10 uždavinys

a) Įvertinkite sumą: $\sum_{i=a}^{b} \frac{c}{i}$
čia a, b, c teigiami sveiki skaičiai, be to $3a < b$.

Pasinaudosime formule

$$\int_{m}^{n+1} f(x)dx \leq \sum_{i=m}^{n} f(i) \leq \int_{m-1}^{n} f(x)dx$$

čia $f(x)$ – monotoniškai mažėjanti f-ja

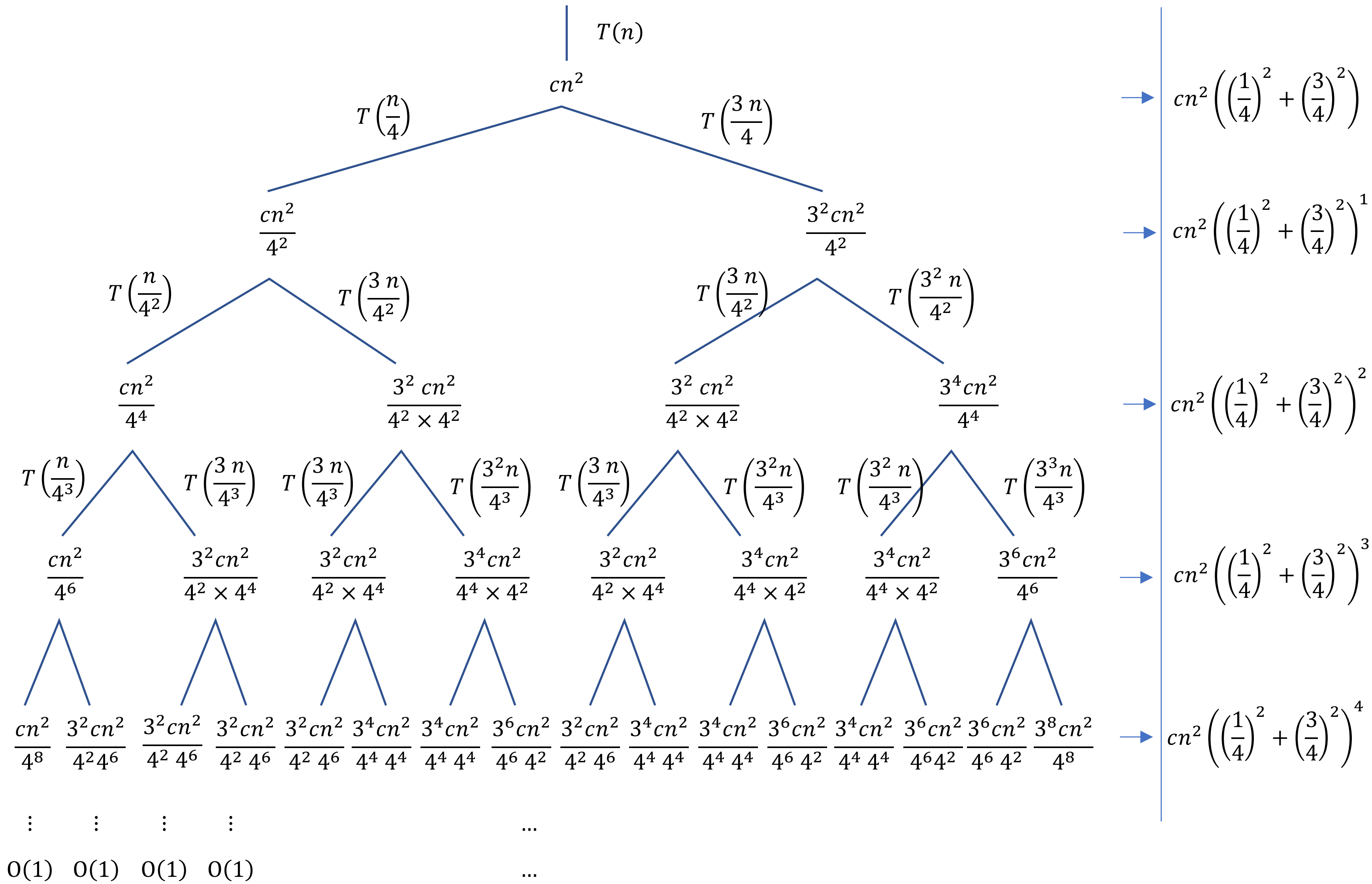
$$\int_{a}^{b+1} \frac{1}{x} dx \leq \sum_{i=m}^{n} f(i) \leq \int_{a-1}^{b} \frac{1}{x} dx$$

$$c \ \ln x|_{a}^{b+1} \leq \sum_{i=a}^{b} \frac{c}{i} \leq c \ln x|_{a-1}^{b}$$

Ats.: $c \ln(b+1) - c \ln a \leq \sum_{i=a}^{b} \frac{c}{i} \leq c \ln b - c \ln(a-1)$

b) Išspręsti rekurentines lygtis: $T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{3n}{4}\right) + cn^2$

