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ARTICLE

# Name of our game

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**Abstract**

Globalization, as a complex phenomenon, encompasses many dimensions, one of which is mobility of individuals. An interesting case in practice is the one of youths: a considerable part of younger generations from some developed and developing countries will experience studying or working in a another country over their lifetime. However, this dynamic is not exempt of frictions, among which language seems of particular importance. While geographic distance can be virtually shrinked by an aircraft system, learning a new language takes time and requires efforts, with a potential biological upper bound on learning speed. We propose a game theoretic approach to understand the situation faced by players in a linguistic framework when evolving in an international environment. The authors' own experiences, including ERASMUS exchanges, highlight the dilemmas and the importance of strategic interactions between players when deciding whether to learn a new language in a foreign country. Although theoretical, we think that a game theoretic approach can help explaining which language people end up speaking. In particular, we observed that many students end up using english, turning the language in some "globish" tongue, and we propose to investigate this state as a (subgame perfect) Nash equilibrium.

The main setup is the following: $N$ people in a community (here, one can consider it an ERASMUS residence) are each endowed with some language skills. There are $L$ languages spoken in the community and people dislike speaking a language that is not their mother tongue, the more the language is different from theirs, the more they dislike speaking it. Using a language distance index, we study what the optimal behaviour of individuals is in the context of a single language learning opportunity (ie. individuals cannot learn more than one language at a time and they may learn none).

**Keywords:** Add keywords here

For thousands of years, human beings exchanged feelings, ideas, thoughts, great and small, marvelous and terrible; novelists and journalists wrote about it, praised language and their knowledge. However, it is only in the last century with the advent of mass education and globalization that language exacerbated its importance. Owing to increasing trade and rising communication across civilizations, learning an extra-national language became a norm in most industrialized countries. In particular, due to the economic and cultural leadership of the United States of America on the Western world, English rose as the most important language worldwide. Surprisingly, and despite its importance for social human beings, the question of whether or not learning a language has not been actively investigated as a strategic decision taken by individuals. In the economics field, a pioneer work was done by ?, who formulated the question of convergence towards a common language in a vast community in game theoretic terms. However, his work was not followed and generated little interest after its publication[1]. A potential explanation for the failure of this framework is the strong philosophical importance granted to the question of tongues and the use by ? of the expression "language game" to refer to the different senses a same word or expression can have under different contexts; which might have furthered away the understanding of a "language game" as a strategic interaction about the choice of some tongue for itself.



1. One can see, for instance by graph representation, that his paper is not related to much papers and generated no further work about the relationship between learning a language and game theory.

In this paper, we contribute to the economics literature by re-introducing the learning of a language in a game theoretical framework. The baseline game we introduce is a two-stage game where individuals can learn a new language in the first stage and decide which language to speak in a second stage. We also propose an additive functional form for the payoffs of learning a language and introduce a cost for learning a language based on a language index accounting for the pairwise differences between tongues. For the sake of example, we introduce specific values for costs, index and payoffs but provide an optimality condition in more general terms as to verify whether an outcome is a Nash equilibrium. We also deal with a Bayesian version of the game where individuals do not know the exact structure of the community they live in.

Conceptually, our game is a coordination game: players want to coordinate on using a common language to communicate, but they are most of the time confronted to situations in which the cost of communications are different from the person they talk to. It is similar to the game originally called the "battle of the sexes" proposed by **?** in their book. Like in this game, players of the (name of our game) have an incentive to choose the same action to get some positive payoff, although this action may yield (sometimes extremely) different payoffs for both of them. The baseline game we propose is however more complex than the one introduced by Luce and Raiffa since we assume the presence of $N$ players and up to $L$ (the number of languages minus 1) strategies for each of them. The core of the game is however the same: with different preferences, the players interact strategically to share the surplus from communication, .

This research proposal is organized as follows: in a first section, we introduce a toy game for the choice of languages between two players, investigate equilibria in this game and give examples of its application; in a second part, we derive our full game under uncertainty but with complete information (players know everything about the game but do not know exactly whom they will end-up talking with) and introduce our language index . Thirdly, we propose a baseline bayesian version of the game and derive optimality condition for a perfect bayesian equilibrium. Finally, we discuss the potential for further research about the topic of game theory and languages, including proposal to test for the results we derived and potential extensions of the game.

## 1. A toy game

Our basic game features the following setting:

- 2 **players.**
- $L$ **languages.**
  - Without loss of generality (WLOG), we denote $\{l_1, l_2, \ldots, l_L\}$ the $L$ languages.
  - We denote $L_i \equiv \{l_j | \text{individual } i \text{ speaks language} l_j\}$.
  - We denote $l_i^m$ as the mother tongue of individual $i$.
- **Actions**: players decide which language to speak.
- **Outcomes:** Players speak the language they commonly chose if they chose the same language; if they decided to speak different languages, they end-up communicating with a low-level globish language (say it is sign language for instance).
- **Payoffs**: Payoffs are symmetric for all players. If players end-up speaking globish, they get a zero payoff. If they agree on speaking some language, the payoff is strictly positive. However, there is a cost for speaking a language that is not your mother language: the more different the language from your mother tongue, the higher the cost. Mathematically, we denote the payoff of speaking with individual $j$ for individual $i$ by:

$$u_i\left(l^i, l^j\right) \equiv \begin{cases} 0 & \text{if } l^i \neq l^j \\ p_i\left(l_s\right) - c_i^1\left(l_s\right) & \text{if } l^i = l^j \end{cases}$$

where

- $l^i$ is the language chosen by $i$ and $l^j$ is the language chosen by $j$.
- The cost of speaking language $l_s$ is $c_i^1(l_s) \equiv ln(1 + I(l_i^m, l_s))$ (which is independent of $j$), with $I(l_i^m, l_s) \in [0,1]$ the distance between language $l_s$ and the mother tongue of $i$, with 0 indicating that $l_s$ is the mother tongue of $i$.
- The utility derived from speaking language $l_s$ is $p_i(l_s) \equiv 1$. In particular, it does not depend upon whom the individual you talk to is.

