L3 - *Big-Step* e Sistema de Tipos

Sintaxe

$$
\begin{array}{rcl}
e & ::= & n \mid b \mid e_1 \ op \ e_2 \mid \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 \\
& \mid & e_1 \texttt{:=}\ e_2 \mid \texttt{!}\ e \mid \mathsf{ref}\ e \\
& \mid & \texttt{skip} \mid e_1\texttt{;}\ e_2 \\
& \mid & \texttt{while}\ e_1\ \texttt{do}\ e_2 \\
& \mid & \texttt{fn}\ x\texttt{:}T \Rightarrow e \mid e_1\ e_2 \mid x \\
& \mid & \texttt{let}\ x\texttt{:}T = e_1\ \texttt{in}\ e_2\ \texttt{end} \\
& \mid & \texttt{let rec}\ f\texttt{:}T_1 \to T_2 = (\texttt{fn}\ y\texttt{:}T_1 \Rightarrow e_1)\ \texttt{in}\ e_2\ \texttt{end}
\end{array}
$$

onde

$$
\begin{array}{rcl}
n & \in & \textit{conjunto de numerais inteiros} \\
b & \in & \{\texttt{true}, \texttt{false}\} \\
op & \in & \{+, \geq\}
\end{array}
$$

Semântica Operacional Big-Step

$$\frac{}{\rho \vdash n, \sigma \ \Downarrow \ n, \sigma} \quad (\textsc{num})$$

$$\frac{}{\rho \vdash b, \sigma \ \Downarrow \ b, \sigma} \quad (\textsc{bool})$$

$$\frac{\rho \vdash e_1, \sigma \ \Downarrow \ n_1, \sigma' \qquad \rho \vdash e_2, \sigma' \ \Downarrow \ n_2, \sigma'' \qquad n = n_1 + n_2}{\rho \vdash e_1 \texttt{+} e_2, \sigma \ \Downarrow \ n, \sigma''} \quad (\textsc{op}+)$$

$$\frac{\rho \vdash e_1, \sigma \ \Downarrow \ n_1, \sigma' \qquad \rho \vdash e_2, \sigma' \ \Downarrow \ n_2, \sigma'' \qquad b = n_1 \geq n_2}{\rho \vdash e_1 \geq e_2, \sigma \ \Downarrow \ b, \sigma''} \quad (\textsc{op}\geq)$$

$$\frac{\rho \vdash e_1, \sigma \ \Downarrow \ \texttt{true}, \sigma' \qquad \rho \vdash e_2, \sigma' \ \Downarrow \ v, \sigma''}{\rho \vdash \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3, \sigma \ \Downarrow \ v, \sigma''} \quad (\textsc{if}1)$$

$$\frac{\rho \vdash e_1, \sigma \ \Downarrow \ \texttt{false}, \sigma' \qquad \rho \vdash e_3, \sigma' \ \Downarrow \ v, \sigma''}{\rho \vdash \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3, \sigma \ \Downarrow \ v, \sigma''} \quad (\textsc{if}2)$$

$$\frac{\rho \vdash e_1, \sigma \ \Downarrow \ l, \sigma' \qquad \rho \vdash e_2, \sigma' \ \Downarrow \ v, \sigma''}{\rho \vdash e_1 \texttt{:=}\ e_2, \sigma \ \Downarrow \ \texttt{skip}, \sigma''[l \mapsto v]} \quad (\textsc{atr})$$

$$\frac{\rho \vdash e, \sigma \ \Downarrow \ l, \sigma' \qquad v = \sigma'(l)}{\rho \vdash \texttt{!}\ e, \sigma \ \Downarrow \ v, \sigma'} \quad (\textsc{deref})$$

$$\frac{\rho \vdash e, \sigma \ \Downarrow \ v, \sigma'}{\rho \vdash \mathsf{ref}\ e, \sigma \ \Downarrow \ l, \sigma'[l \mapsto v]} \quad (\textsc{ref})$$

$$\frac{}{\rho \vdash \texttt{skip}, \sigma \ \Downarrow \ \texttt{skip}, \sigma} \quad (\textsc{skip})$$

$$\frac{\rho \vdash e_1, \sigma \ \Downarrow \ \texttt{skip}, \sigma' \qquad \rho \vdash e_2, \sigma' \ \Downarrow \ v, \sigma''}{\rho \vdash e_1 \texttt{;} e_2, \sigma \ \Downarrow \ v, \sigma''} \quad (\textsc{seq})$$

$$\frac{\rho \vdash e_1, \sigma \ \Downarrow \ \texttt{true}, \sigma' \qquad \rho \vdash e_2 \texttt{;while } e_1 \texttt{ do } e_2, \sigma' \ \Downarrow \ \texttt{skip}, \sigma''}{\rho \vdash \texttt{while } e_1 \texttt{ do } e_2, \sigma \ \Downarrow \ \texttt{skip}, \sigma''} \quad (\textsc{while1})$$

