Guia 5

1 a) $f(x)=5x+3$

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{5(x+h)+3-5x-3}{h}$

$=\lim_{h\to 0}\frac{5x+5h+3-5x-3}{h}$

$=\lim_{h\to 0}\frac{5h}{h}=5$

b) $f(x)=x^3-x^2+2x$

$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{(x+h)^3-(x+h)^2+2(x+h)-x^3-x^2+2x}{h}$

$=\lim_{h\to 0}\frac{(x+h)(x+h)(x+h)-(x^2+2xh+h^2)+2x+2h-x^3-x^2+2x}{h}$

$=\lim_{h\to 0}\frac{(x^2+xh+hx+h^2)(x+h)-x^2-2xh-h^2+2x+2h-x^3-x^2+2x}{h}$

$=\lim_{h\to 0}\frac{\cancel{x^3}+x^2h+x^2h+xh^2+hx^2+h^2x+h^2x+h^3\cancel{-x^2}-2xh-h^2\cancel{+2x}+2h\cancel{-x^3}\cancel{+x^2}\cancel{-2x}}{h}$

$=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3-2xh-h^2+2h}{h}$

$=\lim_{h\to 0}\frac{h(3x^2+3xh+h^2-2x-h+2)}{h}$

$=\lim_{h\to 0}3x^2+3xh+h^2-2x-h+2$

$=\lim_{h\to 0}3x^2-2x+2$ //

c) $f(x)=\sqrt{6-x}$

$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{\sqrt{6-(x+h)}-\sqrt{6-x}}{h}$

$=\lim_{h\to 0}\frac{\sqrt{6-(x+h)}-\sqrt{6-x}}{h}\cdot\frac{\sqrt{6-(x+h)}+\sqrt{6-x}}{\sqrt{6-(x+h)}+\sqrt{6-x}}$

$=\lim_{h\to 0}\frac{6-(x+h)-6-x}{h\left(\sqrt{6-(x+h)}+\sqrt{6-x}\right)}$

$=\lim_{h\to 0}\frac{-h}{h\left(\sqrt{6-(x+h)}+\sqrt{6-x}\right)}$

$=\lim_{h\to 0}\frac{-1}{\sqrt{6-(x+h)}+\sqrt{6-x}}$

$=\lim_{h\to 0}\frac{-1}{2\sqrt{6-x}}$

2) $f(x)=\begin{cases}x^2\,\text{sen}\frac{1}{x} & x\neq 0\\ 0 & x=0\end{cases}$

1. $f(0)=0$
2. $-1\leq x^2\,\text{sen}\frac{1}{x}\leq 1$

$-x^2\leq x^2\,\text{sen}\frac{1}{x}\leq x^2$

$\Rightarrow \lim_{x\to 0}-x^2\leq \lim_{x\to 0}x^2\,\text{sen}\frac{1}{x}\leq \lim_{x\to 0}x^2$

$0\leq \lim_{x\to 0}x^2\,\text{sen}\frac{1}{x}\leq 0$

3. $\lim_{x\to 0}f(x)=0=f(0)=0$

$f'(x)=2x\cdot\text{sen}\frac{1}{x}\cdot -\cos\frac{1}{x}$

$\lim_{h\to 0}\frac{f(h)-f(0)}{h}$

$=\lim_{h\to 0}\frac{h^2\cdot\text{sen}\frac{1}{h}}{h}$

$=\lim_{h\to 0}h\cdot\text{sen}\frac{1}{h}$

$=\lim_{h\to 0}\frac{\text{sen}\frac{1}{h}}{\frac{1}{h}}=1$

3) $f(x)=|5x-3|$

a) $|5x-3|=\begin{cases}5x-3 & x\geq\frac{3}{5}\\ -(5x-3) & x<\frac{3}{5}\end{cases}$

$f'(x)=\begin{cases}5+c & x\geq\frac{3}{5}\\ -5+c & x<\frac{3}{5}\end{cases}$

$f'^{+}(3/5)=5+c$

$f'^{-}(3/5)=-5+c$

b) $f'(x)=\nexists$ porque $f'^{+}\neq f'^{-}$

4 a) $f(x)=\begin{cases}0 & x\leq 0\\ 5-x & 0<x<4\\ \frac{1}{5-x} & x\geq 4\end{cases}$

