Solving Irregular Shape Nesting Problems via Customized Branch-and-Bound

Akang Wang, Christopher L. Hanselman, Chrysanthos E. Gounaris
Dept. of Chemical Engineering and
Center for Advanced Process Decision-making
Carnegie Mellon University

EWO Meeting
September 20-21, 2017

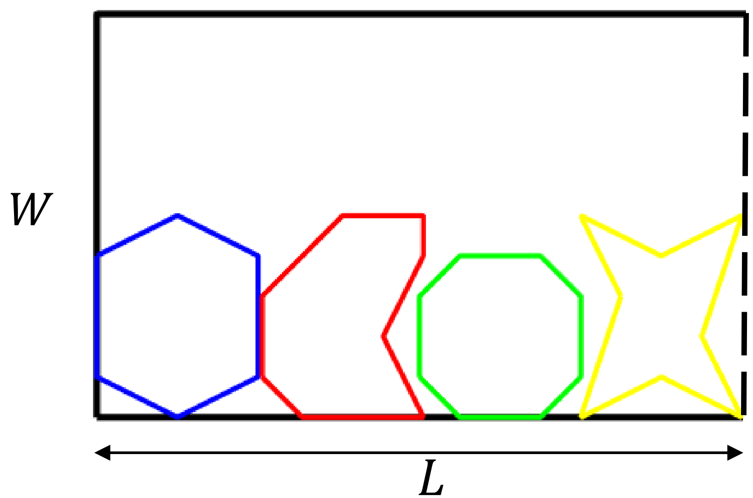
Problem Definition

Given :

- a set of polygons P
- a rectangular sheet of a fixed width W

Determine a physical configuration of these polygons such that:

- all polygons lie within the sheet
- there is no overlap among polygons
- the length of the sheet L is minimized



Non-Convex QP Model

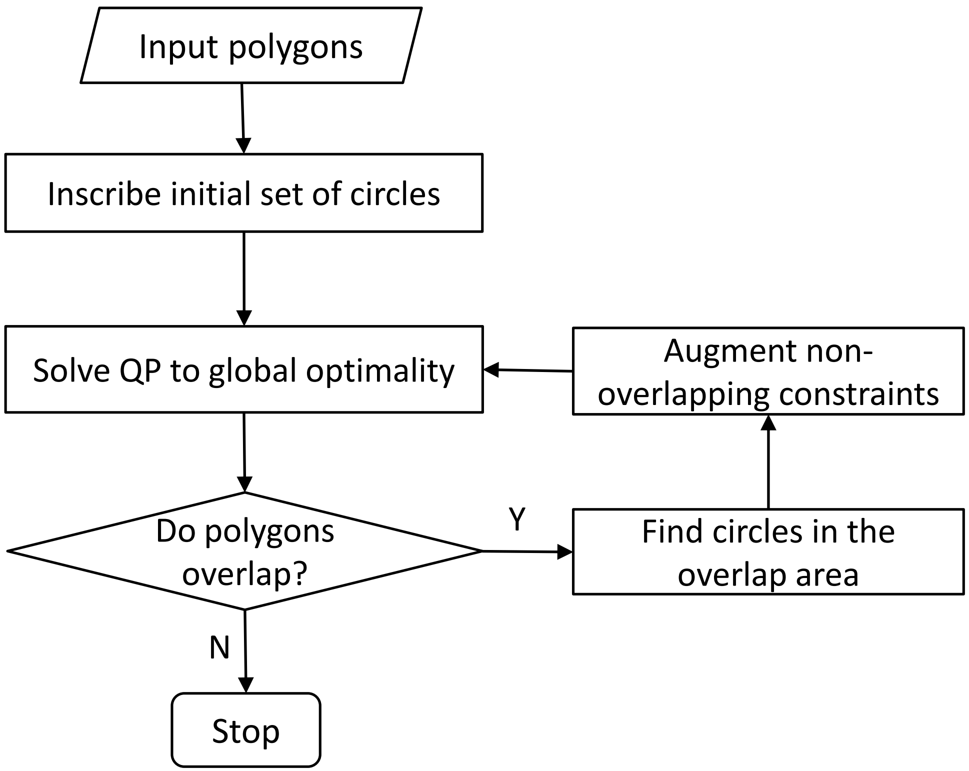
$$
\begin{array}{lll}
\min\limits_{c_p, s_p, h_p, v_p, L} & L & \\
\text{s.t.} & c_p^2 + s_p^2 = 1 & \forall\, p \in P \\
& 0 \leq c_p x_{pi} - s_p y_{pi} + h_p \leq L & \forall\, i \in I_p, \forall\, p \in P \\
& 0 \leq s_p x_{pi} + c_p y_{pi} + v_p \leq W & \forall\, i \in I_p, \forall\, p \in P \\
& \left[(c_p x_{pm} - s_p y_{pm} + h_p) - (c_q x_{qn} - s_q y_{qn} + h_q)\right]^2 + & \\
& \left[(s_p x_{pm} + c_p y_{pm} + v_p) - (s_q x_{qn} + c_q y_{qn} + v_q)\right]^2 \geq (R_m + R_n)^2 & \forall\, (m,n) \in C_p \times C_q, \\
& & \forall\, (p,q) \in \{P \times P : q > p\} \\
& -1 \leq c_p, s_p \leq 1 & \forall\, p \in P
\end{array}
$$

