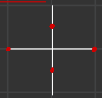
Exercise 1 :



a) C'est une elipse

b) $\frac{\partial \vec{r}}{\partial \rho} = (a\cos(\gamma), b\sin(\gamma))$

$\frac{\partial \vec{r}}{\partial \gamma} = (-a\rho\sin(\gamma), b\rho\cos(\gamma))$

$(a\cos(\gamma), b\sin(\gamma)) \cdot (-a\rho\sin(\gamma), b\rho\cos(\gamma)) = -a^2\rho\cos(\gamma)\sin(\gamma) + b^2\sin(\gamma)\cos(\gamma)$

Ne donne pas 0 donc pas Curviligne orthogonale

c) $$\frac{\partial(x,y)}{\partial(r,\gamma)} = \det \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \gamma} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \gamma} \end{vmatrix} = \begin{vmatrix} a\cos(\gamma) & -a\rho\sin(\gamma) \\ b\sin(\gamma) & b\rho\cos(\gamma) \end{vmatrix} = ab\rho\cos^2(\gamma) + ab\rho\sin^2(\gamma)$$

$$= ab\rho(\cos^2(\gamma) + \sin^2(\gamma))$$

$$= ab\rho \quad d\rho\, d\gamma$$

d) $2 = 3\rho\cos(\gamma) \rightarrow \frac{2}{3} = \rho\cos(\gamma)$

$1 = 2\rho\sin(\gamma) \rightarrow \frac{1}{2} = \rho\sin(\gamma)$

SOH CAH TOA

$\frac{1}{2\sin(\gamma)} = \rho$

$\frac{2}{3} = \frac{1}{2}\frac{\cos(\gamma)}{\sin(\gamma)}$

$\frac{2}{3}\tan(\gamma) = \frac{1}{2}$

$\frac{3}{4} = \tan(\gamma)$

$\gamma = 0.6435$

$\frac{\frac{2}{3}}{\cos(0.6435)} = \rho$

$\rho = \frac{5}{6}$

Rep $= \left(\frac{5}{6}, 0.6435\right)$

Exercise 2:

$$\frac{\partial f}{\partial w} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial w} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial w} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial w}$$

$$\frac{\partial f}{\partial x} = e^{i(\cos(xy)+z)} \cdot -iy\sin(xy) \qquad \frac{\partial x}{\partial w} = \frac{-u^2v}{w^2}$$

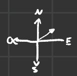
$$\frac{\partial f}{\partial y} = e^{i(\cos(xy)+z)} \cdot -ix\sin(xy) \qquad \frac{\partial y}{\partial w} = uv\cos(uvw)$$

$$\frac{\partial f}{\partial z} = e^{i(\cos(xy)+z)}\, i \qquad \frac{\partial z}{\partial w} = 3w^2$$

$$\frac{\partial f}{\partial w} = e^{i(\cos(xy)+z)} \, -iy\sin(xy)\, \frac{-u^2v}{w^2} + e^{i(\cos(xy)+z)} \, -ix\sin(xy)\, uv\cos(uvw) + e^{i(\cos(xy)+z)}\, i\, 3w^2$$

$$\frac{\partial f}{\partial w} = e^{i(\cos(\frac{u^2v}{w}\sin(uvw))+w^3)} \, -iy\sin(\frac{u^2v}{w}\sin(uvw))\, \frac{-u^2v}{w^2} + e^{i(\cos(\frac{u^2v}{w}\sin(uvw))+w^3)} \, -ix\sin(\frac{u^2v}{w}\sin(uvw))\, uv\cos(uvw) + e^{i(\cos(\frac{u^2v}{w}\sin(uvw))+w^3)}\, i\, 3w^2$$

Exercise 3:



a) $\frac{\partial \vec{H}}{\partial x} = \frac{2x+2y+40}{20\,000} + \frac{2x-2y}{2}\Big|_{(0,0)} = \frac{40}{20\,000}\hat{e}_x$