$f'^{-}(4)=\lim_{h\to 0^-}\frac{f(4+h)-f(4)}{h}$

$=\lim_{h\to 0^-}\frac{\frac{1}{5-(4+h)}-1}{h}$

$=\lim_{h\to 0^-}\frac{\frac{1}{1-h}-1}{h}$

$=\lim_{h\to 0^-}\frac{\frac{1-1+h}{1-h}}{h}$

$=\lim_{h\to 0^-}\frac{\frac{h}{1-h}}{h}$

$=\lim_{h\to 0^-}\frac{h}{h(1-h)}$

$=\lim_{h\to 0^-}\frac{1}{1-h}=1$

$f'^{+}(4)=\lim_{h\to 0^+}\frac{f(4+h)-f(4)}{h}$

$\lim_{h\to 0^+}\frac{\frac{1}{5-4-h}-1}{h}$

$=\lim_{h\to 0^+}\frac{\frac{1}{1-h}-1}{h}$

$=\lim_{h\to 0^+}\frac{\frac{1-1+h}{1-h}}{h}$

$=\lim_{h\to 0^+}\frac{\frac{h}{1-h}}{h}$

$=\lim_{h\to 0^+}\frac{h}{h(1-h)}$

$=\lim_{h\to 0^+}\frac{1}{1-h}=1$

b) $\text{Dom } f=\{x\in\mathbb{R}\mid x\neq 5\}$

d) 1. $f(0)=0$ 1. $f(4)=1$

2. $\lim_{x\to 0^+}f(x)=5$ 2. $\lim_{x\to 4^+}=1$

$\lim_{x\to 0^-}f(x)=0$ $\lim_{x\to 4^-}=1$

$\lim_{x\to 0}f(x)=\nexists$ $f(4)=1=\lim_{x\to 4}f(x)=1$

Es discontinua en $x=0$ y $x=5$

e) No es derivable en $x=0$, $x=4$ y $x=5$

5 a) $f'(x)=7x^6-15x^2$ //

b) $f'(x)=4(x^2-x)^3\cdot(2x-1)$

$=(x^2-x)^3\cdot(8x-4)$ //

c) $f'(x)=\frac{2}{3}x^{-\frac{1}{3}}=\frac{2}{3\sqrt[3]{x}}$ //

d) $f'(x)=\frac{2x\cdot(x+1)^2-x^2\cdot 2(x+1)}{(x+1)^4}$

$=\frac{(x+1)\left(2x\cdot(x+1)-x^2\cdot 2\right)}{(x+1)^4}$

$=\frac{2x^2+2x-2x^2}{(x+1)^3}$

$=\frac{2x}{(x+1)^3}$ //

e) $f'(x)=\frac{1\cdot(x^2+2)^{\frac{1}{2}}-x\cdot\frac{1}{2}(x^2+2)^{-\frac{1}{2}}\cdot 2x}{(x^2+2)}$

$=\frac{\sqrt{x^2+2}-x\cdot\frac{1}{2\sqrt{x^2+2}}\cdot 2x}{(x^2+2)}$

$=\frac{\frac{\sqrt{x^2+2}\cdot 2\sqrt{x^2+2}}{2\sqrt{x^2+2}}-\frac{2x^2}{2\sqrt{x^2+2}}}{(x^2+2)}$

$=\frac{\frac{2(x^2+2)}{2\sqrt{x^2+2}}-\frac{2x^2}{2\sqrt{x^2+2}}}{(x^2+2)}$

$=\frac{\frac{\cancel{2x^2}+4\cancel{-2x^2}}{2\sqrt{x^2+2}}}{(x^2+2)}=\frac{2\cancel{4}}{\cancel{2}\sqrt{x^2+2}\cdot(x^2+2)}$

$=\frac{2}{\sqrt{x^2+2}\cdot(x^2+2)}$ //

f) $f'(x)=\frac{1}{2}\cdot\left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}}\cdot\frac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^2}$

$=\frac{1}{\cancel{2}}\cdot\left(\frac{x+1}{x-1}\right)^{-\frac{1}{2}}\cdot\frac{-\cancel{2}}{(x-1)^2}$

