

2020

1.a Define alphabet and string. Explain the operations of string.

Solution

An alphabet is a finite set of symbols denoted as  $\Sigma$ . For example  $\Sigma = \{0, 1\}$ .

A string is a finite sequence of symbols over an alphabet  $\Sigma$ .

operations of string:

1. Concatenation: The concatenation of two strings  $u$  and  $v$  is denoted by  $u.v$ , the string is formed by the string  $u$  followed by the string  $v$ . For example  $u = \text{man}, v = \text{city}$  the  $u.v = \text{mancity}$

2. Length: The length of a string  $u$  is the number of symbols in  $u$ , denoted by  $|u|$ . For example,  $u = \text{Rodrigo}$  the  $|u| = 7$

3. Reversal operation: Reversal of a string  $u$  (denoted by  $u^R$ ) is a string obtained by reversing the order of symbols in  $u$ . For example  $u = \text{erdogan} \quad u^R = \text{nagodre}$

## Some additional definitions

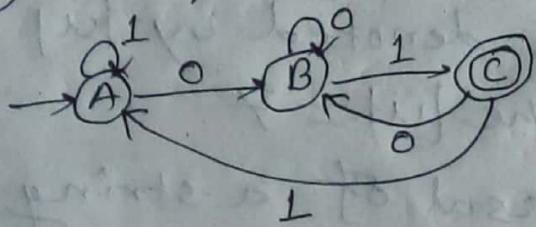
Symbol: Symbols are the basic building blocks of Theory of computation.

Language: Any set of strings (empty or not empty, finite or infinite) is called a language or collection of strings over alphabet ( $\Sigma$ ) under certain condition.

under certain condition.  
 Example:  $L = \{ \text{All strings of length 2 over } \Sigma = \{a, b\} \}$

**2.a** **1.b** construct DFAs for the following regular languages

i)  $L = \{w \in \{0,1\}^* \mid w \text{ ending with '01'}\}$



ii)  $\{w \in \{0,1\}^* \mid w \text{ is a binary number whose decimal equivalent is divisible by } 3\}$

Let  $w$  be any binary number

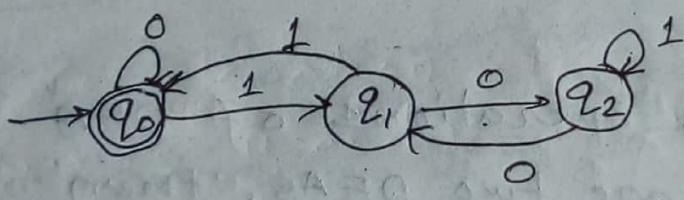
then  $w \bmod 3 = 0, 1, 2$ . So the no. of states  $\exists$

Let them are  $q_0, q_1, q_2$

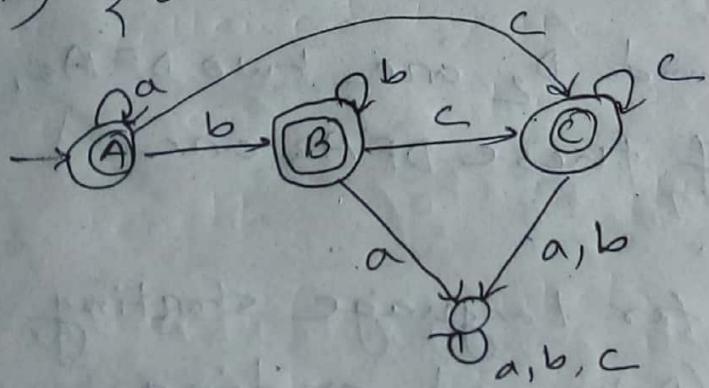
Let's form the state transition table

$\rightarrow q_0$	0	1
$q_0$	$q_0$	$q_1$
$q_1$	$q_2$	$q_0$
$q_2$	$q_1$	$q_2$

Here  $q_0$  is the starting and final or accepting state because we need divisible by 3.



iii)  $\{a^m b^n c^l \mid m, n, l \geq 0\}$



**2.0** Define Finite Automata (FA)

A Finite Automata (FA) is a 5-tuple

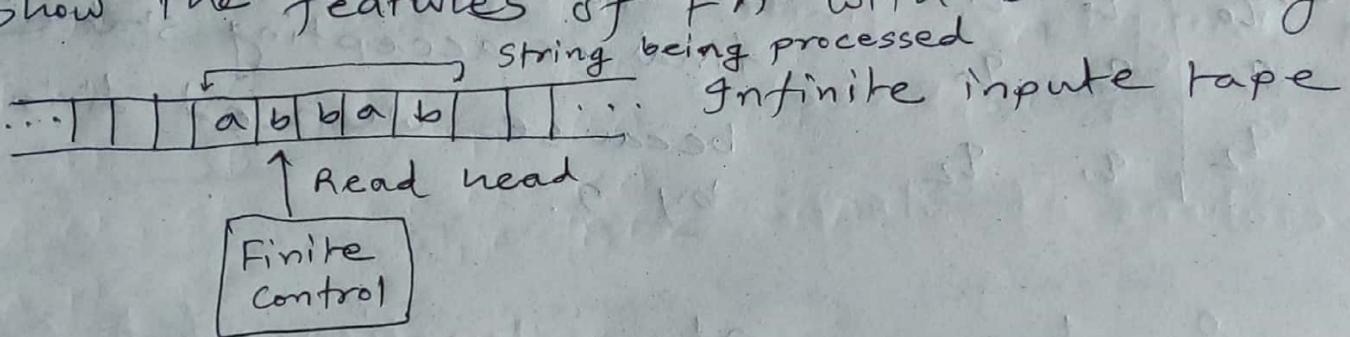
$M = (Q, \Sigma, \delta, q_0, F)$ , where

1.  $Q$  is a finite set of states

2.  $\Sigma$  is a finite set of symbols known as alphabet

3.  $\delta: Q \times \Sigma \rightarrow Q$  is a function, called the transition function.
4.  $q_0$  is an element of  $Q$ ; it is called the starting state.  $q_0 \in Q$
5.  $F$  is a subset of  $Q$ ; the elements of  $F$  are called accept states. ( $F \subseteq Q$ )

Show the features of FA with block diagram



**2.b** Discuss the various operations of DFA

1. Union: If  $D_1$  and  $D_2$  are two DFAs, then union of these DFAs are defined as  $D = D_1 \cup D_2$

2. Concatenation: If  $D_1$  and  $D_2$  are two DFAs, then intersection of these DFAs are denoted by  $D = D_1 \cdot D_2$ .

For example: ~~if~~ DFA for language starting with a ( $D_1$ ) and ending DFA for language ending with b ( $D_2$ )

Then  $D = D_1 \cdot D_2$  is the language starting with a and ending with b

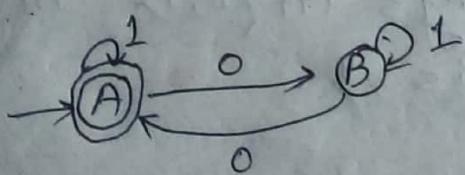
the union is defined as  
 $D_1 = \text{Language starting with } a \text{ and ending with } b$   
 $D_2 = " " " " b " " " " a$   
then  $D = D_1 \cup D_2$  is the DFA for language starting and ending with different symbol.

Complement of DFA: If DFA accepts language  $L$ , then  $\bar{L}$  is accepted by  $\overline{\text{DFA}}$ , a version of DFA where the accepting and non-accepting states are swapped.

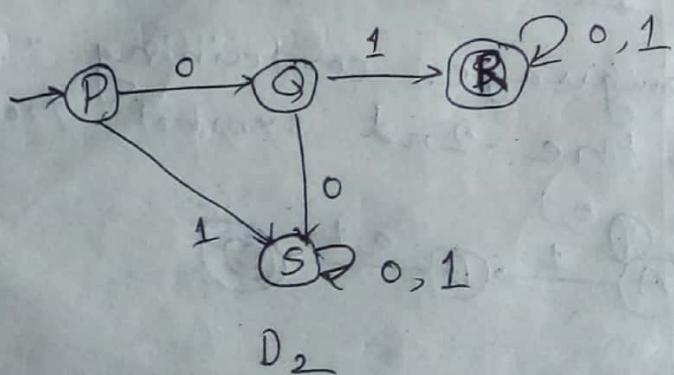
Difference of DFAs. iff  $D_1$  and  $D_2$  are two DFAs then  $D_1 - D_2$  is defined as  $D_1 \cap \overline{D_2}$

Minimization of DFA. It means reducing the number of states from give DFA.

2.c Construct a deterministic machine that accepts the string (say  $w$ ) of language over  $\{0,1\}$  in which no. of 0's of  $w$  is even and  $w$  is started with 01.



