Non Linear Acoustics

Governing Equations

Mass Conservation

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathbf{v}) = 0$$

Momentum Conservation

$$\rho \frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = -\mathbf{grad}p + \eta \boldsymbol{\Delta} \mathbf{v} = \left(\xi + \frac{1}{3}\eta\right) \mathbf{grad}\mathrm{div}\mathbf{v}$$

Energy Conservation

$$\rho T \left(\frac{\partial S}{\partial t} + (\mathbf{v}\nabla)S\right) = \lambda \Delta T + \xi(\mathrm{div}\mathbf{v})^2 + \frac{\eta}{2}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right)$$

State Equation

$$P = P(\rho, S)$$

Simple Wave Equation

$$\frac{\partial v}{\partial x} - \frac{\epsilon}{c_0^2} v \frac{\partial v}{\partial \tau} = 0 \quad \Rightarrow \quad \frac{\partial V}{\partial \Xi} - V\frac{\partial V}{\partial \Theta} = 0$$

$$\epsilon = \frac{\gamma + 1}{2} \quad , \quad \tau = x - \frac{t}{c_0}$$

$$\Theta = \tau \omega_0 \quad , \quad V = \frac{v}{v_0} \quad , \quad \Xi = \frac{x}{x_{NL}}$$

Simple Wave Equation with absorption

$$\zeta, \eta \neq 0 \ , \ \lambda \neq 0 \quad \Rightarrow \quad \frac{\partial^2 v}{\partial t^2} - c_0^2 \frac{\partial^2 v}{\partial x^2} = \frac{b}{c_0}\frac{dr^3 v}{\partial x^2 \partial t}$$

Effective Viscosity

$$b = \zeta + \frac{4}{3}\eta + \lambda\left(\frac{1}{C_V} - \frac{1}{C_P}\right)$$

NL Effects & Absorption – Burger's Equation

$$\frac{\partial v}{\partial x} - \frac{\epsilon}{c_0^2} v \frac{\partial v}{\partial t} - \frac{b}{2\rho_0 c_0^3}\frac{\partial^2 v}{\partial t^2} = 0$$

$$\frac{\partial V}{\partial \Xi} - V\frac{\partial V}{\partial \Theta} - \underbrace{\frac{1}{2\epsilon \mathrm{Re}}}_{G}\frac{\partial^2 V}{\partial \Theta^2} = 0$$

About Re – Acoustical Reynolds

$$\mathrm{Re} = \frac{\rho_0 c_0 v_0}{b \omega_0}$$

- Re << 1 Viscosity hides NL effects ($V \partial V / \partial \Theta$ is small)
- Re >> 1 indicates $\frac{1}{2\epsilon \mathrm{Re}} \partial^2 V / \partial \Theta^2$ is neglectable for short distances

Kordeveg de Vries-Burgers Equation

$$\frac{\partial V}{\partial \Xi} - V\frac{\partial V}{\partial \Theta} - G\frac{\partial^2 V}{\partial \Theta^2} + \underbrace{D\frac{\partial^3 V}{\partial \Theta^3}}_{dissipation} = 0$$