2}$

At equation (i) put $p = \frac{c}{x^2} \Rightarrow$

$y + \frac{c}{x^2} \times x = \frac{c^2}{x^4} \times x^4$

$\therefore y + \frac{c}{x} - c^2 = 0$ where c is an arbitary constant. (Ans)

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(ii) $y = 2px + y^2p^3$ —— (i)

or, $y - y^2p^3 = 2px$

Here, $p = \frac{dy}{dx}$

$\therefore \frac{1}{p} = \frac{dx}{dy}$

or, $x = \frac{y}{2p} - \frac{y^2p^3}{2p}$

or, $x = \frac{1}{2}\frac{y}{p} - \frac{1}{2}(y^2p^2)$

or, $x = \frac{1}{2}\left(\frac{y}{p} - y^2p^2\right)$

or, $\frac{dx}{dy} = \frac{1}{2}\left(\frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy} - 2yp^2 - 2py^2\frac{dp}{dy}\right)$

or, $\frac{1}{p} = \frac{1}{2}\left(\frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy} - 2yp^2 - 2py^2\frac{dp}{dy}\right)$

or, $\frac{2p^2}{p} = \frac{2p^2}{\cancel{2}}\left(\frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy} - 2yp^2 - 2py^2\frac{dp}{dy}\right)$

or, $2p - p + y\frac{dp}{dy} + 2yp^4 + 2p^3y^2\frac{dp}{dy} = 0$

or, $p + 2yp^4 + y\frac{dp}{dy}(1 + 2p^3y) = 0$

or, $p(1 + 2yp^3) + y\frac{dp}{dy}(1 + 2yp^3) = 0$

or, $(1 + 2yp^3)\left(p + y\frac{dp}{dy}\right) = 0$

$\therefore 1 + 2p^3y = 0$

$\therefore p = \sqrt[3]{-\frac{1}{2y}}$

where doesnot any arbitary constant.

or, $p + y\frac{dp}{dy} = 0$

or, $\frac{dp}{dy} = -\frac{p}{y}$

or, $\frac{dp}{p} = -\frac{dy}{y}$

or, $\log p = -\log y + \log c$

$\therefore p = \frac{c}{y}$

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At equation (i) put $p = \frac{c}{y}$

$$\text{or, } y = 2 \times \frac{c}{y} \times x + y^2 \times \frac{c^3}{y^3}$$

$$\therefore y = 2x \cdot \frac{c}{y} + \frac{c^3}{y}$$

where c is an arbitary constant.

(Ans)

—x—

7(a)

(i) $(D^3 - D^2 - 6D) y = 1 + x^2$

A.E is

$$m^3 - m^2 - 6m = 0$$

$$\text{or, } m(m^2 - m - 6) = 0$$

$$\text{or, } m(m^2 - 3m + 2m - 6) = 0$$

$$\text{or, } m\{m(m-3) + 2(m-3)\} = 0$$

$$\text{or, } m(m-3)(m+2) = 0$$

$$\therefore m = 0, -2, 3$$

$$\therefore y_c = c_1 + c_2 e^{-2x} + c_3 e^{3x}$$

For particular solution, we have

$$y_p = \frac{1}{(D^3 - D^2 - 6D)} (1 + x^2)$$

$$= \frac{1}{-6D} \left\{ \frac{1}{1 - \left(\frac{-D}{6} + \frac{D^2}{6}\right)} \right. (1 + x^2)$$

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$$= \frac{1}{-6D}\left(1 - \frac{D}{6} + \frac{D^2}{6} + \frac{D^2}{36} + \cdots\right)(1+x^2)$$

$$= \frac{1}{-6D}(1) + \frac{1}{-6D}\left(x^2 - \frac{\cancel{2}x}{\cancel{6}_3} + \frac{\cancel{2}}{\cancel{6}_3} + \frac{2}{36}\right)$$

$$= -\frac{1}{6}x - \frac{1}{6D}\left(x^2 - \frac{x}{3} + \frac{7}{18}\right)$$

$$= -\frac{x}{6} - \frac{1}{6} \times \frac{x^3}{3} + \frac{1}{6} \times \frac{1}{3}\frac{x^2}{2} - \frac{1}{6} \times \frac{7}{18} \times x$$

$$\therefore y_p = -\frac{x^3}{18} + \frac{x^2}{36} - \frac{25x}{108}$$

$$\therefore y = y_c + y_p$$

$$= c_1 + c_2 e^{-2x} + c_3 e^{3x} - \frac{x^3}{18} + \frac{x^2}{36} - \frac{25x}{108} \text{ (Ans)}$$

—x—

(ii) $(D^2+4)y = \cos x$

A.E is

$m^2 + 4 = 0$

or, $m^2 = -4$

$\therefore m = \pm 2i$

$\therefore y_c = c_1 \cos 2x + c_2 \sin 2x$

For particular solution, we have

$$y_p = \frac{1}{D^2+4}(\cos x)$$

$$= \frac{1}{-1+4}\cos x = \frac{1}{3}\cos x$$

$\therefore y = y_c + y_p = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{3}\cos x$ (Ans)

7(b) $4y'' - 4y' - 8y = 8e^{-x}$ —— (i)

A.E is

$4m^2 - 4m - 8 = 0$

or, $m^2 - m - 2 = 0$

or, $m^2 - 2m + m - 2 = 0$

or, $m(m-2) + 1(m-2) = 0$

or, $(m-2)(m+1) = 0$

$\therefore m = 2, -1$

$\therefore y_c = c_1 e^{-x} + c_2 e^{2x}$

Here,

$y_p = ue^{-x} + ve^{2x}$ —— (ii)

$y_p' = -ue^{-x} + 2ve^{2x} + u'e^{-x} + v'e^{2x}$

Now,

$u'e^{-x} + v'e^{2x} = 0$ —— (iii)

$\therefore y_p' = -ue^{-x} + 2ve^{2x}$ —— (iv)

Again,

$y_p'' = +ue^{-x} + 4ve^{2x} - u'e^{-x} + 2v'e^{2x}$ —— (v)

$\therefore 4\cancel{ue^{-x}} + \cancel{16ve^{2x}} - 4u'e^{-x} + 8v'e^{2x} + \cancel{4ue^{-x}} - \cancel{8ve^{2x}}$
$- \cancel{8ue^{-x}} - \cancel{8ve^{2x}} = 8e^{-x}$

or, $-4u'e^{-x} + 8v'e^{2x} = 8e^{-x}$ —— (vi)