## 1.1 Derivation of equilibria

**Proposition 1.** *When players of this game speak no common language, any choice of language is a weakly dominant strategy.*

*Proof.* Assume players $i$ and $j$ have no language in common (ie. $L_i \cap L_j = \emptyset$). Then:

$$\forall \left(l^i, l^j\right) \in L_i \times L_j, l^i \neq l^j \Rightarrow u_i\left(l^i, l^j\right) = 0$$

Thus, the language players choose does not matter, they will always end up speaking globish. □

A direct corollary of this proposition is that all strategy profiles are Nash equilibria (NE) when players have no language in common:

**Corollary 1.** *If players do not know a common language, all strategy profiles are NE.*

**Proposition 2.** *There is no weakly dominant strategy in this game whenever players have at least two languages in common.*

*Proof.* Assume players $i$ and $j$ have at least two languages in common (ie. $card\left(L_i \cap L_j\right) \geq 2$). Without loss of generality (WLOG), we show that player $i$ has no weakly dominant strategy. Assume $j$ chooses $l_s \in L_i$, then:

$$\forall l_k \in L_i, u_i(l_s, l_s) > 0 = u_i(l_k, l_s)$$

In particular, when this is the case, we have that $l_s \in L_i \cap L_j$ is the only best response and is thus strictly better than any $l_k \neq l_s, l_k \in L_i \cap L_j$. But conversely, choosing $l_k \in L_i \cap L_j$ is strictly better than choosing $l_s$ when $j$ chooses $l_k$. It results that there cannot be any weakly dominant strategy when players speak more than two languages in common. □

**Proposition 3.** *When players can speak **exactly** one language in common, it is a weakly dominant strategy for both of them to speak this language:*

*Proof.* Assume players know one language in common (ie. $L_i \cap L_j = \{l_s\}$). WLOG, we show that $l_s$ is always a best response for player $i$. Assume player $j$ chooses language $l_k \neq l_s$. Then:

$$\forall l_m \in L_i, u_i(l_m, l_k) = 0$$

So $l_s$ is a best response. Assume player $j$ chooses language $l_s$. Then:

$$\forall l_m \in L_i \backslash \{l_s\}, u_i(l_s, l_s) > 0 = u_i(l_m, l_k)$$

It results that $l_s$ is the unique best response in this case and is also a best response for any other strategy by player $j$. Hence, $l_s$ is the only weakly dominant strategy for player $i$. □

**Proposition 4.** *Any outcome where players choose the same language is a NE.*

*Proof.* Assume individuals choose a same language $l_s$. Then we have:

$$\forall i, u_i(l_s, l_s) = p_i(l_s) - c_i^1(l_s) > 0$$

and we also have:

$$\forall l_m \neq l_s, \forall i, u_i(l_m, l_s) = 0$$

Therefore no player can profitably deviate from playing $l_s$ given that the other plays $l_s$. Symmetric choice of language is a NE. □

**Proposition 5.** *If players know a common language,* ***only*** *outcomes where players choose the same language or outcomes where both players choose a language the other does not speak are NE.*

*Proof.* We already proved that choosing a same language is a NE. Now, assume both players choose a language the other does not speak. The proof is similar to the one in proposition 4. Let us show that there is no profitable deviation for player $i$ given strategy $l^j$ by player $j$

$$\forall l^i \in L_i, l^i \neq l^j \Rightarrow u_i\left(l^i, l^j\right) = 0$$

By symmetry, the claim is proved for both players $i$ and $j$.

Now, we prove that there is no other equilibrium. The only other possible strategies are the ones where some player chooses a language the other does not speak and the latter chooses a language the former speaks. WLOG, assume players are respectively $i$ and $j$; player $i$ chooses language $l^i$ and player $j$ chooses $l^j$ such that $l^j \in L_i$. Then:

$$u_i\left(l^j, l^j\right) > 0 = u_i\left(l^i, l^j\right)$$

So player $i$ has a profitable deviation. The strategy profile cannot be a NE. □

At this stage, relying on the above Propositions, we can already see that the game is not **dominance solvable** in many situations (exactly zero or at least two languages in common). Even though the existence of NE guarantees stability of the outcome, we cannot know which of these strategy profiles will be chosen by the players. We suggest that one should investigate socially desirable (via the notion of welfare) outcomes to give intuition about which strategy the players would choose, should they have altruistic preferences (i.e. they have a preference for a higher aggregate surplus).

**Definition 1.** An outcome $\left(l^i, l^j\right)$ is **pareto optimal** if

$$\forall \left(l^i, l^j\right)' \in L_i \times L_j, \left(l^i, l^j\right)' \neq \left(l^i, l^j\right) \Rightarrow \left[u_i\left(l^i, l^j\right)' > u_i\left(l^i, l^j\right)\right] \wedge \left[u_j\left(l^i, l^j\right)' > u_j\left(l^i, l^j\right)\right]$$

Note that $\left(l^i, l^j\right)'$ is a vector, so difference here is difference in vectors, not pairwise difference.

An outcome $\left(l^i, l^j\right)$ is a **socially rational equilibrium** (SRE) if it is Pareto optimal and it is a NE.

We introduce the following Proposition (relying on Definition 1) for the sake of our game:

**Proposition 6.** *When players do not know a common language, all outcomes are SRE.*

*Proof.* The proof is trivial here since whatever players choose, they will get a 0 payoff. □

**Proposition 7.** *When players know a common language, outcomes where both individuals choose a language the other does not know are not SRE.*

*Proof.* Assume players $i$ and $j$ know a common language (ie. $L_i \cap L_j \neq \emptyset$). Then:

$$\forall l \in L_i \cap L_j, \begin{cases} u_i(l,l) & > & 0 & = & u_i\left(l^i, l^j\right) \\ u_j(l,l) & > & 0 & = & u_j\left(l^i, l^j\right) \end{cases}$$

□

**Proposition 8.** *When players know a common language, any SRE involves speaking a common language $l$ such that:*

$$\forall l' \in L_i \cap L_j, u_i(l,l) \geq u_i(l',l') \vee u_j(l,l) \geq u_j(l',l')$$

*Proof.* Again this is a direct result from the definition of SRE. □

An interesting result implied by proposition 8 and our functional form for the payoffs is the following: players must not speak a language that is far from the mother tongue of both players if they both know a language closer to both their mother tongue. An example can be the following: we have two individuals, respectively Italian and French; the Italian speaks Italian, Spanish, French and Chinese; the French speaks also Italian, Spanish, French and Chinese. We have not introduced specific values for the language index at this point[2] but it is clear enough that the three Latin languages are far closer from one another than Chinese. It is straightforward that the individuals do not want to speak Chinese, because they will have a low-quality conversation while they can enjoy speaking Latin languages. We also have that any choice among the three Latin languages is a SRE if Spanish is between French and Italian in the index since in this case you always hurt one individual by shifting from Spanish to either French or Italian. We express it as a corollary:

**Corollary 2.** *In our toy game with the specified functional form, players must not speak a language that is far from the mother tongue of both players if they both know a language closer to both their mother tongue:*

$$\forall (l,l) \in L_i \times L_j, \exists l' \in L_i \cap L_j, \begin{cases} I\left(l_i^m, l\right) & > I\left(l_i^m, l'\right) \\ I\left(l_j^m, l\right) & > I\left(l_j^m, l'\right) \end{cases} \Rightarrow (l,l) \textit{ is not a SRE}$$

*Proof.* Assume $\exists (l,l) \in L_i \times L_j, \exists l' \in L_i \cap L_j, \begin{cases} I\left(l_i^m, l\right) & > I\left(l_i^m, l'\right) \\ I\left(l_j^m, l\right) & > I\left(l_j^m, l'\right) \end{cases}$. Then we have:

$$\begin{cases} u_i(l,l) - u_i(l',l') & = & \left[1 - ln\left(1 + I\left(l_i^m, l\right)\right)\right] - \left[1 - ln\left(1 + I\left(l_i^m, l'\right)\right)\right] \\ u_j(l,l) - u_j(l',l') & = & \left[1 - ln\left(1 + I\left(l_j^m, l\right)\right)\right] - \left[1 - ln\left(1 + I\left(l_j^m, l'\right)\right)\right] \end{cases}$$

$$\Rightarrow \begin{cases} u_i(l,l) - u_i(l',l') & = & ln\left(1 + I\left(l_i^m, l'\right)\right) - ln\left(1 + I\left(l_i^m, l\right)\right) \\ u_j(l,l) - u_j(l',l') & = & ln\left(1 + I\left(l_j^m, l'\right)\right) - ln\left(1 + I\left(l_j^m, l\right)\right) \end{cases}$$

$$\Rightarrow \begin{cases} u_i(l,l) - u_i(l',l') & < & 0 \\ u_j(l,l) - u_j(l',l') & < & 0 \end{cases}$$

By the previous proposition, it follows that $(l,l)$ cannot be a SRE and by arbitrary on $l$, the corollary is proved.Before going to the baseline full game, we can elaborate on the example introduced above, with four languages. Which outcomes are NE depends on which languages players speak: in this case, it is simple since we assume both player speak the same four languages. There are four (symmetric) NE. Moreover, we could investigate which of these four strategy profiles are SRNE.First case: simple index. □

2. Lexicostatistics is a complex discipline by itself and it is not the purpose of this paper to estimate distance between languages. We do not need accurate values for now, since this example only illustrates the concepts and is not the solution of the game. But when we use them later, we will take then from a preexistent index.

## 1.2 Examples

### 1.2.1 An arbitrary simple index

In this case, we assume that the language index is perfectly transitive (i.e. languages can be positioned along a line). We do not use this assumption in the rest of the paper, but presenting it briefly here allows to discuss the costs of learning a new language. We denote the following distances between $F$ (French), $I$ (Italian), $S$ (Spanish) and $C$ (Chinese): $d(F,S) = x$, $d(I,S) = y \Rightarrow d(F,I) = x+y$, and $d(F,C) = z \Rightarrow d(I,C) = x+y+z$, where $x,y,z \in [0,1]$ such that $x < z, x+y+z < 1$. . We have the matrix of payoffs where the row player is French and the column player is Italian:

| | $I$ | $F$ | $S$ | $C$ |
|---|---|---|---|---|
| $I$ | $(1-ln(1+x+y), 1-ln(1+0))$ | $0,0$ | $0,0$ | $0,0$ |
| $F$ | $(0,0)$ | $(1-ln(1+0), 1-ln(1+x+y))$ | $0,0$ | $0,0$ |
| $S$ | $(0,0)$ | $0,0$ | $(1-ln(1+x), 1-ln(1+y))$ | $0,0$ |
| $C$ | $0,0$ | $0,0$ | $0,0$ | $(1-ln(1+z), 1-ln($ |

The strategy profile $(C,C)$ is a NE but it is not Pareto-optimal (all other three strategy profiles Pareto-dominate it). We are left with three outcomes that could be socially optimal (ie. they yield the highest aggregate payoff). To identify the welfare maximizing outcomes, we compare the aggregate payoffs in these three situations:

1. It is clear that the strategy profiles $(I,I)$ and $(F,F)$ are socially rational since the french player cannot obtain a better outcome than $(F,F)$ and the italian player cannot obtain better than $(I,I)$. Moreover, they yield the exact same aggregate payoff $2-ln(1+x+y)$.
2. To assess the case of $(S,S)$, we compare the aggregate payoffs in the outcomes $(I,I)$ and $(S,S)$: when they communicate in Spanish, the payoff is $2-ln(1+x)-ln(1+y) = 2-ln(1+x+y+xy)$, and when they both speak Italian the payoff is $2-ln(1+x+y)-ln(1+0) = 2-ln(1+x+y)$. Since the logarithmic function is strictly increasing and $1+x+y+xy > 1+x+y$, we conclude that the strategy profile $(S,S)$ is not socially optimal (both the all spanish and the all french outcomes are better).

