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Optimizing the placement of irregular shapes 2D Nesting

A. DJILALI, M. FOUAD and S. KOULOUGHLI
Manufacturing engineering Laboratory of Tlemcen MELT.
University of Tlemcen
Tlemcen, Algeria
E-mail: ahmed.djilali@univ-tlemcen.dz

Abstract— **The problems of placement include a wide range of problems, namely the problems of cutting and packing and the problems of layout, which have cultivated a very high attention by numerous industries (textile, wood, ship-building ...). In our work we address the problem of placing 2D shapes in a well-defined rectangular surface so-called Nesting. Many previous works approached this problem for regular shapes such as rectangle, triangle, square, circle. In our study we focused on the optimization of placement of irregular shapes named by 2-dimensional irregular bin packing problems (2DIBPP), the particularity of irregular shapes compared to other regular types is the inability to calculate their surfaces by an exact mathematical equation. In this regard, we give interest to the mathematical modeling of this type of placement problem and develop a heuristic algorithm able to solve this NP- Hard problem while respecting a set of non-overlapping constraints, the ability to pivot, and the confinement precise shapes in the bin.**

Keywords—2DIBPPshapes, Nesting; Irregular.

I. INTRODUCTION

Cutting and packing problems are tackled by many researchers and industries in an attempt to reduce their production costs which represents the major problem in manufacturing industries in a competitive world where products are available almost instantly around the world.

The word placement has been used several times in the literature. Wäscheret al. [4] use this term to define a category of problems, for which a set of weakly heterogeneous components must be assigned to well-defined bins.

Zhang et al. [3] use the term object to designate a component to be placed inside a bin. In large-scale integration problems, the term module refers to a rectangular component [8].

The frequent use of materials is of particular interest for mass production industries, as small improvements in layout can lead to significant material savings and considerably lower production costs. Dyckhoff [11] shows that these problems are both the sectors of the cutting industry and the production systems. Edmund K. Burke [5] solved the problem of one-dimensional and two-dimensional cutting by heuristics, on the other hand in [2] they based on exact methods using dynamic programming and column generation. There may also be placement problems where the bin serves as a support for the components to be positioned. This is the case of a truck where these components must be placed on the chassis [6]. In the meantime [9] as well as [10] offered an overview of cutting and packing problems. Harald Dyckhoff [7] Proposed an annotated bibliography in 1997.

We take the example of the company ACME which had a monthly material expense of about \$ 50 000 however 25% (\$ 12 500) of these costs come from the waste of the raw material, ACME has managed to reduce their production cost by 10% (\$ 1250) with Nesting software.

Cutting and packing problems are encountered in many industries. Whereas the wood, glass and paper industry are mainly concerned with the cutting of regular shapes, mostly arbitrary and irregularly shaped objects must be packed in the shipbuilding, textile and leather industries.

To our knowledge there have not been enough works treating this subject thus we intend to treat the case of cutting forms that are irregular. The duration of generation of the storage area is also relevant in industrial applications. Since manual generation can easily require several days, packing time is an important economic factor. It is therefore understandable that industries are looking for ways to automate the packing process such as CNC machines for cutting and AS / RS for storing packages.

II. MATHEMATICAL MODEL DESCRIPTION

Cutting and packing problems are NP-hard and they are similar to the backpack problem [12], for which the components are only geometrically related to one another. In other words, there is no explicit interaction between the components. All components have the same types of attributes. Objectives and constraints can always be modeled as mathematical functions and are defined globally, that is, they involve all the attributes of the components.

Given a rectangular paper sheet B with a width W and height H, a set of **n** irregular shapes P with Pi the defined area of each part. The purpose of the 2D nesting problem is to find a location that maximizes the use of the area (sheet) and the number of shapes placed in the sheet (the fill rate).

The packing must meet the following constraints:

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1 / Each packaged part must be completely wrapped in the sheet (bin).
2 / the shapes are rotating.
3 / Packaged shapes must not overlap

A. Seed point:

It is a tool that helps developers calculate the area of an irregular region or shape by a very simple function than of a mathematical model that requires more analysis.