$=\frac{\sqrt{x+1}}{\sqrt{x-1}}\cdot\frac{-1}{(x-1)^2}$

$=\frac{-\sqrt{x+1}}{\sqrt{x-1}\cdot(x-1)^2}$ //

h) $f'(x)=2\cos(x)\cdot\cos(x)+2\text{sen}(x)\cdot-\text{sen}(x)$

$=2\cos^2(x)-2\text{sen}^2(x)$ //

i) $f'(x)=\sec^2(x)$ //

j) $f'(x)=2\,\text{sen}\left(\frac{1}{x}\right)\cdot\cos\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)$ //

k) $f'(x)=\frac{1}{tg\left(\frac{x}{2}+\frac{\pi}{4}\right)}\cdot\sec^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\cdot\frac{1\cdot\cancel{2}-x\cdot 0}{\cancel{4}\,2}+0$

$=\frac{\sec^2\left(\frac{x}{2}+\frac{\pi}{4}\right)}{2\,tg\left(\frac{x}{2}+\frac{\pi}{4}\right)}$ //

l) $f'(x)=\frac{1}{\sqrt{1-x^4}}\cdot 2x=\frac{2x}{\sqrt{1-x^4}}$ //

m) $f'(x)=\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2}\cdot\frac{1-x+1+x}{(1-x)^2}-\frac{1}{1+x^2}$

$=\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2}\cdot\frac{2}{(1-x)^2}-\frac{1}{1+x^2}$

$=\frac{2}{1+\left(\frac{1+x}{\cancel{1-x}}\right)^2\cdot\cancel{(1-x)^2}}-\frac{1}{1+x^2}$

$=\frac{2}{1+(1+x)^2}-\frac{1}{1+x^2}$ //

n) $f'(x)=\ln(x)+1$ //

ñ) $f'(x)=e^{\text{sen}(x)}\cdot\cos(x)$ //

o) $f'(x)=(2x-1)\cdot e^{-x}+(x^2-x)\cdot-\frac{1}{e^x}$

$=(2x-1)\cdot-\frac{1}{e^x}\cdot(-1)+(x^2-x)\cdot-\frac{1}{e^x}$

$=\frac{2x-1}{e^x}+\frac{-x^2+x}{e^x}=\frac{-x^2+3x-1}{e^x}$ //

p) $f'(x)=\frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2x}$ //

q) $f'(x)=\frac{1}{\sqrt{x}+\sqrt{x+1}}\cdot\left(\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{x+1}}\right)$

$=\frac{1}{\sqrt{x}+\sqrt{x+1}}\cdot\frac{2\sqrt{x+1}+2\sqrt{x}}{2\sqrt{x}\cdot 2\sqrt{x+1}}$ //

r) $f'(x)=e^{\text{sen}\,x^2}\cdot\cos x^2\cdot 2x$ //

s) $f(x)=e^{x\ln(x)}=x^x$

$f'(x)=e^{x\ln(x)}\cdot\ln(x)+1$ //

t) $f(x)=x^{tg\,x}=e^{\ln(x)\cdot tg(x)}$

$f'(x)=e^{\ln(x)\cdot tg(x)}\cdot\left(\frac{1}{x}\cdot tg(x)+\ln(x)\cdot\sec^2(x)\right)$

$=e^{\ln(x)\cdot tg(x)}\cdot\frac{tg(x)+\ln(x)\cdot\sec^2(x)\cdot x}{x}$ //

u) $f(x)=\log_x e=\frac{\ln(e)}{\ln(x)}=\frac{1}{\ln(x)}$

$f'(x)=\frac{-1\cdot\frac{1}{x}}{\ln^2(x)}=\frac{-1}{x\ln^2(x)}$ //

g) $f'(x)=\frac{0\cdot x\sqrt{1-x^2}-1\cdot\left(1\cdot\sqrt{1-x^2}+x\cdot\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot-2x\right)}{\left(x\sqrt{1-x^2}\right)^2}=\frac{-1\cdot\sqrt{1-x^2}+x\cdot\frac{1}{\cancel{2}}(1-x^2)^{-\frac{1}{2}}\cdot\cancel{-2}x}{\left(x\sqrt{1-x^2}\right)^2}$

