L3 - Resumo

Sintaxe

$$
\begin{array}{rcl}
e & ::= & n \mid b \mid e_1 \; op \; e_2 \mid \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \\
& \mid & e_1 \texttt{:=} e_2 \mid \texttt{!} \; e \mid \mathsf{ref} \; e \mid l \\
& \mid & \texttt{skip} \mid e_1 \texttt{;} \; e_2 \\
& \mid & \texttt{while } e_1 \texttt{ do } e_2 \\
& \mid & fn \; x{:}T \Rightarrow e \mid e_1 \; e_2 \mid x \\
& \mid & \texttt{let } x{:}T = e_1 \texttt{ in } e_2 \texttt{ end} \\
& \mid & \texttt{let rec } f{:}T_1 \rightarrow T_2 = (fn \; y{:}T_1 \Rightarrow e_1) \texttt{ in } e_2 \texttt{ end} \\
& \mid & (e_1, e_2) \mid \sharp 1 \; e \mid \sharp 2 \; e \\
& \mid & \{lab_1 = e_1, \ldots lab_k = e_k\} \mid \sharp lab \; e \\
& \mid & \texttt{raise} \mid \texttt{try } e_1 \texttt{ with } e_2
\end{array}
$$

$$
v \; ::= \; n \mid b \mid \texttt{skip} \mid fn \; x{:}T \Rightarrow e \mid (v_1, v_2) \mid \{lab_1 = v_1, \ldots lab_n = v_n\} \mid l
$$

onde

$$
\begin{array}{lcl}
b & \in & \{\texttt{true}, \texttt{false}\} \\
n & \in & \textit{conjunto de numerais inteiros} \\
l & \in & \textit{conjunto de endereços} \\
op & \in & \{+, \geq\} \\
lab & \in & \textit{conjunto de rótulos}
\end{array}
$$

$$
T \; ::= \; \mathsf{int} \mid \mathsf{bool} \mid \mathsf{unit} \mid T_1 \rightarrow T_2 \mid T_1 * T_2 \mid \{lab_1 : T_1, \ldots lab_n : T_n\} \mid T \; \mathsf{ref}
$$

Semântica Operacional

$$
\frac{[\![n]\!] = [\![n_1 + n_2]\!]}{\langle n_1 + n_2, \; \sigma \rangle \longrightarrow \langle n, \; \sigma \rangle} \quad (\textsc{op}+)
$$

$$
\frac{[\![b]\!] = [\![n_1 \geq n_2]\!]}{\langle n_1 \geq n_2, \; \sigma \rangle \longrightarrow \langle b, \; \sigma \rangle} \quad (\textsc{op}\geq)
$$

$$
\frac{\langle e_1, \; \sigma \rangle \longrightarrow \langle e_1', \; \sigma' \rangle}{\langle e_1 \; op \; e_2, \; \sigma \rangle \longrightarrow \langle e_1' \; op \; e_2, \; \sigma' \rangle} \quad (\textsc{op}1)
$$

$$
\frac{\langle e_2, \; \sigma \rangle \longrightarrow \langle e_2', \; \sigma' \rangle}{\langle v \; op \; e_2, \; \sigma \rangle \longrightarrow \langle v \; op \; e_2', \; \sigma' \rangle} \quad (\textsc{op}2)
$$

$$
\langle \texttt{if true then } e_2 \texttt{ else } e_3, \; \sigma \rangle \longrightarrow \langle e_2, \; \sigma \rangle \quad (\textsc{if}1)
$$

$$
\langle \texttt{if false then } e_2 \texttt{ else } e_3, \; \sigma \rangle \longrightarrow \langle e_3, \; \sigma \rangle \quad (\textsc{if}2)
$$

$$
\frac{\langle e_1, \; \sigma \rangle \longrightarrow \langle e_1', \; \sigma' \rangle}{\langle \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3, \; \sigma \rangle \longrightarrow \langle \texttt{if } e_1' \texttt{ then } e_2 \texttt{ else } e_3, \; \sigma' \rangle} \quad (\textsc{if}3)
$$

