Procedural 1:

1) $\frac{1}{z}e^{\sin(xy)} + \frac{x}{z}e^{\sin(xy)}\cos(xy)\,y$

$\frac{1}{z}e^{\sin(xy)}\left(1 + xy\cos(xy)\right)$

Vert = à se rappeler comment faire

2) a) un cercle

b) $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dx}{dt}$

$\frac{df}{dt} = (-2x)(-\sin(t)) + (-2y)(\cos(t))$

$\frac{df}{dt} = 2x\sin(t) - 2y\cos(t)$

$\frac{df}{dt} = 2\cos(t)\sin(t) - 2\sin(t)\cos(t)$

$\frac{df}{dt} = 0$

3) a) $\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}$

$\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v}$

b) $\boxed{\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial u} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial u}}$

$\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial v}$

4) a) $\frac{\partial T}{\partial x} = -2x + 4$

$\frac{\partial T}{\partial y} = -2y + 8$

$\frac{\partial T}{\partial z} = -2z + 2$

$\vec{\nabla}T = (-2x+4)\hat{e}_x + (-2y+8)\hat{e}_y + (-2z+2)\hat{e}_z$

$\vec{\nabla}T|_{P=(2,6,1)} = (-2(2)+4)\hat{e}_x + (-2(6)+8)\hat{e}_y + (-2(1)+2)\hat{e}_z = (0,-4,0)$

$\boxed{\hat{v} = \frac{1}{|\vec{v}|}\vec{v}}$

$\hat{v} = \frac{1}{\sqrt{1^2+1^2+1^2}}(1,1,1)$

$\hat{v} = \frac{1}{\sqrt{3}}(1,1,1)$

$\boxed{D_{\hat{v}}\vec{T} = \vec{\nabla}T\cdot\hat{v}} = (0,-4,0)\cdot\frac{(1,1,1)}{\sqrt{3}} = \left(0, \frac{-4}{\sqrt{3}}, 0\right)$

5) $\vec{E} = -\vec{\nabla}\vec{V}$

$$\frac{\partial V}{\partial x} = \frac{-q}{8\pi\varepsilon_0}\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{-3/2}\cdot 2(x-x_0)\cdot 1$$

$$\frac{\partial V}{\partial y} = \frac{-q}{8\pi\varepsilon_0}\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{-3/2}\cdot 2(y-y_0)\cdot 1$$

$$\frac{\partial V}{\partial z} = \frac{-q}{8\pi\varepsilon_0}\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{-3/2}\cdot 2(z-z_0)\cdot 1$$

$$\vec{E} = -\vec{\nabla}V = \left[\frac{-\partial V}{\partial x}, \frac{-\partial V}{\partial y}, \frac{-\partial V}{\partial z}\right] = \frac{2q}{8\pi\varepsilon_0}\frac{1}{\|\vec{r}-\vec{r}_0\|^3}\cdot\left[x-x_0,\ y-y_0,\ z-z_0\right]$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{1}{\|\vec{r}-\vec{r}_0\|^2}$$

6) Cartesiane $\to$ spherique
$(-2,2,-3)$

$$r = \sqrt{(2)^2+(2)^2+(3)^2} = \sqrt{17}$$

$$\theta = \arctan\frac{\sqrt{(2)^2+(2)^2}}{-3} = -43.31 + 180 = 136^\circ$$

$$\varphi = \arctan\left(\frac{2}{-2}\right) = 135^\circ$$

$$(r,\theta,\varphi) = (4.123,\ 136,\ 135)$$

7) $V = \frac{qa}{4\pi\varepsilon_0}\frac{\cos(\theta)}{r^2}$

$$\frac{\partial V}{\partial r} = \frac{-qa}{2\pi\varepsilon_0}\frac{\cos(\theta)}{r^3}$$

$$\frac{\partial V}{\partial \theta} = \frac{-qa}{4\pi\varepsilon_0}\frac{\sin(\theta)}{r^2}$$

$$\frac{\partial V}{\partial \varphi} = 0$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \varphi}\hat{\varphi}$$

$$\nabla V = \frac{-qa}{2\pi\varepsilon_0}\frac{\cos(\theta)}{r_3}\hat{e}_r + \frac{1}{r}\frac{-qa}{4\pi\varepsilon_0}\frac{\sin(\theta)}{r^2}\hat{e}_\theta + 0\hat{e}_\varphi$$

$$\vec{E} = -\nabla\vec{V} = \frac{qa}{4\pi\varepsilon_0 r^3}\left(2\cos(\theta)\hat{e}_r + \sin(\theta)\hat{e}_\theta\right)$$

