

Q1)

| انبار \ مشتری | 1 | 2 | 3 | 4 | |
|---------------|----|----|----|----|----|
| 1 | 6 | 5 | 1 | 10 | 40 |
| 2 | 2 | 4 | 8 | 10 | 50 |
| 3 | 11 | 7 | 9 | 6 | 40 |
| | 50 | 20 | 30 | 30 | |

Reducing cost values (in red):

- Row 1: -40 (under 6), 0 (under 10)
- Row 2: 10 (under 2), 20 (under 4), 20 (under 8), 40 (under 10)
- Row 3: 0 (under 11), 10 (under 7), 30 (under 9), 0 (under 6)
- Column 1: 10 (under 50), 0 (under 10)
- Column 2: 0 (under 20)
- Column 3: 10 (under 30)
- Column 4: 0 (under 30)

reducing cost of x_{ij} is written with Blue color

| انبار \ مشتری | 1 | 2 | 3 | 4 | |
|---------------|----|----|----|----|----|
| 1 | 6 | 5 | 1 | 10 | 40 |
| 2 | 2 | 4 | 8 | 10 | 50 |
| 3 | 11 | 7 | 9 | 6 | 40 |
| | 50 | 20 | 30 | 30 | |

Reducing cost values (in blue):

- Row 1: 3 (under 5), 11 (under 1), -1 (under 10)
- Row 2: 20 (under 4), -5 (under 8)
- Row 3: -8 (under 11), -2 (under 7), 30 (under 9)

Based on last picture entering variable is X_{13} and leaving var is X_{23}

| انبار \ مشتری | 1 | 2 | 3 | 4 | |
|---------------|----|----|----|----|----|
| 1 | 6 | 5 | 1 | 10 | 40 |
| 2 | 2 | 4 | 8 | 10 | 50 |
| 3 | 11 | 7 | 9 | 6 | 40 |
| | 50 | 20 | 30 | 30 | |

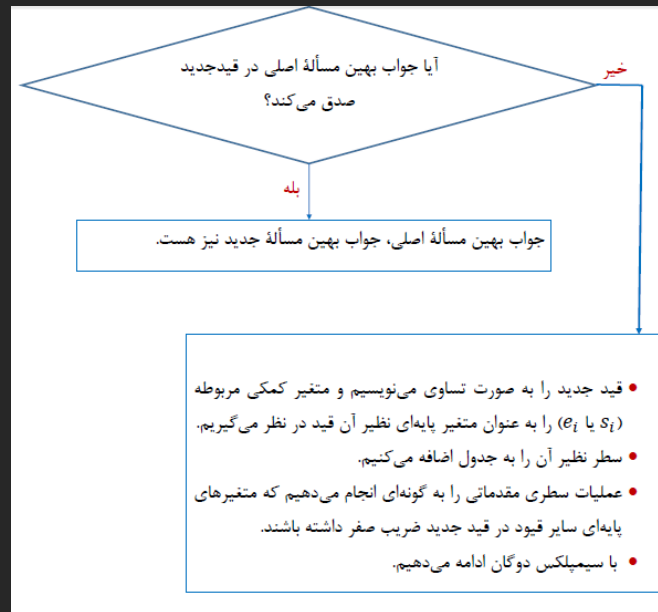
Reducing cost values (in red):

- Row 1: 20 (under 6), 20 (under 1)
- Row 2: 30 (under 2), 20 (under 4)
- Row 3: 10 (under 7), 30 (under 9)

Q2)

We should follow this algorithm.

Because our old optimal solution does not satisfy new constraint that we've added we should add Artificial variable and then solve it by Dual simplex method as follows :



we have Problem here

minimum test : $\{ |3/4|, |3/-1|, |2/-1| \} = 3/4$ for Dual simplex

| | x_1 | x_2 | x_3 | s_1 | s_2 | A_3 | RHS |
|-------|-------|-------|-------|-------|-------|-------|-----|
| Z | 0 | 3 | 3 | 2 | 0 | 0 | 16 |
| m_1 | 1 | 2 | 1 | 1 | 0 | 0 | 8 |
| s_2 | 0 | 3 | -1 | 1 | 1 | 0 | 12 |
| A_3 | 1 | -2 | 0 | 0 | 0 | 1 | 4 |
| | 0 | -4 | -1 | -1 | 0 | 1 | -4 |

| | m_1 | m_2 | m_3 | s_1 | s_2 | A_3 | RHS |
|-------|-------|-------|-------|-------|-------|-------|-------------|
| Z | 0 | 0 | 9/4 | 5/4 | 0 | 3/4 | 13 = z^* |
| m_1 | | | | | | | 6 = x_1^* |
| s_2 | | | | | | | 9 = s_2^* |
| m_2 | | | | | | | 1 = x_2^* |

→ This is an opt table

Q2) Part 2 :

Dual price

جدول بهین:

| BV/متغیرها | x_1 | x_2 | x_3 | s_1 | s_2 | RHS |
|------------|-------|-------|-------|-------|-------|-----|
| z | 0 | 3 | 3 | 2 | 0 | 16 |
| x_1 | 1 | 2 | 1 | 1 | 0 | 8 |
| s_2 | 0 | 3 | -1 | 1 | 1 | 12 |

الف) فرض کنید قید جدید $x_1 - 2x_2 = 4$ به مسأله اضافه شود، با استفاده از تحلیل حساسیت جواب بهین مسأله جدید را پیدا کنید.