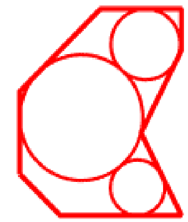
- Rotational angle θ_p is represented via sine and cosine
- Non-convexities stem from trigonometric and non-overlap constraints
- Remaining constraints are linear

Jones, Donald R. "A fully general, exact algorithm for nesting irregular shapes." *Journal of Global Optimization* 59.2-3 (2014): 367-404

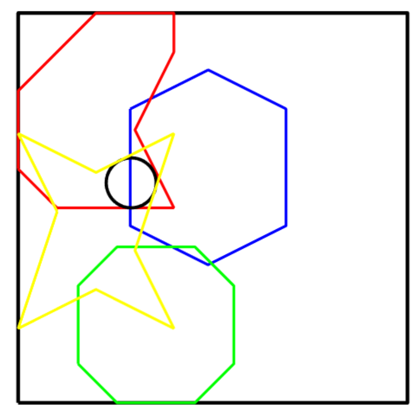
Literature Approach

(based on Iterative Global Optimization)





Use circles to approximate a polygon



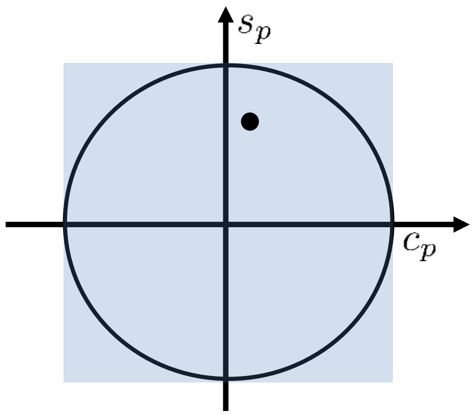
Find the largest circle in the overlap

Jones, Donald R. "A fully general, exact algorithm for nesting irregular shapes." *Journal of Global Optimization* 59.2-3 (2014): 367-404

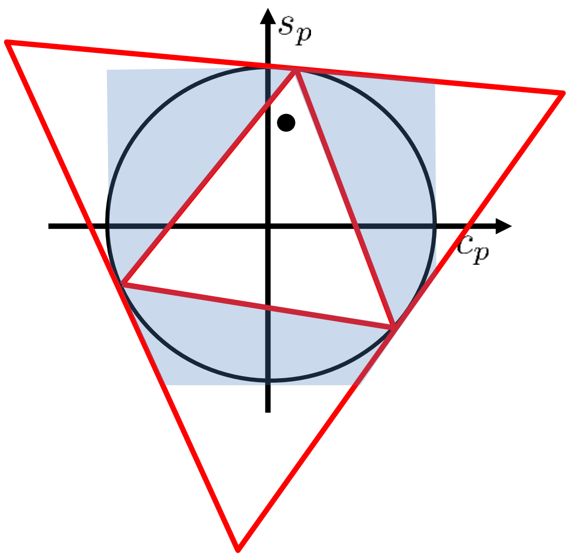
Novel Approach

(based on LP Relaxation and Supervisory B&B)

