Relational Reasoning
(Relationel ræsonnement)

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Abstract

►in English…◄

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Contents

Chapter 1

Introduction

►motivate and explain the problem to be addressed◄
►example of a citation: [1]◄ ►get your bibtex entries from `https://dblp.org/`◄

Chapter 2

Definition of Language

►create draft◄

Syntax

$$
\begin{array}{lll}
e ::= () \mid & & \text{(unit value)} \\
\quad x \mid & & \text{(variables)} \\
\quad \overline{n} \mid e + e \mid e - e \mid e \leq e \mid e < e \mid e = e \mid & & \text{(integers)} \\
\quad \mathsf{true} \mid \mathsf{false} \mid \mathsf{if}\ e\ \mathsf{then}\ e\ \mathsf{else}\ e \mid & & \text{(booleans)} \\
\quad (e,e) \mid \mathsf{fst}\ e \mid \mathsf{snd}\ e \mid & & \text{(products)} \\
\quad \mathsf{inj}_1\ e \mid \mathsf{inj}_2\ e \mid \mathsf{match}\ e\ \mathsf{with}\ \mathsf{inj}_1\ x \Rightarrow e \mid \mathsf{inj}_2\ x \Rightarrow e\ \mathsf{end} \mid & & \text{(sums)} \\
\quad \mathsf{rec}\ f(x) := e \mid e\ e & & \text{(recursive functions)} \\
\quad \Lambda\, e \mid e\ \_ & & \text{(polymorphism)}
\end{array}
$$

$$
v ::= () \mid \overline{n} \mid \mathsf{true} \mid \mathsf{false} \mid (v,v) \mid \mathsf{inj}_1\ v \mid \mathsf{inj}_2\ v \mid \mathsf{rec}\ f(x) := e \mid \Lambda\, e \qquad \text{(values)}
$$

$$
\tau ::= \mathsf{Unit} \mid \mathbb{Z} \mid \mathbb{B} \mid \tau \times \tau \mid \tau + \tau \mid \tau \to \tau \mid \forall X.\, \tau \qquad \text{(types)}
$$

$$
\begin{array}{lll}
K ::= [] \mid K + e \mid v + K \mid K - e \mid v - K \mid K \leq e \mid v \leq K \mid K < e \mid v < K \mid & & \text{(evaluation context)} \\
\quad K = e \mid v = K \mid \mathsf{if}\ K\ \mathsf{then}\ e\ \mathsf{else}\ e \mid (K,e) \mid (v,K) \mid \mathsf{fst}\ K \mid \mathsf{snd}\ K \mid & & \\
\quad \mathsf{inj}_1\ K \mid \mathsf{inj}_2\ K \mid \mathsf{match}\ K\ \mathsf{with}\ \mathsf{inj}_1\ x \Rightarrow e \mid \mathsf{inj}_2\ x \Rightarrow e\ \mathsf{end} \mid K\ e \mid v\ K \mid K\ \_ & &
\end{array}
$$

Typing rules

$$\text{T-VAR}\ \frac{(x:\tau)\in\Gamma}{\Xi\mid\Gamma\vdash x:\tau} \qquad \text{T-UNIT}\ \frac{}{\Xi\mid\Gamma\vdash():\mathsf{Unit}} \qquad \text{T-INT}\ \frac{}{\Xi\mid\Gamma\vdash\overline{n}:\mathbb{Z}}$$

$$\text{T-ADD}\ \frac{\Xi\mid\Gamma\vdash e_1:\mathbb{Z}\qquad\Xi\mid\Gamma\vdash e_2:\mathbb{Z}}{\Xi\mid\Gamma\vdash e_1+e_2:\mathbb{Z}} \qquad \text{T-SUB}\ \frac{\Xi\mid\Gamma\vdash e_1:\mathbb{Z}\qquad\Xi\mid\Gamma\vdash e_2:\mathbb{Z}}{\Xi\mid\Gamma\vdash e_1-e_2:\mathbb{Z}}$$

