



7) $y = \sqrt{3-x}$ $(-1, 2), (2, 1)$ y $(-6, 3)$

$$f'(x) = \frac{-1}{2\sqrt{3-x}}$$

$t(x) = ax + b$
 $a = f'(x)$

$$f'(-1) = \frac{-1}{2\sqrt{4}} = -\frac{1}{4}$$

$$2 = -\frac{1}{4}(-1) + b$$

$$2 = \frac{1}{4} + b$$

$$2 - \frac{1}{4} = b$$

$$\frac{8}{4} - \frac{1}{4} = b$$

$$\frac{7}{4} = b$$

$$t(x) = -\frac{1}{4}x - \frac{7}{4}$$

$$f'(2) = \frac{-1}{2\sqrt{3-2}} = -\frac{1}{2}$$

$$1 = -\frac{1}{2} \cdot 2 + b$$

$$2 = b$$

$$t(x) = -\frac{1}{2}x + 2$$

$$f'(-6) = \frac{-1}{2\sqrt{3-6}} = -\frac{1}{9}$$

$$3 = -\frac{1}{9}(-6) + b$$

$$3 = \frac{2}{3} + b$$

$$3 - \frac{2}{3} = b$$

$$\frac{7}{3} = b$$

$$t(x) = -\frac{1}{9}x + \frac{7}{3}$$

8) para que valores son paralelas las tangentes de $y = x^2$ e $y = x^3$

$$f'(x) = 2x$$

$$g(x) = 3x^2$$

$$2x = 3x^2$$

$$0 = 3x^2 - 2x$$

$$\frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 3 \cdot 0}}{6} = \frac{2 \pm \sqrt{4}}{6} = \begin{cases} x_1 = \frac{1}{3} \\ x_2 = 0 \end{cases}$$

Rta: las tangentes son paralelas

en $x = \frac{2}{3}$ y $x = 0$

9) $f'(x) = -\frac{1}{x^2}$ La recta tangente $(a, 1/a)$ no corta la grafica de f mas que en el punto $(a, 1/a)$

$$f'(a) = -\frac{1}{a^2}$$

$$\frac{1}{a} = -\frac{1}{a^2}x + b$$

$$\frac{1}{a} = -\frac{1}{a} + b$$

$$\frac{2}{a} = b$$

$$t(x) = -\frac{1}{a^2}x + \frac{2}{a}$$

$$\frac{1}{x} = -\frac{1}{a^2}x + \frac{2}{a}$$

$$\frac{1}{x} \cdot x = -\frac{1}{a^2}x^2 + \frac{2}{a} \cdot x$$

$$\Delta = \left(\frac{2}{a}\right)^2 - 4 \cdot \left(-\frac{1}{a^2}\right) \cdot (-1)$$

$$= \frac{4}{a^2} - \frac{4}{a^2} = 0 \rightarrow \text{Una sola raiz doble}$$

10) a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$ $(-5, \frac{9}{4})$ (hiperbola)

$$-\frac{y^2}{9} = 1 - \frac{x^2}{16} \quad f(x) = \pm \sqrt{-9 + \frac{9x^2}{16}} = \sqrt{\frac{-144 + 9x^2}{16}} = \frac{\sqrt{-144 + 9x^2}}{4}$$

$$y = 3^2 \left(1 - \frac{x^2}{16}\right)$$

$$f'(x) = \frac{1}{4} \cdot \frac{1}{2\sqrt{-144 + 9x^2}} \cdot 18x$$

$$y^2 = (-4)3^2 \left(1 - \frac{x^2}{16}\right)$$

$$y^2 = -9 \left(1 - \frac{x^2}{16}\right)$$

$$y = \pm \sqrt{-9 \left(1 - \frac{x^2}{16}\right)}$$

$$x = -5$$

$$\Rightarrow y = \pm \sqrt{-9 \left(1 - \frac{25}{16}\right)}$$

$$y = \pm \sqrt{-9 \left(\frac{16}{16} - \frac{25}{16}\right)}$$

$$y = \pm \sqrt{-9 \left(-\frac{9}{16}\right)}$$

$$y = \pm \sqrt{\frac{81}{16}}$$

$$y = \frac{9}{4} \rightarrow \text{Verifica}$$

$$t(x) = -\frac{5}{4}x - 4$$

$$= \frac{18x}{8\sqrt{-144 + 9x^2}} = \frac{18x}{8\sqrt{9(-16 + x^2)}}$$

$$= \frac{18x}{8 \cdot 3\sqrt{-16 + x^2}} = \frac{3x}{4\sqrt{-16 + x^2}}$$

$$f'(-5) = \frac{3 \cdot (-5)}{4\sqrt{-16 + (-5)^2}} = \frac{-15}{12} = -\frac{5}{4}$$

