① ⓐ $\begin{cases} 2x - y = 9 \\ 3x + 2y = 17 \end{cases} \longrightarrow 2x - 9 = y$

$3x + 2(2x-9) = 17$ $\qquad 2\cdot 5 - y = 9$

$3x + 4x - 18 = 17$ $\qquad -y = -1$

$7x = 35$ $\qquad y = 1$

$x = \frac{35}{7}$

$x = 5$ $\qquad \boxed{x=5\ ,\ y=1}$

ⓑ $\begin{cases} 3x + 2y - 2z = -7 \\ x + 2z = 7 \\ x + 3y + z = -2 \end{cases} \longrightarrow x = 7 - 2z$

$\longrightarrow 3(7-2z) + 2y - 2z = -7$

$21 - 6z + 2y - 2z = -7$

$-8z + 2y = -28$

$2y = -28 + 8z$

$y = -14 + 4z$

$(7-2z) + 3(4z-14) + z = -2$

$7 - 2z + 12z - 42 + z = -2$

$11z = 33$

$z = 3$

$x = 7 - 2\cdot 3$

$x = 1$

$y = -14 + 4\cdot 3$

$y = -2$

$\boxed{x=1 \quad y=-2 \quad z=3}$

② ⓐ $\begin{cases} x - y = 0 \\ 2x + y = 0 \end{cases}$ $\qquad x = y$

$2y + y = 0$ $\qquad x - 0 = 0$

$3y = 0$ $\qquad x = 0$

$y = 0$

$\boxed{x=0,\ y=0}$

ⓑ $\begin{cases} 2x + 5y + 7z = 0 \\ -x + 6y + 3z = 0 \\ 7x - 8y + 5z = 0 \end{cases}$

$$\left(\begin{array}{ccc|c} 2 & 5 & 7 & 0 \\ -1 & 6 & 3 & 0 \\ 7 & -8 & 5 & 0 \end{array}\right) \xrightarrow{2f_2+f_1} \left(\begin{array}{ccc|c} 2 & 5 & 7 & 0 \\ 0 & 17 & 13 & 0 \\ 7 & -8 & 5 & 0 \end{array}\right) \xrightarrow{2f_3-7f_1} \left(\begin{array}{ccc|c} 2 & 5 & 7 & 0 \\ 0 & 17 & 13 & 0 \\ 0 & -54 & -39 & 0 \end{array}\right) \xrightarrow{f_3\cdot(-\frac{1}{3})} \left(\begin{array}{ccc|c} 2 & 5 & 7 & 0 \\ 0 & 17 & 13 & 0 \\ 0 & 17 & 13 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 2 & 5 & 7 & 0 \\ 0 & 17 & 13 & 0 \end{array}\right) \xrightarrow{13f_1-7f_2} \left(\begin{array}{ccc|c} 26 & -54 & 0 & 0 \\ 0 & 17 & 13 & 0 \end{array}\right) \therefore \begin{cases} 26x - 54y = 0 \\ 17y + 13z = 0 \end{cases} \longrightarrow 26x = 54y \longrightarrow \frac{26}{54}x = y$$

$\frac{13}{27}x = y$

$17\left(\frac{13}{27}x\right) + 13z = 0$

$\frac{221}{27}x + 13z = 0$

$\frac{221}{27}x = -13z$

$221x = -351z$

$x = -\frac{27}{17}z$

③ ⓐ $\begin{cases} 2x - 3y = -19 \\ 4x + 5y = 17 \end{cases}$

$2x = -19 + 3y$ $\qquad 4\left(-\frac{19}{2} + \frac{3}{2}y\right) + 5y = 17$

$x = -\frac{19}{2} + \frac{3}{2}y$ $\qquad -38 + 6y + 5y = 17$

$11y = 55$

$y = 5$

$2x - 15 = -19$

$2x = -4$

$x = -2$ $\qquad \boxed{x=-2,\ y=5}$

ⓑ $\begin{cases} 2x + 3y + 2z = 3 \\ 4x - 5y + 5z = 7 \\ -3x + 7y - 2z = 5 \end{cases}$

