

# Introduction to volumetric visualization

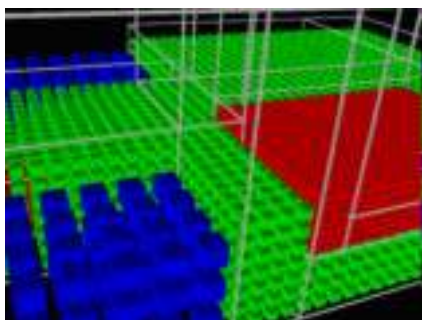
Incluindo slides disponibilizados por  
Chuck Hansen e Luciana Nedel

C. Freitas (2009)

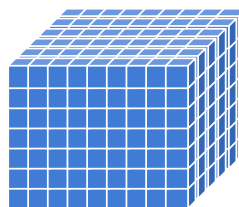


## Volumetric data

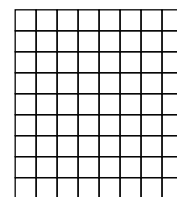
- Data associated to positions or regions in a 3D space
  - Structured grids



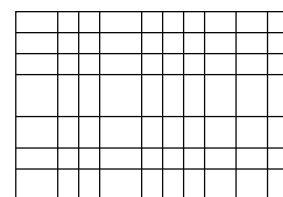
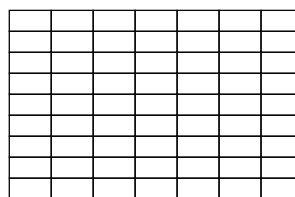
Wikipedia (Berkeley)



regular



cartesian



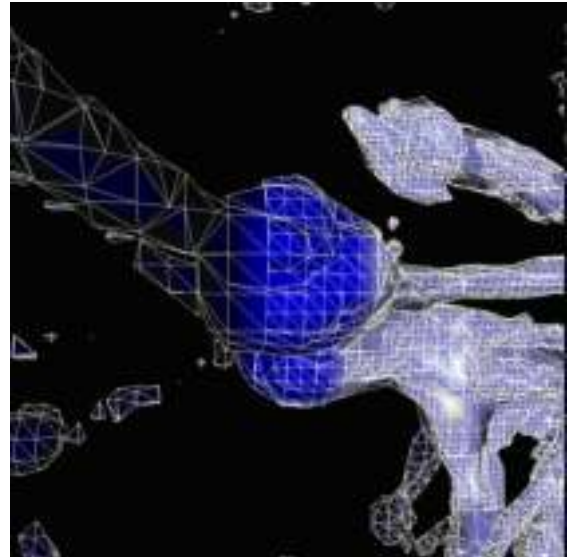
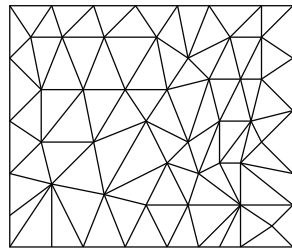
rectilinear

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## Volumetric data

- Data associated to positions or regions in a 3D space
  - Unstructured grids



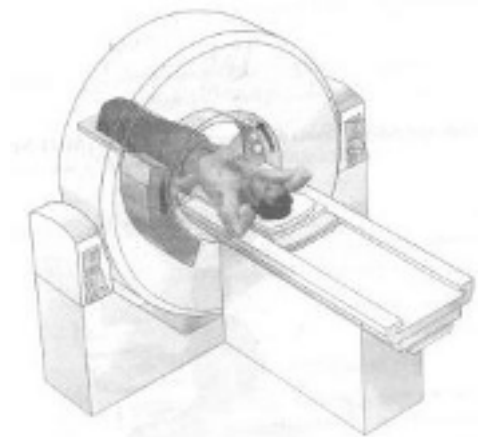
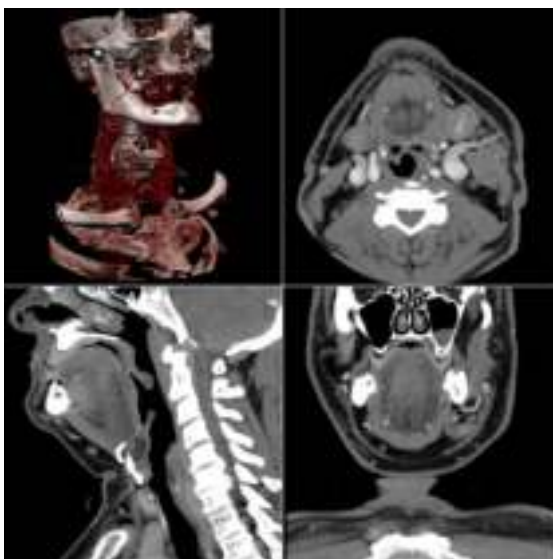
Wikipedia (Berkeley)

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## Acquisition: structured

- CT - Computed tomography



Hounsfield 1967

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## Acquisition: structured

- MRI - Magnetic Resonance Imaging
  - A large magnetic field is applied to the object, which aligns hydrogen nuclei
  - Water content in the tissue is measured



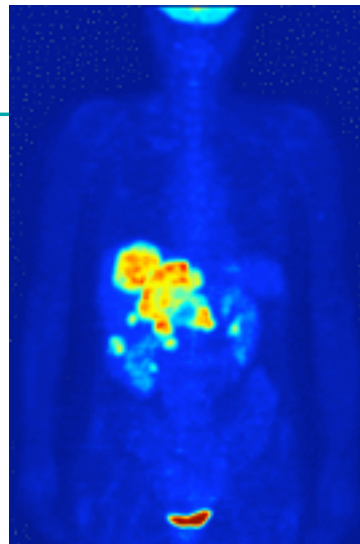
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Lauterbur 73 <sup>1A</sup>

