GUIA 3

① ⓐ $\lim_{x\to 4}(5x^2-2x+3) =$

X	Y
3,8	67,6
3,99	74,62
3,999	74,96
4,1	78,85
4,01	75,38
4,001	75,038

ⓑ $\lim_{x\to -1}\frac{x^3+1}{x+1} =$

X	Y
-1,1	3,31
-1,01	3,03
-1,001	3,003
-0,9	2,71
-0,99	2,97
-0,999	2,997

ⓒ $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x} =$

X	Y
0,1	0,48
0,01	0,49
0,001	0,499
-0,1	0,51
-0,01	0,501
-0,001	0,5001

③ $f(x)\begin{cases} |x| & \text{si } x\leq 0 \\ 9-(x-3)^2 & \text{si } 0<x<4 \\ -1 & \text{si } x\geq 4 \end{cases}$

ⓐ $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 9-(0-3)^2 = 9-9=0$

ⓑ $\lim_{x\to 0^-} f(x) = |x| = 0$

ⓒ $\lim_{x\to 0} f(x) = 0$

ⓓ $\lim_{x\to 4^+} f(x) = -1$

ⓔ $\lim_{x\to 4^-} f(x) = 9-(4-3)^2 = 8$

ⓕ $\lim_{x\to 4} f(x) = \nexists$

④ $g(x)\begin{cases} \frac{1}{x^2} & \text{si } |x|>1 \\ -x & \text{si } |x|<1 \\ 2 & \text{si } |x|=1 \end{cases}$

ⓐ $\lim_{x\to 1^+} g(x) = \frac{1}{1^2} = 1$

ⓑ $\lim_{x\to 1^-} g(x) = -1$

ⓒ $\lim_{x\to 1} g(x) = \nexists$

ⓓ $\lim_{x\to -1^+} g(x) = 1$

ⓔ $\lim_{x\to -1^-} g(x) = \frac{1}{(-1)^2} = 1$

ⓕ $\lim_{x\to 1} g(x) = 1$

⑤ ⓐ $\lim_{h\to 0^-}\frac{\sqrt{h^2}}{h} = \lim_{h\to 0^-}\frac{|h|}{h} = \lim_{h\to 0^-}\frac{-h}{h} = -1$ $\quad |h|\begin{cases} h & \text{si } x\geq 0 \\ -h & \text{si } x<0 \end{cases}$

ⓑ $\lim_{x\to 2^+}\frac{x-2}{|x-2|} = \lim_{x\to 2^+}\frac{x-2}{x-2} = 1$ $\quad |x-2|\begin{cases} x-2 & \text{si } x\geq 2 \\ -(x-2) & \text{si } x<2 \end{cases}$

ⓒ $\lim_{x\to -2}\frac{2-|x|}{2+x} = \lim_{x\to -2}\frac{2+x}{2+x} = 1$ $\quad |x|\begin{cases} x & \text{si } x\geq 0 \\ -x & \text{si } x<0 \end{cases}$

ⓓ $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{|x|}\right) = \lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x}\right) = 0$ $\quad |x|\begin{cases} x & \text{si } x\geq 0 \\ -x & \text{si } x<0 \end{cases}$

ⓔ $\lim_{x\to 0^-}\left(\frac{1}{x}-\frac{1}{|x|}\right) = \lim_{x\to 0^-}\left(\frac{1}{x}-\frac{1}{-x}\right) = \lim_{x\to 0^-}\left(\frac{1}{x}+\frac{1}{x}\right) = \lim_{x\to 0^-}\frac{2}{x} = -\infty$

ⓕ $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{|x|}\right) = \nexists$

⑧ ⓐ $y=\frac{x}{x+4}$ $\quad \lim_{x\to\infty}\frac{x}{x+4} = \lim_{x\to\infty}\frac{x}{x\left(1+\frac{4}{x}\right)} = \lim_{x\to\infty}\frac{1}{1+\frac{4}{x}} = 1$

$x+4=0$

$x=-4$

Posibles AV: -4 $\quad$ AH: 1

ⓑ $y=\frac{x^2+4}{x^2-1}$ $\quad \lim_{x\to\infty}\frac{x^2+4}{x^2-1} = \lim_{x\to\infty}\frac{x^2\left(1+\frac{4}{x^2}\right)}{x^2\left(1-\frac{1}{x^2}\right)} = \lim_{x\to\infty}\frac{1+\frac{4}{x^2}}{1-\frac{1}{x^2}} = 1$