$$\text{T-LE}\ \frac{\Xi\mid\Gamma\vdash e_1:\mathbb{Z}\qquad\Xi\mid\Gamma\vdash e_2:\mathbb{Z}}{\Xi\mid\Gamma\vdash e_1\leq e_2:\mathbb{B}} \qquad \text{T-LT}\ \frac{\Xi\mid\Gamma\vdash e_1:\mathbb{Z}\qquad\Xi\mid\Gamma\vdash e_2:\mathbb{Z}}{\Xi\mid\Gamma\vdash e_1<e_2:\mathbb{B}}$$

$$\text{T-EQ}\ \frac{\Xi\mid\Gamma\vdash e_1:\mathbb{Z}\qquad\Xi\mid\Gamma\vdash e_2:\mathbb{Z}}{\Xi\mid\Gamma\vdash e_1=e_2:\mathbb{B}} \qquad \text{T-TRUE}\ \frac{}{\Xi\mid\Gamma\vdash\mathsf{true}:\mathbb{B}} \qquad \text{T-FALSE}\ \frac{}{\Xi\mid\Gamma\vdash\mathsf{false}:\mathbb{B}}$$

$$\text{T-IF}\ \frac{\Xi\mid\Gamma\vdash e_1:\mathbb{B}\qquad\Xi\mid\Gamma\vdash e_2:\tau\qquad\Xi\mid\Gamma\vdash e_3:\tau}{\Xi\mid\Gamma\vdash\mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3:\tau}$$

$$\text{T-PAIR}\ \frac{\Xi\mid\Gamma\vdash e_1:\tau_1\qquad\Xi\mid\Gamma\vdash e_2:\tau_2}{\Xi\mid\Gamma\vdash(e_1,e_2):\tau_1\times\tau_1} \qquad \text{T-FST}\ \frac{\Xi\mid\Gamma\vdash e:\tau_1\times\tau_2}{\Xi\mid\Gamma\vdash\mathsf{fst}\ e:\tau_1} \qquad \text{T-SND}\ \frac{\Xi\mid\Gamma\vdash e:\tau_1\times\tau_2}{\Xi\mid\Gamma\vdash\mathsf{snd}\ e:\tau_2}$$

$$\text{T-INJ1}\ \frac{\Xi\mid\Gamma\vdash e:\tau_1}{\Xi\mid\Gamma\vdash\mathsf{inj}_1\ e:\tau_1+\tau_2} \qquad \text{T-INJ2}\ \frac{\Xi\mid\Gamma\vdash e:\tau_2}{\Xi\mid\Gamma\vdash\mathsf{inj}_2\ e:\tau_1+\tau_2}$$

$$\text{T-MATCH}\ \frac{\Xi\mid\Gamma\vdash e_1:\tau_1+\tau_2\qquad\Xi\mid\Gamma,x:\tau_1\vdash e_2:\tau\qquad\Xi\mid\Gamma,x:\tau_2\vdash e_3:\tau}{\Xi\mid\Gamma\vdash\mathsf{match}\ e_1\ \mathsf{with}\ \mathsf{inj}_1\ x\Rightarrow e_2\mid\mathsf{inj}_2\ x\Rightarrow e_3\ \mathsf{end}:\tau}$$

$$\text{T-REC}\ \frac{\Xi\mid\Gamma,f:\tau_1\to\tau_2,x:\tau_1\vdash e:\tau_2}{\Xi\mid\Gamma\vdash\mathsf{rec}\ f(x):=e:\tau_1\to\tau_2} \qquad \text{T-APP}\ \frac{\Xi\mid\Gamma\vdash e_1:\tau_1\to\tau_2\qquad\Xi\mid\Gamma\vdash e_2:\tau_1}{\Xi\mid\Gamma\vdash e_1\ e_2:\tau_2}$$

$$\text{T-TLAM}\ \frac{\Xi,X\mid\Gamma\vdash e:\tau}{\Xi\mid\Gamma\vdash\Lambda\,e:\forall X.\,\tau} \qquad \text{T-TAPP}\ \frac{\Xi\mid\Gamma\vdash e:\forall X.\,\tau}{\Xi\mid\Gamma\vdash e\ \_:\tau\left[\tau'/X\right]}$$

