Date: 21/10/24

Math-2231

Numerical Methods.

- Pollinomial.
- System of inner equation.
- Differentiation
- Integration.
- Matrix
- Python.

Error → 4 type

1. Modelling.
2. Inherical — Data Conversion.
3. Numerical
4. Blunders error.

Date: 23/10/24

SS. Shastri → Introductory method. of Numerical Analysis.

1.1423 → significant figure 5.

1.01423 → significant figure 6.

0.01423 → significant figure 4.

1) All no-zero digits.

2) All leading zero after decimal are non significant.

3) All zeros between non-zero. digits are significant.

4) After decimal followings zeros after digits are significa

→ Accuracy and precession.

1.6583 → 1.658

দশমিকের পর তিনঘর

30.0567 → 30.06. (4 significant)

0.859378 → 0.8594. (4 significant)

3.14159 → 3.142

Round করার নিয়মঃ—

>5 হলে পরবর্তী digit +1 হবে

<5 হলে increment হবে না

$=5$ হলে odd হলে increment হবে even হলে increment হবে না।

Measurement of Errors:—

* Absolute Error : $E_A = X - X_1$
* Relative Error : $E_R = \frac{E_A}{X} = \frac{\Delta X}{X}$
* Percentage Error : $E_P = E_R \times 100\%$

If x is rounded to N decimal places then.

$\boxed{\Delta x = \frac{1}{2} \cdot 10^{-N}}$ → Important

$x = 0.51$ then the parcentage value?

$\Delta x = \frac{1}{2} \cdot 10^{-N}$ | $x = 0.51$, $x = 8.6$

$= \frac{1}{2} \cdot 10^{-2}$

$= 0.005$.

$E_R = \frac{0.005}{0.51}$.

$E_P = \frac{0.005}{0.51} \times 100\%$

1/3 → 0.30, 0.33, 0.34.

$\frac{1}{3} \leftarrow 0.30 =$

up to 4 significant digit.

38.46235 → 38.46

0.700029 → 0.7000

0.0022218 → 0.002222

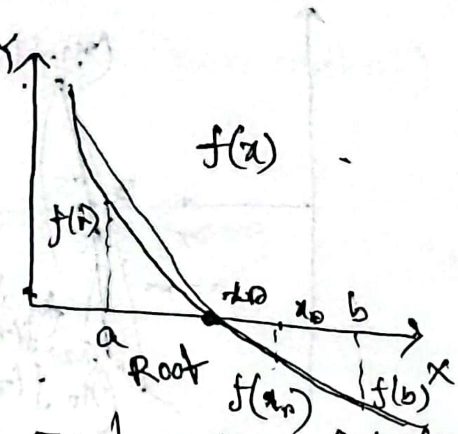
19.235101 → 19.24

2.36425 → 2.364

significant digit থেকে 1 টি Question হয়—

(*)

$f(x) = 0$



$$\frac{|b-a|}{2^n} < \epsilon.$$

$$n \geq \frac{\ln\left(\frac{|b-a|}{\epsilon}\right)}{\ln 2}.$$

How to find root value for $f(x)$

→ Closed Method. (Bracket)

→ Open Method.

$f(x) = 0$ if $f(a) * f(b) = -ve$
$f(a) = ?$ then it is closed method.
$f(b) = ?$

Bracket → Bisection.
→ False.

Bisection Method:

Guess a & b.

$f(a) * f(b) < 0$.

then our guess value is right.

$$x_n = \frac{a+b}{2}$$

~~$x_0 = 0$~~

$$\frac{|b-a|}{2^n}$$

1st iteration → }
2nd → } Difference = tollerence.
3rd →
n →

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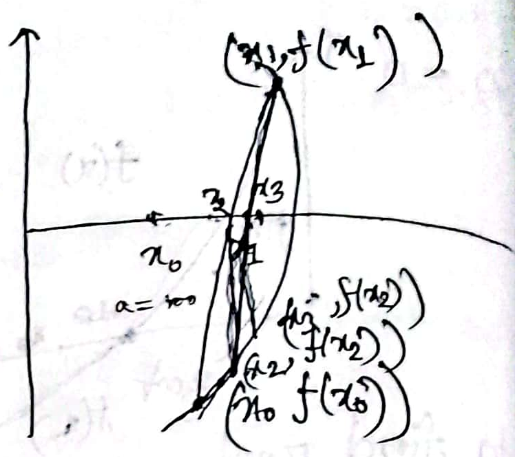
False-Position method:

$$\frac{y - f(x_0)}{x - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$x_2 = x_0 - \frac{f(x_0)}{f(x_1) - f(x_0)}(x_1 - x_0) \quad [y = 0]$$



$f(x) = x^3 - 2x - 5 = 0$

$f(2) = -1$
$f(3) = 16$ } Root 2 and 3 এর মাঝে।

$$a = \frac{(up - 100 + 1)}{}$$

$x_2 = 2 + \frac{1}{17} = 2.059$.

$f(x_2) = -0.386$

$0 \rightarrow$

2000 + 1

$x_3 = 2.059 + \frac{0.386}{16.386} = 2.0812$.

0 — 1000

−1000

tollerence value এর সমান বা কম হলে stop করে দিব।

$\boxed{\text{For exam} \rightarrow \text{theory} + 1\ \text{math}}$

$a = +ve$
$b = -ve$

$$\text{root} = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$c = \frac{a+b}{2}$

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Iteration method: (Open method)

$f(x) = 0$

$x = \varphi(x)$ (Another form)

$x^3 + x^2 - 1 = 0$

$\Rightarrow x^2(x+1) = 1$

$\Rightarrow x = \frac{1}{\sqrt{1+x}}$

$= \varphi(x)$

let initial root x_0

$x_1 = \varphi(x_0)$

$x_2 = \varphi(x_1)$

$\vdots$

$x_{n+1} = \varphi(x_n)$

$x = \varphi(x)$ or $x_1 = \varphi(x_0)$

$\varepsilon = \varphi(\varepsilon)$

$\varepsilon - x_1 = \varphi(\varepsilon) - \varphi(x_0)$

$= (\varepsilon - x_0)\,\varphi(\varepsilon_0) \quad x < \varepsilon_0 < \varepsilon$