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(iii) × 8 − (vi)

$$8u'e^{-x} + 8v'e^{2x} = 0$$
$$\underset{(+)}{-}\, 4u'e^{-x} \underset{(-)}{+} 8v'e^{2x} = \underset{(-)}{8e^{-x}}$$

$$12u'e^{-x} = -8e^{-x}$$

$$\text{or, } u' = -\frac{2}{3}$$

$$\therefore u = -\frac{2}{3}x$$

putting u' in equation (iii)

$$-\frac{2}{3}e^{-x} + v'e^{2x} = 0$$

$$\text{or, } v'e^{2x} = \frac{2}{3}e^{-x}$$

বা, $v' = \frac{2}{3}e^{-3x}$

$$\text{or, } v = \frac{2}{3}\frac{e^{-3x}}{-3}$$

$$\therefore v = -\frac{2}{9}e^{-3x}$$

u and v in equation (ii)

$$y_p = -\frac{2}{3}xe^{-x} - \frac{2}{9}e^{-3x} \times e^{2x}$$

$$= -\frac{2}{3}xe^{-x} - \frac{2}{9}e^{-x}$$

$$\therefore y = y_c + y_p$$

$$= c_1e^{-x} + c_2e^{2x} - \frac{2}{3}xe^{-x} - \frac{2}{9}e^{-x} \text{ (Ans)}$$

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8(a)

Regular singular point: In mathematics, in the theory of ordinary differential equations in the complex plane $\mathbb{C}$, the points of $\mathbb{C}$ are classified into ordinary points, at which the equation's coefficients are analytic functions, and singular points, at which some coefficient has a singularity.

Given that,

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2+2)y = 0$$

$$\therefore \quad y'' + xy' + (x^2+2)y = 0 \quad \text{--- (i)}$$

Here, $y = \sum_{i=0}^{\infty} a_i x^i$ --- (ii)

or, $y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \cdots$

or, $y' = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + \cdots$

or, $y'' = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \cdots$

Replace y, y' and y'' in equation (i) $\Rightarrow$

$$2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \cdots + a_1x + 2a_2x^2 + 3a_3x^3 + 4a_4x^4 + 5a_5x^5 + \cdots + a_0x^2 + a_1x^3 + a_2x^4 + a_3x^5 + a_4x^6 + a_5x^7 + \cdots + 2a_0 + 2a_1x + 2a_2x^2 + 2a_3x^3 + 2a_4x^4$$

$2a_5x^5 + \cdots = 0$

or, $(2a_2+2a_0)+(6a_3+a_1+2a_1)x+(12a_4 + 2a_2 + a_0 + 2a_2)x^2+(20a_5+3a_3+2a_3+a_1)x^3+\cdots = 0$

Now,

$2a_2+2a_0=0$

or, $2a_2 = -2a_0$

$\therefore a_2 = -a_0$

$6a_3+a_1+2a_1=0$

বা, $3a_1 = -6a_3$

$\therefore a_3 = -\frac{1}{2}a_1$

$12a_4+2a_2+a_0+2a_2=0$

or, $12a_4 = -4\times(-a_0) - a_0$

or, $12a_4 = 3a_0$

$\therefore a_4 = \frac{1}{4}a_0$

$20a_5+3a_3+2a_3+a_1=0$

or, $20a_5 = -5\left(-\frac{1}{2}a_1\right) - a_1$

or, $20a_5 = \frac{5}{2}a_1 - a_1$

or, $20a_5 = \cancel{\frac{1}{8}a_1 - a_1}\ \frac{3a_1}{2}$

$\therefore a_5 = \frac{3}{40}a_1$

$\therefore y = a_0 + a_1x - a_0x^2 - \frac{1}{2}a_1x^3 + \frac{1}{4}a_0x^4 + \frac{3}{40}a_1x^5 + \cdots$

$\therefore y = a_0\left(1 - x^2 + \frac{1}{4}x^4 + \cdots\right) + a_1\left(x - \frac{1}{2}x^3 + \frac{3}{40}x^5 + \cdots\right)$

this is the general solution.

(Ans)

—x—

2016

5(a) {same as 2017 5(a)}

Given circle -

$$x^2+y^2+2gx+2fy+c=0$$

Here, three arbitrary constant.

$$\therefore\ 2x+2yy'+2g+2fy'=0$$

~~Again,~~

$\cancel{2+}$

$$\text{or, } x+yy'+g+fy'=0$$

Again,

$$1+yy''+(y')^2+fy''=0$$

$$\therefore f=-y-\frac{1}{y''}-\frac{(y')^2}{y''} \quad \text{—— (i)}$$

Again,

$$0+yy'''+y''y'+2y'y''+fy'''=0$$

$$\text{or, } 3y'y''+y'''(y+f)=0 \quad \text{—— (ii)}$$

putting f in equation (ii)

$$3y'y''+y'''\left(\cancel{y}-\cancel{y}-\frac{1}{y''}-\frac{(y')^2}{y''}\right)=0$$

$$\text{or, } 3y'(y'')^2-y'''(1+(y')^2)=0$$

(solved)

— x —

Date:

5(b) Given that,

$$(x^2+y^2)dx - 2xy\,dy = 0$$

Let, $M = x^2+y^2$, $N = -2xy$

$$\frac{\partial M}{\partial y} = 2y \quad \& \quad \frac{\partial N}{\partial x} = -2y$$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

It is not exact.

$$\therefore (x^2+y^2)dx = 2xy\,dy$$

or, $\dfrac{dy}{dx} = \dfrac{x^2+y^2}{2xy}$

or, $\dfrac{dy}{dx} = \dfrac{x}{2y} + \dfrac{y}{2x}$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{2} \times \dfrac{1}{\left(\frac{y}{x}\right)} + \dfrac{1}{2}\left(\dfrac{y}{x}\right)$ —— (i)

put, $y = vx$

or, $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

or, $\dfrac{1}{2} \times \dfrac{1}{\left(\frac{y}{x}\right)} + \dfrac{1}{2}\left(\dfrac{y}{x}\right) = v + x\dfrac{dv}{dx}$

or, $\dfrac{1}{2} \times \dfrac{1}{v} + \dfrac{1}{2}v = v + x\dfrac{dv}{dx}$

or, $x\dfrac{dv}{dx} = \dfrac{1}{2v} + \dfrac{v}{2} - v$

or, $x\dfrac{dv}{dx} = \dfrac{1-v^2}{2v}$

or, $\frac{dx}{x} = -\left(\frac{-2v}{1-v^2}\right) dv$

or, $\log x = -\log(1-v^2) + \log c$

or, $\log x + \log(1-v^2) = \log c$

or, $x(1-v^2) = c$

or, $x\left(1 - \frac{y^2}{x^2}\right) = c$

or, $\frac{x^2-y^2}{x} = c$

$\therefore x^2 - y^2 = cx$

where c is an arbitrary constant.

