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Assignment No7

With Extra explanation ☺

Q1)

MAX z = -5x1 + 2x2 + 4x3
subject to
3x1 + x2 - x3 >= 2
2x1 + 2x2 + x3 = 5
x1 - x2 <= 1
and x1,x2,x3 >= 0

-->Phase-1<--

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint-1 is of type '≥' we should subtract surplus variable S1 and add artificial variable A1

2. As the constraint-2 is of type '=' we should add artificial variable A2

3. As the constraint-3 is of type '≤' we should add slack variable S2

After introducing slack,surplus,artificial variables

Max z = - A1 - A2

subject to

$$3x_1 + x_2 - x_3 - S_1 + A_1 = 2$$
$$2x_1 + 2x_2 + x_3 + A_2 = 5$$
$$x_1 - x_2 + S_2 = 1$$

and x1,x2,x3,S1,S2,A1,A2≥0

Iteration-1		Cj	0	0	0	0	0	-1	-1	
B	**CB**	**XB**	**x1**	**x2**	**x3**	**S1**	**S2**	**A1**	**A2**	**MinRatio**
A1	-1	2	**(3)**	1	-1	-1	0	1	0	2/3=0.6667→
A2	-1	5	2	2	1	0	0	0	1	5/2=2.5
S2	0	1	1	-1	0	0	1	0	0	1/1=1
z=-7		**Zj**	**-5**	**-3**	**0**	**1**	**0**	**-1**	**-1**	

		Zj-Cj	-5↑	-3	0	1	0	0	0	

Negative minimum Zj-Cj is -5 and its column index is 1. So, the entering variable is x1.

Minimum ratio is 0.6667 and its row index is 1. So, the leaving basis variable is A1.

∴ The pivot element is 3.

Entering =x1, Departing =A1, Key Element =3

Iteration-2		Cj	0	0	0	0	0	-1	
B	**CB**	**XB**	**x1**	**x2**	**x3**	**S1**	**S2**	**A2**	**MinRatio**
x1	0	23	1	13	-13	-13	0	0	---
A2	-1	113	0	43	53	23	0	1	11/5=2.2
S2	0	13	0	-43	**(13)**	13	1	0	13/13=1→
z=-11/3		**Zj**	**0**	**-43**	**-53**	**-23**	**0**	**-1**	
		Zj-Cj	0	-43	-53↑	-23	0	0	

Negative minimum Zj-Cj is -53 and its column index is 3. So, the entering variable is x3.

Minimum ratio is 1 and its row index is 3. So, the leaving basis variable is S2.

∴ The pivot element is 13.

Entering =x3, Departing =S2, Key Element =13

Iteration-3		Cj	0	0	0	0	0	-1	
B	**CB**	**XB**	**x1**	**x2**	**x3**	**S1**	**S2**	**A2**	**MinRatio**
x1	0	1	1	-1	0	0	1	0	---
A2	-1	2	0	**(8)**	0	-1	-5	1	2/8=0.25→
x3	0	1	0	-4	1	1	3	0	---
z=-2		**Zj**	**0**	**-8**	**0**	**1**	**5**	**-1**	
		Zj-Cj	0	-8↑	0	1	5	0	

Negative minimum Zj-Cj is -8 and its column index is 2. So, the entering variable is x2.

Minimum ratio is 0.25 and its row index is 2. So, the leaving basis variable is A2.

∴ The pivot element is 8.

Entering =x2, Departing =A2, Key Element =8

Iteration-4		Cj	0	0	0	0	0	
B	**CB**	**XB**	**x1**	**x2**	**x3**	**S1**	**S2**	**MinRatio**
x1	0	54	1	0	0	-18	38	
x2	0	14	0	1	0	-18	-58	
x3	0	2	0	0	1	12	12	
z=0		**Zj**	**0**	**0**	**0**	**0**	**0**	
		Zj-Cj	0	0	0	0	0	

Since all Zj-Cj≥0
Hence, optimal solution is arrived with value of variables as :
x1=54,x2=14,x3=2
Max z=0