$$\left(\begin{array}{ccc|c} 2 & 3 & 2 & 3 \\ 4 & -5 & 5 & 7 \\ -3 & 7 & -2 & 5 \end{array}\right) \xrightarrow{f_2=f_2+(-2)f_1} \left(\begin{array}{ccc|c} 2 & 3 & 2 & 3 \\ 0 & -11 & 1 & 1 \\ -3 & 7 & -2 & 5 \end{array}\right) \xrightarrow{f_3=\frac{3}{2}f_1+f_3} \left(\begin{array}{ccc|c} 2 & 3 & 2 & 3 \\ 0 & -11 & 1 & 1 \\ 0 & \frac{23}{2} & 1 & \frac{19}{2} \end{array}\right)$$

$$\xrightarrow{f_1=-2f_2+f_1} \left(\begin{array}{ccc|c} 2 & 25 & 0 & 1 \\ 0 & -11 & 1 & 1 \\ 0 & \frac{23}{2} & 1 & \frac{19}{2} \end{array}\right) \xrightarrow{f_3=-f_2+f_3} \left(\begin{array}{ccc|c} 2 & 25 & 0 & 1 \\ 0 & -11 & 1 & 1 \\ 0 & \frac{45}{2} & 0 & \frac{17}{2} \end{array}\right)$$

$\frac{45}{2}y = \frac{17}{2}$

$45y = 17$

$y = \frac{17}{45}$

$-11\cdot\frac{17}{45} + z = 1$

$-\frac{187}{45} + z = 1$

$z = \frac{232}{45}$

$2x + 3\cdot\frac{17}{45} + 2\cdot\frac{232}{45} = 3$

$2x + \frac{51}{45} + \frac{464}{45} = 3$

$2x = \frac{135}{45} - \frac{51}{45} - \frac{464}{45}$

$2x = -\frac{380}{45}$

$x = -\frac{190}{45}$

$x = -\frac{38}{9}$

$\boxed{x=-\frac{38}{9},\ y=\frac{17}{45},\ z=\frac{232}{45}}$

④ ⓐ $\begin{cases} 2x - y = 9 \\ 3x + 2y = 6 \end{cases}$ $\quad \mathbb{Z}_{11}$

$-y = 9 - 2x \ (11)$ $\qquad 3x + 2(2+2x) = 6 \ (11)$

$y = -9 + 2x \ (11)$ $\qquad 3x + 4 + 4x = 6 \ (11)$

$\Rightarrow y = 2 + 2x \ (11)$ $\qquad 7x + 4 = 6 \ (11)$

$2 + 2\cdot 5 = y \ (11)$ $\qquad 7x = 2 \ (11)$

$1 = y \ (11)$ $\qquad 7x = 2 \ (11)$ $\qquad 7\cdot b \equiv 1 \ (11)$

$7\cdot 8 \equiv 1 \ (11)$

$\boxed{x=5,\ y=1}$ $\qquad 7\cdot 8\cdot x = 16 \ (11)$

$x = 5 \ (11)$

ⓑ $\begin{cases} x + 3y = 2 \\ 2x + y = 7 \end{cases}$ $\quad \mathbb{Z}_{11}$

$x = 2 - 3y \ (11)$ $\qquad 2(2-3y) + y = 7 \ (11)$

$x = 2 - 3\cdot 6 \ (11)$ $\qquad 4 - 6y = 7 \ (11)$

$x = -16$ $\qquad 5y = 3 \ (11)$ $\qquad 5\cdot b \equiv 1 \ (11)$

2 13
69
-23
08

$x = 6$ $\qquad 9\cdot 5\, y = 3\cdot 9 \ (11)$ $\qquad b = 9$

$9\cdot 5\, y = 27 \ (11)$

$\boxed{x=6,\ y=6}$ $\qquad y = 6 \ (11)$

5) a) $\begin{cases} x + y - 6z = 21 \\ 5x - 2y + 2z = 2 \\ 3x + 7y + z = 10 \end{cases}$

