Q1)

Min z = $2x_1 - x_2 + 3x_3 + x_4$

$x_1 - x_2 + 3x_3 - x_4 = 1$

$x_1 + x_2 - 2x_3 - x_4 = 1$ (I assumed: All Variables >=0)

Part A:

BV	NVB	System of linear equations	Solution(X1,X2,X3,X4)	Descriptions	Corresponding Z value
X1,X2	X3,X4	$x_1 - x_2 = 1$ $x_1 + x_2 = 1$	(1,0,0,0)	Basic feasible degenerate	2
X1,X3	X2,X4	$x_1 + 3x_3 = 1$ $x_1 - 2x_3 = 1$	(1,0,0,0)	Basic feasible degenerate	2
X1,X4	X2,X3	$x_1 - x_4 = 1$ $x_1 - x_4 = 1$	(a,0,0,(a-1)) infinite Number	Not exist any basic solution	`
X2,X3	X1,X4	$-x_2 + 3x_3 = 1$ $+x_2 - 2x_3 = 1$	(0,5,2,0)	Basic feasible solution	1
X2,X4	X1,X3	$-x_2 - x_4 = 1$ $+x_2 - x_4 = 1$	(0,0,0,-1)	Basic non feasible	`

Part B:

Z*=1

Part C:

No. Although (2,0,0,1) is an element of (a,0,0,(a-1)) but (a,0,0,(a-1)) is Not an unique Point , cause a is an element of R and can change from -∞ to +∞ So (2,0,0,1) is not basic feasible solution.

Q2)

Max z = $2x_1 + 3x_2$

s.t.

$x_1 + x_2 \leq 2$

$4x_1 - 3x_2 \leq 6$

$x_1, x_2 \geq 0$

First step: writing the problem in canonical Form

Max z = $2x_1 + 3x_2$

s.t.

$x_1 + x_2 + S_1 = 2$

$4x_1 - 3x_2 + S_2 = 6$

$x_1, x_2, S_1, S_2 \geq 0$

BV \ All Vars	Z	X1	X2	S1	S2	RHS
Z	1	-2	-3	0	0	0
S1	0	1	1	1	0	2
S2	0	4	-3	0	1	6

we start by defining the decision variable that would inter the basis in the next table.
In problem maximization, this variable is a variable with the most negative coefficient in the row of Z. In this example, X2 has the most negative coefficient, So the column related to it is a Pivot column. Now that we defined what is the variable that would inter the basis, we have to find the variable that would leave the basis in the next table and X2 will replace that variable in the next table. To be able to define the variable that leave the basis, we have a Test which is called "Minimum Test ".
In the minimum test, we divide the right hand side of our problem by the Positive value of pivot column and find the minimum value among those. The Row associated with the minimum value is called "Pivot Row" and define the exiting variable in the next simplex table.

Minimum Test: Min{2/1} = 2/1 = 2
In this example, The minimum value of minimum test is related to the second row of the simplex table.
Notice the intersection of Pivot Row and Pivot Column is called, "Pivot value" which is in this example 1.
In the next table we need to change the coefficient of X2 to one and other coefficient in its column must be zero. Because in the next table X2 becomes a base variable and according to the definition of the basic variable, it only appears in the one constrain of the problem.
To be able to do this transformation, we need to use ERO(Elementary Row Operations).
In the next table X2 replaced S1 and other basic variables stay the same.

BV \ All Vars	Z	X1	X2	S1	S2	RHS	
Z	1	1	0	3	0	6	=Z*
X2	0	1	1	1	0	2	=X2*
S2	0	7	0	3	1	12	=S2*

After each iteration of simplex method, we need to check to the "Optimality condition".
For maximization problem, we are add the optimal table if all the values in the row of Z are positive or zero.
Now ass you see, all the values in the row of Z are non-negative. So we are at to optimal solution.
The final value for the objective function is 6 .The value of decision variables in the finial solutions is given in the right hand side(Yellow part).However the value of decision variables that are not appearing in the basis, are equal to zero.

Now we will check the accuracy of our solution by replacing the value of the decisions variables in the objective function.
6 =? Z , Z = $2x_1+3x_2$ == 2*0 + 3*2 = 6 ●

Kind regards .

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