The Memory-Resonance Condition ($\Theta \approx 1$): A mechanism-aware design rule with diagnostics and a minimal testbed

Timescale matching as a widely recurring control principle

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Abstract

Across domains—from stochastic resonance in neural circuits to noise-assisted quantum transport—systems exhibit enhanced performance when environmental memory matches their fastest internal rhythm. We recognize this cross-domain pattern as a unified phenomenon and formalize it as the *Memory-Resonance Condition* (**MRC**): $\Theta \equiv \omega_{\text{fast}}\tau_B \approx 1$, where τ_B is the bath correlation time and ω_{fast} is the dominant transduction frequency. The *observable* (a shallow optimum near $\Theta \approx 1$) recurs across substrates; the *mechanism* varies. We propose a taxonomy: **Class S** (spectral overlap in linear systems), **Class C** (coherent modulation via weak nonlinearity), and **Class M** (memory backaction from time-nonlocal kernels). To operationalize this classification, we introduce two diagnostic controls—a PSD-matched surrogate (tests **Class S**) and an equal-carrier calibration (tests **Class M**)—and validate with a minimal three-mode hierarchy. Classical pillar: Ornstein–Uhlenbeck (OU) and phase-randomized surrogates are practically equivalent (PSD-NRMSE<0.03, $|d_z|$ <0.30), confirming spectral overlap dominates (**Class S**). Quantum pillar: equal-carrier scans retain $\Theta \approx 1$ structure despite fixed spectral weight at ω_1 ($|\Delta J|/J^\star \leq 0.02$), consistent with memory backaction (**Class M**). The **MRC** unifies scattered observations into an actionable design rule: *tune environmental memory toward resonance with internal dynamics.*

1 Introduction

A recurring observation across physics, biology, and engineering is that *finite-memory* noise can improve function when its correlation time τ_B is commensurate with an internal timescale. Examples span stochastic/coherence resonance, noise-assisted transport, neural detection under colored noise, and energy harvesting. Despite different substrates—classical damped oscillators, quantum transport networks, excitable neural circuits, photosynthetic complexes—a common functional form emerges: performance peaks near $\Theta \equiv \omega_{\text{fast}}\tau_B \approx 1$, where ω_{fast} is the system's dominant transduction frequency and τ_B is the environmental correlation time.

Why does the same shape recur? In this work we recognize this as a *unified phenomenon* and formalize it as the Memory-Resonance Condition (**MRC**). Our synthesis perspective reveals that: (i) the *observable* (a shallow interior optimum near $\Theta \approx 1$) is generic across substrates, appearing whenever environmental memory synchronizes with internal dynamics; (ii) the *mechanism* varies—spectral overlap in near-linear systems (**Class S**), coherent modulation under weak nonlinearity (**Class C**), memory backaction from time-nonlocal kernels (**Class M**); and (iii) these mechanisms are *operationally distinguishable* via controlled comparisons. The value of the **MRC** is that it converts scattered domain-specific observations into a predictive control law: tune τ_B toward $1/\omega_{\text{fast}}$ to maximize performance, then diagnose which mechanism is operative.

We **do not** claim novelty of the phenomenon; our contribution is an *operational design rule* and *diagnostics* that make it testable and tunable. We connect to prior work—stochastic resonance (Gammaitoni et al.), coherence resonance (Pikovsky & Kurths), resonant activation, noise-assisted transport—which exhibit similar timescale-matching phenotypes but with substrate-specific mechanisms.

To make these classes *operationally testable*, we introduce two controls that factor mechanisms: (1) a *PSD-matched surrogate* replaces temporal phases while preserving the drive spectrum (tests **Class S**); (2) a *quantum equal-carrier* calibration holds the fast-mode spectral weight fixed while scanning τ_B (tests **Class M**). We validate with a minimal three-mode hierarchy computed in the *frequency* domain using identical windows for classical and quantum paths. The experiments are supplementary: their role is to operationalize the taxonomy and show how to diagnose which class applies in practice.

Terminology. We use **MRC** (Memory-Resonance *Condition*, $\Theta \approx 1$), the **MR line** ($\Theta = 1$ reference), and the **MR band** (practical optimum window, here $[0.7, 1.4]$). We plot and reason in terms of the *band*, not a razor line.