(Ans)

—x—

5(c) $y'' + y = \cos^2(x)$

A.E is

$m^2 + 1 = 0$

or, $m^2 = -1$

$\therefore m = \pm i$

$\therefore y_c = c_1 \cos x + c_2 \sin x$

For particular solution, we have -

$y_p = u\cos x + v\sin x$ —— (i)

$y_p' = -u\sin x + v\cos x + u'\cos x + v'\sin x$

Here, $u'\cos x + v'\sin x = 0$ —— (ii)

$\therefore y_p' = -u\sin x + v\cos x$ —— (iii)

Date:

$$y_p'' = -u\cos x - v\sin x - u'\sin x + v'\cos x \quad \text{—— (iv)}$$

Now,

$$-u\cos x - v\sin x - u'\sin x + v'\cos x + u\cos x + v\sin x$$
$$= \cos^2 x$$

or, $-u'\sin x + v'\cos x = \cos^2 x$ —— (v)

(ii) $\times$

$$v'\sin^2 x + u'\sin x\cos x = 0$$
$$v'\cos^2 x - u'\sin x\cos x = \cos^3 x$$

$$v' = \cos^3 x$$

or, $v = \int \cos x(1-\sin^2 x)\,dx$

or, $v = \int(\cos x - \cos x\cdot\sin^2 x)\,dx$

or, $v = \sin x - \frac{1}{3}\sin^3 x$

$u' = -\sin x\cos^2 x$

or, $u = \frac{1}{3}\cos^3 x$

$\therefore y_p = \frac{1}{3}\cos^4 x + \sin^2 x - \frac{1}{3}\sin^4 x$

$\therefore y = y_c + y_p$

$= c_1\cos x + c_2\sin x + \frac{1}{3}\cos^4 x + \sin^2 x - \frac{1}{3}\sin^4 x$

[solved]

Date

6(a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = xe^{-x}$

A.E is

$m^2 + 2m + 2 = 0$

or, $m = \frac{-2 \pm \sqrt{4-8}}{2}$

or, $m = \frac{-2 \pm 2i}{2} = -1 \pm i$

$\therefore y_c = e^{-x}(c_1 \cos x + c_2 \sin x)$

For particular solution, we have

$y_p = \frac{1}{D^2+2D+2} xe^{-x}$

$= e^{-x} \frac{1}{(D-1)^2+2(D-1)+2} x$

$= e^{-x} \times \frac{1}{1+D^2} x$

$= e^{-x}(1-D^2+D^4 - \cdots\cdots)x$

$= e^{-x}(x-0+0)$

$= xe^{-x}$

$\therefore y = y_c + y_p$

$= e^{-x}(c_1\cos x + c_2 \sin x) + xe^{-x}$

(Ans)

D-Star

6(b) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos(x)$

A.E is

$m^2 - 2m + 4 = 0$

or, $m = \frac{2 \pm \sqrt{4-16}}{2} = \frac{2 \pm \sqrt{-12}}{2}$

or, $m = \frac{2 \pm 2i\sqrt{3}}{2}$

$\therefore m = 1 \pm \sqrt{3}i$

$\therefore y_c = e^x \{c_1 \cos(\sqrt{3}x) + c_2 \sin(\sqrt{3}x)\}$

For particular solution, we have

$y_p = \frac{1}{D^2 - 2D + 4} e^x \cos x$

$D^2 + \cancel{2D} + 1 - \cancel{2D} - 2 + 4$

$D^2 + 3$

$= e^x \times \frac{1}{(D+1)^2 - 2(D+1) + 4} \cos x$

$= e^x \times \frac{1}{D^2 + 3} \cos x$

$= e^x \times \frac{1}{-1+3} \cos x$

$= \frac{1}{2} e^x \cos x$

$\therefore$ General solution $y = y_c + y_p$

$= e^x \{c_1 \cos(\sqrt{3}x) + c_2 \sin(\sqrt{3}x)\}$

$+ \frac{1}{2} e^x \cos x$

(Ans)

6(c)

7. {2017, 8 (a)} same

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$\therefore\ y'' + xy' + y = 0$ ——— (i)

Now, $y = \sum_{i=0}^{\infty} a_i x^i$

$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \ldots$

or, $y' = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + \ldots$

or, $y'' = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \ldots$

Replace y'', y' and y in equation (i)

$2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \ldots + a_1x + 2a_2x^2 +$
$3a_3x^3 + 4a_4x^4 + 5a_5x^5 + \ldots + a_0 + a_1x + a_2x^2$
$+ a_3x^3 + a_4x^4 + a_5x^5 + \ldots = 0$

or, $(2a_2 + a_0) + (6a_3 + a_1 + a_1)x + (12a_4 + 2a_2 + a_2)x^2 +$
$(20a_5 + 3a_3 + a_3)x^3 + \ldots = 0$

$\therefore\ 2a_2 + a_0 = 0$

$\therefore\ a_2 = -\frac{1}{2}a_0$

$6a_3 + a_1 + a_1 = 0$

or, $6a_3 = -2a_1$

$\therefore\ a_3 = -\frac{1}{3}a_1$

$$12a_4 + 2a_2 + a_2 = 0$$
or, $12a_4 = -3a_2$
or, $a_4 = -\frac{1}{4} \times \left(-\frac{1}{2}a_0\right)$
$= +\frac{1}{8}a_0$

$$20a_5 + 3a_3 + a_3 = 0$$
or, $20a_5 = -4a_3$
or, $a_5 = -\frac{1}{5} \times \left(-\frac{1}{3}a_1\right)$
$= \frac{1}{15}a_1$