$x^2-1=0$

$x^2=1$

$x=\pm\sqrt{1}$

$x=\pm 1$

Posibles AV, ± 1 $\quad$ AH: 1

ⓒ $y=\frac{x^3+1}{x^3+x}$ $\quad \lim_{x\to\infty}\frac{x^3+1}{x^3+x} = \lim_{x\to\infty}\frac{x^3\left(1+\frac{1}{x^3}\right)}{x^3\left(1+\frac{1}{x^2}\right)} = \lim_{x\to\infty}\frac{1+\frac{1}{x^3}}{1+\frac{1}{x^2}} = 1$

$x^3+x=0$

$x=0$

Posibles AV: 0 $\quad$ AH: 1

⑪ ⓐ $\lim_{x\to\infty} 5^{2x+5} = \infty$

ⓑ $\lim_{x\to -\infty}\pi^{-x} = \infty$

ⓒ $\lim_{x\to 2^+} 4^{\frac{x}{2-x}} = 4^{-\infty}$

ⓓ $\lim_{x\to 0^-} 3^{\frac{1}{x}}$

ⓔ $\lim_{x\to\infty}\ln\left(\frac{x}{x^2+3}\right) = \lim_{x\to\infty}\ln\left(\frac{x}{x\left(x+\frac{3}{x}\right)}\right) = \lim_{x\to\infty}\ln\left(\frac{1}{x+\frac{3}{x}}\right) = \lim_{x\to\infty}\ln(0) = -\infty$

ⓕ $\lim_{x\to 3}\ln\left(\frac{9-x^2}{x-3}\right)$

⑫ ⓐ $\lim_{x\to\infty}\left(1+\frac{1}{4x}\right)^{4x+3} = \lim_{x\to\infty}\left(1+\frac{1}{4x}\right)^{4x}\cdot\lim_{x\to\infty}\left(1+\frac{1}{4x}\right)^{3} = e\cdot 1 = e$ //

t amo

ⓑ $\lim_{x\to 0}(1+2x)^{1/3x} = \lim_{x\to 0}\left((1+2x)^{1/2x}\right)^{2/3} = e^{2/3}$

ⓒ $\lim_{x\to +\infty}\left(1+\frac{7}{5x}\right)^{2x} = \lim_{x\to\infty}\left(1+\frac{\frac{1}{7}\cdot 7}{5x\cdot\frac{1}{7}}\right)^{2x} = \lim_{x\to\infty}\left(1+\frac{1}{\frac{5x}{7}}\right)^{\frac{5x}{7}\cdot\frac{14}{5}} = \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{t\,\frac{14}{5}} = e^{\frac{14}{5}}$

ⓓ $\lim_{x\to 0}(1+34x)^{3+\frac{2}{x}} = \lim_{x\to 0}(1+34x)^{\frac{3x+2}{x}} = \lim(1+34x)^{\frac{1}{x}\cdot 3x+2} =$

ⓔ $\lim_{x\to 0^+}\ln(\sqrt[4]{x}) = \lim_{x\to 0^+}\ln\left(x^{\frac{1}{4}}\right) = \lim_{x\to 0^+}\frac{1}{4}\ln(x) = -\infty$

ⓕ $\lim_{x\to 0}\ln\left((1+x)^{3/x}\right) = \lim_{x\to 0}\frac{3}{x}\ln(1+x) = \frac{3}{x}\cdot x = 3$

② ⓐ $\lim_{x\to 4}(5x^2-2x+3) = 5\cdot 4^2-2\cdot 4+3 = 80-8+3 = 75$

ⓑ $\lim_{s\to 2}(s^2+1)(s^2+4s) = (4+1)(4+8) = 5\cdot 12 = 60$

ⓒ $\lim_{t\to -1}\frac{\sqrt{t^3+3t^2+7}}{t+2} = \frac{\sqrt{-1+3+7}}{-1+2} = 3$

ⓓ $\lim_{x\to 1}\frac{x^3+1}{x+1} = 1$

ⓔ $\lim_{x\to -1}\frac{x^3+1}{x+1} = \lim_{x\to -1}(x^2-x+1) = (-1)^2-(-1)+1 = 3$

$x^3+0x^2+0x+1 \;\lfloor\; x+1$

$-x^3+x^2 \qquad x^2-x+1$

$0-x^2$

$-\;-x^2-x$

$0+0+x+1$

$-\;x+1$

$0+0$

ⓕ $\lim_{x\to 4}\frac{x^2-6x+8}{x^2-5x+4} = \lim_{x\to 4}\frac{(x-4)(x-2)}{(x-4)(x-1)} = \lim_{x\to 4}\frac{x-2}{x-1} = \frac{2}{3} \to$

$\frac{6\pm\sqrt{(-6)^2-4\cdot 1\cdot 8}}{2} = \frac{6\pm 2}{2} = \begin{cases} x_1=4 \\ x_2=2 \end{cases}$