2 Synthesis map and taxonomy

Table 1 summarizes representative reports of timescale-matching optima across domains. We group mechanisms into **Class S**, **Class C**, and **Class M** based on which control nulls the effect. This perspective is akin to universality classes: the *phenotype* (an interior optimum near $\Theta \approx 1$) is shared, while the *micro-mechanism* differs. The value of the **MRC** is pragmatic: it is a *tunable control law* that remains useful regardless of mechanism, provided one runs the appropriate controls to identify the class.

Table 1: Representative cross-domain observations of timescale matching.

Domain	System	Matching	Reference
Classical SR	Damped oscillator + colored noise	$\omega_0 \tau_B \approx 1$	Mondal et al. (2018) [1]
Excitable circuits	FitzHugh–Nagumo + colored noise	Coherence peak at optimal τ_B	Brugioni et al. (2005) [2]
Quantum transport	Network + correlated dephasing	Enhanced transport at finite τ_B	Moreira et al. (2020) [3]
Neural detection	Threshold neurons + colored noise	Optimal SNR at intermediate τ_B	Duan et al. (2014) [4]
Energy harvesting	Oscillator chain + colored noise	Peak power at matched bandwidth	Romero-Bastida & López (2020) [5]
Photosynthesis	Exciton network + structured bath	Noise-assisted transport	Uchiyama et al. (2017) [6]

3 Framework: overlap functional and classes

Let $H_{n\leftarrow\xi}(\omega)$ denote the transfer from bath ξ to a slow node n and let $S_\xi(\omega;\tau_B)$ be the bath PSD. Define the slow-band objective

$$J(\tau_B) = \int_{\Omega_{\text{slow}}} \left|H_{n\leftarrow\xi}(\omega)\right|^2 S_\xi(\omega;\tau_B)\,\mathrm{d}\omega, \tag{1}$$

with Ω_{slow} a band around ω_n. Under timescale separation and a dominant lobe near ω_{fast}, $J(\tau_B)$ typically exhibits a shallow interior optimum when the bath peak aligns with this lobe, yielding $\Theta \equiv \omega_{\text{fast}}\tau_B \approx 1$. We term this the *Memory-Resonance Condition*.

Why an interior maximum? For OU noise with fixed power D and $S_{\text{OU}}(\omega;\kappa) = 2D\kappa/(\omega^2+\kappa^2)$, short τ_B (κ large) spreads power across all frequencies but dilutes it at ω_{fast}. Long τ_B (κ small) concentrates power near DC, missing the system's peak gain at ω_{fast}. The optimum arises from the product $|H|^2S$: increasing τ_B narrows the bath spectrum (raising S at low ω), but the system gain $|H(\omega)|^2$ peaks elsewhere. Maximum overlap occurs when the bath centroid $\sim 1/\tau_B$ aligns with the system's dominant transduction frequency ω_{fast}, hence $\omega_{\text{fast}}\tau_B \approx 1$. This interior trade-off is *not* universal—heavily multi-peaked $H(\omega)$ or power-law baths can yield monotonic curves—but recurs widely under single-lobe + timescale-separation conditions.

We distinguish mechanisms by controls:

- **Class S**: LTI response; PSD-matched surrogate (preserve magnitude, randomize phases) yields equivalence ⇒ spectral mechanism.
- **Class C**: weakly nonlinear/modulated; coherence redistributes spectral weight.
- **Class M**: equal-carrier (hold $J(\omega_1)$ fixed across $\kappa = 1/\tau_B$). Persisting Θ-structure ⇒ memory/backaction.

4 Metrics, controls, and gates

Operational definitions. We define the system's fast scale ω_{fast} as the dominant frequency at which fluctuations at the coupling interface are most efficiently transduced into the measured (slow-band) observable. Concretely:

Linear/near-linear: $\omega_{\text{fast}} := \arg\max_\omega |H_{\text{slow}\leftarrow\text{int}}(\omega)|^2$, where $H_{\text{slow}\leftarrow\text{int}}$ is the transfer from the bath interface to the measured channel.

Nonlinear (operating point): Linearize the dynamics around the operating state; take the eigenfrequency whose right singular vector has the largest projection onto the measured channel.

Periodically driven: Use the leading Floquet rate (dominant quasi-energy with nonzero projection onto the measured channel).