8) $\nabla \times (\nabla \times \vec{v}) = \nabla(\nabla \cdot \vec{v}) - \nabla^2 \vec{v}$

Procedural 2:

1) a) $r=1$ $x=\cos(\theta)$
$y=\sin(\theta)$

$\vec{r}(\theta)=[\cos(\theta), \sin(\theta), 0]$ $0\leq\theta\leq 2\pi$

b) $\vec{r}(\theta,r)=[u\cdot\cos(\theta), u\sin(\theta), 0]$ $0\leq\theta\leq 2\pi$
$0\leq r\leq 1$

c) $x=a\cdot\cos(t)$
$y=b\cdot\cos(t)$

$\vec{r}(\theta)=[a\cdot\cos(\theta), b\cdot\sin(\theta), 0]$ $0\leq\theta\leq 2\pi$

d) $y=f(x)$
$x=t$

$\vec{r}(t)=[t, f(t)]$

2) a) Cercle de rayon 1

b)

Procedural 1a

4) $D_{\hat v}\vec T = \nabla\cdot\hat v$

$\frac{\partial T}{\partial x} = -2x+4$

$\frac{\partial T}{\partial y} = -2y+8$

$\frac{\partial T}{\partial z} = -2z+2$

$\nabla\vec T = (-2x+4)\hat e_x + (-2y+8)\hat e_y + (-2z+2)\hat e_z$

$\nabla\vec T|_{(2,6,1)} = (0,-4,0)$

$\hat V = \frac{1}{|\vec v|}\vec v = \frac{1}{\sqrt3}(1,1,1)$

$D_{\hat v}\vec T = \nabla\vec T\cdot\hat v = (0,-4,0)\cdot\frac{(1,1,1)}{\sqrt3}$

$D_{\hat v}\vec T = (0,\frac{-4}{\sqrt3},0)$

b) le maximum c'est $\nabla\vec T$, donc le minimum $= -\nabla\vec T$ donc c'est $(0,4,0)$

$\nabla\vec T$ = vector vers maximum

c) le maximum c'est lorsque c'est 0

$-2x+4=0$ $x=2$

$-2y+8=0$ $y=4$

$-2z+2=0$ $z=1$

Maximum au point $(2,4,1)$

Derivé partielle $=0$ c'est un maximum

5) a) $\vec E = -\nabla V$

$\frac{\partial V}{\partial x} = \frac{-q}{4\pi\varepsilon_0}((x-x_0)^2+(y-y_0)^2+(z-z_0)^2)^{-3/2}(x-x_0)$

$\frac{\partial V}{\partial y} = \frac{-q}{4\pi\varepsilon_0}((x-x_0)^2+(y-y_0)^2+(z-z_0)^2)^{-3/2}(y-y_0)$

$\frac{\partial V}{\partial z} = \frac{-q}{4\pi\varepsilon_0}((x-x_0)^2+(y-y_0)^2+(z-z_0)^2)^{-3/2}(z-z_0)$

$\nabla V = \frac{-q}{4\pi\varepsilon_0}\cdot\frac{1}{\|r-r_0\|^3}\cdot(x-x_0,y-y_0,z-z_0)$

$\nabla V = \frac{-q}{4\pi\varepsilon_0}\frac{1}{\|r-r_0\|^2}$

$\vec E = -\nabla V$

$\vec E = \frac{q}{4\pi\varepsilon_0}\frac{1}{\|r-r_0\|^2}$

Procedural 3:

1) $\int_0^1 \int_0^2 x + x^2 - yx + zx \Big|_0^3$

$\int_0^1 \int_0^2 3 + 9 - 3y + 3z$

$\int_0^1 3y + 9y - \frac{3}{2}y^2 + 3zy \Big|_0^2$

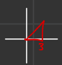
$\int_0^1 6 + 18 - 6 + 6z$

$18z + 3z^2 \Big|_0^1$

$18 + 3$

21

2)