$D_1$



$D_2$

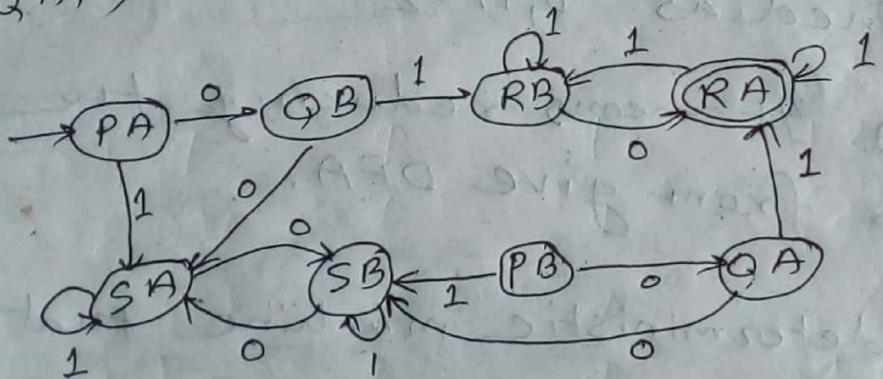
$$D_1 = \{\{A, B\}, \{0, 1\}, A, \delta_{D_1}, \{A\}\}$$

$$D_2 = \{\{P, Q, R, S\}, \{0, 1\}, P, \delta_{D_2}, \{R\}\}$$

$$\text{Now } D = D_1 \cdot D_2$$

$$D = \{\{PA, PB, QA, QB, RA, RB, SA, SB\}, \{0, 1\}, AP, \delta_D, \{RA\}\}$$

$$\begin{array}{lll}
 (PA, 0) = QB & (QB, 0) = SA & (SA, 0) = SB \\
 (PA, 1) = SA & (QB, 1) = RB & (SA, 1) = RA \\
 (PB, 0) = QA & (RA, 0) = RB & (SB, 0) = SA \\
 (PB, 1) = SB & (RA, 1) = RA & (SB, 1) = SB \\
 (QA, 0) = SB & (RB, 0) = RA & \\
 (QA, 1) = RA & (RB, 1) = RB &
 \end{array}$$

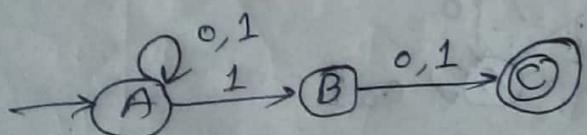


Note: PB and QA are unreachable states

The formula is

$$A \cap B = \{Q_A \times Q_B, \Sigma, \delta, q_A \times q_B, F_A \times F_B\}$$

**3. d)** Construct convert an NFA to DFA for the language containing "all strings in  $\{0, 1\}^*$  in which the 2nd symbol from RHS is 1.



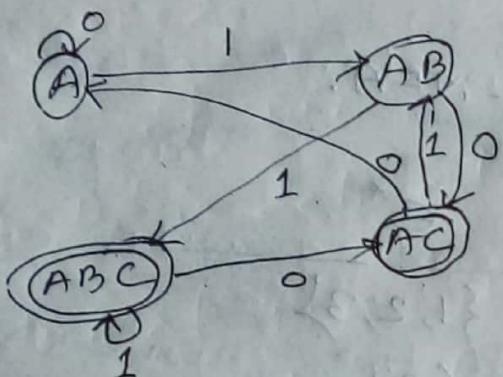
The transition table for NFA is

	0	1
$\rightarrow A$	$\{A\}$ , $\{A, B\}$	$\{C\}$
B	$\{C\}$	$\{C\}$
*	$\emptyset$	$\emptyset$

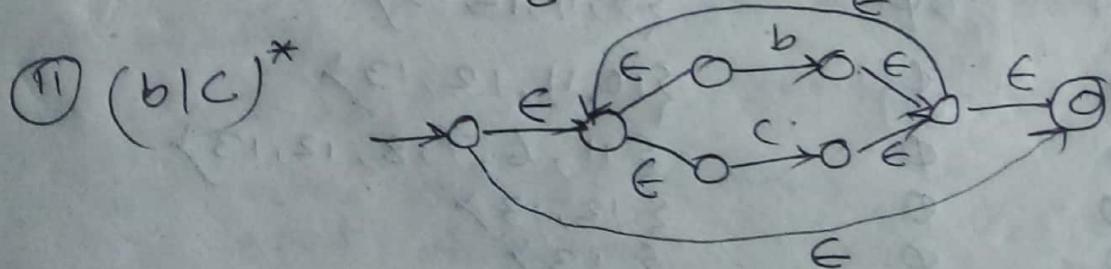
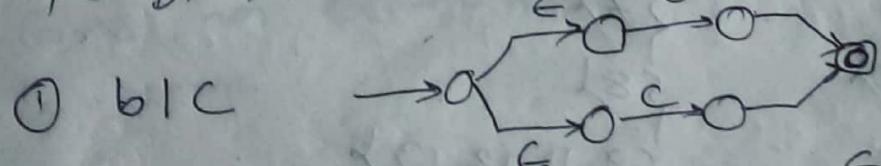
state transition table for DFA

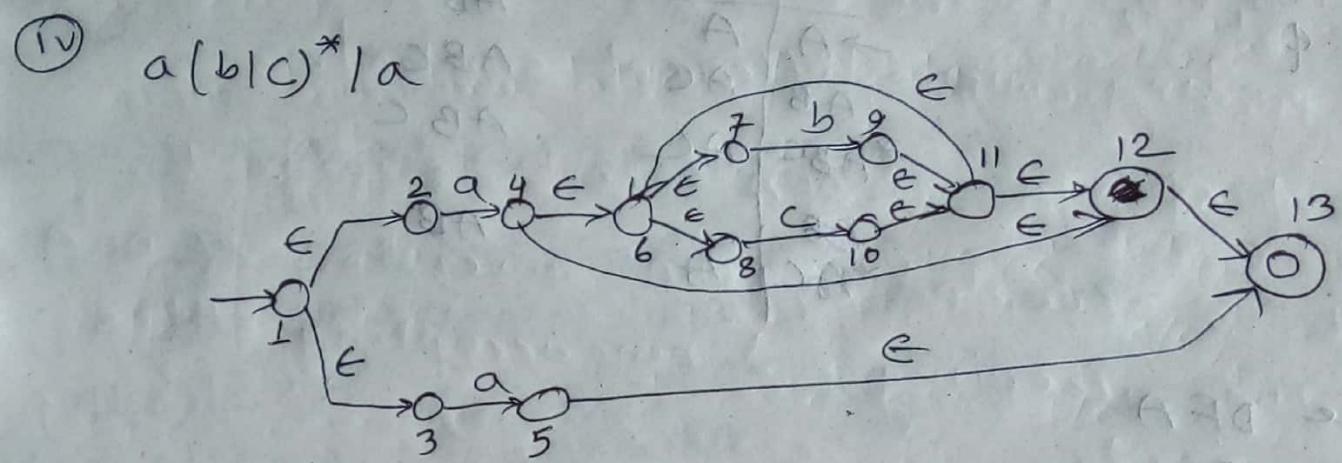
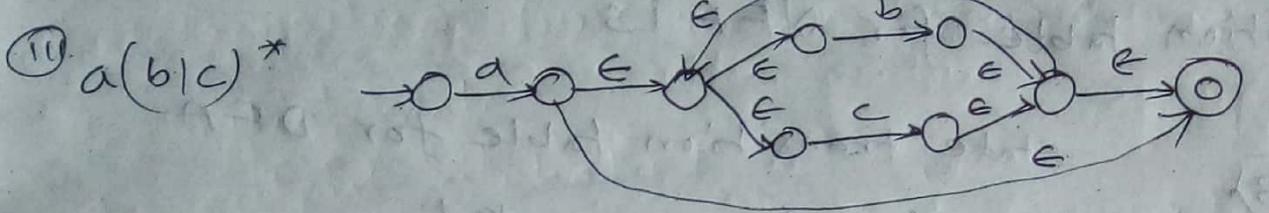
	0	1
$\rightarrow A$	A	
AB	AC	AB
*ABC	AC	ABC
AB		AB
*AC	A	ABC

Draw the DFA



- 3.e) Given a regular expression  $a(b|c)^*a$ . convert it to an  $\epsilon$ -NFA using Thomsom construction. Then convert the  $\epsilon$ -NFA directly to DFA.





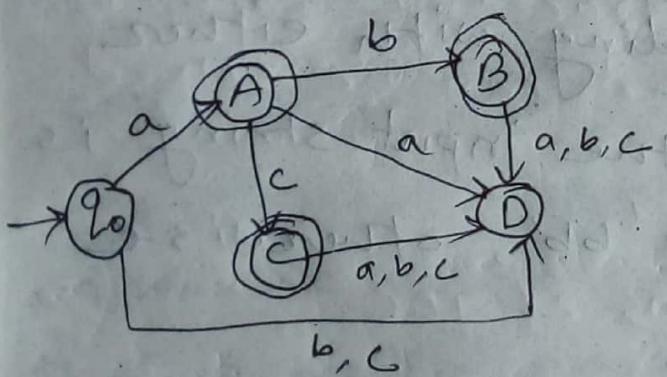
state transition table for  $\epsilon$ -NFA.

