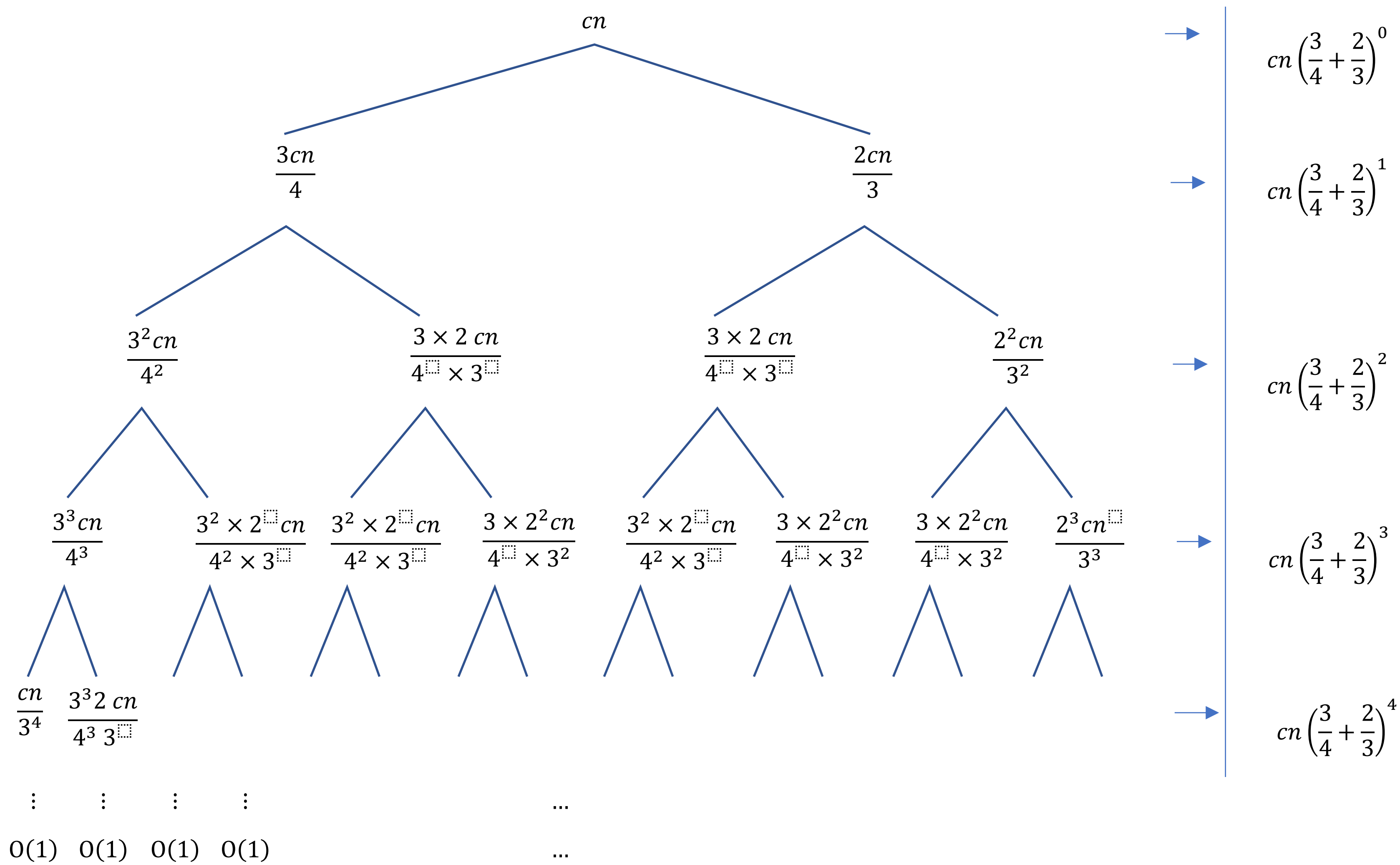
15 uždavinys

Išspręsti lygtį, taikydami sprendinių medžio metodą: $T(n) = T\left(\frac{3n}{4}\right) + T\left(\frac{2n}{3}\right) + cn$

