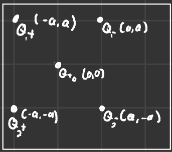
Exercise 1:



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 \|\vec{r}-\vec{r}'\|^2} \frac{(\vec{r}-\vec{r}')}{\|\vec{r}-\vec{r}'\|} \qquad V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\|\vec{r}-\vec{r}'\|}$$

$$\vec{F}_{ab} = \vec{E}_a Q_b$$

a)

$$\vec{E}_{Q_{1+}} = \frac{Q}{8\pi\varepsilon_0 a^2} \frac{(a,-a)}{\sqrt{2}\,a} = \frac{1}{8\pi\varepsilon_0 a^2} \frac{(1,-1)}{\sqrt{2}}$$ → Car pointe vers le centre donc $-r'$

$$\vec{E}_{Q_{2+}} = \frac{Q}{8\pi\varepsilon_0 a^2} \frac{(a,a)}{\sqrt{2}\,a} = \frac{1}{8\pi\varepsilon_0 a^2} \frac{(1,1)}{\sqrt{2}}$$

$$\vec{E}_{Q_{1-}} = \frac{-Q}{8\pi\varepsilon_0 a^2} \frac{(-a,-a)}{\sqrt{2}\,a} = \frac{1}{8\pi\varepsilon_0 a^2} \frac{(-1,-1)}{\sqrt{2}}$$

$$\vec{E}_{Q_{2-}} = \frac{-Q}{8\pi\varepsilon_0 a^2} \frac{(-a,a)}{\sqrt{2}\,a} = \frac{1}{8\pi\varepsilon_0 a^2} \frac{(-1,1)}{\sqrt{2}}$$

$$\vec{E}_{tot} = \frac{Q}{8\pi\varepsilon_0 a^2\sqrt{2}} \left((1+1--1--1), (-1+1--1-1)\right)$$

$$\vec{E}_{tot} = \frac{Q}{8\pi\varepsilon_0 a^2\sqrt{2}} (4,0)$$

$$\vec{F}_{ab} = \frac{Q}{8\pi\varepsilon_0 a^2\sqrt{2}} (4,0)\, Q_+$$

$$\boxed{F_{ab} = \frac{Q^2}{8\pi\varepsilon_0 a^2\sqrt{2}} (4,0)}$$

b)

$$V_{Q_{1+}} = \frac{Q}{4\pi\varepsilon_0\sqrt{2}\,a} = \frac{Q}{4\pi\varepsilon_0\sqrt{2}\,a}$$

$$V_{Q_{2+}} = \frac{Q}{4\pi\varepsilon_0\sqrt{2}\,a} = \frac{Q}{4\pi\varepsilon_0\sqrt{2}\,a}$$

$$V_{Q_{1-}} = \frac{-Q}{4\pi\varepsilon_0\sqrt{2}\,a} = \frac{-Q}{4\pi\varepsilon_0\sqrt{2}\,a}$$

$$V_{Q_{2-}} = \frac{-Q}{4\pi\varepsilon_0\sqrt{2}\,a} = \frac{-Q}{4\pi\varepsilon_0\sqrt{2}\,a}$$

$$V_{tot} = \frac{Q}{4\pi\varepsilon_0\sqrt{2}\,a} (1+1-1-1)$$

$$\boxed{V_{tot} = 0}$$

Exercice 2 :

$V = \frac{4}{3}\pi r^3$

$A_{sphère} = 4\pi r^2$

$C_{ir} = 2\pi r$

$A_{cercle} = \pi r^2$

rayon $= a$

densité $=$ $\rho_Q = \frac{3Q}{4\pi a^3}$

a) $Q = \int \rho_Q \, dv$

$$Q = \frac{3Q}{4\pi a^3} \int_0^{2\pi} \int_0^{\pi} \int_0^{a} r^2 \sin(\theta) \, dr \, d\theta \, d\phi$$

$$Q = \frac{3Q}{4\pi a^3} \int_0^{2\pi} \int_0^{\pi} \frac{r^3}{3}\Big|_0^a \sin(\theta) \, d\theta \, d\phi$$