## Acquisition: structured

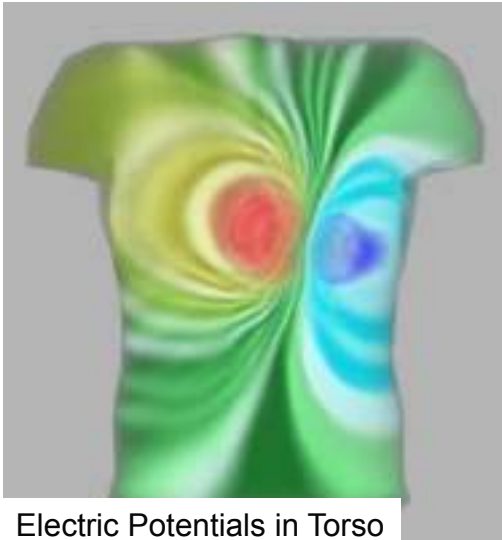
- PET - Positron Emission Tomography
  - A radioactive isotope (into a medium) is injected into the body
  - Positron emission from decay interacts with electrons to create gamma rays
  - Gamma rays are detected to find position of isotopes in body



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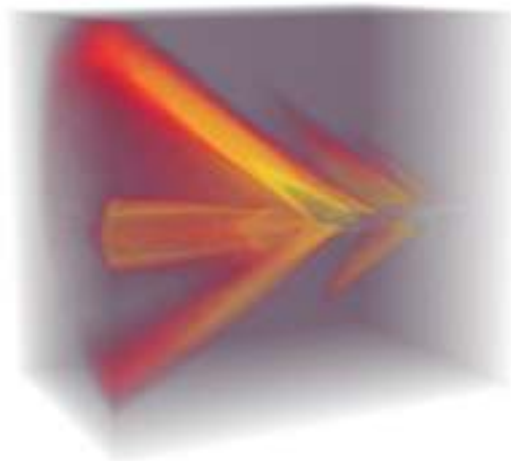
## Computational simulation: (un)structured

- Computational fluid dynamics
- Structural mechanics



Electric Potentials in Torso  
[MacLeod et al 94]

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Turbulence around fighter jet  
[Neely and Batina 92]

## Visualization

- Each voxel can contribute to the image
- Voxels can be **discarded** from the image depending on user interest on specific structures within the volume

Ray casting [Levoy 88]

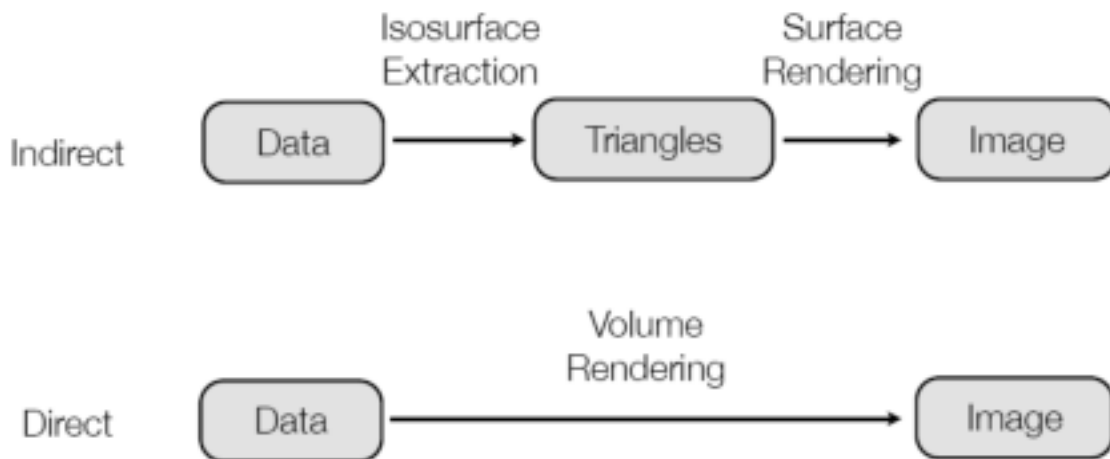


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<http://www.fovia.com/gallery.php>

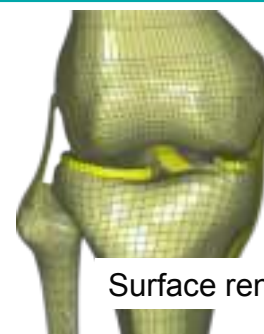
# Volume visualization approaches



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# Volume visualization approaches