-->Phase-2<--

we eliminate the artificial variables and change the objective function for the original,
Max z=-5x1+2x2+4x3+0S1+0S2

Iteration-1		Cj	-5	2	4	0	0	
B	**C/B**	**X/B**	**x1**	**x2**	**x3**	**S1**	**S2**	**MinRatio**
x1	-5	54	1	0	0	-18	**(38)**	10/3=3.3333→
x2	2	14	0	1	0	-18	-58	---
x3	4	2	0	0	1	12	12	4
z=9/4		**Zj**	**-5**	**2**	**4**	**198**	**-98**	
		Zj-Cj	0	0	0	198	-98↑	

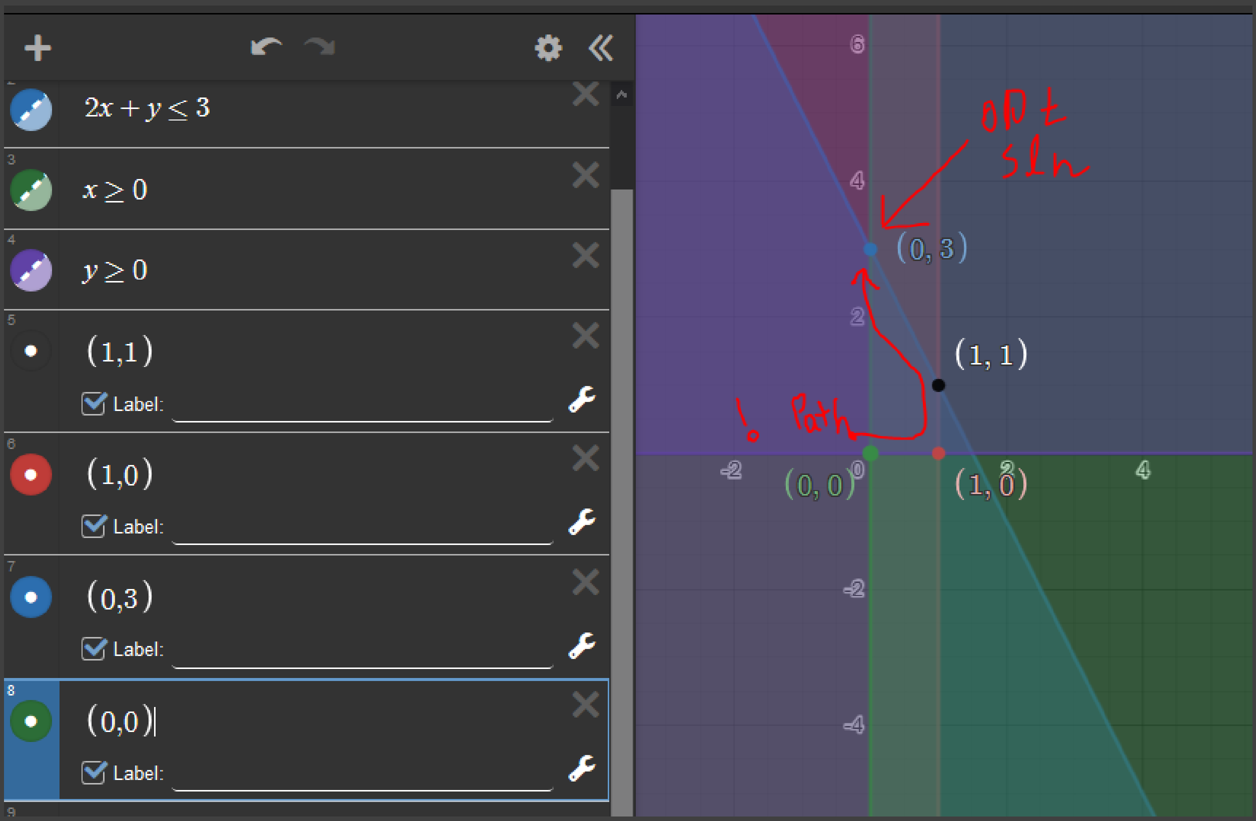
Negative minimum Zj-Cj is -98 and its column index is 5. So, the entering variable is S2.
Minimum ratio is 3.3333 and its row index is 1. So, the leaving basis variable is x1.