$\frac{\partial \vec{H}}{\partial y} = \frac{2y+2x+40}{20\,000} + \frac{2y-2x}{2}\Big|_{(0,0)} = \frac{40}{20\,000}\hat{e}_y$

$N\text{-}O = (-1,1) = \frac{1}{\sqrt{2}}(-1,1) = \hat{u}$

$D_{\vec{H}}\,\hat{u} = \frac{1}{500}(1,1)\cdot\frac{1}{\sqrt{2}}(-1,1)$

$D_{\vec{H}}\,\hat{u} = 0$

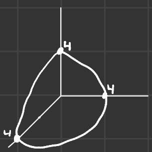
Vû que c'est egale à 0 on sait que cela ne l'aidera pas du tout par sa souffrance

b) Sud Ouest, car c'est le contraire de gradient

c) $x = -10$
$y = -10$

Exercise 4:

a)



b) $\text{Constante} = 4-(x^2+y^2)$

$x^2+y^2 = 4 - \text{Constante}$

c) i) $V = 4\cdot\int_{x=0}^{2}\int_{y=0}^{\sqrt{4-x^2}} 4-(x^2+y^2)\,dy\,dx$

ii) $V = 4\cdot\int_{x=0}^{2}\int_{y=0}^{\sqrt{4-x^2}}\int_{z=0}^{(4-(x^2+y^2))} 1\,dz\,dy\,dx$

ii) b) Oui

Exercise 5:

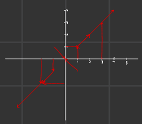
$$\int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1} z \; dz\, dy\, dx$$

$$\int_{x=0}^{1}\int_{y=0}^{1} \frac{1}{2} \; dy$$

Exercise 6:

Cons bale = $4x - 211y + 909z + 8$

Exercise 7:



Exercise 8:

a) tangent c'est la droite $\quad \overrightarrow{P_1P_2} = (1,1,2)$

b) $\vec{r} = (t, t, 2t+1)$

$T(\vec{r}) = \cos(t + t + 2t + 1)$

$T(\vec{r}) = \cos(4t+1)$

$u = 4t+1$

$dt = \frac{du}{4}$

$T(\vec{r}) = \frac{\sin(u)}{4}$

$T(\vec{r}) = \frac{\sin(4t+1)}{4}$

$\vec{r} = (t, t, 2t+1)$

$$\vec{f} = \frac{1}{1-0}\int_0^1 \frac{\sin(4t+1)}{4}\, dt$$

$\vec{f} = -0{,}239 - 0{,}21036 \quad = -0{,}4493$

C) $\vec{F}(x,y,z) = (\cos(z), \sin(x), y^2)$

$\vec{r} = (t, t, 2t-1)$

$\vec{F}(\vec{r}) = (\cos(2t-1), \sin(t), t^2)$

$$\int_0^1 \cos(2t-1) + \sin(t) + t^2(2) \quad dt$$

$u = 2t+1$

$du = 2\,dt$

$dt = \frac{du}{2}$

$$\frac{\sin(u)}{2} - \cos(t) + \frac{2}{3}t^3 \Big|_0^1$$

$0{,}1969 \; - - 0{,}57926$

$0{,}77616$

Exercise 9 :

$$\frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x} \Rightarrow 6xy^2z^4 = 6xy^2z^4$$

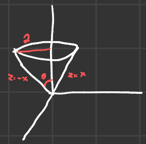
$$\frac{\partial^2}{\partial x \partial z} = \frac{\partial^2}{\partial z \partial x} \Rightarrow 8xy^3z^3 = 8xy^3z^3$$

$$\frac{\partial^2}{\partial y \partial z} = \frac{\partial^2}{\partial z \partial y} \Rightarrow 12x^2y^2z^3 = 12x^2y^2z^3$$

Puisque toute les équations sont vrai cela veut dire que l'integrale ne dépend pas du chemin choisi entre P et Q

Exercise 10 :