$$\left(\begin{array}{ccc|c} 1 & 1 & -6 & 21 \\ 5 & -2 & 2 & 2 \\ 3 & 7 & 1 & 10 \end{array}\right) \xrightarrow{f_2 = f_2 \cdot \frac{1}{5}} \left(\begin{array}{ccc|c} 1 & 1 & -6 & 21 \\ 1 & -\frac{2}{5} & \frac{2}{5} & \frac{2}{5} \\ 3 & 7 & 1 & 10 \end{array}\right) \xrightarrow{f_2 = f_2 - f_1} \left(\begin{array}{ccc|c} 1 & 1 & -6 & 21 \\ 0 & -\frac{7}{5} & \frac{32}{5} & -\frac{103}{5} \\ 3 & 7 & 1 & 10 \end{array}\right) \xrightarrow{f_3 = f_3 \cdot \frac{1}{3}} \left(\begin{array}{ccc|c} 1 & 1 & -6 & 21 \\ 0 & -\frac{7}{5} & \frac{32}{5} & -\frac{103}{5} \\ 1 & \frac{7}{3} & \frac{1}{3} & \frac{10}{3} \end{array}\right) \xrightarrow{f_3 = f_1 - f_3} \left(\begin{array}{ccc|c} 1 & 1 & -6 & 21 \\ 0 & -\frac{7}{5} & \frac{32}{5} & -\frac{103}{5} \\ 0 & -\frac{4}{3} & -\frac{19}{3} & \frac{53}{3} \end{array}\right)$$

$$\xrightarrow[f_2 = f_2 \cdot 5]{f_3 = f_3 \cdot (-3)} \left(\begin{array}{ccc|c} 1 & 1 & -6 & 21 \\ 0 & -7 & 32 & -103 \\ 0 & 4 & 19 & -53 \end{array}\right) \xrightarrow{f_2 \rightleftarrows f_3} \left(\begin{array}{ccc|c} 1 & 1 & -6 & 21 \\ 0 & 4 & 19 & -53 \\ 0 & -7 & 32 & -103 \end{array}\right) \xrightarrow{f_2 = f_2 \cdot \frac{1}{4}} \left(\begin{array}{ccc|c} 1 & 1 & -6 & 21 \\ 0 & 1 & \frac{19}{4} & -\frac{53}{4} \\ 0 & -7 & 32 & -103 \end{array}\right) \xrightarrow{f_3 = 7f_2 + f_3} \left(\begin{array}{ccc|c} 1 & 1 & -6 & 21 \\ 0 & 1 & \frac{19}{4} & -\frac{53}{4} \\ 0 & 0 & \frac{261}{4} & -\frac{783}{4} \end{array}\right) \xrightarrow{f_3 = f_3 \cdot \frac{4}{261}} \left(\begin{array}{ccc|c} 1 & 1 & -6 & 21 \\ 0 & 1 & \frac{19}{4} & -\frac{53}{4} \\ 0 & 0 & 1 & -3 \end{array}\right)$$

$x + 1 + 18 = 21$ $\quad$ $y + \frac{19}{4} \cdot (-3) = -\frac{53}{4}$ $\quad$ $z = -3$

$x = 2$ $\quad$ $y - \frac{57}{4} = -\frac{53}{4}$

$y = 1$

b) $\begin{cases} x - 4y = -2 \\ -5x + 3y = -7 \end{cases}$

$$\left(\begin{array}{cc|c} 1 & -4 & -2 \\ -5 & 3 & -7 \end{array}\right) \xrightarrow{f_2 = f_2 \cdot \left(-\frac{1}{5}\right)} \left(\begin{array}{cc|c} 1 & -4 & -2 \\ 1 & -\frac{3}{5} & \frac{7}{5} \end{array}\right) \xrightarrow{f_2 = f_1 - f_2} \left(\begin{array}{cc|c} 1 & -4 & -2 \\ 0 & -\frac{17}{5} & -\frac{17}{5} \end{array}\right)$$

$-\frac{17}{5} y = -\frac{17}{5}$ $\quad$ $x - 4 = -2$ $\quad$ $\boxed{x = 2, \ y = 1}$

