Mandat 1:

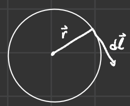
On veut $\vec{N} = \vec{m} \times \vec{B}$

On a: $d\vec{F} = I\, d\vec{l} \times \vec{B}$

$\vec{r} = \vec{OP} \longrightarrow \vec{OP} = \|\vec{r}\|$

$\|\vec{N}\| = \|\vec{r}\| \cdot F_\perp$

$\vec{N} = \vec{r} \times \vec{F}$



$\vec{r} = r(\cos(\theta)\hat{e}_x + \sin(\theta)\hat{e}_y)$

$d\vec{l} = r\hat{e}_\theta = r(-\sin(\theta)\hat{e}_x + \cos(\theta)\hat{e}_y)$

$d\vec{F} = I\, d\vec{l} \times \vec{B}$

$$d\vec{l} \times \vec{B} = \det \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ -r\sin(\theta) & r\cos(\theta) & 0 \\ B_x & B_y & B_z \end{vmatrix} = B_z r\cos(\theta)\hat{e}_x + B_z r\sin(\theta)\hat{e}_y + (-B_y r\sin(\theta) - B_x r\cos(\theta))\hat{e}_z$$

$$d\vec{F} = IB_z r\cos(\theta)\hat{e}_x + IB_z r\sin(\theta)\hat{e}_y + (-IB_y r\sin(\theta) - IB_x r\cos(\theta))\hat{e}_z$$

$\vec{N} = \vec{r} \times \vec{F}$

$$d\vec{N} = \vec{r} \times d\vec{F} = \det \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ r\cos(\theta) & r\sin(\theta) & 0 \\ IB_z r\cos(\theta) & IB_z r\sin(\theta) & -IB_y r\sin(\theta) - IB_x r\cos(\theta) \end{vmatrix}$$

$$d\vec{N} = \vec{r} \times d\vec{F} = (-IB_y r^2\sin^2(\theta) - IB_x r^2\sin(\theta)\cos(\theta))\hat{e}_x + (IB_y r^2\sin(\theta)\cos(\theta) + IB_x r^2\cos^2(\theta))\hat{e}_y + 0\hat{e}_z$$

$$\vec{N} = \int \vec{r} \times d\vec{F}\, d\theta$$

$$\vec{N} = \int ((-IB_y r^2\sin^2(\theta) - IB_x r^2\sin(\theta)\cos(\theta))\hat{e}_x + (IB_y r^2\sin(\theta)\cos(\theta) + IB_x r^2\cos^2(\theta))\hat{e}_y + 0\hat{e}_z)\, d\theta$$

$$\vec{N} = Ir^2 \int_0^{2\pi} ((-B_y\sin^2(\theta) - B_x\sin(\theta)\cos(\theta))\hat{e}_x + (B_y\sin(\theta)\cos(\theta) + B_x\cos^2(\theta))\hat{e}_y)$$

$\sin^2(\theta) = \frac{1}{2}\theta - \frac{\sin(2\theta)}{4}$

$\cos^2(\theta) = \frac{1}{2}\theta + \frac{\sin(2\theta)}{4}$

$\sin(\theta)\cos(\theta) = \frac{\sin^2(\theta)}{2}$

$$\vec{N} = Ir^2 \left(-B_y\left(\frac{1}{2}\theta - \frac{\sin(2\theta)}{4}\right) - B_x\frac{\sin^2(\theta)}{2}\right)\hat{e}_x + \left(B_y\frac{\sin^2(\theta)}{2} + B_x\left(\frac{1}{2}\theta + \frac{\sin(2\theta)}{4}\right)\right)\hat{e}_y \Big|_0^{2\pi}$$

$$\vec{N} = Ir^2\pi(-B_y\hat{e}_x + B_x\hat{e}_y)$$

$\vec{N} = I r^2 \pi \left(-B_y \hat{e}_x + B_x \hat{e}_y\right)$

$\vec{a} = r^2 \pi$ Car l'aire d'un cercle c'est $\pi \cdot r^2$

$\vec{N} = I \vec{a} \left(-B_y \hat{e}_x + B_x \hat{e}_y\right)$

$\vec{N} = I \vec{a} \times \vec{B}$

$\vec{m} = I \vec{a}$ Car on sait que dipole magnétique $\vec{m} = I \vec{a}$

$\boxed{\vec{N} = \vec{m} \times \vec{B}}$

Donc ici on a réussi à prouver que $d\vec{F} = I\, d\vec{l} \times \vec{B}$ est équivalent à $\vec{N} = \vec{m} \times \vec{B}$

Mandat 2 :

On veut $\frac{d\vec{m}}{dt} = \gamma \vec{m} \times \vec{B}$

On a : $\vec{J} = I\vec{\Omega}$

$\vec{m} = \gamma \vec{J}$

$\vec{N} = \vec{m} \times \vec{B} = \frac{d\vec{J}}{dt}$

Si on part de $\vec{m} = \gamma \vec{J}$

$$\frac{d\vec{m}}{dt} = \frac{d\gamma\vec{J}}{dt}$$

$$\frac{d\vec{m}}{dt} = \gamma \frac{d\vec{J}}{dt}$$ car on sait que γ est une constante

$$\frac{d\vec{J}}{dt} = \vec{N}$$

$$\frac{d\vec{m}}{dt} = \gamma \vec{N}$$

$$\vec{N} = \vec{m} \times \vec{B}$$

$$\boxed{\frac{d\vec{m}}{dt} = \gamma \vec{m} \times \vec{B}}$$

Grâces aux formules données soit $\vec{N} = \vec{m} \times \vec{B} = \frac{d\vec{J}}{dt}$ et $\vec{m} = \gamma \vec{J}$ il est possible de

Prouver que $\frac{d\vec{m}}{dt} = \gamma \vec{m} \times \vec{B}$

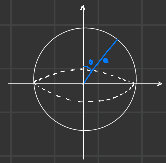
Mandat 3:

On Cherche
- Vecteur moment magnétique du proton
- moment cinétique du proton

On sait
rayon = a
fréquence = Ω
Charge = q
densité de charge uniforme = ρ_q
masse = m
densité de volume uniforme = ρ_m

$\vec{m} = I\vec{a}$

$dI = dq \cdot f$



$$\vec{m} = I\vec{a}$$

$$d\vec{m} = dI \cdot \vec{a}$$

$$d\vec{m} = \frac{r^2 \rho_q \Omega}{2} \, dv$$

$dv = r^2 \sin(\theta)\, dr\, d\theta\, d\varphi$

$$\vec{m} = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \frac{r^4 \rho_q \Omega}{2} \sin(\theta) \, dr\, d\theta\, d\varphi$$

$$\vec{m} = \frac{\rho_q \Omega}{2}\int_0^{2\pi}\int_0^{\pi} \frac{r^5}{5}\Big|_0^a \, d\theta\, d\varphi$$

$$\vec{m} = \frac{\rho_q \Omega a^5}{10}\int_0^{2\pi} -\cos(\theta)\Big|_0^{\pi} \, d\varphi$$

$$\vec{m} = \frac{\rho_q \Omega a^5 \cdot 2}{10} \; \varphi\Big|_0^{2\pi}$$

$$\vec{m} = \frac{2\pi \rho_q \Omega a^5}{5}\hat{e}_z$$

$V = \frac{4\pi r^3}{3}$

$\rho_q = \frac{Q}{V}$

$$\boxed{\vec{m} = Q\,\frac{3a^2 \Omega}{10}}$$

$$dI = dq f$$

$Q = \rho_q \frac{4\pi r^3}{3}$

$dq = \rho_q \, dv$

$\vec{a} = \pi r^2 \hat{e}_z$

$\vec{m} = \rho_m \frac{4\pi r^3}{3}$

$dm = \rho_m \, dv$

$dI = \frac{2}{5} r^2 dm$

$dI = \frac{dq\, \Omega}{2\pi}$

$dI = \frac{\rho_q \Omega}{2\pi} dv$

$$d\vec{J} = dI\, \vec{\Omega}$$

$$d\vec{J} = r^2 \Omega \, dm$$

$$d\vec{J} = r^2 \Omega \rho_m \, dv$$

$$\vec{J} = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^2 \Omega \rho_m \, r^2 \sin(\theta)\, dr\, d\theta\, d\varphi$$

$$\vec{J} = \rho_m \Omega \int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^4 \sin(\theta)\, dr\, d\theta\, d\varphi$$

$$\vec{J} = \rho_m \Omega \int_0^{2\pi}\int_0^{\pi} \frac{r^5}{5}\Big|_0^a \sin(\theta)\, d\theta\, d\varphi$$

$$\vec{J} = \frac{\rho_m a^5 \Omega}{5}\int_0^{2\pi} -\cos(\theta)\Big|_0^{\pi} \, d\varphi$$

$$\vec{J} = \frac{2\rho_m a^5 \Omega}{5} \; \varphi\Big|_0^{2\pi}$$

$$\vec{J} = \frac{4\pi \rho_m a^5 \Omega}{5}\hat{e}_z$$

$$\boxed{\vec{J} = m\,\frac{3a^2 \Omega}{5}}$$

$$\gamma = \frac{\vec{m}}{\vec{J}}$$

$$\vec{m} = \frac{Q\,3a^2\,\Omega}{10}$$

$$\vec{J} = \frac{\vec{m}\,3a^2\,\Omega}{5}$$

$$\gamma = \frac{\frac{Q\,3a^2\,\Omega}{10}}{\frac{m\,3a^2\,\Omega}{5}} = \frac{\frac{Q\cdot 3}{10}}{\frac{m\cdot 3}{5}}$$

$$\gamma = \frac{Q}{m\,2}$$

$$\boxed{\gamma = \frac{Q}{2m}}$$

Comporaison:

$$\text{Comporaison} = \frac{1.602176565 \times 10^{-19}\ C}{2 \cdot 1.67262 \times 10^{-27}\ Kg}$$

Coulomb = As

Tesla = Kg / As²

$$\text{Comporaison} = 48{,}18 \times 10^{6}\ C/Kg$$

$$= 48{,}18 \times 10^{6}\ \frac{As}{Kg} = \frac{Hz}{T}$$

$$\frac{Hz}{T} = \frac{\frac{1}{s}}{\frac{Kg}{As^2}} = \frac{\frac{As^2}{s}}{Kg} = \frac{As}{Kg}$$

$$= 48{,}18 \times 10^{6}\ \frac{Hz}{T}$$

$$= 48.18\ \frac{MHz}{T}$$

$$\boxed{\frac{48{,}18}{42{,}577} = 1{,}13}$$ Comporaison

$\boxed{\text{Mandat 4:}}$

On Cherche

On Sait

Mandat 5:

On cherche l'axe de précession

On sait $\vec{B_0} = B_0 \hat{e}_z$

$\xi_{xy} = m_x + j m_y$ où $j = \sqrt{-1}$

$w_0 = \gamma B_0$

$m_y - j m_x = -j(m_x + j m_y)$

$\frac{d\vec{m}}{dt} = \gamma \vec{m} \times \vec{B}$

$$\frac{d\vec{m}}{dt} = \gamma \vec{m} \times \vec{B}$$

$$\frac{d\vec{m}}{dt} = \gamma \vec{m} \times B_0 \hat{e}_z + 0 \hat{e}_z$$

$$\frac{d\vec{m}}{dt} = \gamma \det \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ m_x & m_y & m_z \\ 0 & 0 & B_0 \end{vmatrix}$$

$$\frac{d\vec{m}}{dt} = \gamma \left(B_0 m_y \hat{e}_x - B_0 m_x \hat{e}_y + 0 \hat{e}_z \right)$$

$$\frac{d\vec{m}}{dt} = \gamma B_0 m_y \hat{e}_x - \gamma B_0 m_x \hat{e}_y + 0 \hat{e}_z$$

