① ⓐ $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}} = \sum_{n=1}^{\infty} (x-0)^n \cdot \frac{1}{\sqrt{n}}$ $\quad \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n+1}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n(1+\frac{1}{n})}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}\cdot\sqrt{1+\frac{1}{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}} = 1$

R.C = 1

$I = [(a-R, a+R)] \Rightarrow I = [-1, 1)$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{\sqrt{n}}$ ∴ converge $\qquad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ ∴ diverge (serie $p = \frac{1}{2}$)

$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

ⓑ $\sum_{n=1}^{\infty} \sqrt{n}\, x^n = \sum_{n=1}^{\infty} \sqrt{n}\,(x-0)^n$ $\quad \lim_{n\to\infty} \frac{\sqrt{n+1}}{\sqrt{n}} = \lim_{n\to\infty} \frac{\sqrt{n}\cdot\sqrt{1+\frac{1}{n}}}{\sqrt{n}} = \lim_{n\to\infty} \sqrt{1+\frac{1}{n}} = 1$

R.C = 1

$I = [(a-R, a+R)] \Rightarrow I = (-1, 1)$

$\sum_{n=1}^{\infty} \sqrt{n}\,(-1)^n$ $\quad \lim_{n\to\infty} \frac{\sqrt{n+1}}{\sqrt{n}} = \lim_{n\to\infty} \frac{\sqrt{n}\cdot\sqrt{1+\frac{1}{n}}}{\sqrt{n}} = \lim_{n\to\infty} \sqrt{1+\frac{1}{n}} = 1$ ∴ diverge

$\sum_{n=1}^{\infty} \sqrt{n}\; 1^n = \sum_{n=1}^{\infty} n^{\frac{1}{2}}$ ∴ diverge por serie geom ($|r| \geq 1$)

ⓒ $\sum_{n=1}^{\infty} \frac{x^n}{n!} = \sum_{n=1}^{\infty} \frac{1}{n!}(x-0)^n$ $\quad \lim_{n\to\infty} \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} = \lim_{n\to\infty} \frac{n!}{(n+1)!} = \lim_{n\to\infty} \frac{n!}{n!\,(n+1)} = \lim_{n\to\infty} \frac{1}{n+1} = 0$

R.C = ∞

$I = \mathbb{R}$

ⓓ $\sum_{n=1}^{\infty} n^n x^n = \sum_{n=1}^{\infty} n^n (x-0)^n$ $\quad \lim_{n\to\infty} \frac{(n+1)^{n+1}}{n^n} = \lim_{n\to\infty} \frac{(n+1)\,(n+1)^n}{n^n} = \lim_{n\to\infty} (n+1)\left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty} (n+1)\underbrace{\left(1+\frac{1}{n}\right)^n}_{\to e} = \infty$ $\quad$ R.C = 0 $\quad I = \{0\}$

ⓔ $\sum_{n=1}^{\infty} (-1)^n \frac{n^2 x^n}{2^n} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{n^2}{2^n}(x-0)^n = \sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^n \cdot n^2 \cdot (x-0)^n$

$\lim_{n\to\infty} \left| \frac{\left(-\frac{1}{2}\right)^{n+1}\cdot(n+1)^2}{\left(-\frac{1}{2}\right)^n \cdot n^2} \right| = \lim_{n\to\infty} \left| \frac{\left(-\frac{1}{2}\right)^n\cdot\left(-\frac{1}{2}\right)\cdot(n^2+2n+1)}{\left(-\frac{1}{2}\right)^n\cdot n^2} \right| = \lim_{n\to\infty} \left| \frac{\left(-\frac{1}{2}\right)(n^2+2n+1)}{n^2} \right| = \lim_{n\to\infty} \left| \frac{-\frac{1}{2}n^2 - n - \frac{1}{2}}{n^2} \right| = \lim_{n\to\infty} \left| \frac{n^2\left(-\frac{1}{2} - \frac{1}{n} - \frac{1}{2n^2}\right)}{n^2} \right| = \left|-\frac{1}{2}\right| = \frac{1}{2}$