$$\frac{\rho \vdash e_1, \sigma \ \Downarrow \ \texttt{false}, \sigma'}{\rho \vdash \texttt{while } e_1 \texttt{ do } e_2, \sigma \ \Downarrow \ \texttt{skip}, \sigma'} \quad (\textsc{while2})$$

$$\rho \vdash \texttt{fn } \ x\texttt{:}T \Rightarrow e, \sigma \ \Downarrow \ < x, e, \rho >, \sigma \quad (\textsc{fn})$$

$$\frac{\rho \vdash e_1, \sigma \ \Downarrow \ < x, e, \rho' >, \sigma' \qquad \rho \vdash e_2, \sigma' \ \Downarrow \ v_2, \sigma'' \qquad \rho'[x \mapsto v_2] \vdash e, \sigma'' \ \Downarrow \ v, \sigma'''}{\rho \vdash e_1 \ e_2, \sigma \ \Downarrow \ v, \sigma'''} \quad (\textsc{apl})$$

$$\frac{v = \rho(x)}{\rho \vdash x, \sigma \ \Downarrow \ v, \sigma} \quad (\textsc{id})$$

$$\frac{\rho \vdash e_1, \sigma \ \Downarrow \ v_1, \sigma' \qquad \rho[x \mapsto v_1] \vdash e_2, \sigma' \ \Downarrow \ v_2, \sigma''}{\rho \vdash \texttt{let } x\texttt{:}T = e_1 \texttt{ in } e_2 \texttt{ end}, \sigma \ \Downarrow \ v_2, \sigma''} \quad (\textsc{let})$$

Sistema de Tipos

$$\Gamma \vdash n : \mathsf{int} \quad (\textsc{Tint})$$

$$\Gamma \vdash b : \mathsf{bool} \quad (\textsc{Tbool})$$

$$\frac{\Gamma \vdash e_1 : \mathsf{int} \qquad \Gamma \vdash e_2 : \mathsf{int}}{\Gamma \vdash e_1 + e_2 : \mathsf{int}} \quad (+)$$

$$\frac{\Gamma \vdash e_1 : \mathsf{int} \qquad \Gamma \vdash e_2 : \mathsf{int}}{\Gamma \vdash e_1 \geq e_2 : \mathsf{bool}} \quad (\mathrm{T}\geq)$$

$$\frac{\Gamma \vdash e_1 : \mathsf{bool} \qquad \Gamma \vdash e_2 : T \qquad \Gamma \vdash e_3 : T}{\Gamma \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 : T} \quad (\textsc{Tif})$$

$$\frac{\Gamma \vdash e_1 : T \ \mathsf{ref} \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1\texttt{:=}e_2 : \mathsf{unit}} \quad (\textsc{Tatr})$$

$$\frac{\Gamma \vdash e : T \ \mathsf{ref}}{\Gamma \vdash \texttt{!} \ e : T} \quad (\textsc{Tderef})$$

$$\frac{\Gamma \vdash e : T}{\Gamma \vdash \mathsf{ref} \ e : T \ \mathsf{ref}} \quad (\textsc{Tref})$$

$$\Gamma \vdash \texttt{skip} : \mathsf{unit} \qquad \text{(TSKIP)}$$

$$\frac{\Gamma \vdash e_1 : \mathsf{unit} \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1 \texttt{;} e_2 : T} \qquad \text{(TSEQ)}$$

$$\frac{\Gamma \vdash e_1 : \mathsf{bool} \qquad \Gamma \vdash e_2 : \mathsf{unit}}{\Gamma \vdash \texttt{while}\ e_1\ \texttt{do}\ e_2 : \mathsf{unit}} \qquad \text{(TWHILE)}$$

$$\frac{\Gamma, x : T \vdash e : T'}{\Gamma \vdash \texttt{fn} x : T \Rightarrow e : T \to T'} \qquad \text{(TFN)}$$

$$\frac{\Gamma \vdash e_1 : T \to T' \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1\ e_2 : T'} \qquad \text{(TAPP)}$$

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T} \qquad \text{(TVAR)}$$

$$\frac{\Gamma \vdash e_1 : T \qquad \Gamma, x : T \vdash e_2 : T'}{\Gamma \vdash \texttt{let}\ x\texttt{:}T = e_1\ \texttt{in}\ e_2\ \texttt{end} : T'} \qquad \text{(TLET)}$$

$$\frac{\Gamma, f : T_1 \to T_2, y : T_1 \vdash e_1 : T_2 \qquad \Gamma, f : T_1 \to T_2 \vdash e_2 : T}{\Gamma \vdash \texttt{let rec}\ f\texttt{:}T_1 \to T_2 = (\texttt{fn}\ y\texttt{:}T_1 \Rightarrow e_1)\ \texttt{in}\ e_2\ \texttt{end} : T} \qquad \text{(TLETREC)}$$