Sudarykime sprendinių medį:



cn
$\frac{3cn}{4}$
$\frac{2cn}{3}$
$\frac{3^2cn}{4^2}$
$\frac{3\times 2\ cn}{4^{\square}\times 3^{\square}}$
$\frac{3\times 2\ cn}{4^{\square}\times 3^{\square}}$
$\frac{2^2cn}{3^2}$
$\frac{3^3cn}{4^3}$
$\frac{3^2\times 2^{\square}cn}{4^2\times 3^{\square}}$
$\frac{3^2\times 2^{\square}cn}{4^2\times 3^{\square}}$
$\frac{3\times 2^2cn}{4^{\square}\times 3^2}$
$\frac{3^2\times 2^{\square}cn}{4^2\times 3^{\square}}$
$\frac{3\times 2^2cn}{4^{\square}\times 3^2}$
$\frac{3\times 2^2cn}{4^{\square}\times 3^2}$
$\frac{2^3cn^{\square}}{3^3}$
$\frac{cn}{3^4}$
$\frac{3^3 2\ cn}{4^3\ 3^{\square}}$
O(1) O(1) O(1) O(1) ...
$cn\left(\frac{3}{4}+\frac{2}{3}\right)^0$
$cn\left(\frac{3}{4}+\frac{2}{3}\right)^1$
$cn\left(\frac{3}{4}+\frac{2}{3}\right)^2$
$cn\left(\frac{3}{4}+\frac{2}{3}\right)^3$
$cn\left(\frac{3}{4}+\frac{2}{3}\right)^4$

$$T(n) = \sum_{i=0}^{h} cn\left(\frac{3}{4}+\frac{2}{3}\right)^{i} = cn\sum_{i=0}^{h}\left(\frac{17}{12}\right)^{i} = \frac{12}{5}cn\left(\left(\frac{17}{12}\right)^{h+1} - 1\right)$$

Sprendinių medžio aukštis h

$$\left\lfloor \log_{\frac{3}{2}} n \right\rfloor \le h \le \left\lfloor \log_{\frac{4}{3}} n \right\rfloor$$

$$T(n) \ge \frac{12}{5}cn\left(\left(\frac{17}{12}\right)^{\left\lfloor \log_{\frac{3}{2}} n \right\rfloor + 1} - 1\right) \ge \frac{12}{5}cn\left(\frac{17}{12}\left(\frac{17}{12}\right)^{\log_{\frac{3}{2}} n} - 1\right) = \frac{12}{5}cn\left(\frac{17}{12}n^{\log_{\frac{3}{2}}\frac{17}{12}} - 1\right)$$

$$= \frac{17}{5}cn^{1+\log_{\frac{3}{2}}\frac{17}{12}} - \frac{12}{5}cn = cn^{1+\log_{\frac{3}{2}}\frac{17}{12}} + \frac{12}{5}cn^{1+\log_{\frac{3}{2}}\frac{17}{12}} - \frac{12}{5}cn \ge cn^{1+\log_{\frac{3}{2}}\frac{17}{12}}$$

$$T(n) \le \frac{12}{5}cn\left(\left(\frac{17}{12}\right)^{\left\lfloor \log_{\frac{4}{3}} n \right\rfloor + 1} - 1\right) \le \frac{12}{5}cn\frac{17}{12}\,n^{\log_{\frac{4}{3}}\frac{17}{12}} = \frac{17}{5}cn^{1+\log_{\frac{4}{3}}\frac{17}{12}}$$

Ats.: $T(n) = \Omega\left(n^{1+\log_{\frac{3}{2}}\frac{17}{12}}\right)$ ir $T(n) = O\left(n^{1+\log_{\frac{4}{3}}\frac{17}{12}}\right)$.