$\vec{F} = (4x, 3z, 5y)$



$z^2 \geq x^2 + y^2 \qquad 0 \leq z \leq 2$

$x = P\cos(\varphi)$
$y = P\sin(\varphi)$
$z = x^2 + y^2 = P^2$

sur surface de tour $P = z$

$x = z\cos(\varphi)$
$y = z\sin(\varphi)$
$z = z$

$\vec{r}(\varphi, z) = [z\cos(\varphi); z\sin(\varphi), z]$

$\frac{\partial \vec{r}}{\partial \varphi} = [-z\sin(\varphi), z\cos(\varphi), 0]$

$\frac{\partial \vec{r}}{\partial z} = [\cos(\varphi), \sin(\varphi), 1]$

$$\vec{N} = \det \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ -z\sin(\varphi) & z\cos(\varphi) & 0 \\ \cos(\varphi) & \sin(\varphi) & 1 \end{vmatrix} = [z\cos(\varphi), z\sin(\varphi), -z\sin^2(\varphi) - z\cos^2(\varphi)] = [z\cos(\varphi), z\sin(\varphi), -z]$$

Pour surface top le rayon est de 2

$x = P\cos(\varphi)$
$y = P\sin(\varphi)$
$z = 2$

$r(P, \varphi) = [P\cos(\varphi), P\sin(\varphi), 2]$

$\frac{\partial \vec{r}}{\partial P} = [\cos(\varphi), \sin(\varphi), 0]$

$\frac{\partial \vec{r}}{\partial \varphi} = [-P\sin(\varphi), P\cos(\varphi), 0]$

$$\vec{N} = \det \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \cos(\varphi) & \sin(\varphi) & 0 \\ -P\sin(\varphi) & P\cos(\varphi) & 0 \end{vmatrix} = [0, 0, P\cos^2(\varphi) + P\sin^2(\varphi)] = [0, 0, P]$$

donc $\hat{n} = \hat{e}_z$

$$\Phi = \int_0^2 \int_0^{2\pi} [4z\cos(\varphi), 3z, 5z\sin(\varphi)] \cdot [z\cos(\varphi), z\sin(\varphi), -z] \, d\varphi \, dz + \int_0^2 \int_0^{2\pi} [4P\cos(\varphi), 6, 5P\sin(\varphi)] \cdot [0, 0, P] \, dP \, d\varphi$$

$$\Phi = \int_0^2 \int_0^{2\pi} 4z^2\cos^2(\varphi) + \underbrace{3z^2\sin(\varphi) + -5z^2\sin(\varphi)}_{0} \, d\varphi \, dz + \int_0^{2\pi}\int_0^2 5P^2\sin(\varphi) \, dP \, d\varphi$$

$$\Phi = 4\int_0^2 z^2 \, dz \cdot \int_0^{2\pi} \cos^2(\varphi) \, d\varphi \qquad \cos^2(\varphi) = \frac{1 + \cos(2\varphi)}{2} \qquad + \int_0^{2\pi} \frac{5}{3}P^3 \sin(\varphi) \Big|_0^2 \, d\varphi$$

$$\Phi = 4\left(\frac{z^3}{3}\Big|_0^2 \cdot \frac{1}{4}\sin(2x) + \frac{1}{2}x \Big|_0^{2\pi}\right) + \underbrace{\int_0^{2\pi} \frac{40}{3}\sin(\varphi) \, d\varphi}_{0}$$

$$\Phi = 32\pi / 3$$

b) $\vec{F} = (4x, 3z, 5y)$

$\vec{\nabla} \cdot \vec{F} = \frac{\partial}{\partial x}(4x) + \frac{\partial}{\partial y} 3z + \frac{\partial}{\partial z}(5y)$