$\varepsilon - x_2 = (\varepsilon - x_1)\,\varphi(\varepsilon_1) \quad x_1 < \varepsilon_1 < \varepsilon$

$\vdots$

$\varepsilon - x_{n+1} = (\varepsilon - x_n)\,\varphi'(\varepsilon_n) \quad x_n < \varepsilon_n < \varepsilon$

$(\varepsilon - x_1)(\varepsilon - x_2) \cdots (\varepsilon - x_{n+1}) = (\varepsilon - x_0)(\varepsilon - x_1) \cdots (\varepsilon - x_n)\,\varphi'(\varepsilon_0)\,\varphi'(\varepsilon_1)\,\varphi'(\varepsilon_n)$

$\varepsilon - x_{n+1} = (\varepsilon - x_0)\,\varphi'(\varepsilon_0)\,\varphi'(\varepsilon_1) \cdots \varphi'(\varepsilon_n)$

$|(\varepsilon - x_{n+1})| \leq (\varepsilon - x_0)\,k^{n+1}$.

$\varphi'(x) < 1$.

converse to root otherwise diverse the root.

(*) $x = 10^x + 1$ — converse form.

$x_0 = 0$

$x_1 = 1 + 1 = 2$.

$x_2 = 10^2 + 1 = 101$

$x_3 = 10^{101} + 1 \ldots$

$x = \varphi(x)$

$\varphi(x)$ and $\varphi'(x)$ continuous $f(x) = 0$

$\varphi'(x) < 1$

$$|\varepsilon - x_n| = |\varphi(\varepsilon) - \varphi(x_{n-1})| \le K(\varepsilon - x_{n-1}).$$

or $|\varepsilon - x_n| \le K|\varepsilon - x_n + x_n - x_{n-1}|$

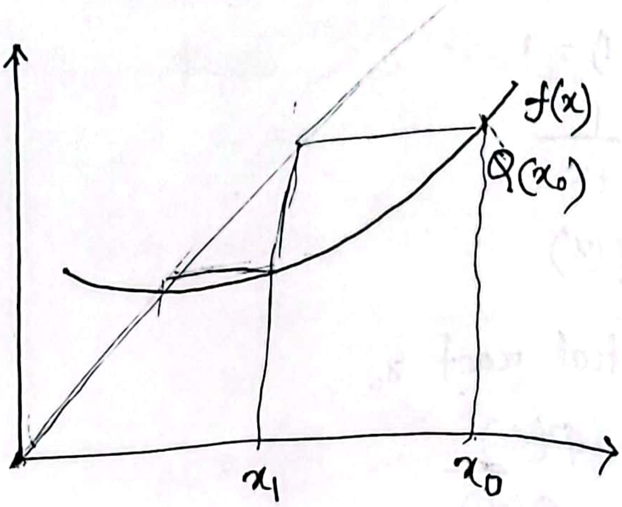
$$\le K|\varepsilon - x_n| + K|x_n - x_{n-1}|$$

$$|\varepsilon - x_n| \le \frac{K}{1-K}|x_n - x_{n-1}| \le \frac{K^n}{1-K}|x_1 - x_0|$$

$$|\varepsilon - x_n| \le \epsilon_p$$

$$\boxed{|x_n - x_{n-1}| \le \frac{1-K}{K} \cdot \epsilon_p}$$

↑ For termination



$f(x)$

$f'(x) \ge 1$

$x_1 = \varphi(x_0)$

Aitken's Process to accelerate the convergence of iteration method

$$(\varepsilon - x_i) = K(\varepsilon - x_{i-1})$$

$$\varepsilon - x_{i+1} = K(\varepsilon - x_i)$$

$$\boxed{\frac{\varepsilon - x_i}{\varepsilon - x_{i+1}} = \frac{\varepsilon - x_{i-1}}{\varepsilon - x_i}}$$

$$\varepsilon = x_{i+1} - \frac{(x_{i+1} - x_i)^2}{x_{i+1} - 2x_i + x_{i-1}}$$

$$= x_{i+1} - \frac{(\Delta x_i)^2}{\Delta^2 x_{i-1}}$$

$$\Delta^2 x_i = \Delta(\Delta x_i)$$

$$\Delta^2 x_{i-1} = \Delta(\Delta x_{i-1})$$

$$= \Delta(x_i - x_{i-1})$$

$$= \Delta x_i - \Delta x_{i-1}$$

x_{i-1}, x_i → Δx_{i-1}

x_i, x_{i+1} → Δx_i

Δx_{i-1}, Δx_i → $\Delta^2 x_{i-1}$

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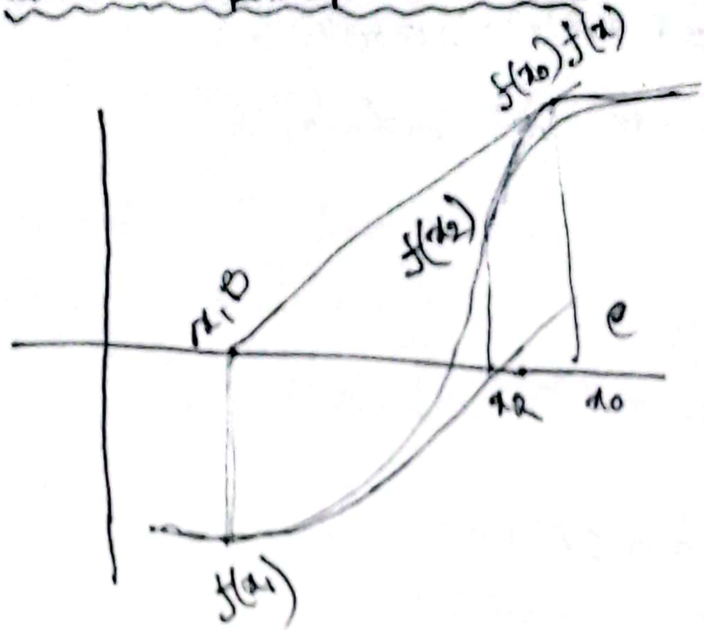
Newton - Raphson method:



$$\tan ABC = \frac{AC}{BC}$$

$$f'(x_0) = \frac{f(x_0)}{h}$$

$$h = \frac{f(x_0)}{f'(x_0)}$$

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

$$x_p = x_0 + h = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$f(x_1) = f(x_0+h)$$

$$= f(x_0) + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \dots = 0$$

$$f(x_n) + (\varepsilon - x_n) f'(x_n) + \frac{1}{2}(\varepsilon - x_n)^2 f''(x_n) + \dots = 0$$

$$x_{n+1} = \phi(x_n)$$

$$= x - \frac{f(x)}{f'(x)}$$

$$\phi'(x) = \frac{f(x) f''(x)}{|f'(x)|^2}$$

$$\phi'(x) < 1$$

$$\epsilon_n = x_n - \varepsilon$$

$$\epsilon_{n+1} = x_{n+1} - \varepsilon$$

$$\epsilon_{n+1} = \frac{1}{2} \epsilon_n^2 \frac{f''(\varepsilon)}{f'(\varepsilon)}$$

$x^3 - 2x - 5 = 0$

$f(x) = x^3 - 2x - 5$

$f'(x) = 3x^2 - 2$

$x_1 = 2 + \frac{1}{10} = 2.1$

$f'(x_1) \simeq \frac{f(x_1) - f(x_0)}{x_1 - x_0}$

(approximate derivation)

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n) - f(x_{n-1})}(x_n - x_{n-1})$

$\boxed{\text{Secant method}}$

$x_1 = F(x_0, y_0),\ y_1 = G(x_0, y_0)$

$x_2 = F(x_1, y_1),\ y_2 = G(x_1, y_1)$

$\vdots$

$x_{n+1} = F(x_n, y_n),\ y_{n+1} = G(x_n, y_n)$

$x = 0.2x^2 + 0.8$

$y = 0.3xy^2 + 0.7$

$F(x,y) = 0.2x^2 + 0.8$

$G(x,y) = 0.3xy^2 + 0.7$

Solution of systems of nonlinear Equation :-

$f(x,y) = 0 \qquad x = F(x,y)$

$g(x,y) = 0 \qquad y = G(x,y)$

$$\left|\frac{\partial F}{\partial x}\right| + \left|\frac{\partial F}{\partial y}\right| < 1$$

$$\left|\frac{\partial G}{\partial x}\right| + \left|\frac{\partial G}{\partial y}\right| < 1$$

$x_1 = F(x_0, y_0)$

$y_1 = G(x_0, y_0)$

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$f(x_0, y_0) \rightarrow$ Initial guess

$f(x_0+h,\ y_0+k)$

Expansion of taylor series.

$$= \underbrace{f(x_0, y_0)}_{f_0} + h\frac{\partial f}{\partial x_0} + k\frac{\partial f}{\partial y_0} + \dots = 0$$

$g(x_0+h, y_0+k)$

$$= \underbrace{g(x_0, y_0)}_{g_0} + h\frac{\partial g}{\partial x_0} + k\frac{\partial g}{\partial y_0} + \dots = 0$$

$$\frac{\partial f}{\partial x_0} = \left|\frac{\partial f}{\partial x}\right|$$

$$h\frac{\partial f}{\partial x_0} + k\frac{\partial f}{\partial y_0} = -f_0$$

$$h\frac{\partial g}{\partial x_0} + k\frac{\partial g}{\partial y_0} = -g_0$$

$$J(f,g) = \begin{vmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{vmatrix}$$

$x_1 = x_0 + h$

$y_1 = y_0 + k$.

$$h = \begin{vmatrix} -f & \frac{\partial f}{\partial y} \\ -g & \frac{\partial g}{\partial y} \end{vmatrix} \Big/ J(f,g)$$

$$k = \begin{vmatrix} \frac{\partial f}{\partial x} & -f \\ \frac{\partial g}{\partial x} & -g \end{vmatrix} \Big/ J(f,g)$$

Interpolation:

$y = f(x)$ $\quad x_0 \le x \le x_n$

$\rightarrow$ unknown polynomial.

$\phi(x) \rightarrow$ Interpolating.

$x_0 \quad x_1 \quad x_2 \quad \dots \quad x_n$

$y_0 \quad y_1 \quad y_2 \quad \dots \quad y_n$

$\rightarrow$ Interpolating polynomial.

$\rightarrow$ Interpolation.

$y = f(x)$

$y = \phi(x)$

$y(x) \quad \phi_n(x) \quad \phi_n(x_i) = y_i, \ i=0,1 \dots n$

$x_0 \dots x_n \quad y(x) - \phi_n(x) = L\,\Pi_{n+1}(x')$

$y_0 \dots x_n \quad \Pi_{n+1}(x) = (x-x_0)(x-x_1)\dots(x-x_n)$

$x_0 < x' < x_n$

$$L = \frac{y(x') - \phi_n(x')}{\Pi_{n+1}(x')}$$

$F(x) = y(x) - \phi_n(x) - L\,\Pi_{n+1}(x) = 0$

$F(x_0) = F(x_1) = \dots = F(x_n) = F(x') = 0$

$F^{(n+1)}(x)$ times differentiate

$0 = y^{n+1}(\varepsilon) - L(n+1)?$

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$$L = \frac{y^{n+1}(\xi)}{(n+1)!}$$

$$\underbrace{y(x) - \phi_n(x)}_{\text{Errorr of Interepolating polynomial.}} = \frac{\Pi_{n+1}(x)}{(n+1)!} y^{n+1}(\xi)$$

$\Delta y_0 = y_1 - y_0$

$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$

$= y_2 - y_1 - y_1 - y_0$

$= y_2 - 2y_1 + y_0$

→ Forroward difference (Δ)

→ Backward difference (∇)

→ Central differena (δ)

Forroward

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0	Δy_0			
			$\Delta^2 y_0$		
x_1	y_1	Δy_1			
			$\Delta^2 y_1$		
x_2	y_2	Δy_2			
			$\Delta^2 y_2$		
x_3	y_3	Δy_3			
x_4	y_4	Δy_4			

