Article

Experimental Verification of Quantum Entanglement via CHSH Inequality Using IBM Quantum Simulator

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Abstract: Quantum entanglement represents one of the most fundamental aspects of quantum mechanics, distinguishing quantum systems from classical ones. The conducted study verifies quantum entanglement through measurements on a Bell state using the Clauser-Horne-Shimony-Holt (CHSH) inequality [1]. For the sake of the experiment, a two-qubit system was prepared in the Bell state $|\psi-\rangle$ and correlation measurements were performed along different basis directions using the Qiskit framework [2] with and an AER simulator [3]. Four distinct measurement configurations ($X \otimes W$, $X \otimes V$, $Z \otimes W$, $Z \otimes V$) were implemented with 2048 shots each, repeated five times to calculate the standard deviation. The experimental CHSH parameter S was determined to be -2.810 ± 0.034, confirming the quantum mechanical prediction of $-2\sqrt{2} \approx -2.828$. The measured value demonstrates clear evidence of quantum entanglement, confirming the non-local nature of quantum correlations.

Keywords: quantum entanglement; CHSH inequality; Bell states; quantum correlation; quantum simulation; quantum computing; quantum information

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1. Introduction

One of the most important yet counterintuitive examples of quantum mechanics is the phenomena of quantum entanglement [4]. Described by Einstein as "spooky action at a distance" [5], it describes a situation in which the particles become dependent in such a way that they cannot be fully described independently. Quantum entanglement has profound implications for our understanding of nature and serves as a fundamental resource for quantum information processing, quantum cryptography, and quantum computing.

The experimental verification of quantum entanglement relies on Bell's theorem [6], which establishes that no local hidden variable theory can reproduce all predictions of quantum mechanics. CHSH inequality provides a practical framework for testing Bell's theorem through measurable correlations [1]. For classical systems obeying local realism, the CHSH parameter S must satisfy $|S| \leq 2$, while quantum mechanics permits violations up to $|S| \leq -2\sqrt{2} \approx -2.828$ for maximally entangled states. It should be noted however that due to noise inherent to real quantum computers, and simulated in the experiments, the calculated value may differ slightly from the theoretical threshold and may include some uncertainty expressed as standard deviation.

Bell states represent the maximally entangled two-qubit states and serve as ideal candidates for exploring quantum correlations. Among these, the state $|\psi-\rangle$ represents the perfect anti-correlation in any measurement basis, making it particularly suitable for CHSH inequality tests [7,8].

The objective of this work is to experimentally verify quantum entanglement by measuring the CHSH parameter [1] for a Bell singlet state using quantum circuit simulations. The experiment involves using the $|\psi-\rangle$ state specifically designed quantum circuit with 2048 shots and 5 experiment repetitions to evaluate the aforementioned inequality and compare the values against theoretical thresholds while including the standard deviation into the measurement results.

2. Materials and Methods

2.1. Theoretical Framework

The CHSH inequality [1] involves correlation measurements between two spatially separated observers, Alice and Bob, each performing measurements on their respective qubits. The CHSH parameter can be defined as: $S = X \otimes W - X \otimes V + Z \otimes W + Z \otimes V$. These transformations can be implemented as quantum gates within a circuit. In our implementation, Alice performs measurements along two directions corresponding to the Pauli X and Z operators, while Bob measures along two directions defined by:

- $W = (1/2)(X + Z)$
- $V = (1/2)(-X + Z)$

These choices yield theoretical dot products: $X \cdot W = 1/\sqrt{2}, X \cdot V = -1/\sqrt{2}, Z \cdot W = 1/\sqrt{2}, andZ \cdot V = 1/\sqrt{2}$, leading to the theoretical CHSH parameter: $S = -1/\sqrt{2} - (-(-1/\sqrt{2})) + (-1/\sqrt{2}) + (-1/\sqrt{2}) = -2\sqrt{2} - 2.828$

2.2. Experimental Evaluation

The formula can be verified by utilizing the circuits shown in Figures 1. The circuits were implemented by utilizing the Qiskit framework in Python.

