L1 - Resumo

Sintaxe

$$
\begin{array}{rcl}
e & ::= & n \mid b \mid e_1 \ op \ e_2 \mid \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 \\
& \mid & l\texttt{:=}\ e \mid \texttt{!}\ l \\
& \mid & \texttt{skip} \mid e_1\texttt{;}\ e_2 \\
& \mid & \texttt{while}\ e_1\ \texttt{do}\ e_2
\end{array}
$$

$$
v ::= n \mid b \mid \texttt{skip}
$$

onde

$$
\begin{array}{rcl}
b & \in & \{\texttt{true}, \texttt{false}\} \\
n & \in & \textit{conjunto de numerais inteiros} \\
l & \in & \textit{conjunto de endereços} \\
op & \in & \{+, \geq\}
\end{array}
$$

Semântica Operacional

$$
\frac{[\![n]\!] = [\![n_1\texttt{+}n_2]\!]}{\langle n_1\texttt{+}n_2,\ \sigma\rangle \longrightarrow \langle n,\ \sigma\rangle} \quad (\textsc{op}+)
$$

$$
\frac{[\![b]\!] \models [\![n_1 \geq n_2]\!]|}{\langle n_1 \geq n_2,\ \sigma\rangle \longrightarrow \langle b,\ \sigma\rangle} \quad (\textsc{op}\geq)
$$

$$
\frac{\langle e_1,\ \sigma\rangle \longrightarrow \langle e_1',\ \sigma'\rangle}{\langle e_1\ op\ e_2,\ \sigma\rangle \longrightarrow \langle e_1'\ op\ e_2,\ \sigma'\rangle} \quad (\textsc{op}1)
$$

$$
\frac{\langle e_2,\ \sigma\rangle \longrightarrow \langle e_2',\ \sigma'\rangle}{\langle v\ op\ e_2,\ \sigma\rangle \longrightarrow \langle v\ op\ e_2',\ \sigma'\rangle} \quad (\textsc{op}2)
$$

$$
\langle \texttt{if true then}\ e_2\ \texttt{else}\ e_3,\ \sigma\rangle \longrightarrow \langle e_2,\ \sigma\rangle \quad (\textsc{if}1)
$$

$$
\langle \texttt{if false then}\ e_2\ \texttt{else}\ e_3,\ \sigma\rangle \longrightarrow \langle e_3,\ \sigma\rangle \quad (\textsc{if}2)
$$

$$
\frac{\langle e_1,\ \sigma\rangle \longrightarrow \langle e_1',\ \sigma'\rangle}{\langle \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3,\ \sigma\rangle \longrightarrow \langle \texttt{if}\ e_1'\ \texttt{then}\ e_2\ \texttt{else}\ e_3,\ \sigma'\rangle} \quad (\textsc{if}3)
$$

$$
\langle \texttt{skip;}e_2,\ \sigma\rangle \longrightarrow \langle e_2,\ \sigma\rangle \quad (\textsc{seq}1)
$$

$$
\frac{\langle e_1,\ \sigma\rangle \longrightarrow \langle e_1',\ \sigma'\rangle}{\langle e_1\texttt{;}e_2,\ \sigma\rangle \longrightarrow \langle e_1'\texttt{;}e_2,\ \sigma'\rangle} \quad (\textsc{seq}2)
$$

$$
\frac{l \in Dom(\sigma)}{\langle l\texttt{:=}\ n,\ \sigma\rangle \longrightarrow \langle \texttt{skip},\ \sigma[l \mapsto n]\rangle} \quad (\textsc{atr}1)
$$

$$\frac{\langle e,\ \sigma\rangle \longrightarrow \langle e',\ \sigma'\rangle}{\langle l\texttt{:=}\ e,\ \sigma\rangle \longrightarrow \langle l\texttt{:=}\ e',\ \sigma'\rangle} \quad (\text{ATR2})$$

$$\frac{l \in Dom(\sigma)\quad \sigma(l) = n}{\langle \texttt{!}\ l,\ \sigma\rangle \longrightarrow \langle n,\ \sigma\rangle} \quad (\text{DEREF})$$

$$\langle \texttt{while}\ e_1\ \texttt{do}\ e_2,\ \sigma\rangle \longrightarrow \langle \texttt{if}\ e_1\ \texttt{then}\ (e_2\texttt{;while}\ e_1\ \texttt{do}\ e_2)\ \texttt{else skip},\ \sigma\rangle \quad (\text{WHILE})$$

Sistema de Tipos

$$\Delta \vdash n : \mathsf{int} \quad (\text{TINT})$$

$$\Delta \vdash b : \mathsf{bool} \quad (\text{TBOOL})$$

$$\frac{\Delta \vdash e_1 : \mathsf{int} \qquad \Delta \vdash e_2 : \mathsf{int}}{\Delta \vdash e_1 + e_2 : \mathsf{int}} \quad (\text{T}+)$$

$$\frac{\Delta \vdash e_1 : \mathsf{int} \qquad \Delta \vdash e_2 : \mathsf{int}}{\Delta \vdash e_1 \geq e_2 : \mathsf{bool}} \quad (\text{T}\geq)$$

$$\frac{\Delta \vdash e_1 : \mathsf{bool} \qquad \Delta \vdash e_2 : T \qquad \Delta \vdash e_3 : T}{\Delta \vdash \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 : T} \quad (\text{TIF})$$

$$\frac{\Delta \vdash e : \mathsf{int} \qquad \Delta(l) = \mathsf{int\ ref}}{\Delta \vdash l\texttt{:=}e : \mathsf{unit}} \quad (\text{TATR})$$

$$\frac{\Delta(l) = \mathsf{int\ ref}}{\Delta \vdash \texttt{!}\ l : \mathsf{int}} \quad (\text{TDEREF})$$

$$\Delta \vdash \texttt{skip} : \mathsf{unit} \quad (\text{TSKIP})$$

$$\frac{\Delta \vdash e_1 : \mathsf{unit} \qquad \Delta \vdash e_2 : T}{\Delta \vdash e_1\texttt{;}e_2 : T} \quad (\text{TSEQ})$$

$$\frac{\Delta \vdash e_1 : \mathsf{bool} \qquad \Delta \vdash e_2 : \mathsf{unit}}{\Delta \vdash \texttt{while}\ e_1\ \texttt{do}\ e_2 : \mathsf{unit}} \quad (\text{TWHILE})$$

Propriedades

Teorema 1 (Determinismo) Se $\langle e,\ \sigma\rangle \longrightarrow \langle e',\ \sigma'\rangle$ e se $\langle e,\ \sigma\rangle \longrightarrow \langle e'',\ \sigma''\rangle$ então $\langle e',\ \sigma'\rangle = \langle e'',\ \sigma''\rangle$.

Teorema 2 (Progresso) Se $\Delta \vdash e : T$ e $Dom(\Delta) \subseteq Dom(\sigma)$ então (i) e é valor, ou (ii) existe $\langle e',\ \sigma'\rangle$ tal que $\langle e,\ \sigma\rangle \longrightarrow \langle e',\ \sigma'\rangle$

Teorema 3 (Preservação) Se $\Delta \vdash e : T$ e $Dom(\Delta) \subseteq Dom(\sigma)$ e $\langle e,\ \sigma\rangle \longrightarrow \langle e',\ \sigma'\rangle$ então $\Delta \vdash e' : T$ e $Dom(\Delta) \subseteq Dom(\sigma')$

Teorema 4 (Decidibilidade da Tipabilidade) Dados ambiente Δ e expressão e, existe algoritmo que decide se existe tipo T tal que $\Delta \vdash e : T$ é verdadeiro ou não.