Szalontai Panna GSACF9

③

8	8	1	4	$\boxed{9}$
7	9	8	8	3
3	9	7	1	4
4	9	5	6	2
7	8	6	6	6

1	1	8	5	0	
2	0	1	1	6	
6	0	2	8	5	
5	0	4	3	7	
2	1	3	3	3	(-1)

1	1	8	5	0
2	0	1	1	6
6	0	2	8	5
5	0	4	3	7
1	0	2	2	2
(-1)	(-1)	(-1)		

(-1)

0	1	7	4	0	
1	0	0	0	6	4.
5	0	$\boxed{1}$	7	4	
1	0	3	2	6	
0	0	1	1	1	
2.	1.			3.	

0	1	6	4	0*	4.
2	1	0	0*	7	
5	0	0*	6	4	
4	0*	3	1	6	
0*	0	0	0	1	5.
1.	2.	3.			

5 vonallal kell lefedni az összes 0-át. A csillaggal jelölt helyek jelölik az optimális hozzárendelést.
A max összpontszám.

$9+8+7+9+7 = \underline{\underline{40}}$

②

	F_1	F_2	F_3	k
T_1	7	2	8	34
T_2	6	4	8	17
i	26	14	17	57 / 51

$\Rightarrow$

	F_1	F_2	F_3	k
T_1	7	2	8	34
T_2	6	4	8	17
T_3	0	0	0	6
i	26	14	17	57

26	8	–	~~34~~ ~~8~~ 0
–	6	11	~~17~~ ~~11~~ 0
–	–	6	~~6~~ 0
~~26~~ 0	~~14~~ ~~6~~ 0	~~17~~ ~~6~~ 0	

$\Rightarrow$

	V_1 7		V_2 2		V_3 6	
u_1 0	26	7	8	2	–	8
u_2 2	–	6	6	4	11	8
u_3 -6	–	0	–	0	6	0

$u_1 + v_1 = 7$

$u_1 + v_2 = 2$

$u_2 + v_2 = 4$

$u_2 + v_3 = 8$

Szalontai Panna BSACE9

(2) Folytatása

$\overline{C_{13}} = C_{13} - u_1 - v_3 = 2$

$\overline{C_{21}} = C_{21} - u_2 - v_1 = -3$

$\overline{C_{31}} = C_{31} - u_3 - v_1 = 1$

$\overline{C_{32}} = C_{32} - u_3 - v_2 = 4$

$7 - 0 = v_1 = 2$

$6 - 7 = u_2 = 1$

$u_3 = 9 = 8 + 1$

$0 - 9 = u_3 = -9$

$2 - 0 = v_2 = 2$

	v_1 7	v_2 2	v_3 9
u_1 0	20 / 7	14 / 2	– / 8
u_2 1	6 / 6	– / 4	11 / 8
u_3 -9	– / 0	– / 0	6 / 0

$\overline{C_{13}} = C_{13} = (u_1 + v_3) = 8 - (0 + 9) = 1$

$\overline{C_{22}} = 4 - (-1 + 2) = 3$

$\overline{C_{31}} = 0 - (-9 + 7) = 2$

$\overline{C_{32}} - 0 - (-9 + 2) = 7$

$7 - 0 = v_1 = 7$

$6 - 7 = u_2 = 1$

$2 - 0 = v_2 = 2$

$8 - 8 = v_3 = 8$

$0 - 8 = u_3 = -8$

	$v_1 = 7$	$v_2 = 2$	v_3 8
u_1 0	9 / 7	14 / 2	11 / 8
u_2 1	17 / 6	– / 4	– / 8
u_3 -8	– / 0	– / 0	6 / 0

$\overline{C_{22}} = C_{22} - (u_2 + v_2) = 4 - (-1 + 2) = 5$

$\overline{C_{23}} = C_{23} - (u_2 + v_3) = 8 - (1 + 8) = 1$

$\overline{C_{31}} = C_{31} - (u_3 + v_1) = 0 - (-8 + 7) = 1$

$\overline{C_{32}} = C_{32} - (u_3 + v_2) = 0 - (-8 + 2) = 6$

$(7 \cdot 9) + (2 \cdot 14) + (8 \cdot 11) + (6 \cdot 17) \cdot (0 6) = 281$

Szalontai Panna G9ACF9

1.

$$-x_1+2x_2\le 12$$
$$8x_1+6x_2\le 91$$
$$9x_1+6x_2\le 99$$
$x_1, x_2, x_3 \ge 0$, egész számok
$$7x_1+5x_2\to max$$

$$\Rightarrow \begin{aligned}-x_1+2x_2+x_3 &= 12\\ 8x_1+6x_2+x_4 &= 91\\ 9x_1+6x_2+x_5 &= 99\end{aligned}$$

	x_1	x_2	x_3	x_4	x_5	b	t
x_3	-1	2	1	0	0	12	
x_4	8	6	0	1	0	91	91/8
x_5	9	6	0	0	1	99	11
Z	-7	-5	0	0	0	0	

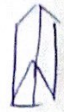
	x_1	x_2	x_3	x_4	x_5	b	t
x_3	0	8/3	1	0	1/9	23	69/8
x_4	0	2/3	0	1	-8/9	3	9/2
x_1	1	2/3	0	0	1/9	11	33/2
Z	0	-1/3	0	0	7/9	77	

	x_1	x_2	x_3	x_4	x_5	b
x_3	0	0	1	-4	11/3	11
x_2	0	1	0	3/2	-4/3	9/2
x_1	1	0	0	-1	1	8
Z	0	0	0	1/2	1/3	157/2

$$X=\left(8, \frac{9}{2}, 11, 0, 0\right)$$
$$Z=157/2$$

Szalontai Panna GSACE9

5. $V=\frac{a\cdot m}{2}\cdot h=\frac{1}{4}m^3$ $\qquad m=\frac{\sqrt{3}}{2}\cdot a$