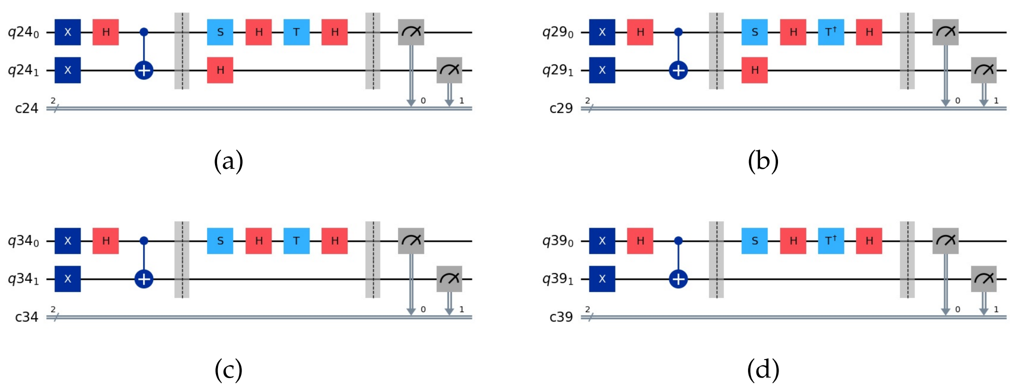

Figure 1. Quantum circuit implementations for the four CHSH measurement configurations: (a) $X \otimes W$, (b) $X \otimes V$, (c) $Z \otimes W$, and (d) $Z \otimes V$. Each circuit begins with Bell state preparation followed by basis rotation gates and measurements.

Each circuit was run 5 times with 2048 shots. For each outcome, the values were assigned with 0 being changed to 1, and 1 to -1. Standard deviations of probabilities of each outcome, as well as mean probability across multiple runs, were collected. When adding, the standard deviations were taken into account as if they were completely independent, that is as a square root of the sum of squares of standard deviations. This is, because the noise on one circuit is assumed to not influence noise on another circuit on an independent run.

3. Results

3.1. Correlation Measurements

The experimental results for all four measurement configurations are presented in Table 1. Each configuration yielded a distribution of measurement outcomes across the four possible two-qubit states (00, 01, 10, 11), from which correlation functions were extracted.

The probability distributions in Figures 2 demonstrate clear asymmetry in outcome frequencies, reflecting the correlations encoded in the entangled Bell state. For $X \otimes W$ and $Z \otimes W$ measurements, outcomes 01 and 10 dominate with probabilities of approximately 43% each, while 00 and 11 occur with roughly 7% probability. Conversely, for $X \otimes V$ measurements, outcomes 00 and 11 become dominant, indicating the basis-dependent nature of quantum correlations.

Table 1. Measured correlation values for different operator combinations with statistical uncertainties from five repeated measurements.

Configuration	Correlation Value	Deviation (%)
$\langle X \otimes W \rangle$	-0.710	± 0.022
$\langle X \otimes V \rangle$	0.696	± 0.013
$\langle Z \otimes W \rangle$	-0.697	± 0.013
$\langle Z \otimes V \rangle$	-0.706	± 0.018