R.C = 2

$I = (-2, 2)$

$\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^n \cdot n^2 \cdot (-2)^n = \sum_{n=1}^{\infty} n^2$ ∴ diverge por ser serie geom ($|r| \geq 1$)

$\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^n \cdot n^2 \cdot 2^n = \sum_{n=1}^{\infty} (-1)^n \cdot n^2$ ∴ diverge por serie p ($p = -2$)

ⓕ $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{4^n \ln n} = \sum_{n=1}^{\infty} \left(-\frac{1}{4}\right)^n \cdot \frac{1}{\ln(n)} \cdot (x-0)^n$

$\lim_{n\to\infty} \left| \frac{\left(-\frac{1}{4}\right)^{n+1} \cdot \frac{1}{\ln(n+1)}}{\left(-\frac{1}{4}\right) \cdot \frac{1}{\ln(n)}} \right| = \lim_{n\to\infty} \left| \frac{\left(-\frac{1}{4}\right)^n \cdot \left(-\frac{1}{4}\right) \cdot \frac{1}{\ln(n+1)}}{\left(-\frac{1}{4}\right)^n \cdot \frac{1}{\ln(n)}} \right| = \lim_{n\to\infty} \left| \frac{\left(-\frac{1}{4}\right) \cdot \frac{1}{\ln(n+1)}}{\frac{1}{\ln(n)}} \right| = \lim_{n\to\infty} \left| -\frac{1}{4} \frac{\ln(n)}{\ln(n+1)} \right| = \frac{1}{4} \lim_{n\to\infty} \left| \frac{\ln(n)}{\ln(n+1)} \right| \overset{L'H}{=} \frac{1}{4}$

R.C = 4 $\quad I = [(a-R, a+R)] \Rightarrow I = [-4, 4]$ $\qquad f(x) = f(n) = a_n$ ∴ $\lim f(x) = \lim a_n$

$x = -4$ $\quad \sum_{n=1}^{\infty} \left(-\frac{1}{4}\right)^n \cdot \frac{1}{\ln(n)} \cdot (-4)^n = \sum_{n=1}^{\infty} \frac{1}{\ln(n)}$ $\quad \lim_{n\to\infty} \frac{1}{\ln(n)} = 0$ ∴ converge

$x = 4$ $\quad \sum_{n=1}^{\infty} \left(-\frac{1}{4}\right)^n \cdot \frac{1}{\ln(n)} \cdot 4^n = \sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln(n)}$ $\quad \lim_{n\to\infty} \frac{1}{\ln(n)} \to 0 \wedge a_n \geq a_{n+1}\ \forall n - \{1\}$ ∴ converge

ⓖ $\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x+2}{2}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n\,2^n} \cdot (x+2)^n$

$\lim_{n\to\infty} \left| \frac{\frac{1}{(n+1)\cdot 2^{n+1}}}{\frac{1}{n\,2^n}} \right| = \lim_{n\to\infty} \frac{n\,\cancel{2^n}}{(n+1)\cdot\cancel{2^n}\cdot 2} = \lim_{n\to\infty} \frac{n}{2n+2} = \lim_{n\to\infty} \frac{n}{n\left(2+\frac{2}{n}\right)} = \lim_{n\to\infty} \frac{1}{2+\frac{2}{n}} = \frac{1}{2}$

R.C = 2

$I = [-4, 0)$

$x = -4$ $\quad \sum_{n=1}^{\infty} \frac{1}{n\,2^n} \cdot (-2)^n = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ ∴ converge por CSA

$x = 0$ $\quad \sum_{n=1}^{\infty} \frac{1}{n\,2^n} \cdot 2^n = \sum_{n=1}^{\infty} \frac{1}{n}$ ∴ diverge por serie armonica