$=\frac{-\sqrt{1-x^2}-\frac{x}{\sqrt{1-x^2}}}{x^2-x^4}=\frac{\frac{-\sqrt{1-x^2}\cdot\sqrt{1-x^2}-x}{\sqrt{1-x^2}}}{x^2-x^4}$

6 a)

$f'(x)=(0+2x)\cdot arctg(x)+\frac{1}{1+x^2}$

$=2x\cdot arctg(x)+(1+x^2)\cdot\frac{1}{1+x^2}$

$=2x\cdot arctg(x)+\frac{1+x^2}{1+x^2}$

$=2x\cdot arctg(x)+1$

$f''(x)=2\cdot arctg(x)+2x\cdot\frac{1}{1+x^2}$

$=2arctg(x)+\frac{2x}{1+x^2}$ //

b) $f'(x)=\frac{1\cdot\sqrt{1-x^2}-x\cdot\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot-2x}{(1-x^2)}$

$=\frac{\sqrt{1-x^2}+\frac{x^2}{\sqrt{1-x^2}}}{(1-x^2)}$

$=\frac{\frac{1-x^2+x^2}{\sqrt{1-x^2}}}{(1-x^2)}$

$=\frac{1}{\sqrt{1-x^2}\cdot(1-x^2)}$ //

$f''(x)=\frac{-\frac{x(1-x^2)}{\sqrt{1-x^2}}+\sqrt{1-x^2}\cdot-2x}{(1-x^2)(1-x^2)^2}$

⑦ $y=\sqrt{3-x}$ $(-1,2),(2,1)$ y $(-6,3)$

$$\boxed{f'(x)=\frac{-1}{2\sqrt{3-x}}}$$

$t(x)=ax+b$
$a=f'(x)$

$f'(-1)=\frac{-1}{2\sqrt{4}}=-\frac{1}{4}$

$2=-\frac{1}{4}(-1)+b$
$2=\frac{1}{4}+b$
$2-\frac{1}{4}=b$
$\frac{8}{4}-\frac{1}{4}=b$
$\frac{7}{4}=b$

$t(x)=-\frac{1}{4}x-\frac{7}{4}$

$f'(2)=\frac{-1}{2\sqrt{3-2}}=-\frac{1}{2}$

$1=-\frac{1}{2}\cdot 2+b$

$2=b$

$t(x)=-\frac{1}{2}x+2$

$f'(-6)=\frac{-1}{2\sqrt{3-6}}=-\frac{1}{9}$

$3=-\frac{1}{9}\cdot(-6)+b$
$3=\frac{2}{3}+b$
$3-\frac{2}{3}=b$
$\frac{7}{3}=b$ $t(x)=-\frac{1}{9}x+\frac{7}{3}$

⑧ para que valores son paralelas las tangentes de $y=x^2$ e $y=x^3$

$f'(x)=2x$
$g(x)=3x^2$

$2x=3x^2$
$0=3x^2-2x$

$\frac{2\pm\sqrt{(-2)^2-4\cdot3\cdot0}}{6}=\frac{2\pm\sqrt{4}}{6}=$ $x_1=\frac{4}{6}=\frac{2}{3}$, $x_2=0$

Rta: las tangentes son paralelas en $x=\frac{2}{3}$ y $x=0$

⑨ $f'(x)=-\frac{1}{x^2}$
$f'(a)=-\frac{1}{a^2}$

La recta tangente $(a,1/a)$ no corta la grafica de $f(x)$ mas que en el punto $(a,1/a)$

$\frac{1}{a}=-\frac{1}{a^{\cancel{2}}}\cdot\cancel{a}+b$
$\frac{1}{a}=-\frac{1}{a}+b$
$\frac{2}{a}=b$

$t(x)=-\frac{1}{a^2}x+\frac{2}{a}$

$\frac{1}{x}=-\frac{1}{a^2}x+\frac{2}{a}$
$\frac{1}{x}\cdot x=-\frac{1}{a^2}x^2+\frac{2}{a}\cdot x$
$\Delta=\left(\frac{2}{a}\right)^2-4\cdot-\frac{1}{a^2}\cdot(-1)$
$=\frac{4}{a^2}-\frac{4}{a^2}=0$ → Una sola raiz doble