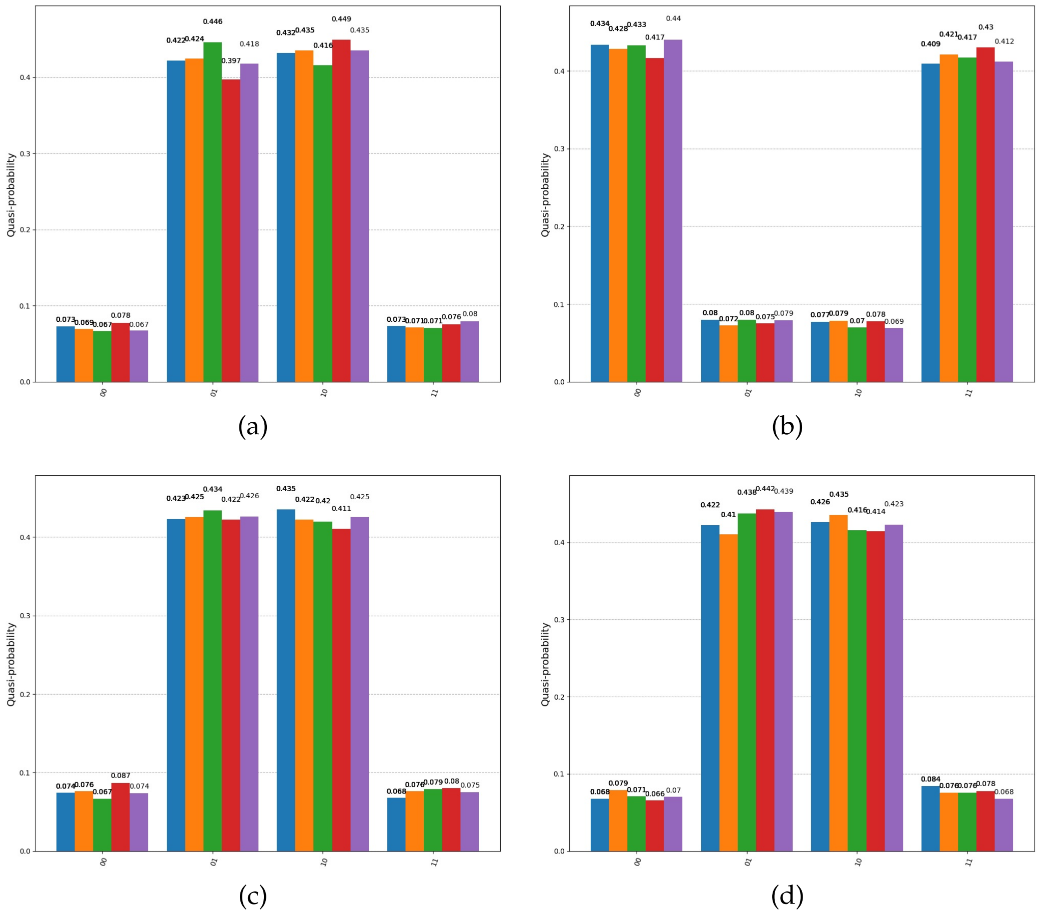

Figure 2. Probability histograms for the four CHSH measurement configurations: (a) $X \otimes W$, (b) $X \otimes V$, (c) $Z \otimes W$, and (d) $Z \otimes V$. The distributions show clear asymmetry, reflecting quantum correlations in the Bell singlet state.

3.2. Inequality Calculation

The CHSH parameter was calculated by combining the four correlation measurements according to the CHSH formula. The experimental value $S = -2.810 \pm 0.034$ clearly violates the classical CHSH bound of $|S| \leq 2$ by more than 23 standard deviations. This violation provides strong evidence for quantum entanglement and the non-local nature of quantum correlations. Moreover, the measured CHSH parameter exhibits excellent agreement with the quantum mechanical prediction of $-2\sqrt{2} \approx -2.828$.

4. Discussion

The experimental results demonstrate successful violation of the CHSH inequality, confirming the presence of quantum entanglement in the prepared Bell singlet state. The measured CHSH parameter of -2.810 significantly exceeds the classical bound while remaining close to the quantum mechanical maximum, indicating near-ideal entanglement quality in the simulated system.

Several factors contribute to the observed deviation from the theoretical value. First, the finite number of measurement shots (2048 per configuration) introduces statistical fluctuations in the estimated probabilities. The standard deviations observed across the five repetitions reflect this sampling variability. Second, the simulators often reflect the noise which can be found in real quantum computers.

The asymmetric probability distributions observed in the measurement outcomes directly reflect the correlation structure of the Bell singlet state. For configurations where Alice and Bob measure in similar bases, the anti-correlated outcomes (01 and 10) dominate. In contrast, when measurements are performed in maximally different bases, the correlation pattern inverts, with correlated outcomes (00 and 11) becoming more probable.

The high statistical significance of the CHSH violation provides robust evidence against local hidden variable theories. This result would remain significant even after accounting for potential systematic errors or loopholes that might affect real hardware implementations.

5. Conclusions

This study successfully verified quantum entanglement through experimental violation of the CHSH inequality [1] using a simulated two-qubit Bell singlet state. The measured CHSH parameter of $S = -2.810$ substantially exceeds the classical bound of $|S| \leq 2$, demonstrating the non-local correlations characteristic of quantum entangled systems.

The experimental framework implemented here provides a robust and replicable methodology for entanglement verification. The statistical analysis through repeated measurements quantifies the precision achievable with current quantum simulations, and proves the consistency of the results.

Future work could extend this analysis by running the simulation on a real quantum computer, or by attempting to utilize other states. Furthermore, more thorough analysis of the uncertainty and distribution of standard deviations could be conducted.

The demonstrated CHSH violation confirms the fundamental quantum nature of entanglement and validates quantum simulators as a robust, controllable and reproducible way to simulate this behavior. In further proves the potential of quantum information in tasks where to classical information theory fails.

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