$$\langle \texttt{skip;}e_2,\ \sigma\rangle \quad\longrightarrow\quad \langle e_2,\ \sigma\rangle \tag{SEQ1}$$

$$\frac{\langle e_1,\ \sigma\rangle \quad\longrightarrow\quad \langle e_1',\ \sigma'\rangle}{\langle e_1\texttt{;}e_2,\ \sigma\rangle \quad\longrightarrow\quad \langle e_1'\texttt{;}e_2,\ \sigma'\rangle} \tag{SEQ2}$$

$$\langle \texttt{while}\ e_1\ \texttt{do}\ e_2,\ \sigma\rangle \quad\longrightarrow\quad \langle \texttt{if}\ e_1\ \texttt{then}\ (e_2\texttt{;while}\ e_1\ \texttt{do}\ e_2)\ \texttt{else skip},\ \sigma\rangle \tag{WHILE}$$

$$\langle (fn\ x\texttt{:}T \Rightarrow e)\ v,\ \sigma\rangle \quad\longrightarrow\quad \langle \{v/x\}e,\ \sigma\rangle \tag{β}$$

$$\frac{\langle e_2,\ \sigma\rangle \quad\longrightarrow\quad \langle e_2',\ \sigma'\rangle}{\langle v\ e_2,\ \sigma\rangle \quad\longrightarrow\quad \langle v\ e_2',\ \sigma'\rangle} \tag{APP1}$$

$$\frac{\langle e_1,\ \sigma\rangle \quad\longrightarrow\quad \langle e_1',\ \sigma'\rangle}{\langle e_1\ e_2,\ \sigma\rangle \quad\longrightarrow\quad \langle e_1'\ e_2,\ \sigma'\rangle} \tag{APP2}$$

$$\langle \texttt{let}\ x\texttt{:}T = v\ \texttt{in}\ e_2\ \texttt{end},\ \sigma\rangle \quad\longrightarrow\quad \langle \{v/x\}e_2,\ \sigma\rangle \tag{LET1}$$

$$\frac{\langle e_1,\ \sigma\rangle \quad\longrightarrow\quad \langle e_1',\ \sigma'\rangle}{\langle \texttt{let}\ x\texttt{:}T = e_1\ \texttt{in}\ e_2\ \texttt{end},\ \sigma\rangle \quad\longrightarrow\quad \langle \texttt{let}\ x\texttt{:}T = e_1'\ \texttt{in}\ e_2\ \texttt{end},\ \sigma'\rangle} \tag{LET2}$$

$$\begin{array}{c}\langle \texttt{let rec}\ f\texttt{:}T_1 \to T_2 = (fn\ y\texttt{:}T_1 \Rightarrow e_1)\ \texttt{in}\ e_2\ \texttt{end},\ \sigma\rangle \\ \longrightarrow \\ \langle \{(fn\ y\texttt{:}T_1 \Rightarrow \texttt{let rec}\ f\texttt{:}T_1 \to T_2 = (fn\ y\texttt{:}T_1 \Rightarrow e_1)\ \texttt{in}\ e_1\ \texttt{end})/f\}e_2,\ \sigma\rangle\end{array} \tag{LETREC}$$

$$\frac{\langle e_2,\ \sigma\rangle \quad\longrightarrow\quad \langle e_2',\ \sigma'\rangle}{\langle \texttt{(}v, e_2\texttt{)},\ \sigma\rangle \quad\longrightarrow\quad \langle \texttt{(}v, e_2'\texttt{)},\ \sigma'\rangle} \tag{PAR1}$$