$\gamma B_0 = w_0$

$$\frac{d\vec{m}}{dt} = w_0 \left(m_y \hat{e}_x - m_x \hat{e}_y + 0 \hat{e}_z \right)$$

$$\frac{dm_x}{dt} = w_0 m_y \qquad \frac{dm_z}{dt} = 0$$

$$\frac{dm_y}{dt} = -w_0 m_x$$

$\xi_{xy} = m_x + j m_y$

$$\frac{d\xi_{xy}}{dt} = \frac{dm_x}{dt} + j \frac{dm_y}{dt}$$

$$\frac{d\xi_{xy}}{dt} = w_0 m_y - w_0 m_x j$$

$$\frac{d\xi_{xy}}{dt} = w_0 (m_y - m_x j)$$

$m_y - j m_x = -j(m_x + j m_y)$

$$\frac{d\xi_{xy}}{dt} = w_0 \left(-j(m_x + j m_y) \right)$$

$$\frac{d\xi_{xy}}{dt} = -j w_0 \xi_{xy}$$

$$\frac{d\xi_{xy}}{dt} + j w_0 \xi_{xy} = 0$$

$$\frac{1}{j w_0} \frac{d\xi_{xy}}{dt} + \xi_{xy} = 0$$

$$\frac{1}{j\omega_0}\frac{d\xi_{xy}}{dt} + \xi_{xy} = 0$$

$$\tau = \frac{1}{j\omega_0}$$

$$\tau \frac{d\xi_{xy}}{dt} + \xi_{xy} = 0$$

$$\xi_{xy}(t) = \xi_{xy\,c}(t) + \xi_{xy\,p}(t)$$

$$\xi_{xy\,c}(t) = Ae^{\lambda t}$$

$$\tau \frac{d\xi_{xy\,c}}{dt} + \xi_{xy\,c} = 0$$

$$\tau \frac{d(Ae^{\lambda t})}{dt} = -\frac{1}{\tau}Ae^{\lambda t}$$

$$\lambda Ae^{\lambda t} = -\frac{1}{\tau}Ae^{\lambda t}$$

$$\lambda = -\frac{1}{\tau}$$

$$\xi_{xy\,c}(t) = Ae^{-t/\tau}$$

$$\tau = \frac{1}{j\omega_0}$$

$$\boxed{\xi_{xy\,c}(t) = Ae^{-j\omega_0 t}}$$ ξ_{xy} complementaire

$$\xi_{xy\,p}(t) = 0$$

$$\xi_{xy}(t) = Ae^{-j\omega_0 t} + 0$$

$$e^{jt} = \cos(t) + j\sin(t)$$
$$e^{-jt} = \cos(t) - j\sin(t)$$

$$m_x + jm_y = A(\cos(\omega_0 t) - j\sin(\omega_0 t))$$

$$\boxed{\begin{aligned} m_x &= A\cos(\omega_0 t) \\ m_y &= A\sin(\omega_0 t) \\ m_z &= \text{Constante} \end{aligned}}$$ équations différentiel

$\boxed{\text{Mandat 6 :}}$

On cherche

On sait

Mandat 7 :

On cherche $\vec{B}_1(t)$

On sait $m_x = A\cos(w_0 t)$

$m_y = -A\cos(w_0 t)$

$\frac{d\vec{m}}{dt} = \gamma \vec{m} \times B_0$

$$\vec{B}_1(t) = B_1\cos(w_0 t)\hat{e}_x - B_1\sin(w_0 t)\hat{e}_y$$

On peut conclure que $w_{RF} = w_0$ car on veut qu'il tourne selon la même fréquence que le B_0 donc $\vec{B}_{tot} = \vec{B}_0 + \vec{B}_1(t)$

$$\vec{B}_{tot} = \vec{B}_0 + \vec{B}_1(t)$$
$$\vec{B}_{tot} = \left(\|\vec{B}_1\|\cos(w_0 t),\ -\|\vec{B}_1\|\sin(w_0 t),\ \|\vec{B}_0\|\right)$$

$$\boxed{\frac{d\vec{m}}{dt} = \gamma \vec{m} \times \vec{B}_{tot}}$$

Mandat 8 :

On cherche à déduire le premier terme de l'équation de Bloch

On sait $\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{B} - \frac{(M_z - M_z^0)\vec{e}_z}{T_1} - \frac{\vec{M}_{xy}}{T_2}$

$\vec{M} = n\vec{m}$ $\quad$ $\vec{M} = \frac{1}{V}\sum_{n=1}^{N} \vec{m}_n$ $\quad$ $\frac{d\vec{m}}{dt} = \gamma m \times B$

$$\frac{d\vec{m}}{dt} = \gamma \vec{m} \times \vec{B}$$

$\vec{M} = \vec{m} n$

$\vec{m} = \frac{\vec{M}}{n}$

$$\frac{d\vec{M}/n}{dt} = \gamma \frac{\vec{M}}{n} \times \vec{B}$$

$$\boxed{\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{B}}$$

$$\frac{d\vec{m}}{dt} \cdot n = \gamma \vec{m} \times \vec{B} \cdot n$$

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{B}$$

Mandat 9 :

On cherche ratio de 1 Tesla sur le champs magnetique de la terre

On sait champs magnetisme terreste moyen = 50 mT

ratio = 1T / champ terestre

$$\text{ratio} = \frac{1\,T}{50 \times 10^{-6}\,T}$$

$\boxed{\text{ratio} = 20\,000}$

On trouve que 1 Tesla est 20 000 fois le champ magnétique terrestre si on prends la moyenne du champ électrique terestre trouvé en ligne au https://www.universalis.fr/encycloipedie/magnetisme-notions-de-base/2-les-lois-classiques-du-magnetisme/#:~:text=L'ordre%20de%20grandeur%20du,produits%20par%20des%20%C3%A9lectro%2Daimants.

