Euler Totient

ফাই (phi)

$\phi(n) = x$

1 থেকে n পর্যন্ত কতগুলো number n এর সাথে co-prime

$\rightarrow n = 10$ $\qquad \phi(10) = 4$

$$\frac{1}{10} \quad \frac{2}{10} \quad \frac{3}{10} \quad \frac{4}{10} \quad \frac{5}{10} \quad \frac{6}{10} \quad \frac{7}{10} \quad \frac{8}{10} \quad \frac{9}{10} \quad \frac{10}{10}$$

$\gcd(a, p) = 1$

$a < p$

p হচ্ছে prime

$$\boxed{\phi(p) = p - 1}$$

$$\overbrace{\boxed{\underbrace{1 \ldots\ldots\ldots p-1}_{1}}}^{p}, \boxed{p}$$

$p - 1$

$$\boxed{\phi(p^k) = p^k - p^{k-1}}$$

$$\boxed{f(ab) = f(a) \cdot f(b)}$$

$N = p_1^{\alpha_1} \times p_2^{\alpha_2} \times p_3^{\alpha_3} \ldots \times p_k^{\alpha_k}$

$\phi(N) = \phi(\underline{p_1^{\alpha_1}} \times \underline{p_2^{\alpha_2}} \times \underline{p_3^{\alpha_3}} \times \ldots \times \underline{p_k^{\alpha_k}})$

$= \boxed{\phi(p_1^{\alpha_1})} \times \phi(p_2^{\alpha_2}) \ldots \phi(p_k^{\alpha_k})$

$= (p_1^{\alpha_1} - p_1^{\alpha_1 - 1})(p_2^{\alpha_2} - p_2^{\alpha_2 - 1}) \ldots (p_k^{\alpha_k} - p_k^{\alpha_k - 1})$

$= \left(p_1^{\alpha_1} - \frac{p_1^{\alpha_1}}{p_1}\right) \ldots$

$= p_1^{\alpha_1}\left(1 - \frac{1}{p_1}\right) p_2^{\alpha_2}\left(1 - \frac{1}{p_2}\right) \ldots p_k^{\alpha_k}\left(1 - \frac{1}{p_k}\right)$

$$= N\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\left(1-\frac{1}{p_3}\right)\cdots\left(1-\frac{1}{p_k}\right)$$

$$\boxed{\phi(N) = N \cdot \prod_{i=1}^{k}\left(1-\frac{1}{p_i}\right)}$$

$p_1 = 2$ $\alpha_1 = 2$

$p_2 = 3$ $\alpha_2 = 1$

$p_3 = 5$ $\alpha_3 = 1$

— o —

Euler's Theorem

$$a^{p-1} \% p = 1$$

$\boxed{\gcd(a, p) = 1}$

congruence

$$a^{p-1} \% p = 1 \Leftrightarrow a^{p-1} \equiv 1 \pmod p$$

$$77^{996} \% 997$$

$a = 77$ $p = 997$

$$\boxed{77^{\boxed{13949}} \% \boxed{997}} = \left(77^{(996\times 14)+5}\right) \% 997$$

$$= \left(77^{\boxed{996}\times\boxed{14}} \cdot 77^5\right) \% 997$$

$\boxed{x}$

$$\phi(60) = 60 \boxed{\left(\frac{2-1}{2}\right)\left(\frac{3-1}{3}\right)\left(\frac{5-1}{5}\right)}$$

$=$

$=$

$= 77^{5} \% 997$

$$a^{n} \% m \ne a^{\boxed{n \% \phi(m)}} \% m$$

$$a^{[n \% \phi(m)] + \phi(m)} \% m$$

$$\boxed{n \geq \log_2 m}$$

$\phi(1) = 1$

Exponial

$n! = n(n-1)(n-2) \cdots 1$

$$f(n,m) = n^{(n-1)^{(n-2)^{\cdot^{\cdot^{1}}}}} \% m$$

$n = 5 \rightarrow 5^{\boxed{4^{3^{2^{1}}}}} \% 997$

$5^{196} \% 997$

$a^{b} \% M$	$O(\log_2(b))$

$1 \leq a \leq 10^9$

$0 \leq b \leq 10^9$

$\boxed{4^{3^{2^{1}}}}$

5

262144

5

for (mult

ϕ

ϕ

$(3) = 3\left(\frac{3-1}{3}\right)$

$\phi(6) = 6\left(\frac{2-1}{2}\right)\left(\frac{3-1}{3}\right)$

$\phi(9) = 9\left(\frac{3-1}{3}\right)$

$\phi(12) = 12\left(\frac{2-1}{2}\right)\left(\frac{3-1}{3}\right)$

t = i+i ; mult ≤ n ; mult += i)

$$\boxed{\begin{array}{l} \text{(mult) /= i ;} \\ \phi\text{(mult) *= (i-1) ;} \end{array}}$$

$\boxed{4^{5}}$ $\boxed{\begin{matrix}262144\\5\end{matrix}}$ $\boxed{994}$

$7^{6^{5}} \xrightarrow{\checkmark} 7^{6^{\boxed{5}}} \xrightarrow{\times} 7^{6} \ \%997$

$7^{6^{5^{4^{3^{2^{1}}}}}} \longrightarrow f(7) \longrightarrow \underbrace{7^{f(6)}}$

$\boxed{f(n) =}$

$f(n)\%m = n^{f(n-1)} \% m$

$= n^{[f(n-1) \% \phi(m)] + \phi(m)} \% m$

$f(7)\%997 = 7^{[\ 984\ ] + 996} \% 997$

$f(6) \% \phi(997) = 6^{(\ 297\ ) + 328} \% 996$

$\Rightarrow f(6) \% 996 = 6 \ldots$

$= 984$

$f(5) \% 328 = 297$

$f(4) \% 160 = 64$

$n^{f(n-1)}$

1) $n \longrightarrow \phi(n)$

$1 \leq n \leq 10^{15}$

2) $n \longrightarrow \phi[1, n]$

$1 \leq n \leq 10^{6}$

$f(4)\%160 = 64$

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$= \frac{fact[n]}{fact[r] \times fact[n-r]}$$

$$= \left(fact[n] \times \boxed{\left(x^{M-2} \% M\right)}\right) \% M$$

1)

=

$1 \leq n \leq 10$

$1 \sim 10^6 \ldots\ldots$

$fact[i] \bmod (10^6+3)$

$= \dfrac{fact[n]}{x}$

$\left(fact[n] \times \dfrac{1}{x}\right) \% M$

1

$$a^{p-1} \equiv 1 \pmod{p}$$

$$\Rightarrow a^{p-1} \times a^{-1} \equiv a^{-1} \pmod{p}$$

$$\Rightarrow a^{p-2} \equiv a^{-1} \pmod{p}$$

$$\Downarrow$$

$$a^{p-2} \% p \equiv a^{-1} \% p$$