2022

1.(a) Inherent error: Error which exist in the problem either due to approximate given data, limitations of computing aids, are called as inherent errors.

Example: Using 3.14 instead of the true value of pi(π) because we cant exactly represent pi with a finite number of digits.

Round-off errors: This errors happens when numbers are rounded to a certain number of digits due to limited computer precision.

Example: If you round $1/3$ to 0.333 instead of keeping the full decimal you introduce a small round-off error.

Truncation Errors: These errors arise due to use of approximate formula in computation or by truncating the infinite series to some approximate terms.

Example: Using only the first few terms of a series to approximate a value instead of including all terms.

We know.

$$\Delta x = \frac{1}{2}(10^{-N})$$

For 8.6 $N=1$.

$$\Delta x = \frac{1}{2} \cdot 10^{-1}$$

$$\therefore \Delta x = 0.05$$

$$E_A \text{ (Absolute error)} = 0.05$$

Again Relative error $E_R = \dfrac{\Delta x}{x} = \dfrac{0.05}{8.6} = 0.0058$.

(b) Exact number: An exact number is a value that is represented perfectly without any error on approximation. It is the true value usually expressed in its complete form.

Example:

Pi(π): The exact value of pi is an irrational number, which means it can not be fully written down with finite digits. However for mathematical purposes, its exact value refers to the infinite.

Square root of 2 ($\sqrt{2}$): The exact value is also an irrational number with an infinite number of decimal places.

Approximation Number: An approximation number is a value that is close to the exact number, but rounded ~~or~~ within a certain precision.

Pi(π) approximated: 3.14159 is an approximation of pi, rounded to 5 decimal places.

square root of 2 ($\sqrt{2}$) approximate: 1.414 is a rounded approximation of $\sqrt{2}$, typically rounded to 3 decimal places.

(c) Absolute ~~error~~ error is the difference between exact value and the approximated value.

$E_A = x - \bar{x}$ where x = exact value and $\bar{x}$ = approximated value.

Relative error is the absolute error divided by the exact value.

$$\therefore E_R = \frac{E_A}{x}.$$

As we can see. relative error is the division of absolute error and. exact value. so it will smaller. than relative error.

For example,

let if the exact value is 10 and the approximate value is 9.8 then

$$\text{absolute error}(E_A) = 10 - 9.8 = 0.2.$$

$$\text{and relative error}(E_R) = \frac{0.2}{10} = 0.02.$$

$E_A > E_R$

2021

1.(a). Significant figures refer to the digits in a number that carry meaningful information about its precision.

Rules for significant figures:

1. Non-zero digits are always significant. example 123 has 3 significant
2. Any zeros between significant digits are significant. (Example: 2004)
3. leading zeros are not significant. (Example: 0.0025 has 2 significant)
4. Trailing zeros in a decimal number are significant (Example: 12.2300

The given measurement 0.006606 has 4 significant figures.

(b) There are 3 types of errors encountered in performing numerical calculations.

(i) Inherent error

(ii) Round-off error.

(iii) Truncation error.

(c) We know

$$\Delta x = \frac{1}{2}(10^{-N})$$

Here $x = 0.52$ and $N = 2$

$$\Delta x = \frac{1}{2}(10^{-2})$$

$$= 0.005$$

$$E_R = \frac{\Delta x}{x} = \frac{0.005}{0.51}$$

$$E_P = E_R \times 100 = \frac{0.005}{0.51} \times 100 = 0.98\%$$

2020

1.(a) Accuracy refers to how close a measured or calculated value is to the true or accepted value.

Precision is about how repeatable or consistent the results are when you measure something multiple times.

(b)

(c) Absolute error is the difference between Real True value or exact value and approximated value. Mathematically if x is true value and $\bar{x}$ is approximate value then absolute error is define by.

$$E_A = (x - \bar{x}) = \Delta x.$$

Relative error is the absolute error divided by exact value.

$$E_R = \frac{E_A}{x}.$$

And percentage error is define by.

$$E_p = 100 \times E_R.$$

(d) Here $x = \frac{1}{3}$ and $\bar{x} = 0.30, 0.33, 0.34$.