Surface rendering



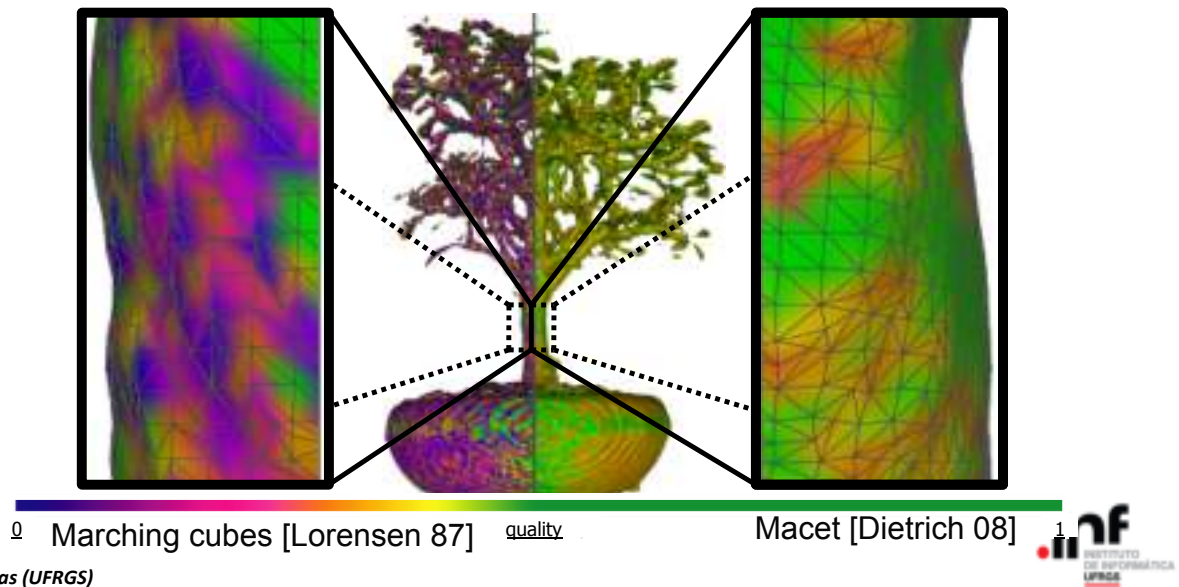
Direct rendering

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## Surface rendering

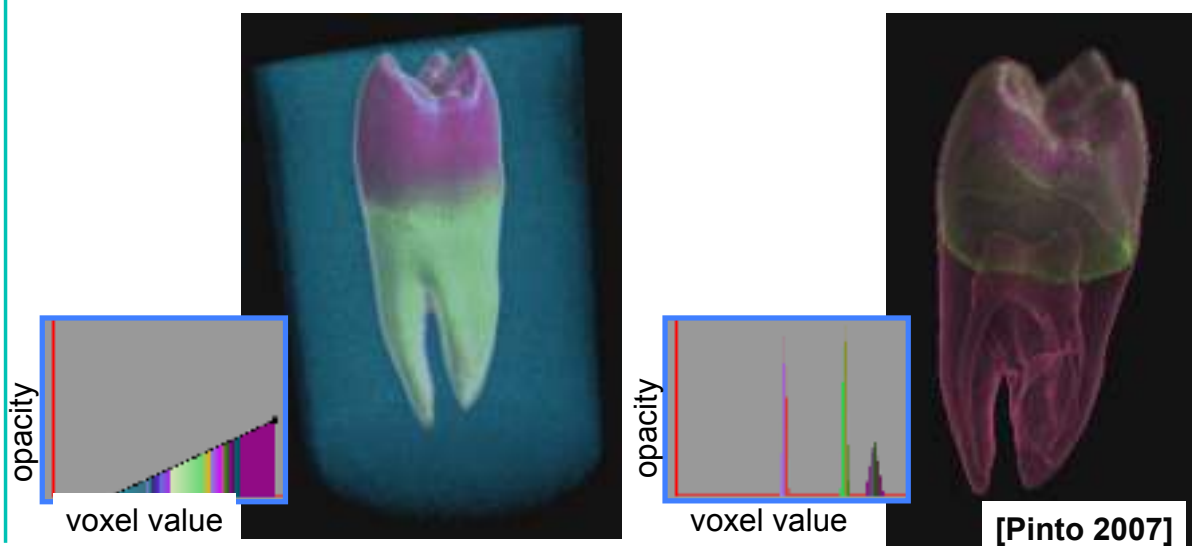
- An isosurface is extracted from the data set
- Different methods extract different meshes



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## Direct volume rendering

- Mapping from value to opacity and/or color
- Different mappings produce different images



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## Isosurface extraction

- How do we specify the part of the volume corresponding to the desired inner structure?



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### A little math

- Dataset:  $v = f(x, y, z)$
- $f: \mathbb{R}^3 \rightarrow \mathbb{R}$
- Want to find  $S_v = \{(x, y, z) \mid f(x, y, z) = v\}$
- All the locations where the value of  $f$  is  $v$
- $S_v$ : isosurface of  $f$  at  $v$ 
  - In 2D: isocontours (some path)
  - In 3D: isosurface
- Why is this useful?

## Surface extraction

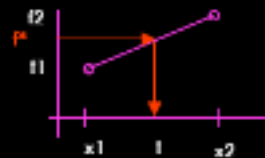
### Note on Inverse Linear Interpolation

- The linear interpolation formula gives value of  $f$  at specified point  $t$ :

$$f(x^*) = f_1 + t(f_2 - f_1)$$

- Inverse linear interpolation gives value of  $t$  at which  $f$  takes a specified value  $f^*$

$$t = (f^* - f_1) / (f_2 - f_1)$$

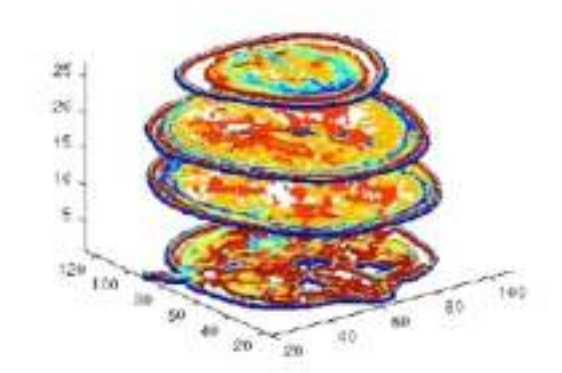


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## Surface extraction

- Take a slice through the 3D volume, often orthogonal to one of the axes
- Obtain the 2D contour of the structures in each slice
- Connect adjacent contours forming polygons