Backward

$\nabla y_1 = y_1 - y_0$

Central:

$\delta y_{1/2} = y_1 - y_0$

$\delta y_{3/2} = y_2 - y_1$

$\delta y_{n-\frac{1}{2}} = y_n - y_{n-1}$

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Newton's forward difference interpolation :-

$(x_0, y_0), (x_1, y_1), (x_2, y_2) \ldots\ldots (x_n, y_n)$

$x_i = x_0 + ih \; ; \; i = 0, 1, 2 \ldots n.$

$$y_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2) + \cdots$$
$$\cdots + a_n(x-x_0)(x-x_1)\cdots(x-x_{n-1})$$

$x = x_0 \; , \; x = x_1$

$y_0 = a_0 \, , \quad y_1 = a_0 + a_1(x_1 - x_0) \rightarrow y_1 = y_0 + a_1 h.$

$$a_1 = \frac{y_1 - y_0}{h}$$

$$a_1 = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y_0}{h} \, , \quad a_2 = \frac{\Delta^2 y_0}{2!\,h^2} \, ; \quad a_3 = \frac{\Delta^3 y_0}{3!\,h^3}$$

$x = x_0 + ph.$

$h = \dfrac{x - x_0}{p}$

$$\boxed{y_n(x) \Rightarrow y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots + \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!}\Delta^n y_0}$$

$$y(x) - y_n(x) = \frac{(x-x_0)(x-x_1)\cdots\cdots(x-x_n)}{(n+1)!} y^{n+1}(\xi)$$

$$y(x) - y_n(x) = \frac{p(p-1)(p-2)\cdots(p-n)}{(n+1)!}\Delta^{n+1} y(\xi)$$

Error of newton's forward diff. interpolation.

(1895)

x	1891	1901	1911	1921	1931
y	46	66	81	93	101.

Here, $x = 1895$

$x_0 = 1891$

$h = 10$

$x = x_0 + ph.$ $\quad \therefore p = \dfrac{1895 - 1891}{10} = 0.4$

		Δ	Δ^2	Δ^3	Δ^4
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

$$y_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}$$

$$y(1895) = 46 + 0.4 \times 20 + \frac{0.4(0.4-1)}{2}(-5) +$$

Data যদি অর্ধেকের চেয়ে কম হলে তাহলে forward.

Data যদি অর্ধেকের চেয়ে বেশি হয় তাহলে Backward.

$$y_n(x) = a_0 + a_1(x - x_n) + a_2(x - x_n)(x - x_{n-1}) + a_n(x - x_n)(x - x_{n-1}) \cdots (x$$

$x = x_0 + ph$

$p = \frac{x - x_n}{h}$ $\quad y_n(x) = y_n + p$

$$y_n(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \cdots\cdots + \frac{p(p+1)(p+2)\cdots(p+n-1)}{n!}\nabla^n y_n$$

Newton's Backward diff. Interpolation formula.

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Interpolation with unequal spaced data points:—

(B) Lagrange interpolation

$y(x)$

$L_n(x_i) = y(x_i) = y_i \,;\; i = 0, 1, 2, \ldots n.$

$(x_0, y_0), (x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$

$$L_1(x) = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y$$

$$= l_0(x) y_0 + l_1(x) y_1$$

$$= \sum_{i=0}^{1} l_i(x_i) y_i$$

$$l_0(x) = \frac{x - x_1}{x_0 - x_1}$$

$l_0(x_0) = 1$

$l_0(x_1) = 0$

$l_1(x_0) = 0$

$l_1(x_1) = 1.$

$$L_2(x) = \sum_{i=0}^{2} l_i(x) y_i$$

$$= l_0(x) y_0 + l_1(x) y_1 + l_2(x) y_2.$$

$$= \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1$$

$$+ \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2.$$

$L_n(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n.$

$y_0 = a_0 + a_1 x_0 + a_2 x_0^2 + a_3 x_0^3 + \ldots + a_n x_0^n.$

$y_1 = a_0 + a_1 x_1 + a_2 x_1^2 + \ldots + a_n x_1^n$

$\vdots$

$y_n = a_0 + a_1 x_n + a_2 x_n^2 + \ldots + a_n x_n^n$

$$\begin{vmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^n \\ 1 & x_1 & x_1^2 & \cdots & x_1^n \\ \vdots & & & & \\ 1 & x_n & x_n^2 & \cdots & x_n^n \end{vmatrix} \neq 0.$$

$$= \cancel{\frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1} + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2$$

$$\begin{vmatrix} L_n(x) & 1 & x & \cdots & x^n \\ y_0 & 1 & x_0 & & x_0^n \\ y_1 & 1 & x_1 & & x_1^n \\ \vdots & \vdots & \vdots & & \vdots \\ y_n & 1 & x_n & & x_n^n \end{vmatrix} = 0$$

$$L_n(x) = \sum_{i=0}^{n} l_i(x) y_i$$

$$l_i(x) = \frac{(x - x_0)(x - x_1) \ldots (x - x_{i-1})(x - x_{i+1}) \ldots (x - x_n)}{(x_i - x_0)(x_i - x_1) \ldots (x_i - x_{i-1})(x_i - x_{i+1}) \ldots (x_i - x_n)}$$

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$$\Pi_{n+1}(x) = (x-x_0)(x-x_1)\cdots(x-x_{i-1})(x-x_i)(x-x_{i+1})\cdots(x-x_n)$$

$$\Pi'_{n+1}(x) = \frac{d}{dx}\left[\Pi_{n+1}(x)\right]_{x=x_i}$$

$$= (x_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})$$

<u>Date: 02/12/24</u>

$$L_x(x) = \sum_{i=0}^{n} \frac{\Pi_{n+1}(x)}{(x-x_i)\Pi'_{n+1}(x_i)} y_i$$

Newton's Divided Difference Interpolation formula:-

(General Newton's interpolation formula)

$$[x_0, x_1] = \frac{y_1-y_0}{x_1-x_0}$$

$$[x_0, x_1, x_2] = \frac{[x_1, x_2] - [x_0, x_1]}{x_2-x_0}$$

$$[x_0, x_1, x_2 \cdots x_n] = \frac{[x_1, x_2 \cdots x_n] - [x_0, x_1 \cdots x_{n-1}]}{x_n-x_0}$$