$\frac{5\pm\sqrt{(-5)^2-4\cdot 1\cdot 4}}{2} = \frac{5\pm 3}{2} = \begin{cases} x_1=4 \\ x_2=1 \end{cases}$

ⓖ $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x} = \lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}\cdot\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} = \lim_{x\to 0}\frac{(\sqrt{1+x}-1)\cdot(\sqrt{1+x}+1)}{x(\sqrt{1+x}+1)} = \lim_{x\to 0}\frac{1+x-\sqrt{1+x}+\sqrt{1+x}-1}{x(\sqrt{1+x}+1)}$

$= \lim_{x\to 0}\frac{1+x-1}{x(\sqrt{1+x}+1)} = \lim_{x\to 0}\frac{1}{\sqrt{1+x}+1} = \frac{1}{2}$

⑥ ⓐ $\lim_{x\to -1} f(x)$ sabiendo que $1\leq f(x)\leq x^2+2x+2$

$\lim_{x\to -1} 1 \leq \lim_{x\to -1} f(x) \leq \lim_{x\to -1} x^2+2x+2$

$1\leq \lim_{x\to -1} f(x) \leq 1$

ⓑ $\lim_{x\to 1} f(x)$ sabiendo que $3x\leq f(x)\leq x^2+x$

$\lim_{x\to 1} 3x \leq \lim_{x\to 1} f(x) \leq \lim_{x\to 1} x^2+2$

$3\leq \lim_{x\to 1} f(x) \leq 3$

⑦ ⓐ $\lim_{x\to\infty}\frac{x^3+1}{x+1} = \lim_{x\to\infty}\frac{x(x^2+1)}{x\left(1+\frac{1}{x}\right)} = \lim_{x\to\infty}\frac{x^2+1}{1+\frac{1}{x}} = \frac{x^2+1}{1+0} = \infty$

ⓑ $\lim_{x\to\infty}\frac{x^2+x}{1+3x^2} = \lim_{x\to\infty}\frac{x^2\left(1+\frac{1}{x}\right)}{x^2\left(\frac{1}{x^2}+3\right)} = \lim_{x\to\infty}\frac{1+\frac{1}{x}}{\frac{1}{x^2}+3} = \frac{1+0}{0+3} = \frac{1}{3}$

ⓒ $\lim_{x\to\infty}\frac{x^2+x+1}{2+2x^2+9x^4} = \lim_{x\to\infty}\frac{x^4\left(\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}\right)}{x^4\left(\frac{2}{x^4}+\frac{2}{x^2}+9\right)} = \lim_{x\to\infty}\frac{\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}}{\frac{2}{x^4}+\frac{2}{x^2}+9} = 0$

ⓓ $\lim_{x\to\infty}\left(\sqrt{18x^2+1}\;\frac{1}{\sqrt{32x^2-3}}\right) = \lim_{x\to\infty}\frac{\sqrt{18x^2+1}}{\sqrt{32x^2-3}} = \lim_{x\to\infty}\frac{x\sqrt{18+\frac{1}{x^2}}}{x\sqrt{32-\frac{3}{x^2}}} = \frac{\sqrt{18}}{\sqrt{32}}$

⑨ ⓐ $\lim_{x\to 1} g(x) = 1$ ⓔ $\lim_{x\to 2} g(x) = 0$ ⓘ $\lim_{x\to 6} g(x) = \nexists$ ⓜ $\lim_{x\to -2} g(x) = \nexists$ ⓟ $\lim_{x\to 0} g(x) = \nexists$

ⓑ $\lim_{x\to 1^+} g(x) = 1$ ⓕ $\lim_{x\to 2^+} g(x) = 0$ ⓙ $\lim_{x\to 6^+} g(x) = 2$ ⓝ $\lim_{x\to -2^+} g(x) = 4$ ⓠ $\lim_{x\to 0^+} g(x) = -\infty$

ⓒ $\lim_{x\to 1^-} g(x) = 1$ ⓖ $\lim_{x\to 2^-} g(x) = 0$ ⓚ $\lim_{x\to 6^-} g(x) = 4$ ⓝ $\lim_{x\to -2^-} g(x) = 4$ ⓡ $\lim_{x\to 0^-} g(x) = \infty$

ⓓ $g(1) = 1$ ⓗ $g(2) = 0$ ⓛ $g(6) = 2$ ⓞ $g(-2) = 2$ ⓢ $g(0) = \nexists$

⑩ ⓐ $\lim_{x\to 0}\frac{(1+x)^2-1}{x} = \lim_{x\to 0}\frac{x^2+2x+1-1}{x} = \lim_{x\to 0}\frac{x^2+2x}{x} = \lim_{x\to 0}\frac{x(x+2)}{x} = \lim_{x\to 0} x+2 = 2$

