

① $A + 3B - C$

$$\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} + 3 \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} - \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} + \begin{pmatrix} 3 & 3 & 6 \\ -6 & 0 & -3 \\ 3 & 9 & 15 \end{pmatrix} - \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 1 & 6 \\ -5 & -2 & -2 \\ 4 & 7 & 14 \end{pmatrix} - \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 5 \\ -7 & -2 & -3 \\ 1 & 7 & 13 \end{pmatrix}$$

$-A + B + 2C$

$$\begin{pmatrix} -1 & 2 & 0 \\ -1 & 2 & -1 \\ -1 & 2 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} + 2 \begin{pmatrix} 2 & -2 & 2 \\ 4 & 0 & 2 \\ 6 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 2 \\ -3 & 2 & -2 \\ 0 & 5 & 6 \end{pmatrix} + \begin{pmatrix} 2 & -2 & 2 \\ 4 & 0 & 2 \\ 6 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 4 \\ 1 & 2 & 0 \\ 6 & 5 & 8 \end{pmatrix}$$

AB

$$\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} 5 & 1 & 4 \\ 6 & 4 & 9 \\ 4 & -2 & -1 \end{pmatrix}$$

BA

$$\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} \times \begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 4 & -8 & -1 \\ -3 & 6 & 1 \\ 9 & -18 & -2 \end{pmatrix}$$

AC

$$\begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -3 & -1 & -1 \\ 0 & -1 & 0 \\ -6 & -1 & -2 \end{pmatrix}$$

CA

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -2 \\ 3 & -6 & -1 \\ 4 & -8 & -1 \end{pmatrix}$$

BC

$$\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} \times \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 9 & -1 & 4 \\ -5 & 2 & -3 \\ 22 & -1 & 9 \end{pmatrix}$$

CB

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 8 \\ 3 & 5 & 9 \\ 4 & 6 & 11 \end{pmatrix}$$

ABC

$$\begin{pmatrix} 5 & 1 & 4 \\ 6 & 4 & 9 \\ 4 & -2 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 19 & -5 & 10 \\ 41 & -6 & 19 \\ -3 & -4 & 1 \end{pmatrix}$$

ACB

$$\begin{pmatrix} -3 & -1 & -1 \\ 0 & -1 & 0 \\ -6 & -1 & -2 \end{pmatrix} \times \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} -2 & -6 & -10 \\ 2 & 0 & 1 \\ -6 & -12 & -21 \end{pmatrix}$$

BAC

$$\begin{pmatrix} 4 & -8 & -1 \\ -3 & 6 & 1 \\ 9 & -18 & -2 \end{pmatrix} \times \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -15 & -4 & -9 \\ 12 & 3 & 4 \\ -33 & -9 & -11 \end{pmatrix}$$

BCA

$$\begin{pmatrix} 9 & -1 & 4 \\ -5 & 2 & -3 \\ 22 & -1 & 9 \end{pmatrix} \times \begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 12 & -24 & -5 \\ -6 & 12 & 5 \\ 30 & -60 & -10 \end{pmatrix}$$

CAB

$$\begin{pmatrix} 1 & -2 & -2 \\ 3 & -6 & -1 \\ 4 & -8 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} 3 & -5 & -6 \\ 14 & 0 & 7 \\ 19 & 1 & 11 \end{pmatrix}$$

CBA

$$\begin{pmatrix} 4 & 4 & 8 \\ 3 & 5 & 9 \\ 4 & 6 & 11 \end{pmatrix} \times \begin{pmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -2 & -1 \end{pmatrix} = \begin{pmatrix} 16 & -32 & -4 \\ 17 & -34 & -4 \\ 21 & -42 & -5 \end{pmatrix}$$

2) $A = 3B - C, -A + B + 2C, \textcircled{AB}, \textcircled{BA}, \textcircled{CA}, \textcircled{BC}, \textcircled{CB}, \textcircled{ABC}, \textcircled{ACB}, \textcircled{BCA}, \textcircled{CAB} \text{ y } \textcircled{CBA}$

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \quad C = (1, -1) \quad A^{2 \times 3} \quad B^{3 \times 1} \quad C^{1 \times 2}$$