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## Surface extraction

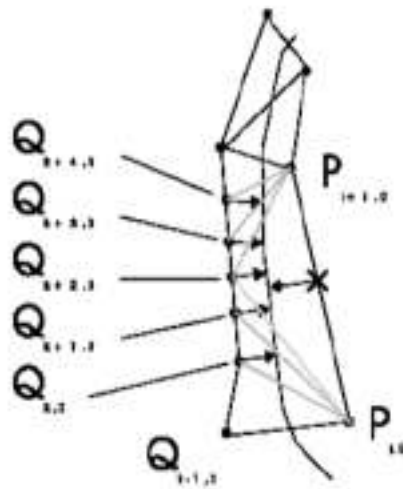
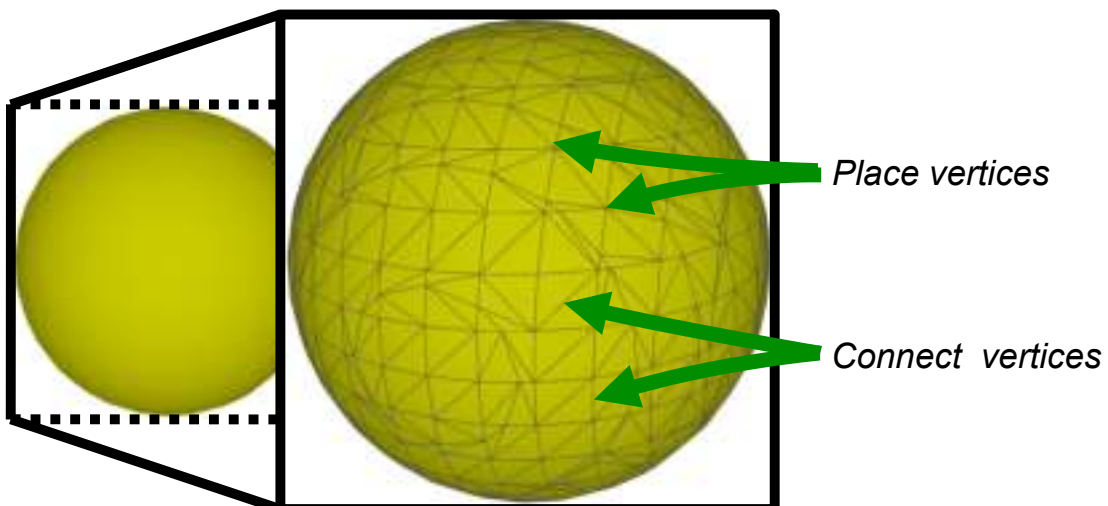
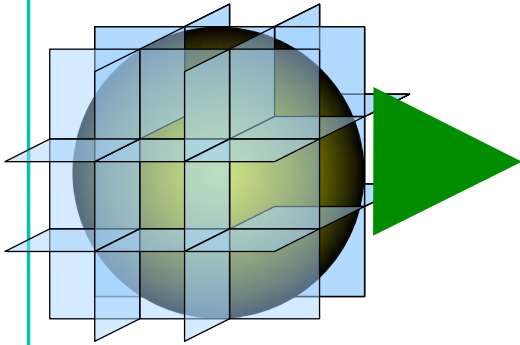


Fig. 6. Vertices  $Q_{k-4,j}$  through  $Q_{k,j}$  are connected to  $P_{i,j}$  or  $P_{i+1,j}$  resp. due to their correspondences on the medial axis.