(x_0, y_0) (x, y)

(x_1, y_1)

(x_2, y_2)

$\vdots$

$$[x, x_0] = \frac{y-y_0}{x-x_0}$$

$$y = y_0 + (x-x_0)[x, x_0]$$

$$[x, x_0, x_1] = \frac{[x, x_0] - [x_0, x_1]}{x-x_1}$$

$$[x, x_0] = [x_0, x_1] + (x-x_1)[x, x_0, x_1]$$

$$y = y_0 + (x-x_0)[x_0, x_1] + (x-x_0)(x-x_1)[x, x_0, x_1]$$

$$[x, x_0, x_1, x_2] = \frac{[x, x_0, x_1] - [x_0, x_1, x_2]}{x-x_2}$$

$$y = y_0 + (x-x_0)[x_0, x_1] + (x-x_0)(x-x_1)[x, x_0, x_1] + (x-x_0)(x-x_1)(x-x_2)[x, x_0, x_1, x_2].$$

$$y = y_0 + (x-x_0)[x_0, x_1) + (x-x_0)(x-x_1)[x_0, x_1, x_2] + \cdots \underbrace{+ (x-x_0)(x-x_1)\cdots(x-x_n)[x, x_0, x_1 \cdots x_n]}_{\text{Reminder.}}$$

x	$\log_{10} x$			
300	2.4771			
301 → ?		0.00145		
304	2.4829		-0.00001	
		0.00140		
305	2.4843		0	
		0.00140		
307	2.4871			

0.00145 $y = 2.4771 + 1 * 0.00145 +$

0.00140 $1 * (-3) * (0.00001)$

0.00140 $\therefore y = 2.4786.$

$$\Delta_{01}(x) = y_0 + (x-x_0)[x_0, x_1]$$

$$= \frac{1}{x_1 - x_0}\begin{vmatrix} y_0 & x_0 - x \\ y_1 & x_1 - x \end{vmatrix}.$$

$$\Delta_{012}(x) = \frac{1}{x_2 - x_1}\begin{vmatrix} \Delta_{01}(x) & x_1 - x \\ \Delta_{02}(x) & x_2 - x \end{vmatrix}$$

Date: 09/12/24

Inverse interpolation :-

$$y_k = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

$$x = x_0 + uh$$

$$u = \frac{1}{\Delta y_0}\left[y_k - y_0 - \frac{u(u-1)}{2}\Delta^2 y_0 - \frac{u(u-1)(u-2)}{6}\Delta^3 y_0 - \cdots\right]$$

$$u_1 = \frac{1}{\Delta y_0}[y_u - y_0]$$

$$u_2 = \frac{1}{\Delta y_0}\left[y_k - y_0 - \frac{u(u-1)}{2}\Delta^2 y_0\right]$$

$$u_3 = \frac{1}{\Delta y_0}\left[y_k - y_0 - \frac{u(u-1)}{2}\Delta^2 y_0 - \frac{u(u-1)(u-2)}{6}\Delta^3 y_0\right]$$

$y = x^3$

$y = 10$

$x = ?$

x	y
2	8
4	64
5	125

x	y	Δ	Δ^2	Δ^3
2	8			
		19		
3	27		18	
		37		6
4	64		24	
		61		
5	125			

$y_K = 10$

$y_0 = 8$

$\Delta y_0 = 19$

$\Delta^2 y_0 = 18$

$\Delta^3 y_0 = 6$

$$u_1 = \frac{1}{19}[2] = 0.1$$

$$u_2 = \frac{1}{19}\left[2 - \frac{0.1(0.1-1)}{2}(18)\right]$$

$$u_2 = 0.15$$

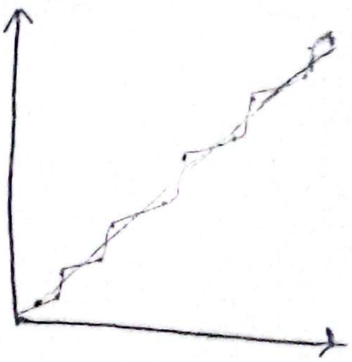
$$u_3 = \frac{1}{19}\left[2 - \frac{0.15(0.15-1)}{2}(18) - \frac{0.15(0.15-1)}{6} \cdots\right]$$

$$u_3 = 0.1532$$

$$x = 2 + 0.154 \times 1$$
$$= 2.154$$

Curve fitting :-

linear + non-linear.



Least square curve fitting Method

(x_i, y_i); $i = 1, 2, 3 \ldots$

$Y = f(x)$

$e_i = y_i - f(x_i)$

$$S = [y_1 - f(x_1)]^2 + [y_2 - f(x_2)]^2 + \cdots + [y_m - f(x_m)]^2$$
$$= e_1^2 + e_2^2 + \cdots + e_m^2$$

General formula for error for curve fitting.

Straight line:

$$Y = a_0 + a_1 x$$

$$S = [y_1 - (a_0 + a_1 x_1)]^2 + [y_2 - (a_0 + a_1 x_2)]^2 + \cdots$$

$$\frac{\partial S}{\partial a_0} = 0 = -2[y_1 - (a_0 + a_1 x_1)] - 2[y_2 - (a_0 + a_1 x_2)] \cdots - 2[y_m - (a_0 + a_1 x_m)]$$

$$\frac{\partial S}{\partial a_1} = 0 = -2x_1[y_1 - (a_0 + a_1 x_1)] - 2x_2[y_2 - (a_0 + a_1 x_2)] \cdots - 2x_m[y_m - (a_0 + a_1 x_m)]$$

$$m a_0 + a_1 \sum_{i=1}^{m} x_i = \sum_{i=1}^{m} y_i$$

$$a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} x_i^2 = \sum_{i=1}^{m} x_i y_i$$

Power function:-

$$y = a x^c$$

or $\log y = \log a + c \log x$.

