# L2 - Resumo

Sintaxe

$$
\begin{array}{rcl}
e & ::= & n \mid b \mid e_1\ op\ e_2 \mid \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 \\
 & \mid & l\texttt{:=}\ e \mid \texttt{!}\ l \\
 & \mid & \texttt{skip}\ \mid e_1\texttt{;}\ e_2 \\
 & \mid & \texttt{while}\ e_1\ \texttt{do}\ e_2 \\
 & \mid & fn\ x\texttt{:}T \Rightarrow e \mid e_1\ e_2 \mid x \\
 & \mid & \texttt{let}\ x\texttt{:}T = e_1\ \texttt{in}\ e_2\ \texttt{end} \\
 & \mid & \texttt{let rec}\ f\texttt{:}T_1 \rightarrow T_2 = (fn\ y\texttt{:}T_1 \Rightarrow e_1)\ \texttt{in}\ e_2\ \texttt{end}
\end{array}
$$

$$
v \ ::= \ n \mid b \mid \texttt{skip} \mid fn\ x\texttt{:}T \Rightarrow e
$$

onde

$$
\begin{array}{rcl}
b & \in & \{\texttt{true}, \texttt{false}\} \\
n & \in & \textit{conjunto de numerais inteiros} \\
l & \in & \textit{conjunto de endereços} \\
op & \in & \{+, \geq\}
\end{array}
$$

$$
\begin{array}{rcl}
T & ::= & \mathsf{int} \mid \mathsf{bool} \mid \mathsf{unit} \mid T_1 \rightarrow T_2 \\
T_{loc} & ::= & \mathsf{int\ ref}
\end{array}
$$

Semântica Operacional

$$
\frac{[\![n]\!] = [\![n_1\texttt{+}n_2]\!]}{\langle n_1\texttt{+}n_2,\ \sigma\rangle \ \longrightarrow \ \langle n,\ \sigma\rangle} \tag{OP+}
$$

$$
\frac{[\![b]\!] \models [\![n_1 \geq n_2]\!]|}{\langle n_1 \geq n_2,\ \sigma\rangle \ \longrightarrow \ \langle b,\ \sigma\rangle} \tag{OP$\geq$}
$$

$$
\frac{\langle e_1,\ \sigma\rangle \ \longrightarrow \ \langle e_1',\ \sigma'\rangle}{\langle e_1\ op\ e_2,\ \sigma\rangle \ \longrightarrow \ \langle e_1'\ op\ e_2,\ \sigma'\rangle} \tag{OP1}
$$

$$
\frac{\langle e_2,\ \sigma\rangle \ \longrightarrow \ \langle e_2',\ \sigma'\rangle}{\langle v\ op\ e_2,\ \sigma\rangle \ \longrightarrow \ \langle v\ op\ e_2',\ \sigma'\rangle} \tag{OP2}
$$

$$
\langle \texttt{if true then}\ e_2\ \texttt{else}\ e_3,\ \sigma\rangle \ \longrightarrow \ \langle e_2,\ \sigma\rangle \tag{IF1}
$$

$$
\langle \texttt{if false then}\ e_2\ \texttt{else}\ e_3,\ \sigma\rangle \ \longrightarrow \ \langle e_3,\ \sigma\rangle \tag{IF2}
$$

$$
\frac{\langle e_1,\ \sigma\rangle \ \longrightarrow \ \langle e_1',\ \sigma'\rangle}{\langle \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3,\ \sigma\rangle \ \longrightarrow \ \langle \texttt{if}\ e_1'\ \texttt{then}\ e_2\ \texttt{else}\ e_3,\ \sigma'\rangle} \tag{IF3}
$$

$$
\langle \texttt{skip;}e_2,\ \sigma\rangle \ \longrightarrow \ \langle e_2,\ \sigma\rangle \tag{SEQ1}
$$

$$\frac{\langle e_1,\ \sigma\rangle \ \longrightarrow\ \langle e_1',\ \sigma'\rangle}{\langle e_1\texttt{;}e_2,\ \sigma\rangle \ \longrightarrow\ \langle e_1'\texttt{;}e_2,\ \sigma'\rangle} \tag{SEQ2}$$