## Surface extraction

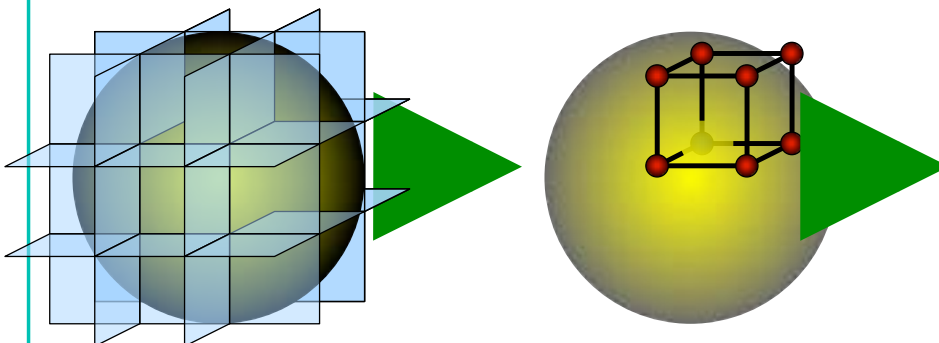


## Marching Cubes (MC) (Lorensen e Cline, 1987)



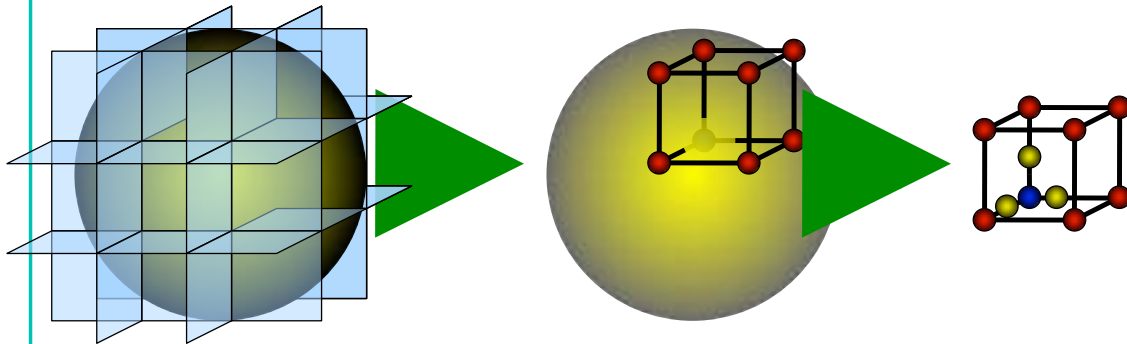
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## Marching Cubes (MC) (Lorensen e Cline, 1987)



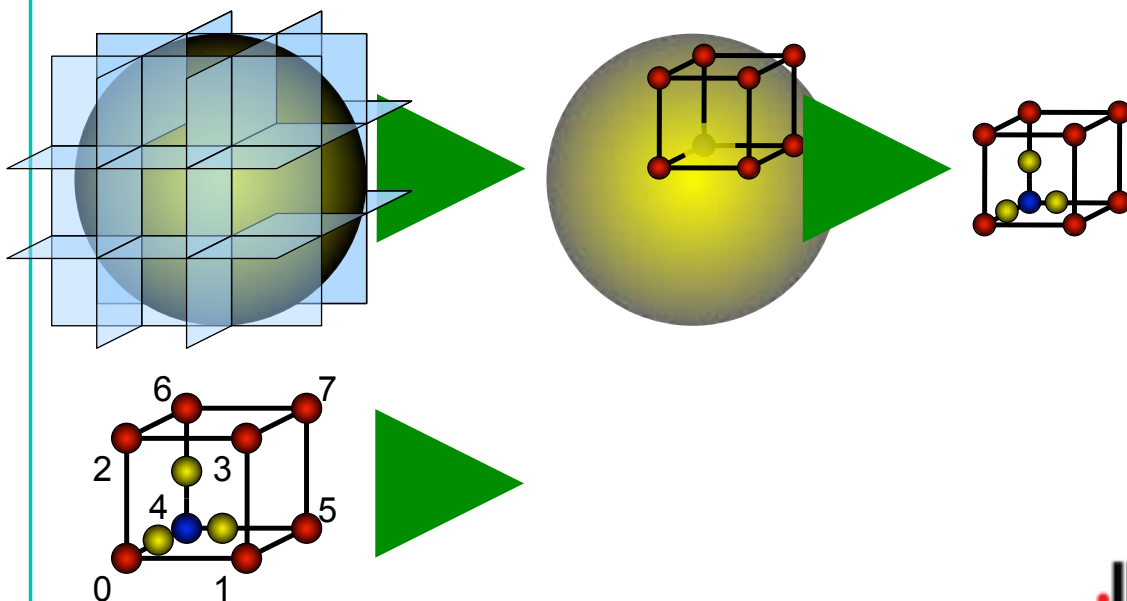
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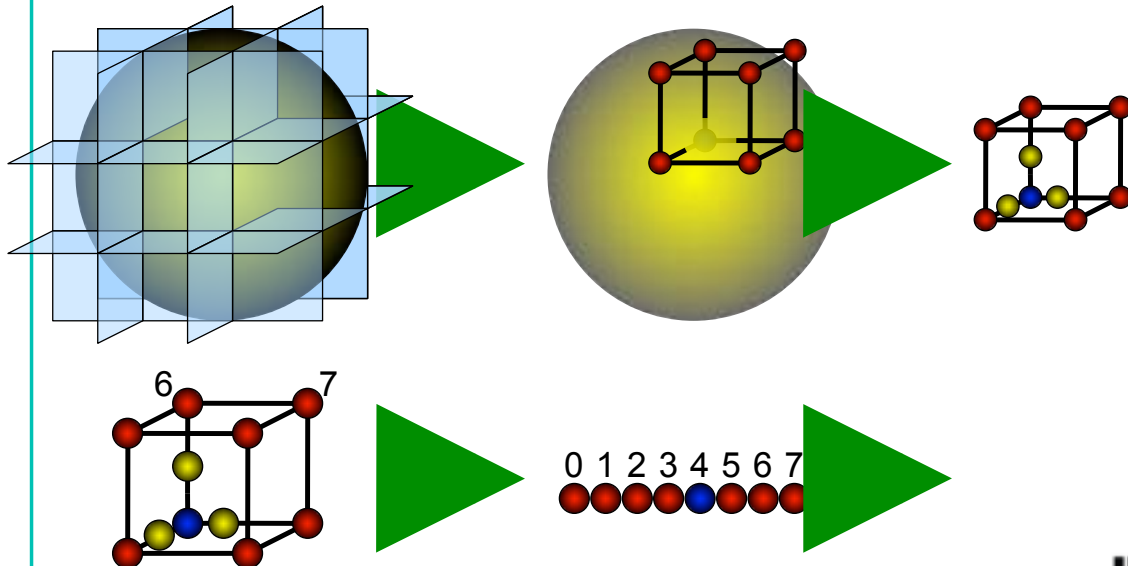
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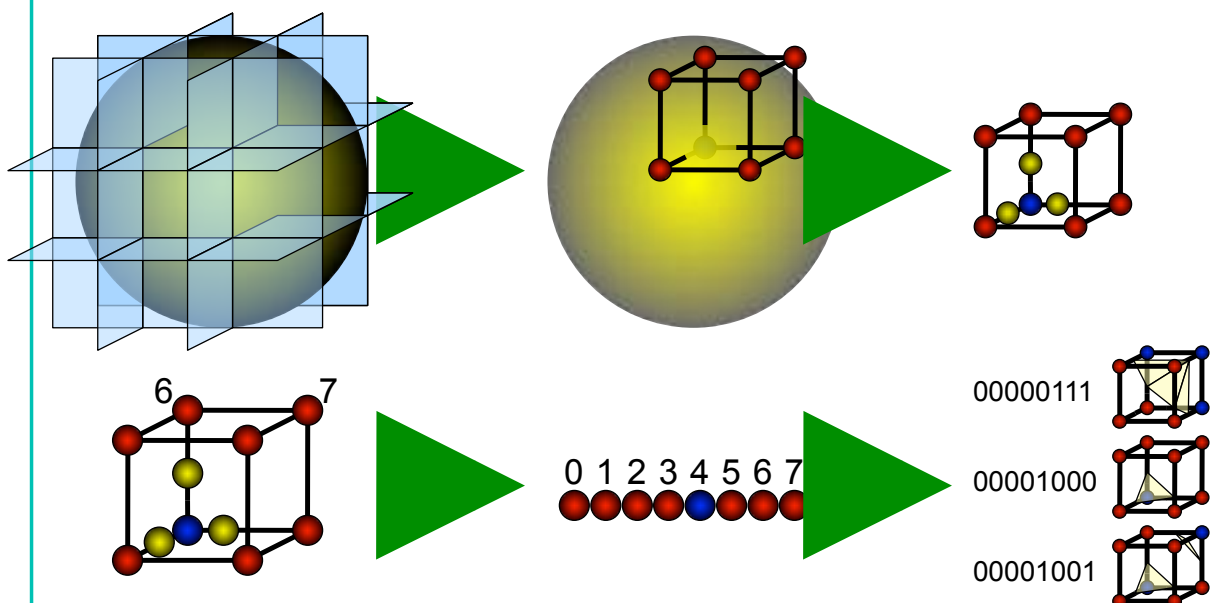
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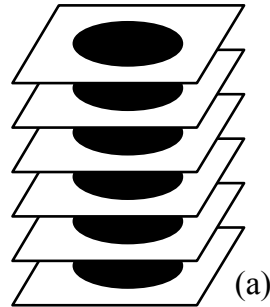
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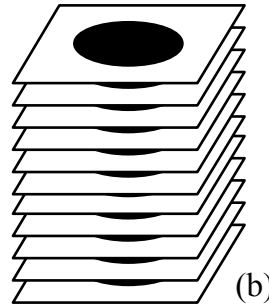
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## Surface extraction