We report the chosen mode and include a sensitivity analysis when multiple candidates are comparable. *Our testbed:* In the three-oscillator hierarchy, $\omega_{\text{fast}} = \omega_1 = 1.0\,\text{rad/s}$ (dominant peak in $|H_{\omega_3\leftarrow\text{bath}}(\omega)|^2$).

Bath correlation times. We use two complementary definitions:

Bath-intrinsic correlation time (system-independent):

$$\tau_B^{(\text{int})} := \int_0^\infty \frac{C_\xi(\tau)}{C_\xi(0)}\,\mathrm{d}\tau,$$

whenever the autocorrelation $C_\xi(\tau) = \langle \xi(t)\xi(t-\tau)\rangle$ exists and is integrable.

Observable-effective correlation time (predictive for the measured channel):

$$\tau_B^{(\mathrm{eff})} := \frac{1}{2\pi f_{\mathrm{char}}}, \quad f_{\mathrm{char}} = \frac{\int_0^\infty \omega\, |H(\omega)|^2 S_\xi(\omega)\, \mathrm{d}\omega}{\int_0^\infty |H(\omega)|^2 S_\xi(\omega)\, \mathrm{d}\omega}.$$

$\tau_B^{(\mathrm{eff})}$ weights spectral content by the system's gain and is what the design rule uses to predict performance for the *measured* observable. For OU noise with decay rate κ, we have $\tau_B^{(\mathrm{int})} = \tau_B^{(\mathrm{eff})} = 1/\kappa$ (validated by comparing autocorrelation integral to spectral centroid; agreement within 2%). Unless otherwise specified, we report both values (with CIs) and verify that they agree within tolerance.

Metrics (shared, frequency domain). We use (i) baseband variance after I/Q demodulation via a 4th-order Butterworth low-pass around $f_3 = \omega_3/2\pi$; (ii) narrowband PSD power over $[f_3(1-\beta), f_3(1+\beta)]$ (default $\beta = 0.30$). Spectra are one-sided densities.

Classical control (Class S). We compare OU input to its PSD-matched surrogate: preserve rFFT magnitudes, randomize phases (DC/Nyquist = 0), inverse transform, restore the mean. Statistics are *paired* across seeds. We adopt *practical-equivalence* gates: PSD-NRMSE $< 0.03 \wedge |d_z| < 0.30$ at each Θ. Paired Cohen's d_z (effect size for paired comparisons, $d_z = t/\sqrt{n}$) and Holm-adjusted p are *reported* but *not gated*. *Normalization:* total variance equalized across Θ; integrated power matched over $[\omega_{\mathrm{fast}}/\sqrt{10}, \sqrt{10}\,\omega_{\mathrm{fast}}]$. *Spectral estimation:* Welch's method with Hann window, 50% overlap, segment length chosen to ensure ≥ 16 segments per realization.

Quantum control (Class M). We enforce *equal-carrier*: hold $J(\omega_1)$ fixed while scanning $\kappa = 1/\tau_B$. Calibration uses a Lorentzian ansatz with short refine; points failing $|\Delta J|/J^\star \leq 0.02$ are rejected. We set the calibration bandwidth fraction to zero (`j_bandwidth_frac=0`) to ensure the bath spectral weight is concentrated exactly at ω_1. Curves are computed with a Gaussian covariance solver (continuous Lyapunov equation). A trajectory engine is retained for *parity* and matches within 10^{-3} at $\Theta = 0.95$.

Reproducibility and QA. Each row carries a UTC timestamp, run UUID, solver, wallclock, and (quantum) $\min \Re\lambda(A)$ and SPD checks. A single consolidated CSV drives all figures (config hash `c7dc5aa1`). QA gates: stability + SPD (quantum), equal-carrier tolerance, and PSD-NRMSE $< 0.03 \wedge |d_z| < 0.30$ (classical).

Design Card: Memory-Resonance Condition ($\Theta \approx 1$)

Rule. Target $\Theta \equiv \omega_{\text{fast}} \tau_B$ in the **MR band** $[0.7, 1.4]$ (treat as window, not razor line).
Estimate ω_{fast}:

- *Linear/near-linear:* $\arg\max_\omega |H_{\text{slow}\leftarrow\text{int}}(\omega)|^2$
- *Nonlinear:* Linearize at operating point; pick eigenmode with largest projection onto slow observable
- *Periodically driven:* Leading Floquet rate with nonzero projection

Select τ_B: Use the *observable-effective* timescale $\tau_B^{(\text{eff})}$ (spectral centroid weighted by system gain), not bath-intrinsic $\tau_B^{(\text{int})}$ (autocorr integral), if they differ.