$\uparrow_0^x \quad \overrightarrow{0 \quad 3}$

$\uparrow dy \quad \overrightarrow{dx}$

$\int_0^3 \int_0^x (x^2 + 2xy + 3)\, dy\, dx$

$\int_0^3 x^2y + xy^2 + 3y \Big|_0^x$

$\int_0^3 x^3 + x^3 + 3x$

$\frac{x^4}{4} + \frac{x^4}{4} + \frac{3}{2}x^2 \Big|_0^3$

54

3) $x^2 + y^2 = 2$

$x^2 + y^2 = 1$

$\int_0^2 \int_0^{\sqrt{2-y^2}} (x+y)\, dx\, dy - \int_0^1 \int_0^{\sqrt{1-y^2}} (x+y)\, dx\, dy$

$\int_0^2 \left(\frac{x^2}{2} + yx\right) \Big|_0^{\sqrt{2-y^2}} - \int_0^1 \left(\frac{x^2}{2} + yx\right) \Big|_0^{\sqrt{1-y^2}}$

$\int_0^2 \left(\frac{2}{2} - \frac{y^2}{2} + y\left(\sqrt{2-y^2}\right)\right.$

Procedural 2:

4) a) $x = a\sin(\theta)\cos(\psi)$ $\boxed{\text{Sphérique}}$

$y = a\sin(\theta)\sin(\psi)$

$z = a\cos(\theta)$

$\vec{r}(\theta,\psi) = (a\sin(\theta)\cos(\psi), a\sin(\theta)\sin(\psi), a\cos(\theta))$ $0 \leq \theta \leq \pi$

$0 \leq \psi \leq 2\pi$

b) $\vec{r}(\theta,\psi) = (a\sin(\theta)\cos(\psi) + x_0,\ a\sin(\theta)\sin(\psi) + y_0,\ a\cos(\theta) + z_0)$ $0 \leq \theta \leq \pi$

$0 \leq \psi \leq 2\pi$

c) $\vec{r}(\theta,\psi) = (a\sin(\theta)\cos(\psi), a\sin(\theta)\sin(\psi), a\cos(\theta))$ $0 \leq \theta \leq \pi/2$

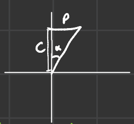
$0 \leq \psi \leq \pi/2$

d) $\vec{r}(r,\theta,\psi) = (r\sin(\theta)\cos(\psi),\ r\sin(\theta)\sin(\psi),\ r\cos(\theta))$ $0 \leq \theta \leq \pi/2$

$0 \leq \psi \leq \pi/2$

$0 \leq r \leq a$

e) Le mieux est cylindrique



$x = P\cos(\psi)$

$y = P\sin(\psi)$

$z = z$

$\tan\alpha = \frac{P}{z}$

$P = C\tan(\alpha)$

on veut juste surface donc α varie pas

$x = z\tan(\alpha)\cos(\psi)$

$y = z\tan(\alpha)\sin(\psi)$

$z = z$

$\vec{r}(z,\psi) = (z\tan(\alpha)\cos(\psi), z\tan(\alpha)\sin(\psi), z)$ $0 \leq \psi \leq 2\pi$

$0 \leq z \leq c$

f) Meme chose que en haut mais on varie de 0 à α

$\vec{r}(z,\psi,\theta) = (z\tan(\theta)\cos(\psi), z\tan(\theta)\sin(\psi), z)$ $0 \leq \psi \leq 2\pi$

$0 \leq \theta \leq \alpha$

$0 \leq z \leq c$

5) $\vec{r}(u,v) = (u, u^2 - v, u + v^2)$

a) $\frac{\partial \vec{r}}{\partial u} = (1, 2u, 1)$ $\quad \frac{\partial r}{\partial u}\Big|_{P=(0,0)} = (1,0,1)$

$\frac{\partial \vec{r}}{\partial v} = (0, -1, 2v)$ $\quad \frac{\partial r}{\partial v}\Big|_{P=(0,0)} = (0,-1,0)$

b) $\vec{N} = \vec{r}_u \times \vec{r}_v$

$\vec{N} = (1,0,1) \times (0,-1,0) = \det \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix}$

$= 1\hat{e}_x - 1\hat{e}_y - 1\hat{e}_z$

il peut regarder selon $\vec{N} = (1, 0, -1)$

on veut unitaire donc on fait / norme $\quad \hat{n} = \left(\frac{1}{\sqrt{3}}, 0, -\frac{1}{\sqrt{3}}\right)$

c) $\frac{\partial r}{\partial u}\Big|_{P=(1,1)} = (1,2,1)$

$\vec{N} = (5,-2,1)$

$\frac{\partial r}{\partial v}\Big|_{P=(1,1)} = (0,-1,2)$

$\vec{r}(\lambda) = r_0 + \lambda\vec{N}$ $\quad r_0 = (1, 1^2-1, 1+1^2)$

$\vec{r}(\lambda) = (1,0,2) + \lambda(5,-2,1)$ $\quad r_0 = (1,0,2)$

$\vec{r}(\lambda) = (1+5\lambda, -2\lambda, 2-\lambda)$ $\longrightarrow$ en parametrique