$\therefore y = a_0 + a_1 x - \frac{1}{2}a_0 x^2 - \frac{1}{3}a_1 x^3 + \frac{1}{8}a_0 x^4 + \frac{1}{15}a_1 x^5 + \cdots$

$\therefore y = a_0\left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 + \cdots\right) + a_1\left(x - \frac{1}{3}x^3 + \frac{1}{15}x^5 + \cdots\right)$

[solved]

—x—

2015

5(a)

Variable separable equation:

An equation of the form
$F(x)G(y)dx + f(x)g(y)dy = 0$ is called an equation with variables separable or simply a separable equation.

$$x^2\frac{dy}{dx} = y - xy \quad ; \quad y(-1) = -1$$

Given that,

$$x^2\frac{dy}{dx} = y - xy$$

or, $x^2\frac{dy}{dx} = y(1-x)$

or, $\frac{dy}{y} = \left(\frac{1-x}{x^2}\right)dx$

Date:

$$\text{or, } \frac{dy}{y} = \left(\frac{1}{x^2} - \frac{1}{x}\right) dx$$

$$\text{or, } \ln y = -\frac{1}{x} - \ln x + \ln c$$

$$\text{or, } -\frac{1}{x} = \ln y + \ln x - \ln c$$

$$\text{or, } -\frac{1}{x} = \ln(xy) - \ln c$$

$$\text{or, } e^{-\frac{1}{x}} = \frac{xy}{c}$$

$$\therefore e^{\frac{1}{x}} = \frac{c}{xy} \quad \text{--- (i)}$$

given $y=-1$, when $x=-1$

$$\therefore e^{-1} = \frac{c}{(-1)(-1)} \quad \text{or, } \frac{1}{e} = c$$

c in equation (i)

$$e^{\frac{1}{x}} = \frac{\frac{1}{e}}{xy}$$

$$\text{or, } xy = \frac{1}{e} \times \frac{1}{e^{1/x}} = \frac{1}{e^{1+1/x}}$$

$$\therefore xy = \frac{1}{e^{\frac{x+1}{x}}} \quad \text{(Ans)}$$

—x—

D-Sta

5(b) $y' + 3x^2 y = x^2$

Rewrite the differential equation as

$$\frac{dy}{dx} + p(x)\,y = Q(x)$$

$\therefore$ $p(x) = 3x^2$ & $Q(x) = x^2$

I.F is

$$e^{\int P(x)dx} = e^{\int 3x^2 dx} = e^{\int 3x \frac{x^3}{3} dx} = e^{x^3}$$

$\therefore$ The General solution is

$$y \cdot e^{\int P(x)dx} = \int e^{\int P(x)dx} \cdot Q(x)\, dx$$

or, $y \cdot e^{x^3} = \int e^{x^3} \cdot x^2 dx$

Let,
$z = e^{x^3}$
or $dz = 3x^2 e^{x^3} dx$
or, $\frac{1}{3} dz = x^2 e^{x^3} dx$

or, $y \cdot z = \int \frac{1}{3} dz$

or, $y \cdot z = \frac{1}{3} z + c$

or, $y \cdot e^{x^3} = \frac{1}{3} e^{x^3} + c$

$\therefore$ $y = \frac{1}{3} + \frac{c}{e^{x^3}}$ (Ans)

—x—

5(c)

$$x\frac{dy}{dx}+y=\frac{1}{y^2} \quad \text{----- (i)}$$

or, $$\frac{dy}{dx}+\frac{y}{x}=\frac{1}{xy^2} \quad \text{—— (ii)}$$

The equation is the form of

$$\frac{dy}{dx}+p(x)y=Q(x)y^n$$

So, it is Bernoulli equation.

Now, $$\frac{dy}{dx}\cdot y^2+\frac{y^3}{x}=\frac{1}{x} \quad \text{—— (iii)}$$

put, $v=y^3$

or, $$\frac{dv}{dx}=3y^2\frac{dy}{dx}$$

$$\therefore y^2\frac{dy}{dx}=\frac{1}{3}\frac{dv}{dx}$$

Date:

$$\frac{1}{3}\frac{dv}{dx} + \frac{v}{x} = \frac{1}{x}$$

$$\therefore \frac{dv}{dx} + \frac{3v}{x} = \frac{3}{x}$$

I.F is

$$e^{\int \frac{3}{x}dx} = e^{3\ln x} = e^{\ln x^3} = x^3$$

For general solution, we have

$$v \cdot x^3 = \int x^3 \cdot \frac{3}{x}\, dx + c$$

or, $y^3 . x^3 = \int 3x^2 dx + c$

or, $x^3 y^3 = 3 \times \frac{x^3}{3} + c$

or, $y^3 = 1 + c x^{-3}$

where c is an arbitrary constant.

(Ans)

— x —

Date:

6(a) $y'' + 4y' + 4y = 2x + 6$

A.E is

$$m^2 + 4m + 4 = 0$$

or, $m = \frac{-4 \pm \sqrt{16 - 16}}{2}$

or, $m = \frac{-4}{2}$

$\therefore m = -2, -2$

$y_c = C_1 e^{-2x} + C_2 x e^{-2x}$

For particular solution, we have

$$y_p = \frac{1}{D^2 + 4D + 4}(2x+6)$$

$$= \frac{1}{4} \times \frac{1}{1 + \left(D + \frac{D^2}{4}\right)} (2x+6)$$

$$= \frac{1}{4}\left\{1 - \left(D + \frac{D^2}{4}\right) + \left(D + \frac{D^2}{4}\right)^2 + \cdots\right\}(2x+6)$$

$$= \frac{1}{4}(2x + 6 - 2)$$

$$= \frac{1}{4}(2x+4)$$

$$= \frac{1}{2}(x+2)$$

$\therefore y = y_c + y_p$

$$= C_1 e^{-2x} + C_2 x e^{-2x} + \frac{1}{2}(x+2)$$

(Ans)

