① Calcular las siguientes integrales e indicar el metodo utilizado

ⓐ $\int_0^{\pi/4} \cos(2x)\operatorname{sen}(2x)$

$f(x) = \cos(2x) \quad f'(x) = -2\operatorname{sen}(2x)$ utilizo integracion por partes

$g(x) = -\frac{\cos(2x)}{2} \quad g'(x) = \operatorname{sen}(2x)$

$$\int \cos(2x)\cdot\operatorname{sen}(2x)\,dx = \cos(2x)\cdot\frac{-\cos(2x)}{2} - \int -2\operatorname{sen}(2x)\cdot -\frac{\cos(2x)}{2}\,dx$$

$$\int \cos(2x)\cdot\operatorname{sen}(2x)\,dx = -\frac{\cos(2x)^2}{2} - \int \operatorname{sen}(2x)\cdot\cos(2x)\,dx$$

$$2\int \cos(2x)\cdot\operatorname{sen}(2x)\,dx = -\frac{\cos(2x)^2}{2}$$

$$\int \cos(2x)\cdot\operatorname{sen}(2x)\,dx = -\frac{\cos(2x)^2}{4}$$

$$\frac{-\cos(2x)^2}{4}\Bigg|_0^{\pi/4} - \frac{\cos(\frac{\pi}{2})^2}{4} + \frac{\cos(0)^2}{4} = \frac{1}{4}$$

ⓑ $\int \frac{\ln(x)}{x^2}\,dx$ $\quad f(x) = \ln(x) \quad f'(x) = \frac{1}{x}$

$g(x) = -\frac{1}{x} \quad g'(x) = \frac{1}{x^2}$ utilice integracion por partes

$$\ln(x)\cdot -\frac{1}{x} + \int \frac{1}{x}\cdot\frac{1}{x}\,dx$$

$$\frac{-\ln(x)}{x} + \int \frac{1}{x^3}\,dx = \frac{-\ln(x)}{x} + \int x^{-2}\,dx = \ln(x)\cdot\left(-\frac{1}{x}\right) - \frac{1}{x} = -\frac{\ln(x)+1}{x} \;//$$

② ⓐ $\int_0^3 \frac{1}{\sqrt{3-x}}\,dx$

$$\lim_{t\to 3^-}\left(\int_0^t \frac{1}{\sqrt{3-x}}\right) = \lim_{t\to 3^-}\left(-2\sqrt{3-x}\Big|_0^t\right) = \lim_{t\to 3^-}\left(-2\sqrt{3-t} + 2\sqrt{3-0}\right) = 2\sqrt{3} \quad \therefore \int_0^3 \frac{1}{\sqrt{3-x}}\,dx \text{ converge}$$

$$\int (3-x)^{-\frac{1}{2}} = -2(3-x)^{\frac{1}{2}}$$

ⓑ $\int_\pi^\infty \operatorname{sen}(x)\,dx$

$$\lim_{t\to\infty}\left(\int_\pi^t \operatorname{sen}(x)\,dx\right) = \lim_{t\to\infty}\left(-\cos(x)\Big|_\pi^t\right) = \lim_{t\to\infty}\left(-\cos(t) + \cos(\pi)\right) = \lim_{t\to\infty}\left(-\cos(t) - 1\right) = \nexists \quad \therefore \int_\pi^\infty \operatorname{sen}(x)\,dx \text{ diverge}$$

③ Dar un ejemplo de

ⓐ Una sucesion $a_n$ estrictamente creciente y convergente

ⓑ Una sucesion $b_n$ alternante

ⓐ $a_n = 1 - \frac{1}{n}$

ⓑ $b_n = (-1)^n$

(4) Determinar si la siguiente serie es abs. convergente, condicionalmente o divergente

(a) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{2n^2+3n+5}$

$a_k \geq a_{k+1}$

$\frac{1}{2k^2+3k+5} \geq \frac{1}{2(k+1)^2+3(k+1)+5}$

$2(k+1)^2+3(k+1)+5 \geq 2k^2+3k+5$

$2(k^2+2k+1)+3k+3+5 \geq 2k^2+3k+5$

$\cancel{2k^2}+4k+2+\cancel{3k}+3+\cancel{5} \geq \cancel{2k^2}+\cancel{3k}+\cancel{5}$

$4k+2 \geq 0$

$\lim_{n\to\infty} \frac{1}{2n+3n+5} = 0$ //

$\sum_{n=1}^{\infty} \frac{1}{2n^2+3n+5}$

$a_n = \frac{1}{2n^2+3n+5}$ $\qquad b_n = \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{\frac{1}{2n^2+3n+5}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{2n^2+3n+5} = \lim_{n\to\infty} \frac{n^2}{n^2\left(2+\frac{3}{n}+\frac{5}{n^2}\right)} = \frac{1}{2}$

$\therefore$ por Teo comp. límites, $\sum \frac{1}{2n^2+3n+5}$ converge

$\therefore \sum^{\infty} \frac{1}{2n^2+3n+5}$ converge absolutamente

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{3n^2+4}{2n^2+3n+5}$

$a_k \geq a_{k+1}$

$\frac{3n^2+4}{2n^2+3n+5} \geq \frac{3(n+1)^2+4}{2(n+1)^2+3(n+1)+5}$

$(3n^2+4)(2n^2+7n+10) \geq 3n^2+6n+10$

$6n^4+21n^3+38n^2+28n+40 \geq 3n^2+6n+10$

$6n^4+21n^3+35n^2+22n+30 \geq 0$

$\lim_{n\to\infty} \frac{3n^2+4}{2n^2+3n+5} = \lim_{n\to\infty} \frac{n^2\left(3+\frac{4}{n^2}\right)}{n^2\left(2+\frac{3}{n}+\frac{5}{n^2}\right)} = \lim_{n\to\infty} \frac{3}{2} = \frac{3}{2}$

$\therefore \sum_{n=1}^{\infty} (-1)^n \frac{3n^2+4}{2n^2+3n+5}$ diverge