ⓗ $\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt[4]{n}} = \sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt[4]{n}}(x-0)^n$ $\quad \lim_{n\to\infty} \left| \frac{\frac{(-2)^{n+1}}{\sqrt[4]{n+1}}}{\frac{(-2)^n}{\sqrt[4]{n}}} \right| = \lim_{n\to\infty} \left| \frac{\cancel{\sqrt[4]{n}} \cdot \cancel{(-2)^n} \cdot (-2)}{\cancel{\sqrt[4]{n}}\,\sqrt[4]{1+\frac{1}{n}} \cdot \cancel{(-2)^n}} \right| = \lim_{n\to\infty} \left| -\frac{2}{\sqrt[4]{1+\frac{1}{n}}} \right| = 2$

R.C = $\frac{1}{2}$

$I = \left(-\frac{1}{2}, \frac{1}{2}\right]$

$x = -\frac{1}{2}$ $\quad \sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt[4]{n}} \left(-\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}}$ ∴ es una se $\qquad$ diverge

$x = \frac{1}{2}$ $\quad \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{\sqrt[4]{n}}$ $\quad a_n \geq a_{n+1} \wedge \lim_{n\to\infty} \frac{1}{\sqrt[4]{n}} = 0$ ∴ converge

① $\sum_{n=1}^{\infty} \frac{e^n}{n^3}(4-x)^n = \sum_{n=1}^{\infty} \frac{e^n \cdot (-1)^n}{n^3}(x-4)^n = \sum_{n=1}^{\infty} (-1)^n \frac{e^n}{n^3}(x-4)^n$

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1} e^{n+1}}{(n+1)^3}}{\frac{(-1)^n e^n}{n^3}} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1} e^{n+1} \cdot n^3}{(-1)^n e^n (n+1)^3} \right| = \lim_{n\to\infty} \left| \frac{(-1)\cdot e \cdot n^3}{(n+1)^3} \right| = e \lim \left(\frac{n}{n+1}\right)^3 = e \cdot \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}(1+\frac{1}{n})} = e$$

$R.C = \frac{1}{e}$ $\qquad I = \left[4-\frac{1}{e}, 4+\frac{1}{e}\right]$

$x = 4-\frac{1}{e}$ $\quad \sum_{n=1}^{\infty} \frac{e^n}{n^3}\left(\frac{1}{e}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n^3}$ $\therefore$ converge por serie p, $p>1$

$x = 4+\frac{1}{e}$ $\quad \sum_{n=1}^{\infty} \frac{e^n}{n^3}\left(\frac{-1}{e}\right)^n = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^3}$ $\quad a_n \geq a_{n+1} \wedge \lim_{n\to\infty} \frac{1}{n^3} = 0$ $\therefore$ converge

② $\sum_{n=0}^{\infty} C_n x^n$ $\quad R.C = 4$

ⓐ $\sum_{n=0}^{\infty} C_n = \sum_{n=0}^{\infty} C_n 1^n$ $\therefore$ la serie converge si $|x| < R \Rightarrow 1 < 4$ $\therefore$ converge

ⓑ $\sum_{n=0}^{\infty} C_n 8^n$ $\therefore$ la serie converge si $|x| < R \Rightarrow 8 < 4$ $\therefore$ diverge

ⓒ $\sum_{n=0}^{\infty} C_n (-3)^n$ $\therefore$ la serie converge si $|x| < R \Rightarrow 3 < 4$ $\therefore$ converge

ⓓ $\sum_{n=0}^{\infty} (-1)^n C_n$ $\therefore$ la serie converge si $|x| < R \Rightarrow 1 < 4$ $\therefore$ converge

③ Usar la expansión $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$ valida en el rango $-1 < x < 1$ para representar las siguientes funciones

