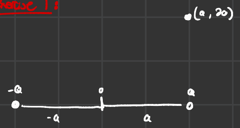
Procedure 1

Exercise 1 :



$$\vec{E}_{Q} = \frac{Q}{4\pi\varepsilon_0 \|r-r'\|^2} \frac{(r-r')}{\|\vec{r}-\vec{r}'\|}$$

$$E_{Q_+} = \frac{Q}{4\pi\varepsilon_0 4a^2} \frac{(0, 2a)}{\sqrt{(2a)^2+0}}$$

$$\vec{E}_{Q_-} = \frac{-Q}{4\pi\varepsilon_0 \left(\sqrt{2a^2+2a^2}\right)^2} \frac{(2a, 2a)}{\sqrt{2a^2+2a^2}}$$

$$E_{Q_+} = \frac{Q}{16\pi\varepsilon_0 a^2} (0, 1)$$

$$E_{Q_-} = \frac{-Q}{32\pi\varepsilon_0 a^2} \frac{(2a, 2a)}{\sqrt{2}\, 2a}$$

$$E_{Q} = \frac{-Q}{32\pi\varepsilon_0 a^2 \sqrt{2}} (1, 1)$$

$$E_{tot} = \frac{Q}{16\pi\varepsilon_0 a^2} \left(\frac{-1}{2\sqrt{2}}, \frac{-1}{2\sqrt{2}} + 1 \right)$$

$$E_{tot} = \frac{Q}{16\pi\varepsilon_0 a^2} \left(\frac{-1}{2\sqrt{2}}, \frac{2\sqrt{2}-1}{2\sqrt{2}} \right)$$

Exercise 2 :

c) $$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_V \rho_e \frac{(\vec{r}-\vec{r}')}{\|\vec{r}-\vec{r}'\|^3} dv'$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_0^{\pi} \int_0^{2\pi} \int_0^{a} K \frac{r^2}{a^2} \sin(\theta)\; r^2 \sin(\theta)\, dr\, d\theta\, d\varphi$$

$$\vec{E} = \frac{K}{4\pi\varepsilon_0 a^2} \int_0^{\pi} \int_0^{2\pi} \int_0^{a} r^4 \sin^2(\theta)\, dr\, d\theta\, d\varphi$$

$$E = \frac{K a^5}{4\pi\varepsilon_0 a^2 5} \int_0^{\pi} \left(\frac{1}{2}\theta - \frac{\sin(2\theta)}{4} \right) \Big|_0^{2\pi} d\varphi$$

$$E = \frac{K a^3}{20\pi\varepsilon_0} \pi^2$$

$$\vec{E} = \frac{K a^3 \pi}{20\varepsilon_0} \cdot 4\pi\varepsilon_0 = \frac{K a^3 \pi^2}{5} = Q$$

$$E = \frac{Q}{4\pi\varepsilon_0}$$

donc si on veut Q

C'est $E \cdot 4\pi\varepsilon_0$

Exercise 3 :

a) $\vec{r} = (0, 0, z')$

$\vec{r}' = (a\cos(\theta), a\sin(\theta), 0)$

$\vec{r} - \vec{r}' = (-a\cos(\theta), -a\sin(\theta), z')$

$Q = \int \lambda \, dl$

$Q = \int_0^{\pi} \lambda a \, d\theta$

$Q = \lambda a 2\pi$

Cable Coaxiale :

On cherche $\frac{Q}{l}$

aire interne = $2\pi\rho l$

$\vec{E}\cdot\int da = \frac{Q_{in}}{\varepsilon_0}$ $\lambda = \frac{Q}{l} \rightarrow \lambda l = Q$

$E\cdot 2\pi\rho l = \frac{\lambda l}{\varepsilon_0}$

$E = \lambda / 2\pi\rho\varepsilon_0$

$\vec{E} = -\nabla V$

$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \alpha}\alpha + \frac{\partial f}{\partial \varphi}\phi$

$-\frac{\partial V}{\partial \rho} = \frac{\lambda}{2\pi\rho\varepsilon_0}$

$-\int \frac{\partial V}{\partial \rho} = \int \frac{\lambda}{2\pi\rho\varepsilon_0}$

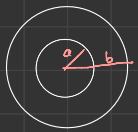
$V = \frac{-\lambda}{2\pi\varepsilon_0}\int \frac{1}{\rho}\, d\rho$

$\Delta V = \frac{-\lambda}{2\pi\varepsilon_0}\int_a^b \frac{1}{\rho}\, d\rho$

$\Delta V = \frac{-\lambda}{2\pi\varepsilon_0} \ln|b| - \ln|a|)$

$\Delta V = \frac{\lambda}{2\pi\varepsilon_0} \ln\left|\frac{a}{b}\right|$

$\frac{Q}{l} = \lambda \Rightarrow \frac{V 2\pi\varepsilon_0}{\ln\left|\frac{a}{b}\right|}$



a) $\frac{Q}{l}$

$Q = \int \lambda \, dl$

$Q = \lambda l$ $\qquad \lambda = Q/l$

$A = 2\pi \rho l$

$E \, 2\pi \rho l = \frac{\lambda l}{\varepsilon_0}$

$\vec{E} = \frac{\lambda}{2\pi \rho \varepsilon_0}$

$E = -\nabla V$

$\vec{E} = \frac{-\partial V}{\partial \rho} \hat{\rho}$

$\frac{-\lambda}{2\pi \rho \varepsilon_0} = \frac{\partial V}{\partial \rho}$

$\frac{-\lambda}{2\pi \varepsilon_0} \int_a^b \frac{1}{\rho} d\rho = V$

$\frac{\lambda}{2\pi \varepsilon_0} \ln\left|\frac{a}{b}\right| = V$ $\qquad \lambda = \frac{Q}{l}$

$\frac{2\pi \varepsilon_0 V}{\ln\left|\frac{a}{b}\right|} = \lambda = \frac{Q}{l}$

b) $\frac{C}{l} =$

$C = \frac{Q}{V}$

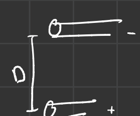
$C = \frac{\lambda l}{\frac{\lambda \ln\left|\frac{a}{b}\right|}{2\pi \varepsilon_0}}$

$\frac{C}{l} = \frac{2\pi \varepsilon_0}{\ln\left|\frac{a}{b}\right|}$

c) $\mathcal{E} = \frac{QV}{2} = \frac{\lambda l \frac{\lambda \ln\left|\frac{a}{b}\right|}{2\pi \varepsilon_0}}{2}$

$\mathcal{E} = \frac{\lambda^2 l \ln\left|\frac{a}{b}\right|}{4\pi \varepsilon_0}$

d)



$D > 5r$