$$\frac{\langle e_1,\ \sigma\rangle \quad\longrightarrow\quad \langle e_1',\ \sigma'\rangle}{\langle \texttt{(}e_1, e_2\texttt{)},\ \sigma\rangle \quad\longrightarrow\quad \langle \texttt{(}e_1', e_2\texttt{)},\ \sigma'\rangle} \tag{PAR2}$$

$$\langle \sharp 1\ \texttt{(}v_1, v_2\texttt{)},\ \sigma\rangle \quad\longrightarrow\quad \langle v_1,\ \sigma\rangle \tag{PRJ1}$$

$$\langle \sharp 2\ \texttt{(}v_1, v_2\texttt{)},\ \sigma\rangle \quad\longrightarrow\quad \langle v_2,\ \sigma\rangle \tag{PRJ2}$$

$$\frac{\langle e,\ \sigma\rangle \quad\longrightarrow\quad \langle e',\ \sigma'\rangle}{\langle \sharp 1\ e,\ \sigma\rangle \quad\longrightarrow\quad \langle \sharp 1\ e',\ \sigma'\rangle} \tag{PRJ3}$$

$$\frac{\langle e,\ \sigma\rangle \quad\longrightarrow\quad \langle e',\ \sigma'\rangle}{\langle \sharp 2\ e,\ \sigma\rangle \quad\longrightarrow\quad \langle \sharp 2\ e',\ \sigma'\rangle} \tag{PRJ4}$$

$$\frac{\langle e_j,\ \sigma\rangle \quad\longrightarrow\quad \langle e_j',\ \sigma'\rangle}{\begin{array}{c}\langle \{lab_i = v_i^{i\in 1\ldots j-1}, lab_j = e_j, lab_k = e_k^{k\in j+1\ldots n}\},\ \sigma\rangle \\ \longrightarrow \\ \langle \{lab_i = v_i^{i\in 1\ldots j-1}, lab_j = e_j', lab_k = e_k^{k\in j+1\ldots n}\},\ \sigma'\rangle\end{array}} \tag{RECORD1}$$

$$\langle \sharp lab_i\ \{lab_1 = v_1, \ldots lab_n = v_n\},\ \sigma\rangle \quad\longrightarrow\quad \langle v_i,\ \sigma\rangle \tag{RECORD2}$$

$$\frac{\langle e,\ \sigma\rangle \quad\longrightarrow\quad \langle e',\ \sigma'\rangle}{\langle \sharp lab_i\ e,\ \sigma\rangle \quad\longrightarrow\quad \langle \sharp lab_i\ e',\ \sigma'\rangle} \tag{RECORD3}$$

$$\frac{l \notin Dom(\sigma)}{\langle \mathsf{ref}\ v,\ \sigma\rangle \quad\longrightarrow\quad \langle l,\ \sigma[l \mapsto v]\rangle} \tag{REF1}$$

$$\frac{\langle e,\ \sigma\rangle \quad\longrightarrow\quad \langle e',\ \sigma'\rangle}{\langle \mathsf{ref}\ e,\ \sigma\rangle \quad\longrightarrow\quad \langle \mathsf{ref}\ e',\ \sigma'\rangle} \tag{REF2}$$

$$\frac{l \in Dom(\sigma) \quad \sigma(l) = v}{\langle \texttt{!}\ l,\ \sigma\rangle \quad\longrightarrow\quad \langle v,\ \sigma\rangle} \tag{DEREF1}$$

$$\frac{\langle e,\ \sigma\rangle \quad\longrightarrow\quad \langle e',\ \sigma'\rangle}{\langle \texttt{!}\ e,\ \sigma'\rangle \quad\longrightarrow\quad \langle \texttt{!}\ e',\ \sigma'\rangle} \tag{DEREF2}$$

$$\frac{l \in Dom(\sigma)}{\langle l \texttt{:=}\ v,\ \sigma\rangle \quad\longrightarrow\quad \langle \texttt{skip},\ \sigma[l \mapsto v]\rangle} \tag{ATR1}$$