ⓐ $f(x) = \frac{1}{1+x}$ en potencias de x

$\frac{1}{1-(-x)} \Rightarrow \sum_{n=0}^{\infty} (-1)^n x^n$

ⓑ $f(x) = \frac{3}{1-x^4}$ en potencias de x

$3 \cdot \frac{1}{1-x^4} \Rightarrow 3 \sum_{n=0}^{\infty} x^{4n}$

ⓒ $f(x) = \ln(x)$ en potencias de $(x-4)$

$$\ln(x) = \int \frac{1}{x}\,dx \cdot \int \frac{1}{4+x-4}\,dx = \frac{1}{4}\int \frac{1}{1+\frac{x-4}{4}}\,dx = \frac{1}{4}\int \frac{1}{1-(-\frac{x-4}{4})} = \frac{1}{4}\int \sum_{n=1}^{\infty} (-1)^n \left(\frac{x-4}{4}\right)^n dx = \frac{1}{4}\sum_{n=1}^{\infty} (-1)^n \int \left(\frac{x-4}{4}\right)^n dx$$

$$= \frac{1}{4}\sum_{n=1}^{\infty} (-1)^n \int (x-4)^n \cdot \left(\frac{1}{4}\right)^n dx = \frac{1}{4}\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{4}\right)^n \cdot \int (x-4)^n\,dx = \frac{1}{4}\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{4}\right)^n \cdot \frac{(x-4)^{n+1}}{n+1} = \sum_{n=1}^{\infty} (-1)^n \cdot \left(\frac{1}{4}\right)^{n+1} \cdot \frac{(x-4)^{n+1}}{n+1}$$ //

ⓓ $f(x) = \frac{1}{x^2}$

$$\frac{1}{x^2} = \left(-\frac{1}{x}\right)'$$

$$= \left(-\frac{1}{x+2-2}\right)' = \left(\frac{1}{2\left(1-\frac{x+2}{2}\right)}\right)' = \frac{1}{2}\left(\frac{1}{1-\frac{x+2}{2}}\right)' = \frac{1}{2}\left(\sum_{n=1}^{\infty}\left(\frac{x+2}{2}\right)^n\right)' = \frac{1}{2}\sum_{n=1}^{\infty}\left(\left(\frac{x+2}{2}\right)^n\right)' = \frac{1}{2}\sum_{n=}^{\infty}\left(\frac{1}{2}\right)^n \cdot \left((x+2)^n\right)' = \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n \cdot \frac{(x+2)^{n+1}}{n+1}$$

ⓔ $f(x) = x \ln(1-x)$

$$x \cdot \int -\frac{1}{1-x}\,dx = x \cdot -\int \frac{1}{1-x}\,dx = x \cdot -\int \sum_{n=1}^{\infty} x^n\,dx = x \cdot -\sum_{n=1}^{\infty} \int x^n\,dx = x \cdot -\sum_{n=1}^{\infty} x^{n+1} = -\sum_{n=1}^{\infty} x^{n+2}$$

④ Expresar las siguientes integrales como una serie de potencias en x

ⓐ $\int \frac{1}{1+x^4}\,dx =$

$$\int \sum_{n=0}^{\infty} (-1)^n x^{4n}\,dx = \sum_{n=0}^{\infty} (-1)^n \int x^{4n}\,dx = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{4n+1} + C$$

ⓑ $\int \frac{x}{1+x^5}\,dx = \int x \frac{1}{1+x^5}$

$$\int x \cdot \sum_{n=0}^{\infty} (-1)^n x^{5n}\,dx = \int \sum_{n=0}^{\infty} (-1)^n x^{5n+1}\,dx = \sum_{n=1}^{\infty} (-1)^n \int x^{5n+1}\,dx = \sum_{n=0}^{\infty} (-1)^n \frac{x^{5n+2}}{5n+2} + C$$

ⓒ $\int \frac{x}{1-x^8}\,dx = \int x \frac{1}{1-x^8}\,dx$

$$\int x \sum_{n=0}^{\infty} x^{8n}\,dx = \int \sum_{n=0}^{\infty} x^{8n+1}\,dx = \sum_{n=0}^{\infty} \int x^{8n+1}\,dx = \sum_{n=0}^{\infty} \frac{x^{8n+2}}{8n+2} + C$$