	$a$	$b$	$c$	$\epsilon^*$
$\rightarrow 1$	$\emptyset$	$\emptyset$	$\emptyset$	$\{1, 2, 3\}$
2	$\{1\}$	$\emptyset$	$\emptyset$	$\{2\}$
3	$\{5\}$	$\emptyset$	$\emptyset$	$\{3\}$
4	$\emptyset$	$\emptyset$	$\emptyset$	$\{4, 6, 7, 8, 12, 13\}$
5	$\emptyset$	$\emptyset$	$\emptyset$	$\{5, 13\}$
6	$\emptyset$	$\emptyset$	$\emptyset$	$\{6, 7, 8\}$
7	$\emptyset$	$\{9\}$	$\emptyset$	$\{7\}$
8	$\emptyset$	$\emptyset$	$\{10\}$	$\{8\}$
9	$\emptyset$	$\emptyset$	$\emptyset$	$\{9, 11, 12, 13\}$
10	$\emptyset$	$\emptyset$	$\emptyset$	$\{10, 11, 12, 13\}$
11	$\emptyset$	$\emptyset$	$\emptyset$	$\{11, 6, 7, 8, 12, 13\}$
12	$\emptyset$	$\emptyset$	$\emptyset$	$\{12, 13\}$
*13	$\emptyset$	$\emptyset$	$\emptyset$	$\{13\}$

state transition table for DFA

	a	b	c
$\rightarrow q_0$	$q_5, q_6, q_7, q_8, q_9, q_{10}$	$\emptyset$	$\emptyset$
$*q_5, q_6, q_7, q_8, q_9, q_{10}$	$\emptyset$	$q_{11}, q_{12}, q_{13}$	$q_{10}, q_{11}, q_{12}, q_{13}$
$*q_{11}, q_{12}, q_{13}$	$\emptyset$	$\emptyset$	$\emptyset$
$*q_{10}, q_{11}, q_{12}, q_{13}$	$\emptyset$	$\emptyset$	$\emptyset$

Let,  $q_5, q_6, q_7, q_8, q_9, q_{10} = A$ ,  $q_{11}, q_{12}, q_{13} = B$ ,  $q_{10}, q_{11}, q_{12}, q_{13} = C$   
and  $\emptyset$  denotes the Dead end (D),  $q_0 = q_0$



4a Define Mealy Machine by 6-tuple.  
Give an example with state table and state diagram.

A mealy machine is a 6-tuple  
 $M(Q, \Sigma, \delta, q_0, \Delta, \lambda)$  consisting of the following  
 $Q$  = finite set of states  
 $\Sigma$  = input alphabet

$\delta$  = Transition function  $Q \times \Sigma \rightarrow Q$

$q_0$  = initial state

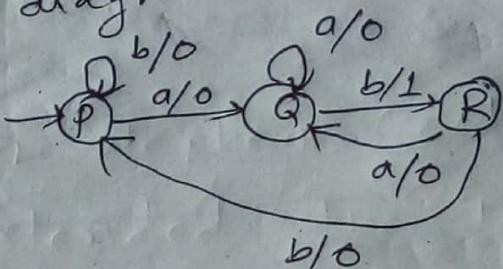
$\Delta$  = output alphabet

$\lambda$  = output function ( $\lambda: Q \times \Sigma \rightarrow \Delta$ )

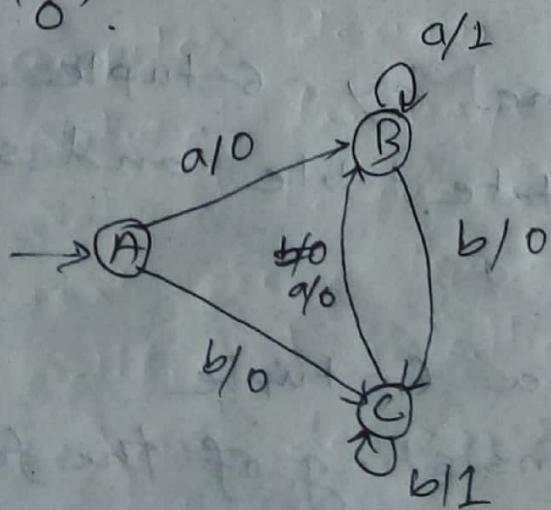
state transition table of Mealy Machine

	a	b
p	q, o p, o	
q	q, o r, i	
r	q, o p, o	

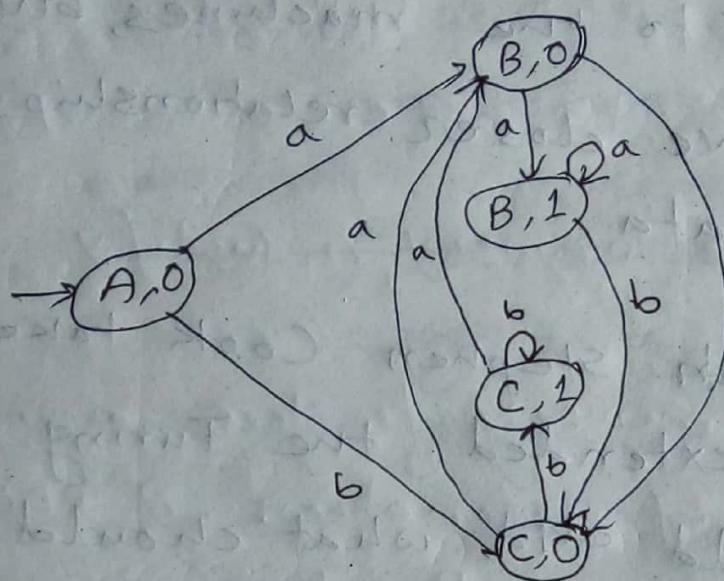
diagram



- 4b** construct a Mealy machine that accepts the language  $eos$  consisting of strings over  $\{a, b\}$  and string should be ending with either 'aa' or 'bb'. Print '1' if the input string is ending with either 'aa' or 'bb', otherwise print '0'.



4c Convert the above Mealy Machine to Moore Machine.



5a What is computation? Briefly discuss the evolution of TOC.

computation: A computation is any type of arithmetic or non-arithmetic calculation that is well defined.

Before 1930's: Alan Turing studied an abstract machine that had all the capabilities of today's computers to solve problems.

1930's to 1950's: Simpler kinds of Machines were used, which we called Finite Automata

Late 1950's to 1960's: Noam Chomsky began the study of formal 'grammar' that are not strictly belongs to the machines, but these grammars have closer relationships to abstract automata.

After 1960's: ~~Stephen~~ Stephen Cook takes the charge and extended the Turing's study of what could and what should not be computed. Finally, in 1971 S. Cook succeeded to separate intractable problems from those problems that can be solved efficiently by computer.

**5b** Write down applications of TOC

1. compiler design
2. Natural language processing
3. Natural language understanding
4. Text processing and searching applications
5. Artificial intelligence
6. circuit design

5c) Why FA can't recognize other than regular languages? Explain with example.

two properties of Finite Automata.

i) the amount of memory is finite and independent of length of string  
~~(i.e. is finite number of states)~~

ii) it can recognize a string as a part of Regular language, if the string contains a non-empty substring such that if it is replaced with an arbitrary number of repetitions of itself, the result must again be in the R. language.

Language like  $\{a^n b^n \mid n \geq 0\}$  can't be recognized by FA. because it need infinite amount of memory. Also if we take a string  $s = aa\underline{abbb}$  and substring of it  $y = ab$ . By replacing this part with two copies of  $y$  yields  $aaababbb$  does not belong to  $\{a^n b^n \mid n \geq 0\}$ . That is FA can't ~~reg~~ recognize other than RL.

**6a** Construct a CFG to generate even and odd set of palindromes over  $\Sigma = \{a, b\}$ .

$S \rightarrow aSa \mid bSb \mid aa \mid bb \mid a \mid b \quad |w| \geq 1$

$S \rightarrow aSa \mid bSb \mid a \mid b \mid \epsilon; |w| \geq 0$

**6b** Does a PDA have memory? Justify.

Yes.  
PDA uses a stack memory to store an unbounded amount of information. A PDA can push an element onto the top of the stack and pop off an element from the stack.

**6c** Write and explain the algorithm for minimization of a DFA.

Algorithm:

Step-1: Divide  
Suppose there is a DFA  $D(Q, \Sigma, Q_0, \delta, F)$  which recognizes a language  $L$ . Then the minimized DFA  $D'(Q', \Sigma, Q'_0, \delta', F')$  can be constructed for Language  $L$  as:

Step-1: Divide  $Q$  into two sets. One set will contain all final states and other set will contain non-final states. This partition is

called  $P_0$  ('0' equivalent)

Step-2: Initialize  $K=1$

Step-3: Find  $P_K$  by partitioning the different sets of  $P_{K-1}$ . In each set of  $P_{K-1}$ , take all possible pair of states. If two states of a set are distinguishable, split the sets into two different sets in  $P_K$ .

Step-4: Stop when  $P_K = P_{K-1}$

Step-5: All states of one set are merged into one. Number of states in minimized DFA will be equal to no. of sets in  $P_K$ .

Note: Two states  $(q_i, q_j)$  are distinguishable in partition  $P_K$  if for any input symbol  $a$ ,  $\delta(q_i, a)$  and  $\delta(q_j, a)$  are in different sets in partition  $P_{K-1}$ .

7a] What is an instantaneous description of a PDA? How will you represent it

The instantaneous description is called as an informal notation and explains how a Pushdown automata computes the given string and makes a decision that the given string is accepted or rejected.

it is represented by 3-tuples  $(q, w, \alpha)$

1.  $q$  is the current state

2.  $w$  is the remaining input

3.  $\alpha$  is the stack contents.

