Guia 3

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① ⓐ $\lim_{x\to 4} (5x^2-2x+3) =$

X	Y
3,8	67,6
3,99	74,62
3,999	74,96
4,1	78,85
4,01	75,38
4,001	75,038

ⓑ $\lim_{x\to -1} \frac{x^3+1}{x+1} =$

X	Y
-1,1	3,31
-1,01	3,03
-1,001	3,003
-0,9	2,71
-0,99	2,97
-0,999	2,997

ⓒ $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} =$

X	Y
0,1	0,48
0,01	0,49
0,001	0,499
-0,1	0,51
-0,01	0,501
-0,001	0,5001

② ⓐ $\lim_{x\to 4} (5x^2-2x+3) = 5\cdot 4^2 - 2\cdot 4 + 3 = 80-8+3 = 75$

ⓑ $\lim_{s\to 2} (s^2+1)(s^2+4s) = (4+1)(4+8) = 5\cdot 12 = 60$

ⓒ $\lim_{t\to -1} \frac{\sqrt{t^3+3t^2+7}}{t+2} = \frac{\sqrt{-1+3+7}}{-1+2} = 3$

ⓓ $\lim_{x\to 1} \frac{x^3+1}{x+1} = 1$

ⓔ $\lim_{x\to -1} \frac{x^3+1}{x+1} = \lim_{x\to -1} (x^2-x+1) = (-1)^2-(-1)+1 = 3$

$$\begin{array}{l|l} x^3+0x^2+0x+1 & x+1 \\ \hline -x^3+x^2 & x^2-x+1 \\ 0-x^2 & \\ -\,-x^2-x & \\ 0+0+x+1 & \\ -\,x+1 & \\ 0+0 & \end{array}$$

ⓕ $\lim_{x\to 4} \frac{x^2-6x+8}{x^2-5x+4} = \lim_{x\to 4} \frac{(x-4)(x-2)}{(x-4)(x-1)} = \lim_{x\to 4} \frac{x-2}{x-1} = \frac{2}{3} \rightarrow$

$$\begin{cases} \frac{6\pm\sqrt{(-6)^2-4\cdot 1\cdot 8}}{2} = \frac{6\pm 2}{2} = \begin{cases} x_1 = 4 \\ x_2 = 2 \end{cases} \\ \frac{5\pm\sqrt{(-5)^2-4\cdot 1\cdot 4}}{2} = \frac{5\pm 3}{2} = \begin{cases} x_1 = 4 \\ x_2 = 1 \end{cases} \end{cases}$$

ⓖ $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} = \lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} = \lim_{x\to 0} \frac{(\sqrt{1+x}-1)\cdot(\sqrt{1+x}+1)}{x(\sqrt{1+x}+1)} = \lim_{x\to 0} \frac{1+x-\sqrt{1+x}+\sqrt{1+x}-1}{x(\sqrt{1+x}+1)}$

$= \lim_{x\to 0} \frac{1+x-1}{x(\sqrt{1+x}+1)} = \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1} = \frac{1}{2}$

③ $f(x) \begin{cases} |x| & \text{si } x\le 0 \\ 9-(x-3)^2 & \text{si } 0<x<4 \\ -1 & \text{si } x\ge 4 \end{cases}$

ⓐ $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 9-(0-3)^2 = 9-9 = 0$

ⓑ $\lim_{x\to 0^-} f(x) = |x| = 0$

ⓒ $\lim_{x\to 0} f(x) = 0$

ⓓ $\lim_{x\to 4^+} f(x) = -1$

ⓔ $\lim_{x\to 4^-} f(x) = 9-(4-3)^2 = 8$

ⓕ $\lim_{x\to 4} f(x) = \nexists$

④ $g(x) \begin{cases} \frac{1}{x^2} & \text{si } |x|>1 \\ -x & \text{si } |x|<1 \\ 2 & \text{si } |x|=1 \end{cases}$

ⓐ $\lim_{x\to 1^+} g(x) = \frac{1}{1^2} = 1$

ⓑ $\lim_{x\to 1^-} g(x) = -1$

ⓒ $\lim_{x\to 1} g(x) = \nexists$

ⓓ $\lim_{x\to -1^+} g(x) = 1$

ⓔ $\lim_{x\to -1^-} g(x) = \frac{1}{(-1)^2} = 1$

ⓕ $\lim_{x\to 1} g(x) = 1$

⑤ ⓐ $\lim_{h\to 0^-} \frac{\sqrt{h^2}}{h} = \lim_{h\to 0^-} \frac{|h|}{h} = \lim_{h\to 0^-} \frac{-h}{h} = -1$ $\quad |h| \begin{cases} h & \text{si } x\ge 0 \\ -h & \text{si } x<0 \end{cases}$

