Class 22

Friday, August 13, 2021 8:59 PM

Sieve

$1 \sim n \rightarrow$

$1 \sim 12 \rightarrow$

1 : 1
2 : 1, 2
3 : 1, 3
4 : 1, 2, 4
5 : 1, 5
6 : 1, 2, 3, 6
7 : 1, 7
8 : 1, 2, 4, 8
9 : 1, 3, 9
10 : 1, 2, 5, 10
11 : 1, 11
12 : 1, 2, 3, 4, 6, 12

vector<int>divisors[1000000];

$O(n\sqrt{n})$

$$\frac{n}{1} + \frac{n}{2} + \frac{n}{3} + \dots + \frac{n}{n}$$

$$\Rightarrow n + \frac{n}{2} + \frac{n}{3} + \dots + 1$$

$$\Rightarrow n\left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}\right)$$

$$\Rightarrow n \cdot \log n$$

$$N = P_1^{\alpha_1} \times P_2^{\alpha_2} \times P_3^{\alpha_3} \times \dots \times P_k^{\alpha_k}$$

$$NOD(N) = (\alpha_1 + 1)(\alpha_2 + 1) \dots (\alpha_k + 1)$$

$$= \boxed{\prod_{i=1}^{k} (\alpha_i + 1)}$$

$$r^0 + r^1 + r^2 + \dots + r^n$$

$$SOD(N) = \frac{(r^{n+1} - 1)}{r - 1}$$

geometric series

$$SOD(16) = 1 + 2 + 4 + 8 + 16 = 2^0 + 2^1 + 2^2 + 2^3 + 2^4$$

$$4 \quad - \quad 2^{4+1} - 1 \quad = 31$$

$SOD(16) = 1 + 2 + 4 + 8 + 16$

$$\boxed{2^4 \longrightarrow = \frac{2^{4+1} - 1}{2-1} = 31}$$

$$SOD(12) = SOD(2^2 \times 3^1)$$

$$= SOD(2^2) \times SOD(3^1)$$

$$= \frac{2^3 - 1}{2-1} \times \frac{3^2 - 1}{3-1}$$

$$= 7 \times 4 = 28$$

$f(ab) = f(a) \times f(b)$

Multiplicative func

Hence,

$$\boxed{SOD(N) = \prod_{i=1}^{k} \left(\frac{P_i^{\alpha_i + 1} - 1}{P_i - 1} \right)}$$

$$\boxed{NOD(N) = \prod_{i=1}^{k} (\alpha_i + 1)}$$

A

$$N = a^2 + b^2 \longrightarrow a^2 + a^2 = 2a^2$$

$$O\left(\sqrt{N/2}\right)$$

$$\Rightarrow a^2 = \frac{N}{2}$$

$$100 = 6^2 + 8^2$$

$$\Rightarrow a = \sqrt{N/2}$$

$$\sqrt{50} \rightarrow 7 \rightarrow 99$$

$$\boxed{\sqrt{N_T} = P} \longleftarrow \boxed{N_T = P^2}$$

$$4 \rightarrow \sqrt{\ } \rightarrow 2$$

$$N \longrightarrow P_1^1 \longrightarrow (1+1) \longrightarrow 2$$

T-prime

$N \to p_1^{(1)} \to (1+1) \to 2 \to$ T-prime

$N^2 \to p_1^{(2)} \to (2+1) \to 3 \to$ T-prime

$25 \to \sqrt{\ } \to 5$

$a \leq b$

$2, 3, 5, 7, \mid 11, 13, 17, 19$

$n = 10$

$cnt = 2$

$3 + \cancel{7}$

$5 + 5$

$O(n \cdot \log(\log(n)))$

$+$

$T \cdot \frac{n}{\lg(n)}$

$n \to \lfloor \log_{10}(n) \rfloor + 1$

$n! \to n \times (n-1) \times (n-2) \times \cdots \times 1$

$\log_{10}(n!) \to \log_{10}(n \times (n-1) \times (n-2) \times \cdots \times 1)$

$\to \log_{10}(n) + \log_{10}(n-1) + \log_{10}(n-2) + \cdots + \log_{10}(1)$

$$\boxed{\log_{10}(n!) = \sum_{i=1}^{n} \log_{10}(i)}$$

$slog[1] = 0$

$slog[i] = slog[i-1] + lg[i]$

$s(4)\ [l(1) + l(2) + l(3)] + l(4)$

$s(4) = \boxed{l(1) + l(2) + l(3)} + l(4)$

$$\boxed{\log_b(n!) = \sum_{i=1}^{n} \log_b(i)} \rightarrow \boxed{\lfloor \log_b(n!) \rfloor} + 1$$

$n = 5$

$b = 2$

$slog[5] = 2.07\ldots$

$\hookrightarrow 3$

$\log_{10}(1) + \log_{10}(2) + \log_{10}(3) + \log_{10}(4) + \log_{10}(5)$

$6 + 1 = 7$

$$\log_b x = \frac{\log_{10} x}{\log_{10} b}$$