7b] Find PDA that accept the give CFG:

i)  $s \rightarrow xaaX$  ii)  $X \rightarrow ax \mid bx \mid \epsilon$

①  $\delta(q, \epsilon, s) = (q, xaaX)$  ④  $\delta(q, b, b) = (q, \epsilon)$

②  $\delta(q, \epsilon, X) = (q, ax), (q, bx), (q, \epsilon)$

③  $\delta(q, a, a) = (q, \epsilon)$

Let's accept  $bbaa$

$\delta(q, bbbaa, s)$  initial

$\delta(q, bbbaa, XaaX) \textcircled{1}$

$\delta(q, bbbaa, bXaaX) \textcircled{2}$

~~$\delta(q, bbbaa, baax) \textcircled{2}$~~

~~$\delta(q, baa, aaX)$~~

$\delta(q, bbbaa, bbXaaX) \textcircled{2}$

$\delta(q, bbaa, bbaaX) \textcircled{3}$

$\delta(q, baa, baaX) \textcircled{4}$

$\delta(q, da, dax) \textcircled{5}$

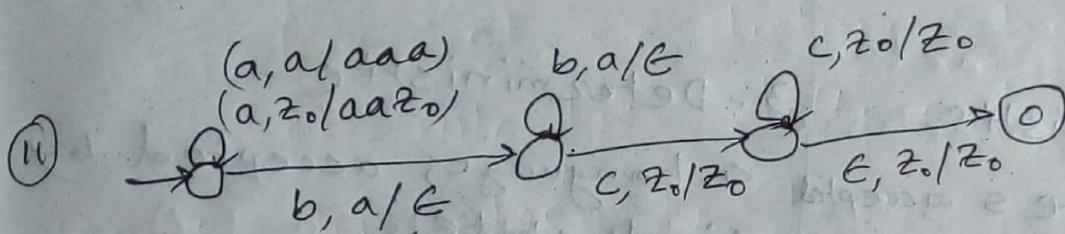
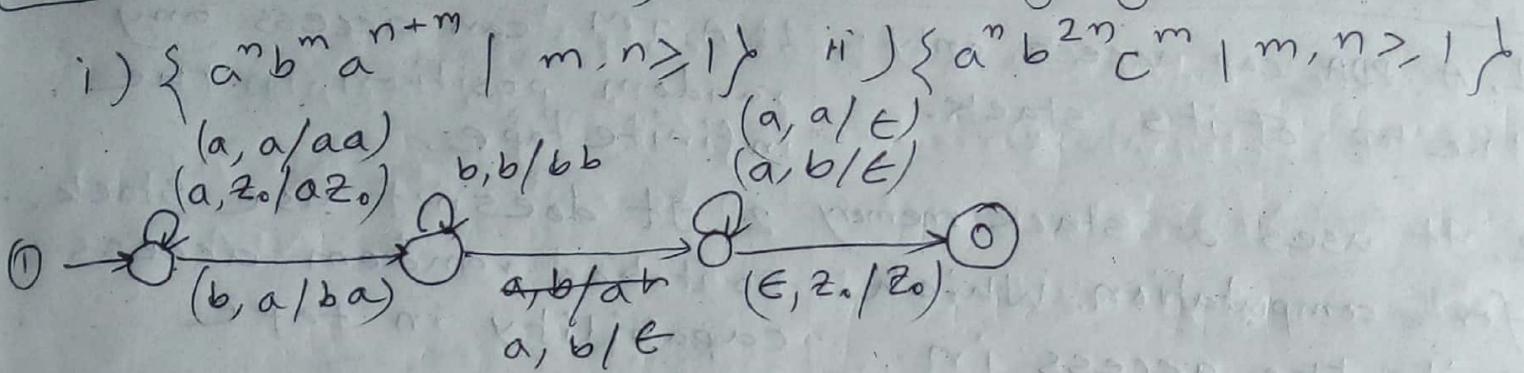
$\delta(q, a, ax) \textcircled{3}$

$\delta(q, \epsilon, x) \textcircled{3}$

$\delta(q, \epsilon, \epsilon) \textcircled{2}$

string is  
as the stack is empty accepted

**7c** Construct PDAs for the languages:



**8a** Define Turing Machine (TM). Differentiate between TM and PDA

TM: A TM is a 7-tuple

$$M = (Q, \Sigma, \delta, \Gamma, q_0, B, F)$$

$Q$ : Non empty finite set of states

$\Sigma$ : input alphabet

$\delta$ : Transition function

$\Gamma$ : ~~Tape~~ Tape alphabet (contains blank symbol)

$q_0$ : initial state

$B$ : special symbol called blank

$F$ : set of final states.

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$

### PDA

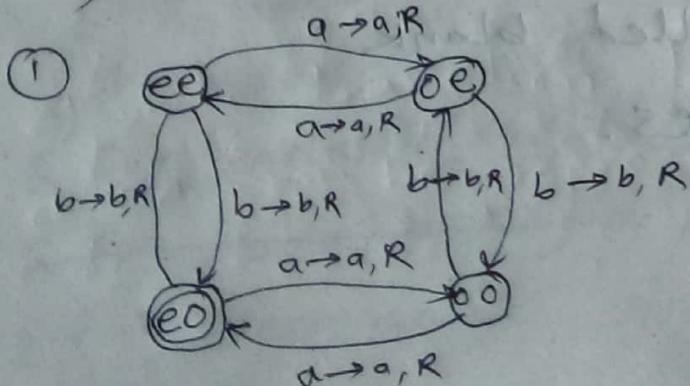
1. A PDA can access only the top of its stack.
2. It uses a stack memory for computation. Which limits the access in memory
3. Non-deterministic
4. The languages accepted by either empty stack or final state
5. Contains a read only head
6. The head can move in only forward direction

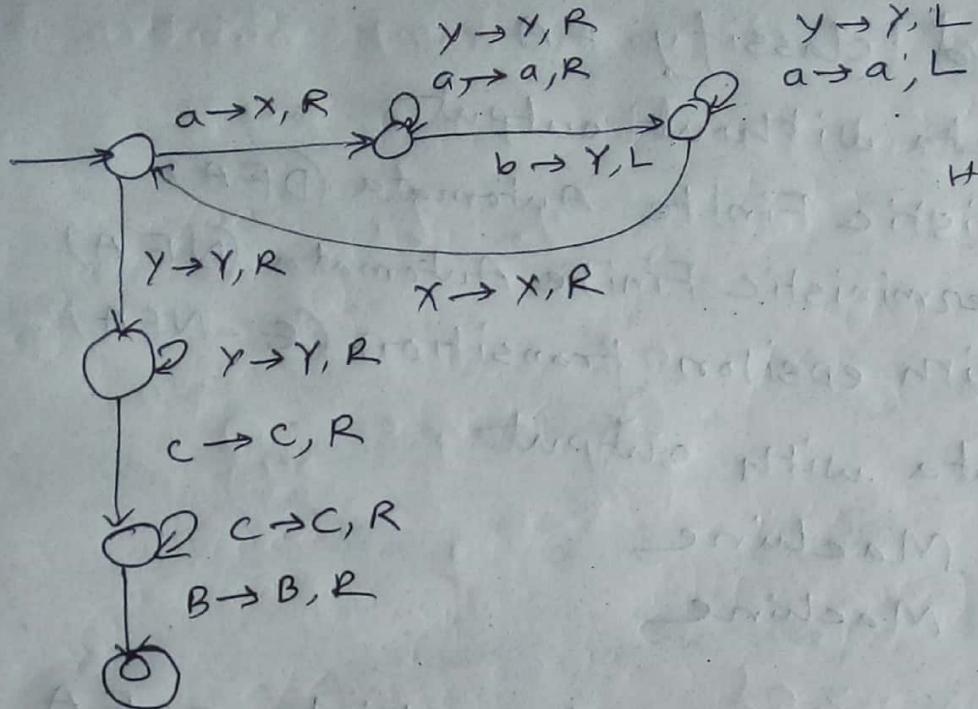
### TM

1. A TM can access any random position in the infinite tape.
2. It doesn't use stack. So it has random access capability in tape
3. Deterministic
4. Languages accepted by final state.
5. Have read/write head
6. The head is bidirectional (forward/backward)

**[8b]** Construct a TM accepting the following language i)  $\{ w \in \{a, b\}^* \mid w \text{ is consist of even number of } a's \text{ and odd number of } b's \}$

ii)  $\{ a^m b^n c^m \mid m, n \geq 1 \}$





Here  $x$  and  $y$   
are written  
in place of  
 $a$  and  $b$  in  
input tape  
and  $B$  is the  
blank symbol

2019

1a) Discuss the basic operations of languages.

1. Union: Let  $L_1$  and  $L_2$  be the two languages

then  $L_1 \cup L_2 = \{x, y \mid x \in L_1 \text{ and } y \in L_2\}$

2. Concatenation:  $L_1 \cdot L_2 = \{x \cdot y \mid x \in L_1 \text{ and } y \in L_2\}$

3. Closure of a language: Let  $L$  be the language

$$L^* = \bigcup_{i=0}^{\infty} L^i$$

4. Intersection: Intersection is defined as

$L_1 \cap L_2$ . that is the language which satisfies both conditions of  $L_1$  and  $L_2$ .  
(string belongs to both  $L_1$  and  $L_2$ )

**1b** Define and classify FA

Finite automata without output

i) Deterministic Finite Automata (DFA)

ii) Non-deterministic Finite Automata (NFA)

iii) NFA with epsilon transition ( $\epsilon$ -NFA)

Finite automata with output

i) Moore Machine

ii) Mealy Machine

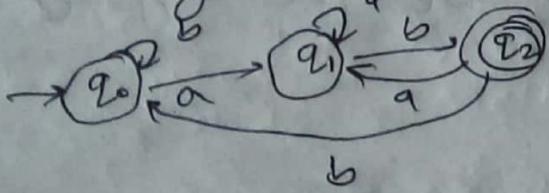
**1c** What are the ways by which a FA can be represented? Explain with example.