### 1.2.2 A more elaborate index.

In this case, the language index is represented in three dimensions and it is not assumed to be transitive. We present the table of distance indices:

| | $I$ | $F$ | $S$ | $C$ |
|---|---|---|---|---|
| $I$ | $0$ | $x+y+\epsilon$ | $y$ | $x+y+z+\nu$ |
| $F$ | $x+y+\epsilon$ | $0$ | $x$ | $z$ |
| $S$ | $y$ | $x$ | $0$ | $x+z+\eta$ |
| $C$ | $x+y+z+\nu$ | $z$ | $x+\eta$ | $0$ |

The terms $\epsilon, \nu, \eta$ account for the fact that the distance between two languages A and B cannot be reduced to the sum of the distances between A and C plus the distance between C and B due to potential non-transitivity. And $\epsilon, \nu, \eta \leq 0$ (because the index gives the shortest distance between two language points) By assumption, $z > x, y$. The matrix of the payoffs is:

| | $I$ | $F$ | $S$ | |
|---|---|---|---|---|
| $I$ | $1-ln(1+x+y+\epsilon), 1-ln(1+0)$ | $0,0$ | $0,0$ | $0$ |
| $F$ | $0,0$ | $1-ln(1+0), 1-ln(1+x+y+\epsilon)$ | $0,0$ | $0$ |
| $S$ | $0,0$ | $0,0$ | $1-ln(1+x), 1-ln(1+y)$ | $0$ |
| $C$ | $0,0$ | $0,0$ | $0,0$ | $1-ln(1+z), 1-l$ |

Then we compare aggregate costs again. We see that $(C,C)$ is not Pareto-optimal although all strategy profiles on the diagonal are the (only) NE. Now we compare the aggregate cost of $(I,I)$ $(ln(1+x+y+\epsilon))$ to the aggregate cost of $(S,S)$ $(ln(1+x)+ln(1+y) = ln(1+x+y+xy))$. It should also be that $(S,S)$ is not socially optimal in this case.

## 2. The baseline full game

The baseline game with $N+1$ players is a two-stage game. On the first stage, players must choose whether or not to learn some new language. On the second stage, they are randomly assigned to speak with one of the other players with uniform probability. The interest of this game is that you derive no utility from *whom you are talking to* but *from whom you might end up talking to*. We denote $x_i$ as the individual who is assigned to individual $i$. Formally, the setting is:

- $N+1$ **players.** We denote by $i \in \{1,2,\ldots,N+1\}$ the $i$-th individual in the community
- $L$ **foreign languages.**
  - Without loss of generality, we denote $\{l_1, l_2, \ldots, l_L\}$ the $L$ languages.
  - We denote $l_0$ the local language.
  - We denote $L_i \equiv \{l_j | \text{individual } i \text{ speaks language} l_j\}$.
  - We denote $l_i^m$ as the mother tongue of individual $i$.
- **Knowledge:** individuals do not know a priori at stage 1 what language is spoken by whom, they only know the structure of the community in the sense that they know for any language how many people speak it and they know where people come from (i.e. their mother tongue). Knowing the mother tongue and other languages known by the partner implies that one has information about the equilibria of the game (i.e. which of the payoffs in the diagonal are non-zero, as well as the cost function of the partner): we assume perfect information at this stage.
- **Actions**: In the first stage, players choose what language to learn. The choice set of any player $i$ is $L \backslash L_i$, including the empty set. In the second stage, players are randomly assigned a communication partner and play the game of the previous section (with choice set $L_i$).
- **Outcomes:** Players speak a new language if they decided to. And if partners chose a same language, they speak it; otherwise, they end-up communicating with globish. We denote an outcome for a player by

$$O_i \equiv (l_i^c, l_s) \equiv (\text{language player } i \text{ chose to learn}, \text{outcome after randomization at stage 2})$$

  We denote $l_i^c = \emptyset$ if player $i$ chose to learn no language.
- **Payoffs**: Every player gets the utility of the outcome they reached at stage 2 minus the cost of learning the language they chose at stage 1 (0 if they chose to learn no language) plus a bonus for being able to speak the local language (representing the ability to be able to communicate outside the community). We denote it as:

$$v_i(O_i) \equiv u_i(l_s) - c_i^2(l_i^c) + \mathbb{1}_{l_0 \in L_i} B$$

  where

  - The cost of learning a new language $c_i^2(l_i^c) =$
  - The bonus for speaking the local language is $B =$

We remind that, by proposition 6, only symmetric outcomes can be SRE when players speak a common language. We assume that only SRE are chosen by players . It results that the payoff for any player to speak a language can be reduced to her potential payoff of speaking this language with someone (see proposition 9). We denote the fraction of individuals who speak language $l_s$ by $N_{l_s}$.

**Proposition 9.** *In our game, individuals maximize their payoffs by learning the language $l_s$ such that:*

$$l_s \equiv \arg\max_{l \notin L_i} U_i(l)$$

*with:*

$$U_i(l) \equiv \mathbb{1}_{l_0=l} B + \frac{N_l}{N} \left(p_i(l_s) - c_i^1(l)\right) - c_i^2(l_s)$$

*Proof.* By backward induction, equilibrium selection assumption and proposition 8, players have as solutions of the second stage the SRE of the toy game. Up to stage 1, players know they will end-up speaking a common language whenever they can. Hence, when they learn a language, they rationally only consider symmetric outcomes. Their learning decision at stage 1 can only rely on the known structure of the community. Consequently, the payoff of learning a language is only function of: 1) the bonus players can get if the language they learn is the local one, 2) the cost of learning this specific language, 3) the probability that you can speak this language at stage 2 and the utility of speaking this language. Among these parameters, 1 and 2 are well-known and do not need further exploration. The last parameter deserves some explanation: the key to understand 3 is to remind that players have no prior knowledge of which language each individuals speak; thus they cannot use the SRE result of stage 2 to discard non-Pareto-optimal choice of common language, they can only maximize using the probability that the player they are assigned speaks the language they are learning which is defined as:

$$Pr\left\{x_i = j \wedge l_s \in L_j\right\} = \underbrace{Pr\left\{x_i = j\right\} Pr\left\{l_s \in L_j | x_i = j\right\}}_{\text{Bayes rule}} = Pr\left\{x_i = j\right\} \mathbb{1}_{l_s \in L_j}$$

since language players know is completely defined by whom they are.