ⓓ $\int \frac{\ln(1-x)}{x}\,dx$

$\ln(1-x) = (-1)\sum_{n=1}^{\infty} \frac{x^n}{n}$, convergente para $|x|<1$

$$\Rightarrow \frac{\ln(1-x)}{x} = \frac{1}{x}\cdot(-1)\sum_{n=1}^{\infty}\frac{x^n}{n} = (-1)\sum_{n=1}^{\infty}\frac{x^{n-1}}{n}$$

$$\Rightarrow \int \frac{\ln(1-x)}{x}\,dx = (-1)\int \sum_{n=1}^{\infty} \frac{x^{n-1}}{n}\,dx$$

$$= (-1)\sum_{n=1}^{\infty} \int \frac{x^{n-1}}{n}\,dx$$

$$= (-1)\sum_{n=1}^{\infty} \frac{1}{n}\cdot\frac{x^n}{n} + C$$

$$= (-1)\sum_{n=1}^{\infty} \frac{x^n}{n^2} + C$$

⑤ ⓐ $f(x) = \cos(x)$

$n=0$ $f(x) \longrightarrow f(0) = \cos(0) = 1$

$n=1$ $f^{(1)}(x) \longrightarrow f^{(1)}(x) = -\operatorname{sen}(0) = 0$

$n=2$ $f^{(2)}(x) \longrightarrow f^{(2)}(x) = -\cos(0) = -1$

$n=3$ $f^{(3)}(x) \longrightarrow f^{(3)}(x) = \operatorname{sen}(0) = 0$

$n=4$ $f^{(4)}(x) \longrightarrow f^{(4)}(x) = \cos(0) = 1$

$\Rightarrow f^{(2n)} = (-1)^n$ num par

$f^{(2n+1)} = 0$

$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{3!} - \frac{x^6}{6!} + \dots$

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{(2n)!}$$

falta calcular R.C e I.C

ⓑ $f(x) = \ln(1+x)$

$\ln(x) = \ln(1+x)$

$n=0$ $f(x) \longrightarrow f(0) = \ln(1) = 0$

$n=1$ $f'(x) \longrightarrow f'(x) = \frac{1}{1+x} \longrightarrow f'(0) = 1$

$n=2$ $f''(x) \longrightarrow f''(x) = -\frac{1}{(1+x)^2} \longrightarrow f''(0) = -1$

$n=3$ $f'''(x) \longrightarrow f'''(x) = \frac{2}{(1+x)^3} \longrightarrow f'''(0) = 2$

$n=4$ $f''''(x) \longrightarrow f^{(4)}(x) = -\frac{6}{(1+x)^4} \longrightarrow f^{(4)}(0) = -6$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n \Rightarrow 1\cdot x - \frac{1\cdot x^2}{2!} + \frac{2\cdot x^3}{3!} - \frac{6\cdot x^4}{4!} + \dots$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}(x-0)^n$$

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+2}}{n+1}}{\frac{(-1)^{n+1}}{n}} \right| = \lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+1}}\cdot(-1)\cdot n}{\cancel{(-1)^{n+1}}\cdot(n+1)} \right| = \lim_{n\to\infty} \left| \frac{-n}{n+1} \right| = \lim_{n\to\infty} \left| \frac{-\cancel{n}}{\cancel{n}(1+\frac{1}{n})} \right| = \lim_{n\to\infty} |-1| = 1$$

R.C = 1

$I = [(a-R, a+R)] = [-1, 1]$

$x=-1$ $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}\cdot(-1)^n = \sum_{n=1}^{\infty} (-1)^{n+2} \frac{1}{n} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ ∴ conv. por CSA

$x=1$ $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}\cdot 1^n = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ ∴ conv. por CSA