$\vec{r}(\lambda) = (1,0,2) + \lambda(5,-2,1)$ $\longrightarrow$ en equation

e) $T(x,y,z) = 2(x-1)^2 + (y-1) + (z-1)$

∇ = Plus chaud $\quad -\nabla$ = plus froid

$T(x(u,v), y(u,v), z(u,v)) = 2(u-1)^2 + ((u^2-v) - 1) + ((u+v^2)-1)$

$\frac{\partial T}{\partial u} = 4(u-1) + 2u + 1 = 4u - 4 + 2v + 1$

$\frac{\partial T}{\partial v} = -1 + 2v$

Lorsque c'est égale a 0 c'est un minimum

Le 0 c'est un minimum, maximum donc un point au $\sqcap$ $\checkmark$

$6u - 3 = 0$ $\quad -1 + 2v = 0$

$u = 3/6 = 1/2$ $\quad v = 1/2$

$\vec{r}(1/2, 1/2) = (1/2, (1/2^2 - 1/2), (1/2 + 1/2^2))$

$= (1/2, -1/4, 3/4)$

$\vec{r}(1/2, 1/2) = (1/2, -1/4, 3/4)$

7) $r(u,v) = [u, v, f(u,v)]$

$N = \nabla g = [-f_u, -f_v, 1]$

$\nabla = \frac{\partial}{\partial}$

$\vec{r}(u,v) = [u, v, f(u,v)]$

$\vec{N} = [-\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, 1]$

Prueba

$x(u,v) = u$

$y(u,v) = v$

$z(u,v) = f(u,v)$

$r_u = [1, 0, \frac{\partial f}{\partial u}]$

$r_v = [0, 1, \frac{\partial f}{\partial v}]$

$\vec{N} = r_u \times r_v$

$$\vec{N} = \det \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 1 & 0 & \frac{\partial f}{\partial u} \\ 0 & 1 & \frac{\partial f}{\partial v} \end{vmatrix} = \left[-\frac{\partial f}{\partial u}, -\frac{\partial f}{\partial v}, 1\right]$$

$h(x,y) = \sin(x)\sin(y) + 1$

$z = \sin(x)\sin(y) + 1$

$\frac{\partial h}{\partial y} = \sin(x)\cos(y)$

$\frac{\partial h}{\partial x} = \cos(x)\sin(y)$

$$\vec{N} = \left[-\frac{\partial h}{\partial x}, -\frac{\partial h}{\partial y}, 1\right]\Big|_{(\pi/4, \pi/6)}$$

$$\vec{N} = [-\cos(x)\sin(y), -\sin(x)\cos(y), 1]$$

8) $F(x,y,z) = 0$

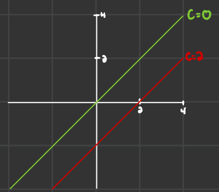
$\vec{r}(t) = (x(t), y(t), z(t))$

$\vec{F}(\vec{r}(t)) = 0$

$$\frac{\partial}{\partial t}\vec{F}(\vec{r}(t)) = \underbrace{\left(\frac{\partial \vec{F}}{\partial x}, \frac{\partial \vec{F}}{\partial y}, \frac{\partial \vec{F}}{\partial z}\right)}_{\vec{\nabla} F} \cdot \underbrace{\left(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}\right)}_{\frac{\partial \vec{r}}{\partial t}}$$

$$\vec{\nabla} F \cdot \frac{\partial \vec{r}}{\partial t} = 0$$

9)



$2x - y = 0$ à 0

$y = 2x$

$2x - y = 2$ à 2

$y = 2x - 2$

10) $f(x,y,z) =$ Constante $= K$

$f(x,y,z) = 9x^2 + 4y^2 + 25z^2$

$K = 9x^2 + 4y^2 + 25z^2$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

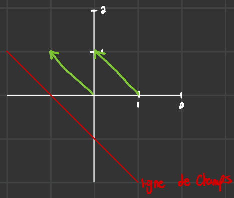
$$1 = \frac{9x^2}{K} + \frac{4y^2}{K} + \frac{25z^2}{K}$$

$$1 = \frac{x^2}{\left(\frac{K}{9}\right)} + \frac{y^2}{\left(\frac{K}{4}\right)^2} + \frac{z^2}{\left(\frac{K}{25}\right)^2}$$

$$1 = \frac{x^2}{\left(\frac{\sqrt{K}}{3}\right)^2} + \frac{y^2}{\left(\frac{\sqrt{K}}{2}\right)^2} + \frac{z^2}{\left(\frac{\sqrt{K}}{5}\right)^2}$$

