Exercise 2:

a) $xs = Ax + Bu$

$xs - Ax = Bu$

$x(Is - A) = Bu$

$x = Bu(Is - A)^{-1}$

$y = (Bu(Is - A)^{-1})C$

$\frac{y}{u} = BC(Is - A)^{-1}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} s - \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} = \begin{bmatrix} s-0 & -1 \\ 6 & s+5 \end{bmatrix}^{-1} = \begin{bmatrix} s+5 & 6 \\ -1 & s \end{bmatrix}$

adjointe

au point de intersection c'est le déterminant du restant

déterminant :

$[s)(s+5)] - [6 \cdot -1) = s^2 + 5s + 6$

↳ ↘ − ↗ si 3x3 c'est (↘ − ↗). intersection

$= \frac{1}{s^2+5s+6} \begin{bmatrix} s+5 & 6 \\ -1 & s \end{bmatrix} \cdot \begin{bmatrix} + & - \\ - & + \end{bmatrix}$ quand addition de les position est impaire

$\frac{1}{s^2+5s+6} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} s+5 & -6 \\ 1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\frac{1}{s^2+5s+6} \begin{bmatrix} s+6 & s-6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \frac{s-6}{s^2+5s+6}$

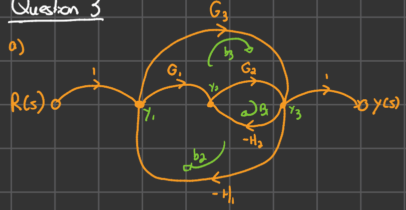
b) $0 = s^2 + 5s + 6 = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 6}}{2} = \frac{-5 \pm 1}{2} = -2 \; -3$ pôles

c) $\frac{y}{u} = \frac{s-6}{s^2+5s+6} \Rightarrow ys^2 + y5s + y6 = us - 6u$

$\ddot{y} + 5\dot{y} + 6y = \dot{u} - 6u$

Question 3

a)



b) $T_1 = G_1G_2$

$T_2 = G_3$

$G_{B_1} = -H_2G_2$

$G_{B_2} = -H_1G_1G_2$

$G_{B_3} = -H_1G_3$

$\Delta = 1 - (G_{B_1} + G_{B_2} + G_{B_3})$

$\Delta_1 = 1 + (G_{B_1} + G_{B_2} + G_{B_3}) = 1$

$G = \frac{1}{\Delta} \sum T_K \Delta_K$

$G = \frac{1}{1 - G_{B_1} - G_{B_2} - G_{B_3}} \cdot (T_1 + T_2) = \frac{G_1G_2 + G_3}{1 + H_2G_2 + H_1G_1G_2 + H_1G_3}$

Question 4:

$$\dddot{x} + 4\ddot{x} + 8\dot{x} + Kx = 3u + 5\dot{u}$$

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K & -8 & -4 \end{bmatrix}$ $B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$

$B_0 = 0$
$B_1 = 0$
$B_2 = 5$
$B_3 = 3 - 4 \cdot 5 = -17$

a) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K & -8 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 5 \\ -17 \end{bmatrix}$ $C = [1\ 0\ 0]$ $D = [0]$

b) $[\lambda I \cdot A]$ = trouver dénominateur

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\lambda - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K & -8 & -4 \end{bmatrix} = \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ K & 8 & \lambda+4 \end{bmatrix} = \lambda(\lambda^2 + 4\lambda) - -1(-K) + 0$$

$$\lambda^3 + 4\lambda - K = 0$$

Question 5:

a) $\dfrac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$A = R(s) - y(s)$

$y(s) = \dfrac{A\, K_p(25)}{s(s+2)}$

$y(s) = \dfrac{(R(s) - y(s))\, K_p(25)}{s(s+2)}$

$y\,(s(s+2)) = RK_p25 - yK_p25$

$y(s^2 + 2s + K_p25) = RK_p25$

$\dfrac{y}{R} = \dfrac{K_p25}{s^2 + 2s + K_p25}$

$K_p = 1$

$2\zeta\omega_n = 2$

$2\zeta\sqrt{K_p25} = 2$

$K_p25 = 4$

$K_p = \dfrac{4}{25} = 0.16$

b) $t_p = \dfrac{\pi}{(0.16 \cdot 25)\sqrt{1 - 0.5^2}}$

$t_p = \dfrac{\pi}{4\,(0.86602)} = 0.906899$

$M_p = 100e^{-\pi/\tan(\cos^{-1}(0.5))} = 16.3033$

c) $A = R - y$

$B = K_pA - K_t\left(\dfrac{25}{s+2}\right)B$

$B\left(K_t\left(\dfrac{25}{s+2}\right) + 1\right) = K_pA$

$B = \dfrac{K_p(R-y)}{K_t\left(\dfrac{25}{s+2}\right) + 1}$

$Y = B\left(\dfrac{25}{s(s+2)}\right)$

$y = \dfrac{K_p(R-y)}{\left(K_T\left(\dfrac{25}{s+2}\right) + 1\right)} \dfrac{25}{s(s+2)}$ (s^2+2s) $\left(\dfrac{K_t25}{s+2} + 1\right)$

$y = \dfrac{25K_pR - 25K_py}{\dfrac{s^2K_T25}{s+2} + s^2 + \dfrac{2sK_t25}{s+2} + 2s}$