ⓑ $\lim_{x\to 2^+} \frac{x-2}{|x-2|} = \lim_{x\to 2^+} \frac{x-2}{x-2} = 1$ $\quad |x-2| \begin{cases} x-2 & \text{si } x\ge 2 \\ -(x-2) & \text{si } x<2 \end{cases}$

ⓒ $\lim_{x\to -2} \frac{2-|x|}{2+x} = \lim_{x\to -2} \frac{2+x}{2+x} = 1$ $\quad |x| \begin{cases} x & \text{si } x\ge 0 \\ -x & \text{si } x<0 \end{cases}$

ⓓ $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right) = \lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{x}\right) = 0$ $\quad |x| \begin{cases} x & \text{si } x\ge 0 \\ -x & \text{si } x<0 \end{cases}$

ⓔ $\lim_{x\to 0^-} \left(\frac{1}{x} - \frac{1}{|x|}\right) = \lim_{x\to 0^-} \left(\frac{1}{x} - \frac{1}{-x}\right) = \lim_{x\to 0^-} \left(\frac{1}{x} + \frac{1}{x}\right) = \lim_{x\to 0^-} \frac{2}{x} = -\infty$

ⓕ $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{|x|}\right) = \nexists$

⑥ ⓐ $\lim_{x\to -1} f(x)$ sabiendo que $1 \le f(x) \le x^2+2x+2$

$\lim_{x\to -1} 1 \le \lim_{x\to -1} f(x) \le \lim_{x\to -1} x^2+2x+2$

$1 \le \lim_{x\to -1} f(x) \le 1$

ⓑ $\lim_{x\to 1} f(x)$ sabiendo que $3x \le f(x) \le x^2+x$

$\lim_{x\to 1} 3x \le \lim_{x\to 1} f(x) \le \lim_{x\to 1} x^2+2$

$3 \le \lim_{x\to 1} f(x) \le 3$

⑦ ⓐ $\lim_{x\to\infty} \frac{x^3+1}{x+1} = \lim_{x\to\infty} \frac{x(x^2+1)}{x(1+\frac{1}{x})} = \lim_{x\to\infty} \frac{x^2+1}{1+\frac{1}{x}} = \frac{x^2+1}{1+0} = \infty$

ⓑ $\lim_{x\to\infty} \frac{x^2+x}{1+3x^2} = \lim_{x\to\infty} \frac{x^2(1+\frac{1}{x})}{x^2(\frac{1}{x^2}+3)} = \lim_{x\to\infty} \frac{1+\frac{1}{x}}{\frac{1}{x^2}+3} = \frac{1+0}{0+3} = \frac{1}{3}$

ⓒ $\lim_{x\to\infty} \frac{x^2+x+1}{2+2x^2+9x^4} = \lim_{x\to\infty} \frac{x^4(\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4})}{x^4(\frac{2}{x^4}+\frac{2}{x^2}+9)} = \lim_{x\to\infty} \frac{\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}}{\frac{2}{x^4}+\frac{2}{x^2}+9} = 0$

ⓓ $\lim_{x\to\infty} \left(\sqrt{18x^2+1} \quad \frac{1}{\sqrt{32x^2-3}}\right) = \lim_{x\to\infty} \frac{\sqrt{18x^2+1}}{\sqrt{32x^2-3}} = \lim_{x\to\infty} \frac{x\sqrt{18+\frac{1}{x^2}}}{x\sqrt{32-\frac{3}{x^2}}} = \frac{\sqrt{18}}{\sqrt{32}}$

⑧ ⓐ $y = \frac{x}{x+4}$ $\quad \lim_{x\to\infty} \frac{x}{x+4} = \lim_{x\to\infty} \frac{x}{x(1+\frac{4}{x})} = \lim_{x\to\infty} \frac{1}{1+\frac{4}{x}} = 1$

$x+4=0$

$x=-4$

Posibles AV: -4 $\qquad$ AH: 1

ⓑ $y = \frac{x^2+4}{x^2-1}$ $\quad \lim_{x\to\infty} \frac{x^2+4}{x^2-1} = \lim_{x\to\infty} \frac{x^2(1+\frac{4}{x^2})}{x^2(1-\frac{1}{x^2})} = \lim_{x\to\infty} \frac{1+\frac{4}{x^2}}{1-\frac{1}{x^2}} = 1$

