Mandat 1 LHI

$\vec{D} = \varepsilon\vec{E}$ $\quad \vec{B} = \mu\vec{H}$ $\quad \nabla\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = \vec{0}$ $\quad \nabla\times\vec{H} - \frac{\partial D}{\partial t} = \vec{J}_f$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times(\nabla\times\vec{E}) = -\frac{\partial}{\partial t}\nabla\times\vec{B}$$

$$\nabla\times\frac{\vec{B}}{\mu} - \frac{\partial\varepsilon\vec{E}}{\partial t} = \vec{J}_f$$

$$\nabla\times\frac{\vec{B}}{\mu} = \vec{J}_f + \frac{\partial}{\partial t}\varepsilon\vec{E}$$

$$\frac{1}{\mu}(\nabla\times\vec{B}) = \vec{J}_f + \frac{\partial}{\partial t}\varepsilon\vec{E}$$

$$\nabla\times\vec{B} = \mu\left(\vec{J}_f + \frac{\partial\varepsilon\vec{E}}{\partial t}\right)$$

$$\nabla\times(\nabla\times\vec{E}) = -\frac{\partial}{\partial t}\left(\mu\left(\vec{J}_f + \frac{\partial\varepsilon\vec{E}}{\partial t}\right)\right)$$

$$\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\mu\left(\frac{\vec{J}_f}{\partial t} + \frac{\partial\varepsilon\vec{E}}{\partial t^2}\right)$$

$$\nabla\left(\frac{\rho_f}{\varepsilon}\right) - \nabla^2\vec{E} = -\mu\frac{\vec{J}_f}{\partial t} - \mu\frac{\partial\varepsilon\vec{E}}{\partial t^2}$$

$$-\nabla^2\vec{E} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = -\mu\frac{\vec{J}_f}{\partial t} - \nabla\left(\frac{\rho_f}{\varepsilon}\right)$$

Pour $\vec{E}$

$$\boxed{\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = \mu\frac{\vec{J}_f}{\partial t} + \nabla\left(\frac{\rho_f}{\varepsilon}\right)}$$

$$\nabla\times\vec{H} - \frac{\partial D}{\partial t} = \vec{J}_f$$

$$\nabla\times(\nabla\times\vec{H}) = \frac{\partial}{\partial t}D\times\nabla + \vec{J}_f\times\nabla$$

$$\nabla\times E = -\frac{\partial B}{\partial t}$$

$$\nabla\times\frac{D}{\varepsilon} = -\frac{\partial B}{\partial t}$$

$$\nabla\times D = -\varepsilon\frac{\partial B}{\partial t}$$

$$\nabla\times(\nabla\times\vec{H}) = \frac{\partial}{\partial t}\left(-\varepsilon\frac{\partial B}{\partial t}\right) + \vec{J}_f\times\nabla$$

$$\frac{1}{\mu}(\nabla\times(\nabla\times\vec{B})) = -\varepsilon\frac{\partial\vec{B}}{\partial t^2} + \vec{J}_f\times\nabla$$

$$\frac{1}{\mu}\left(\nabla(\nabla\cdot\vec{B}) - \nabla^2\vec{B}\right) = -\varepsilon\frac{\partial\vec{B}}{\partial t^2} + \vec{J}_f\times\nabla$$

$$-\frac{1}{\mu}\nabla^2\vec{B} = -\varepsilon\frac{\partial\vec{B}}{\partial t^2} + \vec{J}_f\times\nabla$$

$$-\frac{1}{\mu}\nabla^2\vec{B} + \varepsilon\frac{\partial^2\vec{B}}{\partial t^2} = \vec{J}_f\times\nabla$$

Pour $\vec{B}$

$$\boxed{\nabla^2\vec{B} - \varepsilon\mu\frac{\partial^2\vec{B}}{\partial t^2} = -\mu(\vec{J}_f\times\nabla)}$$

