

## Trajectory function

### Goal

Generate a five-minute, physically plausible 3D drone trajectory sampled at 100 Hz that:

- Stays inside a bounded airspace,
- Is smooth (continuous position, velocity, and acceleration),
- Respects motion limits on speed, acceleration, and jerk (rate of change of acceleration),
- Is randomized every run to provide diverse test cases for filters and intercept algorithms.

Output:  $\text{traj} \in \mathbb{R}^{N \times 3}$  with columns  $[x \ y \ z]$  in meters, where  $N \approx 5 \text{ min} \times 100 \text{ Hz} = 30,000$ .

### Setup & Key Parameters

- Sampling:  $dt = 0.01 \text{ s}$  (100 Hz); duration = 5 min  $\rightarrow N = \text{round}(\text{duration}/dt)$ .
- Bounds:  $x, y \in [-1200, 1200] \text{ m}$ ,  $z \in [40, 700] \text{ m}$  (keeps the path inside a safe “box”).
- Kinematic limits:  $v_{\text{max}} = 25 \text{ m/s}$ ,  $a_{\text{max}} = 8 \text{ m/s}^2$ ,  $j_{\text{max}} = 50 \text{ m/s}^3$ .
- Randomness: `rng('shuffle')` seeds the RNG from system time so every run produces a new path.

### Algorithm (high level)

1. Waypoint sampling & spline path
  - Draw 10–20 random waypoints over the 5-minute horizon (including start and end times).
  - For each axis ( $x, y, z$ ), form a shape-preserving cubic spline through those waypoints using `interp1(...,'pchip')`.

- Result: a smooth reference position path  $\text{raw}(t)$  without any dynamics limits yet.
2. Velocity, acceleration, and jerk limiting (online profile shaping)
- For each time step  $k$ :
    - Compute desired velocity from the reference path:  

$$v_{\text{des}} = (\text{raw}(k) - \text{raw}(k-1)) / dt.$$
    - Clip speed: if  $\|v_{\text{des}}\| > v_{\text{max}}$ , scale it back along its direction.
    - Compute desired acceleration to move current velocity toward  $v_{\text{des}}$ :  

$$a_{\text{des}} = (v_{\text{des}} - v(k-1)) / dt, \text{ then clip to } a_{\text{max}}.$$
    - Enforce jerk limit:  

$$j = (a_{\text{des}} - a(k-1)) / dt; \text{ if } \|j\| > j_{\text{max}}, \text{ scale } j \text{ and recompute } a_{\text{des}} = a(k-1) + j \cdot dt.$$
    - Integrate to update dynamics:
      - $a(k) = a_{\text{des}}$
      - $v(k) = v(k-1) + a(k) \cdot dt$

This loop converts an arbitrary spline into a feasible motion profile that respects  $v/a/jerk$  constraints—i.e., smooth and flyable.

### 3. Position recomputation by integration

- Starting from the initial spline point, integrate the limited velocity:
  - $\text{traj}(k) = \text{traj}(k-1) + v(k) \cdot dt$
- This guarantees position is consistent with the limited velocity/acceleration.

### 4. Clamping to domain

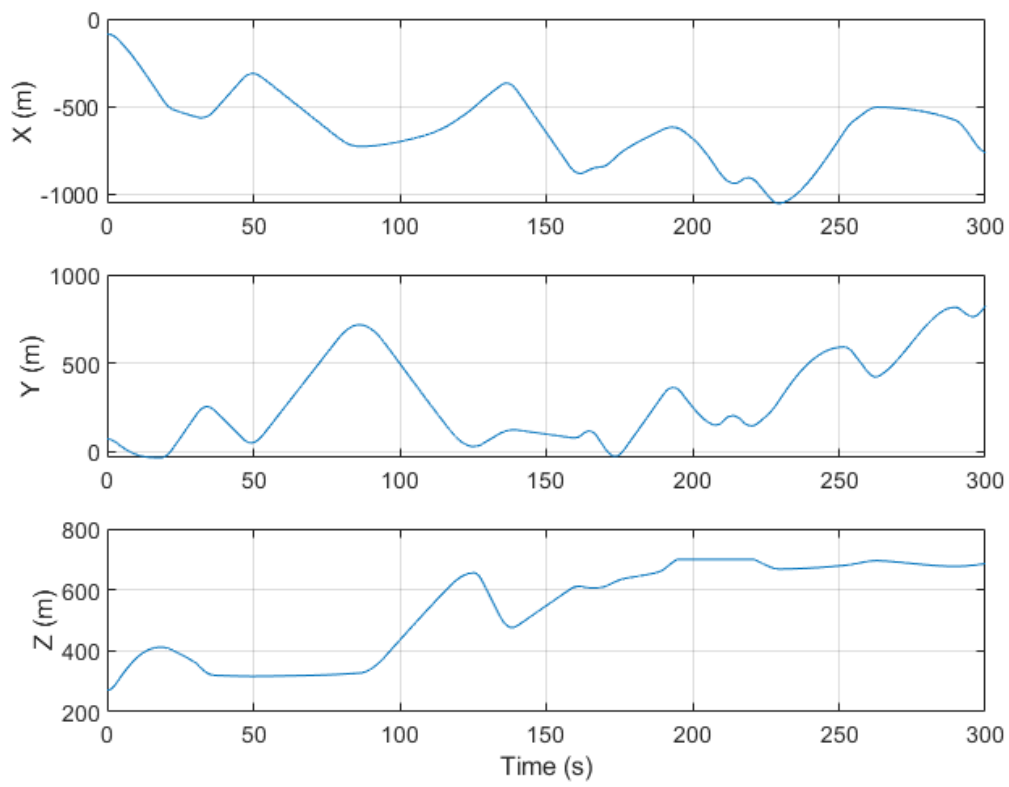
- After integration, clamp each axis to the allowed bounds to avoid numerical drift outside the airspace.

### 5. Metrics & plots

- Compute  $\|v\|$  and  $\|a\|$  over time with `vecnorm` and print averages.
- Produce quick-look plots: 3D path, positions vs. time, and velocity/acceleration profiles.

### **Why these design choices?**

- PCHIP ('pchip'): shape-preserving cubic interpolation avoids overshoot common in standard cubic splines, producing realistic paths between random waypoints.
- Sequential limiting (speed  $\rightarrow$  accel  $\rightarrow$  jerk): mirrors real flight controllers where commands are saturated hierarchically for flyability and to reduce excitation of the estimator.
- Jerk limiting: ensures continuous acceleration, reducing sharp changes that can destabilize filters or cause unrealistic actuator demands.
- Integrate velocity to position: keeps states physically consistent after limiting.
- Random waypoints + shuffled RNG: creates diverse trajectories for robust testing of filters and learning-based tuning.



**Randomized Smooth Trajectory**

