① Considera la siguiente expresion formal. $h\ n = \langle \exists k : 0 \leq k < n : n = k! \rangle$ (HI)

ⓑ PASO INDUCTIVO

$h\ (m+1)$

$\equiv$ {especificacion}

$\langle \exists k : 0 \leq k < m+1 : (m+1) = k! \rangle$

$\equiv$ {Logica}

$\langle \exists k : 0 \leq k < m \lor k = m : (m+1) = k! \rangle$

$\equiv$ {Particion de Rango}

$\langle \exists k : 0 \leq k < m : (m+1) = k! \rangle \lor \langle \exists k : k = m : (m+1) = k! \rangle$

No podemos llegar a la HI, vamos a generalizar

$h :: Int \to Bool$

$h\ n = gh\ n\ 0$

$gh\ 0\ y = False$

$gh\ (x+1)\ y = gh\ x\ (y+1) \lor ((x+1)+y = x!)$

$gh\ m\ p = \langle \exists k : 0 \leq k < m : m+p = k! \rangle$

$h\ n$

$\equiv$ {especificacion}

$\langle \exists k : 0 \leq k < n : n = k! \rangle$

$\equiv$ {aritmetica}

$\langle \exists k : 0 \leq k < n : n+0 = k! \rangle$

$\equiv$ {especificacion gh}

$gh\ n\ 0$

$h\ n = gh\ n\ 0$

CASO BASE

$gh\ 0\ y$

$\equiv$ {especificacion}

$\langle \exists k : 0 \leq k < 0 : 0 = k! \rangle$

$\equiv$ {Logica}

$\langle \exists k : False : 0 = k! \rangle$

$\equiv$ {Rango vacio del exist

alse

PASO INDUCTIVO

$g\ (x+1)\ y$

$\equiv$ {especificacion}

$\langle \exists k : 0 \leq k < x+1 : x+1+y = k! \rangle$

$\equiv$ {Logica}

$\langle \exists k : 0 \leq k < x \lor k = x : (x+1)+y = k! \rangle$

$\equiv$ {Particion de Rango}

$\langle \exists k : 0 \leq k < x : (x+1)+y = k! \rangle \lor \langle \exists k : k = x : (x+1)+y = k! \rangle$

$\equiv$ {Asociatividad}

$\langle \exists k : 0 \leq k < x : x+(1+y) = k! \rangle \lor \langle \exists k : k = x : (x+1)+y = k! \rangle$

$\equiv$ {HI}

$gh\ x\ (y+1) \lor \langle \exists k : k = x : (x+1)+y = k! \rangle$

$\equiv$ {Rango unitario}

$gh\ x\ (y+1) \lor ((x+1)+y = x!)$

$! = \langle \Pi i : 1 \leq i \leq k : i \rangle$

(se puede modularizar, pero el ejercicio no puede)

② Considera $h.xs = \langle \exists\, as, bs : xs = as ++ bs : prod\ as < prod\ bs \rangle$

ⓐ PASO INDUCTIVO

$h\ (x \triangleright xs)$

$\equiv$ { especificacion }

$\langle \exists\, as, bs : (x \triangleright xs) = as ++ bs : prod\ as < prod\ bs \rangle$

$\equiv$ { Neutro $\wedge$ y Tercero excluido }

$\langle \exists\, as, bs : (x \triangleright xs) = as ++ bs \wedge (as = [\,] \vee as \neq [\,]) : prod\ as < prod\ bs \rangle$

$\equiv$ { distrivutibidad }

$\langle \exists\, as, bs : (x \triangleright xs) = as ++ bs \wedge as = [\,] : prod\ as < prod\ bs \rangle \vee \langle \exists\, as, bs : (x \triangleright xs) = as ++ bs \wedge as \neq [\,] : prod\ as < prod\ bs \rangle$

$\equiv$ { Cambio de variable $(a \triangleright as) \leftarrow as$ }

$\langle \exists\, as, bs : (x \triangleright xs) = as ++ bs \wedge as = [\,] : prod\ as < prod\ bs \rangle \vee \langle \exists\, a, as, bs : (x \triangleright xs) = (a \triangleright as) ++ bs \wedge (a \triangleright as) \neq [\,] : prod\ (a \triangleright as) < prod\ bs \rangle$

$\equiv$ { Concatenacion y neutro de disyuncion }

$\langle \exists\, as, bs : (x \triangleright xs) = as ++ bs \wedge as = [\,] : prod\ as < prod\ bs \rangle \vee \langle \exists\, a, as, bs : (x \triangleright xs) = (a \triangleright (as ++ bs)) : prod\ (a \triangleright as) < prod\ bs \rangle$

$\equiv$ { Prop listas }

$\langle \exists\, as, bs : (x \triangleright xs) = as ++ bs \wedge as = [\,] : prod\ as < prod\ bs \rangle \vee \langle \exists\, a, as, bs : x = a \wedge xs = as ++ bs : prod\ (a \triangleright as) < prod\ bs \rangle$

$\equiv$ { Anidado y Rango Unitario }

$\langle \exists\, as, bs : (x \triangleright xs) = as ++ bs \wedge as = [\,] : prod\ as < prod\ bs \rangle \vee \langle \exists\, as, bs : xs = as ++ bs : prod\ (x \triangleright as) < prod\ bs \rangle$

$\equiv$ { def prod }

$\langle \exists\, as, bs : (x \triangleright xs) = as ++ bs \wedge as = [\,] : prod\ as < prod\ bs \rangle \vee \langle \exists\, as, bs : xs = as ++ bs : x \cdot prod\ as < prod\ bs \rangle$

NO PODEMOS LLEGAR A LA HI. VAMOS A GENERALIZAR

$h\_gen\ y\ xs = \langle \exists\, as, bs : xs = as ++ bs : y \cdot prod\ as < prod\ bs \rangle$

$h\ xs$ PASO INDUCTIVO

$\equiv$ { especificacion }

$\langle \exists\, as, bs : xs = as ++ bs : prod\ as < prod\ bs \rangle$

$\equiv$ { Aritmetica }

$\langle \exists\, as, bs : xs = as ++ bs : 1 \cdot prod\ as < prod\ bs \rangle$

$\equiv$ { especificacion de h_gen }

$h\ n\ 1\ xs$

$h\ xs = h\_gen\ 1\ xs$

CASO BASE

$h\_gen\ y\ [\,]$

$\equiv$ { especificacion }

$\langle \exists\, as, bs : [\,] = as ++ bs : y \cdot prod\ as < prod\ bs \rangle$

$\equiv$ { Prop listas }

$\langle \exists\, as, bs : as = [\,] \wedge bs = [\,] : y \cdot prod\ as < prod\ bs \rangle$

$\equiv$ { Anidado y Rang Unitario }

$\langle \exists\, bs : bs = [\,] : y \cdot prod\ [\,] < prod\ bs \rangle$

$\equiv$ { Rango unitario }

$y \cdot prod\ [\,] < prod\ [\,]$

$\equiv$ { def prod }

$y < 1$