$\boxed{\text{Mandat 1}}$ ⊗ LHI

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \qquad \vec{B} = \mu_0(\vec{H} + \vec{M}) \qquad \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = \vec{0} \qquad \nabla \times \vec{H} - \frac{\partial D}{\partial t} = \vec{J}_f$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t} \nabla \times \vec{B}$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) - \frac{\partial}{\partial t}(\varepsilon_0 \vec{E} + \vec{P}) = \vec{J}_f$$

$$\frac{1}{\mu_0}(\nabla \times \vec{B}) - (\nabla \times \vec{M}) = \vec{J}_f + \frac{\partial}{\partial t}(\varepsilon_0 \vec{E} + \vec{P})$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}_f + \frac{\mu_0 \partial}{\partial t}(\varepsilon_0 \vec{E} + \vec{P}) + \mu_0 (\nabla \times \vec{M})$$

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}\left(\mu_0 \vec{J}_f + \frac{\mu_0 \partial}{\partial t}(\varepsilon_0 \vec{E} + \vec{P}) + \mu_0 (\nabla \times \vec{M})\right)$$

$$\vec{E} = \frac{\vec{D} - \vec{P}}{\varepsilon_0} \Rightarrow \nabla \cdot \vec{E} = \frac{\nabla \cdot \vec{D}}{\varepsilon_0} - \frac{\nabla \cdot \vec{P}}{\varepsilon_0}$$

$$\nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t} J_f - \frac{\partial^2}{\partial t^2}\mu_0 (\varepsilon_0 \vec{E} + \vec{P}) - \mu_0 \frac{d}{dt}(\nabla \times \vec{M})$$

$$\nabla \cdot \vec{D} = \rho_f$$

Loi de Gauss

$$\nabla\left(\frac{\rho_f}{\varepsilon_0} - \frac{\Delta \cdot \vec{P}}{\varepsilon_0}\right) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t}\vec{J}_f - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\vec{E} - \mu_0 \frac{\partial^2}{\partial t^2}\vec{P} - \mu_0 \frac{d}{dt}(\nabla \times \vec{M})$$

$$\nabla\left(\frac{\rho_f}{\varepsilon_0}\right) - \frac{1}{\varepsilon_0}\nabla(\nabla \cdot \vec{P}) + \mu_0 \frac{\partial}{\partial t}\vec{J}_f + \mu_0 \frac{\partial^2}{\partial t^2}\vec{P} + \mu_0 \frac{d}{dt}(\nabla \times \vec{M}) = \nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\vec{E}$$

$$\boxed{\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\vec{E} = \frac{1}{\varepsilon_0}\nabla \rho_f - \frac{1}{\varepsilon_0}\nabla(\nabla \cdot \vec{P}) + \mu_0 \frac{\partial}{\partial t}\vec{J}_f + \mu_0 \frac{\partial^2}{\partial t^2}\vec{P} + \mu_0 \frac{\partial}{\partial t}(\nabla \times \vec{M})}$$

pour $\vec{E}$

Mandat 1 ⊗ LHI

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \qquad \vec{B} = \mu_0(\vec{H} + \vec{M}) \qquad \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = \vec{0} \qquad \nabla \times \vec{H} - \frac{\partial D}{\partial t} = \vec{J}_f$$

$$\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J}_f$$

$$\nabla \times (\nabla \times \vec{H}) = \nabla \times \vec{J}_f + \frac{\partial}{\partial t} \nabla \times \vec{D}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \left(\frac{\vec{D} - \vec{P}}{\varepsilon_0}\right) = -\frac{\partial \vec{B}}{\partial t}$$

$$\frac{1}{\varepsilon_0}(\nabla \times \vec{D}) - \frac{1}{\varepsilon_0}(\nabla \times \vec{P}) = -\frac{\partial}{\partial t}\vec{B}$$

$$\nabla \times \vec{D} = -\varepsilon_0 \frac{\partial}{\partial t}\vec{B} + \nabla \times \vec{P}$$