FA can be represented in 3 ways.

1) State transition diagram: A state transition diagram for FA is a diagram that is used to illustrate the operation of a FA.

For example: state transition diagram for

ending with 'ab', i.e



2) State transition table: The transitions of a FA can be represented as tabular form.

some example

	a	b
$q_0$	$q_1$	$q_0$
$q_1$	$q_1$	$q_2$
$q_2$	$q_1$	$q_0$

3) Using formal definition: The formal definition

of FA is  $FA = \{Q, \Sigma, q_0, \delta, F\}$

For example (ending with ab)

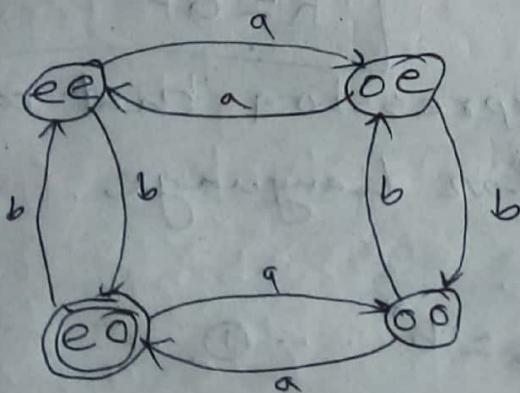
$$FA = \{\{q_0, q_1, q_2\}, \{a, b\}, q_0, \delta, \{q_2\}\}$$

$$\delta(q_0, a) = q_1, \quad \delta(q_1, a) = q_1, \quad \delta(q_1, b) = q_2, \quad \delta(q_2, a) = q_1,$$

$$\delta(q_2, b) = q_0, \quad \delta(q_0, b) = q_0, \quad \delta(q_2, b) = q_0.$$

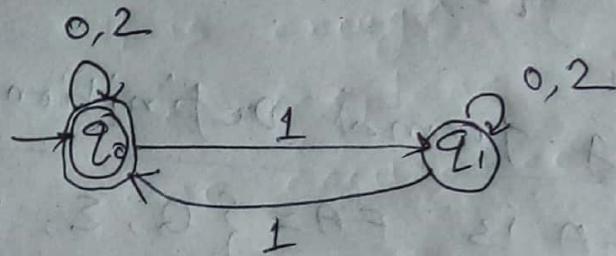
2a minimal DFAs for the following languages

i) even number of a's and odd number of b's

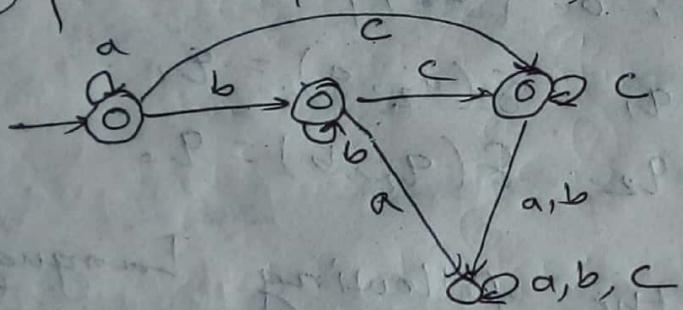


ii)  $\{w \in \{0, 1, 2\}^* \mid w \text{ is a ternary number whose decimal equivalent is divisible by 2}\}$   
 remainders: 0, 1, states:  $q_0, q_1$

	0	1	2
$\rightarrow q_0$	$q_0$	$q_1$	$q_0$
$q_1$	$q_1$	$q_0$	$q_1$



iii)  $\{a^n b^m c^l \mid n, m, l \geq 0\}$



2b Define RE. Derive a RE from the given FA



$$R = Q + RP$$

$$R = QP^*$$

RE: RE is a mathematical expression that can be used to represent regular language.  
 Using Arden's method:

$$q_0 = \epsilon + q_0 b \text{ or } q_0 = \epsilon b^* \text{ or } q_0 = b^* \dots \textcircled{1}$$

$$q_1 = q_0 a + q_1 b + q_2 a \dots \textcircled{11}$$

$$q_2 = q_1 b + q_2 a \text{ or } q_2 = q_1 b a^* \dots \textcircled{111}$$

from ① ⑪ and ⑯

$$q_1 = b^* a + q_1 b + q_1 b a^* a$$

$$\text{or, } q_1 = b^* a + q_1 (b + b a^* a)$$

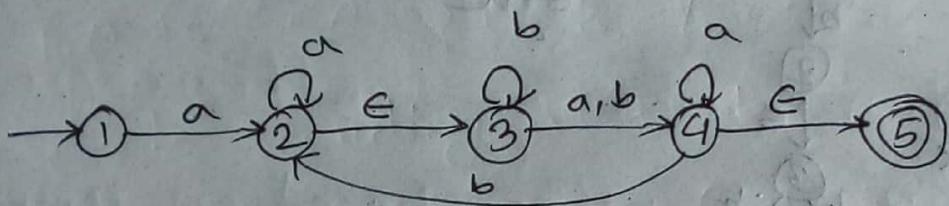
$$\text{or, } q_1 = b^* a (b + b a^* a)^*$$

We are using

$$R = Q + RP$$

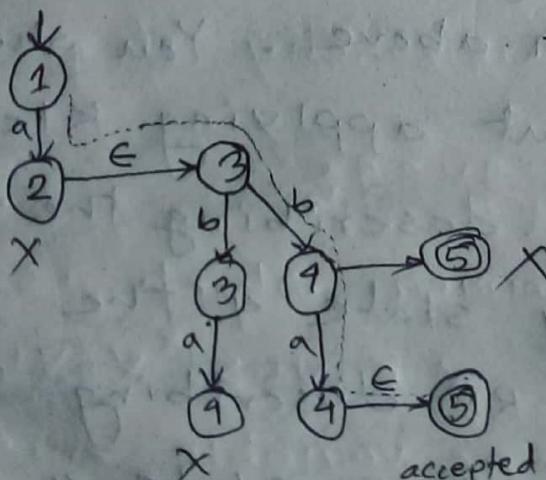
$$R = QP^*$$

3a

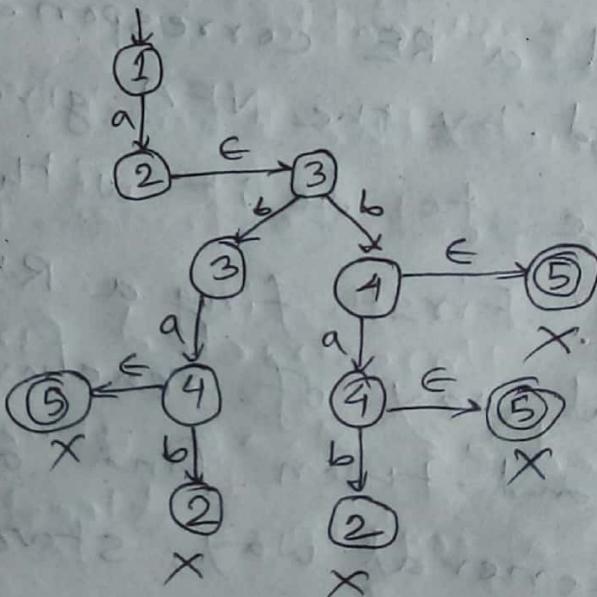


The figure shows a transition diagram for an NFA. For each string below, say whether the NFA accepts it.

