Relational Reasoning
(Relationel ræsonnement)

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Abstract

►in English... ◄

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Contents

Chapter 1

Introduction

▶motivate and explain the problem to be addressed◀
 ▶example of a citation: [1]◀ ▶get your bibtex entries from `https://dblp.org/`◀

Chapter 2

Definition of Language

►**create draft**◄

Syntax

$$
\begin{aligned}
e ::= {}& () \mid & \text{(unit value)} \\
& x \mid & \text{(variables)} \\
& \bar{n} \mid e + e \mid e - e \mid e \le e \mid e < e \mid e = e \mid & \text{(integers)} \\
& \mathsf{true} \mid \mathsf{false} \mid \mathsf{if}\ e\ \mathsf{then}\ e\ \mathsf{else}\ e \mid & \text{(booleans)} \\
& (e,e) \mid \mathsf{fst}\ e \mid \mathsf{snd}\ e \mid & \text{(products)} \\
& \mathsf{inj}_1\ e \mid \mathsf{inj}_2\ e \mid \mathsf{match}\ e\ \mathsf{with}\ \mathsf{inj}_1\ x \Rightarrow e \mid \mathsf{inj}_2\ x \Rightarrow e\ \mathsf{end} \mid & \text{(sums)} \\
& \mathsf{rec}\ f(x) := e \mid e\ e & \text{(recursive functions)} \\
& \Lambda\ e \mid e\ \_ & \text{(polymorphism)}
\end{aligned}
$$

$$
x ::= () \mid \bar{n} \mid \mathsf{true} \mid \mathsf{false} \mid (v,v) \mid \mathsf{inj}_1\ v \mid \mathsf{inj}_2\ v \mid \mathsf{rec}\ f(x) := e \mid \Lambda\ e \qquad \text{(values)}
$$

$$
\tau ::= \mathsf{Unit} \mid \mathbb{Z} \mid \mathbb{B} \mid \tau \times \tau \mid \tau + \tau \mid \tau \to \tau \mid \forall X.\ \tau \qquad \text{(types)}
$$

$$
\begin{aligned}
K ::= {}& [] \mid K + e \mid v + K \mid K - e \mid v - K \mid K \le e \mid v \le K \mid K < e \mid v < K \mid & \text{(evaluation context)} \\
& K = e \mid v = K \mid \mathsf{if}\ K\ \mathsf{then}\ e\ \mathsf{else}\ e \mid (K,e) \mid (v,K) \mid \mathsf{fst}\ K \mid \mathsf{snd}\ K \mid \\
& \mathsf{inj}_1\ K \mid \mathsf{inj}_2\ K \mid \mathsf{match}\ K\ \mathsf{with}\ \mathsf{inj}_1\ x \Rightarrow e \mid \mathsf{inj}_2\ x \Rightarrow e\ \mathsf{end} \mid K\ e \mid v\ K \mid K\ \_
\end{aligned}
$$

Typing rules

$$\frac{\text{T-VAR}}{(x:\tau)\in\Gamma}{\Xi\mid\Gamma\vdash x:\tau}\qquad \frac{\text{T-UNIT}}{\Xi\mid\Gamma\vdash ():\mathsf{Unit}}\qquad \frac{\text{T-NAT}}{\Xi\mid\Gamma\vdash \overline{n}:\mathbb{Z}}$$

$$\frac{\text{T-ADD}}{\Xi\mid\Gamma\vdash e_1:\mathbb{Z}\qquad \Xi\mid\Gamma\vdash e_2:\mathbb{Z}}{\Xi\mid\Gamma\vdash e_1+e_2:\mathbb{Z}}\qquad \frac{\text{T-SUB}}{\Xi\mid\Gamma\vdash e_1:\mathbb{Z}\qquad \Xi\mid\Gamma\vdash e_2:\mathbb{Z}}{\Xi\mid\Gamma\vdash e_1-e_2:\mathbb{Z}}$$

$$\frac{\text{T-LE}}{\Xi\mid\Gamma\vdash e_1:\mathbb{Z}\qquad \Xi\mid\Gamma\vdash e_2:\mathbb{Z}}{\Xi\mid\Gamma\vdash e_1\le e_2:\mathbb{B}}\qquad \frac{\text{T-LT}}{\Xi\mid\Gamma\vdash e_1:\mathbb{Z}\qquad \Xi\mid\Gamma\vdash e_2:\mathbb{Z}}{\Xi\mid\Gamma\vdash e_1<e_2:\mathbb{B}}$$

$$\frac{\text{T-EQ}}{\Xi\mid\Gamma\vdash e_1:\mathbb{Z}\qquad \Xi\mid\Gamma\vdash e_2:\mathbb{Z}}{\Xi\mid\Gamma\vdash e_1=e_2:\mathbb{B}}\qquad \frac{\text{T-TRUE}}{\Xi\mid\Gamma\vdash \mathsf{true}:\mathbb{B}}\qquad \frac{\text{T-FALSE}}{\Xi\mid\Gamma\vdash \mathsf{false}:\mathbb{B}}$$

$$\frac{\text{T-IF}}{\Xi\mid\Gamma\vdash e_1:\mathbb{B}\qquad \Xi\mid\Gamma\vdash e_2:\tau\qquad \Xi\mid\Gamma\vdash e_3:\tau}{\Xi\mid\Gamma\vdash \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3:\tau}$$

$$\frac{\text{T-PAIR}}{\Xi\mid\Gamma\vdash e_1:\tau_1\qquad \Xi\mid\Gamma\vdash e_2:\tau_2}{\Xi\mid\Gamma\vdash (e_1,e_2):\tau_1\times\tau_1}\qquad \frac{\text{T-FST}}{\Xi\mid\Gamma\vdash e:\tau_1\times\tau_2}{\Xi\mid\Gamma\vdash \mathsf{fst}\ e:\tau_1}\qquad \frac{\text{T-SND}}{\Xi\mid\Gamma\vdash e:\tau_1\times\tau_2}{\Xi\mid\Gamma\vdash \mathsf{snd}\ e:\tau_2}$$