$\vec{\nabla} \cdot \vec{F} = 4$

$$\int_0^{2\pi} \int_0^{2} \int_0^{z} 4 \rho \, d\rho \, d\varphi \, dz$$

$$8\pi \int_0^2 \int_0^z \rho \, d\rho \, d\varphi$$

$$8\pi \int_0^2 \frac{\rho^2}{2} \Big|_0^z \, d\varphi$$

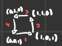
$$8\pi \int_0^2 \frac{z^2}{2} \, d\varphi$$

$$8\pi \frac{z^3}{2 \cdot 3} \Big|_0^2$$

$$\frac{32\pi}{3}$$

Exercise 118

$\vec{F} = (\cos(\pi y), \sin(\pi x), 0)$



a) ① $\vec{r}(t) = (0,0,1) + t((1,0,1) - (0,0,1))$

$r(\vec{t}) = (t, 0, 1)$

② $\vec{r}(t) = (1,0,1) + t((1,1,0) - (1,0,1))$

$\vec{r}(t) = (1, t, 1-t)$

③ $\vec{r}(t) = (1,1,0) + t((0,1,0) - (1,1,0))$

$\vec{r}(t) = (1-t, 1, 0)$

④ $\vec{r}(t) = (0,1,0) + t((0,0,1) - (0,1,0))$

$\vec{r}(t) = (0, 1-t, t)$

$\cos(\pi y), \sin(\pi x), 0$

$$\int_0 \vec{F} \cdot d\vec{r} = \int_{①}^1 (\cos(\pi 0), \sin(\pi t), 0)(1,0,1) + \int_{②}^1 (\cos(\pi t), \sin(\pi), 0)(0,1,-1)$$
$$\int_{③}^1 (\cos(\pi), \sin(\pi(1-t)), 0)(-1,0,0) + \int_{④}^1 (\cos(\pi(1-t)), \sin(\pi 0), 0)(0,-1,1)$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 1 \, dt + \int_0^1 0 \, dt + \int_0^1 1 \, dt + \int_0^1 0 \, dt$$

$$\int_C \vec{F} \cdot d\vec{r} = 1 + 1 = 2$$

b) $dA = xy$

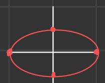
$\hat{n} = \hat{e}_x + \hat{e}_y$

$\int_0^1 \int_0^1$

Exercice 1:

$x = a\rho\cos(\gamma)$
$y = b\rho\sin(\gamma)$

a)



ellipse

b) Curviligne orthogonale veut dire que $\frac{d}{d\rho} \cdot \frac{d}{d\gamma}$

$\frac{\partial}{\partial\rho} = (a\cos(\gamma), b\sin(\gamma))$

$\frac{\partial}{\partial\gamma} = (-a\rho\sin(\gamma), b\rho\cos(\gamma))$

ne sont pas égale à zero donc ce n'est pas curviligne orthogonale

c) $\frac{\partial(x,y)}{\partial(\rho,\gamma)} = \det \begin{vmatrix} \frac{\partial x}{\partial\rho} & \frac{\partial x}{\partial\gamma} \\ \frac{\partial y}{\partial\rho} & \frac{\partial y}{\partial\gamma} \end{vmatrix} d\rho\, d\gamma = \begin{vmatrix} a\cos(\gamma) & b\sin(\gamma) \\ -a\rho\sin(\gamma) & b\rho\cos(\gamma) \end{vmatrix} = ab\rho\cos^2(\gamma) + ab\rho\sin^2(\gamma)\, d\rho\, d\gamma$

$= ab\rho\, d\rho\, d\gamma$

d) $x = a\rho\cos(\gamma)$
$y = b\rho\sin(\gamma)$

$2 = 3\rho\cos(\gamma)$

$1 = 2\rho\sin(\gamma) \rightarrow \frac{1}{2\sin(\gamma)} = \rho$

$2 = 3\frac{1}{2\sin(\gamma)}\cos(\gamma) \Rightarrow \frac{3}{4} = \frac{\sin(\gamma)}{\cos(\gamma)}$

$\tan^{-1}(3/4) = \gamma$

$\gamma = 0{,}6435$

$2 = 3\rho\cos(0{,}6435)$

$\rho = 5/6$

Exercise 2:

$\frac{\partial f}{\partial x} = e^{i(\cos(xy)+z^3)} \cdot -iy\sin(xy)$

$\frac{\partial f}{\partial x} = e^{i(\cos(xy)+z^3)} \cdot -ix\sin(xy)$

$\frac{\partial f}{\partial x} = e^{i(\cos(xy)+z^3)} \cdot i$

$\frac{\partial x}{\partial u} = \frac{2uv}{w}$ $\quad$ $\frac{\partial y}{\partial u} = \cos(uvw)vw$ $\quad$ $\frac{\partial z}{\partial u} = 0$

