

1)

First step: decision variables is x_1 , x_2 , x_3 that is the number of produced basket type one and type two and type three respectively.

Second step is finding our objective function.

$\max z = 50x_1 + 30x_2 + 60x_3$ and z is our sales profit.

Third step is finding all constrains over our variables:

$$30x_1 + 10x_2 + 20x_3 \leq 100$$

$$20x_1 + 40x_2 + 15x_3 \leq 800$$

$$4x_1 + 3x_2 + 10x_3 \leq 100$$

Forth step is finding our restrictions over type and sing of our variable (real or integer, positive or negative)

$$x_1, x_2, x_3 \geq 0$$

gin x_1 x_2 x_3

! if we write gin we don't need " $x_1, x_2, x_3 \geq 0$ " any more, but there is no problem with solution

Lindo Model - Lingo1		Solution Report - Lingo1		
<pre> max 50x1 + 30x2 + 60x3 s.t. 30x1 + 10x2 + 20x3 <= 100 20x1 + 40x2 + 15x3 <= 800 4x1 + 3x2 + 10x3 <= 100 end gin x1 gin x2 gin x3 </pre>		<p>Global optimal solution found.</p> <p>Objective value: 300.0000</p> <p>Objective bound: 300.0000</p> <p>Infeasibilities: 0.000000</p> <p>Extended solver steps: 0</p> <p>Total solver iterations: 0</p> <p>Elapsed runtime seconds: 0.99</p> <p>Model Class: PILP</p> <p>Total variables: 3</p> <p>Nonlinear variables: 0</p> <p>Integer variables: 3</p> <p>Total constraints: 4</p> <p>Nonlinear constraints: 0</p> <p>Total nonzeros: 12</p> <p>Nonlinear nonzeros: 0</p>		
		Variable	Value	Reduced Cost
		X1	0.000000	-50.00000
		X2	10.00000	-30.00000
		X3	0.000000	-60.00000
		Row	Slack or Surplus	Dual Price
		1	300.0000	1.000000
		2	0.000000	0.000000
		3	400.0000	0.000000
		4	70.00000	0.000000

I find another solution for this problem intuitively that is $x_3 = 5$ but Lingo didn't tell about that. Why ???

12) if each cutting had a cost for us we should consider all possible cutting model and if the cost of 20 foot and 17 foot was different we must multiplied their cutting model number with corresponding cost in objective function

Our cutting models:

نوع برش	تعداد قطعات ۲ فوتی	تعداد قطعات ۵ فوتی	تعداد قطعات ۹ فوتی	میزان ضایعات (فوت)
۱	۵	۰	۰	۲
۲	۴	۱	۰	۰
۳	۲	۲	۰	۱
۴	۲	۰	۱	۲
۵	۱	۱	۱	۰
۶	۰	۳	۰	۲

Cutting model	Num 3 foot	Numb of 5 foot	Numb of 9 foot	Exact wasted wood
Y1	6	0	0	2
Y2	5	1	0	0
Y3	3	2	0	1
Y4	3	0	1	2
Y5	2	1	1	0
Y6	1	3	0	2
Y7	0	4	0	0
Y8	0	2	1	1
Y9	0	0	2	2

because each cut haven't any cost for us, I intuitively considered models that have a "exact waste" less than our smallest piece of wood (means 3)

Min $X_1 + x_2 + x_3 + x_4 + x_5 + x_6 + Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + Y_7 + Y_8 + Y_9$

s.t.

$5X_1 + 4x_2 + 2x_3 + 2x_4 + 1x_5 + 6Y_1 + 5Y_2 + 3Y_3 + 3Y_4 + 2Y_5 + 1Y_6 \geq 25$

$x_2 + 2x_3 + x_5 + 3x_6 + 1Y_2 + 2Y_3 + 1Y_5 + 3Y_6 + 4Y_7 + 2Y_8 \geq 20$

$x_4 + x_5 + Y_4 + Y_5 + Y_8 + 2Y_9 \geq 15$

end

gin $X_1, x_2, x_3, x_4, x_5, x_6, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9$

Global optimal solution found.