Seed point functioning. The seed point uses the image processing to detect the placement area (hole) where we want to place a part, if the first point of hole is detected it is noted by a different color than the non-hole one. This is then reproduced (with the same color) until the limit where it reaches the neighboring shapes or the limits of the bin. After having finished all the reproductions possible in the hole, the summation of all the new points results in the new hole surface.

$$\text{Area(hole)}= \text{SP}.\sum_{i=1}^{n} Ki \tag{1}$$

with

SP: seed point

K: reproduced point

$$\text{SP}= \begin{cases} 1 & if\ SP\epsilon hole \\ 0 & elsewhere \end{cases} \tag{2}$$

B. Mathematical model for irregular shapes

In our work we have encountered a problem of calculating the surface of irregular shapes by a well-defined mathematical equation. To solve this challenge, we have defined a method to calculate this surface by the subdivision of the irregular form into regular sub-shapes (triangle, rectangle, square, circle, sectorial circular, trapezium). We then deduce the global mathematical model by the summation of all the models found for each of the sub-shapes in order to optimize the surface of the material sheet in a case of 2D Nesting with shapes of irregular shapes.

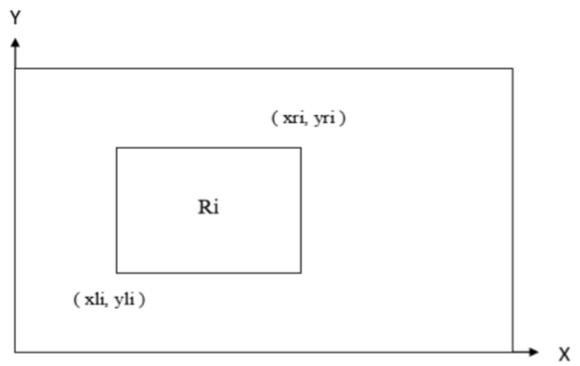

Fig. 1. the important geometric point represented by the mathematical model.

Notations:

TABLE I. MEANING OF THE VARIABLES AND INDICES

Notation	**meaning**	**Interval**
I	Part index	i=1….ni#j
J	Index of the next part	j=2…. nj#i
F	Decision variable	{0,1}
W	the part coast	/
X, Y	coordinates of the shapes (the more left xl and the more right xr)	/
K	sectoral circular	%

In this section we will put the general model of irregular forms problem.

Objectif function

$$Max \sum_{i=1}^{n} Fi.Wi.hi + \sum_{i=1}^{n} Fi.Wi^2 + \sum_{i=1}^{n}(Fi.Wi.hi) \div 2 + \sum_{i+1}^{n}(Fi.\pi ri^2) \div \frac{1}{k} \tag{3}$$

Under the following constraints:

Each part must be packed completely in the bin (the rectangle)

$$\text{Fi} = 0 \cup (0 \leq \text{xli} < \text{xri} \leq \text{w} \cap 0 \leq \text{yli} < \text{yri} \leq \text{H}) \tag{4}$$

Pivoting possibilities set of the part:

$$\text{Fi} = 0 \cup (\text{xri} - \text{xli} = \text{wi} \cap \text{yri} - \text{yli} = \text{hi}) \cup (\text{xri} - \text{xli} = \text{hi} \cap \text{yri} - \text{yli} = \text{wi}) \tag{5}$$

$$\text{Fi} = 0 \cup (\text{Xri} - \text{XLi} = \text{Wi} \cap \text{Yri} - \text{YLi} = \text{Wi}) \cup (\text{Xri} - \text{XLi} = \text{Wi} \cap \text{Yri} - \text{YLi} = \text{Wi}) \tag{6}$$

$$\text{Fi} = 0 \cup \left(\text{Xri} - \text{XLi} = \text{Wi} \cap \text{Yri} - \text{YLi} = -\frac{\text{hi}}{\text{wi}}.\text{X}' + \text{hi}\right) \cup \left(\text{Xri} - \text{XLi} = -\frac{\text{hi}}{\text{wi}}.\text{X}' \cap \text{Yri} - \text{YLi} = \text{Wi}\right) \cup \text{X}' = \text{Xri} - \text{XLi} \tag{7}$$

$$\text{Fi} = 0 \cup (\text{Xri} - \text{XLi} = \text{ri}.\cos\theta \cap \text{Yri} - \text{YLi} = \text{ri}.\sin\theta) \cup (\text{Xri} - \text{XLi} = \text{ri}.\sin\theta \cap \text{Yri} - \text{YLi} = \text{ri}.\cos\theta) \quad 0 \leq \theta \leq 2\text{p} \tag{8}$$

No-overlapping between shapes:

$$\text{Fi} = 0 \cup \text{Fj} = 0 \cup (\text{xli} \geq \text{xrj} \cup \text{xlj} \geq \text{xri} \cup \text{yli} \geq \text{yrj} \cup \text{ylj} \geq \text{yri}) \tag{9}$$

$$Fi = 0 \cup \quad Fj = 0 \cup XLi \geq Xrj \cup XLj \geq Xri \cup YLi \geq \frac{-hj}{wj}.XLj + hj \cup YLj \geq \frac{-hi}{wi}.Xri + hi \tag{10}$$

$$Fi \in \{0,1\}, i = 1,2, \dots, n \tag{11}$$

III. THE ALGORITHM DESCRIPTION

In this section we give the formal description of the problem.

