① Calcule la siguiente integral indefinida

ⓐ $\int \frac{\ln(x)}{x^2}\,dx$

$f(x) = \ln(x)$ $\quad f'(x) = \frac{1}{x}$

$g(x) = -\frac{1}{x}$ $\quad g'(x) = \frac{1}{x^2}$

$$\ln(x)\cdot -\frac{1}{x} + \int \frac{1}{x}\cdot\frac{1}{x}\,dx$$

$$\frac{-\ln(x)}{x} + \int \frac{1}{x^3}\,dx = \frac{-\ln(x)}{x} + \int x^{-2}\,dx = \ln(x)\cdot\left(-\frac{1}{x}\right) - \frac{1}{x} = -\frac{\ln(x)+1}{x}$$

ⓑ $\int_0^1 \frac{x}{x^2-1}\,dx$

$$\lim_{t\to1^-}\left(\int_0^t \frac{x}{x^2-1}\,dx\right) = \lim_{t\to1^-}\left(\frac{1}{2}\ln|x-1|\,\Big|_0^t\right) = \lim_{t\to1^-}\left(\frac{1}{2}\ln|t-1| - \frac{1}{2}\ln|0-1|\right) = \lim_{t\to1^-}\frac{1}{2}\ln|t-1| = -\infty \quad \therefore \int_0^1 \frac{x}{x^2-1}\,dx \text{ diverge}$$

$$\int \frac{x}{x^2-1}\,dx = \int \frac{\cancel{x}}{x^2-1}\,\frac{du}{2\cancel{x}} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| = \frac{1}{2}\ln|x-1| + C,\ C\in\mathbb{R}$$

$u = x^2-1$

$du = 2x\,dx$

$\frac{du}{2x} = 1\,dx$

② Determine si las siguientes sucesiones tienen límite

ⓐ $a_n = \frac{\operatorname{sen}(2n)}{n^2+1}$ $\quad \lim_{n\to\infty}\frac{\operatorname{sen}(2n)}{n^2+1} = 0 \quad \therefore \{a_n\}$ converge a 0

ⓑ $b_n = \frac{n}{e^n}$

encontramos $f(x) = \frac{x}{e^x}$ tal que $f(n) = a_n\ \forall n\in\mathbb{N}$ $\therefore \lim_{x\to\infty}\frac{x}{e^x} \overset{L'H}{=} \lim_{x\to\infty}\frac{1}{e^x} = 0 \quad \therefore \lim_{x\to\infty} f(x) = 0 = \lim_{n\to\infty} a_n \quad \therefore \left\{\frac{n}{e^n}\right\}$ Converge

③ Utilice algún criterio de convergencia y determine si las siguientes series convergen o divergen

ⓐ $\sum_{n=1}^{\infty}\frac{n^{\frac{1}{3}}}{n^3+3n}$

$$\lim_{n\to\infty}\frac{\frac{\sqrt[3]{n}}{n^3+3n}}{\frac{1}{n^{\frac{8}{3}}}} = \lim_{n\to\infty}\frac{n^{\frac{1}{3}}\cdot n^{\frac{8}{3}}}{n^3+3n} = \lim_{n\to\infty}\frac{n^3}{n^3+3n} = \lim_{n\to\infty}\frac{n^3}{n^3\left(1+\frac{3}{n^2}\right)} = 1$$

ⓑ $\sum_{n=1}^{\infty}\frac{2^n}{(n+1)!}$

$$\lim_{n\to\infty}\left(\left|\frac{\frac{2^{n+1}}{(n+2)!}}{\frac{2^n}{(n+1)!}}\right|\right) = \lim_{n\to\infty}\frac{2^{n+1}(n+1)!}{2^n\cdot(n+2)!} = \lim_{n\to\infty}\frac{\cancel{2^n}\cdot 2\cdot\cancel{(n+1)!}}{\cancel{2^n}\cdot\cancel{(n+1)!}\cdot(n+2)} = \lim_{n\to\infty}\frac{2}{n+2} = 0$$

$\therefore$ Por teorema (crit. del cociente) $\sum_{n=1}^{\infty}\frac{2^n}{(n+1)!}$ Converge