$$\frac{l \in Dom(\sigma)}{\langle l\texttt{:=}\ n,\ \sigma\rangle \ \longrightarrow\ \langle \texttt{skip},\ \sigma[l \mapsto n]\rangle} \tag{ATR1}$$

$$\frac{\langle e,\ \sigma\rangle \ \longrightarrow\ \langle e',\ \sigma'\rangle}{\langle l\texttt{:=}\ e,\ \sigma\rangle \ \longrightarrow\ \langle l\texttt{:=}\ e',\ \sigma'\rangle} \tag{ATR2}$$

$$\frac{l \in Dom(\sigma) \quad \sigma(l) = n}{\langle \texttt{!}\ l,\ \sigma\rangle \ \longrightarrow\ \langle n,\ \sigma\rangle} \tag{DEREF}$$

$$\langle \texttt{while}\ e_1\ \texttt{do}\ e_2,\ \sigma\rangle \ \longrightarrow\ \langle \texttt{if}\ e_1\ \texttt{then}\ (e_2\texttt{;}\texttt{while}\ e_1\ \texttt{do}\ e_2)\ \texttt{else}\ \texttt{skip},\ \sigma\rangle \tag{WHILE}$$

$$\langle (fn\ x\texttt{:}T\ \Rightarrow\ e)\ v,\ \sigma\rangle \ \longrightarrow\ \langle \{v/x\}e,\ \sigma\rangle \tag{$\beta$}$$

$$\frac{\langle e_2,\ \sigma\rangle \ \longrightarrow\ \langle e_2',\ \sigma'\rangle}{\langle v\ e_2,\ \sigma\rangle \ \longrightarrow\ \langle v\ e_2',\ \sigma'\rangle} \tag{APP1}$$

$$\frac{\langle e_1,\ \sigma\rangle \ \longrightarrow\ \langle e_1',\ \sigma'\rangle}{\langle e_1\ e_2,\ \sigma\rangle \ \longrightarrow\ \langle e_1'\ e_2,\ \sigma'\rangle} \tag{APP2}$$

$$\langle \texttt{let}\ x\texttt{:}T = v\ \texttt{in}\ e_2\ \texttt{end},\ \sigma\rangle \ \longrightarrow\ \langle \{v/x\}e_2,\ \sigma\rangle \tag{LET1}$$

$$\frac{\langle e_1,\ \sigma\rangle \ \longrightarrow\ \langle e_1',\ \sigma'\rangle}{\langle \texttt{let}\ x\texttt{:}T = e_1\ \texttt{in}\ e_2\ \texttt{end},\ \sigma\rangle \ \longrightarrow\ \langle \texttt{let}\ x\texttt{:}T = e_1'\ \texttt{in}\ e_2\ \texttt{end},\ \sigma'\rangle} \tag{LET2}$$

$$\begin{array}{c} \langle \texttt{let rec}\ f\texttt{:}T_1 \to T_2 = (fn\ y\texttt{:}T_1\ \Rightarrow\ e_1)\ \texttt{in}\ e_2\ \texttt{end},\ \sigma\rangle \\ \longrightarrow \\ \langle \{(fn\ y\texttt{:}T_1\ \Rightarrow\ \texttt{let rec}\ f\texttt{:}T_1 \to T_2 = (fn\ y\texttt{:}T_1\ \Rightarrow\ e_1)\ \texttt{in}\ e_1\ \texttt{end})/f\}e_2,\ \sigma\rangle \end{array} \tag{LETREC}$$

Sistema de Tipos

$$\Gamma; \Delta \vdash n : \mathsf{int} \tag{TINT}$$

$$\Gamma; \Delta \vdash b : \mathsf{bool} \tag{TBOOL}$$

$$\frac{\Gamma; \Delta \vdash e_1 : \mathsf{int} \qquad \Gamma; \Delta \vdash e_2 : \mathsf{int}}{\Gamma; \Delta \vdash e_1 + e_2 : \mathsf{int}} \tag{T+}$$

$$\frac{\Gamma; \Delta \vdash e_1 : \mathsf{int} \qquad \Gamma; \Delta \vdash e_2 : \mathsf{int}}{\Gamma; \Delta \vdash e_1 \geq e_2 : \mathsf{bool}} \tag{T$\geq$}$$