$y = 1$ $\quad$ $x = 2$

c) $$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ -1 & 2 & 1 & -1 \\ -1 & 4 & 5 & 2 \end{array}\right) \xrightarrow{f_2 = f_2 + f_1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ -1 & 4 & 5 & 2 \end{array}\right) \xrightarrow{f_3 = f_3 + f_1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & 6 & 4 \end{array}\right) \xrightarrow{f_3 = f_3 \cdot \frac{1}{3}} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & \frac{4}{3} \end{array}\right)$$

NO HAY SOLUCION

YA QUE $y + 2z = 1 \ \wedge \ y + 2z = \frac{4}{3}$

ABS!!!

Sistema incompatible

d) $$\left(\begin{array}{ccc|c} 1 & -1 & 2 & -3 \\ 4 & 4 & 2 & 1 \\ -2 & 2 & -4 & 6 \end{array}\right) \xrightarrow{f_2 = f_2 \cdot \frac{1}{4}} \left(\begin{array}{ccc|c} 1 & -1 & 2 & -3 \\ 1 & 1 & \frac{1}{2} & \frac{1}{4} \\ -2 & 2 & -4 & 6 \end{array}\right) \xrightarrow{f_2 = f_2 - f_1} \left(\begin{array}{ccc|c} 1 & -1 & 2 & -3 \\ 0 & 2 & -\frac{3}{2} & -\frac{11}{4} \\ -2 & 2 & -4 & 6 \end{array}\right) \xrightarrow{f_3 = 2f_1 + f_2} \left(\begin{array}{ccc|c} 1 & -1 & 2 & -3 \\ 0 & 2 & -\frac{3}{2} & -\frac{11}{4} \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{f_2 = f_2 \cdot \left(-\frac{1}{2}\right)} \left(\begin{array}{ccc|c} 1 & -1 & 2 & -3 \\ 0 & -1 & \frac{3}{4} & \frac{11}{8} \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & -3 \\ 0 & -1 & \frac{3}{4} & \frac{11}{8} \end{array}\right) \xrightarrow{f_1 = f_1 - f_2} \left(\begin{array}{ccc|c} 1 & 0 & \frac{5}{4} & \frac{13}{8} \\ 0 & -1 & \frac{3}{4} & \frac{11}{8} \end{array}\right)$$

$x + \frac{5}{4} z = \frac{13}{8}$ $\quad$ $-y + \frac{3}{4} z = \frac{11}{8}$ $\quad$ $S = \left\{ \left(\frac{13}{8} - \frac{5}{4} z, -\frac{11}{8} + \frac{3}{4} z, z \right) \middle| \ z \in \mathbb{Q} \right\}$

$x = \frac{13}{8} - \frac{5}{4} z$ $\quad$ $-y = \frac{11}{8} - \frac{3}{4} z$

$y = -\frac{11}{8} + \frac{3}{4} z$

6) a)

$$\left(\begin{array}{cccc|c} 1 & 0 & 3 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 3 & 3 & 0 \end{array}\right) \xrightarrow{f_3 = f_3 \cdot \frac{1}{3}} \left(\begin{array}{cccc|c} 1 & 0 & 3 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \xrightarrow{f_2 = f_2 - f_3} \left(\begin{array}{cccc|c} 1 & 0 & 3 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \xrightarrow{f_1 = f_1 - 3f_3} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right)$$

$x_1 - 2x_4 = 0$ $\quad$ $x_2 + x_4 = 0$ $\quad$ $x_3 + x_4 = 0$ $\quad \Rightarrow \quad$ $\{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 - 2x_4 = 0 \ \wedge \ x_2 + x_4 = 0 \ \wedge \ x_3 + x_4 = 0\}$

$x_1 = 2x_4$ $\quad$ $x_2 = -x_4$ $\quad$ $x_3 = -x_4$

si tomamos $x_4 = t$, $t \in \mathbb{R}$ $\Rightarrow \{(2t, -t, -t, t) \in \mathbb{R}^4 \mid t \in \mathbb{R}\}$