i) aba

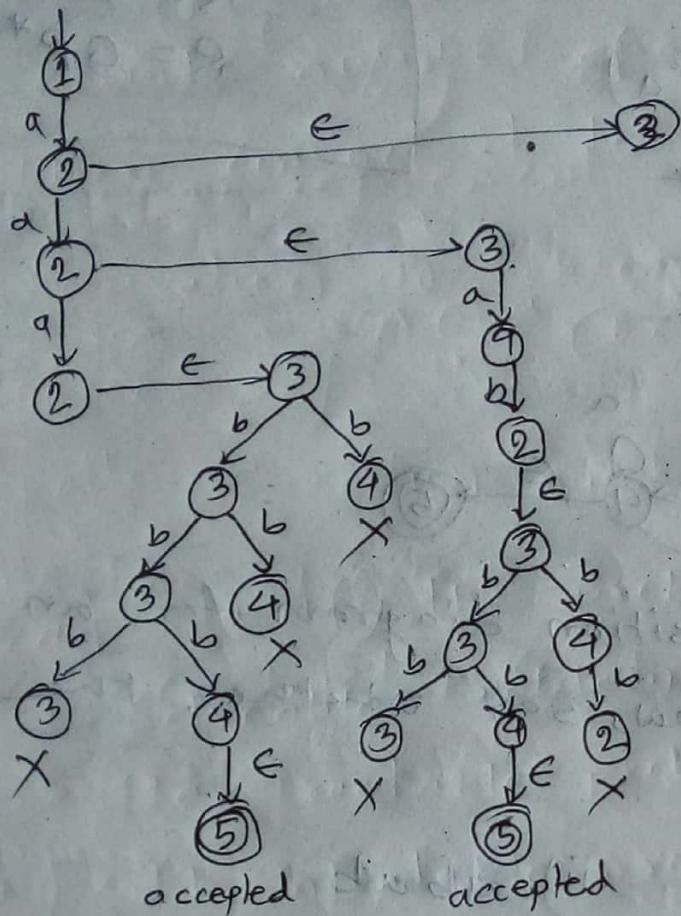


ii) abab



not accepted

iii) aaabb



It is enough to show  
the acceptance in only  
one occurrence

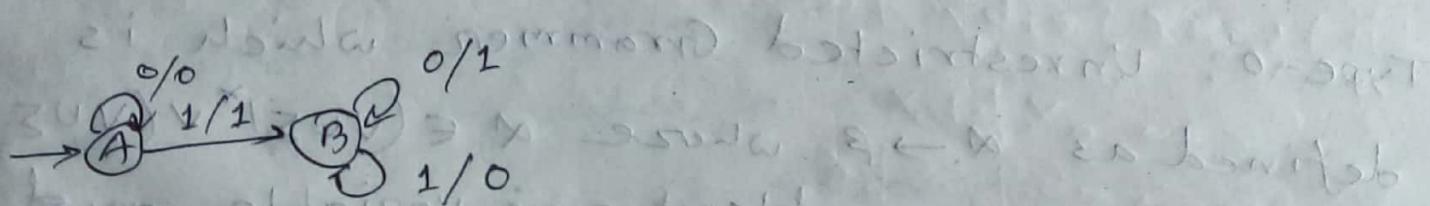
**[3b]** Find a RE corresponding to the language accepted by the NFA given above. You should be able to do it without applying Kicen's theorem; First find a RE describing the most general way of reaching state 4 the first time, and then find a RE describing the most general way, starting in state 4, of moving to state 4 the next time.

$$aa^*b^*(a+b)a^*(ba^*b^*(a+b)a^*)^*$$

**[4a]** Define Mealy Machine. Distinguish between Moore machine and mealy Machine.

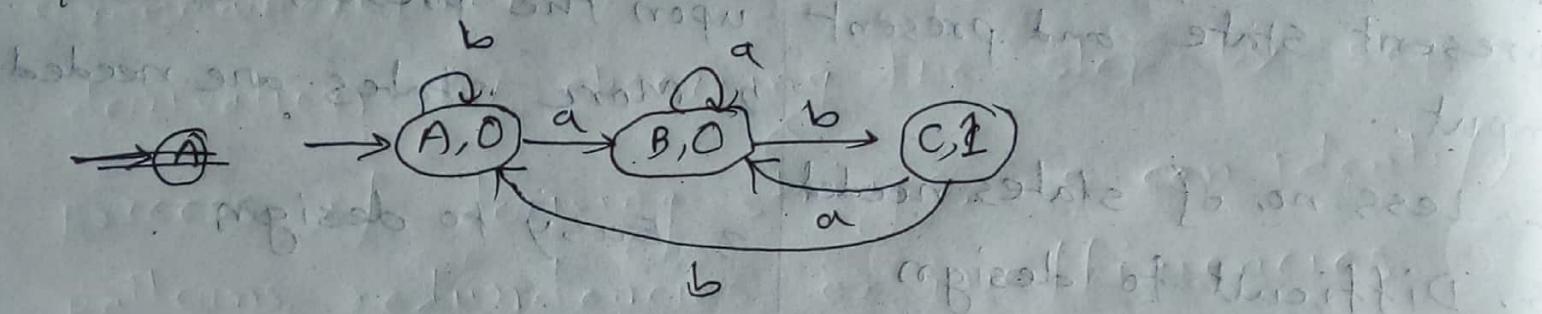
Mealy Machine	Moore Machine
1. Output depends on the present state and present input.	1. Output depends only upon the present state
2. less no. of states needed	2. More states are needed
3. Difficult to design	3. Easy to design
4. It reacts comparatively faster to inputs.	4. Slower
5. If input changes, o/p also changes	5. If input changes, o/p doesn't change.

**[4b]** Construct a Mealy machine that takes binary numbers as input and produces 2's complement number as output. Assume the string is read LSB to MSB and end carry is discarded.



4c construct a Moore machine that takes strings over alphabet  $\{a, b\}$  as input and counts number of 'ab' as a substring.

Then convert it to a Mealy machine



5a classify the problems based on computation

1. Solvable problem

1a. Tractable problem (solvable in polynomial time)

1b. Intractable problem (takes too long time)

2. Unsolvaeble

5b Discuss the Chomsky hierarchy of grammars with respect to formal languages.

Type-0: Unrestricted Grammar which is defined as  $\alpha \rightarrow \beta$  where  $\alpha \in (V \cup \Sigma)^*$   $\vee$   $(V \cup \Sigma)^*$  that is  $\alpha$  contains at least one variable and  $\beta \in (V \cup \Sigma)^*$

Type-1: Context sensitive Grammar which is defined as  $\alpha \rightarrow \beta$  where,  $\alpha, \beta \in (V \cup \Sigma)^+$  and  $|\alpha| \leq |\beta|$

Type-2: Context free Grammar is  $A \rightarrow^* \alpha$  where  $A \in V$ ,  $\alpha \in (V \cup \Sigma)^*$

Type-3: Regular Grammar is  $A \rightarrow aB$ ,  $A \rightarrow \epsilon$  where,  $A, B \in V$ ,  $a \in \Sigma$ . And the rules must be left or right linear/ regular.

[6 a] What is derivation? What is the function of it?

Derivation is a sequence of grammar rule applications that starts with an initial symbol and generates a string of symbols following the production rules of the grammar.

Derivation: The sequence of substitutions to obtain a string is called a derivation

Parse tree: The pictorial representation of derivation of a string is a parse tree.

The prime function of derivation is to generate a language. As well as it can be used for language recognition.

Grammar refinement is also possible by derivation. By observing the step-by-step process of deriving strings, we can identify patterns, redundancies, or inefficiencies in the grammar.

Q6b Construct CFG that generate the following languages

i)  $\{w \in \{a,b\}^* \mid w \text{ is any string starting and ending with same symbols}\}$

$$S \rightarrow aXa \mid bXb \mid a \mid b \mid \epsilon$$

$$X \rightarrow aX \mid bX \mid \epsilon$$

ii)  $\{w \in \{a,b\}^* \mid w \text{ is odd length string}\}$

$$S \rightarrow aX \mid bX$$

$$X \rightarrow aS \mid bS \mid \epsilon$$

iii)  $\{0^l 1^l 2^n 3^n \mid l, n > 0\}$

**6c** construct a machine that accepts the language generated from the following grammar. What is the language for this grammar?  $A \rightarrow Aa \mid Ab \mid B \epsilon, B \rightarrow Cb, C \rightarrow \epsilon$

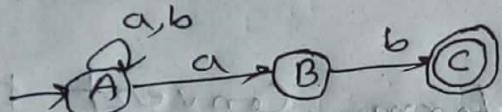
First Reverse the Grammar (Left to Right linear)

$$A \rightarrow aA \mid bA \mid aB$$

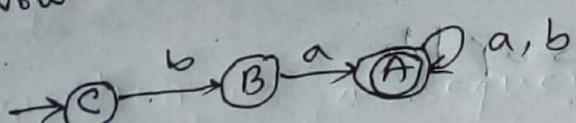
$$B \rightarrow bC$$

$$C \rightarrow \epsilon$$

Now draw finite automata



Now reverse the FA

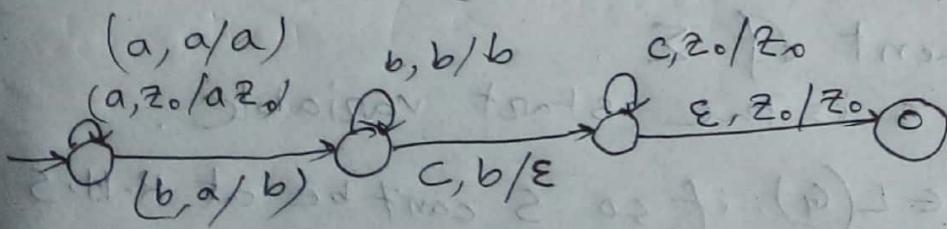
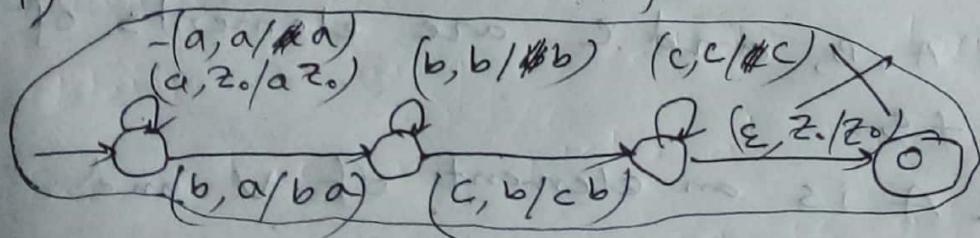