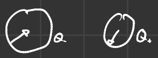
$$Q = \frac{Q}{4\pi} \int_0^{2\pi} -\cos(\theta)\Big|_0^{\pi}$$

$$Q = \frac{2Q}{4\pi} \psi\Big|_0^{2\pi}$$

$$Q = \frac{4\pi Q}{4\pi}$$

$$Q = Q$$

b) Dans une sphère c'est toujours radiale



c) $\oint \vec{E} \cdot dA = \frac{Q_{in}}{\varepsilon_0}$

$$\vec{E} \cdot A = \frac{Q}{\varepsilon_0}$$

$Q = \rho_v \cdot V \quad \rightarrow Q = \rho_v \cdot \frac{4}{3}\pi r^3 \rightarrow \frac{3Q}{4\pi a^3} \cdot \frac{4}{3}\pi r^3 = \frac{Q r^3}{a^3}$

$A = 4\pi r^2$

$$\vec{E} = \frac{\frac{Q r^3}{a^3}}{4\pi r^2 \varepsilon_0}$$

$$\vec{E} = \frac{Q r}{a^3 4\pi \varepsilon_0} \hat{e}_r$$

Exercise 3:

ρ_v = uniforme

rayon = b

Cylindre

Poisson en-dedans

Laplace dehors

dehors = 0

a) $\nabla^2 V = -\frac{\rho_v}{\varepsilon_0}$

$\nabla^2 V = 0$

$\cancel{\frac{1}{\rho}} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0$

$\frac{\partial V}{\partial \rho} = \frac{\text{constante}}{\rho}$

$V = \int \frac{\text{constante}}{\rho} \, d\rho$

$V = \text{const} \int \frac{1}{\rho} \, d\rho$

$V = K_1 \ln|\rho| + K_2$ K_1 et K_2 sont des constantes

b) $\nabla^2 V = -\frac{\rho_v}{\varepsilon_0}$

$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right)$ → seul qui nous interesse

$-\frac{\rho_f}{\varepsilon_0} = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right)$

$-\frac{\rho_f \rho}{\varepsilon_0} + K_1 = \frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right)$

$-\frac{\rho_f \rho^2}{2\varepsilon_0} + K_1 = \rho \frac{\partial V}{\partial \rho}$

$-\frac{\rho_f \rho}{2\varepsilon_0} + \frac{K_1}{\rho} = \frac{\partial V}{\partial \rho}$

$-\frac{\rho_f \rho^2}{4\varepsilon_0} + K_1 \ln|\rho| + K_2$ où K_1 et K_2 sont des constante

Exercise 4:

σ

$$\vec{J} = \frac{J_0}{a^2 b^2}\left(2xy^2z\,\hat{e}_x + 2x^2yz\,\hat{e}_y + x^2y^2\,\hat{e}_z\right)$$

a) Surface element on z $\hat{e}_z$

$$d\hat{A} = (0,0,1)$$

$$I = \int_s \vec{J}\cdot d\hat{A}$$

$$I = \frac{J_0}{a^2b^2}\int_s \left(2xy^2z\,\hat{e}_x + 2x^2yz\,\hat{e}_y + x^2y^2\,\hat{e}_z\right)(0,0,1)\,dx\,dy$$

$$I = \frac{J_0}{a^2b^2}\int_{-b/2}^{b/2}\int_{-a/2}^{a/2} x^2y^2\,dx\,dy$$

$$I = \frac{J_0}{a^2b^2}\int_{-b/2}^{b/2} \frac{x^3}{3}y^2\Big|_{-a/2}^{a/2}\,dy$$

$$I = \frac{J_0}{a^2b^2}\int_{-b/2}^{b/2} \frac{a^3}{24}y^2 - -\frac{a^3}{24}y^2\,dy$$