50 mT est aussi iéquavalent de ½ gauss, un autre unité pouvant decrire le champs magnétique.

Mon dst 10:

On veut trouver une formule analytique pour le champ dans un solénoide

On connait $B = \frac{u_0}{2} \int_{-L/2}^{+L/2} \frac{R^2 N' I \, dz}{(R^2+z^2)^{3/2}}$

où N': nombre spires
I: courant
R: Rayon
L: longueur
z: position du centre

$$\vec{B} = \frac{u_0}{2} \int_{-L/2}^{+L/2} \frac{R^2 N' I \, dz}{(R^2+z^2)^{3/2}}$$

$$\vec{B} = \frac{u_0 R^2 N' I}{2} \int_{-L/2}^{L/2} \frac{1}{(R^2+z^2)^{3/2}} dz$$

on peut sortir les constantes

$z = R\tan(t)$
$dz = R\sec^2(t)\, dt$

$$\vec{B} = \frac{u_0 R^2 N' I}{2} \int_{-L/2}^{L/2} \frac{1}{(R^2+(R\tan(t))^2)^{3/2}} R\sec^2(t)\, dt$$

$$\vec{B} = \frac{u_0 R^2 N' I}{2} \int_{-L/2}^{L/2} \frac{R\sec^2(t)}{(R^2+R^2\tan^2(t))^{3/2}} dt$$

$$\vec{B} = \frac{u_0 R^2 N' I}{2} \int_{-L/2}^{L/2} \frac{R\sec^2(t)}{(R^2(1+\tan^2(t)))^{3/2}} dt$$

$$\vec{B} = \frac{u_0 R^2 N' I}{2} \int_{-L/2}^{L/2} \frac{R\sec^2(t)}{R^3 \cdot (1+\tan^2(t))^{3/2}} dt$$

$$\vec{B} = \frac{u_0 R^2 N' I}{2} \cdot \frac{R}{R^3} \int_{-L/2}^{L/2} \frac{\sec^2(t)}{(1+\tan^2(t))^{3/2}} dt$$

$\tan^2(t) = \sec^2(t) - 1$

$$\vec{B} = \frac{u_0 N' I}{2} \int_{-L/2}^{L/2} \frac{\sec^2(t)}{(1+\sec^2(t)-1)^{3/2}} dt$$

$$\vec{B} = \frac{u_0 N' I}{2} \int_{-L/2}^{L/2} \frac{\sec^2(t)}{(\sec^2(t))^{3/2}} dt$$

$$\vec{B} = \frac{u_0 N' I}{2} \int_{-L/2}^{L/2} \frac{\sec^2(t)}{\sec^3(t)} dt$$

$$\vec{B} = \frac{u_0 N' I}{2} \int_{-L/2}^{L/2} \frac{1}{\sec(t)} dt$$

$\cos(t) = \frac{1}{\sec(t)}$

$$\vec{B} = \frac{u_0 N' I}{2} \int_{-L/2}^{L/2} \cos(t)\, dt$$

$$\vec{B} = \frac{u_0 N' I}{2} \sin(t) \Big|_{-L/2}^{L/2}$$

$$\vec{B} = \frac{\mu_0 N' I}{2} \sin(t) \Big|_{-L/2}^{L/2}$$

$z = R\tan(t)$

$\hookrightarrow t = \arctan\left(\frac{z}{R}\right)$

$$\vec{B} = \frac{\mu_0 N' I}{2} \sin\left(\arctan\left(\frac{z}{R}\right)\right) \Big|_{-L/2}^{L/2}$$

$$\sin(\arctan(x)) = \frac{x}{\sqrt{1+x^2}}$$

$$\vec{B} = \frac{\mu_0 N' I}{2} \frac{\frac{z}{R}}{\sqrt{1+\left(\frac{z}{R}\right)^2}} \Bigg|_{-L/2}^{L/2}$$

$$\vec{B} = \frac{\mu_0 N' I}{2} \frac{z}{R\sqrt{1+\left(\frac{z}{R}\right)^2}} \Bigg|_{-L/2}^{L/2}$$

$$\vec{B} = \frac{\mu_0 N' I}{2} \frac{z}{R\sqrt{\frac{R^2+z^2}{R_2}}} \Bigg|_{-L/2}^{L/2}$$

$$\vec{B} = \frac{\mu_0 N' I}{2} \frac{z}{\sqrt{R^2+z^2}} \Bigg|_{-L/2}^{L/2}$$

on ajoute $(z-z_0)$ parce qu'il ne commence pas à zéro

$$\vec{B} = \frac{\mu_0 N' I}{2} \frac{(z-z_0)}{\sqrt{R^2+(z-z_0)^2}}$$

$$\vec{B} = \frac{\mu_0 N' I}{2} \left(\frac{(L/2 - z_0)}{\sqrt{R^2+(L/2-z_0)^2}} - \frac{(-L/2 - z_0)}{\sqrt{R^2+(-L/2-z_0)^2}} \right) \Rightarrow \frac{\mu_0 N' I}{2} \left(\frac{L - z_0}{2\sqrt{R^2+(L/2-z_0)^2}} + \frac{L+z_0}{2\sqrt{R^2+(-L/2-z_0)^2}} \right)$$

$$\boxed{\vec{B} = \frac{\mu_0 N' I}{2} \left(\frac{L - 2z_0}{2\sqrt{R^2+(L/2-z_0)^2}} + \frac{L+2z_0}{2\sqrt{R^2+(-L/2-z_0)^2}} \right)}$$

Mandat 11 :
Partie 2

On cherche pour plusieur longueur

On soit
$$B = \frac{u_0 N I}{2}\left[\frac{\frac{L}{2} - z_0}{\sqrt{R^2 + (\frac{L}{2} - z_0)^2}} + \frac{\frac{L}{2} + z_0}{\sqrt{R^2 + (-\frac{L}{2} - z_0)^2}}\right]$$