$T_\Delta=\frac{\frac{\sqrt{3}}{2}\cdot a\cdot a}{2}=\frac{\frac{\sqrt{3}}{2}a^2}{2}=\frac{\sqrt{3}}{4}\cdot a^2$

$A=3\cdot a\cdot h+a\cdot m\to min$

$\downarrow$

$\frac{\sqrt{3}}{4}a^2\cdot h=\frac{1}{4}\to h=\frac{1}{4}:\frac{\sqrt{3}a^2}{4}$

$\downarrow$

$3\cdot a\cdot h+\frac{\sqrt{3}}{2}\cdot a^2\to min$ $\qquad h=\frac{\cancel{4}}{\cancel{4}\cdot\sqrt{3}\cdot a^2}=\frac{1}{\sqrt{3}\cdot a^2}$

$f(a)=\frac{3\cdot a}{\sqrt{3}\cdot a^2}+\frac{\sqrt{3}}{2}\cdot a^2=\frac{3\cdot a}{\sqrt{3}\cdot a^2}+\frac{\sqrt{3}a^2}{2}+\frac{6a+3a^4}{2\sqrt{3}a^2}$

$f'(a)=-\frac{\sqrt{3}}{a^2}+\sqrt{3}=0$

$\sqrt{3}a=+\frac{\sqrt{3}}{a^2}$

$\sqrt{3}a^3=\sqrt{3}$

$a^3=1$

$a=1$

$f''(a)=\frac{\sqrt{3}(a^3+2)}{a^3}=\sqrt{3}\cdot 3>0$ így $a=1$ esetén minimum van.

$a=1m$

$h=\frac{\sqrt{3}}{4}m$

A háromszög oldala 1m, a hasáb magassága $\frac{\sqrt{3}}{4}m$

$A=3\cdot 1\cdot\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{2}\cdot 1^2=\frac{\sqrt{3}\cdot 3}{4}+\frac{\sqrt{3}}{2}=\frac{\sqrt{3}\cdot 3+\sqrt{3}\cdot 2}{4}=\frac{5\cdot\sqrt{3}}{4}=2{,}1651m^2$

Szalontai Panna G9ACE9

6

$$x_1 + x_2 - 2 \le 0$$
$$x_1^2 + x_2^2 - 4 \le 0$$
$$6x_1 - 2x_2 + 5 \to \min$$

$$F(x_1, x_2, \mu_1, \mu_2) = 6x_1 - 2x_2 + 5 + \mu_1(x_1 + x_2 - 2) + \mu_2(x_1^2 + x_2^2 - 4)$$

1. $6 + \mu_1 + 2\mu_2 x_1 = 0$
2. $(-2) + \mu_1 + 2\mu_2 x_2 = 0$
3. $\mu_1 (x_1 + x_2 - 2) = 0$
4. $\mu_2 (x_1^2 + x_2^2 - 4) = 0$
5. $\mu_1, \mu_2 = 0$
6. $x_1 + x_2 - 2 \le 0$
7. $x_1^2 + x_2^2 - 4 \le 0$

I. μ_1 és μ_2 egyszerre nem lehet 0 az 1. és 2. miatt

Ha $\mu_2 = 0$, akkor az 1. pontnál és a 2. pontnál μ_1 nem ugyanaz a szám, így μ_2 nem lehet 0.

$\mu_2 > 0 \quad \mu_1 = 1$

II. $\mu_1 > 0$ és $\mu_2 > 0$

$$\left.\begin{aligned} x_1 + x_2 - 2 &= 0 \\ x_1^2 + x_2^2 - 4 &= 0 \end{aligned}\right\} \to x_1 = 2 - x_2$$

$$(2 - x_2)^2 + x_2 - 4 = 0$$
$$x^2 + x_2^2 - 4x_2 + 4 - 4 = 0$$
$$2x_2^2 - 4x_2 = 0$$
$$2x_2(x_2 - 4) = 0$$

$$\left.\begin{aligned} x_2 = 0 &\to x_1 - 2 \ (2;0) \\ x_2 = 4 &\to x_1 = -2 \ (-2;4) \end{aligned}\right\}$$

Nem KKT pont mert az 1. és 2. pont feltétele nem teljesül.

$1 + 3 \cdot 2.$

$$\left.\begin{aligned} 6 + 2\mu_2 x_1 &= 0 \\ -6 + \mu_2 x_2 &= 0 \end{aligned}\right\} +$$

$$2\mu_2 x_1 + 6\mu_2 x_2 = 0$$

$$\left.\begin{aligned} x_1 + 3x_2 &= 0 \\ x_1^2 + x_2^2 - 4 &= 0 \end{aligned}\right\} \to x_1 = (-3x_2)$$

$$(-3x_2)^2 + x_2^2 - 4 = 0$$
$$9x^2 + x_2^2 - 4 = 0$$
$$10x^2 = 4$$
$$x_2^2 = \frac{4}{10}$$

$x_1 = -3 \cdot \sqrt{\frac{4}{10}} = -1{,}8975 \quad x_2 = \sqrt{\frac{4}{10}} = 0{,}6325$

$(-1{,}8975; 0{,}6325)$

Megfelel minden pontnak így ez KKT pont.