$$t(x) = ax + b \rightarrow a = -\frac{5}{4}$$

$$\frac{9}{4} = -\frac{5}{4}(-5) + b$$

$$\frac{9}{4} = \frac{25}{4} + b$$

$$-\frac{16}{4} = b$$

$$-4 = b$$

b) $\frac{x^2}{9} + \frac{y^2}{36} = 1$ $(-1, 4\sqrt{2})$ (elipse)

$$\frac{y^2}{36} = 1 - \frac{x^2}{9}$$

$$y^2 = 36 \left(1 - \frac{x^2}{9}\right)$$

$$y = \pm \sqrt{36 \left(1 - \frac{x^2}{9}\right)}$$

$$x = -1$$

$$y = \pm \sqrt{36 \left(1 - \frac{1}{9}\right)}$$

$$y = 6 \cdot \sqrt{1 - \frac{1}{9}}$$

$$y = \frac{6}{3} \cdot \sqrt{8} = 2\sqrt{8}$$

$$y = 2\sqrt{2 \cdot 4} = 4\sqrt{2}$$

$$t(x) = \frac{\sqrt{2}}{2}x + \frac{9\sqrt{2}}{2}$$

$$f(x) = \frac{6\sqrt{9-x^2}}{3} = 2\sqrt{9-x^2}$$

$$f'(x) = 2 \cdot \frac{1}{2\sqrt{9-x^2}} \cdot -2x$$

$$= \frac{-2x}{\sqrt{9-x^2}} = \frac{-2x}{\sqrt{9-x^2}}$$

$$f'(-1) = \frac{-2(-1)}{\sqrt{9-(-1)^2}} = \frac{2}{\sqrt{8}} = \frac{2\sqrt{8}}{8} = \frac{2 \cdot \sqrt{2 \cdot 4}}{8} = \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{2}$$

$$a = \frac{\sqrt{2}}{2}$$

$$4\sqrt{2} = \frac{\sqrt{2}}{2}(-1) + b$$

$$4\sqrt{2} + \frac{\sqrt{2}}{2} = b$$

$$\frac{8\sqrt{2} + \sqrt{2}}{2} = b$$

$$\frac{9\sqrt{2}}{2} = b$$

Material Extra

$$\textcircled{11} \textcircled{a} f(x) = (1+3x^3)^5$$

$$f'(x) = 5(1+3x^3)^4 \cdot 12x^2$$

$$= 60x^2(1+3x^3)^4 //$$

$$\textcircled{b} f(x) = (1+x+x^2)^3$$

$$f'(x) = 3(1+x+x^2)^2 \cdot (1+2x) //$$

$$\textcircled{c} f(x) = \frac{1}{(x^2-1)^5}$$

$$f'(x) = \frac{-1 \cdot 5(x^2-1)^4 \cdot 2x}{(x^2-1)^{10}}$$

$$= \frac{-10x}{(x^2-1)^6} //$$

$$\textcircled{d} f(x) = (3x^2+3)(2x^2+1)$$

$$f'(x) = 6x \cdot (2x^2+1) + (3x^2+3) \cdot 4x$$

$$= 12x^3 + 6x + 12x^3 + 12x$$

$$= 24x^3 + 18x //$$

$$\textcircled{e} f(x) = \frac{2x^3+5}{4x^2+7}$$

$$f'(x) = \frac{6x^2(4x^2+7) - (2x^3+5) \cdot 8x}{(4x^2+7)^2}$$

$$= \frac{24x^4 + 42x^2 - 16x^4 - 40x}{(4x^2+7)^2}$$

$$= \frac{8x^4 + 42x^2 - 40x}{(4x^2+7)^2} //$$

$$\textcircled{f} f(x) = \frac{x}{x-1} + \frac{2}{(x-1)^2} + \frac{3}{(x-1)^3}$$

$$f'(x) = \frac{x-1-x}{(x-1)^2} + \frac{4(x-1)}{(x-1)^3} + \frac{-9(x-1)^2}{(x-1)^4}$$

$$= \frac{x}{(x-1)^2} + \frac{4}{(x-1)^2} + \frac{-9}{(x-1)^2} //$$

$$\textcircled{g} f(x) = \sqrt{1-x^2}$$

$$f'(x) = \frac{-2x}{2\sqrt{1-x^2}}$$

$$= -\frac{x}{\sqrt{1-x^2}} //$$

$$\textcircled{h} f(x) = (2+5x^2)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3} \cdot (2+5x^2)^{-\frac{2}{3}} \cdot 10x$$

$$= \frac{10x}{3\sqrt[3]{(2+5x^2)^2}} //$$

$$\textcircled{12} f(x) = \frac{1}{\sqrt{(x^2-2)}} \cdot \frac{1}{(x^2-2)^{\frac{1}{3}}}$$

$$f'(x) = -\frac{\frac{1}{3} \cdot (x^2-2)^{-\frac{4}{3}} \cdot 2x}{(x^2-2)^{\frac{1}{3}}}$$

$$= -\frac{\frac{1}{3} \cdot \frac{1}{(x^2-2)^{\frac{4}{3}}} \cdot 2x}{(x^2-2)^{\frac{1}{3}}}$$