$\{t(2,-1,-1,1) \mid t \in \mathbb{R}\}$

b) $$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 3 & 4 & -8 & 0 \end{array}\right) \xrightarrow{f_2 - 3f_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right) \xrightarrow{f_1 - f_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right)$$

$x_1 = 0$ $\quad$ $x_2 - 2x_3 = 0$ $\quad$ $\{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 = 0 \ \wedge \ x_2 - 2x_3 = 0\}$

$x_2 = 2x_3$ $\quad$ tomamos $x_3 = t$, $t \in \mathbb{R}$ $\Rightarrow \{(0, 2t, t) \in \mathbb{R}^3 \mid t \in \mathbb{R}\}$

$\{t(0,2,1) \mid t \in \mathbb{R}\}$

c) $$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -1 & -2 & 3 & 0 \\ 1 & 4 & 9 & 0 \end{array}\right) \xrightarrow{f_2 + f_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 4 & 0 \\ 1 & 4 & 9 & 0 \end{array}\right) \xrightarrow{f_3 - f_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 3 & 8 & 0 \end{array}\right) \xrightarrow{f_3 - 3f_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -4 & 0 \end{array}\right) \xrightarrow{f_1 - f_2} \left(\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -4 & 0 \end{array}\right) \xrightarrow{f_3 \cdot \left(-\frac{1}{4}\right)} \left(\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$\xrightarrow[f_1 + 3f_3]{f_2 - 4f_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$\boxed{x_1 = 0 \quad x_2 = 0 \quad x_3 = 0}$

$\{(0,0,0) \in \mathbb{R}^3\}$

(d)

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & -1 & 0 \\ 2 & 3 & -1 & -1 & 0 \\ 5 & 7 & -2 & -3 & 0 \\ 1 & 2 & -1 & 0 & 0 \end{array}\right) \xrightarrow{f_4-f_1} \left(\begin{array}{cccc|c} 1 & 1 & 0 & -1 & 0 \\ 2 & 3 & -1 & -1 & 0 \\ 5 & 7 & -2 & -3 & 0 \\ 0 & 1 & -1 & 1 & 0 \end{array}\right) \xrightarrow[f_2-2f_1]{f_3-5f_1} \left(\begin{array}{cccc|c} 1 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 2 & -2 & 2 & 0 \\ 0 & 1 & -1 & 1 & 0 \end{array}\right) \xrightarrow[f_3-2f_2]{f_1-f_2} \left(\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \end{array}\right) \xrightarrow{f_4-f_2} \left(\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

∴ Hay 2 variables libres (T: hay tantas variables libres como filas nulas)

$x_1 + x_3 - 2x_4 = 0$ $\qquad$ $x_2 - x_3 + x_4 = 0$

$x_1 = -x_3 + 2x_4$ $\qquad$ $x_2 = x_3 - x_4$

$\{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 + x_3 - 2x_4 = 0 \wedge x_2 - x_3 + x_4 = 0\}$

tomamos $x_3 = t$, $x_4 = s$ $t,s \in \mathbb{R} \Rightarrow \{(-t+2s, t-s, t, s) \in \mathbb{R}^4 \mid t,s \in \mathbb{R}\}$

$\{(-t, t, t, 0) + (2s, -s, 0, s) \mid t,s \in \mathbb{R}\}$

$\{t(-1,1,1,0) + s(2,-1,0,1) \mid t,s \in \mathbb{R}\}$

(e)

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 3 & 1 & -1 & -1 & 7 \\ 5 & 0 & -3 & -1 & 5 \end{array}\right) \xrightarrow[f_3-5f_1]{f_2-3f_1} \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 0 & -5 & -4 & 2 & 1 \\ 0 & -10 & -8 & 4 & -5 \end{array}\right) \xrightarrow{f_2 \leftrightarrow f_3} \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 0 & -10 & -8 & 4 & -5 \\ 0 & -5 & -4 & 2 & 1 \end{array}\right) \xrightarrow{f_2\cdot(-\frac{1}{10})} \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 0 & 1 & \frac{8}{10} & -\frac{4}{10} & \frac{5}{10} \\ 0 & -5 & -4 & 2 & 1 \end{array}\right) \xrightarrow[f_3+5f_2]{f_1-2f_2} \left(\begin{array}{cccc|c} 1 & 0 & -\frac{6}{10} & -\frac{2}{10} & 1 \\ 0 & 1 & \frac{8}{10} & -\frac{4}{10} & \frac{5}{10} \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Sist incomp