$$\nabla \times (\nabla \times \vec{H}) = \nabla \times \vec{J}_f + \frac{\partial}{\partial t}\left(-\varepsilon_0 \frac{\partial}{\partial t}\vec{B} + \nabla \times P\right)$$

$$\nabla \times \left(\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)\right) = \nabla \times \vec{J}_f - \varepsilon_0 \frac{\partial^2}{\partial t^2}\vec{B} + \frac{\partial}{\partial t}(\nabla \times P)$$

$$\nabla\left(\nabla \cdot \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)\right) - \nabla^2\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = \nabla \times \vec{J}_f - \varepsilon_0 \frac{\partial^2}{\partial t^2}\vec{B} + \frac{\partial}{\partial t}(\nabla \times P) \qquad \nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

$$\frac{1}{\mu_0}\nabla(\nabla \cdot \vec{B}) - \nabla(\nabla \cdot \vec{M}) - \frac{1}{\mu_0}(\nabla^2 \vec{B}) + \nabla^2 \vec{M} = \nabla \times \vec{J}_f - \varepsilon_0 \frac{\partial^2}{\partial t^2}\vec{B} + \frac{\partial}{\partial t}(\nabla \times P)$$

$\nabla \cdot \vec{B} = 0$

$$-\nabla(\nabla \cdot \vec{M}) - \frac{1}{\mu_0}(\nabla^2 \vec{B}) + \nabla^2 \vec{M} = \nabla \times \vec{J}_f - \varepsilon_0 \frac{\partial^2}{\partial t^2}\vec{B} + \frac{\partial}{\partial t}(\nabla \times P)$$

$$\frac{1}{\mu_0}(\nabla^2 \vec{B}) + \varepsilon_0 \frac{\partial^2}{\partial t^2}\vec{B} = -\nabla(\nabla \cdot \vec{M}) + \nabla^2 \vec{M} - \nabla \times \vec{J}_f - \frac{\partial}{\partial t}(\nabla \times P)$$

$$\nabla^2 \vec{B} - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2}\vec{B} = -\mu_0 \nabla(\nabla \cdot \vec{M}) + \mu_0 \nabla^2 \vec{M} - \mu_0(\nabla \times \vec{J}_f) - \mu_0 \frac{\partial}{\partial t}(\nabla \times \vec{P})$$

$$\boxed{\nabla^2 \vec{B} - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2}\vec{B} = -\mu_0 \nabla(\nabla \cdot \vec{M}) + \mu_0 \nabla^2 \vec{M} - \mu_0(\nabla \times \vec{J}_f) - \mu_0 \frac{\partial}{\partial t}(\nabla \times \vec{P})}$$

Pour $\vec{B}$

Mandat 2

$\vec{B} = \nabla \times \vec{A}$ $\quad \nabla \cdot \vec{B} = 0$ $\quad \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = \vec{0}$ $\quad \vec{E} = -\nabla \psi - \frac{\partial \vec{A}}{\partial t}$

---

a) $\nabla \cdot \vec{B} = 0$ Loi de Gauss pour le magnetisme

$\nabla \cdot (\nabla \times \vec{A}) = 0$ Identité vectoriel

$$\frac{\nabla \cdot (\nabla \times \vec{A})}{\nabla} = \frac{\nabla \cdot \vec{B}}{\nabla}$$

Il est possible de voir que si on divise par ∇ de chaque côté la formule restante est $\nabla \times \vec{A} = \vec{B}$ ce qui est l'équation rechercher

$$\nabla \times \vec{A} = \vec{B}$$

Donc on peut prouver que $\boxed{\vec{B} = \nabla \times \vec{A}}$

b) $\vec{0} = \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t}$ loi de Gauss Magnetisme

$\vec{0} = \nabla \times \vec{E} + \frac{\partial}{\partial t}(\nabla \times \vec{A})$ grâce à la formule trouvé plus haut

