departamento de matemática **universidade de aveiro**

1. No espaço vectorial real indicado, determine as coordenadas do vector u na base ordenada $\mathcal{B}$ dada.

   (a) $u = (3, -7)$ e $\mathcal{B}_{\mathbb{R}^3} = ((1, 0), (0, 1))$, em $\mathbb{R}^2$;

   (b) $u = (1, 1)$ e $\mathcal{B} = ((2, -4), (3, 3))$, em $\mathbb{R}^2$;

   (c) $u = (2, -1, 3)$ e $\mathcal{B} = ((1, 0, 0), (2, 2, 0), (3, 3, 3))$, em $\mathbb{R}^3$;

   (d) $u = 2 - x + x^2$ e $\mathcal{B} = (1 + x, 1 + x^2, x + x^2)$, em $P_2[x]$;

   (e) $u = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$ e $\mathcal{B} = \left(\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right)$, em $M_{2\times 2}(\mathbb{R})$.

2. Considere os vectores $u = (1, 1, -1)$, $v = (2, 1, 0)$ e $w = (-1, 0, 1)$ do espaço vectorial real $\mathbb{R}^3$.

   (a) Mostre que $\mathcal{B} = (u, v, w)$ é uma base ordenada de $\mathbb{R}^3$.

   (b) Determine as coordenadas do vector $a = (2, -1, 2)$ na base $\mathcal{B}$.

   (c) Determine $S = \langle u, v \rangle$ e averigúe que o vector $b = (0, 1, -2) \in S$.

3. No espaço vectorial real $\mathbb{R}^2$, considere as bases ordenadas

$$\mathcal{B} = ((2, 0), (1, 3))\,, \quad \mathcal{B}' = ((1, -3), (2, 4)) \quad \text{e} \quad \mathcal{B}'' = ((1, -1), (1, 1)).$$

   Determine as coordenadas dos vectores $u = (8, 6)$ e $v = (1, -2)$ relativamente às bases $\mathcal{B}$, $\mathcal{B}'$ e $\mathcal{B}''$.

4. No espaço vectorial real $\mathbb{R}^3$, considere o subespaço vectorial S gerado pelos vectores $u = (1, 0, -2)$ e $v = (2, -1, 3)$.

   (a) Determine $k \in \mathbb{R}$ de modo que o vector $w = (1, -2, k)$ pertença a S.

   (b) Fazendo $k = 0$, mostre que $\mathcal{B} = (u, v, w)$ é uma base ordenada de $\mathbb{R}^3$ e determine as coordenadas de $b = (0, 1, 0)$ em relação a essa base.

5. Considere, no espaço vectorial real $\mathbb{R}^3$, o subconjunto de $\mathbb{R}^3$:

$$\mathcal{B} = \{(1, 0, 1), (1, 1, 0), (k, 1, -1)\}.$$

   (a) Determine os valores de $k \in \mathbb{R}$ para os quais $\mathcal{B}$ é uma base de $\mathbb{R}^3$.

   (b) Considere $k = 1$.

      i. Indique as coordenadas do vector $u = (5, 4, 2)$ relativamente à base $\mathcal{B}$.

      ii. Determine o vector v cujas coordenadas relativamente à base $\mathcal{B}$ são $(2, -3, 4)$, isto é, $v = (2, -3, 4)_{\mathcal{B}}$.

6. Determine uma base $\mathcal{B}$ e indique a dimensão do subespaço vectorial S, no espaço vectorial indicado. Determine ainda as coordenadas de u em relação à base $\mathcal{B}$.

   (a) $S = \{(x, y) \in \mathbb{R}^2 : 2x + 3y = 0\}$ e $u = (-9, 6)$, em $\mathbb{R}^2$

   (b) $S = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + 3z = 0\}$ e $u = (-3, 3, -1)$, em $\mathbb{R}^3$;

   (c) $S = \{(x, y, z) \in \mathbb{R}^3 : x + y = 0 \ \wedge\ 2y - z = 0\}$ e $u = (1, -1, -2)$, em $\mathbb{R}^3$;

   (d) $S = \langle (1, 0, 1), (1, 1, 1), (1, -1, 1)\rangle$ e $u = (5, 2, 5)$, em $\mathbb{R}^3$;

   (e) $S = \langle (1, 0, 1), (1, 1, 2), (0, -1, -1), (3, 1, 4)\rangle$ e $u = (4, 1, 5)$, em $\mathbb{R}^3$;

   (f) $S = \{(a, b, c, d) \in \mathbb{R}^4 : a + b + c = 0\}$ e $u = (1, -1, 0, 4)$, em $\mathbb{R}^4$;

