

departamento de matemática



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1. No espaço vectorial real indicado, determine as coordenadas do vector u na base ordenada \mathcal{B} dada.

(a) $u = (3, -7)$ e $\mathcal{B}_{\mathbb{R}^3} = ((1, 0), (0, 1))$, em \mathbb{R}^2 ;

(b) $u = (1, 1)$ e $\mathcal{B} = ((2, -4), (3, 3))$, em \mathbb{R}^2 ;

(c) $u = (2, -1, 3)$ e $\mathcal{B} = ((1, 0, 0), (2, 2, 0), (3, 3, 3))$, em \mathbb{R}^3 ;

(d) $u = 2 - x + x^2$ e $\mathcal{B} = (1 + x, 1 + x^2, x + x^2)$, em $P_2[x]$;

(e) $u = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$ e $\mathcal{B} = \left(\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right)$, em $M_{2 \times 2}(\mathbb{R})$.

2. Considere os vectores $u = (1, 1, -1)$, $v = (2, 1, 0)$ e $w = (-1, 0, 1)$ do espaço vectorial real \mathbb{R}^3 .

(a) Mostre que $\mathcal{B} = (u, v, w)$ é uma base ordenada de \mathbb{R}^3 .

(b) Determine as coordenadas do vector $a = (2, -1, 2)$ na base \mathcal{B} .

(c) Determine $S = \langle u, v \rangle$ e averigüe que o vector $b = (0, 1, -2) \in S$.

3. No espaço vectorial real \mathbb{R}^2 , considere as bases ordenadas

$$\mathcal{B} = ((2, 0), (1, 3)), \quad \mathcal{B}' = ((1, -3), (2, 4)) \quad \text{e} \quad \mathcal{B}'' = ((1, -1), (1, 1)).$$

Determine as coordenadas dos vectores $u = (8, 6)$ e $v = (1, -2)$ relativamente às bases \mathcal{B} , \mathcal{B}' e \mathcal{B}'' .

4. No espaço vectorial real \mathbb{R}^3 , considere o subespaço vectorial S gerado pelos vectores $u = (1, 0, -2)$ e $v = (2, -1, 3)$.

(a) Determine $k \in \mathbb{R}$ de modo que o vector $w = (1, -2, k)$ pertença a S .

(b) Fazendo $k = 0$, mostre que $\mathcal{B} = (u, v, w)$ é uma base ordenada de \mathbb{R}^3 e determine as coordenadas de $b = (0, 1, 0)$ em relação a essa base.

5. Considere, no espaço vectorial real \mathbb{R}^3 , o subconjunto de \mathbb{R}^3 :

$$\mathcal{B} = \{(1, 0, 1), (1, 1, 0), (k, 1, -1)\}.$$

(a) Determine os valores de $k \in \mathbb{R}$ para os quais \mathcal{B} é uma base de \mathbb{R}^3 .

(b) Considere $k = 1$.

i. Indique as coordenadas do vector $u = (5, 4, 2)$ relativamente à base \mathcal{B} .

ii. Determine o vector v cujas coordenadas relativamente à base \mathcal{B} são $(2, -3, 4)$, isto é, $v = (2, -3, 4)_{\mathcal{B}}$.

6. Determine uma base \mathcal{B} e indique a dimensão do subespaço vectorial S , no espaço vectorial indicado. Determine ainda as coordenadas de u em relação à base \mathcal{B} .

- (a) $S = \{(x, y) \in \mathbb{R}^2 : 2x + 3y = 0\}$ e $u = (-9, 6)$, em \mathbb{R}^2
- (b) $S = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + 3z = 0\}$ e $u = (-3, 3, -1)$, em \mathbb{R}^3 ;
- (c) $S = \{(x, y, z) \in \mathbb{R}^3 : x + y = 0 \wedge 2y - z = 0\}$ e $u = (1, -1, -2)$, em \mathbb{R}^3 ;
- (d) $S = \langle (1, 0, 1), (1, 1, 1), (1, -1, 1) \rangle$ e $u = (5, 2, 5)$, em \mathbb{R}^3 ;
- (e) $S = \langle (1, 0, 1), (1, 1, 2), (0, -1, -1), (3, 1, 4) \rangle$ e $u = (4, 1, 5)$, em \mathbb{R}^3 ;
- (f) $S = \{(a, b, c, d) \in \mathbb{R}^4 : a + b + c = 0\}$ e $u = (1, -1, 0, 4)$, em \mathbb{R}^4 ;
- (g) $S = \{(a, b, c, d) \in \mathbb{R}^4 : a - 2b = 0 \wedge c = 3d\}$ e $u = (-2, -1, 6, 2)$, em \mathbb{R}^4 ;
- (h) $S = \langle (1, 0, 1, 1), (1, 1, 0, 1), (1, -1, 2, 1) \rangle$ e $u = (3, 0, 3, 3)$, em \mathbb{R}^4 ;
- (i) $S = \langle (1, 1, 1, 1, 0), (1, 1, -1, -1, -1), (2, 2, 0, 0, -1), (1, 1, 5, 5, 2) \rangle$ e $u = (2, 2, 0, 0, -1)$, em \mathbb{R}^5 ;
- (j) $S = \{ax^3 + bx^2 + cx + d \in P_3[x] : b = 0 \wedge d = 2c\}$ e $u = -5x^3 + 3x + 6$, em $P_3[x]$;
- (k) $S = \left\{ \begin{bmatrix} x + y - 2z & 2x - y \\ 0 & 3x + 5z \end{bmatrix} : x, y, z \in \mathbb{R} \right\}$ e $u = \begin{bmatrix} 0 & 1 \\ 0 & 8 \end{bmatrix}$, em $M_{2 \times 2}(\mathbb{R})$.