$$\frac{\langle e,\ \sigma\rangle \quad\longrightarrow\quad \langle e',\ \sigma'\rangle}{\langle l \texttt{:=}\ e,\ \sigma\rangle \quad\longrightarrow\quad \langle l \texttt{:=}\ e',\ \sigma'\rangle} \tag{ATR2}$$

$$\frac{\langle e_1,\ \sigma\rangle \quad\longrightarrow\quad \langle e_1',\ \sigma'\rangle}{\langle e_1 \texttt{:=}\ e_2,\ \sigma\rangle \quad\longrightarrow\quad \langle e_1' \texttt{:=}\ e_2,\ \sigma'\rangle} \tag{ATR3}$$

Exceções

todas as regras para propagação de **`raise`** <u>*mais*</u> *as seguintes regras:*

$$\langle \texttt{try}\ v_1\ \texttt{with}\ e_2,\ \sigma\rangle \ \rightarrow\ \langle v_1,\ \sigma\rangle \tag{TRY1}$$

$$\langle \texttt{try raise with}\ e_2,\ \sigma\rangle \ \rightarrow\ \langle e_2,\ \sigma\rangle \tag{TRY2}$$

$$\frac{\langle e_1,\ \sigma\rangle \ \rightarrow \langle e_1',\ \sigma'\rangle}{\langle \texttt{try}\ e_1\ \texttt{with}\ e_2,\ \sigma\rangle \rightarrow \langle \texttt{try}\ e_1'\ \texttt{with}\ e_2,\ \sigma'\rangle} \tag{TRY3}$$

Sistema de Tipos

$$\Gamma \vdash n : \mathsf{int} \tag{TINT}$$

$$\Gamma \vdash b : \mathsf{bool} \tag{TBOOL}$$

$$\frac{\Gamma \vdash e_1 : \mathsf{int} \qquad \Gamma \vdash e_2 : \mathsf{int}}{\Gamma \vdash e_1 + e_2 : \mathsf{int}} \tag{+}$$

$$\frac{\Gamma \vdash e_1 : \mathsf{int} \qquad \Gamma \vdash e_2 : \mathsf{int}}{\Gamma \vdash e_1 \geq e_2 : \mathsf{bool}} \tag{T$\geq$}$$

$$\frac{\Gamma \vdash e_1 : \mathsf{bool} \qquad \Gamma \vdash e_2 : T \qquad \Gamma \vdash e_3 : T}{\Gamma \vdash \mathtt{if}\ e_1\ \mathtt{then}\ e_2\ \mathtt{else}\ e_3 : T} \tag{TIf}$$

$$\Gamma \vdash \mathtt{skip} : \mathsf{unit} \tag{TSkip}$$

$$\frac{\Gamma \vdash e_1 : \mathsf{unit} \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1 \mathtt{;} e_2 : T} \tag{TSeq}$$

$$\frac{\Gamma \vdash e_1 : \mathsf{bool} \qquad \Gamma \vdash e_2 : \mathsf{unit}}{\Gamma \vdash \mathtt{while}\ e_1\ \mathtt{do}\ e_2 : \mathsf{unit}} \tag{TWhile}$$

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T} \tag{TVar}$$

$$\frac{\Gamma, x : T \vdash e : T'}{\Gamma \vdash fn x : T \Rightarrow e : T \rightarrow T'} \tag{TFn}$$

$$\frac{\Gamma \vdash e_1 : T \rightarrow T' \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1\ e_2 : T'} \tag{TApp}$$

$$\frac{\Gamma \vdash e_1 : T \qquad \Gamma, x : T \vdash e_2 : T'}{\Gamma \vdash \mathtt{let}\ x\mathtt{:}T = e_1\ \mathtt{in}\ e_2\ \mathtt{end} : T'} \tag{TLet}$$