   (g) $S = \{(a, b, c, d) \in \mathbb{R}^4 : a - 2b = 0 \ \wedge\ c = 3d\}$ e $u = (-2, -1, 6, 2)$, em $\mathbb{R}^4$;

   (h) $S = \langle (1, 0, 1, 1), (1, 1, 0, 1), (1, -1, 2, 1)\rangle$ e $u = (3, 0, 3, 3)$, em $\mathbb{R}^4$;

   (i) $S = \langle (1, 1, 1, 1, 0), (1, 1, -1, -1, -1), (2, 2, 0, 0, -1), (1, 1, 5, 5, 2)\rangle$ e $u = (2, 2, 0, 0, -1)$, em $\mathbb{R}^5$;

   (j) $S = \{ax^3 + bx^2 + cx + d \in P_3[x] : b = 0 \ \wedge\ d = 2c\}$ e $u = -5x^3 + 3x + 6$, em $P_3[x]$;

   (k) $S = \left\{ \begin{bmatrix} x + y - 2z & 2x - y \\ 0 & 3x + 5z \end{bmatrix} : x, y, z \in \mathbb{R} \right\}$ e $u = \begin{bmatrix} 0 & 1 \\ 0 & 8 \end{bmatrix}$, em $M_{2\times 2}(\mathbb{R})$.

1. (a) $u = (3, -7)_{\mathcal{B}}$;
   (b) $u = \left(0, \frac{1}{3}\right)_{\mathcal{B}}$;
   (c) $u = (3, -2, 1)_{\mathcal{B}}$;
   (d) $u = (0, 2, -1)_{\mathcal{B}}$;
   (e) $u = (-1, 1, -1, 3)_{\mathcal{B}}$.

2. (b) $a = (-3, 2, -1)_{\mathcal{B}}$;
   (c) $S = \{(x, y, z) \in \mathbb{R}^3 : z + 2y - x = 0\}$.

3. $u = (3, 2)_{\mathcal{B}}$, $u = (2, 3)_{\mathcal{B}'}$ e $u = (1, 7)_{\mathcal{B}''}$,
   $v = \left(\frac{5}{6}, -\frac{2}{3}\right)_{\mathcal{B}}$, $v = \left(\frac{4}{5}, \frac{1}{10}\right)_{\mathcal{B}'}$, $v = \left(\frac{3}{2}, -\frac{1}{2}\right)_{\mathcal{B}''}$.

4. (a) $k = 12$; (b) $b = \left(\frac{1}{4}, -\frac{1}{6}, -\frac{7}{12}\right)_{\mathcal{B}}$.

5. (a) $k \in \mathbb{R} \setminus \{0\}$; (b) $u = (1, 5, -1)_{\mathcal{B}}$; (c) $v = (3, 1, -2)$.

6. (a) $\mathcal{B} = \left(\left(-\frac{3}{2}, 1\right)\right)$, $\dim S = 1$ e $u = (6)_{\mathcal{B}}$;
   (b) $\mathcal{B} = ((-2, 1, 0), (-3, 0, 1))$, $\dim S = 2$ e $u = (3, -1)_{\mathcal{B}}$;
   (c) $\mathcal{B} = ((-1, 1, 2))$, $\dim S = 1$ e $u = (-1)_{\mathcal{B}}$;
   (d) $\mathcal{B} = ((1, 0, 1), (1, 1, 1))$, $\dim S = 2$ e $u = (3, 2)_{\mathcal{B}}$;
   (e) $\mathcal{B} = ((3, 1, 4), (1, 1, 2))$, $\dim S = 2$ e $u = \left(\frac{3}{2}, \frac{1}{2}\right)_{\mathcal{B}}$;
   (f) $\mathcal{B} = ((-1, 1, 0, 0), (-1, 0, 1, 0), (0, 0, 0, 1))$, $\dim S = 3$ e $u = (-1, 0, 4)_{\mathcal{B}}$;
   (g) $\mathcal{B} = ((2, 1, 0, 0), (0, 0, 3, 1))$, $\dim S = 2$ e $u = (-1, 2)_{\mathcal{B}}$;
   (h) $\mathcal{B} = ((1, 0, 1, 1), (1, 1, 0, 1))$, $\dim S = 2$ e $u = (3, 0)_{\mathcal{B}}$;
   (i) $\mathcal{B} = ((1, 1, 1, 1, 0), (1, 1, -1, -1, -1))$, $\dim S = 2$ e $u = (1, 1)_{\mathcal{B}}$;
   (j) $\mathcal{B} = (x^3, x + 2)$, $\dim S = 2$ e $u = (-5, 3)_{\mathcal{B}}$;
   (k) $\mathcal{B} = \left(\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -2 & 0 \\ 0 & 5 \end{bmatrix} \right)$, $\dim S = 3$ e $u = (1, 1, 1)_{\mathcal{B}}$.