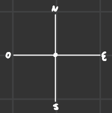
$\frac{\partial x}{\partial v} = \frac{u^2}{w}$ $\quad$ $\frac{\partial y}{\partial v} = \cos(uvw)uw$ $\quad$ $\frac{\partial z}{\partial v} = 0$

$\frac{\partial x}{\partial w} = -\frac{u^2v}{w^2}$ $\quad$ $\frac{\partial z}{\partial w} = \cos(uvw)uv$ $\quad$ $\frac{\partial z}{\partial w} = 3w^2$

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial w} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial w} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial w}$$

$$\frac{\partial f}{\partial u} = e^{i(\cos(\frac{u^2v}{w}\sin(uvw))+w^3)} \cdot -\frac{u^2v}{w^2} + e^{i(\cos(\frac{u^2v}{w}\sin(uvw))+w^3)}\cos(uvw)uv + e^{i(\cos(\frac{u^2v}{w}\sin(uvw))+w^3)} \cdot 3w^2$$

Exercise 3:



a) trouver gradient

$$\nabla = \left(\frac{\partial H}{\partial x}, \frac{\partial H}{\partial y}\right)$$

$$\frac{\partial \vec{H}}{\partial x} = \frac{2x+2y+40}{20\,000} + \frac{2x-2y}{2} \qquad \frac{0+0+40}{20\,000} + \frac{0-0}{2} = \frac{1}{500}$$

Elle est au point (0,0)

$$\frac{\partial \vec{H}}{\partial y} = \frac{2y+2x+40}{20\,000} + \frac{2y-2x}{2} \qquad \frac{0+0+40}{20\,000} + \frac{0-0}{2} = \frac{1}{500}$$

$$\nabla = \left(\frac{1}{500}, \frac{1}{500}\right)$$

$$\hat{u} = \frac{(-1,1)}{\sqrt{2}}$$

$$D_{\hat{u}} f = \vec{\nabla} f \cdot \hat{u}$$

$$D_{\hat{u}} f = \frac{1}{500}(1,1) \cdot (-1,1)\frac{1}{\sqrt{2}}$$

$$D_{\hat{u}} f = \frac{1}{500} \cdot \frac{-1}{\sqrt{2}} + \frac{1}{500} \cdot \frac{1}{\sqrt{2}} = 0$$

la direction Nord ouest l'cidera donc pas

b) Ce sera au sud-ouest car c'est $-\nabla$

c) le minimum c'est lorsque $\frac{\partial H}{\partial x} = 0$ et $\frac{\partial H}{\partial y} = 0$

Donc on voit ici que c'est $x = -10$ et $y = -10$

Exercise 4:

i) $x = \sqrt{4-y^2}$

$y = \sqrt{4-x^2}$

$$V = 4 \cdot \int_{x=0}^{2} \int_{y=0}^{\sqrt{4-y^2}} (4-(x^2+y^2))\, dx\, dy$$

ii) $x = \sqrt{4-y^2}$

$y = \sqrt{4-x^2}$

$$V = 4 \cdot \int_{0}^{2} \int_{y=0}^{\sqrt{4-x^2}} \int_{x=0}^{\sqrt{4-y^2}} 1 \, dz\, dy\, dx$$

Exercise 5:

$$P = \int_0^1 \int_0^1 \int_0^1 z \, dz \, dy \, dx$$

$$P = \int_0^1 \int_0^1 \frac{z^2}{2} \Big|_0^1 \, dy \, dx$$

$$P = \int_0^1 \frac{1}{2} y \Big|_0^1 \, dx$$

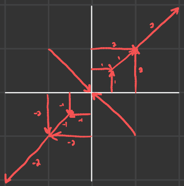
$$P = \frac{1}{2} x \Big|_0^1$$

$$P = \frac{1}{2}$$

Exercise 6:

Constante $= 4x - 211y + 4092z + 8$

Exercise 7:



Exercise 8:

a) $\overrightarrow{P_1P_2} = (1,1,2)$ car $(1,1,3) - (0,0,1)$

b) $\vec{r}(t) = r_0 + t(r_1 - r_0)$

$\vec{r}(t) = (0,0,1) + t((1,1,3) - (0,0,1))$

$\vec{r}(t) = (t, t, 1+2t)$

$T(x,y,z) = \cos(t + t + 1 \cdot 2t)$

$T(t) = \cos(4t+1)$

$$\vec{f} = \frac{1}{1-0} \int_0^1 \cos(4t+1) \, dt$$

$$\vec{f} = \int_0^1 \cos(u)$$

$u = 4t+1$

$dt = 4du$

$\frac{1}{4} dt = du$

$$\vec{f} = \sin(u) \, du \Big|_0^1$$

$$\vec{f} = \frac{\sin(4t+1)}{4} \Big|_0^1$$

$$\vec{f} = -0{,}23973 - 0{,}21036$$

$$\vec{f} = -0{,}45$$

b) $\vec{F}(x,y,z) = (\cos(z), \sin(x), y^2)$

$P_1 = (0,0,1)$
$P_2 = (1,1,3)$

$\vec{r} = r_0 + t(r_1 - r_0)$

$\vec{r} = (t, t, 1+2t)$

$\frac{dr}{dt} = (1, 1, 2)$

$F(t) = (\cos(1+2t), \sin(t), t^2)$

$W = \int_C (\vec{F} \cdot \frac{d\vec{r}}{dt})\, dt$

$w = \int_0^1 (\cos(1+2t), \sin(t), t^2) \cdot (1,1,2)\, dt$

$W = \int_0^1 \cos(1+2t) + \sin(t) + 2t^2\, dt$

$u = 1+2t$
$du = 2dt$
$\frac{1}{2}du = dt$

$W = \frac{\sin(1+2t)}{2}\Big|_0^1 - \cos(t)\Big|_0^1 + \frac{2}{3}t^3\Big|_0^1$

$W = (0{,}070 - 0{,}54030 + 2/3) - (0{,}4207 - 1 + 0)$

$W = 0{,}77566$

Exercise 4:

$\frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x} \Rightarrow 6xy^2z^4 = 6xy^2z^4$

$\frac{\partial^2}{\partial x \partial z} = \frac{\partial^2}{\partial z \partial x} \Rightarrow 8xy^3z^3 = 8xy^3z^3$

$\frac{\partial^2}{\partial y \partial z} = \frac{\partial^2}{\partial z \partial y} \Rightarrow 12x^2y^2z^3 = 12x^2y^2z^3$

Donc non la valeur ne dépend pas du chemin suivi en P et Q

Exercise 10:

$\vec{F} = (4x, 3z, 5y)$ $\qquad 0 \leq z \leq 2$

Cone donc cylindrique

$\int_S F \cdot \hat{n}\, dA$

$x = \rho\cos(\psi)$

$y = \rho\sin(\psi)$

$z = z$

Pour gros côté

$x = z\cos(\psi)$

$y = z\sin(\psi)$

$z = z$

$$\vec{r}(\psi, z) = [z\cos(\psi), z\sin(\psi), z]$$

$$\frac{\partial \vec{r}}{\partial \psi} = [-z\sin(\psi), z\cos(\psi), 0]$$

$$\frac{\partial \vec{r}}{\partial z} = [\cos(\psi), \sin(\psi), 1]$$

$$\vec{N} = \det \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ -z\sin(\psi) & z\cos(\psi) & 0 \\ \cos(\psi) & \sin(\psi) & 1 \end{vmatrix} = [z\cos(\psi), z\sin(\psi), -z\sin^2(\psi) - z\cos^2(\psi)]$$

$$= [z\cos(\psi), z\sin(\psi), -z]$$

$$\int_0^2 \int_0^{2\pi} [4z\cos(\psi), 3z, 5z\sin(\psi)][z\cos(\psi), z\sin(\psi), -z]\, d\psi\, dz$$

$$\int_0^2 \int_0^{2\pi} 4z^2\cos^2(\psi) + 3z^2\sin(\psi) - 5z^2\sin(\psi) \quad d\psi\, dz$$