$$I = \frac{J_0}{a^2b^2}\int_{-b/2}^{b/2} \frac{a^3}{12}y^2\,dy$$

$$I = \frac{J_0\,a^3}{a^2b^2\,12}\,\frac{y^3}{3}\Big|_{-b/2}^{b/2}$$

$$I = \frac{J_0\,a^3}{a^2b^2\,12}\left(\frac{b^3}{24} - -\frac{b^3}{24}\right)$$

$$\boxed{I = \frac{J_0\,ab}{144}}$$

b) $\vec{J} = \sigma\vec{E}$

$$\vec{E} = \frac{\vec{J}}{\sigma}$$

c) $\vec{E} = -\nabla V$ $\quad\quad \int \frac{\partial V}{\partial x} = V$

$$V = \frac{-J_0}{a^2b^2}\left(\int 2xy^2z\,dx,\ \int 2x^2yz\,dy,\ \int x^2y^2\,dz\right)$$

$$V = \frac{-J_0}{a^2b^2\sigma}\left(x^2y^2z + x^2y^2z + x^2y^2z\right)$$

$$V = \frac{-J_0}{a^2b^2\sigma}\,x^2y^2z$$

Exercise 5:

rayon = b
$\rho_q = Kz/L$

a) il y en aura un centré à O car il y a deux charge de force opposé et sont égal
définition du moment dipolaire

c) $\vec{P} = \int_V r \rho_q \, dV$ $\qquad \vec{r} = (\rho\cos(\theta), \rho\sin(\theta), z)$

$$\vec{P} = \int_{-L/2}^{L/2} \int_0^{\pi} \int_0^{a} (\rho\cos(\theta), \rho\sin(\theta), z) \frac{Kz}{L} \rho \, d\rho \, d\theta \, dz$$

$$\vec{P} = \frac{K}{L} \int_{-L/2}^{L/2} \int_0^{\pi} \int_0^{a} (z\rho^2\cos(\theta), z\rho^2\sin(\theta), \rho z^2 \; d\rho \, d\theta \, dz$$

Les $\hat{e}_x$ et $\hat{e}_y$ sont de 0

$$\vec{P} = \frac{K}{L} \int_{-L/2}^{L/2} \int_0^{2\pi} \frac{\rho^2}{2} z^2 \Big|_0^a \, d\theta \, dz$$

$$\vec{P} = \frac{Ka^2}{2L} \int_{-L/2}^{L/2} z^2 \Big|_0^{2\pi}$$

$$\vec{P} = \frac{\pi K b^2}{L} \frac{z^3}{3}\Big|_{-L/2}^{L/2}$$

$$\vec{P} = \frac{\pi K b^2 L^2}{12} \hat{e}_z$$

Exercice 6 :

Masse = m

Particule Charge = q

Vitesse = V

$$x = V_0 t + \frac{at^2}{2}$$

$$a = \frac{V_f - V_0}{2}$$

$$x = \frac{V_f + V_0}{2} t$$

a) $V = \frac{Q_0}{C}$

b) $V = \frac{Q}{C}$

$\frac{\partial v}{\partial x} = 0$ car bouge pas

$\frac{\partial v}{\partial y} = 0$ car bouge pas

$\frac{\partial v}{\partial z} = \frac{\partial Q/C}{\partial z} \hat{o}_z$

$$\vec{E} = -\frac{\partial Q/C}{\partial z} \hat{o}_z$$

$F = Q \cdot \vec{E}$

$F = Q \cdot -\frac{\partial Q/C}{\partial z} \hat{o}_z$

$F = ma$

$a = \frac{E}{m}$

$$x = V_0 t + \frac{at^2}{2}$$

$T = \frac{V}{L}$

$$\Delta x = \frac{-Q \frac{\partial Q/C}{\partial z} \hat{o}_z T^2}{2 \cdot m \cdot L^2}$$

Deflexion vers le bas car c'est negatif

$$\Delta x = \frac{-Q \frac{\partial Q/C}{\partial z} \hat{o}_z T^2}{2 m L^2}$$

Exercise 7 :

$Q = \int \lambda \, dl$

$Q = \lambda l$

$A = 2\pi r l$

$\oint E \cdot d\vec{a} = \frac{Q_{in}}{\varepsilon}$

$\vec{E} \cdot 2\pi r l = \frac{\lambda \quad l}{\varepsilon}$

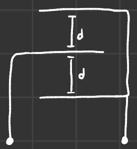
$\vec{E} = \frac{\lambda}{2\pi r \varepsilon} \hat{e}_r$

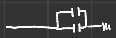
$\nabla \cdot E = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \vec{E})$

$\nabla \cdot \vec{E} = 0$

On ne peut pas
dériver une constante
donc cela ne marche
pas. Car r est au dénominateur
et la fois x r après en devient
une constante

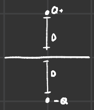
Exercise 8:





Exercise 9 :

a)



b) il s'annule car ils ont la même charge mais de force opposé.

