Class 23

Tuesday, August 17, 2021 10:19 PM

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$n! \leftarrow$ fact(n)

$r! \leftarrow$ fact(r)

$(n-r)! \leftarrow$ fact(n-r)

$a = 50 \rightarrow a--$

$b = 1 \rightarrow b++$

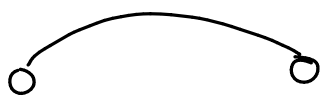
$^{5}C_{2} = 10$

$$^{50}C_{47} = \frac{50!}{47!\,3!} = \frac{50 \times 49 \times 48 \times \cancel{47!}}{\cancel{47!} \times 3!} = \frac{50 \times 49 \times 48}{1 \times 2 \times 3}$$

$n-r, r$

$^{50}C_{1} = {}^{50}C_{49}$

$^{n}C_{r} = {}^{n}C_{n-r}$



1

1 1

1 2 1

1 3 3 1

$\boxed{1\ 4\ 6\ 4\ 1}$

$^{n}C_{r}$ $\quad$ $^{n}C_{n-r}$

$$\frac{50 \times 49}{1 \times 2} \times \frac{48}{3}$$

$$\frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3}$$

$$\boxed{\frac{\overset{25}{\cancel{50}}}{1} \times \frac{49}{\cancel{2}}} \times \frac{48}{\cancel{3}}$$

$\frac{50}{1} \% 97$

$50 \times \frac{49}{2} \% 97 \rightarrow \frac{61 \times 48}{\boxed{3}}$

-o-

Stars and bars:

$a + b + c = 2$

$a, b, c \in \mathbb{N}$

$a, b, c \geq 0$

2 0 0 $\rightarrow$ $\underbrace{*\ \ *}\ |\ |$

0 2 0 $\rightarrow$ $\overset{0}{\frown}\ |\ \overset{2}{\underset{*\ \ *}{\frown}}\ |\ \overset{0}{\frown}$

$\underbrace{*\ *}_{2}\ \underbrace{|}_{0}\ \underbrace{|}_{0}$

$$
\begin{array}{ccccl}
2 & 0 & 0 & & \\
0 & 2 & 0 & \rightarrow & |*\ *| \\
0 & 0 & 2 & \rightarrow & ||*\ * \\
1 & 0 & 1 & \rightarrow & *||* \\
1 & 1 & 0 & \rightarrow & *|*| \\
0 & 1 & 1 & \rightarrow & |*|*
\end{array}
$$

$*\ *\ |\ |$

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$$\frac{4!}{2!\,2!} = \frac{24}{4} = 6$$

$$\binom{2+2}{2} = \binom{4}{2} = 6$$

$$\frac{10!}{3!\,7!} \qquad \frac{10!}{7!\,3!}$$

$$a+b+c+d = 7$$

number of star = 7

number of bar = 3 $\rightarrow$ $\dfrac{\overset{n}{10!}}{7!\,\underset{r}{3!}} = \dbinom{10}{3} = \dbinom{10}{7}$

$$\binom{\#\text{star} + \#\text{bar}}{\#\text{star}}$$

real world $\longrightarrow$ scenario + solve $\longrightarrow$ stars & bars