(f)

$$\left(\begin{array}{cc|c} 1 & 2 & 15 \\ 2 & 4 & -5 \end{array}\right) \xrightarrow{f_2-2f_1} \left(\begin{array}{cc|c} 1 & 2 & 15 \\ 0 & 0 & -35 \end{array}\right)$$

No HAY solucion $\quad$ Sistema :compatible

$0 = -35$, ABS!!!

(g)

$$\left(\begin{array}{ccc|c} 2 & -1 & 1 & 1 \\ 3 & 2 & -4 & 4 \\ -6 & 3 & -3 & 2 \end{array}\right) \xrightarrow{f_1\cdot\frac{1}{2}} \left(\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 3 & 2 & -4 & 4 \\ -6 & 3 & -3 & 2 \end{array}\right) \xrightarrow[f_3+6f_1]{f_2-3f_1} \left(\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{11}{4} & \frac{5}{2} \\ 0 & 0 & 0 & 5 \end{array}\right)$$

No HAY solucion $\quad$ Sistema :compatible

$0 = 5$, ABS!!!

(h)

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & -1 & 1 \\ 2 & 3 & -1 & -1 & 3 \\ 5 & 7 & -2 & -3 & 7 \\ 1 & 2 & -1 & 0 & 2 \end{array}\right) \xrightarrow{\substack{f_2-2f_1 \\ f_3-5f_1 \\ f_4-f_1}} \left(\begin{array}{cccc|c} 1 & 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 2 & -2 & 2 & 2 \\ 0 & 1 & -1 & 1 & 1 \end{array}\right) \xrightarrow{\substack{f_1-f_2 \\ f_3-2f_2 \\ f_4-f_2}} \left(\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 + x_3 - 2x_4 = 0$ $\qquad$ $x_1 = -x_3 + 2x_4$

$x_2 - x_3 + x_4 = 1$ $\qquad$ $x_2 = x_3 - x_4 + 1$

$\{(x_1, x_2, x_3, x_4) \mathbb{R}^4 \mid x_1 + x_3 - 2x_4 = 0 \wedge x_2 - x_3 + x_4 = 1\}$

tomamos $x_3 = t$, $x_4 = s$ $t,s \in \mathbb{R} \Rightarrow \{(-t+2s, t-s+1, t, s) \mid t,s \in \mathbb{R}\}$

$\{(-t, t, t, 0) + (2s, -s, 0, s) + (0,1,0,0) \mid t,s \in \mathbb{R}\}$

$\{t(-1,1,1,0) + s(2,-1,0,1) + (0,1,0,0) \mid t,s \in \mathbb{R}\}$

7 (a)

$$\begin{cases} 2x + 2y + 9z = 4 \\ x + 3y + 10z = 6 \end{cases} \qquad \mathbb{Z}_{11}$$

$$\left(\begin{array}{ccc|c} 2 & 2 & 9 & 4 \\ 1 & 3 & 10 & 6 \end{array}\right) \xrightarrow{f_1\cdot 6} \left(\begin{array}{ccc|c} 1 & 1 & 10 & 2 \\ 1 & 3 & 10 & 6 \end{array}\right) \xrightarrow{f_2-f_1} \left(\begin{array}{ccc|c} 1 & 1 & 10 & 2 \\ 0 & 2 & 0 & 4 \end{array}\right) \xrightarrow{f_2\cdot 6} \left(\begin{array}{ccc|c} 1 & 1 & 10 & 2 \\ 0 & 1 & 0 & 2 \end{array}\right) \xrightarrow{f_1-f_2} \left(\begin{array}{ccc|c} 1 & 0 & 10 & 0 \\ 0 & 1 & 0 & 2 \end{array}\right)$$