⑩ ⓐ $\frac{x^2}{16}-\frac{y^2}{9}=1$ $(-5,\frac{9}{4})$ (hiperbola)

$-\frac{y^2}{3^2}=1-\frac{x^2}{4^2}$
$\cdot y^2=3^2\left(1-\frac{x^2}{4^2}\right)$
$y^2=(-1)3^2\left(1-\frac{x^2}{4^2}\right)$
$y^2=-9\left(1-\frac{x^2}{16}\right)$
$\boxed{y=\pm\sqrt{-9\left(1-\frac{x^2}{16}\right)}}$
$x=-5$
$\Rightarrow y=\pm\sqrt{-9\left(1-\frac{(-5)^2}{16}\right)}$
$y=\pm\sqrt{-9\left(\frac{16}{16}-\frac{25}{16}\right)}$
$y=\pm\sqrt{-9\left(\frac{9}{16}\right)}$
$y=\pm\sqrt{\frac{81}{16}}$
$y=\frac{9}{4}$ → Verifica

$t(x)=-\frac{5}{4}x-4$

$f(x)=\pm\sqrt{-9+\frac{9x^2}{16}}=\sqrt{\frac{-144+9x^2}{16}}=\frac{\sqrt{-144+9x^2}}{4}$

$f'(x)=\frac{1}{4}\cdot\frac{1}{2\sqrt{-144+9x^2}}\cdot 18x$

$=\frac{18x}{8\sqrt{-144+9x^2}}=\frac{18x}{8\sqrt{9(-16+x^2)}}$

$=\frac{\cancel{18}^{3}x}{4\cancel{24}\sqrt{-16+x^2}}=\frac{3x}{4\sqrt{-16+x^2}}$

$f'(-5)=\frac{3\cdot(-5)}{4\sqrt{-16+(-5)^2}}=\frac{-15}{12}=-\frac{5}{4}$

$t(x)=ax+b$ → $a=-\frac{5}{4}$

$\frac{9}{4}=-\frac{5}{4}\cdot(-5)+b$
$\frac{9}{4}=\frac{25}{4}+b$
$-\frac{16}{4}=b$
$-4=b$

ⓑ $\frac{x^2}{9}+\frac{y^2}{36}=1$ $(-1,4\sqrt{2})$ (elipse)

$\frac{y}{6^2}=1-\frac{x^2}{3^2}$
$y^2=6^2\left(1-\frac{x^2}{3^2}\right)$
$\boxed{y=\pm\sqrt{6^2\left(1-\frac{x^2}{3^2}\right)}}$
$x=-1$
$y=\pm\sqrt{6^2\left(1-\frac{(-1)^2}{9}\right)}$
$y=6\cdot\sqrt{1-\frac{1}{9}}$
$y=\frac{\cancel{6}\sqrt{8}}{\cancel{3}}=2\sqrt{8}$
$y=2\sqrt{2\cdot4}=4\sqrt{2}$

$t(x)=\frac{\sqrt{2}}{2}\cdot x+\frac{9\sqrt{2}}{2}$

$f(x)=\frac{6\sqrt{9-x^2}}{3}=2\sqrt{9-x^2}$

$f'(x)=2\cdot\frac{1}{2\sqrt{9-x^2}}\cdot-2x$

$=\frac{\cancel{-4}^{-2}x}{\cancel{4}\sqrt{9-x^2}}=\frac{-2x}{\sqrt{9-x^2}}$

$f'(-1)=\frac{-2(-1)}{\sqrt{9-(-1)^2}}=\frac{2}{\sqrt{8}}=\frac{2\sqrt{8}}{8}=\frac{2\cdot\sqrt{2\cdot4}}{8}=\frac{4\sqrt{2}}{8}=\frac{\sqrt{2}}{2}$

$a=\frac{\sqrt{2}}{2}$

$4\sqrt{2}=\frac{\sqrt{2}}{2}(-1)+b$
$4\sqrt{2}+\frac{\sqrt{2}}{2}=b$
$\frac{8\sqrt{2}+\sqrt{2}}{2}=b$
$\frac{9\sqrt{2}}{2}=b$