When $\bar{x} = 0.30$.

$$\Delta x = \frac{1}{3} - 0.30 = 0.03333$$

$$E_R = \frac{0.03333}{1/3} = 0.09999$$

$$E_p = 100 \times E_R = 9.999\%$$

When $\bar{x} = 0.33$

$$\Delta x = \frac{1}{3} - 0.33 = 0.003333$$

$$E_R = \frac{0.003333}{1/3} = 0.009999$$

$$E_p = E_R \times 100 = 0.9999\%$$

When $\bar{x} = 0.34$

$$\Delta x = \left|\frac{1}{3} - 0.34\right| = 0.0066$$

$$E_R = \frac{0.0066}{1/3} = 0.0198$$

$$E_p = E_R \times 100 = 1.98\%$$

$\therefore$ When $\bar{x} = 0.33$ the the error is less than the others so 0.33 is the best approximation.

2019

1.(a) Numerical methods are mathematical techniques that use mathematical ~~tolls~~ tools or procedures to find approximation solutions to problem.

Numerical methods are used to solve problems that are difficult or impossible to solve analytically. They are important because they can –

1. Solve complex problems.
2. Model real world solutions.
3. Provide approximate solutions.

(b) 2022 1(a).

(c). Here the general rules for rounding:

1. Identify the place ~~real~~ value. (Decide which digit you want to round to. For example nearest tens, hundreds, decimal places etc.
2. Check the digit immediately to the right of the target place value.
3. Apply rounding ~~at~~ Rule.

(i) If the digit is 0, 1, 2, 3 or 4, keep the target digit unchanged and replace all digits to its right with zeros and remove them.

(ii) If the digit is 5, 6, 7, 8, or 9 increase the target digit by 1 and replace all digits to its right with zeros or remove them.

(c). The given true value $x = \frac{2}{3}$

approximated value $\bar{x} = 0.6667$ (4 significant digits).

$\therefore$ Absolute error $E_A = |x - \bar{x}| = |\frac{2}{3} - 0.6667)| = 0.00003333$

Relative error, $E_R = \frac{E_A}{x} = \frac{0.00003333}{\frac{2}{3}} = 0.00005$

percentage error $E_p = 0.00005 \times 100 = 0.005\%$

<u>2018</u>

1 (a). 2019 1(a).

(b) Bias ~~is~~ reflects the error from systematic simplifications or assumptions, and it can be minimized but not always eliminated entirely.

(c) 2021 1 (a).

(d) <u>Algebraic equation</u>: An equation that involves only algebraic operations (addition, subtraction, multiplication, division ~~etc~~ etc.)
<u>form</u>: $P(x) = 0$
<u>example</u>: $x^2 - 4 = 0$, $3x^3 - 2x + 1 = 0$
Root may be real or complex.

<u>Trancendental Equations</u>: An equation that involves at least one transcendental function such as exponential, logarithmic trigonometric or their combinations.
<u>form</u>: Contains terms like e^x, $\ln(x)$, $\sin(x)$, etc.
<u>examples</u>: $e^x - x = 0$, $\sin x - x^2 = 0$, $\ln x + x = 2$.
Roots can be finite or infinite in a number.

2022

2.(a) The bisection method is a numerical technique to find the root of a continuous function $f(x)$ on a closed interval $[a,b]$ where $f(a)$ and $f(b)$ have oposite signs. $(f(a) \cdot f(b) < 0)$

Derivation:

1. If $f(x)$ is continuous on $[a,b]$ and $f(a) f(b) < 0$ there exists at least one root $x^* \in (a,b)$ such that $f(x^*) = 0$

2. Compute the midpoint of the interval $[a,b]$.

$$c = \frac{a+b}{2}.$$

3. Evaluate $f(c)$ [Update the interval]
   (i) If $f(c) f(a) < 0$, set $b = c$
   (ii) If $f(c) f(b) < 0$ set $a = c$

4. Repeat until the interval $B \cdot |f(c)| < \epsilon$, where ϵ = tolerance.

(b) .