ⓑ $\lim_{x\to 3}\frac{(x^2-9)}{x^2-5x+9} = \frac{0}{3} = 0$

ⓒ $\lim_{x\to 0}\left(\frac{\operatorname{sen}(x)}{\operatorname{sen}(5x)}\right)^2 = \lim_{x\to 0}\frac{\operatorname{sen}(x)}{1}\cdot\frac{x}{x}\cdot\frac{1}{\operatorname{sen}(5x)}\cdot\frac{5}{1} = \lim_{x\to 0}\overbrace{\frac{\operatorname{sen}(x)}{x}}^{1}\cdot\overbrace{\frac{5x}{\operatorname{sen}(5x)}}^{1}\cdot\frac{1}{5} = \frac{1}{5}$

ⓓ $\lim_{r\to\infty} r\cdot\operatorname{sen}\frac{1}{r} = \lim_{u\to 0}\frac{1}{u}\cdot\operatorname{sen}(u) = \lim_{u\to 0}\frac{\operatorname{sen}(u)}{u} = 1$

ⓔ $\lim_{x\to 0}\frac{\tan 3x}{x} = \lim_{x\to 0}\frac{\tan 3x}{3x}\cdot 3 = 3$

ⓕ $\lim_{x\to 0^+}\frac{1-\cos(x)}{x^2} =$

Material Extra

(13) (a) $\lim_{x\to a} f(x) = \nexists$ y $\lim_{x\to a} g(x) = \nexists \Rightarrow \lim (f+g)(x) = \exists$, $\Rightarrow \lim (f\cdot g)(x) = \exists$

$\lim_{x\to a} (f+g)(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = \nexists$

$\lim (f\cdot g)(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = \nexists$

(b) $\lim_{x\to a} f(x) = L$ y $\lim_{x\to a} (f+g)(x) = M \Rightarrow \lim_{x\to a} g(x) = \exists$

Si, $\lim_{x\to a} g(x) = \lim_{x\to a} (f+g)(x) - \lim_{x\to a} f(x)$

(c) $\lim_{x\to a} f(x) = L$ y $\lim_{x\to a} g(x) = \nexists \Rightarrow \lim (f+g)(x) = \exists$

No, porque incluye la suma de un termino que no existe

(d) $\lim_{x\to a} f(x) = L$ y $\lim_{x\to a} (f\cdot g)(x) = M \Rightarrow \lim_{x\to a} g(x) = \exists$

No necesariamente, puede ser que $g(x)$ tenga un comportamiento que el producto tenga un limite pero g no.

Ejemplo: $f(x) = x$ $g(x) = \frac{1}{x}$

(14) $V(t) = 1000 \cdot \frac{\sqrt{t+3} - 2}{t-1}$

$\lim_{t\to 1} V(t) = 1000 \cdot \frac{\sqrt{t+3} - 2}{t-1} = \lim_{t\to 1} 1000 \cdot \lim_{t\to 1} \frac{\sqrt{t+3} - 2}{t-1}$

$= 1000 \lim_{t\to 1} \frac{\sqrt{t+3} - 2}{t-1} \overset{L'H}{=} 1000 \cdot \lim_{t\to 1} \frac{\sqrt{t+3} - 2}{t-1} \cdot \frac{\sqrt{t+3} + 2}{\sqrt{t+3} + 2} =$

$1000 \cdot \lim_{t\to 1} \frac{t+3-4}{(t-1)(\sqrt{t+3}+2)} = 1000 \lim_{t\to 1} \frac{\cancel{t-1}}{\cancel{(t-1)}(\sqrt{t+3}+2)} =$

$1000 \cdot \lim_{t\to 1} \frac{1}{\sqrt{t+3}+2} = 1000 \cdot \frac{1}{4} = 250$

Rta: El valor se aproxima a $250 m^3$

(16) $y(x) = \frac{4}{2+8e^{-2x}}$

(a) $y(0) = \frac{4}{10} \cdot 1.000.000$

$= \frac{4.000.000}{10}$

$= 400.000$

Rta: La poblacion inicial es de 400.000

(b) $\lim_{x\to\infty} y(x) = \lim_{x\to\infty} \frac{4}{2+8e^{-2x}} = \lim_{x\to\infty} \frac{4}{2+8\frac{1}{e^{2x}}}$

$= \frac{4}{2} = 2$

Rta: La poblacion tiende a estabilizarse en el valor 2.000.000

(15) $f(t) = \frac{10t}{t^2+1}$

$\lim_{x\to\infty} f(t) = \lim_{x\to\infty} \frac{10t}{t^2+1} = \lim_{x\to\infty} \frac{10t}{t(t+\frac{1}{t})} = \lim \frac{10}{t+\frac{1}{t}} = 0$