$\vec{0} = \nabla \times \vec{E} + \frac{\partial}{\partial t}(\nabla \times \vec{A}) + \vec{\nabla} \times (\vec{\nabla} \psi)$ on peut ajouter $\vec{\nabla} \times (\vec{\nabla} \psi)$, car cela $= 0$

$\vec{0} = \vec{E} + \frac{\partial \vec{A}}{\partial t} + \vec{\nabla} \psi$ Car ils étaient tout $\nabla \times$ donc il est possible de simplifier

$$\boxed{\vec{E} = -\vec{\nabla} \psi - \frac{\partial \vec{A}}{\partial t}}$$

$\boxed{\text{Mandat 3}}$

$$\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J}_f \qquad \vec{B} = \mu_0(\vec{H} + \vec{M}) \qquad \nabla \cdot \vec{A} + \varepsilon_0 \mu_0 \frac{\partial \psi}{\partial t} = 0 \qquad \vec{E} = -\nabla\psi - \frac{\partial \vec{A}}{\partial t}$$

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a) $\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J}_f$

$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$
$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$
$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\nabla \times \left(\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)\right) = \nabla \times J_f + \frac{\partial}{\partial t} \nabla \times (\varepsilon_0 \vec{E} + \vec{P})$$

$$\nabla \times \left(\left(\nabla \times \frac{\vec{B}}{\mu_0}\right) - (\nabla \times \vec{M})\right) = \nabla \times \vec{J}_f + \varepsilon_0 \frac{\partial}{\partial t} \nabla \times \vec{E} + \frac{\partial}{\partial t} \nabla \times \vec{P}$$

$\vec{B} = \nabla \times \vec{A}$

$$\nabla \times \left(\frac{1}{\mu_0}(\nabla \times (\nabla \times \vec{A})) - (\nabla \times \vec{M})\right) = \nabla \times \vec{J}_f + \varepsilon_0 \frac{\partial}{\partial t} \nabla \times \left(-\nabla\psi - \frac{\partial \vec{A}}{\partial t}\right) + \frac{\partial}{\partial t} \nabla \times \vec{P}$$

On peut simplifier le ∇

$$\frac{1}{\mu_0}(\nabla \times (\nabla \times \vec{A})) - (\nabla \times \vec{M}) = \vec{J}_f + \varepsilon_0 \frac{\partial}{\partial t}(-\nabla\psi) - \varepsilon_0 \frac{\partial^2}{\partial t^2} \vec{A} + \frac{\partial}{\partial t} \vec{P}$$

$$\frac{1}{\mu_0}(\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}) - (\nabla \times \vec{M}) = \vec{J}_f - \varepsilon_0 \frac{\partial}{\partial t} \nabla\psi - \varepsilon_0 \frac{\partial^2}{\partial t^2} \vec{A} + \frac{\partial}{\partial t} \vec{P}$$

$$(\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}) - \mu_0(\nabla \times \vec{M}) = \mu_0 \vec{J}_f - \varepsilon_0 \mu_0 \frac{\partial}{\partial t} \nabla\psi - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \vec{A} + \mu_0 \frac{\partial}{\partial t} \vec{P}$$

$$\nabla(\nabla \cdot \vec{A}) + \varepsilon_0 \mu_0 \frac{\partial}{\partial t} \nabla\psi = \mu_0 \vec{J}_f - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \vec{A} + \mu_0 \frac{\partial}{\partial t} \vec{P} + \mu_0(\nabla \times \vec{M}) + \nabla^2 \vec{A}$$

$$\nabla\left((\nabla \cdot \vec{A}) + \varepsilon_0 \mu_0 \frac{\partial}{\partial t} \psi\right) = \mu_0 \vec{J}_f - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \vec{A} + \mu_0 \frac{\partial}{\partial t} \vec{P} + \mu_0(\nabla \times \vec{M}) + \nabla^2 \vec{A}$$