11) $\vec{v} = -\hat{e}_x + \hat{e}_y$

a)



b) ligne de Champs

c) $\frac{\partial x}{\partial s} = V_x(x,y) = -1 \implies x = \int_0^{x} -1\, ds = -s + cte$

$\frac{\partial y}{\partial s} = V_y(x,y) = 1 \implies y = \int_0^{x} 1\, ds = s + cte$

$(x,y) = s(-1,1) + (x_0, y_0)$

$y = s + y_0$

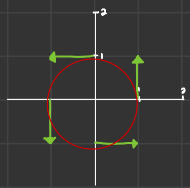
$x = -s + x_0$

$s = -x + x_0$

$y = -x + (x_0 + y_0)$

12) a) $\vec{v} = -y\hat{e}_x + x\hat{e}_y$

$\vec{v}(0,1) = (-1, 0)$
$\vec{v}(0,-1) = (1, 0)$
$\vec{v}(1,0) = (0, 1)$
$\vec{v}(-1,0) = (0,-1)$



b) ligne de Champs

c) $y = r\sin(\theta)$ $\qquad -r\sin(\theta)\left(\cos(\theta)\hat{e}_r - \sin(\theta)\hat{e}_\theta\right)$

$-y = -r\sin(\theta)$

$x = r\cos(\theta)$ $\qquad r\cos(\theta)\left(\sin(\theta)\hat{e}_r + \cos(\theta)\hat{e}_\theta\right)$

$-r\left(\cancel{\sin(\theta)\cos(\theta)\hat{e}_r} - \sin^2(\theta)\hat{e}_\theta\right) + r\left(\cancel{\cos(\theta)\sin(\theta)\hat{e}_r} + \cos^2(\theta)\hat{e}_\theta\right)$

$r\sin^2(\theta)\hat{e}_\theta + r\cos^2(\theta)\hat{e}_\theta + 0\hat{e}_r$

$r(1)\hat{e}_\theta + 0\hat{e}_r$

$0\hat{e}_r + r\hat{e}_\theta$

Procedural 3:

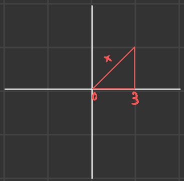
1) $\int_0^1 \int_0^2 \int_0^3 (1 + 2x - y + z)\, dx\, dy\, dz$

$\int_0^1 \int_0^2 x + x^2 - yx + zx \Big|_0^3 = 12 - 3y + 3z$

$\int_0^1 12y - \frac{3}{2}y^2 + 3yz \Big|_0^2 = 24 - 6 + 6z = 18 + 6z$

$18z + \frac{6}{2}z^2 \Big|_0^1 = 18 + 3 = 21$

2)



$\int_0^3 \int_0^x (x^2 + 2xy + 3)\, dy\, dx$

$\int_0^3 (x^2y + xy^2 + 3y)\Big|_0^x dx = x^3 + x^3 + 3x$

$\frac{x^4}{4} + \frac{x^4}{4} + \frac{3}{2}y^2 \Big|_0^3 = 20{,}25 + 20{,}25 + 13{,}5$

$= 54$

3) Carlesienne $= (x, y, z)$

$x^2 + y^2 = r^2$

$x = \sqrt{r^2 - y^2}$

$y = \sqrt{r^2 - x^2}$

$\int_0^2 \int_0^{\sqrt{r^2 - y^2}} (x + y)\, dx\, dy$

$\int_0^2 \left(\frac{x^2}{2} + yx\right)\Big|_0^{\sqrt{r^2 - y^2}} dy$

$\int_0^2 \frac{\left(\sqrt{r^2 - y^2}\right)^2}{2} + \left(\sqrt{r^2 - y^2}\right)y$