- Marching cubes [Lorensen and Cline 87]

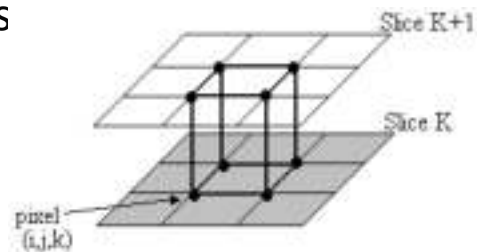


(a)



(b)

- Two adjacent slices form cubes

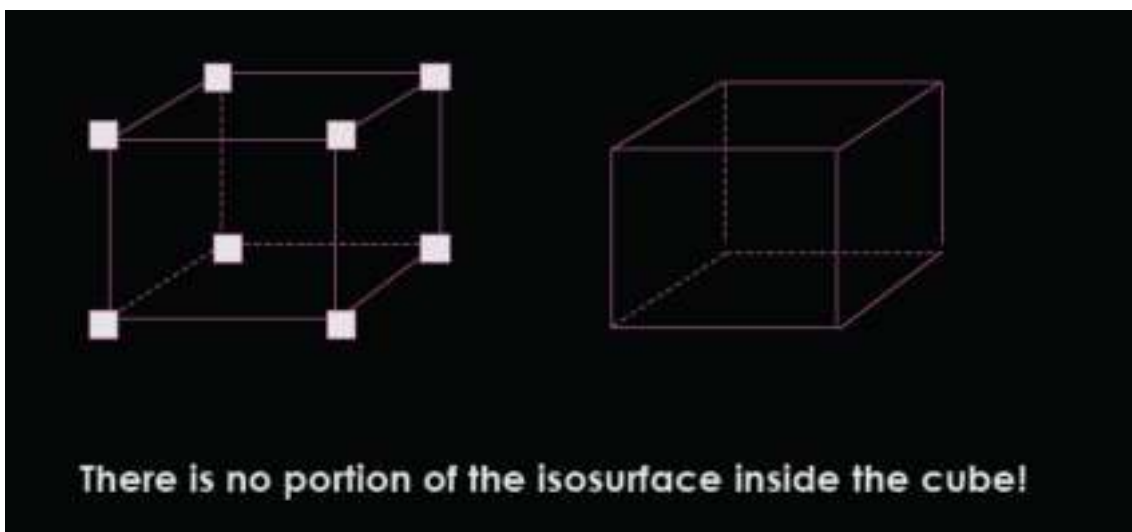


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## Marching cubes (Lorensen e Cline, 1987)