Lorenz $= 0$

$$0 = \mu_0 \vec{J}_f - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \vec{A} + \mu_0 \frac{\partial}{\partial t} \vec{P} + \mu_0(\nabla \times \vec{M}) + \nabla^2 \vec{A}$$

$$\boxed{\nabla^2 \vec{A} - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \vec{A} = -\mu_0 \vec{J}_f - \mu_0 \frac{\partial}{\partial t} \vec{P} - \mu_0(\nabla \times \vec{M})}$$

Pour $\vec{A}$

$\boxed{\text{Mandat 3}}$

$\nabla\cdot\vec{D} = \rho_f$ $\quad\quad$ $\vec{B} = \mu_0(\vec{H}+\vec{M})$ $\quad\quad$ $\nabla\cdot\vec{A} + \varepsilon_0\mu_0\frac{\partial\psi}{\partial t} = 0$ $\quad\quad$ $\vec{E} = -\nabla\psi - \frac{\partial\vec{A}}{\partial t}$

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b) $\nabla\cdot\vec{D} = \rho_f$

$$\nabla\cdot(\varepsilon_0\vec{E}+\vec{P}) = \rho_f$$

$$\varepsilon_0(\nabla\cdot\vec{E}) + (\nabla\cdot\vec{P}) = \rho_f$$

$$\varepsilon_0\left(\nabla\cdot\left(-\nabla\psi - \frac{\partial\vec{A}}{\partial t}\right)\right) + \nabla\cdot\vec{P} = \rho_f$$

$$-\varepsilon_0\left(\nabla\cdot\nabla\psi + \frac{\partial}{\partial t}\nabla\cdot\vec{A}\right) + \nabla\cdot\vec{P} = \rho_f$$

$$\nabla\cdot\vec{A} + \varepsilon_0\mu_0\frac{\partial\psi}{\partial t} = 0$$

$$\nabla\cdot\vec{A} = -\varepsilon_0\mu_0\frac{\partial\psi}{\partial t}$$

$$\frac{\partial}{\partial t}\nabla\cdot\vec{A} = -\varepsilon_0\mu_0\frac{\partial^2\psi}{\partial t^2}$$

$$-\varepsilon_0\left(\nabla^2\psi - \varepsilon_0\mu_0\frac{\partial^2\psi}{\partial t}\right) + \nabla\cdot\vec{P} = \rho_f$$ grace à la condition de Lorenz et $\nabla\cdot\nabla f = \nabla^2 f$

$$-\varepsilon_0\nabla^2\psi + \varepsilon_0^2\mu_0\frac{\partial^2\psi}{\partial t^2} + \nabla\cdot\vec{P} = \rho_f$$

$$\varepsilon_0\nabla^2\psi - \varepsilon_0^2\mu_0\frac{\partial^2\psi}{\partial t^2} = \nabla\cdot\vec{P} - \rho_f$$

$$\boxed{\nabla^2\psi - \varepsilon_0\mu_0\frac{\partial^2\psi}{\partial t^2} = \frac{\nabla\cdot\vec{P}}{\varepsilon_0} - \frac{\rho_f}{\varepsilon_0}}$$

Pour ψ

$\boxed{\text{Mandat 3}}$

$$\nabla^2\psi - \varepsilon_0 u_0 \frac{\partial^2\psi}{\partial t^2} = \frac{\nabla\cdot\vec{P}}{\varepsilon_0} - \frac{\rho_f}{\varepsilon_0} \qquad \nabla^2\vec{A} - \varepsilon_0 u_0 \frac{\partial^2}{\partial t^2}\vec{A} = -u_0\vec{J}_f - u_0\frac{\partial}{\partial t}\vec{P} - u_0(\nabla\times\vec{M})$$

$$\underbrace{\frac{\partial \rho_f}{\partial t}} + \underbrace{\nabla\cdot\vec{J}_f} = 0$$