$\int_0^2 \frac{r^2 - y^2}{2} + \sqrt{r^2 - y^2}\, y$

$u = r^2 - y^2$ $\qquad \frac{du}{dy} = -2y$

$-\frac{1}{2} du = y\, dy$

Nouvelles bornes

$y = 0 \to a^2$

$y = a \to 0$

$K_1 = \int_{a^2}^0 \sqrt{u}\left(-\frac{1}{2} du\right)$

$K_1 = -\frac{1}{2} \frac{u^{3/2}}{3/2} \Big|_{a^2}^0$

$K_1 = -\frac{1}{2} \frac{2}{3} \; -a^{2 \cdot 3/2}$

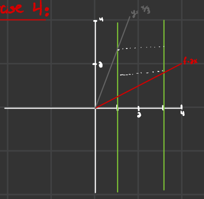
$K_1 = \frac{1}{3} a^3$

$K_2 = \frac{1}{2} \int_0^a a^2 - y^2\, dy$

$K_2 = \frac{1}{2}\left(a^2 y - \frac{y^3}{3}\right)\Big|_0^a$

$K_2 = \frac{1}{2}\left(a^3 - \frac{a^3}{3}\right) = \frac{a^3}{2} - \frac{a^3}{6} = a^3\left(\frac{1}{2} - \frac{1}{6}\right) = \frac{a^3}{3}$

Exercise 4:



$$\int_1^3 \int_{x/3}^{2x} 1 \, dy \, dx$$

$$\int_1^3 y \Big|_{x/3}^{2x}$$

$$\int_1^3 2x - x/3$$

$$x^2 - \frac{x^2}{6} \Big|_1^3$$

$$\frac{15}{2} - \frac{5}{6} = \frac{20}{3}$$

Exercise 5:

$$x^2 + y^2 + z^2 = 1$$

$$z = \sqrt{a^2 - x^2 - y^2}$$

$$y = \sqrt{a^2 - x^2}$$

$$x = a^2$$

$$V = 8 \cdot \int_0 \int_0^{\sqrt{1-x^2}} \sqrt{1 - x^2 - y^2} \, dz \, dy \, dx$$

Exercise 6:

$$\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a-x^2-y^2}} 1 \, dz \, dy \, dx$$

Exercise 7:

$$\iiint_V x \, dx \, dy \, dz$$

$$x^2 + y^2 + z^2 = a^2$$

$$x = \sqrt{a^2 - z^2 - y^2}$$

$$y = \sqrt{a^2 - z^2}$$

$$z = \sqrt{a^2}$$

$$\int_{-a}^{a} \int_{-\sqrt{a^2-z^2}}^{\sqrt{a^2-z^2}} \int_{-\sqrt{a^2-z^2-y^2}}^{\sqrt{a^2-z^2-y^2}} x \, dx \, dy \, dz$$

$$\int_{-a}^{a} \int_{-\sqrt{a^2-z^2}}^{\sqrt{a^2-z^2}} \frac{x^2}{2} \Big|_{-\sqrt{a^2-z^2-y^2}}^{\sqrt{a^2-z^2-y^2}}$$

$$\int_{-a}^{a} \int_{-\sqrt{a^2-z^2}}^{\sqrt{a^2-z^2}} \left(\frac{a^2 - z^2 - y^2}{2} - \frac{a^2 - z^2 - y^2}{2} \right) = 0$$

$$\frac{a^2 - a^2}{2} = 0$$

$$\frac{-z^2 - -z^2}{2} = 0$$

$$\frac{-y^2 - -y^2}{2} = 0$$

Exercise 8:

a) $I = \int_V r_\perp^2 \, dm = \int_V r^2 \rho_m \, dv$ $\rho_m = r \frac{\rho_{max}}{a}$

on change en sphérique

$$I = \int_V r_\perp^2 \frac{\rho_{max}}{a} dv$$

$\sin(\theta) = \frac{r_\perp}{r} \rightarrow r_\perp = r\sin(\theta)$

$$I = \int_V (r \cdot \sin(\theta))^2 \frac{r\rho_{max}}{a} r^2 \sin(\theta) \, dr \, d\theta \, d\varphi$$

$$I = \int_0^{2\pi} \int_0^{\pi} \int_0^{a} r^2 \cdot r \cdot r^2 \cdot \sin^2(\theta) \cdot \sin(\theta) \cdot \frac{\rho_{max}}{a} \, dr \, d\theta \, d\varphi$$

$$I = \int_0^{2\pi} \int_0^{\pi} \int_0^{a} r^5 \sin^3(\theta) \frac{\rho_{max}}{a} \, dr \, d\theta \, d\varphi$$

$$I = \int_0^{2\pi} \int_0^{\pi} \frac{a^6}{6} \sin^3(\theta) \frac{\rho_{max}}{a} \Big|_0^a$$

$$I = \int_0^{2\pi} \int_0^{\pi} \frac{(a)^6}{6} \sin^3(\theta) \frac{\rho_{max}}{a}$$

$$I = \int_0^{2\pi} \frac{(2\pi)^6}{6} \frac{\rho_{max}}{a}$$

$\sin^3(\theta)$

$u = \cos(\theta)$ $du = \sin(\theta)$

Procedural 4 :

