Assignment 3

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Q.1)

1) $\exists x(I(x) \land \forall y(y \neq x \rightarrow I(y))$
2) $\exists x(\neg I(x) \land \forall y(I(y) \rightarrow y = x))$
3) $\forall x(I(x) \rightarrow \exists y(C(x,y) \land x \neq y))$
4) $\exists x(F(x) \land \forall y \neg C(x,y))$
5) $\exists x \exists y (x \neq y \land \neg C(x,y))$
6) $\exists x \forall y\ C((x,y)$
7) $\exists x \exists y (x \neq y \land \exists z(\neg C(x,y) \land \neg C(y,z))$
8) $\exists x \exists y (x \neq y \land \forall z(C(x,z) \lor C(y,z))$

Q.2)

1) Big v Dumb v (Big -> Dumb)

Big	Dumb	Big v Dumb	Big -> Dump	Big v Bump v (Big -> Dump)
F	F	F	T	T
F	T	T	T	T
T	F	T	F	T
T	T	T	T	T

Vald

2) 2) (Smoke->Fire)->((Smoke ∧ Heat)->Fire)

Smoke	Heat	Fire	Smoke->Fire	Smoke ∧ Heat	(Smoke ∧ Heat)->Fire	(Smoke->Fire)->((Smoke and Heat)->Fire)
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	F	T
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

Valid

Q.3)

1) There are only three models for $(A \wedge B) \vee (B \wedge C)$

A	B	C	$(A \wedge B) \vee (B \wedge C)$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

A=T, B=T, C=T ; A=F, B=T, C=T ; A=T, B=T, C=F

2) There are three models for this sentence

A	B	A ∨ B
T	T	T
T	F	T
F	T	T
F	F	F

A = T, B = T; A = F, B = T; A = T, B = F

3) There are 4 models for this sentence

A	B	C	A <-> B <-> C
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

A = T, B = T, C = T; A = T, B = F, C = F; A = F, B = T, C = F; A = F, B = F, C = T

Q.4) 1) $Q(y, Gee(A,B)), Q(Gee(X,X), y)$

substitute $= y / Gee(X,X)$

$(Q(Gee(X,X), Gee(A,B)), Q(Gee(X,X), Gee(X,X))$

substitute $= X / A$

$Q(Gee(A,A), Gee(A,B)), Q(Gee(A,A), Gee(A,A))$

Unification not possible as you can't unify constant A with constant B

2) Older(Father(y), y), Older(Father(x), John

Substitute = y/x

Older(Father(x), x), Older(Father(x)), John)

Substitute = x/John

Older(Father(John), John), Older(Father(John), John)

mgu for these two expressions is $\{y/x, x/John\}$

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3) Knows(Father(y), y), Knows(x, x)

Substitute = x/Father(y)

Knows(Father(y), y), Knows(Father(y), Father(y))

Unification not possible as you unify variable y with Father(y)

0.5) a)

1) $\forall x \forall y ((Child(x) \land Candy(y)) \rightarrow Loves(x,y))$

2) $\forall x ((\exists y (Candy(y) \land Loves(x,y))) \rightarrow \neg Fanatic(x))$

3) $\forall x ((\exists y (Pumpkin(y) \land Eats(x,y))) \rightarrow Fanatic(x))$

4) $\forall x \forall y ((Pumpkin(y) \land (Buys(x,y)) \rightarrow Carves(x,y) \lor Eats(x,y))$

5) $\exists x (Pumpkin(x) \land Buy(John, x))$

6) Candy(Lifesavers)

b)

1) $\forall x \forall y ((Child(x) \land Candy(y)) \rightarrow Loves(x,y))$

$\forall x \forall y (\neg (Child(x) \land Candy(y)) \lor Loves(x,y))$

$\forall x \forall y ((\neg Child(x) \lor \neg Candy(y)) \lor Loves(x,y))$

$\neg Child(x) \lor \neg Candy(y) \lor Loves(x,y)$

2) $\forall x ((\exists y (Candy(y) \land Loves(x,y))) \rightarrow \neg Fanatic(x))$

$\forall x(\neg(\exists y(Candy(y) \wedge Loves(x,y))) \vee \neg Fanatic(x))$

$\forall x((\forall y(\neg Candy(y) \vee \neg Loves(x,y))) \vee \neg Fanatic(x))$

$\neg Candy(b) \vee \neg Loves(a,b) \vee \neg Fanatic(a)$

3) $\forall x((\exists y(Pumpkin(y) \wedge Eats(x,y))) \rightarrow Fanatic(x))$

$\forall x(\neg(\exists y(Pumpkin(y) \wedge Eats(x,y))) \vee Fanatic(x))$

$\forall x((\forall y(\neg Pumpkin(y) \vee \neg Eats(x,y))) \vee Fanatic(x))$

$\neg Pumpkin(d) \vee \neg Eats(c,d) \vee Fanatic(c)$

4) $\forall x \forall y((Pumpkin(y) \wedge Buy(x,y)) \rightarrow Carves(x,y) \vee Eats(x,y))$

$\forall x \forall y(\neg(Pumpkin(y) \wedge Buy(x,y)) \vee Carves(x,y) \vee Eats(x,y))$

$\forall x \forall y((\neg Pumpkin(y) \vee \neg Buy(x,y)) \vee Carves(x,y) \vee Eats(x,y))$

$\neg Pumpkin(f) \vee \neg Buy(e,f) \vee Carves(e,f) \vee Eats(e,f)$

5) $\exists x(Pumpkin(x) \wedge Buy(John,x))$

$Pumpkin(x) \wedge Buy(John,x)$

$Pumpkin(z)$

$Buy(John,z)$

6) $Candy(Lifesavers)$

c) 1) $\neg Child(x) \vee \neg Candy(y) \vee Loves(x,y)$

2) $\neg Candy(b) \vee \neg Loves(a,b) \vee \neg Fanatic(a)$

3) $\neg Pumpkin(d) \vee \neg Eats(c,d) \vee Fanatic(c)$

4) $\neg Pumpkin(f) \vee \neg Buy(e,f) \vee Carves(e,f) \vee Eats(e,f)$

5) $Pumpkin(v)$

6) $Buy(John,z)$

7) Candy(Lifesavers)

8) Child(John

Child(John) → ∃y (Pumpkin(y) ∧ Carves(John,y

Child(John) ∧ (¬Pumpkin(y) ∨ ¬Carves(John

9) ¬Pumpkin(y) ∨ ¬Carve(John, y)

10) ¬Candy ∨ Loves(John,y) [(1,8), (x/John)]

11) ¬Fanatic(John) ∨ ¬Candy(y) [(2,10), (b/y, a/John)]

12) ¬Pumpkin(y) ∨ ¬Eats(John,y) ∨ ¬Candy(y) [(3,11), (d/y, c/John)]

13) ¬Pumpkin(y) ∨ ¬Candy(y) ∨ ¬Buy(John,y) ∨ Carve(John, y) [(4,12), (f/y, e/John)]

14) ¬Pumpkin(y) ∨ ¬Candy(y) ∨ ¬Buy(John,y) [(9,13), (g/y)]

15) ¬Candy(y) ∨ ¬Buy(John,y) [(5,14), (v/y)]

16) ¬Candy(y) [(6,15), (z/y)]

17) Null [(7,16), (y/Lifesavers)]

Therefore, the sentence has been proved by using resolution by refutation