AB

$$\begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix}$$

CA

$$(1, -1) \times \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix} = (1 \ -3 \ 0)$$

BC

$$\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \times (1, -1) = \begin{pmatrix} 3 & -3 \\ 1 & -1 \\ -1 & 1 \end{pmatrix}$$

ABC

$$\begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix} \times (1, -1) = \begin{pmatrix} 4 & -4 \\ 4 & -4 \\ -1 & 1 \end{pmatrix}$$

BCA

$$\begin{pmatrix} 3 & -3 \\ 1 & -1 \\ -1 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -9 & 0 \\ 1 & -3 & 0 \\ -1 & 3 & 0 \end{pmatrix}$$

CAB

$$(1 \ -3 \ 0) \times \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = 0$$

3) a) $A^2 = 0$

$$A \in K^{2 \times 2} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$A \in K^{3 \times 3} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

b) $A^2 = -I_{d_2}$

$$A \in \mathbb{R}^{2 \times 2} \quad \textcircled{\times} \quad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = -I_{d_2} \Rightarrow \begin{pmatrix} a^2+bc & ab+bd \\ ca+dc & cb+d^2 \end{pmatrix} = -I_{d_2}$$

$$\begin{cases} a^2+bc = -1 \\ cb+d^2 = -1 \\ ab+bd = 0 \\ ca+dc = 0 \end{cases} \Rightarrow \begin{cases} b(a+d) = 0 \\ c(a+d) = 0 \end{cases} \Rightarrow \begin{cases} b \vee (a+d) = 0 \\ c \vee (a+d) = 0 \end{cases}$$

$\begin{matrix} 0 & a \\ -a & 0 \end{matrix}$

Si $b=0 \Rightarrow c \cdot 0 + d^2 = -1$ (No possible en \mathbb{R})
Si $c=0 \Rightarrow a^2 + 0 = -1$ (No possible en \mathbb{R})
 $\therefore a+d=0$

c) $AB \neq BA$ (Cuadradas)

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad BA = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

d) $A \in K^{n \times n}, A \neq 0, A \neq I_{d_n} \text{ y } A^2 = A$

$$A \in K^{2 \times 2}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad A \in K^{n \times n} \begin{pmatrix} 1 & \dots & 0 \\ 0 & \dots & 0 \\ \vdots & & \\ 0 & \dots & 1 \end{pmatrix}$$

$$a+d=0$$

$$a=1 \quad d=-1$$

$$bc = -2 \quad b=2$$

$$b+b= \quad c=-1$$

e) $A, B \in K^{n \times n}$ dar condiciones para que se cumpla:

i) $(A+B)^2 = A^2 + 2AB + B^2$

ii) $(A+B)(A-B) = A^2 - B^2$

i) $(A+B)^2 = (A+B)(A+B) \Rightarrow$

$$(A+B)(A+B) = A^2 + 2AB + B^2$$

$$A^2 + AB + BA + B^2 = A^2 + 2AB + B^2 \Leftrightarrow AB = BA$$

ii) $(A+B)(A-B) = A^2 - B^2$

$$A^2 - AB + BA + B^2 = A^2 - B^2 \Leftrightarrow AB = BA$$

4) a) $\text{tr}(A+B) = \text{tr}(A) + \text{tr}(B)$

$$\sum_{i=1}^n A_{ii} + \sum_{i=1}^n B_{ii}$$

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$$= \text{tr}(A) + \text{tr}(B)$$

b) $\text{tr}(AB) = \text{tr}(BA)$

$$\text{tr}(AB) = \sum_{i=1}^n (AB)_{ii} = \sum_{i=1}^n \left(\sum_{k=1}^n A_{ik} B_{ki} \right)$$

$$= \sum_{k=1}^n \left(\sum_{i=1}^n B_{ki} A_{ik} \right)$$

$$= \sum_{k=1}^n B_{kk} A_{kk}$$

$$= \text{tr}(BA)$$

5) a) Si $A, B \in K^{n \times n}$ son matrices diagonales, entonces $AB = BA$

$$A = \begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & 0 & \dots & 0 \\ 0 & b_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & b_{nn} \end{pmatrix}$$


$$AB = \begin{pmatrix} a_{11}b_{11} & 0 & \dots & 0 \\ 0 & a_{22}b_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{nn}b_{nn} \end{pmatrix} \quad BA = \begin{pmatrix} b_{11}a_{11} & 0 & \dots & 0 \\ 0 & b_{22}a_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & b_{nn}a_{nn} \end{pmatrix}$$