$$\frac{\Gamma, f : T_1 \rightarrow T_2, y : T_1 \vdash e_1 : T_2 \qquad \Gamma, f : T_1 \rightarrow T_2 \vdash e_2 : T}{\Gamma \vdash \mathtt{let\ rec}\ f\mathtt{:}T_1 \rightarrow T_2 = (fn\ y\mathtt{:}T_1 \Rightarrow e_1)\ \mathtt{in}\ e_2\ \mathtt{end} : T} \tag{TLetRec}$$

$$\frac{\Gamma \vdash e_1 : T_1 \qquad \Gamma \vdash e_2 : T_2}{\Gamma \vdash (e_1, e_2) : T_1 *\ T_2} \tag{TPar}$$

$$\frac{\Gamma \vdash e : T_1 * T_2}{\Gamma \vdash \sharp 1\ e : T_1} \tag{TPrj1}$$

$$\frac{\Gamma \vdash e : T_1 * T_2}{\Gamma \vdash \sharp 2\ e : T_2} \tag{TPrj2}$$

$$\frac{\Gamma \vdash e_1 : T_1 \qquad \ldots \qquad \Gamma \vdash e_n : T_n}{\Gamma \vdash \{lab_1 = e_1, \ldots lab_n = e_n\} : \{lab_1 : T_1, \ldots lab_n : T_n\}} \tag{TRcd}$$

$$\frac{\Gamma \vdash e : \{lab_1 : T_1, \ldots lab_n : T_n\}}{\Gamma \vdash \sharp lab_i\ e : T_i} \tag{TPrj}$$

$$\frac{\Gamma \vdash e_1 : T\ \mathsf{ref} \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1 \mathtt{:=} e_2 : \mathsf{unit}} \tag{TAtr}$$

$$\frac{\Gamma \vdash e : T\ \mathsf{ref}}{\Gamma \vdash\ \mathtt{!}\ e : T} \tag{TDeref}$$

$$\frac{\Gamma \vdash e : T}{\Gamma \vdash \textsf{ref}\ e : T\ \textsf{ref}} \qquad (\textsc{TRef})$$

$$\frac{\Gamma(l) = T\ \textsf{ref}}{\Gamma \vdash l : T\ \textsf{ref}} \qquad (\textsc{TL})$$

$$\Gamma \vdash \texttt{raise} : T \qquad (\textsc{TRs})$$

$$\frac{\Gamma \vdash e_1 : T \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash \texttt{try}\ e_1\ \texttt{with}\ e_2 : T} \qquad (\textsc{TTry})$$

Subtipos

$$\frac{\Gamma \vdash e : S \qquad S \ <:\ T}{\Gamma \vdash e : T} \qquad (\textsc{T-Sub})$$

$$S \ <:\ S \qquad (\textsc{S-Refl})$$

$$\frac{S \ <: U \qquad U <: T}{S <: T} \qquad (\textsc{S-Trans})$$

$$\{lab_i : T_i^{i \in 1 \ldots n+k}\} \quad <: \quad \{lab_i : T_i^{i \in 1 \ldots n}\} \qquad (\textsc{S-RcdWidth})$$

$$\frac{S_i <: T_i \quad \textit{para cada } i \in 1 \ldots n}{\{lab_i : S^{i \in 1 \ldots n}\} \quad <: \quad \{lab_i : T^{i \in 1 \ldots n}\}} \qquad (\textsc{S-RcdDepth})$$

$$\frac{\{k_j : S_j^{j \in 1 \ldots n}\} \ \ \textit{é permutação de} \ \ \{l_i : T_i^{i \in 1 \ldots n}\}}{\{k_j : S_j^{j \in 1 \ldots n}\} \quad <: \quad \{l_i : T_i^{i \in 1 \ldots n}\}} \qquad (\textsc{S-RcdPerm})$$

$$\frac{T_1 \ <:\ S_1 \qquad S_2 \ <:\ T_2}{S_1 \to S_2 \quad <: \quad T_1 \to T_2} \qquad (\text{S-ARROW})$$