$u_0 = 4\pi \cdot 10^{-7}$

$I = 500$

L = 20

$$B_{z\_max} = \frac{\mu_0 N I}{2}\left[0 - \frac{10 + 10}{\sqrt{10^2 + (-20)^2}}\right]$$

$$= \frac{4\pi \cdot 10^{-7} \cdot 40 \cdot 500}{2}[0{,}8944]$$

$$= 11{,}24 \times 10^{-3} \text{ T}$$

$$B_{z\_min} = \frac{\mu_0 N I}{2}\left[\frac{10}{\sqrt{200}} + \frac{10}{\sqrt{200}}\right]$$

$$= \frac{4\pi \cdot 10^{-7} \cdot 40 \cdot 500}{2}[\sqrt{2}]$$

$$= 17{,}77 \times 10^{-3} \text{ T}$$

$$\frac{B_{z\_max}}{B_{z\_0}} = \frac{11{,}24 \times 10^{-3}}{17{,}77 \times 10^{-3}} = 63{,}25\%$$

$\boxed{L \text{ de } 20 = 63{,}25\%}$

L = 40

$$B_{z\_max} = \frac{\mu_0 N I}{2}\left[\frac{20 - 10}{\sqrt{10^2 + (20-10)^2}} + \frac{20 + 10}{\sqrt{10^2 + (-20-10)^2}}\right]$$

$$= [1{,}6558] \quad \frac{\mu_0 N I}{2} \text{ constante}$$

$$B_{z\_min} = \frac{\mu_0 N I}{2}\left[\frac{20}{\sqrt{10^2 + 20^2}} + \frac{20}{\sqrt{10^2 + (-20)^2}}\right]$$

$$= [1{,}788] \quad \frac{\mu_0 N I}{2} \text{ constante}$$

$$\frac{B_{z\_max}}{B_{z\_0}} = \frac{1{,}6558}{1{,}788} = 92{,}60\%$$

$\boxed{L \text{ de } 40 = 92{,}6\%}$

L = 60

$$B_{z\_max} = \frac{\mu_0 N I}{2}\left[\frac{20}{\sqrt{10^2 + 20^2}} + \frac{40}{\sqrt{10^2 + (-40)^2}}\right]$$

$$= [1{,}86456] \quad \frac{\mu_0 N I}{2} \text{ constante}$$

$$B_{z\_min} = \frac{\mu_0 N I}{2}\left[\frac{60}{\sqrt{10^2 + 30^2}}\right]$$

$$= [1{,}89737] \quad \frac{\mu_0 N I}{2} \text{ constante}$$

$$\frac{B_{z\_max}}{B_{z\_0}} = \frac{1{,}86456}{1{,}89737} = 98{,}27\%$$

$\boxed{L \text{ de } 60 = 98{,}27\%}$

L = 100

$$B_{z\_max} = \frac{\mu_0 N I}{2}\left[\frac{50 - 10}{\sqrt{10^2 + (50-10)^2}} + \frac{50 + 10}{\sqrt{10^2 + (-50-10)^2}}\right]$$

$$= [1{,}9565] \quad \frac{\mu_0 N I}{2} \text{ constante}$$

$$B_{z\_min} = \frac{\mu_0 N I}{2}\left[\frac{50}{\sqrt{2600}} + \frac{50}{\sqrt{2600}}\right]$$

$$= [1{,}96116] \quad \frac{\mu_0 N I}{2} \text{ constante}$$

$$\frac{B_{z\_max}}{B_{z\_0}} = \frac{1{,}9565}{1{,}96116} = 99{,}76\%$$

$\boxed{L \text{ de } 100 = 99{,}76\%}$

Mandat 12 :

12.1 :

On Cherche nombre de spires pour 7T

On Soit $I = 500$

$u_0 = 4\pi \times 10^{-7}$

$B = 7$

$$B = \frac{u_0 N I}{2}\left[\frac{\frac{L}{2} - z_0}{\sqrt{R^2 + (\frac{L}{2} - z_0)^2}} + \frac{\frac{L}{2} + z_0}{\sqrt{R^2 + (-\frac{L}{2} - z_0)^2}}\right]$$

$z_0 = 0$

$L/2 = 10$

$R = 10$

$$7 = \frac{4\pi \times 10^{-7} \cdot N \cdot 500}{2}\left[\frac{10}{\sqrt{200}} + \frac{10}{\sqrt{200}}\right]$$

$$7 = 4\pi \times 10^{-7} \cdot N \cdot 500 \cdot 0.707$$

$$7 = 0.000444N$$

$$\boxed{N = 15\ 757}$$

12.2 :

On Cherche la longueur de Cuivre

On sait que $N = 15\ 757$ par m

$L = 20, 40, 60, 100$ cm

rayon $= 0,1$ m

$R = 0,1515$

$2\pi \cdot 0,1 \cdot \frac{15757}{5} = 1,98$ Km — diviser par 5 pour donner sur 20 cm

$2\pi \cdot 0,1 \cdot \frac{12456}{2,5} = 3,13$ Km — diviser par 2.5 pour donner sur 40 cm

$2\pi \cdot 0,1 \cdot 3 \cdot \frac{11\ 743}{5} = 4,43$ Km — diviser par $\frac{5}{3}$ pour donner sur 60 cm

$2\pi \cdot 0,1 \cdot 11361 = 7,14$ Km — diviser par 1 pour donner sur 100 cm

Calcul fait sur python

Mandat 13:

On cherche la puissance dissipé

On sait $R = \frac{L}{\sigma A}$ $\quad I = 500A$

$V = RI$

$\sigma = 6 \times 10^8$

$P = VI \longrightarrow P = I^2R$

$R = \frac{L}{\sigma A}$

$R = \frac{1980.08}{5.8 \cdot 10^7 \, \pi (2.5 \cdot 10^{-3})^2} = 1.7387$

$P = I^2R$

$P = 500^2 \cdot 1.7387 = 434\,675\ W$

$\boxed{434\,675 W \text{ pour } L = 1980.08 m}$ $\boxed{L = 20}$

$R = \frac{L}{\sigma A}$

$R = \frac{3130.5}{5.8 \cdot 10^7 \, \pi (2.5 \cdot 10^{-3})^2} = 2.75$

$P = I^2R$

$P = 500^2 \cdot 2.75 = 687\,220 W$

$\boxed{687\,220 W \text{ pour } L = 3130.5 m}$ $\boxed{L = 40}$

$R = \frac{L}{\sigma A}$

$R = \frac{4427}{5.8 \cdot 10^7 \, \pi (2.5 \cdot 10^{-3})^2} = 3.887$

$P = I^2R$

$P = 500^2 \cdot 3.887 = 971\,833 W$

$\boxed{971\,833 W \text{ pour } L = 4427 m}$ $\boxed{L = 60}$

$R = \frac{L}{\sigma A}$

$R = \frac{7138.33}{5.8 \cdot 10^7 \, \pi (2.5 \cdot 10^{-3})^2} = 6,268$

$P = I^2R$

$P = 500^2 \cdot 6.268 = 1\,567\,035\ W$

$\boxed{1\,567\,035 W \text{ pour } L = 7138 m}$ $\boxed{L = 100}$

Mandat 14 :

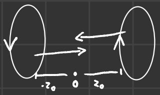
On Cherche

On Sait

Mandat 15 :

On cherche un développement de taylor pour le champ magnétique

On sait : $\vec{B} = \dfrac{\mu_0 r^2 I}{2(r^2+(z\pm z_0)^2)^{3/2}}$



On voit que les champs travail un contre l'autre donc on soustrait

formule générale :

$$\vec{B}_{tot} = \frac{\mu_0 r^2 I}{2(r^2+(z-z_0)^2)^{3/2}} - \frac{\mu_0 r^2 I}{2(r^2+(z+z_0)^2)^{3/2}}$$

$$\boxed{f(z) = \frac{\mu_0 r^2 I}{2}\left(\frac{1}{(r^2+(z-z_0)^2)^{3/2}} - \frac{1}{(r^2+(z+z_0)^2)^{3/2}}\right)}$$ f(z) finale

$$f(0) = \frac{\mu_0 r^2 I}{2}\left(\frac{1}{(r^2+(-z_0)^2)^{3/2}} - \frac{1}{(r^2+(z_0)^2)^{3/2}}\right)$$

$$f(0) = \frac{\mu_0 r^2 I}{2}\left(\frac{1}{(r^2+(z_0)^2)^{3/2}} - \frac{1}{(r^2+(z_0)^2)^{3/2}}\right)$$ car $(-x)^2 = (x)^2$

$$f(0) = \frac{\mu_0 r^2 I}{2}(0)$$

$\boxed{f(0) = 0}$ à $z=0$

Première dérivé :

$$f(z) = \frac{\mu_0 r^2 I}{2}\left(\frac{1}{(r^2+(z-z_0)^2)^{3/2}} - \frac{1}{(r^2+(z+z_0)^2)^{3/2}}\right)$$

$$f'(z) = \frac{\mu_0 r^2 I}{2}\left((r^2+(z-z_0)^2)^{-3/2} - (r^2+(z+z_0)^2)^{-3/2}\right)$$

$$f'(z) = \frac{\mu_0 r^2 I}{2}\left(\frac{-3}{2}(r^2+(z-z_0)^2)^{-5/2}\,2(z-z_0) - -\frac{3}{2}(r^2+(z+z_0)^2)^{-5/2}\,2(z+z_0)\right)$$

$$f'(z) = \frac{\mu_0 r^2 I}{2}\left(-\frac{3\cdot 2\cdot(z-z_0)}{2(r^2+(z-z_0)^2)^{5/2}} + \frac{3\cdot 2\cdot(z+z_0)}{2(r^2+(z+z_0)^2)^{5/2}}\right)$$

$$\boxed{f'(z) = \frac{\mu_0 r^2 I}{2}\left(\frac{-3(z-z_0)}{(r^2+(z-z_0)^2)^{5/2}} + \frac{3(z+z_0)}{(r^2+(z+z_0)^2)^{5/2}}\right)}$$ f'(z) finale

$$f'(0) = \frac{\mu_0 r^2 I}{2}\left(\frac{-3(-z_0)}{(r^2+(-z_0)^2)^{5/2}} + \frac{3z_0}{(r^2+z_0^2)^{5/2}}\right)$$

$$f'(0) = \frac{u_0 r^2 I}{2}\left(\frac{3 z_0}{(r^2+z_0^2)^{5/2}} + \frac{3 z_0}{(r^2+z_0^2)^{5/2}}\right)$$ Car $(-x)^2 = (x)^2$ et $-a \cdot -b = ab$

$$f'(0) = \frac{u_0 r^2 I}{2}\left(\frac{2 \cdot 3 z_0}{(r^2+z_0^2)^{5/2}}\right)$$

$$\boxed{f'(0) = \frac{3 u_0 r^2 I z_0}{(r^2+z_0^2)^{5/2}}}$$ a $z=0$

Deuxième dérivée:

$$f'(z) = \frac{u_0 r^2 I}{2}\left(\frac{-3(z-z_0)}{(r^2+(z-z_0)^2)^{5/2}} + \frac{3(z+z_0)}{(r^2+(z+z_0)^2)^{5/2}}\right)$$

$$f''(z) = \frac{u_0 r^2 I}{2}\left(-3(z-z_0)(r^2+(z-z_0)^2)^{-5/2} + 3(z+z_0)(r^2+(z+z_0)^2)^{-5/2}\right)$$ on sait que $\frac{d f(x) g(x)}{dx} = f'(x) g(x) + f(x) g'(x)$

