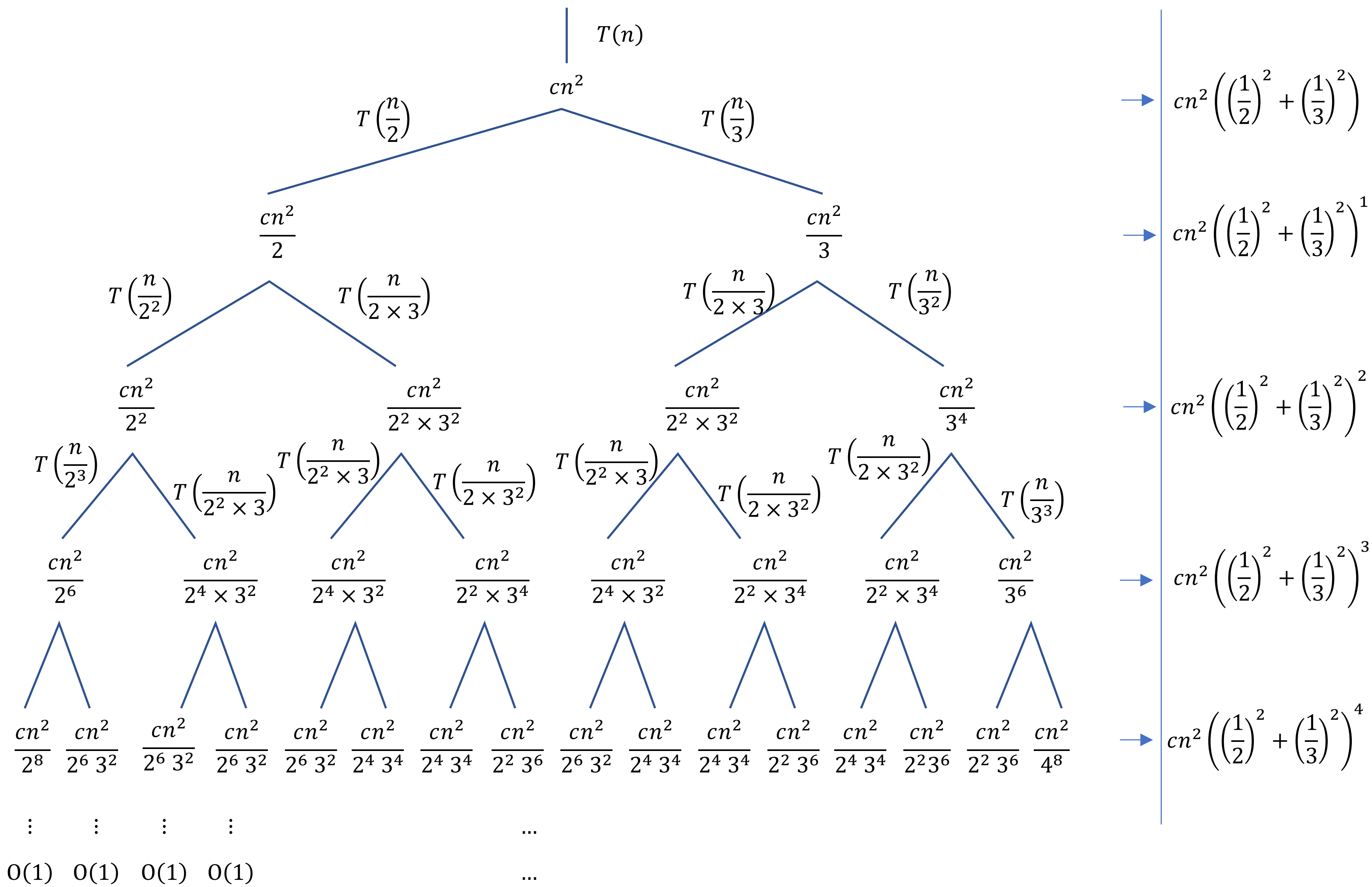
11 uždavinys

Išspręsti lygtį, taikydami sprendinių medžio metodą: $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + cn^2$.