- Let P= {p1, ..., pn} be a set of n polygons to package within a rectangular sheet bin b.
- We note W and L the height and length of the rectangular bin respectively.
- We denote Pi⊆P as the set of packaged shapes.
- n is the number of shapes placed on the sheet.
- each packaged peace pi has an orientation angle oi∈ [0,360°] and a dedicated position within the sheet (xi, yi) from the origin of the part.
- We consider the origin of each part as the right inferior angle of the wrapping sheet for a given rotation angle and the right inferior corner of the sheet.
- The objective is to minimize the waste and maximize the utilization of the sheet.

A. The set of feasible placements

To obtain all feasible placements between the two polygons in a storage sheet one uses the notions of the No Fit polygon (NFP) and Inner Fit Polygon (IFP): such that NFP represents the polygon of non-overlap between two polygons (shapes) see figure 3. For the obtained NFP it is enough to trace a shape around the limit of another polygon. One of the polygons remains attached to the location and the other polygons traverse the edges of the fixed polygon while ensuring that the two polygons always touch but never intersect. We need to choose a B reference point that will be drawn when B moves around A.

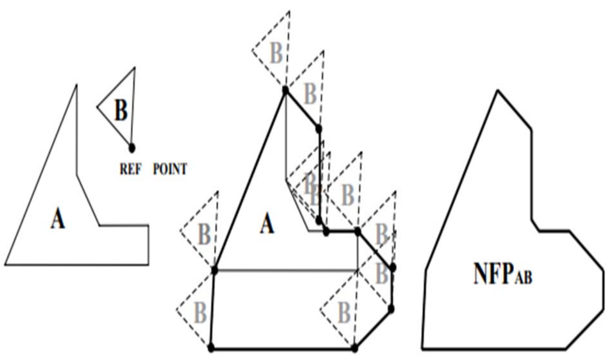

Fig. 2. Constructing an NFP

We then use the NFP to test the overlap between shapes see, Figure 3.

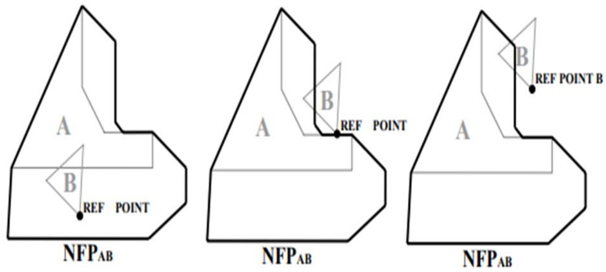

Fig. 3. The overlap between shapes

To define the IFP for the part and the Bin. We follow the same procedure as the NFP except that the orbital polygon is between the stationary polygon or between the polygon and the bin see Figure 4.

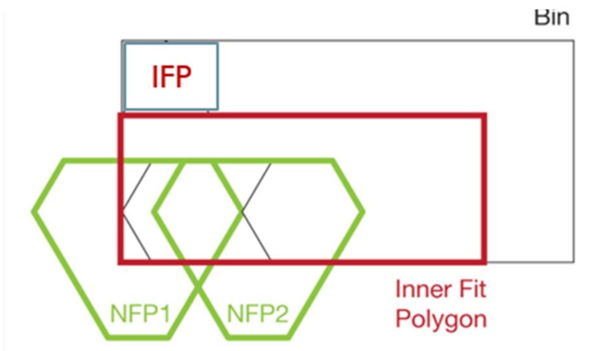

Fig. 4. Example of inner fit polygon

After finding the 2 polygons NFP and IFP we group them to visualize all the feasible placements in a bin see figure 6.

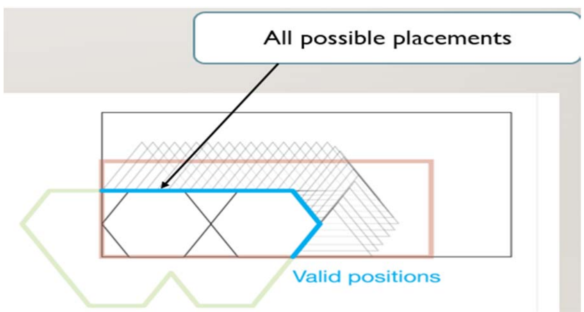

Fig. 5. The set of feasible placements between 2 shapes and a triangle

B. The position selection method

To choose the position that merited more than the candidate positions in the set of feasible placements obtained, three methods were used which have already used in [1] but by a different method that provide a set of achievable placement positions and orientations for the part (pi) with the minimum space possible.