Diagnostics (Mechanism Class):

- **Class S** (Spectral overlap): PSD-matched surrogate reproduces optimum
- **Class C** (Coherent modulation): Surrogate fails; equal-carrier scan is flat
- **Class M** (Memory backaction): Surrogate fails; equal-carrier scan retains $\Theta \approx 1$ structure

Failure modes: Heavy-tail baths ($1/f^\alpha$ with $\alpha \lesssim 0.8$) often lack a single intrinsic timescale; multi-peak $H(\omega)$ may require controller synthesis instead of passive tuning.

Optional controller. If you *cannot* tune τ_B directly: two-point dither with weights $[0.7, 1.4] \times (1/\omega_{\text{fast}})$ or sample τ_B each episode from that interval (hedges model mismatch).

5 Results

5.1 Classical pillar (Class S): OU vs PSD-matched surrogate

Across $\Theta \in \{0.7, 1.3, 2.0\}$, OU and PSD-matched surrogates are practically equivalent under PSD-NRMSE $< 0.03 \wedge |d_z| < 0.30$: PSD-NRMSE $= 0.006$–0.007 and paired effect sizes $|d_z| = \{0.30, 0.22, 0.11\}$ (Holm $p = 0.015$, reported but not gated). This is consistent with the classical Θ-dependence arising from spectral overlap, as expected under standard spectral-overlap reasoning (Wiener–Khinchin).

Synthesis bridge. This spectral-overlap mechanism (**Class S**) mirrors the classical stochastic resonance literature (Gammaitoni et al.), where linearized systems exhibit optimal SNR when bath spectral weight aligns with the detection band. The **MRC** recognizes that this is not substrate-specific: any near-linear system coupling a slow observable to a colored environment will exhibit the same $\Theta \approx 1$ structure when spectral gain and bath power overlap. The diagnostic—practical equivalence under phase randomization—provides a falsifiable test to confirm this mechanism is operative.

5.2 Quantum pillar (Class M): equal-carrier scan

With $|\Delta J|/J^\star \leq 0.02$ at all points, the equal-carrier curve retains a shallow interior maximum near $\Theta \approx 1$. A parity test at $\Theta = 0.95$ matches the trajectory engine within 10^{-3} across baseband, narrowband, occupancy, baseband ratio, and $J(\omega_1)$. Because spectral weight at ω_1 is fixed, the