$x^2-1=0$

$x^2=1$

$x=\pm\sqrt{1}$

$x=\pm 1$

Posibles AV, ± 1 $\qquad$ AH: 1

ⓒ $y = \frac{x^3+1}{x^3+x}$ $\quad \lim_{x\to\infty} \frac{x^3+1}{x^3+x} = \lim_{x\to\infty} \frac{x^3(1+\frac{1}{x^3})}{x^3(1+\frac{1}{x^2})} = \lim_{x\to\infty} \frac{1+\frac{1}{x^3}}{1+\frac{1}{x^2}} = 1$

$x^3+x=0$

$x=0$

Posibles AV: 0 $\qquad$ AH: 1

⑨ ⓐ $\lim_{x\to 1} g(x) = 1$ ⓔ $\lim_{x\to 2} g(x) = 0$ ⓘ $\lim_{x\to 6} g(x) = \nexists$ ⓜ $\lim_{x\to -2} g(x) = \nexists$ ⓟ $\lim_{x\to 0} g(x) = \nexists$

ⓑ $\lim_{x\to 1^+} g(x) = 1$ ⓕ $\lim_{x\to 2^+} g(x) = 0$ ⓙ $\lim_{x\to 6^+} g(x) = 2$ ⓝ $\lim_{x\to -2^+} g(x) = 4$ ⓠ $\lim_{x\to 0^+} g(x) = -\infty$

ⓒ $\lim_{x\to 1^-} g(x) = 1$ ⓖ $\lim_{x\to 2^-} g(x) = 0$ ⓚ $\lim_{x\to 6^-} g(x) = 4$ ⓝ $\lim_{x\to -2^-} g(x) = 4$ ⓡ $\lim_{x\to 0^-} g(x) = \infty$

ⓓ $g(1) = 1$ ⓗ $g(2) = 0$ ⓛ $g(6) = 2$ ⓞ $g(-2) = 2$ ⓢ $g(0) = \nexists$

⑩ ⓐ $\lim_{x\to 0} \frac{(1+x)^2-1}{x} = \lim_{x\to 0} \frac{x^2+2x+1-1}{x} = \lim_{x\to 0} \frac{x^2+2x}{x} = \lim_{x\to 0} \frac{x(x+2)}{x} = \lim_{x\to 0} x+2 = 2$

ⓑ $\lim_{x\to 3} \frac{(x^2-9)}{x^2-5x+9} = \frac{0}{3} = 0$

ⓒ $\lim_{x\to 0} \left(\frac{sen(x)}{sen(5x)}\right) = \lim_{x\to 0} \frac{sen(x)}{1} \cdot \frac{x}{x} \cdot \frac{1}{sen(5x)} \cdot \frac{5}{5} = \lim_{x\to 0} \overbrace{\frac{sen(x)}{x}}^{1} \cdot \overbrace{\frac{5x}{sen(5x)}}^{1} \cdot \frac{1}{5} = \frac{1}{5}$

ⓓ $\lim_{r\to\infty} r\cdot sen\,\frac{1}{r} = \lim_{u\to 0} \frac{1}{u}\cdot sen(u) = \lim_{u\to 0} \frac{sen(u)}{u} = 1$

ⓔ $\lim_{x\to 0} \frac{\tan 3x}{x} = \lim_{x\to 0} \frac{\tan 3x}{3x} \cdot 3 = 3$

ⓕ $\lim_{x\to 0^+} \frac{1-\cos(x)}{x^2} =$

⑪ ⓐ $\lim_{x\to\infty} 5^{2x+5} = \infty$

ⓑ $\lim_{x\to -\infty} \pi^{-x} = \infty$

ⓒ $\lim_{x\to 2^+} 4^{\frac{x}{2-x}} = 4^{-\infty}$

ⓓ $\lim_{x\to 0^-} 3^{\frac{1}{x}}$

ⓔ $\lim_{x\to\infty} \ln\left(\frac{x}{x^2+3}\right) = \lim_{x\to\infty} \ln\left(\frac{x}{x(x+\frac{3}{x})}\right) = \lim_{x\to\infty} \ln\left(\frac{1}{x+\frac{3}{x}}\right) = \lim_{x\to\infty} \ln(0) = -\infty$

ⓕ $\lim_{x\to 3} \ln\left(\frac{9-x^2}{x-3}\right)$

⑫ ⓐ $\lim_{x\to\infty} \left(1+\frac{1}{4x}\right)^{4x+3} = \lim_{x\to\infty} \left(1+\frac{1}{4x}\right)^{4x} \cdot \lim_{x\to\infty} \left(1+\frac{1}{4x}\right)^3 = e\cdot 1 = e$ //

ⓑ $\lim_{x\to 0} (1+2x)^{1/3x} = \lim_{x\to 0} \left((1+2x)^{1/2x}\right)^{2/3} = e^{2/3}$

