**1 uždavinys:** Įvertinti sumą

$$?\leq \sum_{i=1}^{m} i \log_2 i \leq ?$$

Kadangi $f(i) = i \log_2 i$, monotoniškai didėjanti funkcija (patikrinti galima, randant išvestinę ir patikrinant ar ji teigiama visame sumavimo intervale), todėl sumos įvertinimui galima taikyti (Pagrindinė literatūra, A priedas, A.11, A,12 formulės)

$$\int_{m-1}^{n} f(x)dx \leq \sum_{i=m}^{n} f(i) \leq \int_{m}^{n+1} f(x)dx$$

Mūsų atveju reikia rasti pirmykštę funkcijai $f(x) = x \log_2 x$:

$$\begin{aligned}\int f(x)dx = \int x \log_2 x \, dx = [dalimis\ integruosime] = \frac{1}{2}\int \log_2 x \, dx^2 \\ = \frac{1}{2}\left(x^2 \log_2 x - \int x^2 d \log_2 x\right) = \frac{1}{2}\left(x^2 \log_2 x - \frac{1}{\ln 2}\int x \, dx\right) \\ = \frac{1}{2}\left(x^2 \log_2 x - \frac{1}{2 \ln 2} x^2\right) + C\end{aligned}$$

Sustatyti rėžius pagal Niutono-Leibnico formulę, suprastinti jei galima ir gausime sumos įvertinimą.

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**2 uždavinys**: Asimptotiškai palyginti

$$f(n) = \frac{n^2}{(n+5)\ln(n^2+1)}$$

$$g(n) = \frac{n+5}{\ln n}$$

$$\begin{aligned}\lim_{n\to\infty} \frac{\frac{n^2}{(n+5)\ln(n^2+1)}}{\frac{n+5}{\ln n}} = \lim_{n\to\infty} \frac{n^2 \ln n}{(n+5)^2 \ln(n^2+1)} = \lim_{n\to\infty} \frac{n^2}{(n+5)^2} \lim_{n\to\infty} \frac{\ln n}{\ln(n^2+1)} \\ = \lim_{n\to\infty} \frac{1}{\left(1+\frac{5}{n}\right)^2} \lim_{n\to\infty} \frac{(\ln n)'}{(\ln(n^2+1))'} = 1 \times \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n^2+1} 2n} = \lim_{n\to\infty} \frac{n^2+1}{n^2} = \lim_{n\to\infty} \frac{1+\frac{1}{n^2}}{1} \\ = 1\end{aligned}$$

Ats.: $f(n)$ ir $g(n)$ auga konstantos tikslumu vienodai ir $f(n) = \Theta\big(g(n)\big)$ ir $\frac{n^2}{(n+5)\ln(n^2+1)} = \Theta\left(\frac{n+5}{\ln n}\right) = \Theta\left(\frac{n}{\ln n}\right)$

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**3 uždavinys**: Asimptotiškai palyginti

$$f(n) = \frac{n}{\ln n}\sqrt{n}$$

$$g(n) = \sqrt[3]{n^4}\left(16 + \sqrt[6]{n+1}\right)$$

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4}\left(16 + \sqrt[6]{n+1}\right)}{\frac{n}{\ln n}\sqrt{n}} = \lim_{n\to\infty} \frac{\ln n\left(16 + \sqrt[6]{n+1}\right)}{n^{\frac{3}{2}-\frac{4}{3}}} = 16\lim_{n\to\infty} \frac{\ln n}{n^{\frac{1}{6}}} + \lim_{n\to\infty} \frac{\ln n\,\sqrt[6]{n+1}}{n^{\frac{1}{6}}}$$

$$= 16\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{6}n^{-\frac{5}{6}}} + \lim_{n\to\infty} \ln n\left(1 + \frac{1}{n}\right)^{\frac{1}{6}} = 96\lim_{n\to\infty} \frac{1}{n^{\frac{1}{6}}} + \lim_{n\to\infty} \ln n = 0 + \infty = \infty$$

Ats.: $g(n)$ auga greičiau, todėl $f(n) = O\big(g(n)\big)$ ir $\frac{n}{\ln n}\sqrt{n} = O\left(\sqrt[3]{n^4}\left(16 + \sqrt[6]{n+1}\right)\right) = O\left(n^{\frac{3}{2}}\right)$.