Exercise 1 :

$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$ $\quad$ $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial z}$ $\quad$ $\frac{\partial f}{\partial y} = \frac{\partial f}{\partial z}$

ils sont déjà derivé donc on dit dérivé contraire

$y\cos(xy)\,dx + x\cos(xy)\,dy$

$\downarrow$ Par rapport à y $\quad$ $\downarrow$ Par rapport à x

$\cos(xy) + y\sin(xy)x \qquad \cos(xy) + x\sin(xy)y$

$\cos(xy) + xy\sin(xy) \qquad \cos(xy) + xy\sin(xy)$

2) $y^2e^{2x}\,dx + ye^{2x}\,dy$

$\downarrow dy \qquad \downarrow dx$

$2y\,e^{2x} \qquad ye^{2x}2$

$2ye^{2x} = 2ye^{2x}$

3) $xe^{-x^2-y^2}dx + ye^{-x^2-y^2}dy$

$\downarrow dy \qquad \downarrow dx$

$-2y\,xe^{-x^2-y^2} \qquad ye^{-x^2-y^2}\cdot -2x$

$-2xye^{-x^2-y^2} \qquad -2xye^{-x^2-y^2}$

7) $2xy\,dx + x^2dy + \sinh(z)\,dz$

$\frac{\partial^2}{\partial x\partial y} = \frac{\partial^2}{\partial y\partial x} \qquad 2x = 2x$

$\frac{\partial^2}{\partial x\partial z} = \frac{\partial^2}{\partial z\partial x} \qquad 0 = 0$

$\frac{\partial^2}{\partial y\partial z} = \frac{\partial^2}{\partial z\partial y} \qquad 0 = 0$

Exercise 2 :

$(3x^2e^{2y} + x)\,dx + (2x^3e^{2y})\,dy$

$\frac{\partial f}{\partial x\partial y} = \frac{\partial f}{\partial y\partial x}$

$3x^2e^{2y}\cdot 2 = 6x^2e^{2y}$

$6x^2e^{2y} = 6x^2e^{2y}$

14) $2x\sin(y)\,dx + x^2\cos(y)\,dy + y^2\,dz$

$\frac{\partial f}{\partial x\partial y} = \frac{\partial f}{\partial y\partial x} \qquad 2x\cos(y) = 2x\cos(y)$ $\qquad$ Ø Pareille

$\frac{\partial f}{\partial x\partial z} = \frac{\partial f}{\partial z\partial x} \qquad 0 = 0$

$\frac{\partial f}{\partial y\partial z} = \frac{\partial f}{\partial z\partial y} \qquad 0 \neq 2y$

Exercise 3:

$F = [x-z,\ y-x,\ z-y]$

$\vec{r} = [u\cos(v),\ u\sin(v),\ u]$ $\qquad 0 \le u \le 3$, $\ 0 \le v \le \pi$

$\vec{F}(\vec{r}) = [u\cos(v) - u,\ u\sin(v) - u\cos(v),\ u - u\sin(v)]$

$\frac{\partial r}{\partial u} = [\cos(v),\ \sin(v),\ 1]$

$\frac{\partial r}{\partial v} = [-u\sin(v),\ u\cos(v),\ 0]$

$$\vec{N} = \vec{r}_u \times \vec{r}_v = \det \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \cos(v) & \sin(v) & 1 \\ -u\sin(v) & u\cos(v) & 0 \end{vmatrix} = -u\cos(v)\hat{e}_x - u\sin(v)\hat{e}_y + \underbrace{u\cos^2(v) + u\sin^2(v)}_{\underbrace{u(\cos^2(v)+\sin^2(v))}_{u}}\hat{e}_z$$