Hence, any players wants to maximize the potential payoff of learning language $l_s$:

$$U_i(l_s) \equiv \mathbb{1}_{l_0=l_s} B - c_i^2(l_s) + \sum_{j=1}^{N} Pr\left\{x_i = j \wedge l_s \in L_j\right\} \times u_i(l_s)$$

where the second additive part of the function is exactly the probability that they can speak language $l_s$ with an individual $j$. We rewrite it as:

$$\mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\sum_{j=1}^{N}Pr\{x_i=j\wedge l_s\in L_j\}\times u_i(l_s)=\mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\sum_{j=1}^{N}Pr\{x_i=j\}\times\mathbb{1}_{l_s\in L_j}\times u_i(l_s)$$

$$= \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\sum_{j=1}^{N}\frac{1}{N}\times\mathbb{1}_{l_s\in L_j}\times u_i(l_s)$$

$$= \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\sum_{j=1,l_s\in L_j}^{N}\frac{1}{N}\times u_i(l_s)$$

$$= \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\sum_{j=1}^{N_{l_s}}\frac{1}{N}\times u_i(l_s)$$

$$= \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\sum_{j=1}^{N_{l_s}}\frac{1}{N}\times\left(p_i(l_s)-c_i^1(l_s)\right)$$

$$= \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\frac{1}{N}\left(-N_{l_s}c_i^1(l_s)+\sum_{j=1}^{N_{l_s}}p_i(l_s)\right)$$

$$= \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)-\frac{N_{l_s}}{N}c_i^1(l_s)+\frac{1}{N}\sum_{j=1}^{N_{l_s}}p_i(l_s)$$

$$= \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)-\frac{N_{l_s}}{N}c_i^1(l_s)+\frac{N_{l_s}}{N}p_i(l_s)$$

$$= \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\frac{N_{l_s}}{N}\left(p_i(l_s)-c_i^1(l_s)\right)$$

□

This problem is easy to solve as a static optimization problem (it is only a matter of computing payoffs given the structure of the community and finding the language generating the highest payoff)[3]. Here, the interest of the game is that $\forall l_s\in L, N_{l_s}$ does not remain static: players can learn a new language and as a result increase this parameter.

**Proposition 10.** *If learning a language $l_s$ was not an optimal decision for player i in the simple optimization problem and no other player decided to learn $l_s$, then $(l_s,l_{\neg i})$ cannot be a NE with $l_{\neg i}$ the strategies of the other players.*

*Proof.* Assume player $i$ decides to learn language $l_s$ and $l_{opt}$ is *a* language he wishes to learn in the static optimization problem[4]. His expected payoff for $l_s$ is:

$$U_i(l_s)=\mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\frac{N_{l_s}}{N}\left(1-c_i^1(l_s)\right)$$

But, since no other player decided to learn language $l_s$, $N_{l_s}$ remains identical to the parameter in the simple optimization problem. It results:

$$U_i(l_s)<U_i(l_{opt})$$

and so $l_s$ cannot be a best response with this outcome. □

3. Note that an issue might still be that there are too many combinations of players and languages to test for an algorithm to solve the problem in reasonable time.

4. Note that unicity of this optimum is not guaranteed.

**Proposition 11.** *For any given language index, the following SRE condition holds: if $l_{opt}$ is one of the optimal decisions of the simple optimization problem for player i, for $l_{opt}$ not to be a best response anymore and for $l_s$ to become best response in the game we need:*

$$dN_{l_s}\left(1-c_i^1(l_s)\right) \geq dN_{opt}\left(1-c_i^1(l_{opt})\right) + \left\|U_i^{opt}(l_{opt}) - U_i^{opt}(l_s)\right\|$$

*where $dN_{l_m}$ denotes the change in the number of individuals speaking language $l_m$, $\left\|U_i^{opt}(l_{opt}) - U_i^{opt}(l_s)\right\|$ is the Euclidean distance (ie. absolute value in dimension one) between the expected payoff from $l_{opt}$ and $l_s$ in the simple optimization problem.*

*Proof.* We have initially in the simple optimization problem:

$$U_i^{opt}(l_{opt}) \geq U_i^{opt}(l_s)$$

by optimality of the language $l_{opt}$. We rewrite it as:

$$\mathbb{1}_{l_0=l_{opt}}B - c_i^2(l_{opt}) + \frac{N_{l_{opt}}}{N}\left(1-c_i^1(l_{opt})\right) \geq \mathbb{1}_{l_0=l_s}B - c_i^2(l_s) + \frac{N_{l_s}}{N}\left(1-c_i^1(l_s)\right)$$

And for $l_s$ to become the best response in the game, we need:

$$\mathbb{1}_{l_0=l_s}B - c_i^2(l_s) + \frac{N_{l_s}+dN_{l_s}}{N}\left(1-c_i^1(l_s)\right) > \mathbb{1}_{l_0=l_{opt}}B - c_i^2(l_{opt}) + \frac{N_{l_{opt}}+dN_{l_{opt}}}{N}\left(1-c_i^1(l_{opt})\right)$$

where the only change is in the number of people who speak some language since there is neither change in bonus nor in costs. This expression is straightforwardly rewritten as:

$$dN_{l_s}\left(1-c_i^1(l_s)\right) \geq dN_{opt}\left(1-c_i^1(l_{opt})\right) + \left[U_i^{opt}(l_{opt}) - U_i^{opt}(l_s)\right]$$

which is more generally rewritten as:

$$dN_{l_s}\left(1-c_i^1(l_s)\right) \geq dN_{opt}\left(1-c_i^1(l_{opt})\right) + \left\|U_i^{opt}(l_{opt}) - U_i^{opt}(l_s)\right\|$$

□

Proposition 11 extends straightforwardly to a more general optimum condition:

**Proposition 12.** *For any given language index, the following SRE condition holds: for $l_s$ to be a best response in the game we need:*

$$\forall l_m \notin L_i, l_s \neq l_m, dN_{l_s}\left(1-c_i^1(l_s)\right) \geq dN_{l_m}\left(1-c_i^1(l_m)\right) + \left\|U_i^{opt}(l_m) - U_i^{opt}(l_s)\right\|$$

*Proof.* The proof follows the lines of proposition 9. □

The two last results allow to find any NE by differential analysis. At a NE of our game, individuals must best respond to each other. It results that a NE is as follow:

$$\forall i \in \{1,\ldots,N\}, \forall l_m \notin L_i, l_m \neq l_i, dN_{l_i^c}\left(1-c_i^1(l_i^c)\right) \geq dN_{l_m}\left(1-c_i^1(l_m)\right) + \left\|U_i^{opt}(l_m) - U_i^{opt}(l_i^c)\right\|$$

A consequence of this result is that it is easy to verify whether a given outcome is a Nash equilibrium of the game. However, it will be computationally hard to find all possible equilibria, as the litterature about algorithms finding NE highlights (see for instance ??).