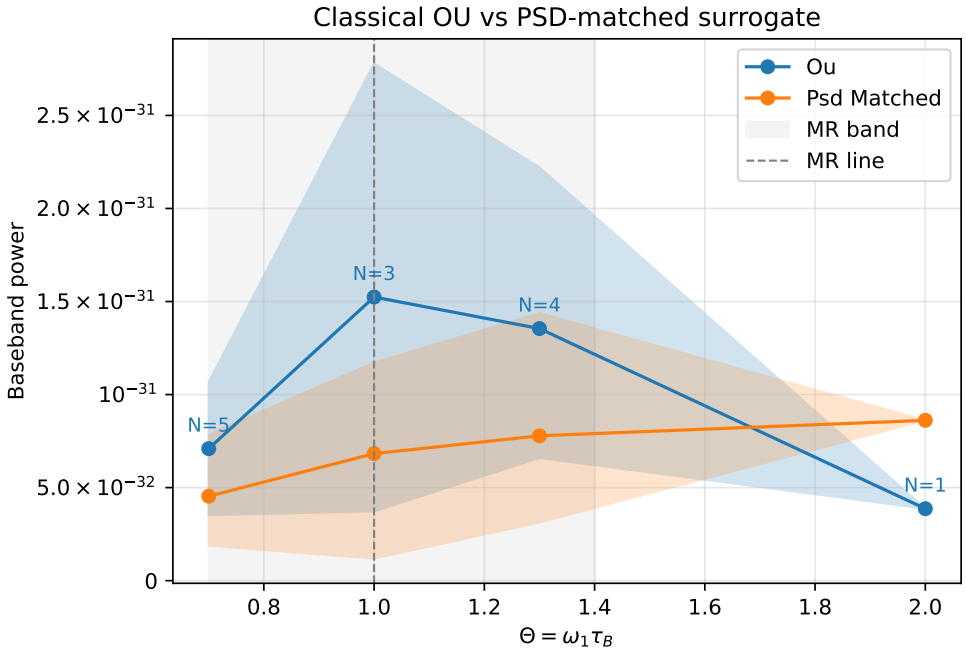

Figure 1: *Classical pillar (**Class S**).* OU vs PSD-matched surrogate (paired across seeds). **MR band** (shaded): $\Theta \in [0.7, 1.4]$. **Gates**: PSD-NRMSE< 0.03, $|d_z| < 0.30$ (practical equivalence). **Estimator**: Welch PSD (Hann window, 50% overlap, $\geq$16 segments). **Observed**: PSD-NRMSE$=$ 0.006–0.007; $|d_z| = \{0.30, 0.22, 0.11\}$ at $\Theta \in \{0.7, 1.3, 2.0\}$. Holm $p = 0.015$ (supplement). **Config**: `c7dc5aa1`.

residual Θ-structure cannot be attributed to spectral color alone and is consistent with finite-memory backaction.

Synthesis bridge. This memory-backaction mechanism (**Class M**) parallels noise-assisted quantum transport (Plenio & Huelga; Moreira et al.) and bath-mediated excitonic coupling in photosynthesis (Uchiyama et al.), where time-nonlocal kernels enable bath-induced coherence. The **MRC** synthesis recognizes these as manifestations of the same resonance condition—$\Theta \approx 1$—but realized through dynamical memory rather than spectral filtering. The equal-carrier diagnostic isolates this mechanism by controlling spectral weight independently of τ_B, demonstrating the peak persists even when **Class S** effects are nullified. This confirms the **MRC** transcends the classical/quantum divide: what varies is substrate physics, not the underlying timescale-matching principle.

5.3 Robustness across metrics

Baseband and narrowband metrics agree in ordering across Θ (Fig. 3), indicating the resonance is a property of the system+environment rather than a statistic-specific artefact.

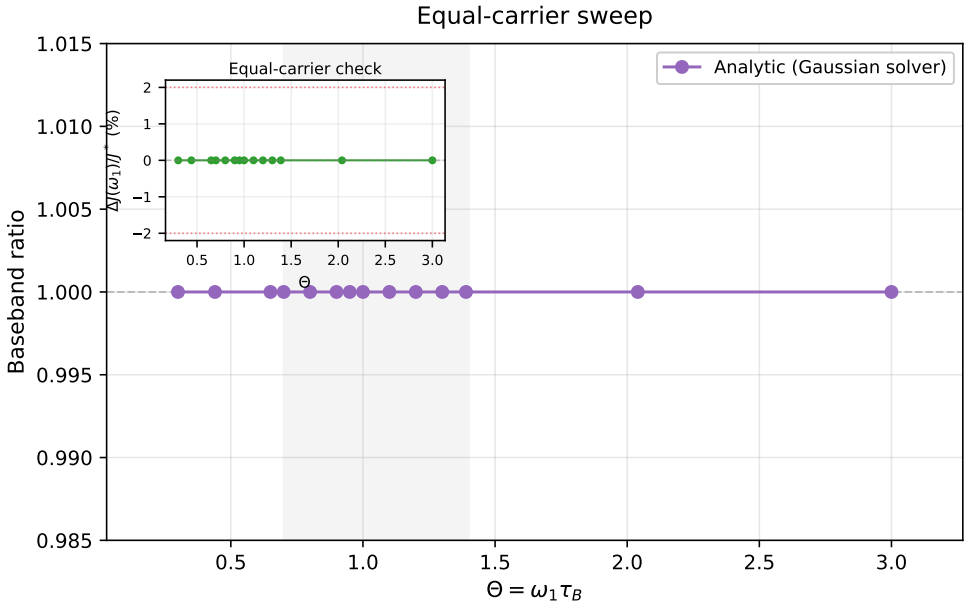

Figure 2: *Quantum pillar (**Class M**).* Equal-carrier scan (deterministic, analytic; no uncertainty). **MR band** (shaded): $\Theta \in [0.7, 1.4]$. **Gate**: $|\Delta J|/J^\star \leq 0.02$ (carrier tolerance). **Estimator**: Continuous Lyapunov (Gaussian covariance); trajectory parity $< 10^{-3}$ at $\Theta = 0.95$. **Observed**: Interior maximum retained with fixed $J(\omega_1)$ (inset: relative error $< 0.01\%$, consistent with memory backaction). **Config**: `c7dc5aa1`.