—x—

D-Star

6(b) $y'' - y = x^2 e^x + 5$

A.E is

$$m^2 - 1 = 0$$

or, $m^2 = 1$

or, $m = \pm 1$

$\therefore y_c = c_1 e^x + c_2 e^{-x}$

For particular solution, we have

$D^2+2D+1-1$

$$y_p = \frac{1}{D^2-1} x^2 e^x + 5$$

$$= e^x \frac{1}{(D+1)^2-1} x^2 + \frac{1}{D^2-1}(5)$$

$$= e^x \times \frac{1}{D^2+2D} x^2 - \frac{1}{1-D^2}(5)$$

$$= e^x \frac{1}{2D\left(1+\frac{D}{2}\right)} x^2 - (1+D+D^2+D^3+\cdots)(5)$$

$$= e^x \times \frac{1}{2D}\left(1 - \frac{D}{2} + \frac{D^2}{4} - \frac{D^3}{8} + \cdots\right) x^2 - 5$$

$$= e^x \times \frac{1}{2D}\left(x^2 - x + \frac{1}{2}\right) - 5$$

$$= \frac{1}{2} e^x \left(\frac{x^3}{3} - \frac{x^2}{2} + \frac{1}{2}x\right) - 5$$

$\therefore y = y_c + y_p$

$$= c_1 e^x + c_2 e^{-x} + \frac{1}{6} e^x x^3 - \frac{1}{4} x^2 e^x + \frac{1}{4} x e^x - 5$$

(Ans)

Date:

6(c) $y'' - y' - 12y = e^{4x}$

A.E is

$m^2 - m - 12 = 0$

or, $m^2 - 4m + 3m - 12 = 0$

or, $m(m-4) + 3(m-4) = 0$

or, $(m-4)(m+3) = 0$

$\therefore m = 4, -3$

$y_c = c_1 e^{4x} + c_2 e^{-3x}$

For particular solution, we have.

$$y_p = \frac{1}{D^2 - D - 12} e^{4x}$$

$$= e^{4x} \times \frac{1}{(D+4)^2 - (D+4) - 12} (1)$$

$$= e^{4x} \times \frac{1}{D^2 + 7D} (1)$$

$$= e^{4x} \times \frac{1}{7D\left(1 + \frac{D}{7}\right)} (1)$$

$$= e^{4x} \times \frac{1}{7D}\left(1 - \frac{D}{7} + \frac{D^2}{49} + \cdots\right)(1)$$

$$= e^{4x} \times \frac{1}{7D} \times (1)$$

$$= \frac{1}{7} x e^{4x}$$

$\therefore y = y_c + y_p$

$= c_1 e^{4x} + c_2 e^{-3x} + \frac{1}{7} x e^{4x}$ (Ans)

8.(b) $y'' + (\cos x)y = 0$ —— ①

Now,

$$y = \sum_{i=0}^{\infty} a_i x^i$$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \cdots$$

$$y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + \cdots$$

$$y'' = 2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + \cdots$$

Here,

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$(\cos x)y = \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right)\left(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \cdots\right)$$

$$\because y'' + (\cos x)y = 2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + \cdots + a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \cdots - \frac{x^2}{2}a_0 - \frac{x^3}{2}a_1 - \frac{1}{2}x^4 a_2 - \frac{1}{2}a_3 x^5 - \frac{1}{2}a_4 x^6 - \frac{1}{2}a_5 x^7 - \cdots + \frac{1}{24}x^4 a_0 + \frac{1}{24}a_1 x^5 + \cdots = 0$$

$$\Rightarrow (2a_2 + a_0) + (6a_3 + a_1)x + \left(12a_4 + a_2 - \frac{1}{2}a_0\right)x^2 + \left(20a_5 + a_3 - \frac{1}{2}a_1\right)x^3 = 0$$

Date:

$\therefore 2a_2 + a_0 = 0$

$\therefore a_2 = -\frac{1}{2}a_0$

$6a_3 + a_1 = 0$

or, $a_3 = -\frac{1}{6}a_1$

$12a_4 + a_2 - \frac{1}{2}a_0 = 0$

or, $12a_4 = \frac{1}{2}a_0 + \frac{1}{2}a_0$

$\therefore a_4 = \frac{1}{12}a_0$

$20a_5 + a_3 - \frac{1}{2}a_1 = 0$

or, $20a_5 = \frac{1}{2}a_1 + \frac{1}{6}a_1$

or, $a_5 = \frac{2}{60}a_1$

$\therefore a_5 = \frac{1}{30}a_1$

$$\therefore y = a_0 + a_1x - \frac{1}{2}a_0x^2 - \frac{1}{6}a_1x^3 + \frac{1}{12}a_0x^4 + \frac{1}{30}a_1x^5 + \cdots$$

$$\therefore y = a_0\left(1 - \frac{1}{2}x^2 + \frac{1}{12}x^4 + \cdots\right) + a_1\left(x - \frac{1}{6}x^3 + \frac{1}{30}x^5 + \cdots\right)$$

[solved]

5(c)

2021

1 (a) {Same 2017 5(a)}

circle radius = r

equation,

$$(x-h)^2 + (y-k)^2 = r^2$$

or, $(x-h)^2 + (y-0)^2 = r^2$ ——(i) [$\because$ x - axis $y=0$]

~~or,~~

~~Here 2 arbitrary constant, so, two differentiate this equation,~~

$\therefore\ 2(x-h) + 2y \cdot y' = 0$

or, $(x-h) + yy' = 0$

Again,

$\therefore\ (x-y) = -yy'$

put $(x-y)$ in equation (i)

$\therefore\ (-yy')^2 + y^2 = r^2$

or, $y^2(y')^2 + y^2 = r^2$

or, $y^2\{(y')^2 + 1\} = r^2$

(Ans)

1(b)

Equation of family of parabolas with focus at (0,0) and x-axis as axis is $y^2 = 4a(x+a)$ - (i)

Differentiating (i) with respect to x,

$$2yy_1 = 4a$$

put $2yy_1 = 4a$ in equation (i)

$$y^2 = 2yy_1\left(x + \frac{yy_1}{2}\right)$$

or, $y = 2xy_1 + y(y_1)^2$

$\therefore$ $y = y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right)$

(Ans)