Exercise 10 :

Exercise 11 :

$$\oint_C \vec{B}\, dl = \mu_0 I_{traverse}$$

$$5 \times 10^{-5} \cdot 2\pi l = \mu_0 I_{travers}$$

$$I_{travers} = \frac{5 \times 10^{-5} \cdot 2\pi\, 0{,}01}{\mu_0}$$

$$I_{travers} = \frac{5 \times 10^{-5} \cdot 2\pi \cdot 1 \times 10^{-2}}{4\pi \cdot 10^{-7}}$$

$$I_{travers} = \frac{5 \cdot 2}{4}$$

$$I_{travers} = 2{,}5\ A$$

Prise 2

Exercise 1:

$$\vec{E}_{Q_1} = \frac{Q}{8\pi\varepsilon_0 a^2}\frac{(1,-1)}{\sqrt{2}}$$

$$\vec{E}_{Q_2} = \frac{Q}{8\pi\varepsilon_0 a^2}\frac{(1,1)}{\sqrt{2}}$$

$$\vec{E}_{Q_3} = \frac{-Q}{8\pi\varepsilon_0 a^2}\frac{(-1,-1)}{\sqrt{2}}$$

$$\vec{E}_{Q_4} = \frac{-Q}{8\pi\varepsilon_0 a^2}\frac{(-1,1)}{\sqrt{2}}$$

$$\vec{E}_{tot} = \frac{Q}{8\pi\varepsilon_0 a^2\sqrt{2}}\left((1+1+1+1),(-1+1+1-1)\right)$$

$$\vec{E}_{tot} = \frac{Q}{8\pi\varepsilon_0 a^2\sqrt{2}}(4,0)$$

a) $\vec{F} = \vec{E}Q$

$$\vec{F} = \frac{Q^2}{2\pi\varepsilon_0 a^2\sqrt{2}}\hat{e}_x$$

b) $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{2}a}$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{2}a}$$

$$V_3 = \frac{1}{4\pi\varepsilon_0}\frac{-Q}{\sqrt{2}a}$$

$$V_4 = \frac{1}{4\pi\varepsilon_0}\frac{-Q}{\sqrt{2}a}$$

$$V_{tot} = \frac{Q}{4\pi\varepsilon_0\sqrt{2}a}(1+1-1-1)$$

$$V_{tot} = 0$$

Exercise 2:

a) $Q_{tot} = \int \rho_a \, dv$

$$Q_{tot} = \int_0^{2\pi}\int_0^{\pi}\int_0^{a}\frac{3Q}{4\pi a^3} r^2\sin(\theta)\,dr\,d\theta\,d\varphi$$

$$Q_{tot} = \frac{3Q}{4\pi a^3}\int_0^{2\pi}\int_0^{\pi}\frac{r^3}{3}\Big|_0^a \,d\theta\,d\varphi$$

$$Q_{tot} = \frac{3Q}{4\pi a^3}\frac{a^3}{3}\int_0^{2\pi} -\cos(\theta)\Big|_0^{\pi}\,d\varphi$$

$$Q_{tot} = \frac{3Q\,2\,a^3}{3\cdot 4\pi a^3}\,\varphi\Big|_0^{2\pi}$$

$$Q_{tot} = \frac{\cancel{3}Q\cancel{2}\cdot\cancel{2\pi}\cdot\cancel{a^3}}{\cancel{3}\cdot\cancel{4}\cdot\cancel{\pi}\cdot\cancel{a^3}}$$

$$Q_{tot} = Q$$

c) $\oint E\,dA = \frac{Q}{\varepsilon_0}$

$$Q = \rho_v \cdot V$$

$$= \frac{3Q}{4\pi a^3}\cdot\frac{4}{3}\pi r^3$$

$$= \frac{Qr^3}{a^3}$$

$$A = 4\pi r^2$$

$$E = \frac{Qr^3}{a^3}\cdot\frac{1}{4\pi r^2}$$

$$\vec{E} = \frac{Qr}{4\pi a^3\varepsilon_0}\hat{e}_r$$

Exercise 4:

a) $I = \int \vec{J}\, dA$

$$I = \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} \frac{J_0}{a^2b^2}\left(2xy^2z,\ 2x^2yz,\ x^2y^2\right)(0,0,1)\, dx\,dy$$

$$I = \frac{J_0}{a^2b^2} \int_{-b/2}^{b/2} \frac{x^3}{3}y^2 \Big|_{-a/2}^{a/2} dy$$

$$I = \frac{J_0 a}{12 b^2} \frac{y^3}{3} \Big|_{-a/2}^{a/2}$$

$$\boxed{I = \frac{J_0 ab}{144}}$$

b) $\vec{J} = \sigma \vec{E}$

$$\vec{E} = \frac{J}{\sigma}$$

$$\vec{E} = \frac{J_0}{a^2b^2\sigma}\left(2xy^2z,\ 2x^2yz,\ x^2y^2\right)$$

c) $\vec{E} = -\nabla V$

$$\left(-\frac{\partial v}{\partial x},\ -\frac{\partial v}{\partial y},\ -\frac{\partial v}{\partial z}\right) = V$$

$$\frac{-J_0}{a^2b^2\sigma}\left(\frac{\partial\, 2xy^2z}{\partial x},\ \frac{\partial\, 2x^2yz}{\partial y},\ \frac{\partial\, x^2y^2}{\partial z}\right) = V$$

$$V = \frac{-J_0}{a^2b^2\sigma}\left(x^2y^2z\right)$$

Exercise 5:

a)

Exercice 5 :

a) Oui, car deux charges seront opposé de même force mais de signe opposé

b) $p = \int_V r \, \rho_a \, dv$

$r = [\rho\cos(\varphi), \rho\sin(\varphi), z]$

$$\vec{p} = \frac{K}{L} \int_{-L/2}^{L/2} \int_0^{2\pi} \int_0^b [\rho^2\cos(\varphi) z, \rho^2\sin(\varphi) z, \rho z^2] \, d\rho \, d\varphi \, dz$$

$$\vec{p} = \frac{K}{L} \int_{-L/2}^{L/2} \int_0^{2\pi} \frac{\rho^2}{2} z^2 \Big|_0^b \, d\varphi \, dz$$

Les x y sont de 0 car juste en z

$$\vec{p} = \frac{Kb^2\pi}{L} \frac{z^3}{3} \Big|_{-L/2}^{L/2}$$

$$\vec{p} = \frac{Kb^2\pi}{L} \frac{L^3}{12}$$

$$\vec{p} = \frac{Kb^2L^2\pi}{12} \hat{e}_z$$

Exercise 7:

$Q = \int \lambda \, dl$

$Q = \lambda l$

$A = 2\pi \rho l$

$\vec{E} \, 2\pi\rho l = \frac{\lambda l}{\varepsilon}$

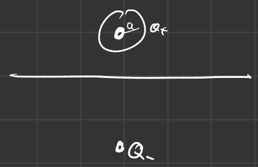
$\vec{E} = \frac{\lambda}{2\pi\rho\varepsilon}$

$\nabla \cdot \vec{E} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\lambda}{2\pi\rho\varepsilon} \right)$

$\nabla \cdot E = \underline{\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\frac{\lambda}{2\pi\varepsilon} \right)}$

no plus de ρ donc se sera 0

a)



b) Elle sera de 0 car même si le rayon vari les forces sont égal à distance égale donc ils s'annulent

Exercise 11:

$\vec{B} = 5 \times 10^{-5}$

$\oint \vec{B} \cdot dl = \mu_0 I_{travers}$

$5 \times 10^{-5} \cdot 2\pi l = 4\pi \times 10^{-7} I_{travers}$

$\frac{5 \times 10^{-5} \, 2\pi \cdot 0.01}{4\pi \times 10^{-7}} = I_{travers}$

$I_{travers} = 2.5$