$$f''(z) = \frac{u_0 r^2 I}{2}\left(-3(r^2+(z-z_0)^2)^{-5/2} + -3(z-z_0) \cdot -\frac{5}{2}(r^2+(z-z_0)^2)^{-7/2} 2(z-z_0) + 3(r^2+(z+z_0)^2)^{-5/2} + 3(z+z_0) \cdot -\frac{5}{2}(r^2+(z+z_0)^2)^{-7/2} 2(z+z_0)\right)$$

$$f''(z) = \frac{u_0 r^2 I}{2}\left(\frac{-3}{(r^2+(z-z_0)^2)^{5/2}} + \frac{15(z-z_0)^2}{(r^2+(z-z_0)^2)^{7/2}} + \frac{3}{(r^2+(z+z_0)^2)^{5/2}} - \frac{15(z+z_0)^2}{(r^2+(z+z_0)^2)^{7/2}}\right)$$

$$f''(z) = \frac{-3 u_0 r^2 I}{2}\left(\left(\frac{1}{(r^2+(z-z_0)^2)^{7/2}}\left((r^2+(z-z_0)^2) - 5(z-z_0)^2\right)\right) - \left(\frac{1}{(r^2+(z+z_0)^2)^{7/2}}\left((r^2+(z+z_0)^2) - 5(z+z_0)^2\right)\right)\right)$$

$$\boxed{f''(z) = \frac{-3 u_0 r^2 I}{2}\left(\frac{1}{(r^2+(z-z_0)^2)^{7/2}}\left(r^2 - 4(z-z_0)^2\right) - \frac{1}{(r^2+(z+z_0)^2)^{7/2}}\left(r^2 - 4(z+z_0)^2\right)\right)}$$ $f''(z)$ finale

$$f''(0) = \frac{-3 u_0 r^2 I}{2}\left(\frac{1}{(r^2+(-z_0)^2)^{7/2}}\left(r^2 - 4(-z_0)^2\right) - \frac{1}{(r^2+(z_0)^2)^{7/2}}\left(r^2 - 4(z_0)^2\right)\right)$$

$$f''(0) = \frac{-3 u_0 r^2 I}{2}\left(\frac{1}{(r^2+(z_0)^2)^{7/2}}\left(r^2 - 4(z_0)^2\right) - \frac{1}{(r^2+(z_0)^2)^{7/2}}\left(r^2 - 4(z_0)^2\right)\right)$$

$$f''(0) = \frac{-3 u_0 r^2 I}{2}(0)$$

$$\boxed{f''(0) = 0}$$ à $z=0$

Troisième dérivée :

$$f''(z) = \frac{-3u_0 r^2 I}{2}\left(\frac{1}{(r^2+(z-z_0)^2)^{7/2}}(r^2-4(z-z_0)^2) - \frac{1}{(r^2+(z+z_0)^2)^{7/2}}(r^2-4(z+z_0)^2)\right)$$

$$f'''(z) = \frac{-3u_0 r^2 I}{2}\left((r^2+(z-z_0)^2)^{-7/2}(r^2-4(z-z_0)^2) - (r^2+(z+z_0)^2)^{-7/2}(r^2-4(z+z_0)^2)\right)$$

$$f'''(z) = \frac{-3u_0 r^2 I}{2}\left(\left(-\frac{7}{2}\cdot 2(z-z_0)(r^2+(z-z_0)^2)^{-9/2}(r^2-4(z-z_0)^2) + (r^2+(z-z_0)^2)^{-7/2}(-8(z-z_0))\right) - \left(-\frac{7}{2}\cdot 2(z+z_0)(r^2+(z+z_0)^2)^{-9/2}(r^2-4(z+z_0)^2) + (r^2+(z+z_0)^2)^{-7/2}(-8(z+z_0))\right)\right)$$

$$f'''(z) = \frac{-3u_0 r^2 I}{2}\left(\left(\frac{-7(z-z_0)(r^2-4(z-z_0)^2)}{(r^2+(z-z_0)^2)^{9/2}} - \frac{8(z-z_0)}{(r^2+(z-z_0)^2)^{7/2}}\right) - \left(\frac{-7(z+z_0)(r^2-4(z+z_0)^2)}{(r^2+(z+z_0)^2)^{9/2}} - \frac{8(z+z_0)}{(r^2+(z+z_0)^2)^{7/2}}\right)\right)$$

On distribue le −

$$f'''(z) = \frac{-3u_0 r^2 I}{2}\left(\frac{-7(z-z_0)(r^2-4(z-z_0)^2)}{(r^2+(z-z_0)^2)^{9/2}} - \frac{8(z-z_0)}{(r^2+(z-z_0)^2)^{7/2}} + \frac{7(z+z_0)(r^2-4(z+z_0)^2)}{(r^2+(z+z_0)^2)^{9/2}} + \frac{8(z+z_0)}{(r^2+(z+z_0)^2)^{7/2}}\right)$$

$$f'''(z) = \frac{-3u_0 r^2 I}{2}\left((r^2+(z-z_0)^2)^{-9/2}(z-z_0)\underbrace{\left(-7(r^2-4(z-z_0)^2) - 8(r^2+(z-z_0)^2)\right)}_{-7r^2+28(z-z_0)^2-8r^2-8(z-z_0)^2} - (r^2+(z+z_0)^2)^{-9/2}(z+z_0)\left(-7(r^2-4(z+z_0)^2) - 8(r^2+(z+z_0)^2)\right)\right)$$

$$\boxed{f'''(z) = \frac{-3u_0 r^2 I}{2}\left(\frac{(z-z_0)(-15r^2+20(z-z_0)^2)}{(r^2+(z-z_0)^{9/2}} - \frac{(z+z_0)(-15r^2+20(z+z_0)^2)}{(r^2+(z+z_0)^2)^{9/2}}\right)}$$