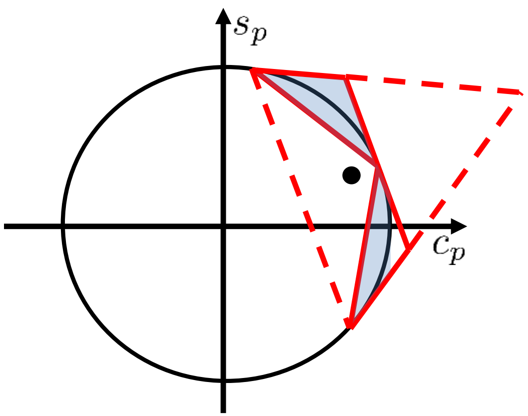
Enforce trigonometric constraints via branching on polygon rotation angles



Before any branching



Three-way branching
(first time only)

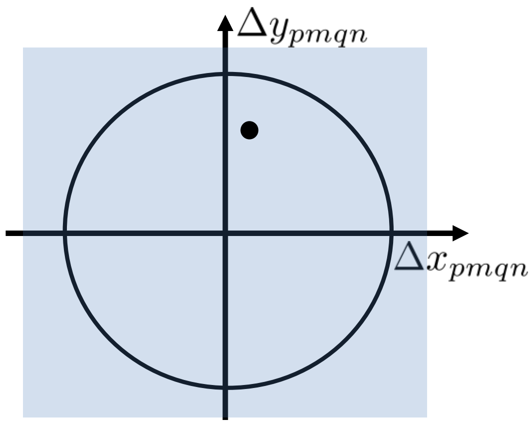


Two-way branching
(subsequently)

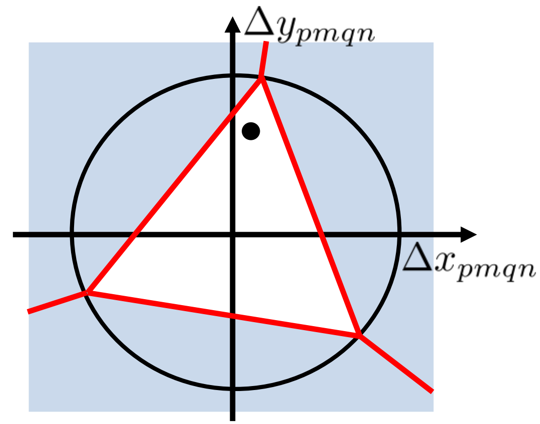
Novel Approach

(based on LP Relaxation and Supervisory B&B)

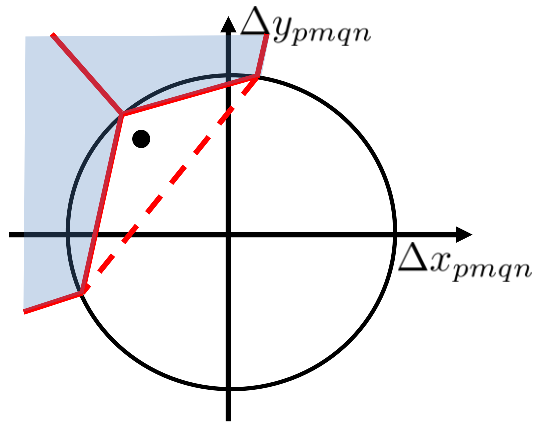
Enforce non-overlap constraints via branching on relative circle-circle placement



Before any branching



Three-way branching
(first time only)



Two-way branching
(subsequently)