ⓒ $f(x) = \operatorname{sen}(5x^2)$

Usamos la serie del $\operatorname{sen}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ reemplazamos x por $5x^2$ $\sum_{n=0}^{\infty} (-1)^n \frac{(5x^2)^{2n+1}}{(2n+1)!}$

Falta int conv

ⓓ $f(x) = xe^x$

Falta int conv

$$1 + e^0 \cdot x + \frac{e^0 \cdot x^2}{2!} + \frac{e^0 \cdot x^3}{3!} + \dots \quad \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow xe^x = x \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$$

⑥ ⓐ $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $e^{0.1}$ ✓

Hay que hallar n tal que

$|f(x) - T_{n,0}(x)| = |R_{n,0}(x)| < 5 \cdot 10^{-5}$

$R_{n,0} = \frac{e^{(t)}}{(n+1)!} (0.1)^{n+1}$ para algun $t \in (0, 0.1)$

$$\leq \frac{e^{0.1}}{(n+1)!}(0.1)^{n+1} < \frac{e^1}{(n+1)!}(0.1)^{n+1} < \frac{5}{(n+1)!}(0.1)^{n+1} < 5 \cdot \frac{1}{10^5}$$

Basta encontrar un n tal que

$$\frac{5}{(n+1)!} \frac{1}{10^{n+1}} < 5 \cdot \frac{1}{10^5}$$

$$\frac{5}{(n+1)!} \frac{1}{10^{n+1}} < 5 \cdot \frac{1}{10^5}$$

$$\frac{1}{(n+1)!\,10^{n+1}} < 5 \cdot \frac{1}{10^5}$$

$\Rightarrow$ $10^5 = 100.000$

$n=1$ $\quad \frac{1}{2!\,10^2 \cdot 5} < \frac{1}{100.000} \Rightarrow \frac{1}{1000} < \frac{1}{100.000}$

$n=2$ $\quad \frac{1}{3!\,10^3 \cdot 5} < \frac{1}{100.000} \Rightarrow \frac{1}{30.000} < \frac{1}{100.000}$

$n=3$ $\quad \frac{1}{4! \cdot 10^4 \cdot 5} < \frac{1}{100.000} \Rightarrow \frac{1}{1.200.000} < \frac{1}{100.000}$

$\Rightarrow$ $n=3$ satisface que $|e^{0.1} - T_{3,0}(0.1)| = |R_{3,0}(0.1)| \leq 5 \cdot 10^{-5}$

(b) $\ln(1{,}4)$ ✓

$1{,}4 = 1 + 0{,}4$

$$\ln(1+x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{k+1}}{(k+1)!}$$

$$f^{(n+1)}(x) = \frac{(-1)^n \cdot n!}{(n+x)^{n+1}}$$

$$T_{n,0}(0{,}4) = \sum_{k=0}^{n} (-1)^k \frac{x^{k+1}}{(k+1)}$$

$$(1+t)^{n+1} > 1$$

$$R_{n,0}(0{,}4) = \frac{f^{(n+1)}(t)}{(n+1)!} \cdot (0{,}4)^{n+1} \qquad t \in (0,\ 0.4)$$

$$\frac{1}{(1+t)^{n+1}} \leq 1$$

$$|R_{n,0}(0{,}4)| < \ldots < 5 \cdot 10^{-5}$$

$$\Rightarrow \left| \frac{n!}{(1+t)^{n+1} \cdot (n+1)!} \cdot (0{,}4)^{n+1} \right| < \left| \frac{\cancel{n!}}{(n+1) \cdot \cancel{n!}} (0{,}4)^{n+1} \right| < \frac{(0{,}4)^{n+1}}{n+1} < 5 \cdot 10^{-5}$$