$\sin(0) = 0$
$\sin(2\pi) = 0$

$$\int_0^2 z^2\, dz \cdot \int_0^{2\pi} 4\cos^2(\psi)$$

$$\left.\frac{z^3}{3}\right|_0^2 \cdot \left. 4 \cdot \frac{1}{2}\psi + \frac{1}{4}\sin(2\psi)\right|_0^{2\pi}$$

$$\frac{8}{3} \cdot 4 \cdot \pi = \frac{32}{3}\pi$$

Pour le top

$x = \rho\cos(\psi)$

$y = \rho\sin(\psi)$

$z = 2$

$\vec{r} = [\rho\cos(\psi), \rho\sin(\psi), 2]$

$\frac{\partial \vec{r}}{\partial \rho} = [\cos(\psi), \sin(\psi), 0]$

$\frac{\partial \vec{r}}{\partial \psi} = [-\rho\sin(\psi), \rho\cos(\psi), 0]$

$$\int_0^2 \int_0^{2\pi} [4\rho\cos(\psi), 6, 5\rho\sin(\psi)][0, 0, \rho]\, d\rho\, d\psi$$

$$\int_0^2 \int_0^{2\pi} 0 + 0 + 5\rho^2\sin(\psi)$$

$$\int_0^2 \rho^2\, d\rho \cdot \int_0^{2\pi} 5\sin(\psi)$$

$\sin(0) = 0$
$\sin(2\pi) = 0$

$$= 0$$

$\vec{N} = [0, 0, \rho]$

b) $\vec{F} = (4x, 3z, 5y)$

$z^2 \geq x^2 + y^2 \qquad 0 \leq z \leq 2$

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial}{\partial x}(4x) + \frac{\partial}{\partial y}(3z) + \frac{\partial}{\partial z}(5y)$$

$$\vec{\nabla} \cdot \vec{F} = 4$$

$$\int_0^{2\pi} \int_0^2 \int_0^z 4\rho \, d\rho \, dz \, d\varphi$$

$$8\pi \int_0^2 \int_0^z \rho \, d\rho \, dz$$

$$8\pi \int_0^2 \frac{\rho^2}{2} \Big|_0^z$$

$$8\pi \quad \frac{z^3}{2 \cdot 3} \Big|_0^2$$

$$8\pi \cdot \frac{8}{6}$$

$$\frac{32\pi}{3}$$

Exercise 12:

$$\nabla\cdot(\nabla\times\vec{F})=0$$

$$\nabla\cdot\ \det\begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$\nabla\cdot\left[\frac{\partial F_3}{\partial y}-\frac{\partial F_2}{\partial z},\ \frac{\partial F_1}{\partial z}-\frac{\partial F_3}{\partial x},\ \frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\right]$$

$$\frac{\partial^2 F_3}{\partial x\partial y}-\frac{\partial^2 F_2}{\partial z\partial x}+\frac{\partial^2 F_1}{\partial z\partial y}-\frac{\partial^2 F_3}{\partial x\partial y}+\frac{\partial^2 F_2}{\partial x\partial z}-\frac{\partial^2 F_1}{\partial z\partial y}$$

$$0$$

$$\vec{\nabla}\times(\vec{\nabla}f)=0$$

$$\vec{\nabla}f=\left[\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial z}\right]$$

$$\nabla\times\left[\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial z}\right]$$

$$\det\begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix}=\left[\frac{\partial^2 f}{\partial y\partial z}-\frac{\partial^2 f}{\partial y\partial z},\ \frac{\partial^2 f}{\partial x\partial z}-\frac{\partial^2 f}{\partial z\partial x},\ \frac{\partial^2 f}{\partial y\partial x}-\frac{\partial^2 f}{\partial x\partial y}\right]$$

$$=[0,0,0]$$

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