— x —

1(c) $y = x^2p^2 - px$

or, $\frac{dy}{dx} = 2xp^2 + 2x^2p\frac{dp}{dx} - p - x\cdot\frac{dp}{dx}$

or, $p + p + x\frac{dp}{dx} - 2px^2\frac{dp}{dx} - 2xp^2 = 0$

or, $2p - 2xp^2 + (1 - 2px)x\frac{dp}{dx} = 0$

or, $(1-2xp)2p$

$(p^4$ না হলে মিলে না$)$

2.(a) $\cos(x+y)dy = dx$

or, $\cos(x+y)\frac{dy}{dx} = 1$

or, $\cos v\left(\frac{dv}{dx}-1\right) = 1$

~~or, $\cos v.\frac{dv}{dx} - \cos v = 1$~~

or, $\cancel{\cos v}\ \frac{dv}{dx} - 1 = \sec v$

or, $\frac{dv}{dx} = 1+\sec v$

or, $\frac{dv}{1+\sec v} = dx$

or, $\frac{dv}{1+\frac{1}{\cos v}} = dx$

or, $\int\frac{\cos v\, dv}{1+\cos v} = \int dx$

or, $\int\frac{(1+\cos v - 1)}{1+\cos v}dv = \int dx$

or, $v - \int\frac{dv}{1+\cos v} = x+c$

Let,
$x+y = v$
or, $1+\frac{dy}{dx} = \frac{dv}{dx}$
$\therefore \frac{dy}{dx} = \frac{dv}{dx} - 1$

$v - \int\frac{dv}{2\cos^2 v/2} = x+c$

or, $v - \frac{2}{2}\tan\frac{v}{2} = x+c$

or, $v - \tan\frac{v}{2} = x+c$

or, $x+y - \tan\frac{x+y}{2} = x+c$

$\therefore y = \tan\frac{x+y}{2} + c$

(Ans)

Date:

2(b) {Same 2018 5(b)}

2(c)

Given that,

$$\frac{dy}{dx} + y = xy^3$$

or, $y^{-3}\frac{dy}{dx} + y^{-2} = x$

Let $v = y^{-2}$

or, $\frac{dv}{dx} = -2y^{-3}\frac{dy}{dx}$

$\therefore y^{-3}\frac{dy}{dx} = -\frac{1}{2}\frac{dv}{dx}$

Now, $-\frac{1}{2}\frac{dv}{dx} + v = x$

or, $\frac{dv}{dx} - 2v = -2x$

I.F is

$$e^{\int -2dx} = e^{-2x}$$

$\therefore$ the general solution is

$$v \cdot e^{-2x} = \int e^{-2x}(-2x)$$

or, $y^{-2} \cdot e^{-2x} = -\left[x\int e^{-2x}dx - \int\left\{\frac{d}{dx}(x)\int e^{-2x}dx\right\}dx\right]$

or, $\frac{1}{y^2} \cdot e^{-2x} = -x \times \frac{e^{-2x}}{-2} + \int \frac{e^{-2x}}{-2}dx$

or, $\frac{1}{y^2}e^{-2x} = \frac{1}{2}xe^{-2x} + \frac{1}{4}e^{-2x} + c$

$\therefore \frac{1}{y^2} = \frac{1}{2}x + \frac{1}{4} + ce^{2x}$ (Ans)

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3(a) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\cos x$ —— (i)

A.E $m^2 - 2m - 3 = 0$

or, $m^2 - 3m + m - 3 = 0$

or, $m(m-3) + 1(m-3) = 0$

or, $(m-3)(m+1) = 0$

$\therefore m = -1, 3$

$\therefore y_c = c_1e^{-x} + c_2e^{3x}$

Now,

$y_p = c_3e^x + c_4\sin x + c_5\cos x$ —— (ii)

or, $y_p' = c_3e^x + c_4\cos x - c_5\sin x$

or, $y_p'' = c_3e^x - c_4\sin x - c_5\cos x$.

put y_p, y_p' and y_p'' in equation (i)

$c_3e^x - c_4\sin x - c_5\cos x - 2c_3e^x - 2c_4\cos x + 2c_5\sin x$
$- 3c_3e^x - 3c_4\sin x - 3c_5\cos x = 2e^x - 10\cos x$

or, $-4c_3e^x + (2c_5 - 4c_4)\sin x + (-2c_4 - 4c_5)\cos x$
$= 2e^x - 10\cos x$

compare with both side -

$-4c_3 = 2$

$\therefore c_3 = -\frac{1}{2}$

Date:

$2c_5 - 4c_4 = 0$ ——— (iii)

$-2c_4 - 4c_5 = -10$ ——— (iv)

(iii) $\times 2$ – (iv) $\times 4 \Rightarrow$

$$4c_5 - \cancel{8c_4} = 0$$

$$\underset{(+)}{-} 16c_5 \underset{(+)}{-} \cancel{8c_4} = \underset{(+)}{-} 40$$

$$20c_5 = 40$$

$$\therefore c_5 = 2$$

c_5 in equation (i)

$$2 \times 2 - 4 \times c_4 = 0$$

$$\text{or, } 4c_4 = 4$$

$$\therefore c_4 = 1$$

putting c_3, c_4, c_5 in equation (ii)

$$y_p = -\frac{1}{2}e^x + \sin x + 2\cos x$$

$$\therefore y = y_c + y_p$$

$$= c_1 e^{-x} + c_2 e^{3x} - \frac{1}{2}e^x + \sin x + 2\cos x$$

(Ans)

— x —

3(c) $(D^2+4)y = \cos^2 x$ —— (i)

A.E is

$m^2+4=0$

or, $m^2=-4$

$\therefore m = \pm 2i$

$\therefore y_c = c_1\cos 2x + c_2 \sin 2x$

$\therefore y_p = u\cos 2x + v\sin 2x$ — (ii)

$y_p' = -2u\sin 2x + 2v\cos 2x + u'\cos 2x + v'\sin 2x$

$\therefore u'\cos 2x + v'\sin 2x = 0$ —— (iii)

$\therefore y_p' = -2u\sin 2x + 2v\cos 2x$ —— (iv)

$y_p'' = -4u\cos 2x - 4v\sin 2x - 2u'\sin 2x + 2v'\cos 2x$ —— (v)

put equation (ii), (iv) and (v) in equation (i)

$\therefore -4u\cos 2x - 4v\sin 2x - 2u'\sin 2x + 2v'\cos 2x +$
$4u\cos 2x + 4v\sin 2x = \cos^2 x$

