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ASSIGNMENT — Banking Contagion

1 Friendly warm-up

It is usual to represent the banking network as a set of nodes (Banks) connected through edges (contracts or inter-dependencies). In the simplest case, we would use a network graph to visualize the network. Such as below:

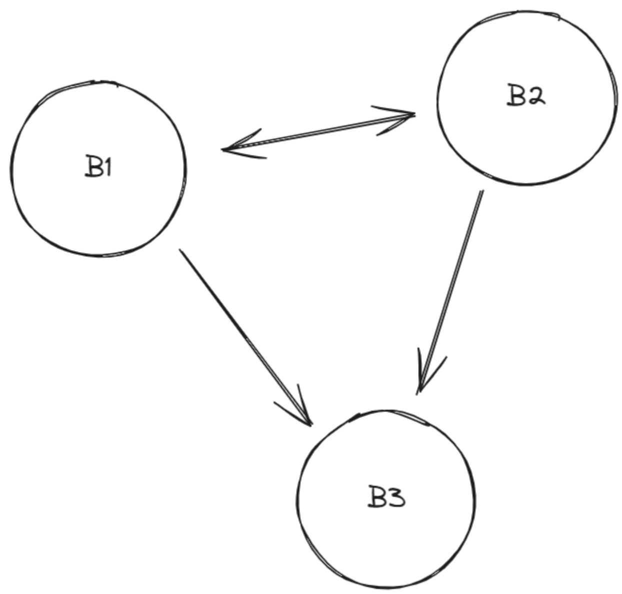


Figure 1: Simple banking network.
Source: the author

In Figure 1, we see a simple banking system composed of 3 banks. Banks are connected via contractual obligations. For example, banks traditionally lend money to each other in the short-term via the interbanking market. This creates networks of exposures like the one represented above. In our simple case, it seems that bank 1 (B1) has an impact on both B2 and B3. B2 has an impact on both B1 and B3. B3 does not impact anyone.

Since it is sometimes difficult to perform complex operations on graphs, economists prefer to use matrices to represent networks.

To illustrate that point, let's consider the following adjacency matrix (matrix of inter-dependencies) and its corresponding graph (see Figure 2).

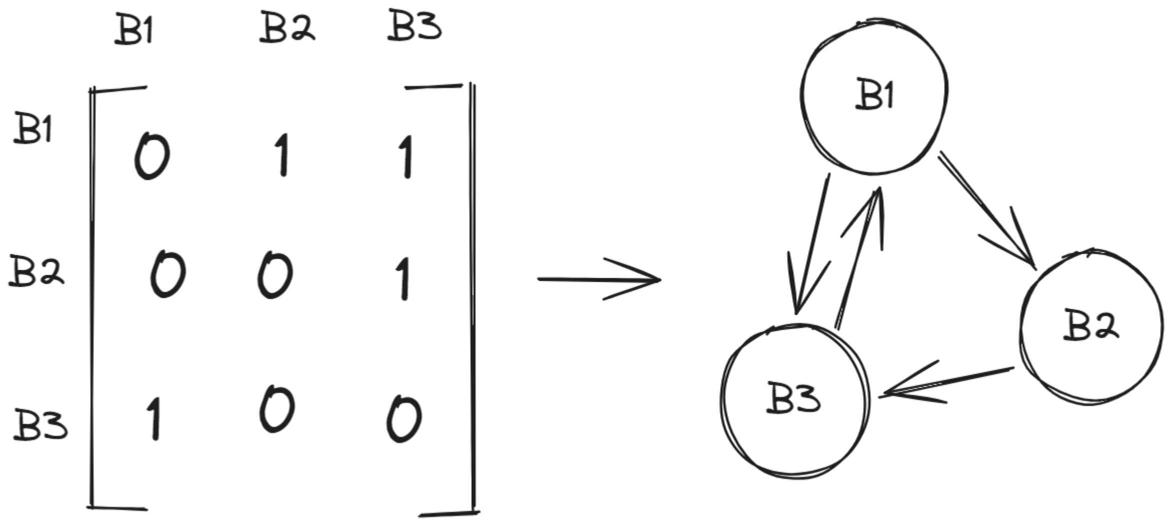


Figure 2: How an adjacency matrix translates into a network.
Source: the author

The adjacency matrix (on the left) is only composed of 0 and 1. Where 0 means that the B_i does not impact B_j, and where 1 says that the B_i impacts B_j.

The right way to read the adjacency matrix is to look at the rows. For instance, consider the first row of this matrix (see Figure 3).