∴ The pivot element is 38.
Entering =S2, Departing =x1, Key Element =38

Iteration-2		Cj	-5	2	4	0	0	
B	**C/B**	**X/B**	**x1**	**x2**	**x3**	**S1**	**S2**	**MinRatio**
S2	0	103	83	0	0	-13	1	
x2	2	73	53	1	0	-13	0	
x3	4	13	-43	0	1	23	0	
z=6		**Zj**	**-2**	**2**	**4**	**2**	**0**	
		Zj-Cj	3	0	0	2	0	

Since all Zj-Cj≥0
Hence, optimal solution is arrived with value of variables as :
x1*=0, x2*=7/3=2.3333 ,x3*=1/3=0.3333 , finally Max z*=6

Q2)



Ok, let's go to show it with Simplex method.

After introducing slack variables

Max Z =

subject to

x1 + S1 = 1

2 x1 + x2 + S2 = 3

and x1,x2,S1,S2≥0

Iteration-1		Cj	1	1	0	0	
B	**C/B**	**X/B**	**x1**	**x2**	**S1**	**S2**	**MinRatio**
S1	0	1	**(1)**	0	1	0	1/1=1→
S2	0	3	2	1	0	1	3/2=1.5
Z=0		**Zj**	**0**	**0**	**0**	**0**	
		Zj-Cj	-1↑	-1	0	0	

Visiting (0,0)

Entering =x1, Departing =S1, Key Element =1

Iteration-2		Cj	1	1	0	0	
B	**C/B**	**X/B**	**x1**	**x2**	**S1**	**S2**	**MinRatio**
x1	1	1	1	0	1	0	---
S2	0	1	0	**(1)**	-2	1	1/1=1→
Z=1		**Zj**	**1**	**0**	**1**	**0**	
		Zj-Cj	0	-1↑	1	0	

visiting(1,0)

Entering =x2, Departing =S2, Key Element =1

Iteration-3		Cj	1	1	0	0	
B	**C/B**	**X/B**	**x1**	**x2**	**S1**	**S2**	**MinRatio**
x1	1	1	1	0	**(1)**	0	1/1=1→
x2	1	1	0	1	-2	1	---
Z=2		**Zj**	**1**	**1**	**-1**	**1**	
		Zj-Cj	0	0	-1↑	1	

Visiting (1,1)

Entering =S1, Departing =x1, Key Element =1

Iteration-4		Cj	1	1	0	0	
B	**C/B**	**X/B**	**x1**	**x2**	**S1**	**S2**	**MinRatio**
S1	0	1	1	0	1	0	
x2	1	3	2	1	0	1	
Z=3		**Zj**	**2**	**1**	**0**	**1**	
		Zj-Cj	1	0	0	1	

Finally visiting (0,3)

Hence, optimal solution is arrived with value of variables as :
x1=0,x2=3 Max Z=3

Q3)

سوال سوم: مسألهٔ غیرخطی زیر را در نظر بگیرید که در آن $x = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix}$ با شرح توضیحات لازم، مسأله را به صورت یک LP بازنویسی کنید.

$$\min z = max\big((2x_1 - 3x_2),\ \ (4x_1 - 2x_3),\ \ \ (2x_2 + x_3)\big)$$
$$s.t.$$
$$Ax = b$$
$$x \geq 0$$

Notation :

$(2x1-3x2) = A1$ & $(4x1-2x3) = A2$ & $(2x2+x3) = A3$

we should consider (A1,A2,A3) as a set of three point and then interpret "max" as a choose maximum number not Max of |(A1,A2,A3)|= sqr(A1^2+A2^2,A3^2).

Representing above question in LP form:

Min z = W

subject to
A1 <= W
A1 <= W
A1 <= W
AX = b
x>=0

Thank you.