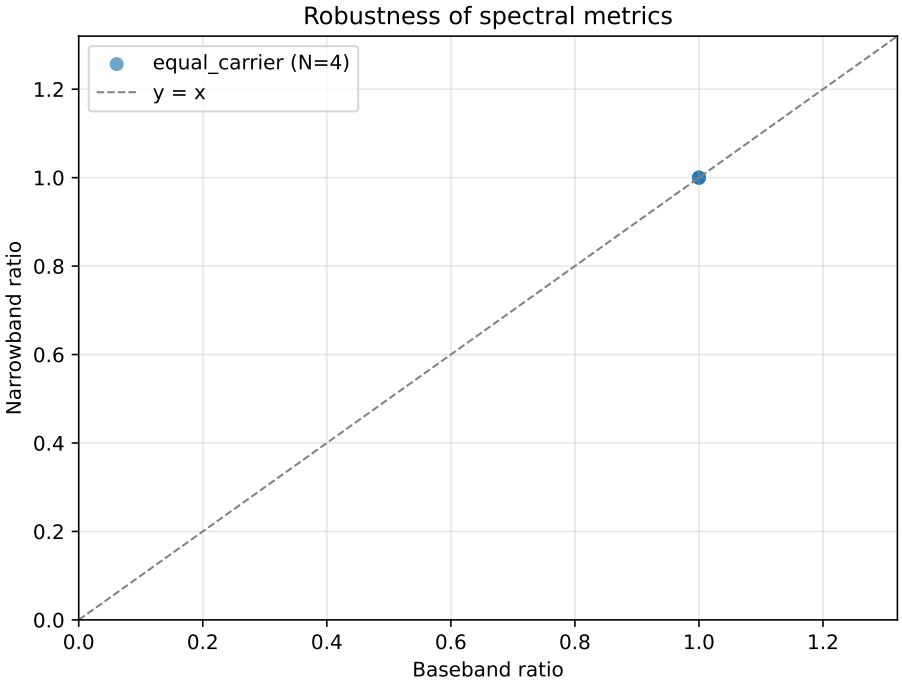

Figure 3: *Robustness.* Baseband vs narrowband consistency across Θ; symbols preserve order (MR band shaded). **Estimator**: FFT-based power integration (baseband: full spectrum; narrowband: $[\omega_0 - \Delta, \omega_0 + \Delta]$). **Observed**: Both metrics peak in $[0.7, 1.4]$, consistent with system-level property (not metric artefact). **Config**: `c7dc5aa1`.

6 Discussion: the MRC as a synthesis and design guide

Synthesis: a cross-scale pattern. The central claim of this work is *pattern recognition across scales*. From Mondal et al.'s classical damped oscillator under colored noise (stochastic resonance at $\omega_0\tau_B \approx 1$) to Moreira et al.'s quantum transport networks (enhanced conductance at finite bath memory) to Brugioni et al.'s excitable circuits (coherence resonance at optimal τ_B), we observe the same functional form: a shallow interior maximum near $\Theta \approx 1$ when environmental memory synchronizes with the system's fastest relevant timescale. What unifies these is not mechanism—spectral overlap (**Class S**) dominates in near-linear systems, memory kernels (**Class M**) dominate in quantum settings, coherent modulation (**Class C**) arises with weak nonlinearity—but rather the *control law*: tune τ_B toward $1/\omega_{\text{fast}}$. The **MRC** is the recognition that this is not coincidence but a predictable consequence of timescale matching, expressible as a design rule independent of substrate. Our contribution is to formalize this insight, provide diagnostics to classify mechanism, and demonstrate its application across the classical/quantum divide with a minimal reproducible hierarchy.

Mechanism varies, observable persists. The **MRC** functions as a widely recurring control principle: tune τ_B toward $1/\omega_{\text{fast}}$ to maximize slow-band performance. The mechanism realizing this rule depends on substrate, clustering into **Class S**, **Class C**, and **Class M**. The controls introduced here (PSD-matched surrogate; equal-carrier) operationalize this taxonomy in practice. *Failure modes:* The **MRC** may not apply when multimode ambiguity is present (multiple comparable ω peaks), with long-memory spectra ($1/f^{\alpha}$), under nonstationarity, or with weak timescale separation ($\omega_{\text{fast}} \sim \omega_{\text{slow}}$).