$$Y = a_0 + a_1 X$$

$$Y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n.$$

$$S = [y_1 - (a_1 x_1 + a_2 x_1^2 + \cdots + a_n x^n)]^2 + [y_2 - (a_0 + a_1 x_2 + \cdots + a_n x_2^n)]^2 + \cdots + [y_m - (a_0 + a_1 x_m + a_2 x_m^2 + \cdots + a_n x_m^n)]^2$$

$$\frac{\partial S}{\partial a_0} = \frac{\partial S}{\partial a_1} = \cdots = \frac{\partial S}{\partial a_n} = 0$$

$$m a_0 + a_1 \sum_{i=1}^{m} x_i + a_2 \sum_{i=1}^{m} x_i^2 + \cdots + a_n \sum_{i=1}^{m} x_i^n = \sum_{i=1}^{m} y_i$$

$$a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} x_i^2 + \cdots + a_n \sum_{i=1}^{m} x_i^{n+1} = \sum_{i=1}^{m} x_i y_i$$

$$\vdots$$

$$a_0 \sum_{i=1}^{m} x_i^n + a_1 \sum_{i=1}^{m} x_i^{n+1} + \cdots + a_n \sum_{i=1}^{m} x_i^{n+n} = \sum_{i=1}^{m} x_i^n y_i$$

x	0	1.0	2.0
y	1.0	6.0	17.0

$$a_0 \sum_{i=1}^{m} w_i + a_1 \sum_{i=1}^{m} w_i x_i = \sum_{i=1}^{m} w_i y_i$$

$$a_0 \sum_{i=1}^{m} w_i x_i + a_1 \sum_{i=1}^{m} w_i x_i^2 = \sum_{i=1}^{m} w_i x_i y_i$$

x	y	x^2	x^3	x^4	xy	x^2y
0	0	0	0	0	0	0
1	6	1	1	1	6	6
2	17	4	8	16	34	68
3	24	5	9	17	40	74

$3a_0 + 3a_1 + 5a_2 = 24$

$3a_0 + 5a_1 + 9a_2 = 40$

$5a_0 + 9a_1 + 17a_2 = 74$

$y = a_0 e^{a_1 x}$

$\log y = \log a_0 + a_1 x.$

$Y = A + Bx.$

Linear Weighted LS method:-

$Y = a_0 + a_1 x$

$$S(a_0, a_1) = \sum_{i=1}^{m} w_i \left[y_i - (a_0 + a_1 x_i)\right]^2$$

$$\frac{\partial S}{\partial a_0} = 0 = -2 \sum_{i=1}^{m} w_i \left[y_i - (a_0 + a_1 x_i)\right]$$

$$\frac{\partial S}{\partial a_0} = 0 = -2 \sum_{i=1}^{m} w_i \left[y_i - (a_0 + a_1 x_i) x_i\right]$$

$$a_0 \sum_{i=1}^{m} w_i + a_1 \sum_{i=1}^{m} w_i x_i = \sum_{i=1}^{m} w_i y_i$$

Date: 18/12/24

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{33} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{vmatrix}$$

0 করতে হবে

Gaussian Elemination $\rightarrow$ Upper Triangular.

Gauss Jordan elemination $\rightarrow$ Upper and lower Triangular.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n = b_1$$

$$\dots\dots\dots\dots\dots$$

$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \dots + a_{nn}x_n = b_n.$$

$$x_1 = \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}}x_2 - \frac{a_{13}}{a_{11}}x_3 - \dots - \frac{a_{1n}}{a_{11}}x_n.$$

$$x_2 = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}}x_1 - \frac{a_{23}}{a_{22}}x_3 - \dots - \frac{a_{2n}}{a_{22}}x_n.$$

$$x_n = \frac{b_n}{a_{nn}} - \frac{a_{n1}}{a_{nn}}x_1 - \frac{a_{n2}}{a_{nn}}x_2 - \dots - \frac{a_{nn-1}}{a_{nn}}x_{n-1}$$

Date: 18/12/24

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{33} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{vmatrix}$$

0 করতে হবে

Gaussian Elemination → Upper Triangular.

Gauss Jordan elimination → Upper and lower Triangular.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_1$$
$$\cdots\cdots\cdots\cdots\cdots$$
$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n = b_n.$$

$$x_1 = \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}}x_2 - \frac{a_{13}}{a_{11}}x_3 - \cdots - \frac{a_{1n}}{a_{11}}x_n.$$

$$x_2 = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}}x_1 - \frac{a_{23}}{a_{22}}x_3 - \cdots - \frac{a_{2n}}{a_{22}}x_n.$$

$$x_n = \frac{b_n}{a_{nn}} - \frac{a_{n1}}{a_{nn}}x_1 - \frac{a_{n2}}{a_{nn}}x_2 - \cdots \frac{a_{nn-1}}{a_{nn}}x_{n-1}$$

Jacobi

$x_1 = x_2 = x_3 = \dots = x_n = 0$

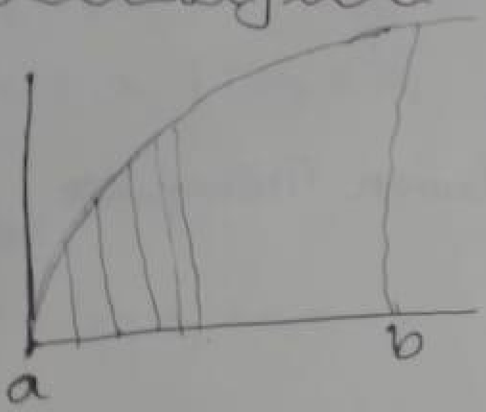
$x_1^{(2)} = \frac{b_1}{a_{11}}$ $\quad x_1^{3} =$