Como el producto de escalares es conmutativo, $a_{ii}b_{ii} = b_{ii}a_{ii}$

6) Si A es un múltiplo escalar de la matriz identidad ($A = c \cdot I_n$ para algún $c \in k$) entonces $AB = BA \quad \forall B \in k^{n \times n}$

$$A = c \cdot I_n \quad AB = c \cdot I_n \cdot B \quad \therefore I_n \cdot B = B \Rightarrow AB = c \cdot B \quad \therefore AB = BA \quad \forall B \in k^{n \times n}$$

$$BA = B \cdot c \cdot I_n$$

7) Si $A \in k^{n \times n}$ cumple $AB = BA$ para toda $B \in k^{n \times n}$, entonces A es múltiplo escalar de I_n 

$$I_{1,0} \in Z(k^{n \times n}) = \{AB = BA \quad \forall B \in k^{n \times n}\} = \{\lambda I_n \mid \lambda \in k\}$$

6) 6)

Suponemos que $AB = I_3$

$B \in k^{3 \times 3}$, expandimos a la id y calculamos.

$$\left(\begin{array}{ccc|ccc} 3 & -1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{array} \right) \xrightarrow{f_1 - f_2} \left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & -1 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{f_2 - 2f_1 \\ f_3 - f_1}} \left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & -1 & 0 \\ 0 & 5 & -1 & -2 & 3 & 0 \\ 0 & -1 & -1 & -1 & 1 & 1 \end{array} \right) \xrightarrow{\substack{f_2 \cdot (-1) \\ f_3 \cdot (-1)}} \left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 1 & -1 & -1 \\ 0 & 5 & -1 & -2 & 3 & 0 \end{array} \right)$$

$$\xrightarrow{\substack{f_1 + 2f_2 \\ f_3 - 5f_2}} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 3 & -3 & -2 \\ 0 & 1 & 1 & 1 & -1 & -1 \\ 0 & 0 & -6 & -7 & 8 & 5 \end{array} \right) \xrightarrow{f_3 \cdot (-\frac{1}{6})} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 3 & -3 & -2 \\ 0 & 1 & 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & \frac{7}{6} & -\frac{4}{3} & -\frac{5}{6} \end{array} \right) \xrightarrow{\substack{f_1 - 3f_3 \\ f_2 - f_3}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \\ 0 & 0 & 1 & \frac{7}{6} & -\frac{4}{3} & -\frac{5}{6} \end{array} \right)$$

Llegamos a que A es una matriz equivalente por filas, denotamos $B = A^{-1}$

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \\ \frac{7}{6} & -\frac{4}{3} & -\frac{5}{6} \end{pmatrix}$$

6) Suponemos que $AB = I_3$

$B \in k^{3 \times 3}$, expandimos a la id y calculamos.

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{f_1 - f_2 \\ f_2 - f_3}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right)$$

Llegamos a que A es una matriz equivalente por filas, denotamos $B = A^{-1}$

$$A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

7) Suponemos que $AB = I_3$

$B \in k^{3 \times 3}$, expandimos a la id y calculamos.

$$\left(\begin{array}{ccc|ccc} 2 & 5 & -1 & 1 & 0 & 0 \\ 4 & -1 & 2 & 0 & 1 & 0 \\ 6 & 4 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{f_1 \cdot (\frac{1}{2})} \left(\begin{array}{ccc|ccc} 1 & \frac{5}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 4 & -1 & 2 & 0 & 1 & 0 \\ 6 & 4 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{f_2 - 4f_1 \\ f_3 - 6f_1}} \left(\begin{array}{ccc|ccc} 1 & \frac{5}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -11 & 4 & -2 & 1 & 0 \\ 0 & -11 & 4 & -3 & 0 & 1 \end{array} \right)$$

Llegamos a un ABS. \therefore no es inv.