1. (a) $u = (3, -7)_{\mathcal{B}}$;
(b) $u = (0, \frac{1}{3})_{\mathcal{B}}$;
(c) $u = (3, -4, 3)_{\mathcal{B}}$;
(d) $u = (0, 2, -1)_{\mathcal{B}}$;
(e) $u = (-1, 1, -1, 3)_{\mathcal{B}}$.
2. (b) $a = (-3, 2, -1)_{\mathcal{B}}$;
(c) $S = \{(x, y, z) \in \mathbb{R}^3 : z + 2y - x = 0\}$.
3. $u = (3, 2)_{\mathcal{B}}$, $u = (2, 3)_{\mathcal{B}'}$ e $u = (1, 7)_{\mathcal{B}''}$,
 $v = (\frac{5}{6}, -\frac{2}{3})_{\mathcal{B}}$, $v = (\frac{4}{5}, \frac{1}{10})_{\mathcal{B}'}$, $v = (\frac{3}{2}, -\frac{1}{2})_{\mathcal{B}''}$.
4. (a) $k = 12$; (b) $b = (\frac{59}{12}, -\frac{13}{6}, -\frac{7}{12})_{\mathcal{B}}$.
5. (a) $k \in \mathbb{R} \setminus \{0\}$; (b) $u = (1, 5, 1)_{\mathcal{B}}$; (c) $v = (3, 1, -2)$.
6. (a) $\mathcal{B} = ((-\frac{3}{2}, 1))$, $\dim S = 1$ e $u = (6)_{\mathcal{B}}$;
(b) $\mathcal{B} = ((-2, 1, 0), (-3, 0, 1))$, $\dim S = 2$ e $u = (3, -1)_{\mathcal{B}}$;
(c) $\mathcal{B} = ((-1, 1, 2))$, $\dim S = 1$ e $u = (-1)_{\mathcal{B}}$;
(d) $\mathcal{B} = ((1, 0, 1), (1, 1, 1))$, $\dim S = 2$ e $u = (3, 2)_{\mathcal{B}}$;
(e) $\mathcal{B} = ((3, 1, 4), (1, 1, 2))$, $\dim S = 2$ e $u = (\frac{3}{2}, \frac{1}{2})_{\mathcal{B}}$;
(f) $\mathcal{B} = ((-1, 1, 0, 0), (-1, 0, 1, 0), (0, 0, 0, 1))$, $\dim S = 3$ e $u = (-1, 0, 4)_{\mathcal{B}}$;
(g) $\mathcal{B} = ((2, 1, 0, 0), (0, 0, 3, 1))$, $\dim S = 2$ e $u = (-1, 2)_{\mathcal{B}}$;
(h) $\mathcal{B} = ((1, 0, 1, 1), (1, 1, 0, 1))$, $\dim S = 2$ e $u = (3, 0)_{\mathcal{B}}$;
(i) $\mathcal{B} = ((1, 1, 1, 1, 0), (1, 1, -1, -1, -1))$, $\dim S = 2$ e $u = (1, 1)_{\mathcal{B}}$;
(j) $\mathcal{B} = (x^3, x + 2)$, $\dim S = 2$ e $u = (-5, 3)_{\mathcal{B}}$;
(k) $\mathcal{B} = \left(\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -2 & 0 \\ 0 & 5 \end{bmatrix} \right)$, $\dim S = 3$ e $u = (1, 1, 1)_{\mathcal{B}}$.