$$\frac{\left(\frac{2}{5}\right)^{n+1}}{n+1} < \frac{1}{500.000}$$

$$n=5 \quad \frac{\left(\frac{2}{5}\right)^6}{6} < \frac{1}{500.000} \Rightarrow \frac{\frac{64}{15.625}}{6} < \frac{1}{500.000} \quad \text{ABS}$$

$$n=7 \quad \frac{\left(\frac{2}{5}\right)^8}{8} < \frac{1}{500.000} \Rightarrow \frac{\frac{256}{390.625}}{8} < \frac{1}{500.000} \Rightarrow \frac{\cancel{256}\ 32}{1\cancel{8} \cdot 390.625} < \frac{1}{500.000} \quad \text{ABS}$$

$$n=8 \quad \frac{\left(\frac{2}{5}\right)^9}{9} < \frac{1}{500.000} \Rightarrow \frac{\frac{512}{1.953.125}}{9} < \frac{1}{500.000} \Rightarrow \frac{512}{9 \cdot 1.953.125} < \frac{1}{500.000} \quad \checkmark$$

$n=8$

⑦ Estimar el error cometido al aproximar $f(x)=\sqrt[3]{x}$ por su polinomio de Taylor de orden 2 centrado en $a=8$, para $7 \leq x \leq 9$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$ $\qquad T_{2,8} = x^{\frac{1}{3}} + \frac{1}{3}x^{-\frac{2}{3}}(-1) - \frac{2}{9}x^{-\frac{5}{3}} \cdot 1$

$f''(x) = -\frac{2}{9}x^{-\frac{5}{3}}$

$f'''(x) = \frac{10}{27}x^{-\frac{8}{3}}$ $\qquad \frac{10}{27}\sqrt[3]{9^8}$

$$R_{2,8}(7) = \left|\frac{f^{(3)}(t)}{3!}(-1)^3\right| = \frac{f^{(3)}(t)}{3!} = \frac{\frac{10\sqrt[3]{7}}{9261}}{6} = \frac{10\sqrt[3]{7}}{55.566} \approx 0{,}00034$$

$$R_{2,8}(9) = \left|\frac{f^{(3)}(t)}{3!}1^3\right| = \frac{f^{(3)}(t)}{3!} = \frac{\frac{10\sqrt[3]{9}}{19683}}{6} = \frac{10\sqrt[3]{9}}{118.098} \approx 0{,}00017$$

$f(7) = \sqrt[3]{7} \approx 1{,}9129$

$f(9) = \sqrt[3]{9} \approx 2{,}0800$

$$T_{2,8}(7) = \sqrt[3]{8} + \frac{1}{3}8^{-\frac{2}{3}}(7-8) - \frac{2}{9}8^{-\frac{5}{3}}(7-8)^2 = 2 - \frac{1}{3}\frac{1}{\sqrt[3]{8^2}} - \frac{2}{9}\frac{1}{\sqrt[3]{8^5}} = 2 - \frac{1}{3}\cdot\frac{1}{4} - \frac{2}{9}\cdot\frac{1}{32} = 2 - \frac{1}{12} - \frac{1}{9\cdot 16} = \frac{275}{144} \approx 1{,}9097$$

$$T_{2,8}(9) = \sqrt[3]{8} + \frac{1}{3}8^{-\frac{2}{3}} - \frac{2}{9}8^{-\frac{5}{3}} = 2 + \frac{1}{3}\cdot\frac{1}{\sqrt[3]{8^2}} - \frac{2}{9}\frac{1}{\sqrt[3]{8^5}} = 2 + \frac{1}{3}\cdot\frac{1}{4} - \frac{2}{9}\cdot\frac{1}{32} = 2 + \frac{1}{12} - \frac{1}{9\cdot 16} = \frac{299}{144} \approx 2{,}0763$$

$|f(7) - T_{2,8}(7)| = |1{,}9129 - 1{,}9097| = 0{,}0032$

✓ Precision hasta el segundo decimal

$|f(9) - T_{2,8}(9)| = |2{,}0800 - 2{,}0763| = 0{,}0037$