Objective value: 16.00000
Objective bound: 16.00000
Infeasibilities: 0.000000
Extended solver steps: 0
Total solver iterations: 7
Elapsed runtime seconds: 0.47

Model Class: MILP

Total variables: 15
Nonlinear variables: 0
Integer variables: 15

Total constraints: 4
Nonlinear constraints: 0

Total nonzeros: 42
Nonlinear nonzeros: 0

Variable	Value	Reduced Cost
X1	0.000000	1.000000
X2	0.000000	1.000000
X3	0.000000	1.000000
X4	0.000000	1.000000
X5	0.000000	1.000000
X6	0.000000	1.000000
Y1	0.000000	1.000000
Y2	0.000000	1.000000
Y3	0.000000	1.000000
Y4	0.000000	1.000000
Y5	13.00000	1.000000
Y6	0.000000	1.000000
Y7	1.000000	1.000000
Y8	2.000000	1.000000
Y9	0.000000	1.000000

Row	Slack or Surplus	Dual Price
1	16.00000	-1.000000
2	1.000000	0.000000
3	1.000000	0.000000
4	0.000000	0.000000

Min $x_1+x_2+x_3+x_4+x_5+x_6+Y_1+Y_2+Y_3+Y_4+Y_5+Y_6+Y_7+Y_8+Y_9$
s.t.

$5X_1+4x_2+2x_3+2x_4+1x_5+6Y_1+5Y_2+3Y_3+3Y_4+2Y_5+1Y_6 \geq 25$

$x_2+2x_3+x_5+3x_6+1Y_2+2Y_3+1Y_5+3Y_6+4Y_7+2Y_8 \geq 20$

$x_4+x_5+Y_4+Y_5+Y_8+2Y_9 \geq 15$

end

gin x1

gin x2

gin x3

gin x4

gin x5

gin x6

gin Y1

gin Y2

gin Y3

gin Y4

gin Y5

gin Y6

gin Y7

gin Y8

gin Y9

3) Global optimal solution found.

Objective value:	5370.000
Objective bound:	5370.000
Infeasibilities:	0.000000
Extended solver steps:	0
Total solver iterations:	9
Elapsed runtime seconds:	1.16
Model Class:	PILP
Total variables:	21
Nonlinear variables:	0
Integer variables:	21
Total constraints:	15
Nonlinear constraints:	0
Total non zeros:	91
Nonlinear non zeros:	0

Front table means we must hire $(8+2+4+5=)$ 19 people for our company and distribute them in a week days in this order 8-0-2-4-0-5-0 means in first day 8 people should start their work and so on. Also extra work for i 'th day is showed by y_i and w_i .
For example $y_1=5$ means the 5 people that started their work in day 1 must go extra work at day six and $w_1=0$ means we don't need any people that started its work in first day to work in day number 7 and so on

Variable	Value	Reduced Cost
X1	8.000000	250.0000
X2	0.000000	250.0000
X3	2.000000	250.0000
X4	4.000000	250.0000
X5	0.000000	250.0000
X6	5.000000	250.0000
X7	0.000000	250.0000
Y1	5.000000	62.000000
Y2	0.000000	62.000000
Y3	0.000000	62.000000
Y4	0.000000	62.000000
Y5	0.000000	62.000000
Y6	5.000000	62.000000
Y7	0.000000	62.000000
W1	0.000000	62.000000
W2	0.000000	62.000000
W3	0.000000	62.000000
W4	0.000000	62.000000
W5	0.000000	62.000000
W6	0.000000	62.000000
W7	0.000000	62.000000

Row	Slack or Surplus	Dual Price
1	5370.000	-1.000000
2	0.000000	0.000000
3	0.000000	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	0.000000	0.000000
7	0.000000	0.000000
8	0.000000	0.000000
9	3.000000	0.000000
10	0.000000	0.000000
11	2.000000	0.000000
12	4.000000	0.000000
13	0.000000	0.000000
14	0.000000	0.000000
15	0.000000	0.000000

Min $250x_1+250x_2+250x_3+250x_4+250x_5+250x_6+250x_7+62y_1+62y_2+62y_3+62y_4+62y_5+62y_6+62y_7+62w_1+62w_2+62w_3+62w_4+62w_5+62w_6+62w_7$

s.t.

$x_1+x_7+x_6+x_5+x_4+y_3+w_2 \geq 17$

$x_2+x_1+x_7+x_6+x_5+w_3+y_4 \geq 13$

$x_3+x_2+x_1+x_7+x_6+w_4+y_5 \geq 15$

$x_4+x_3+x_2+x_1+x_7+w_5+y_6 \geq 19$

$x_5+x_4+x_3+x_2+x_1+w_6+y_7 \geq 14$

$x_6+x_5+x_4+x_3+x_2+w_7+y_1 \geq 16$

$x_7+x_6+x_5+x_4+x_3+w_1+y_2 \geq 11$

$y_1+w_1-x_1 \leq 0$

$y_2+w_2-x_2 \leq 0$

$y_3+w_3-x_3 \leq 0$

$y_4+w_4-x_4 \leq 0$

$y_5+w_5-x_5 \leq 0$

$y_6+w_6-x_6 \leq 0$

$y_7+w_7-x_7 \leq 0$

end

gin x1

gin x2

gin x3

gin x4

gin x5

gin x6

gin x7

gin y1

gin y2

gin y3

gin y4

gin y5

gin y6

gin y7

gin w1

gin w2

gin w3

gin w4

gin w5

gin w6

gin w7

x_i means number of people that start their work at the i 'th day

y_i means number of people that start their work at i 'th day and will work extra at 5 days after their start day