Custom LP Relaxation

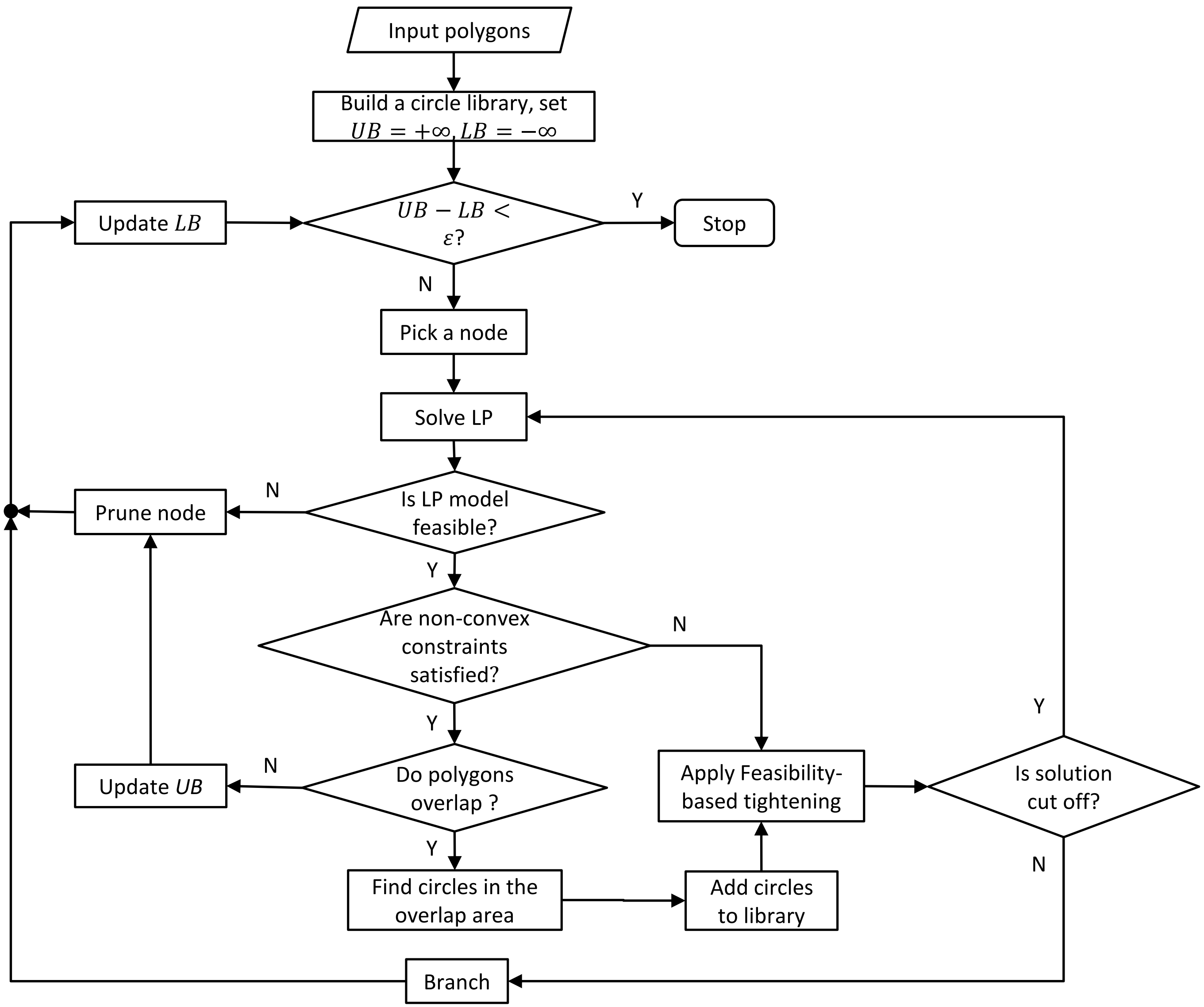
$$
\begin{aligned}
\min_{c_p, s_p, h_p, v_p, L} \quad & L \\
\text{s.t.} \quad & \hat{\alpha}_{pk} c_p + \hat{\beta}_{pk} s_p \geq \hat{\gamma}_{pk} && \forall\, k \in \{1,2,3\}, \forall\, p \in \hat{P} \\
& 0 \leq c_p x_i - s_p y_i + h_p \leq L && \forall\, i \in I_p,\ \forall\, p \in P \\
& 0 \leq s_p x_i + c_p y_i + v_p \leq W && \forall\, i \in I_p, \forall\, p \in P \\
& \alpha_{pmqnk} \left[(c_p x_m - s_p y_m + h_p) - (c_q x_n - s_q y_n + h_q) \right] + \\
& \beta_{pmqnk} \left[(s_p x_m + c_p y_m + v_p) - (s_q x_n + c_q y_n + v_q) \right] \geq \gamma_{pmqnk} && \forall\, k \in \{1,2,3\}, \\
& && \forall\, (m,n) \in C^2_{pq}, \\
& && \forall\, (p,q) \in \{P \times P : q > p\} \\
& -1 \leq c_p, s_p \leq 1 && \forall\, p \in P
\end{aligned}
$$