Sudarykime sprendinių medį:



$T(n)$
cn^2
$T\left(\frac{n}{4}\right)$
$T\left(\frac{3n}{4}\right)$
$\frac{cn^2}{4^2}$
$\frac{3^2cn^2}{4^2}$
$T\left(\frac{n}{4^2}\right)$
$T\left(\frac{3n}{4^2}\right)$
$T\left(\frac{3n}{4^2}\right)$
$T\left(\frac{3^2n}{4^2}\right)$
$\frac{cn^2}{4^4}$
$\frac{3^2cn^2}{4^2\times 4^2}$
$\frac{3^2cn^2}{4^2\times 4^2}$
$\frac{3^4cn^2}{4^4}$
$T\left(\frac{n}{4^3}\right)$
$T\left(\frac{3n}{4^3}\right)$
$T\left(\frac{3n}{4^3}\right)$
$T\left(\frac{3^2n}{4^3}\right)$
$T\left(\frac{3n}{4^3}\right)$
$T\left(\frac{3^2n}{4^3}\right)$
$T\left(\frac{3^2n}{4^3}\right)$
$T\left(\frac{3^3n}{4^3}\right)$
$\frac{cn^2}{4^6}$
$\frac{3^2cn^2}{4^2\times 4^4}$
$\frac{3^2cn^2}{4^2\times 4^4}$
$\frac{3^4cn^2}{4^4\times 4^2}$
$\frac{3^2cn^2}{4^2\times 4^4}$
$\frac{3^4cn^2}{4^4\times 4^2}$
$\frac{3^4cn^2}{4^4\times 4^2}$
$\frac{3^6cn^2}{4^6}$
$\frac{cn^2}{4^8}$
$\frac{3^2cn^2}{4^2 4^6}$
$\frac{3^2cn^2}{4^2 4^6}$
$\frac{3^2cn^2}{4^2 4^6}$
$\frac{3^2cn^2}{4^2 4^6}$
$\frac{3^4cn^2}{4^4 4^4}$
$\frac{3^4cn^2}{4^4 4^4}$
$\frac{3^6cn^2}{4^6 4^2}$
$\frac{3^2cn^2}{4^2 4^6}$
$\frac{3^4cn^2}{4^4 4^4}$
$\frac{3^4cn^2}{4^4 4^4}$
$\frac{3^6cn^2}{4^6 4^2}$
$\frac{3^4cn^2}{4^4 4^4}$
$\frac{3^6cn^2}{4^6 4^2}$
$\frac{3^6cn^2}{4^6 4^2}$
$\frac{3^8cn^2}{4^8}$
$\vdots$ $\vdots$ $\vdots$ $\vdots$ ...
O(1) O(1) O(1) O(1) ...
$cn^2\left(\left(\frac{1}{4}\right)^2+\left(\frac{3}{4}\right)^2\right)$
$cn^2\left(\left(\frac{1}{4}\right)^2+\left(\frac{3}{4}\right)^2\right)^1$
$cn^2\left(\left(\frac{1}{4}\right)^2+\left(\frac{3}{4}\right)^2\right)^2$
$cn^2\left(\left(\frac{1}{4}\right)^2+\left(\frac{3}{4}\right)^2\right)^3$
$cn^2\left(\left(\frac{1}{4}\right)^2+\left(\frac{3}{4}\right)^2\right)^4$

$$T(n) = \sum_{i=0}^{h} cn^2 \left(\left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2\right)^i = cn^2 \sum_{i=0}^{h} \left(\frac{5}{8}\right)^i = \frac{8}{3} cn^2 \left(1 - \left(\frac{5}{8}\right)^{h+1}\right)$$

Sprendinių medžio aukštis h

$$\lfloor \log_4 n \rfloor \leq h \leq \left\lfloor \log_{\frac{4}{3}} n \right\rfloor$$

$$T(n) = \frac{8}{3} cn^2 \left(1 - \left(\frac{5}{8}\right)^{h+1}\right) \leq \frac{8}{3} cn^2,$$

nes $0 \leq 1 - \left(\frac{5}{8}\right)^{h+1} \leq 1$, visiems $h \geq 0$.

$$T(n) = \frac{8}{3} cn^2 \left(1 - \left(\frac{5}{8}\right)^{h+1}\right) \geq \frac{8}{3} cn^2 \left(1 - \left(\frac{5}{8}\right)^{\lfloor \log_4 n \rfloor + 1}\right) \geq cn^2$$

kai $n \geq 1$ ir $\left(\frac{5}{8}\right)^{\lfloor \log_4 n \rfloor + 1} \leq \frac{5}{8}$.

Ats.: $T(n) = \Theta(n^2)$