$$\frac{\Gamma; \Delta \vdash e_1 : \mathsf{bool} \qquad \Gamma; \Delta \vdash e_2 : T \qquad \Gamma; \Delta \vdash e_3 : T}{\Gamma; \Delta \vdash \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 : T} \tag{TIF}$$

$$\frac{\Gamma;\Delta \vdash e : \mathsf{int} \qquad \Delta(l) = \mathsf{int\ ref}}{\Gamma;\Delta \vdash l\texttt{:=}e : \mathsf{unit}} \quad (\textsc{Tatr})$$

$$\frac{\Delta(l) = \mathsf{int\ ref}}{\Gamma;\Delta \vdash \texttt{!}\ l : \mathsf{int}} \quad (\textsc{Tderef})$$

$$\Gamma;\Delta \vdash \texttt{skip} : \mathsf{unit} \quad (\textsc{Tskip})$$

$$\frac{\Gamma;\Delta \vdash e_1 : \mathsf{unit} \qquad \Gamma;\Delta \vdash e_2 : T}{\Gamma;\Delta \vdash e_1\texttt{;}e_2 : T} \quad (\textsc{seq})$$

$$\frac{\Gamma;\Delta \vdash e_1 : \mathsf{bool} \qquad \Gamma;\Delta \vdash e_2 : \mathsf{unit}}{\Gamma;\Delta \vdash \texttt{while}\ e_1\ \texttt{do}\ e_2 : \mathsf{unit}} \quad (\textsc{Twhile})$$

$$\frac{\Gamma(x) = T}{\Gamma;\Delta \vdash x : T} \quad (\textsc{Tvar})$$

$$\frac{\Gamma, x : T;\Delta \vdash e : T'}{\Gamma;\Delta \vdash fn\ x : T \Rightarrow e : T \to T'} \quad (\textsc{Tfn})$$

$$\frac{\Gamma;\Delta \vdash e_1 : T \to T' \qquad \Gamma;\Delta \vdash e_2 : T}{\Gamma;\Delta \vdash e_1\ e_2 : T'} \quad (\textsc{Tapp})$$

$$\frac{\Gamma;\Delta \vdash e_1 : T \qquad \Gamma, x : T;\Delta \vdash e_2 : T'}{\Gamma;\Delta \vdash \texttt{let}\ x\texttt{:}T = e_1\ \texttt{in}\ e_2\ \texttt{end} : T'} \quad (\textsc{Tlet})$$

$$\frac{\Gamma, f : T_1 \to T_2, y : T_1;\Delta \vdash e_1 : T_2 \qquad \Gamma, f : T_1 \to T_2;\Delta \vdash e_2 : T}{\Gamma;\Delta \vdash \texttt{let rec}\ f\texttt{:}T_1 \to T_2 = (fn\ y\texttt{:}T_1 \Rightarrow e_1)\ \texttt{in}\ e_2\ \texttt{end} : T} \quad (\textsc{Tletrec})$$

Propriedades

**Teorema 1 (Determinismo)** Se $\langle e,\ \sigma\rangle \longrightarrow \langle e',\ \sigma'\rangle$ e se $\langle e,\ \sigma\rangle \longrightarrow \langle e'',\ \sigma''\rangle$ então $\langle e',\ \sigma'\rangle = \langle e'',\ \sigma''\rangle$.

**Teorema 2 (Progresso)** Se $e$ é fechado e $\Gamma;\Delta \vdash e : T$ e $Dom(\Delta) \subseteq Dom(\sigma)$ então $e$ é um valor ou existe $e';\sigma'$ tal que $\langle e,\ \sigma\rangle \longrightarrow \langle e',\ \sigma'\rangle$ e $e'$ é fechado.

**Teorema 3 (Preservação)** Se $e$ é fechado e $\Gamma;\Delta \vdash e : T$ , $Dom(\Delta) \subseteq Dom(\sigma)$ e $\langle e,\ \sigma\rangle \longrightarrow \langle e',\ \sigma'\rangle$ então $\Gamma;\Delta \vdash e' : T$ e $Dom(\Delta) \subseteq Dom(\sigma')$ .

**Teorema 4 (Decidibilidade da Tipabilidade)** Dados ambiente $\Gamma$ e expressão $e$, existe algoritmo que decide se existe tipo $T$ tal que $\Gamma;\Delta \vdash e : T$ é verdadeiro ou não.