Exercices

1.1 Weak forms

1.1.1 Basic exercices (MRAC/INPAM/ATVE/IMDEA)

Exercice 1.1. What is the weak form associated to problem

$$u''(x) + u(x) = 0 \text{ on }]0;1[, \quad u(0) = 0, \quad u'(1) = 1. \tag{1.1}$$

Solution 1.1.

$$\forall\, v, \quad \int_\Omega u(x)v(x) - u'(x)v'(x)\ \mathrm{d}\Omega = -v(1). \tag{1.2}$$

Exercice 1.2. What is the weak form associated to problem

$$u''(x) + u'(x) + u(x) = f(x) \text{ on }]0;1[, \quad u(0) = 0, \quad u(L) = 1. \tag{1.3}$$

Solution 1.2.

$$\forall\, v, \quad \int_\Omega u(x)v(v) + u'(x)v(v) - u'(x)v'(x)\ \mathrm{d}\Omega = \int_\Omega f(x)v(x)\ \mathrm{d}\Omega. \tag{1.4}$$

Exercice 1.3. Let consider the 1D acoustic cavity harmonic problem at circular frequency ω:

$$\Omega =]0;L[, \quad p''(x) + k^2 p = 0, \quad k = \frac{\omega}{c}, \quad c = \sqrt{\frac{K}{\rho}}. \tag{1.5}$$

k is the wave number, c is the sound velocity, K is the compressibilty and ρ is the density.

- What is the weak form associated to this problem with boundary conditions

$$p'(0) = 0, \quad p'(L) = -\rho\omega^2. \tag{1.6}$$

- What is the physical significance of the boundary condition in $x = L$

Solution 1.3.

$$\forall\, v, \quad \int_\Omega u(x)v(v) + u'(x)v(v) - u'(x)v'(x)\ \mathrm{d}\Omega = \int_\Omega f(x)v(x)\ \mathrm{d}\Omega. \tag{1.7}$$

Exercice 1.4.

- What is the weak form associated to problem

$$p''(x) + k^2 p(x) = 0 \text{ on }]0; L[, \quad p'(0) = 0, \quad p'(1) = \rho\omega^2. \tag{1.8}$$

- What is the physical significance of the boundary condition in $x = 1$

Solution 1.4.

$$\forall\, v, \quad \int_\Omega u(x)v(v) + u'(x)v(v) - u'(x)v'(x)\ \mathrm{d}\Omega = \int_\Omega f(x)v(x)\ \mathrm{d}\Omega. \tag{1.9}$$

1.1.2 Advanced exercices (MRAC/INPAM)