- Observing how a surface “passes through” a voxel



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## Isosurface Construction - One Positive Vertex - 1

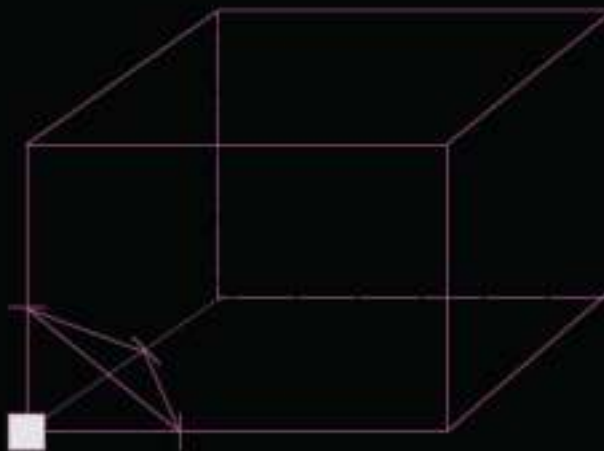


Intersections with edges found by inverse linear interpolation  
(as in contouring)

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## Isosurface Construction - One Positive Vertex - 2



Joining edge intersections across faces forms a triangle  
as part of the isosurface

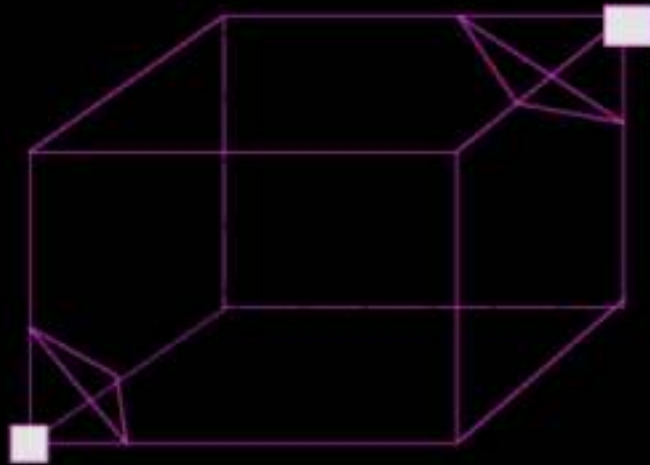
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## Isosurface Construction - Positive Vertices at Opposite Corners

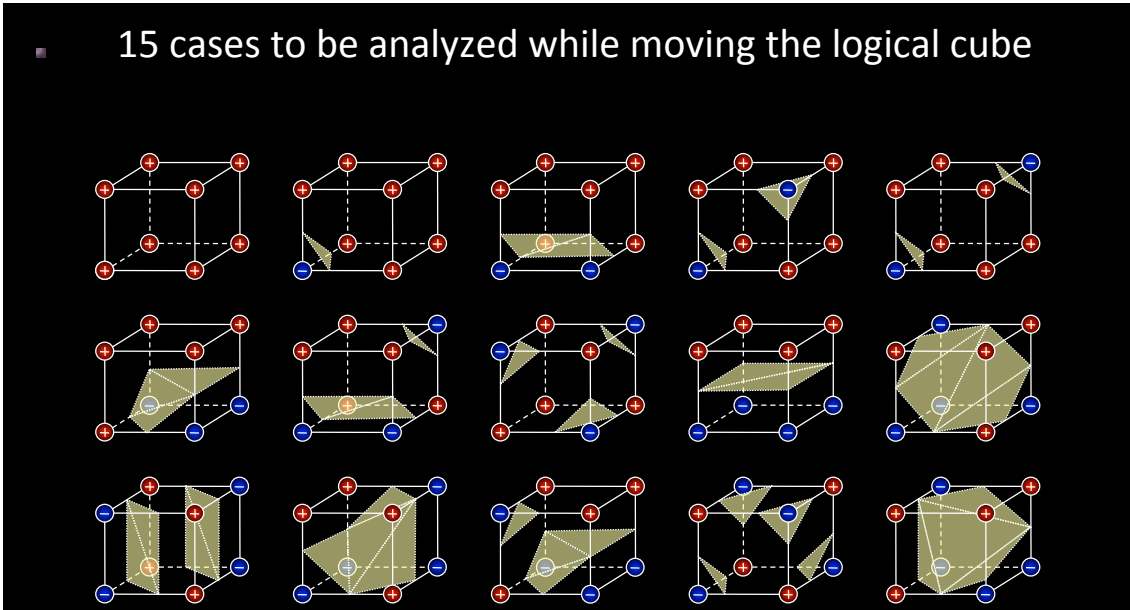


## Isosurface Construction

- One can work through all 256 cases in this way - although it quickly becomes apparent that many cases are similar.
- For example:
  - 2 cases where all are positive, or all negative, give no isosurface
  - 16 cases where one vertex has opposite sign from all the rest
- In fact, there are only 15 topologically distinct configurations

## Marching Cubes (MC) (Lorensen e Cline, 1987)

15 cases to be analyzed while moving the logical cube



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## Isosurface Construction

- In some configurations, just one triangle forms the isosurface
- In other configurations ...
  - ...there can be several triangles
  - ...or a polygon with 4, 5 or 6 points which can be triangulated
- A software implementation will have separate code for each configuration