Material Extra

11) a) $f(x)=(1+3x^4)^5$

$f'(x)=5(1+3x^4)^4\cdot 12x^3$

$=60x^3(1+3x^4)^4$ //

b) $f(x)=(1+x+x^2)^3$

$f'(x)=3(1+x+x^2)^2\cdot(1+2x)$ //

c) $f(x)=\dfrac{1}{(x^2-1)^5}$

$f'(x)=\dfrac{-1\cdot 5(x^2-1)^4\cdot 2x}{(x^2-1)^{10}}$

$=\dfrac{-10x}{(x^2-1)^6}$ //

d) $f(x)=(3x^2+3)(2x^2+1)$

$f'(x)=6x\cdot(2x^2+1)+(3x^2+3)\cdot 4x$

$=12x^3+6x+12x^3+12x$

$=24x^3+18x$ //

e) $f(x)=\dfrac{2x^3+5}{4x^2+7}$

$f'(x)=\dfrac{6x^2\cdot(4x^2+7)-(2x^3+5)\cdot 8x}{(4x^2+7)^2}$

$=\dfrac{24x^4+42x^2-16x^4-40x}{(4x^2+7)^2}$

$=\dfrac{8x^4+42x^2-40x}{(4x^2+7)^2}$ //

f) $f(x)=\dfrac{x}{x-1}+\dfrac{2}{(x-1)^2}+\dfrac{3}{(x-1)^3}$

$f'(x)=\dfrac{x-1-x}{(x-1)^2}+\dfrac{-4(x-1)}{(x-1)^4}+\dfrac{-9(x-1)^2}{(x-1)^6}$

$=\dfrac{x}{(x-1)^2}+\dfrac{-4}{(x-1)^3}+\dfrac{-9}{(x-1)^4}$ //

g) $f(x)=\sqrt{1-x^2}$

$f(x)=\dfrac{-2x}{2\sqrt{1-x^2}}$

$=-\dfrac{x}{\sqrt{1-x^2}}$ //

h) $f(x)=(2+5x^2)^{\frac{1}{3}}$

$f'(x)=\dfrac{1}{3}\cdot(2+5x^2)^{-\frac{2}{3}}\cdot 10x$

$=\dfrac{10x}{3\sqrt[3]{(2+5x^2)^2}}$ //

i) $f(x)=\dfrac{1}{\sqrt[3]{(x^2-2)}}=\dfrac{1}{(x^2-2)^{\frac{1}{3}}}$

$f'(x)=\dfrac{-\frac{1}{3}\cdot(x^2-2)^{-\frac{2}{3}}\cdot 2x}{(x^2-2)^{\frac{2}{3}}}$

$=\dfrac{-\frac{1}{3}\cdot\frac{1}{(x^2-2)^{\frac{2}{3}}}\cdot 2x}{(x^2-2)^{\frac{2}{3}}}$

$=-\dfrac{2x}{3(x^2-2)^{\frac{2}{3}}\cdot(x^2-2)^{\frac{2}{3}}}$

$=-\dfrac{2x}{3(x^2-2)^{\frac{4}{3}}}$

$=-\dfrac{2x}{3(x^2-2)\sqrt[3]{x^2-2}}$ //

j) $f(x)=(5-3\cos(x))^4$

$f'(x)=4(5-3\cos(x))^3\cdot 3\,\mathrm{sen}(x)$

$=12(5-3\cos(x))^3\cdot\mathrm{sen}(x)$ //

k) $f(x)=\mathrm{sen}(x)+\mathrm{sen}^2(x)+\mathrm{sen}^3(x)$

$f'(x)=\cos(x)+2\,\mathrm{sen}(x)\cdot\cos(x)+3\,\mathrm{sen}^2(x)\cdot\cos(x)$ //

l) $f(x)=\dfrac{1}{\mathrm{arctg}(x)}$

$f'(x)=\dfrac{\frac{-1}{1+x^2}}{\mathrm{arctg}^2(x)}=-\dfrac{1}{(1+x^2)\,\mathrm{arctg}^2(x)}$ //

m) $f(x)=\mathrm{sen}^3x-\cos^3x$

$f'(x)=3\,\mathrm{sen}^2(x)\cdot\cos(x)+3\cos^2(x)\cdot\mathrm{sen}(x)$ //

n) $f(x)=\dfrac{1}{3\,\mathrm{sen}^3(x)}-\dfrac{1}{\cos(x)}$

$f'(x)=-\dfrac{9\,\mathrm{sen}^2(x)\cdot\cos(x)}{9\,\mathrm{sen}^6(x)}+\dfrac{\mathrm{sen}(x)}{\cos^2(x)}$