$$\nabla^2\psi - \varepsilon_0 u_0 \frac{\partial^2\psi}{\partial t^2} = \frac{\nabla\cdot\vec{P}}{\varepsilon_0} - \frac{\rho_f}{\varepsilon_0}$$

$$\rho_f = -\varepsilon_0\nabla^2\psi + \varepsilon_0^2 u_0 \frac{\partial^2\psi}{\partial t^2} + \nabla\cdot\vec{P}$$

$$\boxed{\frac{\partial \rho_f}{\partial t} = -\varepsilon_0\frac{\partial}{\partial t}\nabla^2\psi + \varepsilon_0^2 u_0 \frac{\partial^3\psi}{\partial t^3} + \frac{\partial}{\partial t}\nabla\cdot\vec{P}}$$

$$\nabla^2\vec{A} - \varepsilon_0 u_0 \frac{\partial^2}{\partial t^2}\vec{A} = -u_0\vec{J}_f - u_0\frac{\partial}{\partial t}\vec{P} - u_0(\nabla\times\vec{M})$$

$$\nabla^2\vec{A} - \varepsilon_0 u_0 \frac{\partial^2}{\partial t^2}\vec{A} + u_0\frac{\partial}{\partial t}\vec{P} + u_0(\nabla\times\vec{M}) = -u_0\vec{J}_f$$

$$\frac{\nabla^2\vec{A}}{-u_0} + \varepsilon_0\frac{\partial^2}{\partial t^2}\vec{A} - \frac{\partial}{\partial t}\vec{P} - (\nabla\times\vec{M}) = \vec{J}_f$$

$$\boxed{\nabla\cdot\frac{\nabla^2\vec{A}}{-u_0} + \nabla\cdot\varepsilon_0\frac{\partial^2}{\partial t^2}\vec{A} - \nabla\cdot\frac{\partial}{\partial t}\vec{P} - \nabla\cdot(\nabla\times\vec{M}) = \nabla\cdot J_f}$$

$$-\varepsilon_0\frac{\partial}{\partial t}\nabla^2\psi + \varepsilon_0^2 u_0 \frac{\partial^3\psi}{\partial t^3} + \frac{\partial}{\partial t}\nabla\cdot\vec{P} + \nabla\cdot\frac{\nabla^2\vec{A}}{-u_0} + \nabla\cdot\varepsilon_0\frac{\partial^2}{\partial t^2}\vec{A} - \nabla\cdot\frac{\partial}{\partial t}\vec{P} - \nabla\cdot(\nabla\times\vec{M}) = 0$$

$$\varepsilon_0^2 u_0 \frac{\partial^3\psi}{\partial t^3} + \nabla\cdot\varepsilon_0\frac{\partial^2}{\partial t^2}\vec{A} - \varepsilon_0\frac{\partial}{\partial t}\nabla^2\psi + \frac{\partial}{\partial t}\nabla\cdot\vec{P} + \nabla\cdot\frac{\nabla^2\vec{A}}{-u_0} - \nabla\cdot\frac{\partial}{\partial t}\vec{P} - \nabla\cdot(\nabla\times\vec{M}) = 0$$

$$\frac{\partial^2}{\partial t^2}\varepsilon_0\underbrace{\left(\nabla\cdot\vec{A} + \varepsilon_0 u_0\frac{\partial\psi}{\partial t}\right)}_{0} - \varepsilon_0\frac{\partial}{\partial t}\nabla^2\psi + \frac{\partial}{\partial t}\nabla\cdot\vec{P} + \nabla\cdot\frac{\nabla^2\vec{A}}{-u_0} - \nabla\cdot\frac{\partial}{\partial t}\vec{P} - \nabla\cdot(\nabla\times\vec{M}) = 0$$

$$-\varepsilon_0\frac{\partial}{\partial t}\nabla^2\psi + \nabla\cdot\frac{\nabla^2\vec{A}}{-u_0} \underbrace{- \nabla\frac{\partial}{\partial t}\vec{P} + \frac{\partial}{\partial t}\nabla\cdot\vec{P}}_{0} - \nabla\cdot(\nabla\times\vec{M}) = 0$$