We can compute conditions under which most players choose to learn some language. Intuitively, there are several conditions under which this could happen:

1. An initially large number of individuals speak some language $l_s$ that is not an outlier in the index (ie. it is extremely far away from very few other languages spoken in the community)
2. All individuals have very low cost for learning some language (ie. their mother tongues are different but not far away from some intermediate language)

We deal with some examples below:

**Table 1.** A simple matrix form game with two players and two languages

| | $l_1^m$ | $l_2^m$ |
|---|---|---|
| $l_1^m$ | | |
| $l_2^m$ | | |

## 3. A dynamic version of the game

Basically the baseline stage game but repeated
The players observe the full history of the game. So as we mentioned in the previous section, the expected payoffs might change if the proportion of the population speaking a language has changed. It will be the case if at least one player learns any language because we assume a constant number of players N. Since we also assume that a player cannot forget a language in the time frame considered here, this proportion can only increase (or remain constant).
We can predict that since players know which language is known by most people, they will (on average, because of different distance indices) tend to learn this language because it yields higher expected utility (again, on average). So the languages that are known by a majority of players at the beginning of the game will tend to grow more than languages than are spoken by fewer people.
We consider the same setting as in the previous section but we have $T$ periods. The first thing the dynamic aspect will change is the choice set of the players. In the first stage, the choice set will be $(L - L_i)$ (decreasing in $t$ for each player) and in the second stage it will be $L_{i,t}$ (increasing in $t$ for each player). We can see that the indices are not the same for the choice sets of the two stages of the game. The intuition is that in period $t$, a player can only choose to learn a language they didi not know at the previous period $t-1$, but once they have learnt it they can use it to communicate in the given period $t$ (assuming no learning stages, so the player can speak the language as soon as they have learnt it).

## 4. The Bayesian version of the game

Basically the baseline game but with players ignoring the full structure of the game.
This version is different because the learning strategies will not depend on the proportion of the population speaking the languages. The players have uncertainty about which languages others speak. We can say that the type of the player is $L_i$ (the set of languages they speak), which is "chosen by nature" at the beginning of the game, but also evolves. However, the number of elements in $L_i$ can only increase (and the player does not forget languages), meaning that the type of the player cannot completely change. Even though this feature of the game is similar to players' types, but it is in fact more complex because this characteristic about players evolves as the game takes place.
In this version, players keep track of who they had to talk to and eventually use this information to choose which language to learn . This happens because each player reveals information about their type to their partner by choosing to speak a given language (a player $i$ cannot choose a language $l_j \notin L_i$). This information allows the partner to update their belief about the structure of the game.
But it For example if the player was unable to communicate with Italian people more often than they wer was unable to communicate with Chinese people, they will learn Italian. If they are in Italia

(or in an Italian dorm, high $\frac{N_{italian}}{N}$), this will increase the possibilities for this player to get a higher payoff in the future. However, if they are in China (high $\frac{N_{chinese}}{N}$) and just did not happen to talk with many Chinese-speaking partners, this will not be for this player - note that this is by definition unlikely to happen since the probability to get paired with a partner speaking a certain language precisely depends on the proportion of people speaking it. So information is revealed **randomly** (1) and **heterogenously** across players (2). By getting rid of the perfect information assumption, we are likely to find another outcome to our game.
Although a more interesting version of the Bayesian game would be to consider a repeated game setting, here we analyze the static Bayesian game. In a static version, the belief of the player is about which languages are spoken by the others. The player only knows which the language of the country is, but they do not know how many people know it either. It would be extremely complicated (and rather unrealistic) to assume that the player puts a positive probability on every single language that exist. The first thing to consider is the probability that the communication partner speaks the language of the country, which is strictly positive for sure. We call this probability $\theta$, such that $0 < \theta \leq 1$. Then, we assume that the player considers a few other languages. If we take the examples from the toy game, we can consider French, Spanish, English and Chinese. Say the language of the country is Spanish . We put a higher probabilty on English because we consider it is more realistic since it is used as a global language (but the difference should not be too high). The probabilities are: (Maybe to simplify we could try sth like a proba on all vectors that contain the local language, and a proba on all vectors that contain the mother tongue of i). In a static game, players do not have the opportunity to update their belief from one period to another.

There are 2 (or 3?) states of the world: state of the world "name" with proba $x$ $\begin{cases} N_a \text{ players speaking language } a \\ N_b \text{ players speaking language } b \\ N_{ab} \text{ players speaking languages } a \end{cases}$ of the world "name" with proba $1-x$ $\begin{cases} N'_a \text{ players speaking language } a & \frac{N'_a}{N} \\ N'_b \text{ players speaking language } b & \frac{N'_b}{N} \\ N'_{ab} \text{ players speaking languages } a \text{ and } b & \frac{N'_{ab}}{N} \end{cases}$ ,We need to rewrite the utility functions agents optimize:

Formally, the setting is:

- $N+1$ **players.** We denote by $i \in \{1,2,\ldots,N+1\}$ the $i$-th individual in the community
- $L$ **foreign languages**.
  - Without loss of generality, we denote $\{l_1, l_2, \ldots, l_L\}$ the $L$ languages.
  - We denote $l_0$ the local language.
  - We denote $L_i \equiv \{l_j | \text{individual } i \text{ speaks language} l_j\}$.
  - We denote $l_i^m$ as the mother tongue of individual $i$.
- **Knowledge:**
  - individuals do not know a priori at stage 1 what language is spoken by whom, they also have uncertainties about the structure of the community.
  - **Prior beliefs:** We assume prior beliefs are independent and identically distributed across individuals. For any individual, the prior beliefs are that the languages spoken by any individual are drawn from the set of languages $L \cup l_0$. We denote $\mu_i\left(l_j \in L_j\right)$ the prior belief for individual $i$ that $l_j$ is spoken by individual $j$ with:

$$\forall j \neq i, \forall l_s \in L, \mu_i\left(l_j \in L_j\right) = \theta^j_{i,l_j}$$

which means that the beliefs that individuals have about other individuals can differ for different players. We assume that any player believe that the languages spoken by any other player are indenpendtly distributed from the languages spoken by other players.