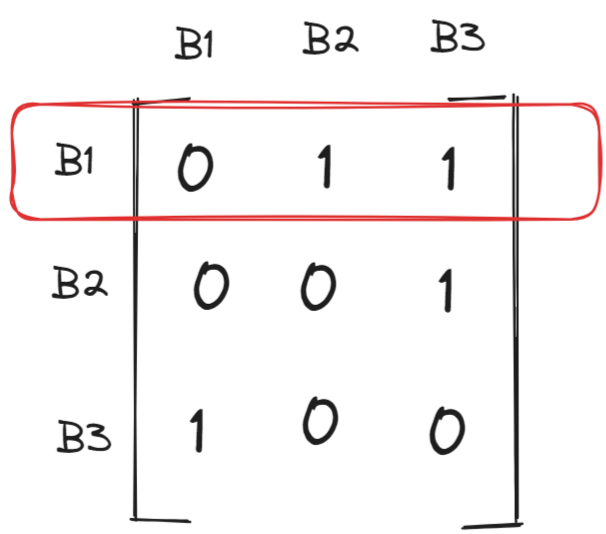


Figure 3: How to read an adjacency matrix
Source: the author

This first row reads in the following way. Bank 1 does not impact itself (the first number is 0). The bank 1 impacts B2 and B3 (as the two other numbers are 1).

Reading in the same way the second line, we see that B2 only impacts B3; and that B3 only impacts B1. You can check in the graph if the connections are well represented!

Modelling banking contagion.

So, how do we use that to model how a contagion propagates through a banking network? Traditionally, we assign equity values for each bank. The idea is that when a bank fails, the money that it borrowed from other banks is lost, and that is directly translated to a decrease in equity in the banks that were owed the money. If the equity level of a bank becomes negative, the bank fails.

So every time a bank fails, it transmits a “shock” to the other banks. If the shock is big enough, another bank fails and propagates in turn another shock, and etc. until the entire system collapses. For instance, let’s consider the network in Figure 2. Let’s say that B1 fails and let’s follow the series of events:

- The shock from B1 transmits to B2 and B3.
- Let’s assume B3 is robust enough to resist but B2 fails.
- B2 propagates a shock to B3.
- B3, already weakened by the shock from B1, fails because of the shock of B2.
- Banking system collapsed.

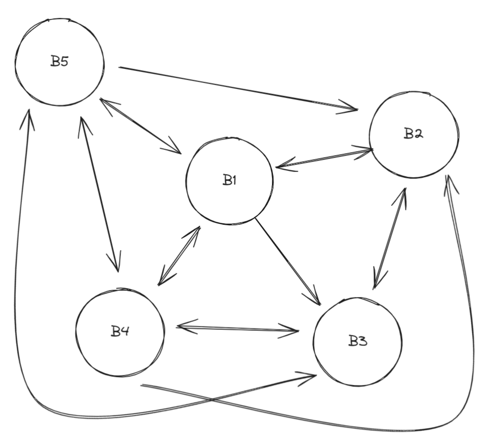
This is, obviously, a simple case, but let’s see how this applies to larger networks and how we can use this to inform policy-making.

2 Context and problem set up

Let’s consider the following adjacency matrix and the corresponding network visualization.

$$
\begin{array}{c}
\\ B1 \\ B2 \\ B3 \\ B4 \\ B5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} B1 & B2 & B3 & B4 & B5 \end{array} \\
\begin{bmatrix}
0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$

(a) Adjacency Matrix



(b) Network representation

Figure 4: Banking System

Let us now assume that the banks have the following equities:

- **B1**: 1000 (Million USD)
- **B2**: 120 (Million USD)
- **B3**: 200 (Million USD)
- **B4**: 100 (Million USD)
- **B5**: 150 (Million USD)

Let's consider that, when a bank fails, it transmits 14.8% of its original equity to the banks it is connected to. We will call this the "transmission rate" or also the "shock size".

The first thing that we can notice, is that B1 is significantly larger than the other banks and also more connected than the other banks. That is very close to the reality, where we have a small number of large and connected banks and a large amount of smaller banks. The failure of the smaller banks is usually not a systemic issue but if a large bank fails, that's another story. This is the "Too Big To Fail" case.

If B1 is the first to fail, how does the banking network react to that failure? Does it survive or collapse?

$\longrightarrow$ Let's do it together :)[1]

3 Assignments

Alright, we have managed to model our first cascade of failure in a banking network. Now, from a policy-making point of view, how can we use the model to make informed decisions about the regulations for the banking system?

There are three ways[2] that we can address the cascade of failure.

1. We can limit the transmission rate (how banks are inter-connected).
2. We can improve the equity of smaller banks to make them more resilient.
3. We can limit the size of the largest banks so that the impact in the case of their failure is smaller.

Your task for this assignment is to evaluate each of these solutions using the model we made together. The questions we need to answer are:

1. What is the highest transmission rate the system can sustain before entirely collapsing?
2. What is the minimum amount we need to increase the smaller banks' equity in order for the system to survive? (Guideline: Increase the equity of banks B2, B4 and B5 from the same amount)
3. What is the maximum equity B1 can have before its failure makes the entire system collapse?

Important note: You might be tempted to try to prevent the first failure and say "If there is no bank failing after B1 fails then the system survives". You'd be right, but it is very expensive to make the entire system resilient. Instead, here, we'll define a success if there is at least 1 bank that is left standing in the system.

Submitted by Mathis Mourey on April 25, 2024.

[1] Solution in the Excel file!

[2] Actually, there are many more but, for the sake of simplicity, we will focus on these three.