$=-\dfrac{\cos(x)}{\mathrm{sen}^4(x)}+\dfrac{\mathrm{sen}(x)}{\cos^2(x)}$

$=-\dfrac{\cos^3(x)+\mathrm{sen}^5(x)}{\mathrm{sen}^4(x)\cdot\cos^2(x)}$ //

o) $f(x)=\sqrt[3]{2^x+x}=\sqrt[3]{e^{x\cdot\ln(2)}+x}$

$f'(x)=\dfrac{e^{x\cdot\ln(2)}\cdot\ln(2)+1}{3\sqrt[3]{(e^{x\cdot\ln(2)}+x)^2}}$ //

p) $f(x)=\ln(\ln(x))$

$f'(x)=\dfrac{1}{\ln(x)\cdot x}$

q) $f(x)=\arccos(\sqrt{x})$

$f'(x)=-\dfrac{1}{\sqrt{1-x}\cdot 2\sqrt{x}}$ //

r) $f(x)=\mathrm{arcsen}\left(\frac{1}{x^2}\right)$

$f'(x)=\dfrac{1}{\sqrt{1-\frac{1}{x^4}}}\cdot\dfrac{-2x}{x^4}$

$=-\dfrac{2x}{\sqrt{1-\frac{1}{x^4}}\cdot x^4}$ //

s) $f(x)=\dfrac{1+\mathrm{sen}^2(x)}{1+\cos^2(x)}$

$f'(x)=\dfrac{2\,\mathrm{sen}(x)\cdot\cos(x)\cdot(1+\cos^2(x))-(1+\mathrm{sen}^2(x))\cdot 2\cos(x)\cdot(-\mathrm{sen}(x))}{(1+\cos^2(x))^2}$

$=\dfrac{2\,\mathrm{sen}(x)\cdot\cos(x)\cdot(1+\cos^2(x))+(1+\mathrm{sen}^2(x))\cdot 2\cos(x)\cdot\mathrm{sen}(x)}{(1+\cos^2(x))^2}$

$=\dfrac{\mathrm{sen}(x)\cdot\cos(x)\left(2(1+\cos^2(x)+2(1+\mathrm{sen}^2(x))\right)}{(1+\cos^2(x))^2}$

$=\dfrac{\mathrm{sen}(x)\cdot\cos(x)\cdot 6}{(1+\cos^2(x))^2}$ //

t) $f(x)=\ln\left(\frac{3x}{4}\right)$

$f'(x)=\dfrac{1}{\frac{3x}{4}}\cdot\dfrac{12}{16}$

$=\dfrac{4}{3x}\cdot\dfrac{3}{4}=\dfrac{1}{x}$ //

u) $f(x)=7\ln(x^{\frac{2}{5}})$

$f'(x)=\dfrac{7}{x^{\frac{2}{5}}}\cdot x^{-\frac{3}{5}}$

$f'(x)=\dfrac{2}{5}\cdot\dfrac{7}{x^{\frac{2}{5}}\cdot x^{\frac{3}{5}}}=\dfrac{14}{5x}$ //

v) $f(x)=4\ln(\mathrm{sen}(x))$

$f'(x)=\dfrac{4\cos(x)}{\mathrm{sen}(x)}=4\,\mathrm{cotg}(x)$ //

w) $f(x)=\ln(\mathrm{arctg}(x))$

$f'(x)=\dfrac{1}{\mathrm{arctg}(x)}\cdot\dfrac{1}{1+x^2}$

$=\dfrac{1}{(\mathrm{arctg}(x))(1+x^2)}$ //