where: $C^2_{pq} \subseteq C_p \times C_q$

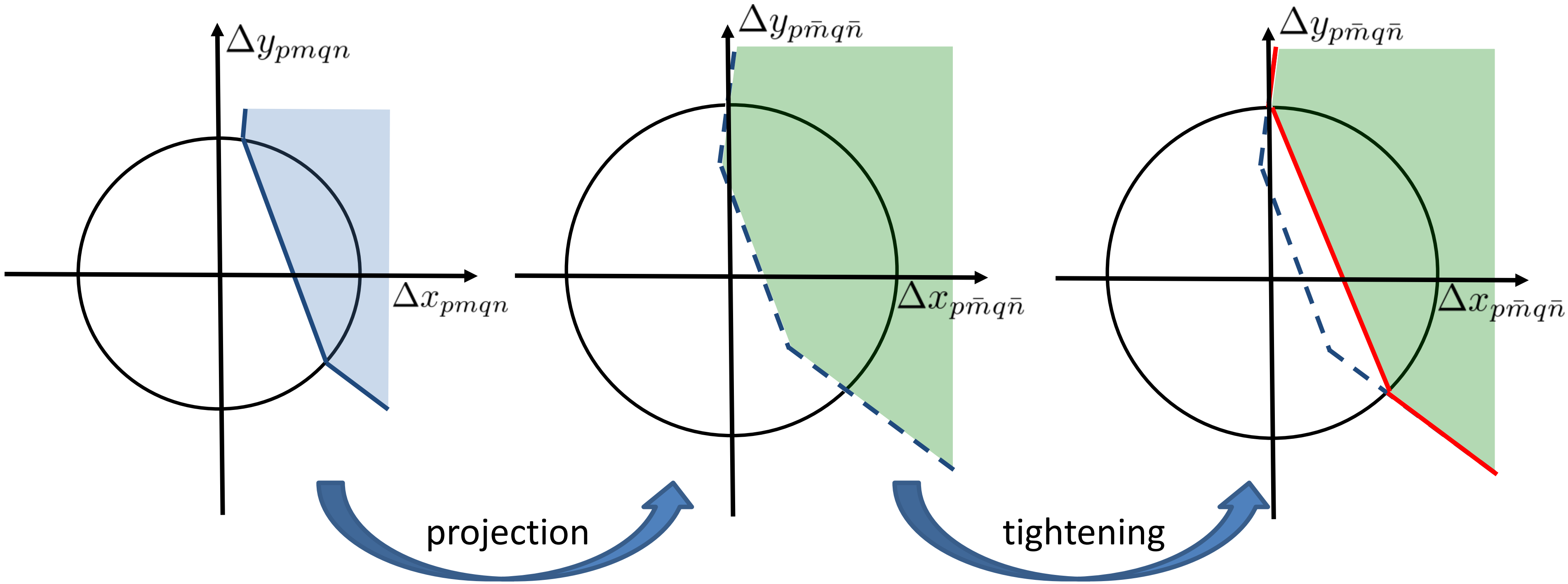
- Relax trigonometric and non-overlap constraints into linear constraints
- C^2_{pq} starts empty, and grows dynamically (to a hopefully small subset)

Novel Approach

(based on LP Relaxation and Supervisory B&B)



Feasibility-Based Tightening



Constraint tightening from $(\Delta x_{pmqn}, \Delta y_{pmqn})$ to $(\Delta x_{p\bar{m}q\,\bar{n}}, \Delta y_{p\bar{m}q\bar{n}})$

Computational Studies

# of polygons	# of instances	Avg. # of non-convex polygons	"Best-of-Four" *QP-Nest* approach			New BB-based Approach		
			# solved instances	Avg. Time (sec)	Avg. Gap (%)	# solved instances	Avg. Time (sec)	Avg. Gap (%)
4	35	1.7	35	65	-	35	2	-
5	21	2.1	16	1,371	1.0	21	55	-
6	7	2.6	1	632	2.2	7	406	-
7	1	3.0	0	-	6.6	1	4,176	-

- Allow for translation and fixed-orientation
- Use an optimal solution as incumbent
- "Best-of-Four" includes the best results from BARON, GloMIQO, LindoGlobal and SCIP
- Time limit: 2 hours