(c) $x^3+x-1=0$

$f(x) = x^3+x-1$

$f(0) = -1$ and $f(1) = 1$.

$\therefore$ the root lies between 0 to 1.

$f(0.60) = -0.182 < 0$

$f(0.70) = 0.043 > 0$.

Let; $a = 0.60$, $b = 0.70$.

our inverval $[0.60, 0.70]$

$$x_1 = \frac{a \times f(b) - b \times f(a)}{f(b) - f(a)}$$

$$= \frac{0.60 \times 0.043 - 0.70 \times -0.182}{0.043 + 0.182}.$$

$$= 0.6809$$

$f(x_1) = -0.00342 < 0$

$a = 0.6809$, $b = 0.70$

$$x_2 = \frac{0.6809 \times 0.043 - 0.70 \times (-0.00342)}{0.043 + 0.00342}$$

$$= 0.6823$$

$f(x_2) = -0.00006663 < 0$.

$a = 0.6823$, $b = 0.70$

$$x_3 = \frac{0.6823 \times 0.043 - 0.70 \times (-0.00006663)}{0.043 + 0.00006663}$$

$$= 0.6823$$

$\therefore$ The real root of the given equation is 0.682 upto 3 demal places.

2021

2.(a). ~~Open~~ Bracketing method:

(i) Require two points $[a,b]$ such that $f(a)f(b)<0$.

(ii) Always converges if $f(x)$ is continuous

(iii) Takes more steps

(iv) Example: Bisection, False position method

~~Brace~~ Open Method

(i) Requires one or two initial guesses.

(ii) May not converge if guesses are poor.

(iii) Fewer steps if it converges.

(iv) Example: Iteration, Newton Raphson method.

(b) We know that Newton Raphson formula is —

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Let us suppose that $x_n = a + \epsilon_n$

$$x_{n+1} = a + \epsilon_{n+1}$$

$$a + \epsilon_{n+1} = a + \epsilon_n - \frac{f(a+\epsilon_n)}{f'(a+\epsilon_n)}$$

$$\epsilon_{n+1} = \epsilon_n - \frac{f(a+\epsilon_n)}{f'(a+\epsilon_n)}$$

Using taylor series

$$\epsilon_{n+1} = \epsilon_n - \cancel{\epsilon}\, \frac{\left[f(a) + \epsilon_n f'(a) + \frac{\epsilon_n^2 f''(a)}{2!}\right]}{\left[f'(a) + \epsilon_n f''(a) + \frac{\epsilon_n^2 f'''(a)}{2!}\right]}$$

$\therefore f(a) = 0$

$$\epsilon_{n+1} = \epsilon_n - \frac{\left[\epsilon_n f'(a) + \frac{\epsilon_n^2 f''(a)}{2!}\right]}{\left[f'(a) + \epsilon_n f''(a) + \frac{\epsilon_n^2 f'''(a)}{2!}\right]}$$

$$= \epsilon_n - \frac{\epsilon_n f'(a)\left[\frac{\epsilon_n f'(a)}{\epsilon_n f'(a)} + \frac{\epsilon_n^2 \cdot f''(a)}{\epsilon_n f'(a) \cdot 2}\right]}{f'(a)\left[\frac{f'(a)}{f'(a)} + \frac{\epsilon_n f''(a)}{f'(a)} + \frac{\epsilon_n^2 f'''(a)}{f'(a) \cdot 2}\right]}$$

ignoring higher derivative.

$$\epsilon_{n+1} = \epsilon_n - \frac{\epsilon_n\left[1 + \frac{\epsilon_n f''(a)}{2f'(a)}\right]}{\left[1 + \frac{\epsilon_n f''(a)}{f'(a)}\right]}$$

$$= \epsilon_n - \epsilon_n\left[1 + \frac{\epsilon_n f''(a)}{2f'(a)}\right]\left[1 + \frac{\epsilon_n f''(a)}{f'(a)}\right]^{-1}$$

$$= \epsilon_n - \epsilon_n\left[1 + \frac{\epsilon_n f''(a)}{2f'(a)}\right]\left[1 - \frac{\epsilon_n f''(a)}{f'(a)}\right]$$

$$= \epsilon_n - \epsilon_n\left[1 - \frac{\epsilon_n f''(a)}{f'(a)} + \frac{\epsilon_n f''(a)}{2f'(a)} - \frac{\epsilon_n^2 f''(a)}{2f'''(a)}\right]$$