$x_2^{(2)} = \frac{b_2}{a_{22}}$

$x_n^{(2)} = \frac{b_n}{a_{nn}}$

Gauss ~~Sched~~ Seidel

Numerical Integration:—



$y = f(x)$

$$I = \int_{x_0}^{x_n} y\,dx = \int_{x_0}^{x_n} \left[y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{6}\Delta^3 y_0 + \dots \right] dx$$

$$\boxed{\begin{aligned} x_i &= x_0 + ph \\ dx &= h\,dp \end{aligned}}$$

$$I = h\int_0 \left[y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{6}\Delta^3 y_0 + \dots \right] dp.$$

$$\int_{x_0}^{x_n} y\,dx = nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24}\Delta^3 y_0 + \dots \right]$$

$n = 1$ Trapezoidal Rule

$$\int_{x_0}^{x_1} y\,dx = h\left[y_0 + \tfrac{1}{2}\Delta y_0 \right] = h\left[y_0 + \tfrac{1}{2}(y_1 - y_0) \right]$$

$$= \frac{h}{2}\left[y_0 + y_1 \right]$$

$$\int_{x_1}^{x_2} y\,dx = \frac{h}{2}\left[y_1 + y_2 \right]$$

$$\int_{x_{n-1}}^{x_n} y\,dx = \frac{h}{2}\left[y_{n-1} + y_n \right]$$

Curve fitting and interpolation

↓

CT (23/12/24)

Date: 06/01/25

$$\int_{x_0}^{x_1} y\,dx = \int_{x_0}^{x_1} \left[y_0 + (x-x_0) y_0' + \frac{(x-x_0)}{2} y_0'' + \cdots \right]$$

$$= h y_0 + \frac{h^2}{2} y_0' + \frac{h^3}{6} y_0'' + \cdots\cdots$$

$\boxed{x_1 - x_0 = h}$

$$\frac{h}{2}[y_0 + y_1] = \frac{h}{2}\left[y_0 + y_0 + h y_0' + \frac{h^2}{2} y_0' + \frac{h^3}{6} y_0'' + \cdots \right]$$

$$= h y_0 + \frac{h^2}{2} y_0' + \frac{h^3}{4} y'' + \cdots\cdots$$

$$\int_{x}^{x_1} y\,dx - \frac{h}{2}[y_0 + y_1] = -\frac{1}{12} h^3 y_0'' + \cdots\cdots$$

$$E = -\frac{1}{12} h^3 \left(y_0'' + y_1'' + y_2'' + \cdots + y_{n-1}'' \right)$$

$$= -\frac{1}{12} h^3 n y''(x^2).$$

$$= -\frac{(b-a)}{12} h^2 y''(\bar{x}) \qquad [\because nh = b-a]$$

$n = 2$

[Samson]

$$\int_{x_0}^{x_2} y\,dx = 2h\left[y_0 + \Delta y_0 + \frac{1}{6}\Delta^2 y_0 \right].$$

$$= \frac{h}{3}\left[y_0 + 4y_0 + y_2 \right].$$

$$\int_{x_2}^{x_4} y\,dx = \frac{h}{3}\left[y_2 + 4y_3 + y_4 \right]$$

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_n) + y_n \right]$$

Romberg Integration:-

$\boxed{I = \int_a^b y\,dx}$ → Analytical method.

$I_1 \to h_1$

$I_2 \to h_2$.

$I_1 - E_1 =$ True value.

$I_2 - E_2 =$ True value.

$I_1 - E_1 = I_2 - E_2$

or $E_2 - E_1 = I_2 - I_1$

$E_1 = -\frac{1}{12}(b-a)h_1^2 y''(\bar{x})$

$E_2 = -\frac{1}{12}(b-a)h_2^2 y''(\bar{x})$

$\frac{E_1}{E_2} = \frac{h_1^2}{h_2^2} \Rightarrow \frac{E_2}{E_2 - E_1} = \frac{h_2^2}{h_2^2 - h_1^2}$

$$E_2 = \frac{h_2^2}{h_2^2 - h_1^2}(E_2 - E_1) = \frac{h_2^2(I_2 - I_1)}{h_2^2 - h_1^2}$$

$I_3 = I_2 - E_2$.

$= I_2 - \frac{h_2^2(I_2 - I_1)}{h_2^2 - h_1^2}$

$= \frac{I_1 h_2^2 - I_2 h_1^2}{h_2^2 - h_1^2}$

$h_2 = \frac{1}{2}h_1 = \frac{1}{2}h$

$I(h)$ $\quad I(h, \frac{1}{2}h)$ $\quad I(h, \frac{1}{2}h, \frac{1}{4}h)$

$I(\frac{1}{2}h)$

$I(\frac{1}{2}h, \frac{1}{4}h)$ $\quad I(\frac{1}{2}h, \frac{1}{4}h, \frac{1}{8}h)$

$I(\frac{1}{4}h)$

$I(\frac{1}{4}h, \frac{1}{8}h)$

$I(\frac{1}{8}h)$

$I = \int_0^1 \frac{1}{1+x}\,dx$.

$h = 0.5, 0.25, 0.125$.

$I(h) = 0.7084$.

$I(\frac{1}{2}h) = 0.6970$

$I(\frac{1}{4}h) = 0.6941$.

$I(h, \frac{1}{2}h) = 0.7084 + \frac{1}{3}$

$I(h, \frac{1}{2}h) = \frac{1}{3}[4I(\frac{1}{2}h) - I(h)]$

$I_1 = I(h)$ $\quad = \frac{1}{3}[3I(\frac{1}{2}h) + I(\frac{1}{2}h) - I(h)]$

$I_2 = I(\frac{1}{2}h)$ $\quad = I(\frac{1}{2}h) + \frac{1}{3}[I(\frac{1}{2}h) - I(h)]$

Date: 8/01/24

→ Ordinary differential equation. (How to solve it.

- ↳ taylor method.
- ↳ Picard method.

এগুলোর মধ্যে অনেক ত্রুটি থাকে যার কারণে wolerr method আসে

→ woler modified rule/method

এই চারটি method + equation solve.

→ Back substitution.

→ Forward elemination.