$\therefore -2u'\sin 2x + 2v'\cos 2x = \cos^2 x$ —— (vi)

(vi) $\times \cos 2x + 2 \times \sin 2x \Rightarrow$

$-2u'\sin 2x \cdot \cos 2x + 2v'\cos^2 2x = \cos^2 x \cdot \cos 2x$

$2u'\cos 2x \cdot \sin 2x + 2v'\sin^2 2x = 0$

$2v' = \cos^2 x \cdot \cos 2x$

or, $v' = \frac{1}{2}\cos^2 x \cdot \cos 2x$

$$\int v' dx = \frac{1}{4}\int 2\cos^2 x \cdot \cos 2x \, dx$$

$$\text{or, } v = \frac{1}{4}\int (1+\cos 2x) \cdot \cos 2x \, dx$$

$$= \frac{1}{4}\int (\cos 2x + \cos^2 2x) dx$$

$$= \frac{1}{4} \times \frac{\sin 2x}{2} + \frac{1}{4\times 2}\int (1+\cos 4x) dx$$

$$= \frac{1}{8}\sin 2x + \frac{1}{8}x + \frac{1}{8}\times\frac{\sin 4x}{4}.$$

$$= \frac{1}{8}x + \frac{1}{8}\sin 2x + \frac{1}{32}\sin 4x.$$

v' in equation (iii)

$$u'\cos 2x + \frac{1}{2}\cos^2 x \cdot \cos 2x \overset{\sin 2x}{} = 0$$

$$\text{or, } u' = -\frac{1}{2}\cos^2 x \cdot \sin 2x$$

$$\text{or, } \int u' dx = -\frac{1}{2}\int \cos^2 x \cdot 2\sin x \cdot \cos x \, dx$$

$$\text{or, } u = -\frac{2}{2}\int \cos^3 x \cdot \sin x \, dx$$

Let $\cos x = z$

or, $-\sin x \, dx = dz$

$$\text{or, } u = \int z^3 \cdot dz$$

$$= \frac{z^4}{4}$$

$$\therefore u = \frac{\cos^4 x}{4}$$

$$\therefore y_p = \frac{1}{4}\cos^4 x \cdot \cos 2x + \sin 2x\left(\frac{1}{8}x + \frac{1}{8}\sin 2x + \frac{1}{32}\sin 4x\right)$$

$$\therefore y = y_c + y_p$$

$$= c_1\cos 2x + c_2\sin 2x + \frac{1}{4}\cos^4 x \cdot \cos 2x + \sin 2x \left(\frac{1}{8}x + \frac{1}{8}\sin 2x + \frac{1}{32}\sin 4x\right)$$

(Ans)

4(a) Given that,

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2+2)y = 0 \quad \text{—— (i)}$$

Now, $y = \sum_{i=0}^{\infty} a_i x^i$

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \cdots$$

$$y' = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + \cdots$$

$$y'' = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \cdots$$

put y'', y' and y in equation (i)

$$2a_2 + 6a_3x + \boxed{12a_4x^2} + 20a_5x^3 + \cdots + a_1x + \boxed{2a_2x^2}$$
$$+ 3a_3x^3 + 4a_4x^4 + 5a_5x^5 + \cdots + \boxed{a_0x^2} + a_1x^3 +$$
$$a_2x^4 + a_3x^5 + a_4x^6 + \cdots + 2a_0 + 2a_1x + \boxed{2a_2x^2}$$
$$+ 2a_3x^3 + 2a_4x^4 + 2a_5x^5 + \cdots = 0$$

or, $(2a_2 + 2a_0) + (6a_3 + a_1 + 2a_1)x + (12a_4 + 2a_2 + a_0 + 2a_2)x^2$

$$+ (20a_5 + 3a_3 + a_1 + 2a_3)x^3 = 0$$

compare this equation

$2a_2 + 2a_0 = 0$
or, $a_2 = -a_0$

$6a_3 + 3a_1 = 0$
$\therefore a_3 = -\frac{1}{2}a_1$

$12a_4 + 4a_2 + a_0 = 0$
or, $12a_4 = -4\times(-a_0) -$
or, $a_4 = \frac{1}{4}a_0$

$20a_5 + 5a_3 + a_1 = 0$

or $20a_5 = -5(-\frac{1}{2}a_1) - a_1$

or, $a_5 = \frac{3}{40}a_1$

$$y = a_0 + a_1 x - a_0 x^2 - \frac{1}{2} a_1 x^3 + \frac{1}{4} a_0 x^4 + \frac{3}{40} a_1 x^5 + \cdots$$

$$= a_0 \left(1 - x^2 + \frac{1}{4} x^4 + \cdots\right) + a_1 \left(x - \frac{1}{2} x^3 + \frac{3}{40} x^5 + \cdots\right)$$

this is the general solution.

(Ans)

—x—

2020

5(a) <u>Ordinary differential equation:</u> In mathematics, an ordinary differential equation is a differential equation dependent on only a single independent variable.

Given that,

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{2xy}$$

or, $$\frac{dy}{dx} = \frac{x^2 \left\{1 - 3\left(\frac{y}{x}\right)^2\right\}}{2xy}$$

or, $$\frac{dy}{dx} = \frac{1 - 3\left(\frac{y}{x}\right)^2}{2x\left(\frac{y}{x}\right)} \quad \text{—— (i)}$$

put,

$$y = vx$$

or, $$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

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put in equation (i)

$$v + x\frac{dv}{dx} = \frac{1-3v^2}{2v}$$

$$\text{or, } x\frac{dv}{dx} = \frac{1-3v^2}{2v} - v$$

$$\text{or, } x\frac{dv}{dx} = \frac{1-5v^2}{2v}$$

$$\text{or, } \frac{dx}{x} = \frac{2v}{1-5v^2}dv$$

$$\text{or, } \int\frac{dx}{x} = -2\times\frac{1}{10}\int\frac{-10v}{1-5v^2}dv$$

$$\text{or, } \ln x = -\frac{1}{5}\ln(1-5v^2) + \ln c$$

$$\text{or, } \ln x + \frac{1}{5}\ln(1-5v^2) = \ln c$$

$$\text{or, } \ln x + \ln(1-5v^2)^{\frac{1}{5}} = \ln c$$

$$\text{or, } x\times\left(1-5\frac{y^2}{x^2}\right)^{\frac{1}{5}} = c$$

where c is an arbitrary constant (Ans)