$$\underbrace{-\frac{\nabla^2}{u_0}\left(\varepsilon_0 u_0\frac{\partial}{\partial t}\psi + \nabla\cdot\vec{A}\right)}_{0} - \nabla\cdot(\nabla\times\vec{M}) = 0$$

$$\underbrace{-\nabla\cdot(\nabla\times\vec{M})}_{0} = 0$$

$$\boxed{0 = 0}$$

Mandat 4

$$\vec{A}' = \vec{A} + \nabla \chi \qquad \varphi' = \varphi - \frac{\partial \chi}{\partial t} \qquad \vec{B} = \nabla \times \vec{A} \qquad \vec{E} = -\nabla \varphi - \frac{\partial \vec{A}}{\partial t}$$

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1) Prouvons que $\vec{B} = \nabla \times \vec{A} = \nabla \times \vec{A}'$

$$\vec{B} = \nabla \times \vec{A}' \qquad \vec{A}' = \vec{A} + \nabla \chi$$

$$\vec{B} = \nabla \times (\vec{A} + \nabla \chi)$$

$$\vec{B} = \nabla \times \vec{A} + \underbrace{\nabla \times (\nabla \chi)}_{0} \qquad \vec{\nabla} \times (\vec{\nabla} f) = 0$$

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

Donc on voit que $\vec{B} = \nabla \times \vec{A}$

2) Prouvons que $\vec{E} = -\nabla \psi - \frac{\partial \vec{A}}{\partial t} = -\nabla \psi' - \frac{\partial \vec{A}'}{\partial t}$

$$\vec{E} = -\nabla \psi' - \frac{\partial \vec{A}'}{\partial t} \qquad \psi' = \psi - \frac{\partial \chi}{\partial t} \qquad \vec{A}' = \vec{A} + \nabla \chi$$

$$\vec{E} = -\nabla\left(\psi - \frac{\partial \chi}{\partial t}\right) - \frac{\partial}{\partial t}\left(\vec{A} + \nabla \chi\right)$$

$$\vec{E} = -\nabla \psi + \nabla \frac{\partial \chi}{\partial t} - \frac{\partial \vec{A}}{\partial t} - \nabla \frac{\partial \chi}{\partial t}$$

$$\vec{E} = -\nabla \psi - \frac{\partial \vec{A}}{\partial t} + \underbrace{\nabla \frac{\partial \chi}{\partial t} - \nabla \frac{\partial \chi}{\partial t}}_{0}$$

$$\boxed{\vec{E} = -\nabla \psi - \frac{\partial \vec{A}}{\partial t}}$$

Mandat 4

$$\vec{A}' = \vec{A} + \nabla \chi \qquad \varphi' = \varphi - \frac{\partial \chi}{\partial t}$$

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3) $\nabla \cdot \vec{A}' + \varepsilon_0 \mu_0 \frac{\partial \varphi'}{\partial t} = 0$

$$\nabla \cdot (\vec{A} + \nabla \chi) + \varepsilon_0 \mu_0 \frac{\partial}{\partial t}\left(\varphi - \frac{\partial \chi}{\partial t}\right) = 0$$

$$\nabla \cdot \vec{A} + \nabla \cdot (\nabla \chi) + \varepsilon_0 \mu_0 \frac{\partial}{\partial t} \varphi - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \chi = 0$$

$$\nabla^2 \chi - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \chi = \underbrace{- \nabla \cdot \vec{A} - \varepsilon_0 \mu_0 \frac{\partial}{\partial t} \varphi}_{0}$$

$\nabla \cdot \vec{A} + \varepsilon_0 \mu_0 \frac{\partial \varphi}{\partial t} = 0$

$$\boxed{\nabla^2 \chi - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \chi = 0}$$

pour χ