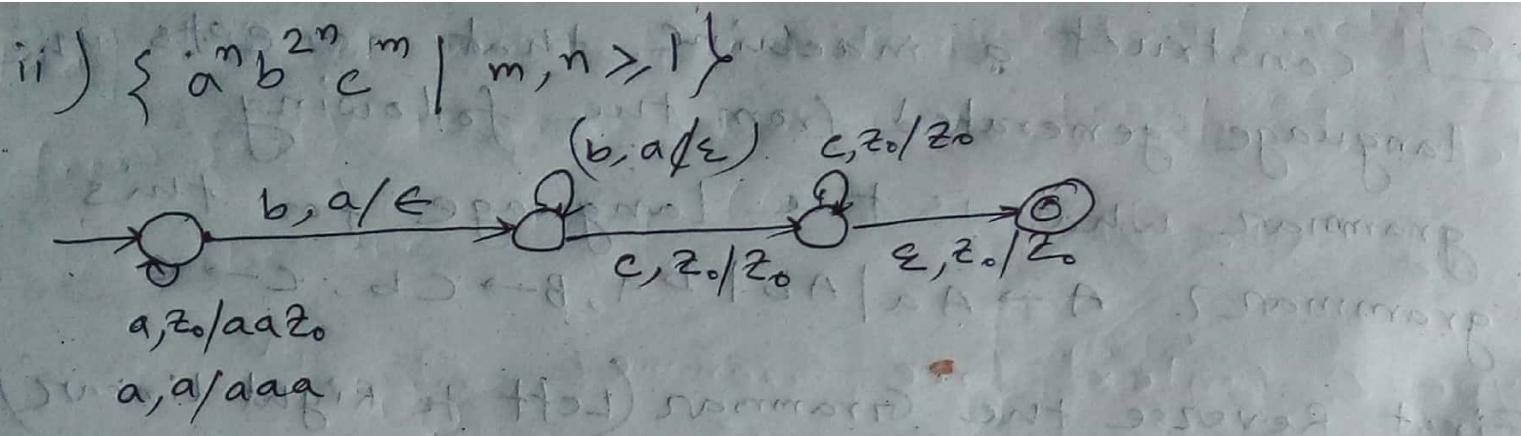
[Initial and final states are interchanged]

Hence the language is  $\{ w \in \{a,b\}^* \mid w \text{ starts with 'ba'} \}$

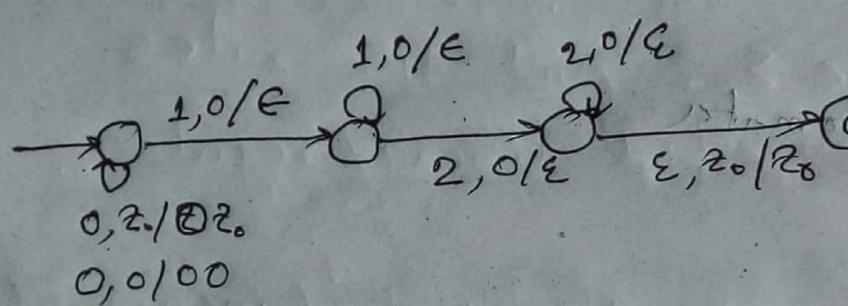
**7a** Construct PDA for the following CFG

$$i) a^n b^l c^m \mid l, m, n > 1 \}$$





iii)  $\{0^{n+m} 1^m 2^n \mid m, n \geq 1\}$



**7b** Define Chomsky normal form. Convert the following CFG to CNF:

$S \rightarrow aXbYb, X \rightarrow S1\varepsilon, Y \rightarrow bY1b$

A CFG  $G = (V, \Sigma, R, S)$  is said to be in Chomsky Normal Form, if every rule in  $R$  has one of the following three forms:

1.  $A \rightarrow BC$ , where  $A, B$ , and  $C$  are elements of  $V$ ,  $B \neq S$  and  $C \neq S$ .
  2.  $A \rightarrow a$ , where  $A$  is an element of  $V$  and  $a$  is an element of  $\Sigma$ .
  3.  $S \rightarrow \varepsilon$ , where  $S$  is the start variable.
- Note:  $S \rightarrow \varepsilon$  iff  $\varepsilon \in L(G)$ . If so  $S$  can't be at R.H.S

~~First remove null production~~ First introduce a new starting state (because  $s$  at RHS)

$$s \rightarrow \alpha X | Y b | \alpha$$

$$X \rightarrow s$$

$$Y \rightarrow b Y | b$$

~~Remove unit production~~

$$s' \rightarrow s$$

$$s \rightarrow \alpha X | Y b | \alpha$$

$$X \rightarrow s | \epsilon$$

$$Y \rightarrow b Y | b$$

⑩ Remove null production

$$s' \rightarrow s$$

$$s \rightarrow \alpha X | Y b | \alpha$$

$$X \rightarrow s$$

$$Y \rightarrow b Y | b$$

Remove the unreachable state

$$s' \rightarrow \alpha X | Y b | \alpha$$

$$X \rightarrow \alpha X | Y b | \alpha$$

$$Y \rightarrow b Y | \alpha$$

Now construct CNF

$$s' \rightarrow M X | Y N | M$$

$$X \rightarrow M X | Y N | M$$

$$Y \rightarrow N Y | M$$

$$M \rightarrow a$$

$$N \rightarrow b$$

Remove unit production

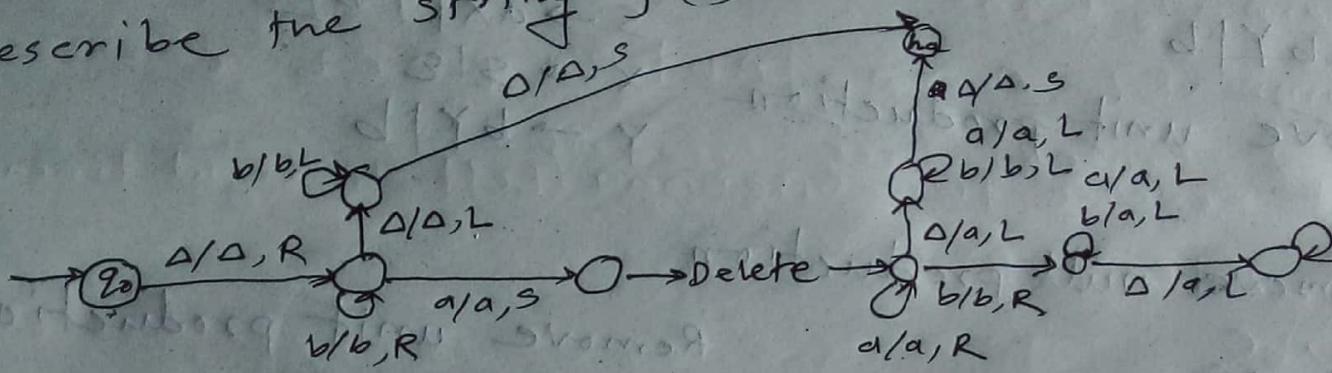
$$s' \rightarrow \alpha X | Y b | \alpha$$

$$s \rightarrow \alpha X | Y b | \alpha$$

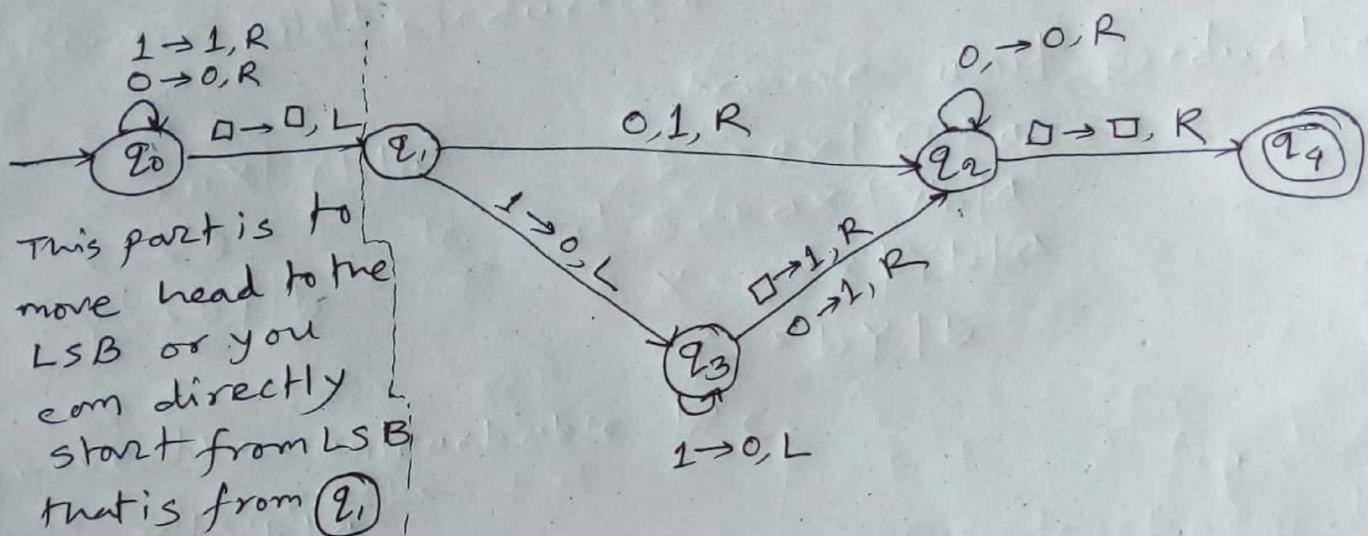
$$X \rightarrow \alpha X | Y b | \alpha$$

$$Y \rightarrow b Y | b$$

8a) The TM shown below computes a function from  $\{a, b\}^*$  to  $\{a, b\}^*$ . For any string  $x \in \{a, b\}^*$ , describe the string  $f(x)$



8b Draw a transition diagram for a TM with input alphabet  $\{0, 1\}$  that interprets the input string as the binary representation of a nonnegative integer and adds 1 to it.