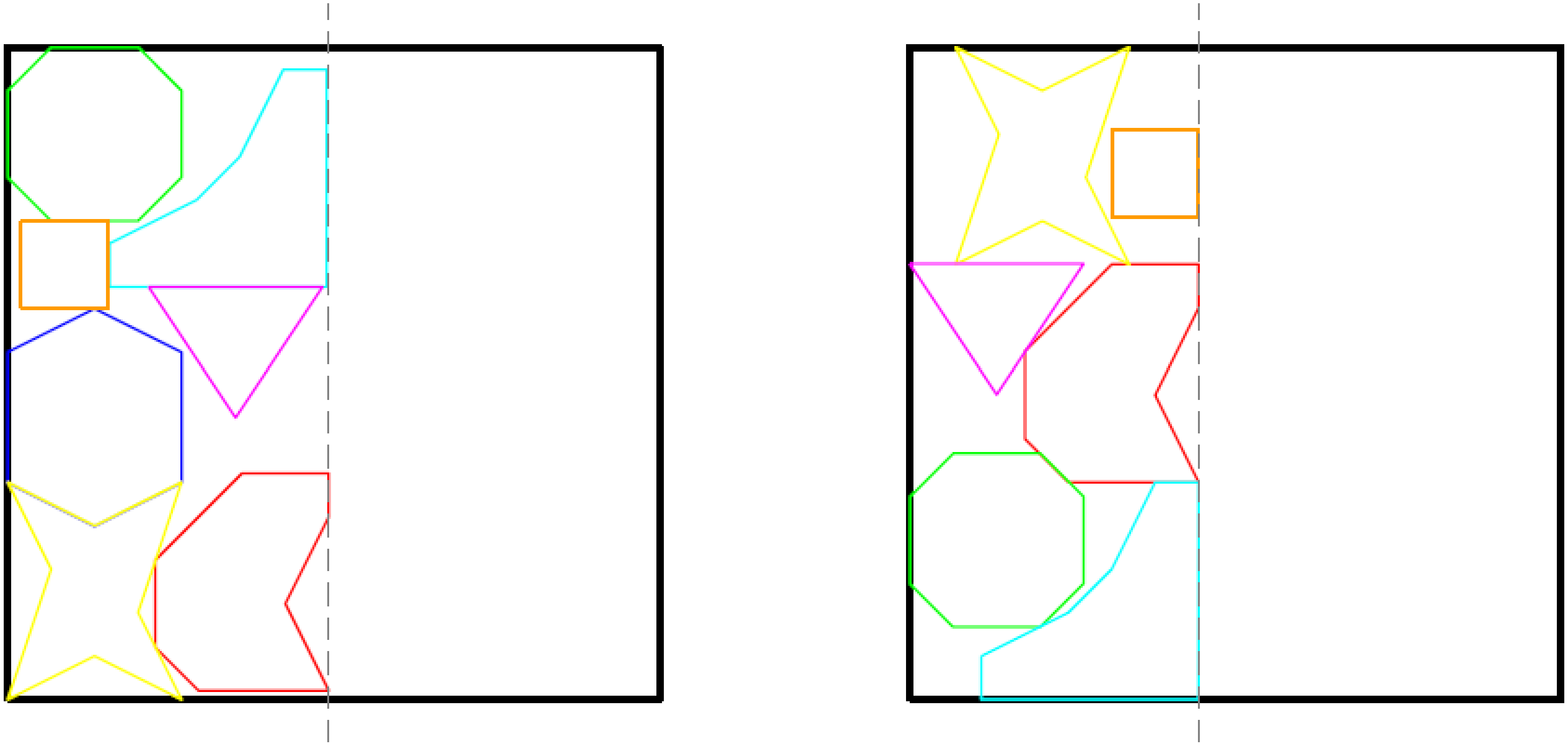
Computational Studies

# of polygons	# of instances	Avg. # of non-convex polygons	"Best-of-Four" *QP-Nest* approach			New BB-based Approach		
			# solved instances	Avg. Time (sec)	Avg. Gap (%)	# solved instances	Avg. Time (sec)	Avg. Gap (%)
4	35	1.7	24	7,181	13.1	35	287	-
5	21	2.1	0	-	22.7	10	17,058	41.6
6	7	2.6	0	-	71.9	0	-	54.8
7	1	3.0	0	-	*	0	-	52.6