$$\frac{\text{T-INJ1}}{\Xi\mid\Gamma\vdash e:\tau_1}{\Xi\mid\Gamma\vdash \mathsf{inj}_1\ e:\tau_1+\tau_2}\qquad \frac{\text{T-INJ2}}{\Xi\mid\Gamma\vdash e:\tau_2}{\Xi\mid\Gamma\vdash \mathsf{inj}_2\ e:\tau_1+\tau_2}$$

$$\frac{\text{T-MATCH}}{\Xi\mid\Gamma\vdash e_1:\tau_1+\tau_2\qquad \Xi\mid\Gamma,x:\tau_1\vdash e_2:\tau\qquad \Xi\mid\Gamma,x:\tau_2\vdash e_3:\tau}{\Xi\mid\Gamma\vdash \mathsf{match}\ e_1\ \mathsf{with}\ \mathsf{inj}_1\ x\Rightarrow e_2\mid \mathsf{inj}_2\ x\Rightarrow e_3\ \mathsf{end}:\tau}$$

$$\frac{\text{T-REC}}{\Xi\mid\Gamma,f:\tau_1\to\tau_2,x:\tau_1\vdash e:\tau_2}{\Xi\mid\Gamma\vdash \mathsf{rec}\ f(x):=e:\tau_1\to\tau_2}\qquad \frac{\text{T-APP}}{\Xi\mid\Gamma\vdash e_1:\tau_1\to\tau_2\qquad \Xi\mid\Gamma\vdash e_2:\tau_1}{\Xi\mid\Gamma\vdash e_1\ e_2:\tau_2}$$

$$\frac{\text{T-TLAM}}{\Xi,X\mid\Gamma\vdash e:\tau}{\Xi\mid\Gamma\vdash \Lambda\ e:\forall X.\ \tau}\qquad \frac{\text{T-TAPP}}{\Xi\mid\Gamma\vdash e:\forall X.\ \tau}{\Xi\mid\Gamma\vdash e\ \_:\tau\left[\tau'/X\right]}$$

Dynamics

$$\text{HEAD-STEP-STEP} \quad \frac{e \to_h e'}{K[e] \to K[e']}$$

$$\text{E-ADD} \quad \frac{}{\overline{n_1} + \overline{n_2} \to_h \overline{n_1 + n_2}}$$

$$\text{E-SUB} \quad \frac{}{\overline{n_1} - \overline{n_2} \to_h \overline{n_1 - n_2}}$$

$$\text{E-EQ} \quad \frac{n_1 = n_2}{\overline{n_1} = \overline{n_2} \to_h \text{true}}$$

$$\text{E-NOT-EQ} \quad \frac{n_1 \neq n_2}{\overline{n_1} = \overline{n_2} \to_h \text{false}}$$

$$\text{E-LE} \quad \frac{n_1 \leq n_2}{\overline{n_1} \leq \overline{n_2} \to_h \text{true}}$$

$$\text{E-NOT-LE} \quad \frac{n_1 \nleq n_2}{\overline{n_1} \leq \overline{n_2} \to_h \text{false}}$$

$$\text{E-LT} \quad \frac{n_1 < n_2}{\overline{n_1} < \overline{n_2} \to_h \text{true}}$$

$$\text{E-NOT-LT} \quad \frac{n_1 \nless n_2}{\overline{n_1} < \overline{n_2} \to_h \text{false}}$$

$$\text{E-IF-TRUE} \quad \text{if true then } e_2 \text{ else } e_3 \to_h e_2$$

$$\text{E-IF-FALSE} \quad \text{if false then } e_2 \text{ else } e_3 \to_h e_3$$

$$\text{E-FST} \quad \text{fst } (v_1, v_2) \to_h v_1$$

$$\text{E-SND} \quad \text{snd } (v_1, v_2) \to_h v_2$$

$$\text{E-MATCH-INJ1}$$
$$\text{match } (\text{inj}_1 \ v) \text{ with inj}_1 \ x \Rightarrow e_2 \mid \text{inj}_2 \ x \Rightarrow e_3 \text{ end} \to_h e_2[v/x]$$

$$\text{E-MATCH-INJ2}$$
$$\text{match } (\text{inj}_2 \ v) \text{ with inj}_1 \ x \Rightarrow e_2 \mid \text{inj}_2 \ x \Rightarrow e_3 \text{ end} \to_h e_3[v/x]$$

$$\text{E-REC-APP}$$
$$(\text{rec } f(x) := e)v \to_h e[\text{rec } f(x) := e/f][v/x]$$

$$\text{E-TAPP-TLAM}$$
$$(\Lambda \ e) \_ \to_h e$$

Chapter 3

Contextual Equivalence

►draft◄

Chapter 4

Logical Relations for Contextual Equivalence

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Chapter 5

Examples of Application of Contextual Equivalence

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Chapter 6

Comparison to Other Work and Ideas for Future Work

▶draft◀

Chapter 7

Conclusion

►conclude on the problem statement from the introduction◄

Acknowledgments

►…◄

Bibliography

[1] Aske Simon Christensen, Anders Møller, and Michael I. Schwartzbach. Precise analysis of string expressions. In Radhia Cousot, editor, *Static Analysis, 10th International Symposium, SAS 2003, San Diego, CA, USA, June 11-13, 2003, Proceedings*, volume 2694 of *Lecture Notes in Computer Science*, pages 1–18. Springer, 2003.

Appendix A

The Technical Details

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