Neural Networks and Backpropagation Lecture

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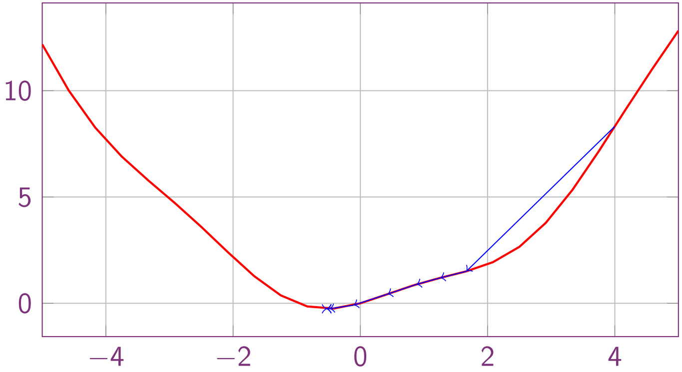
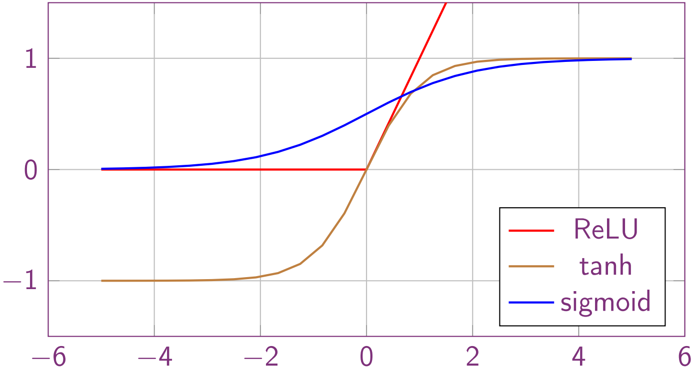
Neural Networks and Function Minimisation



Function of one real-valued variable

$$
\begin{aligned}
f : \mathbb{R} &\to \mathbb{R} \\
x &\mapsto f(x)
\end{aligned}
$$

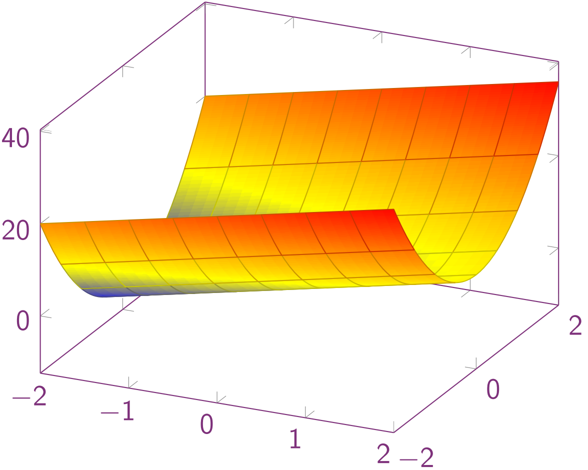
Activation functions



Function of two real-valued variables

$$\begin{aligned} f : \mathbb{R}^2 &\to \mathbb{R} \\ x, y &\mapsto f(x, y) \end{aligned}$$

Example: $f(x, y) = 4x + 7y^2$



Function of n real-valued variables

$$
\begin{aligned}
f : \mathbb{R}^n &\to \mathbb{R} \\
x_1, \ldots, x_n &\mapsto f(x_1, \ldots, x_n)
\end{aligned}
$$

Jacobian

$$\begin{aligned} f : \mathbb{R}^n &\to \mathbb{R}^m \\ x_1, \ldots, x_n &\mapsto (y_1, \ldots, y_m) = f(x_1, \ldots, x_n) \end{aligned}$$

Jacobian

$$J_f = \frac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}\boldsymbol{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

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Chain Rule

Given $h(x) = f(g(x))$

$$h'(x) = f'(g(x)) \times g'(x)$$

Example: A simple network

$$\boldsymbol{x}$$

$$\downarrow$$

$$h(\boldsymbol{x})$$

$$\downarrow$$

$$(g \circ h)(\boldsymbol{x})$$

$$\downarrow$$

$$(f \circ g \circ h)(\boldsymbol{x})$$

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Function of multiple vectors

$$\begin{aligned} f : \mathbb{R}^s \times \mathbb{R}^t &\to \mathbb{R}^u \\ x, y &\mapsto f(x, y) \end{aligned}$$

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Example: Multilinear Map

$$\begin{aligned} f \colon \mathbb{R}^n \times \mathbb{R}^{m \times n} \times \mathbb{R}^m &\to \mathbb{R}^m \\ x, W, b &\mapsto Wx + b \end{aligned}$$

$$\begin{tikzcd}
\mathbf{x} \arrow[d] & \mathbf{w_h} \arrow[dl] \\
h(\mathbf{x}, \mathbf{w_h}) \arrow[d] & \mathbf{w_g} \arrow[dl] \\
g(h(\mathbf{x}, \mathbf{w_f}), \mathbf{w_g}) \arrow[d] & \mathbf{w_f} \arrow[dl] \\
f(g(h(\mathbf{x}, \mathbf{w_h}), \mathbf{w_g}), \mathbf{w_f}) &
\end{tikzcd}$$