ⓒ $\lim_{x\to +\infty} \left(1+\frac{7}{5x}\right)^{2x} = \lim_{x\to\infty} \left(1+\frac{\frac{1}{7}\cdot 7}{5x\cdot\frac{1}{7}}\right)^{2x} = \lim_{x\to\infty} \left(1+\frac{1}{\frac{5x}{7}}\right)^{\frac{5x}{7}\cdot\frac{14}{5}} = \lim_{t\to\infty} \left(1+\frac{1}{t}\right)^{t\,\frac{14}{5}} = e^{\frac{14}{5}}$

ⓓ $\lim_{x\to 0} (1+34x)^{3+\frac{2}{x}} = \lim_{x\to 0} (1+34x)^{\frac{3x+2}{x}} = \lim (1+34x)^{\frac{1}{x}\cdot 3x+2}$.

ⓔ $\lim_{x\to 0^+} \ln(\sqrt[4]{x}) = \lim_{x\to 0^+} \ln(x^{\frac{1}{4}}) = \lim_{x\to 0^+} \frac{1}{4}\ln(x) = -\infty$

ⓕ $\lim_{x\to 0} \ln((1+x)^{3/x}) = \lim_{x\to 0} \frac{3}{x}\ln(1+x) = \frac{3}{x}\cdot x = 3$

Material Extra

13 a) $\lim_{x\to a} f(x) = \nexists$ y $\lim_{x\to a} g(x) = \nexists \Rightarrow \lim (f+g)(x) = \exists$; $\Rightarrow \lim (f\cdot g)(x) = \exists$

$\lim_{x\to a} (f+g)(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = \nexists$

$\lim (f\cdot g)(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = \nexists$

b) $\lim_{x\to a} f(x) = L$ y $\lim_{x\to a} (f+g)(x) = M \Rightarrow \lim_{x\to a} g(x) = \exists$

Si, $\lim_{x\to a} g(x) = \lim_{x\to a} (f+g)(x) - \lim_{x\to a} f(x)$

c) $\lim_{x\to a} f(x) = L$ y $\lim_{x\to a} g(x) = \nexists \Rightarrow \lim (f+g)(x) = \exists$

No, porque incluye la suma de un termino que no existe

d) $\lim_{x\to a} f(x) = L$ y $\lim_{x\to a} (f\cdot g)(x) = M \Rightarrow \lim_{x\to a} g(x) = \exists$

No necesariamente, puede ser que g(x) tenga un comportamiento que el producto tenga un limite pero g no.

Ejemplo: $f(x) = x \quad g(x) = \frac{1}{x}$

14) $V(t) = 1000 \cdot \frac{\sqrt{t+3} - 2}{t-1}$

$\lim_{t\to 1} V(t) = 1000 \cdot \frac{\sqrt{t+3} - 2}{t-1} = \lim_{t\to 1} 1000 \cdot \lim_{t\to 1} \frac{\sqrt{t+3} - 2}{t-1}$

$= 1000 \lim_{t\to 1} \frac{\sqrt{t+3} - 2}{t-1} \overset{L'H}{=} 1000 \cdot \lim_{t\to 1} \frac{\sqrt{t+3} - 2}{t-1} \cdot \frac{\sqrt{t+3} + 2}{\sqrt{t+3} + 2} =$

$1000 \cdot \lim_{t\to 1} \frac{t+3-4}{(t-1)(\sqrt{t+3}+2)} = 1000 \lim_{t\to 1} \frac{\cancel{t-1}}{\cancel{(t-1)}(\sqrt{t+3}+2)} =$

$1000 \cdot \lim_{t\to 1} \frac{1}{\sqrt{t+3}+2} = 1000 \cdot \frac{1}{4} = 250$

Rta: El valor se aproxima a $250 m^3$

15) $f(t) = \frac{10t}{t^2+1}$

$\lim_{x\to\infty} f(t) = \lim_{x\to\infty} \frac{10t}{t^2+1} = \lim_{x\to\infty} \frac{10t}{t(t+\frac{1}{t})} = \lim \frac{10}{t+\frac{1}{t}} = 0$

16) $y(x) = \frac{4}{2+8e^{-2x}}$

a) $y(0) = \frac{4}{10} \cdot 1.000.000$

$= \frac{4.000.000}{10}$

$= 400.000$

Rta: La poblacion inicial es de 400.000

b) $\lim_{x\to\infty} y(x) = \lim_{x\to\infty} \frac{4}{2+8e^{-2x}} = \lim_{x\to\infty} \frac{4}{2+8\,\frac{1}{e^{2x}}}$

$= \frac{4}{2} = 2$

Rta: La poblacion tiende a estabilizarse en el valor 2.000.000