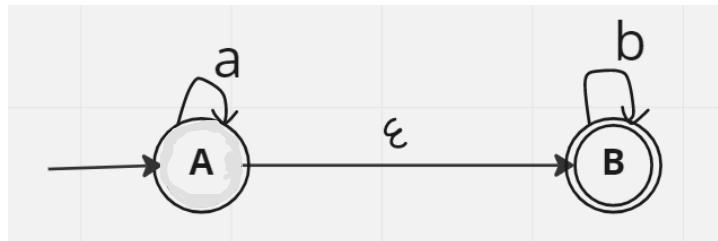
# CT-1

Q1: Define  $\epsilon$ -NFA. Construct an  $\epsilon$ -NFA for the language  $\{a^m b^n \text{ where } m, n \geq 0\}$

**Answer:**

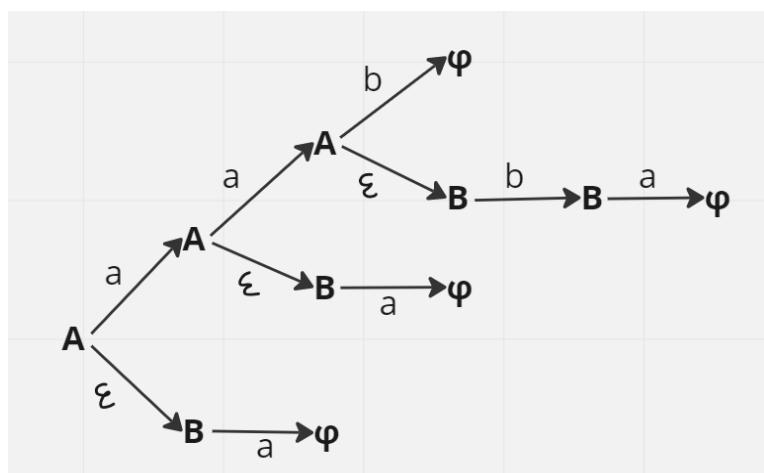
An  $\epsilon$ -NFA is a 5-tuple  $M = (Q, \Sigma, \delta, q, F)$ , where

1.  $Q$  is a finite set, whose elements are called states
2.  $\Sigma$  is a finite set, called the alphabet; the elements of  $\Sigma$  are called symbols
3.  $\delta : Q \times (\Sigma \cup \epsilon) \rightarrow 2^Q$  is a function, called the transition function
4.  $q$  is an element of  $Q$ ; it is called the start state
5.  $F$  is a subset of  $Q$ ; the elements of  $F$  are called accept states



Q2: Prove that “aabab” is rejected by the above  $\epsilon$ -NFA.

**Answer:**



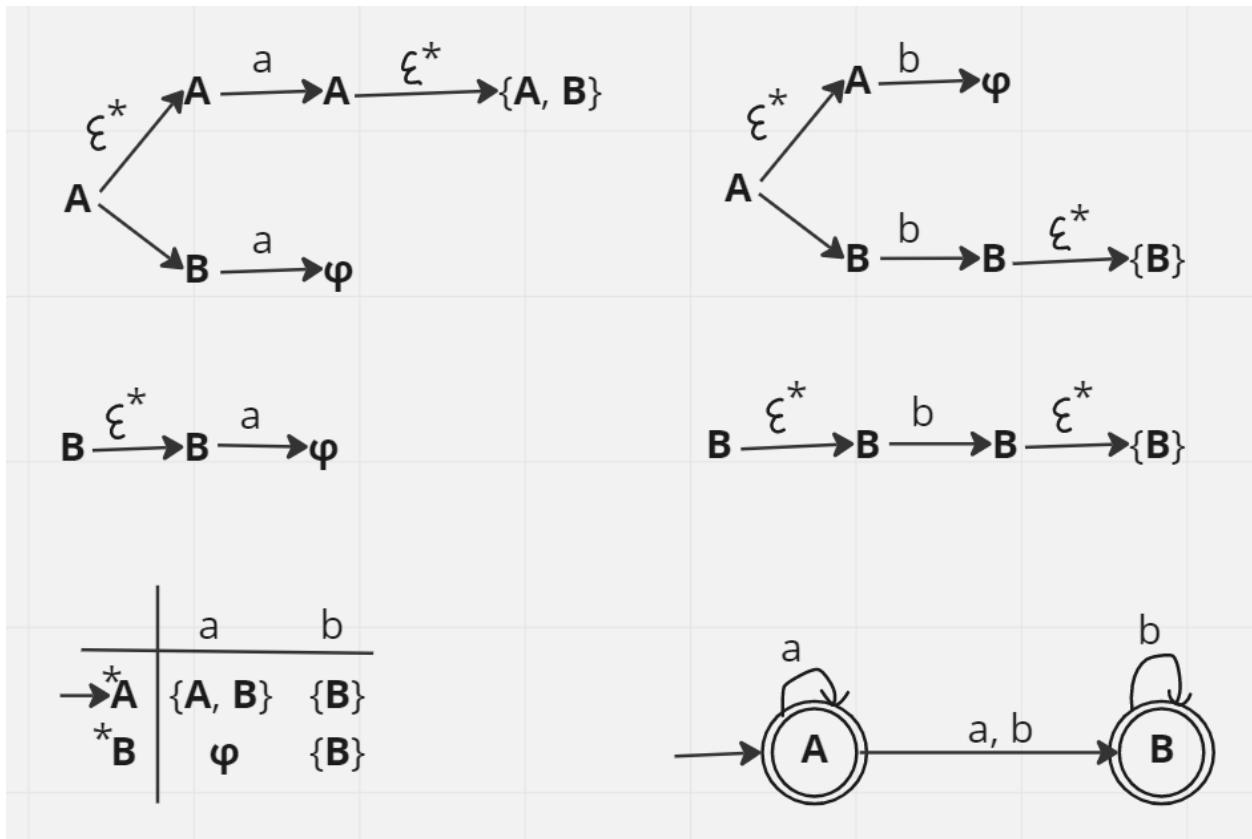
As all the transitions end in a death configuration, the string “aabab” is rejected by the  $\epsilon$ -NFA mentioned in Question number 1.

Q3: Convert the  $\epsilon$ -NFA to NFA

**Answer:**

$\epsilon^*$  (closure):- All the states that can be reached from a particular state only by seeing the  $\epsilon$  symbol.

Here  $\epsilon^*$  of state **A** is  $\{A, B\}$  because both are reachable from **A** without consuming any input symbol.



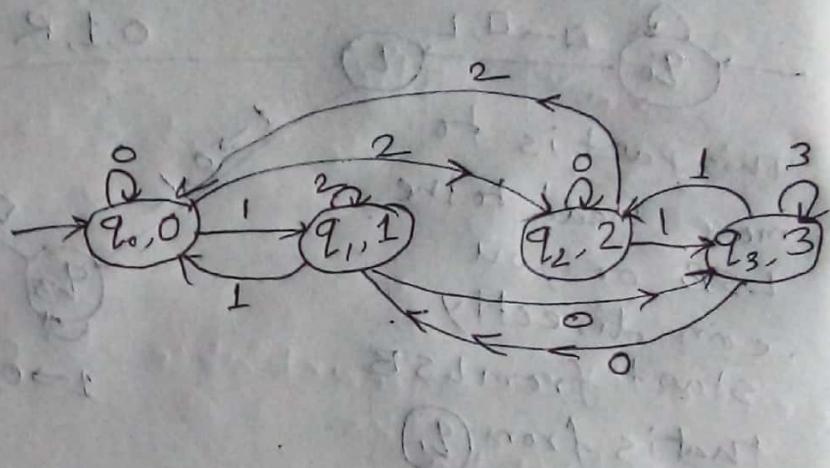
Q-1

Construct a Moore Machine that takes base-3 number as input and produce modulo-4 number of its decimal equiv. as output.

Hence,  $\Sigma = \{0, 1, 2\}$  and  $\Delta = \{0, 1, 2, 3\}$

so, the states are  $q_0, q_1, q_2$  and  $q_3$

	0	1	2	$\Delta$
$\rightarrow q_0$	$q_0$	$q_1$	$q_2$	0
$q_1$	$q_3$	$q_0$	$q_1$	1
$q_2$	$q_2$	$q_3$	$q_0$	2
$q_3$	$q_1$	$q_2$	$q_3$	3



2i) construct grammars for the following languages

{w | w is a string starting and ending with same symbol}

$$S \rightarrow aAa \mid bAb \mid a \mid b$$

$$A \rightarrow aA \mid bA \mid \epsilon$$

ii) {set of all palindromic strings over {a, b}}

$$S \rightarrow aSa \mid bSb \mid a \mid b \mid \epsilon$$

③ construct a FA from the following LLG

①  $A \rightarrow Aa \mid Ab \mid Ba$

$B \rightarrow Cb$

$C \rightarrow \epsilon$

see: 2019 - [6c]