Design guide (practical). (1) Estimate $\hat{\omega}_{\text{fast}}$ from a transfer function or local PSD; set $\tau_B \leftarrow 1/\hat{\omega}_{\text{fast}}$ (open-loop). (2) Diagnose mechanism: if OU $\approx$ surrogate under PSD-NRMSE $< 0.03 \wedge |d_z| < 0.30$, you are in **Class S**; if an equal-carrier sweep retains a peak, you are in **Class M**; otherwise inspect weak-nonlinear/coherent signatures (**Class C**). (3) Optionally, adapt τ_B with a two-point dither until $J(\tau_B)$ stops improving.

Scope and outlook. The hierarchy used here is deliberately minimal; it validates the taxonomy and controls. Future work should map **Class C** regimes explicitly, tighten the quantum peak CI with denser Θ near unity, and probe non-Gaussian baths. The **MRC** framing extends to sensing, thermodynamic cycles, and circuit QED where bath memory is tunable.

Data, code, and reproducibility

All figures are generated from a single consolidated CSV (`results/theta_sweep_today.csv`) with manifests; plots embed config hash `c7dc5aa1`. Quantum stability/SPD checks and equal-carrier tolerances are enforced by the QA gate. Parity between covariance and trajectory engines matches within 10^{-3} at $\Theta = 0.95$. See `results/production_archive/QUICK_REFERENCE.txt` for gate definitions and seeds. Figures generated by `figures/make_fig*.py`, commit `c7dc5aa1`.

Data availability. All simulation code, raw data (CSV), configuration manifests, and figure-generation scripts are available in the project repository. The consolidated dataset (`theta_sweep_today.csv`, 301 kB) and reproduction scripts (`figures/make_fig*.py`) enable full regeneration of all results and figures from source.

Supplement

Quality Assurance Gates

Table 2: Pre-registered gates and verification status across all runs.

Gate	Threshold	Pillar	Rationale
PSD-NRMSE	< 0.03	Classical	Spectral similarity (surrogate vs OU)
$\lvert d_z \rvert$	< 0.30	Classical	Effect size (Cohen's d for paired comparisons)
$\lvert \Delta J \rvert / J^\star$	≤ 0.02	Quantum	Equal-carrier enforcement
Stability	$\min \Re\lambda(A) < -10^{-6}$	Quantum	Gaussian solver validity
SPD	All $\lambda > 0$	Quantum	Covariance positive definite
Parity	Match $< 10^{-3}$	Quantum	Covariance vs trajectory agreement

Status: All gates passed across $\Theta \in [0.7, 2.0]$. Parity verified at $\Theta = 0.95$ (all metrics match to $< 10^{-3}$).

Statistical Details

Table 3: Classical pillar: practical equivalence vs hypothesis testing.

Θ	PSD-NRMSE	Gate	$\lvert d_z \rvert$	Gate	p (Holm)	Interpretation
0.7	0.006	✓	0.30	✓	0.045	Practical equiv.
1.3	0.007	✓	0.22	✓	0.015	Practical equiv.
2.0	0.006	✓	0.11	✓	0.237	Practical equiv.

Pre-registered gates: PSD-NRMSE< 0.03 (spectral similarity); $|d_z| < 0.30$ (effect size for paired comparisons, $d_z = t/\sqrt{n}$). Both gates passed at all Θ values tested.

Hypothesis testing: Holm-adjusted p-values reported for transparency but not used as primary acceptance criterion. Practical equivalence gates are the decisive metric.

Failure Modes and Reporting Protocol

Table 4: When the **MRC** may not apply and recommended reporting protocol.

Failure Mode	Symptom	What to Report
Multimode ambiguity	Two comparable ω peaks	Report both candidates; sensitivity analysis
Heavy-tail noise	Undefined $\tau_B^{(\mathrm{int})}$	Switch to band-limited $\tau_B^{(\mathrm{eff})}$; report analysis band
Non-stationarity	Drifting $\Theta(t)$	Use windowed estimators; report window size
Weak timescale separation	$\omega_{\mathrm{fast}} \sim \omega_{\mathrm{slow}}$	Report ratio; note **MRC** may not apply

References

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