Sudarykime sprendinių medį:



$T(n)$
cn^2
$T\left(\frac{n}{2}\right)$ $T\left(\frac{n}{3}\right)$
$\frac{cn^2}{2}$ $\frac{cn^2}{3}$
$T\left(\frac{n}{2^2}\right)$ $T\left(\frac{n}{2\times 3}\right)$ $T\left(\frac{n}{2\times 3}\right)$ $T\left(\frac{n}{3^2}\right)$
$\frac{cn^2}{2^2}$ $\frac{cn^2}{2^2\times 3^2}$ $\frac{cn^2}{2^2\times 3^2}$ $\frac{cn^2}{3^4}$
$T\left(\frac{n}{2^3}\right)$ $T\left(\frac{n}{2^2\times 3}\right)$ $T\left(\frac{n}{2^2\times 3}\right)$ $T\left(\frac{n}{2\times 3^2}\right)$ $T\left(\frac{n}{2^2\times 3}\right)$ $T\left(\frac{n}{2\times 3^2}\right)$ $T\left(\frac{n}{2\times 3^2}\right)$ $T\left(\frac{n}{3^3}\right)$
$\frac{cn^2}{2^6}$ $\frac{cn^2}{2^4\times 3^2}$ $\frac{cn^2}{2^4\times 3^2}$ $\frac{cn^2}{2^2\times 3^4}$ $\frac{cn^2}{2^4\times 3^2}$ $\frac{cn^2}{2^2\times 3^4}$ $\frac{cn^2}{2^2\times 3^4}$ $\frac{cn^2}{3^6}$
$\frac{cn^2}{2^8}$ $\frac{cn^2}{2^6\,3^2}$ $\frac{cn^2}{2^6\,3^2}$ $\frac{cn^2}{2^6\,3^2}$ $\frac{cn^2}{2^6\,3^2}$ $\frac{cn^2}{2^4\,3^4}$ $\frac{cn^2}{2^4\,3^4}$ $\frac{cn^2}{2^2\,3^6}$ $\frac{cn^2}{2^6\,3^2}$ $\frac{cn^2}{2^4\,3^4}$ $\frac{cn^2}{2^4\,3^4}$ $\frac{cn^2}{2^2\,3^6}$ $\frac{cn^2}{2^4\,3^4}$ $\frac{cn^2}{2^2 3^6}$ $\frac{cn^2}{2^2\,3^6}$ $\frac{cn^2}{4^8}$
$\vdots$ $\vdots$ $\vdots$ $\vdots$ ...
O(1) O(1) O(1) O(1) ...
$cn^2\left(\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^2\right)$
$cn^2\left(\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^2\right)^1$
$cn^2\left(\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^2\right)^2$
$cn^2\left(\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^2\right)^3$
$cn^2\left(\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^2\right)^4$

$$T(n) = \sum_{i=0}^{h} cn^2 \left(\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 \right)^i = cn^2 \sum_{i=0}^{h} \left(\frac{5}{6}\right)^i = 6cn^2 \left(1 - \left(\frac{5}{6}\right)^{h+1}\right)$$

Sprendinių medžio aukštis h

$$\lfloor \log_3 n \rfloor \leq h \leq \lfloor \log_2 n \rfloor$$

$$T(n) = 6cn^2 \left(1 - \left(\frac{5}{6}\right)^{h+1}\right) \leq 6cn^2$$

nes $0 \leq 1 - \left(\frac{5}{6}\right)^{h+1} \leq 1$, visiems $h \geq 0$.

$$T(n) = 6cn^2 \left(1 - \left(\frac{5}{6}\right)^{h+1}\right) \geq 6cn^2 \left(1 - \left(\frac{5}{6}\right)^{\lfloor \log_3 n \rfloor + 1}\right) \geq cn^2$$

kai $n \geq 1$ ir $\left(\frac{5}{6}\right)^{\lfloor \log_3 n \rfloor + 1} \leq \frac{5}{6}$.

Ats.: $T(n) = \Theta(n^2)$