$= u(\cos(v) - \sin^2(v) + \cos^2(v) + \sin^2(v))$

$\vec{N} = [-u\cos(v),\ -u\sin(v),\ u]$

$$\Phi = \int_S \vec{F}\cdot\vec{N}\, du\, dv = \int_0^\pi \int_0^3 [-u\cos(v); u\sin(v), u]\cdot[u\cos(v) - u,\ u\sin(v) - u\cos(v),\ u - u\sin(v)]\, du\, dv$$

$$= \int_0^\pi \int_0^3 -u^2\cos^2(v) + u^2\cos(v) - u\sin^2(v) + u^2\sin(v)\cos(v) + u^2 - u^2\sin(v)\, du\, dv$$

$$= \int_0^\pi \int_0^3 u^2(-\cos^2(v) + \cos(v) - \sin^2(v) + \sin(v)\cos(v) - \sin(v) + 1)\, du\, dv$$

$$= \int_0^\pi \frac{u^3}{3}(\quad + \cos(v) \quad + \sin(v)\cos(v) - \sin(v) \quad)\Big|_0^3 dv$$

$$= \int_0^\pi \frac{u^3}{3}(\cos(v) - \sin(v)\cos(v) - \sin(v))\Big|_0^3 dv$$

$$= 9 \cdot \left(\sin(v) + \frac{\sin^2(v)}{2} + \cos(v)\right)\Big|_0^\pi$$

$= 9\cdot(0 - 0 - 1 - 1)$

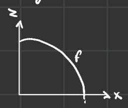
$= 9\cdot(-2)$

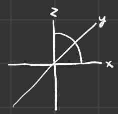
$= -18$

Exercise 4:

$\vec{F} = [0, x, 0]$ Coordonnée sphérique

$x^2 + y^2 + z^2 = a^2$





$\vec{r} = [r\cdot\cos(\theta), 0, r\sin(\theta)]$

$\vec{N} = \frac{\partial r}{\partial \theta} \times \frac{\partial r}{\partial}$

$\frac{\partial \vec{r}}{\partial \theta} = [-r\sin(\theta), 0, r\cos(\theta)]$

$\frac{\partial \vec{r}}{\partial r} = [\cos(\theta), 0, \sin(\theta)]$

$$\vec{N} = \det \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ -r\sin(\theta) & 0 & r\cos(\theta) \\ \cos(\theta) & 0 & \sin(\theta) \end{vmatrix} = 0\,\hat{e}_x - (r\sin^2(\theta) + r\cos^2(\theta))\hat{e}_y + 0\,\hat{e}_z$$

$$= -r(\sin^2(\theta) + \cos^2(\theta))\,\hat{e}_y$$

$$= -r\,\hat{e}_y$$

$$= \int_0^a \int_0^{\pi/2} [0, r\cdot\cos(\theta), 0]\cdot[0, -r, 0]\, dr\, d\theta$$

$$= -\int_0^a r^2 \cdot \int_0^{\pi/2} \cos(\theta)$$

$$= -\frac{r^3}{3}\Big|_0^a \cdot -\sin(\theta)\Big|_0^{\pi/2}$$

$$= \frac{a^3}{3} - 0$$

Exercise 11:

$\vec{F} = [y^3, x^3]$

$y = 5x^2$

$A = (0.0)$
$B = (2.20)$

$x = t$
$y = 5t^2$ $\qquad \vec{r} = [t, 5t^2]$

$0 \leq t \leq 2$

$\vec{F} = [(5t^2)^3, t^3] = [125t^6, t^3]$

$d\vec{r} = [1, 10t]$

$$\int_C F(r) \cdot dr = \int_0^2 [125t^6, t^3] \cdot [1, 10t]\, dt$$

$$= \int_0^2 125t^6 + 10t^4$$

$$= \frac{125}{7}t^7 + \frac{10}{5}t^5 \Big|_0^2$$

$$= 2349,71$$

Exercise 11:

$F = [e^x, e^y, e^z]$

$r = [t, t^2, t^2]$

$\vec{F}_{(r)} = (e^t, e^{t^2}, e^{t^2}) \qquad dr = [1, 2t, 2t]$

$$\int_0^2 (e^t, e^{t^2}, e^{t^2}) \cdot [1, 2t, 2t]$$

$$\int_0^2 e^t + 2te^{t^2} + 2te^{t^2} \quad dt$$

$$e^t + e^{t^2} + e^{t^2} \Big|_0^2$$

$$e^t + 2e^{t^2} \Big|_0^2$$

$116,58 - 3$ ou $e^2 + 2e^4 - 3$