Mathematics for Acousticians

Exercices

1.1 Vector spaces

Exercice 1.1. Show that the set $\mathbb{R}_n[X]$ of polynomials of degree lower than $n \in \mathbb{N}$ is a vector space.

Exercice 1.2. Show that the set of functions y solution of

$$y''(x) + 5y'(x) + 3y(x) = 0 \tag{1.1}$$

is a vector space.

Exercice 1.3. Let $[\mathbf{M}]$ be a $n \times n$ matrix. Show that the solution of

$$[\mathbf{M}]\mathbf{X} = \mathbf{0} \tag{1.2}$$

is a vector space.

Exercice 1.4. What is the dimension of the nullspace[a] of matrix

$$[\mathbf{M}] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}. \tag{1.3}$$

[a] the set of vector $\mathbf{x}$ so that $[\mathbf{M}]\mathbf{x} = \mathbf{0}$

Exercice 1.5. Is the family of three vectors a basis of $\mathbb{R}^3$? If yes, is this basis orthonormal ?

$$[\boldsymbol{\Phi}] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \tag{1.4}$$

Exercice 1.6. Let V be the subspace of $\mathbb{R}^3$ spanned by the two vectors $\mathbf{e}_1 = \begin{Bmatrix} \sqrt{1/2} \\ \sqrt{1/2} \\ 0 \end{Bmatrix}$ and $\mathbf{e}_2 = \begin{Bmatrix} 1 \\ 0 \\ 2 \end{Bmatrix}$

Find an orthonormal basis of V including $\mathbf{e}_1$

Exercice 1.7. Show that the following applications define inner products on the given vector space E.

- $E = \mathbb{R}^n$, $< \mathbf{x}, \mathbf{y} >= \sum_{i=1}^{n} x_i y_i$.

- $E = \mathbb{R}^3 < \mathbf{x}, \mathbf{y} >= \mathbf{x}^t[\mathbf{A}]\mathbf{y}$ with

$$[\mathbf{A}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

- $E = \mathbb{R}^2 < \mathbf{x}, \mathbf{y} >= \mathbf{x}^t[\mathbf{A}]\mathbf{y}$ with

$$[\mathbf{A}] = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Exercice 1.8. Show that the following applications define inner products on the given vector space E.

- $E = \mathbb{R}^n$, $< \mathbf{x}, \mathbf{y} >= \sum_{i=1}^{n} x_i y_i$.
- $E = \mathbb{R}^3 < \mathbf{x}, \mathbf{y} >= \mathbf{x}^t[\mathbf{A}]\mathbf{y}$ with

$$[\mathbf{A}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

- $E = \mathbb{R}^2 < \mathbf{x}, \mathbf{y} >= \mathbf{x}^t[\mathbf{A}]\mathbf{y}$ with

$$[\mathbf{A}] = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Exercice 1.9. Let $u = \{u_n\}_{n \in \mathbb{N}}$ be a numerical sequence on $\mathbb{N}$. u is said square summable (or $u \in l^2(\mathbb{N})$) if:

$$u \in l^2(\mathbb{N}) \Leftrightarrow \sum_{n \in \mathbb{N}} u_n^2 < +\infty$$

- Show that $l^2(\mathbb{N})$ is a vector space.
- Show that $< u, v >= \sum_{n \in \mathbb{N}} u_n v_n$ is an inner product on $l^2(\mathbb{N})$.
- Show that $\mathcal{N}(u) = \sqrt{\sum_{n \in \mathbb{N}} u_n^2}$ is a norm on $l^2(\mathbb{N})$.

<u>Exercice</u> 1.10. Let $\mathbb{P}_3$ be the vector space of polynomials with degree lower than 3. We consider the application:

$$< P, Q >= \int_0^1 P(x)Q(x)\, dx$$

Show that this application is an inner product.
Let $\{P_0 = 1, P_1 = x, P_2 = x^2, P_3 = x^3\}$ be a basis of this space.
Is this basis orthonormal?