$y = 2$

$x - z = 0$

$x = z$

$\{(x, 2, z) \in \mathbb{Z}_{11} \mid x = z\}$

tomamos $z = t$ $t \in \mathbb{Z}_{11} \Rightarrow \{(t, 2, t) \mid t \in \mathbb{Z}_{11}\}$

$\{(t, 0, t) + (0,2,0) \mid t \in \mathbb{Z}_{11}\}$

$\{t(1,0,1) + (0,2,0) \mid t \in \mathbb{Z}_{11}\}$

(b)

$$\begin{cases} x_1 + x_2 + x_3 - x_4 = 2 \\ x_1 + 6x_2 - 2x_3 - 7x_4 = 8 \\ 2x_1 + 2x_2 - 3x_3 - 3x_4 = 5 \\ x_1 + 7x_2 - 10x_3 + 2x_4 = 7 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 2 \\ 1 & 6 & -2 & -7 & 8 \\ 2 & 2 & -3 & -3 & 5 \\ 1 & 7 & -10 & 2 & 7 \end{array}\right) \xrightarrow{\substack{f_2-f_1 \\ f_3-2f_1 \\ f_4-f_1}} \left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 2 \\ 0 & 5 & -3 & -6 & 4 \\ 0 & 0 & -5 & -1 & 1 \\ 0 & 6 & -11 & 3 & 5 \end{array}\right) \xrightarrow{f_2\cdot 9} \left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 2 \\ 0 & 1 & -27 & -54 & 36 \\ 0 & 0 & -5 & -1 & 1 \\ 0 & 6 & 0 & 3 & 5 \end{array}\right) \xrightarrow[f_4-6f_2]{f_1-f_2} \left(\begin{array}{cccc|c} 1 & 0 & 28 & 53 & -34 \\ 0 & 1 & -27 & -54 & 36 \\ 0 & 0 & -5 & -1 & 1 \\ 0 & 0 & -162 & 327 & 216 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 6 & -2 & -1 \\ 0 & 1 & 6 & 1 & 3 \\ 0 & 0 & -5 & -1 & 1 \\ 0 & 0 & 3 & 8 & 7 \end{array}\right) \xrightarrow{f_3\cdot(-9)} \left(\begin{array}{cccc|c} 1 & 0 & 6 & -2 & -1 \\ 0 & 1 & 6 & 1 & 3 \\ 0 & 0 & 1 & 9 & -9 \\ 0 & 0 & 3 & 8 & 7 \end{array}\right) \xrightarrow{\substack{f_1-6f_3 \\ f_2-6f_3 \\ f_4-3f_3}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -56 & 53 \\ 0 & 1 & 0 & -53 & 57 \\ 0 & 0 & 1 & 9 & -9 \\ 0 & 0 & 0 & -19 & 34 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 9 & -9 \\ 0 & 0 & 0 & 3 & 1 \end{array}\right) \xrightarrow{f_3\cdot 4} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 9 & -9 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 9 & -9 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right) \xrightarrow{\substack{f_3-9f_4 \\ f_2-2f_4 \\ f_1+f_4}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -6 \\ 0 & 0 & 1 & 0 & -9 \\ 0 & 0 & 0 & 1 & -45 \end{array}\right)$$

$x_1 = 2$ $\qquad$ $x_1 = 2$

$x_2 = -6$ $\qquad$ $x_2 = -6$

$x_3 = -9 \rightarrow x_3 = 2$

$x_4 = -45 \rightarrow x_4 = -1$

⑧ $\begin{cases} x_1 + x_2 - x_3 + 2x_4 = a \\ 2x_1 + x_2 + x_3 + x_4 = 1 \\ 3x_1 + 2x_2 + 0 + 3x_4 = 0 \\ x_1 - x_2 + x_3 + 2x_4 = 1 \end{cases}$ (*)