The given system of equation can be written as.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix}$$

Now augmented matrix for given system.

$$C = [A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \\ 2 & -1 & 3 & 9 \end{array}\right]$$

$$C \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \\ 0 & -3 & 1 & -3 \end{array}\right]$$

$$C \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \\ 0 & 0 & 2 & 6 \end{array}\right]$$

$x + y + z = 6$

$-2y = -4 \Rightarrow y = 2$

$2z = 6 \Rightarrow z = 3$

$x = \underline{1}$

Gauss-Jordan Method:

1. Consider the system of equations

$Ax = B$ — (1)

2. Find augmented matrix for given system as.

$C = [A:B]$.

3. Transform 'C' into normal form.
4. Find the solution of equation.

Solve by Gauss-Jordan method

$6x - y + z = 13$

$x + y + z = 9$

$10x + y - z = 19$

Soln: The augmented matrix for given system.

$$C = [A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 6 & -1 & 1 & 13 \\ 10 & 1 & -1 & 19 \end{array}\right].$$

$$C \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -7 & -5 & -41 \\ 0 & -9 & -11 & -71 \end{array}\right]$$

$$C \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -2 & -6 & -30 \\ 0 & -7 & -5 & -41 \end{array}\right]$$

* Gauss - Elimination method (working rule)

1. Consider the system of equation.

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_2.$$

$$AX = B$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

2. Find Augmented matrix for given system.

$$C = [A : B]$$

3. Transform of Augmented matrix 'C' into upper triangular form / Echelon form.

4. Find equations corresponding to upper triangular matrix / echelon

5. Using back substitution find solution of given system of equations.

Q. Solve the system of equation by Gauss - Elemination metho.

$$2x - y + 3z = 9.$$
$$x + y + z = 6$$
$$x - y + z = 2.$$

$$C \sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & 3 & 15 \\ 0 & -7 & -5 & -41 \end{bmatrix}$$

$$C \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 3 & 15 \\ 0 & 0 & 16 & 4 \end{array}\right]$$

$$C \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 3 & 15 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$C \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$C \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$x = 2,\ y = 3,\ z = 4.$

ODE:

Standard form

$y_1 = \frac{dy}{dx} = f(x,y)$, with $y(x_0) = y_0$

Find $y(x)$
x is any known value.

$y(x)$ - function

Tylor's method:-

$y(x) = y_0 + (x-x_0)(y_1)_0 + \frac{(x-x_0)^2}{2!}(y_2)_0 + \frac{(x-x_0)^3}{3!}(y_3)_0 + \frac{(x-x_0)^4}{4!}(y_4)$

where

$(y_1)_0 = \left(\frac{dy}{dx}\right)_{x=x_0}$, $(y_2)_0 = \left(\frac{d^2y}{dx}\right)_{x=0}$

$x=0.1, x=0.2$, $\frac{dy}{dx} = x^2y - 1$, $y(0)=1$.

Find $y(x)$.

To find $y(0.1)$ and $y(0.2)$ by taylor's method.

$y(x) = y_0 + (x-x_0)(y_1)_0 + \frac{(x-x_0)^2}{2!}(y_2)_0 + \frac{(x-x_0)^3}{3!}(y_3)_0 + \frac{(x-x_0)^4}{4!}(y_4)_0 +$ — (1)

$x_0 = 0$, $y_0 = 1$.

$y_1 = x^2y - 1 \Rightarrow (y_1)_0 = (x_0)^2 y_0 - 1 = 0 \cdot 1 - 1 = -1$

$y_2 = \frac{dy_1}{dx} = x^2y_1 + 2xy - 0 \Rightarrow (y_2)_0 = (x_0)^2(y_1)_0 + 2(x_0)(y_0) = 0$

$y_3 = \frac{dy_2}{dx} = x^2y_2 + 2xy_1 + 2xy_1 + 2y = x_0^2y_2 + 4xy_1 + 2y = (x_0)^2(y_2)_0 + 4 x_0 (y_1)_0 + 2(y_0)$

$= 2.$

$(y_4$

* Trapezoida Rule: ($n=1$)

$$\int_{a=x_0}^{b=x_n} y\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3+\cdots+y_{n-1})\right]$$

(used for any value of n

* Simpson 1/3 Rule ($n=2$)

$$\int_{a=x_0}^{b=x_n} y\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5+\cdots+y_{n-1}) + 2(y_2+y_4+y_6+\cdots+y_{n-2})\right].$$

[n should be multiple of 2)

(1) $\int_0^6 \frac{dx}{(1+x^2)}$

let $nh = a-b$

$\therefore h = \frac{a-b}{n}$

$\therefore h = \frac{6-0}{6} = 1$

$x_n = x_0 + nh$	$y_n = \frac{1}{1+x_n^2}$
$x_0 = 0$	$y_0 = \frac{1}{1+0} = 1$
$x_1 = x_0 + h = 1$	$y_1 = \frac{1}{1+1^2} = \frac{1}{2}$
$x_2 = x_0 + 2h = 2$	$y_2 = \frac{1}{1+2^2} = \frac{1}{5}$
$x_3 = x_0 + 3h = 3$	$y_3 = \frac{1}{1+3^2} = \frac{1}{10}$
$x_4 = x_0 + 4h = 4$	$y_4 = \frac{1}{1+4^2} = \frac{1}{17}$
$x_5 = x_0 + 5h = 5$	$y_5 = \frac{1}{1+5^2} = \frac{1}{26}$
$x_6 = x_0 + 6h = 6$	$y_6 = \frac{1}{1+6^2} = \frac{1}{37}$

(i) By trapezoidal rule.

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[\left(1+\frac{1}{37}\right) + 2\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{10}+\frac{1}{17}+\frac{1}{26}\right)\right]$$

$= 1.411$.

(ii) By Simpson 1/3 rule.

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left(1+\frac{1}{37}\right) + 4\left(\frac{1}{2}+\frac{1}{10}+\frac{1}{26}\right) + 2\left(\frac{1}{5}+\frac{1}{17}\right)$$

$$(y_4 = \frac{dy_3}{dx} = x^2 y_3 + y_2 2x + 4xy_2 + 4y_1 + yy_1$$

$$= x^2 y_3 + 6xy_2 + 6y_1$$

$$= (x_0)^2 (y_3)_0 + 6x_0 (y_2)_0 + 6(y_1)_0$$

$$= -6$$