

Binary Search → Direct
→ Indirect

10 5 2 -3 4 7

sorted dataset

$O(N \log_2 N)$

2

$-3 \leq 2 \leq 4 \leq 5 \leq 7 \leq 10$

-3 2 4

$n = 1024$

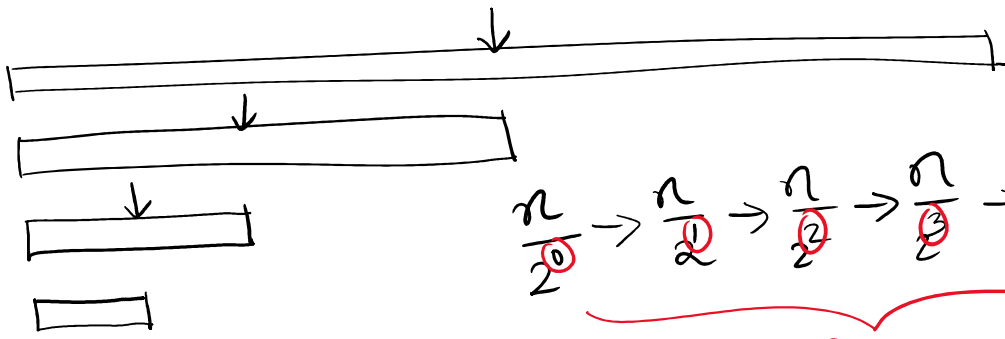
$n = 512$

$n = 256$

$n = 128$

⋮

$n = 1$



$\frac{n}{2^0} \rightarrow \frac{n}{2^1} \rightarrow \frac{n}{2^2} \rightarrow \frac{n}{2^3} \rightarrow \frac{n}{2^4} \rightarrow \dots \rightarrow \frac{n}{2^k}$
k th step

$\frac{n}{2^k} = 1$

$\Rightarrow n = 2^k$

$\Rightarrow \log_2(n) = k$

$x = 50$

1, 4, 6, 7, 12, 14, 16, 17, 20, 29,
30, 34, 39, 47, 48, 64, 73, 76

Right $L = M + 1$

$M = \frac{L+R}{2}$

$x = 50$

1 4 6 7 12 14 16 17 20 29 30 34 39 47 48 64 73 76

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
 1, 4, 6, 7, 12, 14, 16, 17, 20, 29, 30, 34, 39, 41, 40, 67, 72, 70

$L = 0$
 $R = N - 1$

while ($L \leq R$) {

$M = \frac{L+R}{2}$

$M = \frac{L+R+1}{2}$

if ($a_M == x$) return true;

else if ($a_M < x$) $L = M + 1;$

else $R = M - 1;$

} return false;

$L = R = 9 = 16$

$L = M$

1) Lower-bound

2) Upper-bound

$3 - 3 = 0$

lower_bound(arr, L) = 3

upper_bound(arr, R) = 3

arr →

| |
|----|
| 04 |
| 1 |

 08
5 12
5 16
7 20
8 24
11 28
13

$(24 - 04) = \frac{20}{4} = 5$

$[2, 8]$
 $[L, R]$

int idx = it - re.begin() // 5

lower_bound C++

// arr, x

0, 1, 2, 3, 4, 5, 6, 7?

1 5 5 7 8 11 13 Δ UB(arr)

1 5 5 7 8 11 13

$L=0, R=N$

$x=48$ $UB=15$

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| H | H | H | H | H | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 1 | 4 | 6 | 7 | 12 | 14 | 16 | 17 | 20 | 29 | 30 | 34 | 39 | 46 | 48 | 64 | 73 | 77 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |

L M R

i) $a_m \leq x \rightarrow L = M + 1$
 ii) $\text{else} \rightarrow R = M$

$L=0, R=N$
 while ($L < R$) {

$M = (L + R) / 2$

if ($a_m < x$) $L = M + 1$
 else $R = M$

}

return L ;

| | | | |
|----|----|----|----|
| 48 | 48 | 48 | 48 |
| 47 | 47 | 48 | 48 |
| 48 | 48 | 48 | 49 |

48 48 48 48