$$\left(\begin{array}{cccc|c} 1 & 1 & -1 & 2 & a \\ 2 & 1 & 1 & 1 & 1 \\ 3 & 2 & 0 & 3 & 0 \\ 1 & -1 & 1 & 2 & 1 \end{array}\right) \xrightarrow[\substack{f_3-3f_1 \\ f_4-f_1}]{f_2-2f_1} \left(\begin{array}{cccc|c} 1 & 1 & -1 & 2 & a \\ 0 & -1 & 3 & -3 & 1-2a \\ 0 & -1 & 3 & -3 & -3a \\ 0 & -2 & 2 & 0 & 1-a \end{array}\right) \xrightarrow{f_2\cdot(-1)} \left(\begin{array}{cccc|c} 1 & 1 & -1 & 2 & a \\ 0 & 1 & -3 & 3 & -1+2a \\ 0 & -1 & 3 & -3 & -3a \\ 0 & -2 & 2 & 0 & 1-a \end{array}\right) \xrightarrow[\substack{f_3+f_2 \\ f_4+2f_2}]{f_1-f_2} \left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1-a \\ 0 & 1 & -3 & 3 & -1+2a \\ 0 & 0 & 0 & 0 & -1-a \\ 0 & 0 & -4 & 6 & -1+3a \end{array}\right)$$

$$\xrightarrow{f_3 \leftrightarrow f_4} \left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1-a \\ 0 & 1 & -3 & 3 & -1+2a \\ 0 & 0 & -4 & 6 & -1+3a \\ 0 & 0 & 0 & 0 & -1-a \end{array}\right) \xrightarrow{f_3\cdot(-\frac{1}{4})} \left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1-a \\ 0 & 1 & -3 & 3 & -1+2a \\ 0 & 0 & 1 & -\frac{3}{2} & 4-12a \\ 0 & 0 & 0 & 0 & -1-a \end{array}\right) \xrightarrow[f_2+3f_3]{f_1-2f_3} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -7+23a \\ 0 & 1 & 0 & -\frac{3}{2} & 11-34a \\ 0 & 0 & 1 & -\frac{3}{2} & 4-12a \\ 0 & 0 & 0 & 0 & -1-a \end{array}\right)$$

∴ el sistema tiene solución si $a = -1$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -30 \\ 0 & 1 & 0 & -\frac{3}{2} & 45 \\ 0 & 0 & 1 & -\frac{3}{2} & 16 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 + 2x_4 = -30$ $\quad x_2 - \frac{3}{2}x_4 = 45$ $\quad x_3 - \frac{3}{2}x_4 = 16$

$x_1 = -30 - 2x_4$ $\quad x_2 = 45 + \frac{3}{2}x_4$ $\quad x_3 = 16 + \frac{3}{2}x_4$

$\{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 + 2x_4 = -30 \wedge x_2 - \frac{3}{2}x_4 = 45 \wedge x_3 - \frac{3}{2}x_4 = 16\}$

tomamos $x_4 = t$, $t \in \mathbb{R} \Rightarrow \{(-30-2t, 45+\frac{3}{2}t, 16+\frac{3}{2}t, t) \in \mathbb{R}^4 \mid t \in \mathbb{R}\}$

$\{(-30, 45, 16, 0) + t(-2, \frac{3}{2}, \frac{3}{2}, 1) \in \mathbb{R}^4 \mid t \in \mathbb{R}\}$

⑨ $ax^2 + bx + c$ $\quad (1,2); (2,7); (3,14)$

$\begin{cases} a + b + c = 2 \\ 4a + 2b + c = 7 \\ 9a + 3b + c = 14 \end{cases}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 4 & 2 & 1 & 7 \\ 9 & 3 & 1 & 14 \end{array}\right) \xrightarrow[f_3-9f_1]{f_2-4f_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & -3 & -1 \\ 0 & -6 & -8 & -4 \end{array}\right) \xrightarrow{f_2\cdot(-\frac{1}{2})} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & -6 & -8 & -4 \end{array}\right) \xrightarrow[f_3+6f_2]{f_1-f_2} \left(\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 0 & 1 & -1 \end{array}\right) \xrightarrow[f_2-\frac{3}{2}f_3]{f_1+f_3\cdot\frac{1}{2}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$\boxed{a = 1 \quad b = 2 \quad c = -1}$

⑩

Falta 10 y controlar las (*)