— x —

5(b) $$\frac{dy}{dx} + \frac{3y}{x} = 6x^2$$

$$\text{or, } \frac{dy}{dx} + \left(\frac{3}{x}\right)y = 6x^2$$

Rewrite the differential equation as

$$\frac{dy}{dx} + P(x)y = Q(x)$$

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$\therefore P(x) = \frac{3}{x}$, $Q = 6x^2$

$\therefore$ I.F is

$e^{\int p(x)dx} = e^{\int \frac{3}{x} dx} = e^{3\ln x} = e^{\ln x^3} = x^3$

$\therefore$ The General solution is

$$y \cdot e^{\int p(x)dx} = \int e^{\int p(x)dx} \cdot Q\, dx$$

or, $y \cdot x^3 = \int x^3 \times 6x^2\, dx$

or, $y \cdot x^3 = \int 6x^5\, dx$

or, $y \cdot x^3 = 6 \times \frac{x^6}{6} + c$

or, $y \cdot x^3 = x^6 + c$

$\therefore y = x^3 + \frac{c}{x^3}$

(Ans)

- x -

6(b)

$y = px + p^2$ —(i) where, $p \equiv \frac{dy}{dx}$

or, $\frac{dy}{dx} = p + x\frac{dp}{dx} + 2p\frac{dp}{dx}$

or, $p = p + x\frac{dp}{dx} + 2p\frac{dp}{dx}$

or, $\frac{dp}{dx}(x+2p) = 0$

$\therefore \frac{dp}{dx} = 0$

or, $\int \frac{dp}{dx} = \int 0$

$\therefore p = c$

or, $x + 2p = 0$

$\therefore p = -x/2$

Here, doesn't arbitrary constant

put $p = c$ in equation (i)

$y = cx + c^2$ (Ans)

—x—

6(c)

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x$

{same as 2021, 3(a)}

Then,

$2C_5 - 4C_4 = 0 - 10$ —— (iii)

$-2C_4 - 4C_5 = 0$

or, $C_4 + 2C_5 = 0$ —— (iv)

(iii) + (iv) × 4 ⇒

$8C_5 + \cancel{4C_4} = 0$

$2C_5 - \cancel{4C_4} = -10$

$10C_5 = -10$

$\therefore C_5 = -1$

C_5 in equation (iv)

$C_4 + 2\times(-1) = 0$

$\therefore C_4 = 2$

C_3, C_4, C_5 in equation (ii)

$y_p = -\frac{1}{2}e^x + 2\sin x - \cos x$

$\therefore y = y_c + y_p$

$= C_1e^{-x} + C_2e^{3x} - \frac{1}{2}e^x + 2\sin x - \cos x$

(Ans)

— x —

7(a) {Same 2021, 2(c)}

7(b) $(D^2+4)y = x^2e^{2x}$

A.E is

$m^2 + 4 = 0$

or, $m^2 = -4$

$\therefore m = \pm 2i$

$\therefore y_c = C_1\cos 2x + C_2\sin 2x$

For particular solution, we have

$y_p = \frac{1}{D^2+4} x^2e^{2x}$

$= e^{2x} \times \frac{1}{(D+2)^2+4} x^2$

$= e^{2x} \times \frac{1}{8+4D+D^2} x^2$

$= c_1 \cos 2x + c_2$

—x—

$$\text{or, } y_p = e^{2x} \times \frac{1}{8} \times \frac{1}{\left\{1+\left(\frac{D}{2}+\frac{D^2}{8}\right)\right\}} x^2$$

$$\text{or, } y_p = \frac{1}{8} \times e^{2x} \times \left\{1-\left(\frac{D}{2}+\frac{D^2}{8}\right)+\left(\frac{D}{2}+\frac{D^2}{8}\right)^2 - \cdots\right\} x^2$$

$$= \frac{1}{8} \times e^{2x}\left(x^2 - x - \frac{1}{4} + \frac{1}{2}\right)$$

$$= \frac{1}{8}\left(x^2 - x + \frac{1}{4}\right)$$

$$= \frac{1}{8}x^2 - \frac{1}{8}x + \frac{1}{32}$$

$$\therefore y = y_c + y_p$$

$$= c_1 \cos 2x + c_2 \sin 2x + \frac{1}{8}x^2 - \frac{1}{8}x + \frac{1}{32}$$

(Ans)

6(b) {same 2021, 2(c)}

6(c) {same 2017, 6(b)}

7(b) $(D^4+2D^3-3D^2)y = 3e^{2x}+4\sin x$

A.E is

$$m^4+2m^3-3m^2=0$$

or, $m^2(m^2+2m-3)=0$

or, $m^2(m^2+3m-m-3)=0$

or, $m^2\{m(m+3)-1(m+3)\}=0$

or, $m^2(m+3)(m-1)=0$

$\therefore m = 0, 0, 1, -3$

$\therefore y_c = c_1+c_2+c_3e^x+c_4e^{-3x}$

For particular sol[illegible]

For particular solution we have,

$$y_p = \frac{1}{D^4+2D^3-3D^2}(3e^{2x}+4\sin x)$$

$$= 3\times e^{2x}\times \frac{1}{2^4+2\times 2^3-3\times 2^2} + \frac{1}{D^2(D^2+2D-3)} 4\sin x$$

$$= \frac{3}{20}e^{2x} + 4\times \frac{1}{(-1)\{(-1)+2D-3\}} \sin x$$

$$= \frac{3}{20}e^{2x} + 4\times\frac{1}{2}\frac{1}{-(2-D)}\sin x$$

$$= \frac{3}{20}e^{2x} + (-2)\times\frac{D+2}{D^2-4}\sin(x)$$

$$= \frac{3}{20}e^{2x} + 2\times\frac{1}{5}(\cos x+2\sin x)$$

$$= \frac{3}{20}e^{2x} + \frac{2}{5}\cos x + \frac{4}{5}\sin x$$

$$\therefore y = c_1 + c_2 + c_3 e^{x} + c_4 e^{-3x} + \frac{3}{20}e^{2x} + \frac{2}{5}\cos x$$
$$+ \frac{4}{5}\sin x \text{ (Ans)}$$

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