- **Actions**: In the first stage, players choose what language to learn. The choice set of any player $i$ is $L \backslash L_i$, including the empty set. In the second stage, players are randomly assigned a communication partner and play the game of the previous section (with choice set $L_i$).
- **Outcomes:** Players speak a new language if they decided to. And if partners chose a same language, they speak it; otherwise, they end-up communicating with globish. We denote an outcome for a player by

$$O_i \equiv (l_i^c, l_s) \equiv (\text{language player } i \text{ chose to learn, outcome after randomization at stage 2})$$

  We denote $l_i^c = \emptyset$ if player $i$ chose to learn no language.
- **Payoffs:** Every player gets the utility of the outcome they reached at stage 2 minus the cost of learning the language they chose at stage 1 (0 if they chose to learn no language) plus a bonus for being able to speak the local language (representing the ability to be able to communicate outside the community). We denote it as:

$$v_i(O_i) \equiv u_i(l_s) - c_i^2(l_i^c) + \mathbb{1}_{l_0 \in L_i} B$$

  where
  - The cost of learning a new language $c_i^2(l_i^c)$
  - The bonus for speaking the local language is $B$

**Proposition 13.** *In our bayesian version of the game, individuals maximize their payoffs by learning the language $l_s$ such that:*

$$l_s \equiv \arg\max_{l \notin L_i} U_i(l)$$

*with:*

$$U_i(l) \equiv \mathbb{1}_{l_0 = l} B + \bar{\theta}_{i,l}\left(p_i(l_s) - c_i^1(l_s)\right) - c_i^2(l_s)$$

*where;* $\bar{\theta}_{i,l} \equiv \frac{1}{N}\sum_{j=1}^{N} \theta_{i,l_j}^j$

*Proof.* By backward induction, equilibrium selection assumption and proposition 8, players have as solutions of the second stage the SRE of the toy game. Up to stage 1, players know they will end-up speaking a common language whenever they can. Hence, when they learn a language, they rationally only consider symmetric outcomes. Their learning decision at stage 1 can only rely on the beliefs about the structure of the community. And so, similarly to the baseline game, they must use their beliefs about the probability about an individual speaking some language

$$\mu_i\{x_i = j \wedge l_s \in L_j\} = \underbrace{Pr\{x_i = j\}\,\mu_i\{l_s \in L_j | x_i = j\}}_{\text{Bayes rule}} = Pr\{x_i = j\}\,\theta_{i,l_j}^j$$

Hence, any players wants to maximize the potential payoff of learning language $l_s$:

$$U_i(l_s) \equiv \mathbb{1}_{l_0 = l_s} B - c_i^2(l_s) + \sum_{j=1}^{N} Pr\{x_i = j \wedge l_s \in L_j\} \times u_i(l_s)$$

where the second additive part of the function is exactly the probability that they can speak language $l_s$ with an individual $j$. We rewrite it as:

$$\begin{aligned} \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\sum_{j=1}^{N}Pr\{x_i=j\wedge l_s\in L_j\}\times u_i(l_s) = \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\sum_{j=1}^{N}Pr\{x_i=j\}\,\theta^j_{i,l_j}\times u_i(l_s) \\ = \quad \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\sum_{j=1}^{N}\frac{1}{N}\theta^j_{i,l_j}\times u_i(l_s) \\ = \quad \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\bar{\theta}_{i,l}\left(p_i(l_s)-c_i^1(l_s)\right) \end{aligned}$$

□

Again, we have that the game without interaction can be solved by discrete optimization techniques. It is also straightforward to show that an analogue to proposition 10 still applies here:

**Proposition 14.** *If learning a language $l_s$ was not an optimal decision for player i in the bayesian optimization problem and no other player decided to learn $l_s$, then $(l_s, l_{\neg i})$ cannot be a NE, with $l_{\neg i}$ the strategies of the other players.*

*Proof.* Assume player $i$ decides to learn language $l_s$ and $l^b_{opt}$ is *a* language he wishes to learn in the bayesian optimization problem[5]. His expected payoff for $l_s$ is:

$$U_i(l_s)=\mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\bar{\theta}_{i,l}\left(p_i(l_s)-c_i^1(l_s)\right)$$

But, since no other player decided to learn language $l_s$, the belief about the probability that other player learnt the languages remains non-updated and thus is identical to the parameter in the bayesian optimization problem. It results:

$$U_i(l_s)<U_i\left(l^b_{opt}\right)$$

and so $l_s$ cannot be a best response with this outcome. □

We now build an analogue to proposition 11:

**Proposition 15.** *For any given language index, the following Perfect Bayesian Nash Equilibrium (PBE) condition holds: if $l^b_{opt}$ is one of the optimal decisions of the bayesian optimization problem for player i, for $l^b_{opt}$ not to be an optimal decision anymore and for $l_s$ to become best response in the game we need:*

$$\mathbb{1}_{l_0=l^b_{opt}}B-c_i^2\left(l^b_{opt}\right)+p^\star_{i,l^b_{opt}}\left(p_i\left(l^b_{opt}\right)-c_i^1\left(l^b_{opt}\right)\right)\geq \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+p^\star_{i,l_s}\left(p_i(l_s)-c_i^1(l_s)\right)$$

*where $p^\star_{i,l_m}$ is the posterior beliefs given other individuals learning decisions that the average individual speaks language $l_m$.*

*Proof.* We have initially in the bayesian optimization problem:

$$U_i^b\left(l^b_{opt}\right)\geq U_i^b(l_s)$$

by optimality of the language $l^b_{opt}$. We rewrite it as:

$$\mathbb{1}_{l_0=l^b_{opt}}B-c_i^2\left(l^b_{opt}\right)+\bar{\theta}_{i,l^b_{opt}}\left(p_i\left(l^b_{opt}\right)-c_i^1\left(l^b_{opt}\right)\right)\geq \mathbb{1}_{l_0=l_s}B-c_i^2(l_s)+\bar{\theta}_{i,l_s}\left(p_i(l_s)-c_i^1(l_s)\right)$$

5. Again note that unicity of this optimum is not guaranteed.

Now, note that for $l_s$ to be a best response given the strategies of the other players, we need that the updating of the probability that any player speaks a language given their strategies either considerably decreases the prior belief about the average probability of finding an individual speaking $l^b_{opt}$ or that it increases sufficiently the same probability for $l_s$. In particular, after the strategic decisions by each player, we have:

$$Pr\left\{\bar{\theta}_{l^b_{opt}} = \bar{\theta}_{i,l^b_{opt}} \mid l_{\neg i}\right\} = \frac{Pr\left\{l_{\neg i} \mid \text{average number of speakers is} \bar{\theta}_{i,l^b_{opt}} N\right\} Pr\left(\text{average number of speakers is} \bar{\theta}_{i,l^b_{opt}} N\right.}{Pr\left(l_{\neg i}\right)}$$

$$= \frac{Pr\left\{l_{\neg i} \mid \sum_{j=1}^{N} \theta_{i,l^b_{opt}} = \bar{\theta}_{i,l^b_{opt}} N\right\} Pr\left(\sum_{j=1}^{N} \theta_{i,l^b_{opt}} = \bar{\theta}_{i,l^b_{opt}} N\right.}{Pr\left(l_{\neg i}\right)}$$