$$= -\frac{2x}{3(x^2-2)^{\frac{5}{3}}}$$

$$= -\frac{2x}{3(x^2-2)^2 \sqrt{x^2-2}} //$$

$$\textcircled{13} f(x) = (5-3\cos(x))^4$$

$$f'(x) = 4(5-3\cos(x))^3 \cdot 3\sin(x)$$

$$= 12(5-3\cos(x))^3 \cdot \sin(x) //$$

$$\textcircled{14} f(x) = \sin(x) + \sin^2(x) + \sin^3(x)$$

$$f'(x) = \cos(x) + 2\sin(x) \cdot \cos(x) + 3\sin^2(x) \cdot \cos(x) //$$

$$\textcircled{15} f(x) = \frac{1}{\arctan(x)}$$

$$f'(x) = \frac{-1}{\arctan^2(x)} \cdot \frac{1}{(1+x^2)\arctan^2(x)}$$

$$\textcircled{16} f(x) = \sin^3(x) \cdot \cos^2(x)$$

$$f'(x) = 3\sin^2(x) \cdot \cos(x) + 3\cos^2(x) \cdot \sin(x) //$$

$$\textcircled{17} f(x) = \frac{1}{3\sin^2(x)} - \frac{1}{\cos(x)}$$

$$f'(x) = -\frac{1}{3\sin^4(x) \cdot \cos(x)} + \frac{\sin(x)}{\cos^2(x)}$$

$$= -\frac{\cos(x)}{3\sin^4(x)} + \frac{\sin(x)}{\cos^2(x)}$$

$$= \frac{\cos^3(x) + 3\sin^5(x)}{3\sin^4(x) \cdot \cos^2(x)} //$$

$$\textcircled{18} f(x) = \sqrt[3]{2x+x} = \sqrt[3]{e^{\ln(2x+x)}} = \sqrt[3]{e^{2\ln(2x)} + x}$$

$$f'(x) = \frac{e^{2\ln(2x)} \cdot \ln(2x) + 1}{3\sqrt[3]{(e^{2\ln(2x)} + x)^2}} //$$

$$\textcircled{19} f(x) = \ln(\ln(x))$$

$$f'(x) = \frac{1}{\ln(x)} \cdot \frac{1}{x}$$

$$\textcircled{20} f(x) = \arccos(\sqrt{x})$$

$$f'(x) = -\frac{1}{\sqrt{1-x} \cdot 2\sqrt{x}}$$

$$\textcircled{21} f(x) = \arcsin\left(\frac{1}{x^2}\right)$$

$$f'(x) = \frac{1}{\sqrt{1-\frac{1}{x^4}}} \cdot \frac{-2x}{x^4}$$

$$= -\frac{2x}{\sqrt{1-\frac{1}{x^4}} \cdot x^4} //$$

$$\textcircled{22} f(x) = \frac{1+\sin^2(x)}{1+\cos^2(x)}$$

$$f'(x) = \frac{2\sin(x) \cdot \cos(x) \cdot (1+\cos^2(x)) - (1+\sin^2(x)) \cdot 2\cos(x) \cdot (-\sin(x))}{(1+\cos^2(x))^2}$$

$$= \frac{2\sin(x) \cdot \cos(x) \cdot (1+\cos^2(x)) + (1+\sin^2(x)) \cdot 2\cos(x) \cdot \sin(x)}{(1+\cos^2(x))^2}$$

$$= \frac{\sin(x) \cdot \cos(x) \cdot (2(1+\cos^2(x)) + 2(1+\sin^2(x)))}{(1+\cos^2(x))^2}$$

$$= \frac{\sin(x) \cdot \cos(x) \cdot 6}{(1+\cos^2(x))^2} //$$

$$\textcircled{23} f(x) = \ln\left(\frac{3x}{4}\right)$$

$$f'(x) = \frac{1}{\frac{3x}{4}} \cdot \frac{12}{16}$$

$$= \frac{4}{3x} \cdot \frac{3}{4} = \frac{1}{x} //$$

$$\textcircled{24} f(x) = 7\ln(x^{\frac{1}{3}})$$

$$f'(x) = \frac{7}{x^{\frac{2}{3}}} \cdot x^{\frac{1}{3}}$$

$$f'(x) = \frac{7}{5} \cdot \frac{7}{x^{\frac{2}{3}} \cdot x^{\frac{1}{3}}} = \frac{14}{5x} //$$

$$\textcircled{25} f(x) = 4\ln(\sin(x))$$

$$f'(x) = \frac{4\cos(x)}{\sin(x)} = 4\cot(x) //$$

$$\textcircled{26} f(x) = \ln(\arctan(x))$$

$$f'(x) = \frac{1}{\arctan(x)} \cdot \frac{1}{1+x^2}$$

$$= \frac{1}{(\arctan(x))(1+x^2)} //$$