Dynamics

$$\textsc{head-step-step}\ \frac{e \to_h e'}{K[e] \to K[e']}$$

$$\textsc{e-add}\ \overline{n_1} + \overline{n_2} \to_h \overline{n_1 + n_2} \qquad \textsc{e-sub}\ \overline{n_1} - \overline{n_2} \to_h \overline{n_1 - n_2} \qquad \textsc{e-eq}\ \frac{n_1 = n_2}{\overline{n_1} = \overline{n_2} \to_h \mathsf{true}}$$

$$\textsc{e-not-eq}\ \frac{n_1 \neq n_2}{\overline{n_1} = \overline{n_2} \to_h \mathsf{false}} \qquad \textsc{e-le}\ \frac{n_1 \leq n_2}{\overline{n_1} \leq \overline{n_2} \to_h \mathsf{true}} \qquad \textsc{e-not-le}\ \frac{n_1 \not\leq n_2}{\overline{n_1} \leq \overline{n_2} \to_h \mathsf{false}} \qquad \textsc{e-lt}\ \frac{n_1 < n_2}{\overline{n_1} < \overline{n_2} \to_h \mathsf{true}}$$

$$\textsc{e-not-lt}\ \frac{n_1 \not< n_2}{\overline{n_1} < \overline{n_2} \to_h \mathsf{false}} \qquad \textsc{e-if-true}\ \mathsf{if\ true\ then}\ e_2\ \mathsf{else}\ e_3 \to_h e_2 \qquad \textsc{e-if-false}\ \mathsf{if\ false\ then}\ e_2\ \mathsf{else}\ e_3 \to_h e_3$$

$$\textsc{e-fst}\ \mathsf{fst}\ (v_1, v_2) \to_h v_1 \qquad \textsc{e-snd}\ \mathsf{snd}\ (v_1, v_2) \to_h v_2$$

$$\textsc{e-match-inj1}\ \mathsf{match}\ (\mathsf{inj}_1\ v)\ \mathsf{with}\ \mathsf{inj}_1\ x \Rightarrow e_2 \mid \mathsf{inj}_2\ x \Rightarrow e_3\ \mathsf{end} \to_h e_2[v/x]$$

$$\textsc{e-match-inj2}\ \mathsf{match}\ (\mathsf{inj}_2\ v)\ \mathsf{with}\ \mathsf{inj}_1\ x \Rightarrow e_2 \mid \mathsf{inj}_2\ x \Rightarrow e_3\ \mathsf{end} \to_h e_3[v/x]$$

$$\textsc{e-rec-app}\ (\mathsf{rec}\ f(x) := e)v \to_h e[\mathsf{rec}\ f(x) := e/f][v/x] \qquad \textsc{e-tapp-tlam}\ (\Lambda\, e)\ \_ \to_h e$$

Chapter 3

Contextual Equivalence

►draft◄

Chapter 4

Logical Relations for Contextual Equivalence

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Chapter 5

Examples of Application of Contextual Equivalence

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Chapter 6

Comparison to Other Work and Ideas for Future Work

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Chapter 7

Conclusion

►conclude on the problem statement from the introduction◄

Acknowledgments

►…◄

Bibliography

[1] Aske Simon Christensen, Anders Møller, and Michael I. Schwartzbach. Precise analysis of string expressions. In Radhia Cousot, editor, *Static Analysis, 10th International Symposium, SAS 2003, San Diego, CA, USA, June 11-13, 2003, Proceedings*, volume 2694 of *Lecture Notes in Computer Science*, pages 1–18. Springer, 2003.

Appendix A

The Technical Details

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