And w_i is number of people that start their work at i 'th day and will work extra at 6 days after their start day

5)

we have tree layer of decision and in each layer we have 3 and 3 and 5 element respectively. So we must have $3 \times 3 \times 5 = 45$ decision variables. But we are lucky because we can assume that some event (variables) in our models are with probability of zero, in other word (for example we have not any road from W1 to C2 or we have not any road from C1 to receiver 3,4,5):

first layer:

Name of workshop	Total ability for production	Production decision variables name
1	400	W1
2	400	W2
3	400	W3

Second layer for first workshop:

Name of rode	Cost for that	Number of production passed by this center decision variable name
(1,1)	C(1,1)	C1
(1,2)	Infinite*	C2
(1,3)	C(1,3)	C3

(1,1) means from first workshop to first center

C(i,j) is a parametric cost

* Infinite =probability zero for this event , if we want to use literature of probability theory

And in same way we have some non zero probability decision variables in third layer.

And finally we must multiply all of these layers's variables to get a final variable.

So we have 22 decision variables:

W1c1r1 , W1c1r2 , W1c3r3 , W1c3r4, W1c3r5

W2c1r1, W2c1r2, W2c2r1, W2c2r2, W2c2r3, W2c2r4, W2c2r5, W2c3r3, W2c3r4, W2c3r5

W3c1r1, W3c1r2, W3c2r1, W3c2r2, W3c2r3, W3c2r4, W3c2r5

and each decision variable has its own cost coefficient

cost coefficient for xicjrk is (C(i,j)+d(j,k)) and xicjrk means number of production that we must produce

in i'th Workshop and sent to j'th Center and then use k'th road to send to the k'th receiver

our conditions is satisfying that we don't produce more than 400 in each workshop (\leq) and also

checking that we send enough (or exact – means using = instead of \geq) product to each costumer.

Min (C(1,1)+d(1,1)) W1c1r1 + (C(1,1)+d(1,k2)) W1c1r2 + (C(1,3)+d(3,3)) W1c3r3 + (C(1,3)+d(3,4)) W1c3r4 + (C(1,3)+d(3,5)) W1c3r5 + (C(2,1)+d(1,1)) W2c1r1+ (C(2,1)+d(1,2)) W2c1r2+ (C(2,2)+d(2,1)) W2c2r1+ (C(2,2)+d(2,2)) W2c2r2+ (C(2,2)+d(2,3)) W2c2r3+ (C(2,2)+d(2,4)) W2c2r4+ (C(2,2)+d(2,5)) W2c2r5+ (C(2,3)+d(3,3)) W2c3r3+ (C(2,3)+d(3,4)) W2c3r4+ (C(2,3)+d(3,5)) W2c3r5+ (C(3,1)+d(1,1)) W3c1r1, (C(3,1)+d(1,2)) W3c1r2, (C(3,2)+d(2,1)) W3c2r1+(C(3,2)+d(2,2)) W3c2r2+ (C(3,2)+d(2,3)) W3c2r3+ (C(3,2)+d(2,4)) W3c2r4+ (C(3,2)+d(2,5)) W3c2r5

s.t.

$$W1c1r1 + W1c1r2 + W1c3r3 + W1c3r4 + W1c3r5 \leq 400$$

$$W2c1r1 + W2c1r2 + W2c2r1 + W2c2r2 + W2c2r3 + W2c2r4 + W2c2r5 + W2c3r3 + W2c3r4 + W2c3r5 \leq 400$$

$$W3c1r1 + W3c1r2 + W3c2r1 + W3c2r2 + W3c2r3 + W3c2r4 + W3c2r5 \leq 400$$

$$W1c1r1 + W2c1r1 + W3c1r1 + W3c2r1 + W2c2r1 = 100$$

$$W1c1r2 + W2c1r2 + W3c1r2 + W2c2r2 + W3c2r2 = 200$$

$$W1c3r3 + W2c2r3 + W3c2r3 + W2c3r3 = 150$$

$$W1c3r4 + W2c2r4 + W3c2r4 + W2c3r4 = 50$$

$$W1c3r5 + W2c2r5 + W2c3r5 + W3c2r5 = 300$$

End

Gin(All variables)