Q1)

Q1

standard Form:

$$\begin{cases} Z + 2x_1 - x_2 - 4x_3 + 0e + Ma_1 + Ma_2 + 0S = 0 \\ 3x_1 + x_2 - x_3 - e + a_1 + 0a_2 + 0S = 2 \\ 2x_1 + 2x_2 + x_3 + 0e + 0a_1 + a_2 + 0S = 5 \\ x_1 - x_2 + 0x_3 + 0e + 0a_1 + 0a_2 + 1S = 1 \end{cases}$$

$x_1, x_2, x_3 \geq 0$

		Z	x_1	x_2	x_3	e	a_1	a_2	S	RHS
r_1	Z	1	2	-1	-4	0	M	M	0	0
r_2	a_1	0	3	+1	-1	-1	1	0	0	2
r_3	a_2	0	2	2	+1	0	0	1	0	5
r_4	S	0	1	-1	0	0	0	0	1	1

In First table we should get rid of "M" in the base Column (means: a_1 & a_2)

$-Mr_2 + r_1 \to r_1$

$-Mr_3 + r_1 \to r_1$ $\Rightarrow$

	Z	x_1	x_2	x_3	e	a_1	a_2	S	RHS
Z	1	$-5M+2$	$-3M-1$	-4	M	0	0	0	$-7M$
a_1	0	3	+1	-1	-1	1	0	0	2
a_2	0	2	2	+1	0	0	1	0	5
S	0	1	-1	0	0	0	0	1	1

Now checking "optimality condition"

→ most Negative Coeff is for x_1 / Minimum Test: $\min\left\{\frac{2}{3}, \frac{5}{2}, \frac{1}{1}\right\}$ (0.4, 2.5, 1)

so x_1 should inter the table & a_1 leave it.

	z	x_1	x_2	x_3	e	a_1	a_2	S	RHS
z	1	0	$-\frac{4}{3}M-\frac{5}{3}$	$-\frac{5}{3}M-\frac{1}{3}$	$\frac{2}{3}M+\frac{2}{3}$	$\frac{5}{3}M-\frac{2}{3}$	0	0	$= -\frac{1}{3}M+\frac{11}{3}$
x_1	0	1	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{1}{3}$	0	0	$= \frac{2}{3}$
a_2	0	0	$\frac{11}{3}$	$\frac{5}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	1	0	$= \frac{11}{3}$
→ S	0	0	$\frac{11}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{-1}{3}$	0	1	$= \frac{1}{3}$

<based on last table>

$\frac{1}{3}(+5M-2)R_2 + R_1 \rightarrow R_1$

$\frac{-2}{3}R_2 + R_3 \rightarrow R_3$

$\frac{-1}{3}R_2 + R_4 \rightarrow R_4$

"optimality" ✗ → $\min\left\{\frac{11}{3} \times \frac{3}{5}, \frac{1}{3} \times \frac{3}{1}\right\}$

Q1

	z	x_1	x_2	x_3	e	a_1	a_2	S	RHS
z	1	0	$-8M-15$	0	$M+4$		0	$5M+10$	$= -2M+2$
x_1	0	1	-1	0	0		0	1	≤ 1
a_2	0	0	8	0	-1		1	-5	≤ 2
x_3	0	0	-4	1	1		0	3	≤ 1

$3r_4 \to r_4$ /

$\left(\frac{+5}{3}M + \frac{10}{3}\right) r_4 + r_1 \to r_1$ / $+\frac{1}{3} r_4 + r_2 \to r_2$ / $\frac{-5}{3} r_4 + r_3 \to r_3$

Checking "opt Condition" ✗ $\to \min\{\frac{2}{8}\}$

we should inter x_2 & replace it with a_2

	z	x_1	x_2	x_3	e	a_1	a_2	S	RHS
z	1	0	0	0	17/8			5/8	23/4
x_1	0	1		0					1,25
x_2	0	0	1	0	$-1/8$			$-5/8$	1/4
x_3	0	0		1					2

$\frac{1}{8} r_3 \to r_3$ / $8M+15 / r_3 + r_1 \to r_1$ /

"opt Condition" ✓

$z^* = 23/4 = 5.75$, $x_1^* = 1,25$

$x_2^* = 0,25$

$x_3^* = 2$

Correctness: $z^{*?} = -2x_1^* + x_2^* + 4x_3^* \Leftrightarrow 5,75 = 5,75$ ✓

Q2)
set of solutions : (X1 ,X2 ,X3 ,X4) = { (2,3,0,0)+ a(1,2,1,0) s.t a>=0 }

Q3) because we still have an artificial variable (a2) in our final table and we can't get rid of it (and also we can't get rid of M in the right hand side of Z row), so we have **no feasible solution** problem.

Q4) 121 / because each increasing of value of Xj can increase 7 unit of Z value and overall, maximum value that Xj can have (Xj*) is 3 so we will have 121 for our final Z right hand side.

Thank you.

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