Question 6:

$$m\ddot{x} = -K_1 x - K_2 x^2 + \sqrt{F}$$

$$0 = -K_2 x_c^2 - K_1 x_c + \sqrt{F_c}$$ Equation équilibe

$$-K_2 x^2 = -K_2 x^2 - 2K_2 x_c \Delta x$$

On derive

$$\sqrt{F} = \sqrt{F_c} - \frac{1}{2\sqrt{F_c}} \Delta F$$

$$m\frac{d^2x}{dt} = -K_1 x - (-K_1 x_c) - (-K_2 x_c^2 - 2K_2 x_c \Delta x) - \left(\sqrt{F_c} - \frac{1}{2\sqrt{F_c}} \Delta F\right)$$

$$-K_1 x_c \qquad -K_2 x_c^2 \qquad +\sqrt{F_c}$$

On ajoute équation équilibe après avoir changer variable

$$m\frac{d^2x}{dt} = K_1 \Delta x - 2K_2 x_c \Delta x + \frac{1}{2\sqrt{F_c}} \Delta F$$

Question 2:

$\dot{x} = Ax + Bu$

$x - Ax = Bu$

$X(Is - A) = Bu$

$x = Bu\,(Is-A)^{-1}$

$y = C(Is-A)^{-1}Bu$

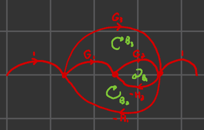
$\frac{y}{u} = C(Is-A)^{-1}B$

$$(Is-A)^{-1} = \begin{bmatrix}1&0\\0&1\end{bmatrix}s - \begin{bmatrix}0&1\\-6&-5\end{bmatrix} = \begin{bmatrix}s&-1\\6&s+5\end{bmatrix}$$

$$\begin{matrix}(-1)^2\, s+5 & 6(-1)^3\\ (-1)^3\, -1 & s\end{matrix} = \begin{bmatrix}s+5&-6\\1&s\end{bmatrix}^T = \begin{bmatrix}s+5&1\\-6&s\end{bmatrix}\frac{1}{s^2+5s+6}$$

$$\frac{y}{u} = \begin{bmatrix}1&1\end{bmatrix}\begin{bmatrix}s+5&-6\\1&s\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = [$$

Question 3:



$G_{B_1} = -H_2G_2$

$G_{B_2} = -H_1G_1G_2$

$G_{B_3} = -H_1G_3$

$T_1 = G_1G_2$

$T_2 = G_3$

$\Delta = 1 - (-H_2G_2 + -H_1G_1G_2 - H_1G_3)$

$\Delta_K = 1 -$ ~~[illegible]~~ $= 1$

$$G = \frac{G_1G_2 + G_3}{1 + H_2G_2 + H_1G_1G_2 + H_1G_3}$$

Question 4:

$\dddot{x} + 4\ddot{x} + 8\dot{x} + Kx = 3u + 5\dot{u}$

$b_0 = 3$ $\quad a_0 = K$

$b_1 = 5$ $\quad a_1 = 8$

$b_2 = 0$ $\quad a_2 = 4$

$b_3 = 0$

$$A = \begin{bmatrix}0&1&0\\0&0&1\\-K&-8&-4\end{bmatrix} \quad B = \begin{bmatrix}0\\5\\-17\end{bmatrix} \quad C = \begin{bmatrix}1&0&0\end{bmatrix} \quad D = [0]$$

$[I\lambda - A] = 0$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\lambda - \begin{bmatrix}0&1&0\\0&0&1\\-K&-8&-4\end{bmatrix} \quad \begin{bmatrix}\lambda&-1&0\\0&\lambda&-1\\K&8&\lambda+4\end{bmatrix} = \lambda(\lambda(\lambda+4)+8) + -1(-K)$$

$\lambda(\lambda^2+4\lambda+8) + K$

$\lambda^3 + 4\lambda^2 + 8\lambda + K$

Question 5:

$A = R_{(s)} - y_{(s)}$

$$y_{(s)} = \frac{A K_p 25}{s(s+2)} = \frac{R_{(s)}K_p25 - y_{(s)}K_p25}{s(s+2)} = y_{(s)}(s(s+2)) = R_{(s)}K_p25 - y_{(s)}K_p25$$

$y_{(s)}(s(s+2)) + y_{(s)}K_p25) = R_{(s)}K_p25$

$y_{(s)}(s^2+2s+25K_p) = R_{(s)}K_p25$

$$\frac{y_{(s)}}{R_{(s)}} = \frac{K_p25}{s^2+2s+25K_p}$$

$2 = 2\zeta\omega_n$ $\qquad \omega_n^2 = 25K_p$

$2 = \omega_n$ $\qquad 2^2 = 25K_p$

$K_p = 0.16$

$$T_p = \frac{\pi}{2\sqrt{1-0.5^2}} \qquad \frac{\pi}{1.73} \qquad T_p = 1.8159$$

$\cos^{-1}(\zeta)$

$M_p = 100e^{-\pi/\tan(\cos^{-1}(\zeta))}$

Question 6:

$$m \frac{d^2x}{dt^2} = -K_1 x - K_2 x^2 + \sqrt{F}$$

$$0 = -K_1 x_{eq} - K_2 x_{eq}^2 + \sqrt{F_{eq}}$$

$$-K_2 x^2 = -K_2 x_{eq}^2 - 2K_2 x_{eq} \Delta x$$

$$\sqrt{F} = \sqrt{F} + \sqrt{F_{eq}} \Delta F$$

$$m \frac{d^2x}{dt^2} = -K_1 x \quad + K_2 x_{eq}^2 + 2K_2 x_{eq} \Delta x + \sqrt{F} + \frac{1}{2\sqrt{F_{eq}}} \Delta F$$

$$= -K_1 x + K_1 x_{eq} + K_2 x_{eq} - K_2 x_{eq}^2 + 2K_2 x_{eq} \Delta x + \sqrt{F} + \frac{1}{2\sqrt{F_{eq}}} \Delta F \cdot \sqrt{F}$$

$$m \Delta \ddot{x} = -K_1 \Delta x + 2K_2 x_{eq} \Delta x + \frac{1}{2\sqrt{F_e}} \Delta F$$