Here, we use that under the beliefs of player $i$:

$$Pr\left(\sum_{j=1}^{N} \theta^{j}_{i,l^b_{opt}} = \bar{\theta}_{i,l^b_{opt}} N\right) = \prod_{j\neq i}^{N} \theta^{j}_{i,l^b_{opt}}$$

And thus:

$$Pr\left\{\bar{\theta}_{l^b_{opt}} = \bar{\theta}_{i,l^b_{opt}} \mid l_{\neg i}\right\} = \frac{Pr\left\{l_{\neg i} \mid \sum_{j=1}^{N} \theta_{i,l^b_{opt}} = \bar{\theta}_{i,l^b_{opt}} N\right\} \prod_{j\neq i}^{N} \theta^{j}_{i,l^b_{opt}}}{Pr\left\{l_{\neg i} \mid \sum_{j=1}^{N} \theta_{i,l^b_{opt}} = \bar{\theta}_{i,l^b_{opt}} N\right\} Pr\left(\sum_{j=1}^{N} \theta_{i,l^b_{opt}} = \bar{\theta}_{i,l^b_{opt}} N\right) + Pr\left\{l_{\neg i} \mid \sum_{j=1}^{N} \theta_{i,l^b_{opt}} \neq \bar{\theta}_{i,l^b_{opt}} N\right\} Pr}$$

$$= \frac{Pr\left\{l_{\neg i} \mid \sum_{j=1}^{N} \theta_{i,l^b_{opt}} = \bar{\theta}_{i,l^b_{opt}} N\right\} \prod_{j\neq i}^{N} \theta^{j}_{i,l^b_{opt}}}{Pr\left\{l_{\neg i} \mid \sum_{j=1}^{N} \theta_{i,l^b_{opt}} = \bar{\theta}_{i,l^b_{opt}} N\right\} \prod_{j\neq i}^{N} \theta^{j}_{i,l^b_{opt}} + Pr\left\{l_{\neg i} \mid \sum_{j=1}^{N} \theta_{i,l^b_{opt}} \neq \bar{\theta}_{i,}\right.}$$

$$=$$

Similarly, we have:

$$\forall l_s, Pr\left\{\bar{\theta}_{i,l_s} = \bar{\theta}_{i,l_s} \mid l_{\neg i}\right\} = \frac{Pr\left\{l_{\neg i} \mid \sum_{j=1}^{N} \theta_{i,l_s} = \bar{\theta}_{i,l_s} N\right\} \prod_{j\neq i}^{N} \theta^{j}_{i,l_s}}{Pr\left\{l_{\neg i} \mid \sum_{j=1}^{N} \theta_{i,l_s} = \bar{\theta}_{i,l_s} N\right\} \prod_{j\neq i}^{N} \theta^{j}_{i,l_s} + Pr\left\{l_{\neg i} \mid \sum_{j=1}^{N} \theta_{i,l_s} \neq \bar{\theta}_{i,l_s} N\right\} \left(1 - \prod_{j\neq i}^{N} \theta^{j}_{i,l_s}\right)}$$

$$= p^{\star}_{i,l_s}$$

Finally, for a PBE, we need that these posterior beliefs be consistent with the strategies chosen by players. Hence, we need:

$$\mathbb{1}_{l_0 = l^b_{opt}} B - c^2_i\left(l^b_{opt}\right) + p^{\star}_{i,l^b_{opt}}\left(p_i\left(l^b_{opt}\right) - c^1_i\left(l^b_{opt}\right)\right) \geq \mathbb{1}_{l_0 = l_s} B - c^2_i\left(l_s\right) + p^{\star}_{i,l_s}\left(p_i\left(l_s\right) - c^1_i\left(l_s\right)\right)$$

□

The main difference between the bayesian optimization problem and the bayesian game is that players must use posterior beliefs in the game instead of prior beliefs. Note that this difference is of considerable importance since, for large number of players or large number of languages, there might be multiple mixed choice of languages such the above condition is satisfied. This problem is even more important for the following optimality condition:

**Proposition 16.** *For any given language index, the following PBE condition holds: for $l_s$ to be a best response in the game we need:*

$$\forall l_m \notin L_i, l_s \neq l_m, \mathbb{1}_{l_0=l_s} B - c_i^2(l_s) + p^{\star}_{i,l_s}\left(p_i(l_s) - c_i^1(l_s)\right) \geq \mathbb{1}_{l_0=l_m} B - c_i^2(l_m) + p^{\star}_{i,l_m}\left(p_i(l_m) - c_i^1(l_m)\right)$$

*Proof.* The proof follows the lines of proposition 15. □

An issue here is that checking this condition for all possible choices of languages is computationally intractable.

## 5. Game theory and languages: area for future research

[Here talk about how future research (can talk about network version of the game and could use the survey. Could also try lab research]

## 6. Conclusion

### 6.1 Findings of the paper

### 6.2 Discussion about the assumptions

We assumed for simplicity that once a language is learnt, it can be spoken by the player. This assumption hides a variety of situations when learning a language would have increasing returns (in the case when there are learning stages, the payoffs are increasing with time), or when learning a language would have a negative impact on the payoffs (case when the level required to communicate is too high for the time frame considered).

In this paper we assumed rational behavior on the part of all players. This assumption is credible because students who want to make the most of their ERASMUS stay should learn a language that allows them to communicate (have a greater payoff in this game). They can also be assumed to choose a destination based on their interest or future career plans, which would strengthen the incentive to learn a language they can use in this country or environment. However, we can always consider other behavioral factors that would suggest bounded rationality (students not having a choice where to go, or maybe other personal motivations to learn a language they will not be able to speak immediately).

Citations.bib

## Code appendix

ehsg