7) Suponemos que $AB = I_4$

$B \in k^{4 \times 4}$, expandimos a la id y calculamos.

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 1 & -3 & 3 & -8 & 0 & 1 & 0 & 0 \\ -2 & 1 & 2 & -2 & 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 4 & 0 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{f_2 - f_1 \\ f_3 + 2f_1 \\ f_4 - f_1}} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & -4 & 2 & -10 & -1 & 1 & 0 & 0 \\ 0 & 3 & 4 & 2 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{f_2 \cdot (-\frac{1}{4})} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 3 & 4 & 2 & 2 & 0 & 1 & 0 \\ 0 & -4 & 2 & -10 & -1 & 1 & 0 & 0 \end{array} \right)$$

$$\xrightarrow{\substack{f_1 - f_2 \\ f_3 - 3f_2 \\ f_4 + 4f_2}} \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & -1 & 1 & -2 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 0 & 4 & -4 & 5 & 0 & 1 & -3 \\ 0 & 0 & 2 & -2 & 3 & -3 & 4 & -8 \end{array} \right) \xrightarrow{f_3 \cdot (\frac{1}{4})} \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & -1 & 1 & -2 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & \frac{5}{4} & 0 & \frac{1}{4} & -\frac{3}{4} \\ 0 & 0 & 2 & -2 & 3 & -3 & 4 & -8 \end{array} \right) \xrightarrow{f_4 \cdot (\frac{1}{2})} \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & -1 & 1 & -2 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & \frac{5}{4} & 0 & \frac{1}{4} & -\frac{3}{4} \\ 0 & 0 & 1 & -1 & \frac{3}{2} & -\frac{3}{2} & 2 & -4 \end{array} \right)$$

Llegamos a un ABS. \therefore No es inv

7)

7) Suponemos que $AB = I_3$

$B \in k^{3 \times 3}$, expandimos a la id y calculamos.

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{f_2 - 2f_1 \\ f_3 - 3f_1}} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -3 & -5 & -2 & 1 & 0 \\ 0 & -6 & -2 & -3 & 0 & 1 \end{array} \right) \xrightarrow{f_2 \cdot (-\frac{1}{3})} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{3} & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & -6 & -2 & -3 & 0 & 1 \end{array} \right) \xrightarrow{\substack{f_1 - 2f_2 \\ f_3 + 6f_2}} \left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{7}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 1 & \frac{5}{3} & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 7 & 4 & -2 & 1 \end{array} \right)$$

$$\xrightarrow{3 \cdot} \left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{7}{3} & -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 1 & \frac{5}{3} & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & \frac{7}{3} & -\frac{2}{3} & \frac{1}{3} \end{array} \right) \xrightarrow{\substack{f_1 + \frac{7}{3}f_3 \\ f_2 - \frac{5}{3}f_3}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{152}{15} + \frac{7}{30}i & \frac{2}{3} & \frac{49}{150} - \frac{7}{150}i \\ 0 & 1 & 0 & \frac{23}{3} - \frac{1}{10}i & -\frac{1}{3} & -\frac{7}{30} + \frac{1}{30}i \\ 0 & 0 & 1 & -\frac{21}{5} + \frac{3}{30}i & 0 & \frac{7}{30} - \frac{1}{50}i \end{array} \right)$$

Llegamos a que A es una matriz equivalente por filas, denotamos $B = A^{-1}$

$$A^{-1} = \begin{pmatrix} -\frac{152}{15} + \frac{7}{30}i & \frac{2}{3} & \frac{49}{150} - \frac{7}{150}i \\ \frac{23}{3} - \frac{1}{10}i & -\frac{1}{3} & -\frac{7}{30} + \frac{1}{30}i \\ -\frac{21}{5} + \frac{3}{30}i & 0 & \frac{7}{30} - \frac{1}{50}i \end{pmatrix}$$

(7+i). $x = 1$

$$x = \frac{1}{7+i} \cdot \frac{7-i}{7-i}$$

$$x = \frac{7-i}{49-i^2}$$

$$x = \frac{7-i}{50} = \frac{7}{50} - \frac{1}{50}i \quad 6)$$

Falla ⑧ ⑨ y ⑩