$f'''(z)$ finale

$$f'''(0) = \frac{-3u_0 r^2 I}{2}\left((r^2+z_0^2)^{-9/2}(z_0)(15r^2-20z_0) - (r^2+z_0^2)^{-9/2}(z_0)(-15r^2+20z_0)\right)$$

$$f'''(0) = \frac{-3u_0 r^2 I}{2}\left((r^2+z_0^2)^{-9/2}(z_0)(15r^2-20z_0+15r^2-20z_0)\right)$$

$$\boxed{f'''(0) = \frac{-3u_0 r^2 I}{2}\left(\frac{(z_0)(30r^2-40z_0^2)}{(r^2+z_0^2)^{9/2}}\right)}$$

à $z=0$

$$0 = 30r^2 - 40z_0^2$$

$$30r^2 = 40z_0^2$$

$$\frac{30}{40}r^2 = z_0^2$$

$$\sqrt{\frac{3}{4}}\,r = z_0$$

$$\boxed{\text{Pour forcer à } 0 \quad z_0 = \sqrt{3/4}\,r}$$

Somme

$f(0) = 0$

$f'(0) = \dfrac{3u_0 r^2 I z_0}{(r^2+z_0^2)^{5/2}}$ on Calcul ō $z_0 = \sqrt{\frac{3}{4}}\, r$

$f''(0) = 0$

$f'''(0) = -\dfrac{3u_0 r^2 I}{2}\left(\dfrac{(z_0)(30r^2 - 40z_0^2)}{(r^2+z_0^2)^{9/2}}\right)$ on Calcul ō $z_0 = \sqrt{\frac{3}{4}}\, r$

$f'(0) = \dfrac{3u_0 r^2 I \sqrt{\frac{3}{4}}\, r}{(r^2 + (\sqrt{\frac{3}{4}}\, r)^2)^{5/2}}$

$f'''(0) = -\dfrac{3u_0 r^2 I}{2}\left(\dfrac{\sqrt{3/4}\,(30r^2 - 40(\sqrt{3/4}\, r)^2)}{(r^2+(\sqrt{\frac{3}{4}}\, r)^2)^{9/2}}\right)$

$f'(0) = \dfrac{\frac{3\sqrt{3}}{2} r^3 I}{(\frac{7}{4} r^2)^{5/2}}$

$f'''(0) = 0$

$f'(0) = \dfrac{48\sqrt{3}\, I u_0}{49\sqrt{7}\, r^2}$

$f(z) = f(0) + f'(0)z + \dfrac{f''(0)z^2}{2!} + \dfrac{f'''(0)z^3}{3!}$

$f(z) = 0 + \dfrac{48\sqrt{3}\, I u_0}{49\sqrt{7}\, r^2} z + \dfrac{0z^2}{2!} + \dfrac{0z^3}{3!}$

$\boxed{f(z) = \dfrac{48\sqrt{3}\, I u_0}{49\sqrt{7}\, r^2} z}$ nombre de taylor de la fonction

Mandat 16 :

On cherche l'amperage pour un gradient de 10 mT

On sait $\mu_0 = 4\pi \cdot 10^{-7}$

$r = 0.1\ m$

$$\nabla \cdot \vec{B} = \frac{\partial f}{\partial z} = \frac{48\sqrt{3}\, I \mu_0}{49\sqrt{7}\, r^2}\ z$$

$$\frac{\partial f}{\partial x} = 0$$

$$\frac{\partial f}{\partial y} = 0$$

$$\frac{\partial f}{\partial z} = \frac{48\sqrt{3}\, I \mu_0}{49\sqrt{7}\, r^2}$$

$$0{,}01 = \frac{48\sqrt{3}\, I \mu_0}{49\sqrt{7}\, r^2}$$

$$I = \frac{0.01 \cdot 49 \cdot \sqrt{7}\, r^2}{48\sqrt{3}\, \mu_0}$$

$$I = \frac{0{,}01 \cdot 49 \cdot \sqrt{3} \cdot 0.1^2}{48\sqrt{3}\ 4\pi \cdot 10^{-7}}$$

$$I = \frac{0{,}012964}{0{,}0001044}$$

$\boxed{I = 124{,}1\ A}$ intensité pour 10 mT

Verification

$$\frac{48\sqrt{3}\, I \mu_0}{49\sqrt{7}\, r^2}\ z$$

$I =$ Ampere

$\mu_0 = kg \cdot m / A^2 s^2$

$r = m$

$T = K / A s^2$

$$\frac{A \cdot Kg \cdot m}{A^2 s^2\ m^2}$$

$\frac{Kg}{s^2 A} = T$ donc c'est OK

$\boxed{\text{Mandat 17 :}}$

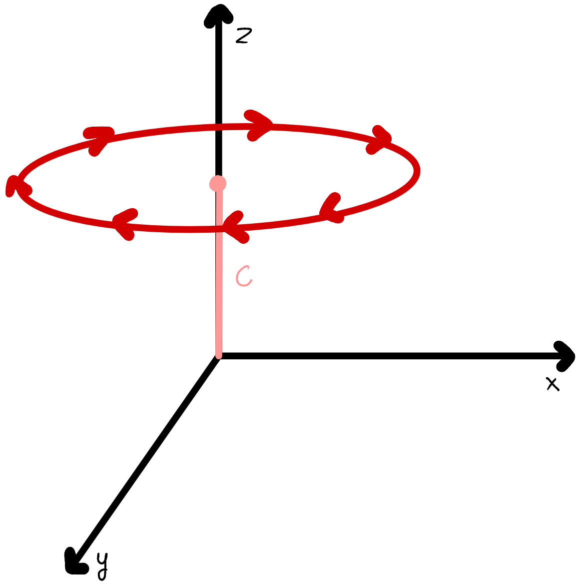
On Cherche

On Sait

Mandat 18:

On Cherche

On Sait


z
C
x
y