Numerical Methods for vibroacoustics

Exercices

1.1 Weak forms

Exercice 1.1. What is the weak form associated to problem

$$u''(x) + u(x) = 0 \text{ on }]0;1[, \quad u(0) = 0, \quad u'(1) = 1. \tag{1.1}$$

Exercice 1.2. What is the weak form associated to problem

$$u''(x) + u'(x) + u(x) = f(x) \text{ on }]0;L[, \quad u(0) = 0, \quad u(L) = 1. \tag{1.2}$$

Exercice 1.3. Let consider the 1D acoustic cavity harmonic problem at circular frequency ω:

$$\Omega =]0;L[, \quad p''(x) + k^2 p = 0, \quad k = \frac{\omega}{c}, \quad c = \sqrt{\frac{K}{\rho}}. \tag{1.3}$$

k is the wave number, c is the sound velocity, K is the compressibilty and ρ is the density.

- What is the weak form associated to this problem with boundary conditions

$$p'(0) = 0, \quad p'(L) = -\rho\omega^2. \tag{1.4}$$

- What is the physical significance of the boundary condition in $x = L$

Exercice 1.4.

- What is the weak form associated to problem

$$p''(x) + k^2 p(x) = 0 \text{ on }]0;L[, \quad p'(0) = 0, \quad p'(L) = \rho\omega^2. \tag{1.5}$$

- What is the physical significance of the boundary condition in $x = L$

1.2 Elementary matrices

Exercice 1.5. What is the elementary matrix associated to the following weak form:

$$\int_0^h u(x)v'(x)\ \mathrm{d}x \tag{1.6}$$

with the following discretisation

$$u(x) = [\boldsymbol{\Phi}_1(x)\,|\boldsymbol{\Phi}_2(x)] \begin{Bmatrix} \mathbf{u}_1 \\ \\ \mathbf{u}_2 \end{Bmatrix} \tag{1.7}$$

with

$$\boldsymbol{\Phi}_1(x) = \frac{x}{h}, \quad \boldsymbol{\Phi}_2(x) = \frac{x}{h} \tag{1.8}$$

Exercice 1.6. What are the interpolation functions for Lagrange elements of degree 2 on an element defined on the $[0; h]$ interval. and associated to nodes in $x = 0$, $x = h/4$, and $x = h$.

Exercice 1.7. What is the elementary matrix associated to the following weak form:

$$\int_0^h u'(x)v''(x)\ \mathrm{d}x \tag{1.9}$$

with the following discretisation

$$u(x) = [\boldsymbol{\Phi}_1(x)\,|\boldsymbol{\Phi}_2(x)|\boldsymbol{\Phi}_3(x)] \begin{Bmatrix} \mathbf{u}_1 \\ \\ \mathbf{u}_2 \\ \\ \mathbf{u}_3 \end{Bmatrix} \tag{1.10}$$

with

$$\mathbf{\Phi}_1(x) = \frac{2x^2}{h^2} - \frac{3x}{h} + 1, \quad \mathbf{\Phi}_2(x) = -\frac{4x^2}{h^2} + \frac{4x}{h}, \quad \mathbf{\Phi}_3(x) = \frac{2x^2}{h^2} - \frac{x}{h}. \tag{1.11}$$

1.3 Assembly of matrices

Exercice 1.8. Let consider the following volumic weak form

$$\forall\, v, \quad \int_0^1 u(x)v(x)\ \mathrm{d}x \tag{1.12}$$

$]0;1[$ is divided in two elements: $]0;1/3[$ and $]1/3;1[$. What is the global matrix associated to this discretization ?

Exercice 1.9. Let consider the following volumic weak form

$$\forall\, v, \quad \int_0^1 u'(x)v'(x)\ \mathrm{d}x \tag{1.13}$$

$]0;1[$ is divided in two elements: $]0;1/2[$ discretized by linear elements and and $]1/2;1[$ discretized by quadratic elements. What is the global matrix associated to this discretization ?

Exercice 1.10. The cubic Hermite basis is the set of four polynomials on $]0;1[$ defined by:

$$\psi_1(x) = 1 - 3x^2 + 2x^3 \tag{1.14}$$
$$\psi_2(x) = x - 2x^2 + x^3 \tag{1.15}$$
$$\psi_3(x) = -x^2 + x^3 \tag{1.16}$$
$$\psi_4(x) = 3x^2 - 2x^3 \tag{1.17}$$