* GO solvers could not complete the first iteration of the QP-Nest approach

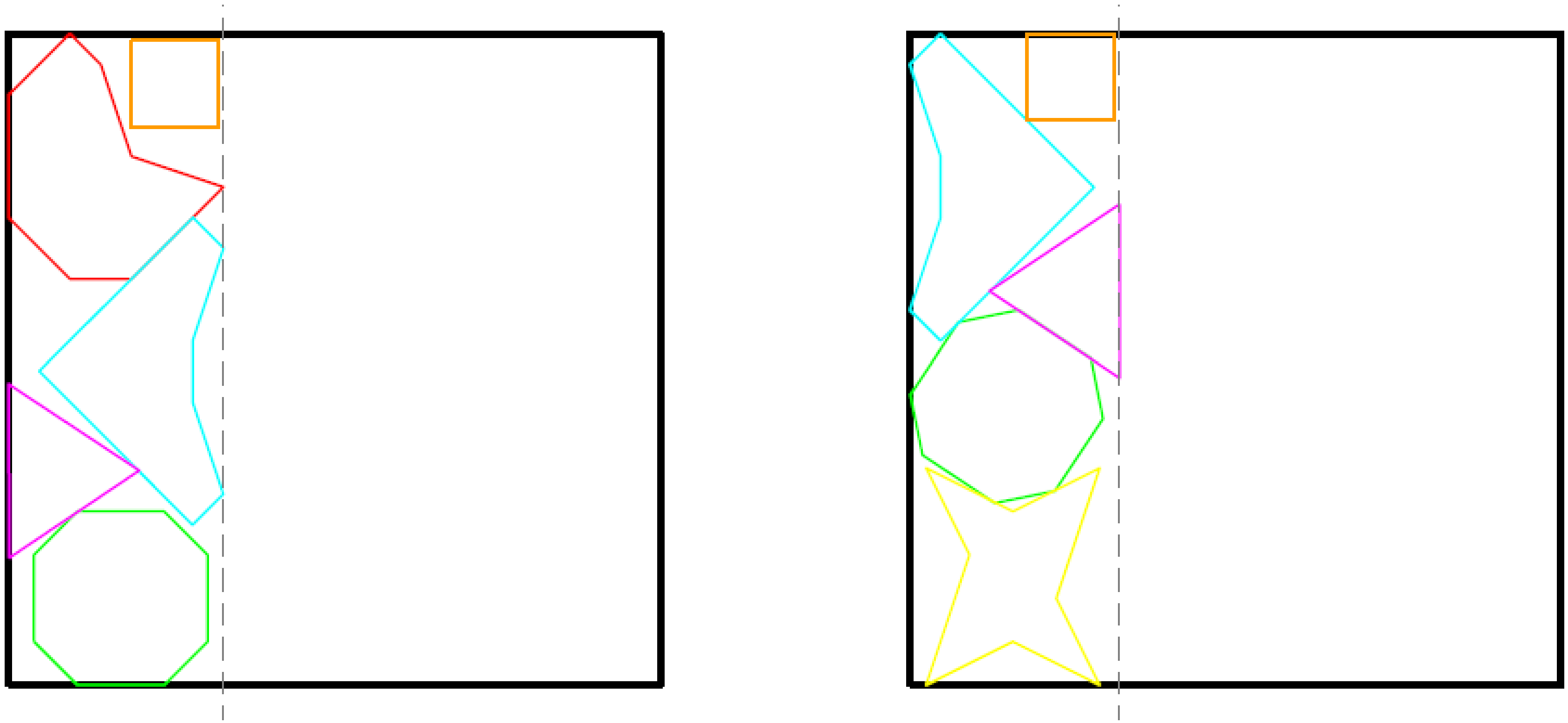
- Allow for free translation and rotation
- Use an optimal fixed-orientation solution as incumbent
- "Best-of-Four" includes the best results from BARON, GloMIQO, LindoGlobal and SCIP
- Time limit: 10 hours

Example Results



Optimal solutions for representative six-polygon (left) and seven-polygon (right) nesting problems under fixed orientation

Example Results



Optimal solutions for representative five-polygon nesting instances under free rotation

Conclusions

- We developed an exact approach to solve the Nesting Problem to global optimality that does not rely on the use of general-purpose global optimization solvers

- We identified a generic approach to dynamically satisfy reverse convex quadratic constraints commonly found in optimization models within the field of cutting and packing

- We conducted a comprehensive computational study that illustrates the competitiveness of our approach compared to the previous state-of-the-art

- We presented, for the first time in the open literature, provably optimal solutions for Nesting Problem instances featuring five polygons under free rotation.