ب) اگر قرار بود بین یک واحد افزایش در سمت راست اولین و دومین قید یکی را انتخاب کنید، کدام را انتخاب می کردید؟ چرا؟ اثر این افزایش روی مقدار بهینه تابع هدف چیست؟

we can answer second part with help of Dual price
 because dual price of first slack variabel is grater than second slack var
 (I mean $2 > 0$) we should increase RHS of First constraint
 in other hand if we increase 8 to 9 opt value of objective function will
 be $16+2 = 18$

Q3)

Part 1) One way for solving this problem is by using "Sensitivity analysis " and "" shadow price" theorems.

Since the Dual price for X1 is Y5 and equal to -20 and we are in a Maximization problem and since 86 is in valid interval of change($85 \leq 86 \leq 90$)then it means if we reduce one unit of X1 it can increase $-(-20)$ of objective function and we will have " 32580 " for the new optimal value of objective function due to the following calculation :

reducing amout increament

| | | |
|---|-----|-------------------|
| 1 | +20 | $? = 2 * 20 = 40$ |
| 2 | ? | |

Part 2) In this part we are adding new activity (variable) :

After introducing slack,surplus,artificial variables and BEFORE ADDING NEW VAR

subject to

and $X_1, X_2, M_1, S_1, S_2, S_3, S_4, S_5, S_6, A_1, A_2 \geq 0$

First table BEFORE ADDING NEW VAR:

[illegible]

Opt table BEFORE ADDING NEW VARIABLE (I mean Jeep machine)

| Opt table | | | | | | | | | | | |
|-----------|--|-----|-----|-----|------|------|-----|-----|------|-----|-------|
| BV | | X_1 | X_2 | M_1 | S_1 | S_2 | S_3 | S_4 | S_5 | S_6 | RHS |
| X_2 | | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0.8 | 0 | 27.6 |
| S_6 | | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0.8 | 1 | 1.6 |
| S_3 | | 0 | 0 | 0 | -0.7 | -0.7 | 1 | 0 | 0.04 | 0 | 0.88 |
| S_4 | | 0 | 0 | 0 | -3 | -3 | 0 | 1 | -0.4 | 0 | 1.2 |
| X_1 | | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 88 |
| M_1 | | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 98 |
| Z= | | 0 | 0 | 0 | 400 | 350 | 0 | 0 | 20 | 0 | 32540 |

$$B = \begin{bmatrix} 0.8 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0.6 & 0.7 & 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= B^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & -0.8 & 0 \\ -0.7 & -0.7 & 1 & 0 & -0.04 & 0 \\ -3 & -3 & 0 & 1 & 0.4 & 0 \\ -1 & -1 & 0 & 0 & 0.8 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{a}_{x_3} = B^{-1} a_{x_3}$$

$$B \cdot \begin{bmatrix} 1.2 \\ 0 \\ 2 \\ 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.2 \\ 1.16 \\ 0.4 \\ -1.2 \\ 0 \end{bmatrix} = -Q_{x_3}$$

$$\bar{C}_{x_3} = \begin{bmatrix} x_1 & x_2 & s_3 & s_4 & s_6 & m_1 \\ 300 & 400 & 0 & 0 & 0 & -50 \end{bmatrix} \begin{bmatrix} 0 \\ 1,2 \\ 1,6 \\ 9,4 \\ -1,2 \\ 0 \end{bmatrix} - 600 = -120$$

480

We have used $C_{BV}^T (B^{-1} \cdot \bar{a}_{x_3}) - C_{x_3}$ Formula

AFTER ADDING NEW VARIABLE :

| Opt table | | | | | | | | | | | |
|-----------|-----|-----|-----|------|------|-----|-----|------|-----|-------------|--------------|
| BV | X_1 | X_2 | M_1 | S_1 | S_2 | S_3 | S_4 | S_5 | S_6 | X_3 | RHS |
| X_2 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0.8 | 0 | 0 | 27.6 |
| S_6 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0.8 | 1 | 1.2 | 1.6 |
| S_3 | 0 | 0 | 0 | -0.7 | -0.7 | 1 | 0 | 0.04 | 0 | 1.16 | 0.88 |
| S_4 | 0 | 0 | 0 | -3 | -3 | 0 | 1 | -0.4 | 0 | 0.4 | 1.2 |
| X_1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | -1.2 | 88 |
| M_1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 98 |
| Z= | 0 | 0 | 0 | 400 | 350 | 0 | 0 | 20 | 0 | -120 | 32540 |

Now since X_3 has a negative reduce cost, we are not in a OPT table. After elementary row operation and Minimum test we will have next table as follows :

| BV | X1 | X2 | M1 | X3 | S1 | S2 | S3 | S4 | S5 | S6 | RHS |
|---------|----|----|----|----|----------|----------|----------|----|---------|----|-----------------|
| X2 | 0 | 1 | 0 | 0 | 1.7241 | 1.7241 | -1.0345 | 0 | 0.7586 | 0 | 26.6897 |
| S6 | 0 | 0 | 0 | 0 | 1.7241 | 1.7241 | -1.0345 | 0 | 0.7586 | 1 | 0.6897 |
| X3 | 0 | 0 | 0 | 1 | -0.6034 | -0.6034 | 0.8621 | 0 | 0.0345 | 0 | 0.7586 |
| S4 | 0 | 0 | 0 | 0 | -2.7586 | -2.7586 | -0.3448 | 1 | -0.4138 | 0 | 0.8966 |
| X1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 88 |
| M1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 98 |
| Z=32631 | 0 | 0 | 0 | 0 | 327.5862 | 277.5862 | 103.4483 | 0 | 24.1379 | 0 | 32631.03 |

32631 is > than 32540 So we should Produce new variable.

Thanks for all of your time that you have spent for teaching us .I really appreciate it. 9513004