Ignore

$$= \epsilon_n - \epsilon_n\left[1 + \frac{-2\epsilon_n f''(a) + \epsilon_n f''(a)}{2f'(a)}\right]$$

$$= \epsilon_n - \epsilon_n\left[1 - \frac{\epsilon_n f''(a)}{2f'(a)}\right]$$

$$= \epsilon_n - \epsilon_n + \frac{\epsilon_n^2 f''(a)}{2f'(a)}$$

$$\epsilon_{n+1} = \epsilon_n^2 \cdot \frac{f''(a)}{2f'(a)} \Rightarrow \epsilon_{n+1} = \epsilon_n^2 \cdot K \Rightarrow \boxed{\epsilon_{n+1} = \epsilon_n^2}$$

$$\left[\because K = \frac{f''(a)}{2f'(a)}\right]$$

$\therefore$ The order of convergence of Newton Raphson method is two.

③ $x^3-3x+1=0 \Rightarrow f'(x) = 3x^2-3$

$f(x) = x^3-3x+1$.

$f(0) = 1$ and $f(1) = -1$.

The root lies between 0 to 1.

let, $x_0 = 0.35$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.35 - \frac{-0.007125}{-2.6325}$$

$$= 0.34729.$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.34729 - \frac{0.00001676}{-2.63382}.$$

$$= 0.34729$$

∴ The real root for the given equation is 0.3473 upto correct to 4 decimal places.

CamScanner

2020

2. (a) Merits and demerits of Bisection method :—

Merits:

1. Works for any continuous function
2. Avoids issues like division by zero.
3. No guesswork for initial values.
4. The method is works even if the function is not differentiable.

Demerits:

1. Takes many iteration.
2. If the interval contains multiple roots it only find one root.
3. If the initial interval [a, b] does not contain a root, the method fails.

(b). $f(x) = x^3 - 3x - 5$

$f(2) = -3$

$f(3) = 13$

$\because$ The root lies between 2 to 3.

$f(2.2) = -0.952$, $f(2.3) = 0.267$.

Let, $a = 2.2$, $b = 2.3$

$c = \frac{a+b}{2} = \frac{2.2+2.3}{2} = 2.25$

$f(c) = -0.359375 < 0$

$a = 2.25$, $b = 2.3$

$c = \frac{2.25+2.3}{2} = 2.275$

$f(c) = -0.0504 < 0$.

$a = 2.275$, $b = 2.3$

$c = \frac{2.275+2.3}{2} = 2.2875$

$f(c) = 0.1072 > 0$

$a = 2.275$, $b = 2.2875$

$c = \frac{2.275+2.2875}{2} = \cancel{2.2781}\ 2.28125$

$f(c) = \cancel{0.0115 < 0}\ 0.0281 > 0$

$a = 2.275$, $b = 2.28125$

$c = \frac{2.275+2.28125}{2}$

$= \cancel{2.2828}\ 2.278125$

$f(c) = -0.0112 < 0$

$a = 2.278125$, $b = 2.28125$

$c = \frac{2.278125+2.28125}{2}$

$= 2.2796$

2019

2.(a). The false position method is a numerical technique for finding the root of an equation $f(x)=0$.

Derivation:

1. select two points a, b such that $f(a)$ and $f(b)$ have oposite signs. $f(a)f(b)<0$.

2. Compute the point of intersection. using the formula.

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

3. Evaluate $f(c)$.

(i) If $f(c)=0$, c is the root and stops the process.

(ii) otherwise update the interval.

if $f(a)f(c)<0$ set $b=c$.

if $f(c)f(b)<0$ set $a=c$.

4. Repeat untill the interval become sufficiently small.

(b).