In what follows, we describe the placement rules in the bin from the left end of the sheet extending to the right. We evaluate three well-known placement rules; bottom left placement, minimum length placement and maximum usage area, to determine the definitive position of each part.

However, these rules are dominated by first trying to fill the holes in the bin.

The first step in placing (pi) is to sort all the holes (Hk) in the box decreasing area, then using the procedure of filling the holes.

If (pi) fits into a given hole, place it in that hole and overlook the others.

If (pi) does not correspond to any hole, the algorithm tries to find a feasible placement position in each of the regions (b) starting with the one equal to the area of pi. Using our placement strategy described below for a partially packaged bin.

- Bottom left (BL): gives priority to placing the reference point of the part in the lowest position and as far left as possible relative to the extremity of bin, once a position is found in the bin region (b) search may stop. If more than one orientation of (pi) gives the same position BL, select the orientation that minimizes the bounding bin width of the part. If not, it selects the position of the part randomly.
- The placement position that minimizes the length (ML): it is the fact of positioning the part with the placement that minimizes the length between the origin of the sheet and the rightest x coordinate of (pi). In case of equality, we select the placement position at the bottom left.
- The maximum usage rule of the storage area of the part (MU) this rule will improve our stockage policy to take advantage of the storage area limited by the IFP and the set of feasible placements from our method, in such a way that we will select the position that maximizes the equation below.

$$U = \mathrm{area(p1)} + \frac{\mathrm{area(p2)}}{\mathrm{area(p1+P2)}} \qquad (12)$$

With:

p1: the part already stored in the bin

P2: the part to be selected is positioned to maximize the use of the storage area.

Algorithm: Placement algorithm for irregular shapes

1. ***input:*** *P0, the placement method // scheduling the order in descending mode and place the shapes in bottom left to right*
2. ***output:*** *P*: all the shapes enter the bin*
3. *α=0, π/2, π,3π/2;*
4. ***For*** *i =1...n* ***do*** *// i number of shapes*
5. ***For*** *L=1,2...q* ***do*** *// L number of holes*
6. *Scheduling the holes in descending mode*
7. ***if*** *HL < Pn* ***do*** *// elimination holes too small*
8. *Consider H as a polygon (usable hole)*
9. ***if*** *area (Pi)<=area (Hi)* ***do*** *// p: shapes H: Holes*
10. *Find the possible locations of the shapes P in the holes L with the corresponding angle and then begin with the most equal hole of its area.*
11. *Initialize H(i-1) // the remaining holes*
12. ***if*** *new (Pi)<= remains (Hi)* ***do//*** *if we receive other shapes can enter the remaining holes*
13. *Place the new Pi in the vacuum most equal to its area*
14. ***Else*** *reject the part*

Fin

Fin

Figure 6 represents results of the heuristic method used to find the optimal placement of irregular shapes drawn using CATIA V5.

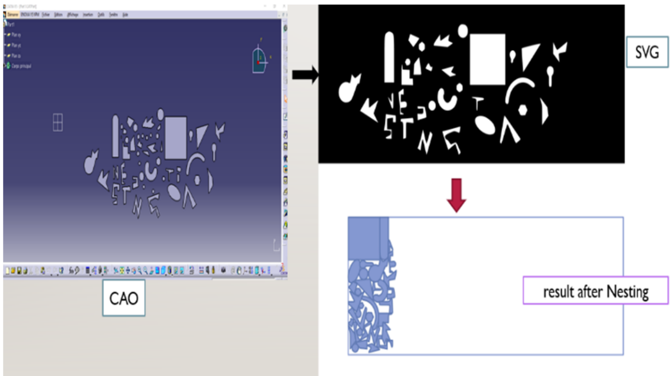

Fig.6. heuristic method results

CONCLUSION:

Our work solves a 2D Nesting problem for irregular shapes that has received little attention in the literature. We have proposed the method of seed point to calculate the surfaces of irregular shapes in a geometric base. The progress of our proposed algorithm is ensured with the results of our method which solves this placement problem. We then created a heuristic to maximize the use of the material for the purpose of minimizing the industry's return price.

As a future job in trying to develop a nesting software contains our algorithm which we believe will certainly be effective as other nesting software used in many industries.

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