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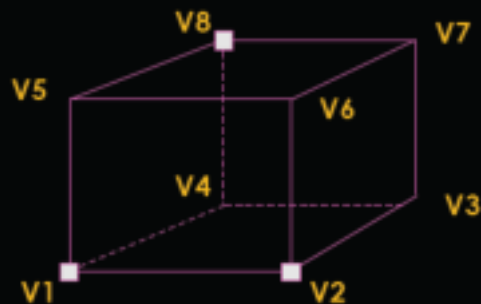
# Marching Cubes Algorithm

- Step 1: Classify the eight vertices relative to the isosurface value

8-bit index ; 1+ve;0 -ve

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
|---|---|---|---|---|---|---|---|

V1 V2 V3 V4 V5 V6 V7 V8



Code identifies edges intersected:  
V1V4; V1V5; V2V3; V2V6; V5V8; V7V8; V4V8

# Marching Cubes Algorithm

- Step 2: Look up table which identifies the canonical configuration
- For example:

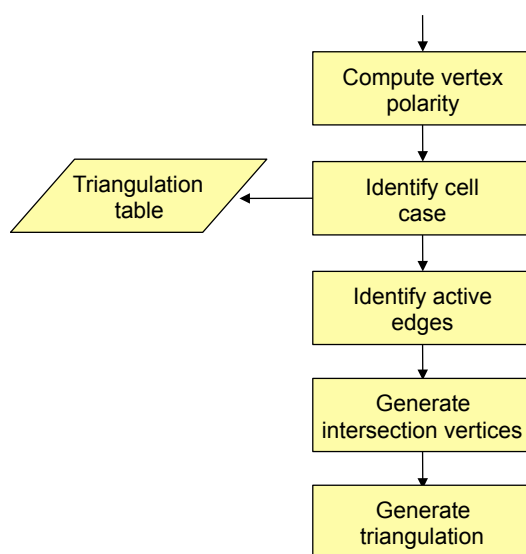
|          |                 |
|----------|-----------------|
| 00000000 | Configuration 0 |
| 10000000 | Configuration 1 |
| 01000000 | Configuration 1 |
| ...      |                 |
| 11000001 | Configuration 6 |
| ...      |                 |
| 11111111 | Configuration 0 |

256 entries in table

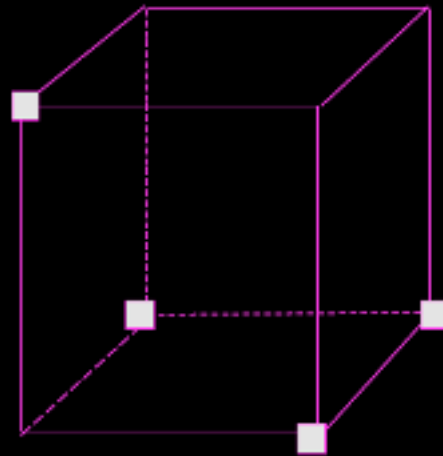
- Step 3: Inverse linear interpolation along the identified edges will locate the intersection points
- Step 4: The canonical configuration will determine how the pieces of the isosurface are created (0, 1, 2, 3 or 4 triangles)
- Step 5: Pass triangles to renderer for display

Algorithm marches from cube to cube between slices, and then from slice to slice to produce a smoothly triangulated surface

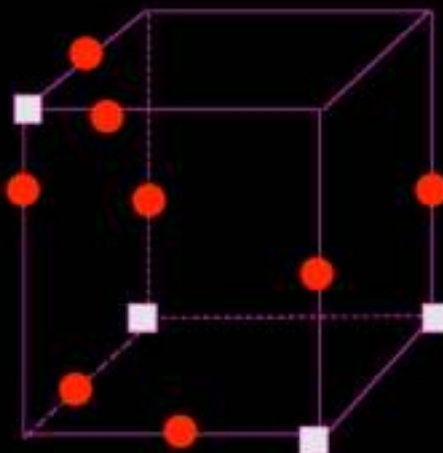
## Marching Cubes (MC) (Lorensen e Cline, 1987)



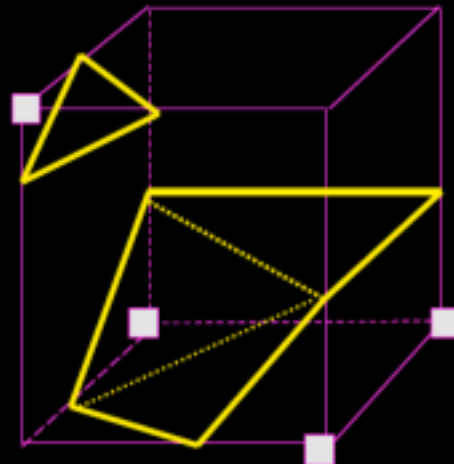
- Case 12 has three positive vertices on the bottom plane; and one positive vertex on the top plane, directly above the single negative on the bottom plane.
- Without looking at the answer.... Try to work out the isosurface!



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- Without looking at the answer.... Try to work out the isosurface!



## Isosurfacing by Marching Cubes Algorithm

- **Advantages**
  - isosurfaces good for extracting boundary layers
  - surface defined as triangles in 3D - well-known rendering techniques available for lighting, shading and viewing ... with hardware support
- **Disadvantages**
  - shows only a slice of data
  - ambiguities?



# Ambiguities

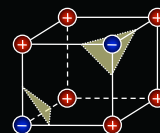
- Marching cubes suffers from exactly the same problems that we saw in contouring



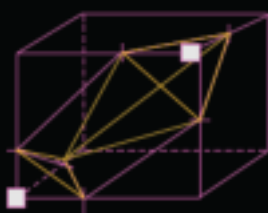
Case 3: Triangles are chosen to slice off the positive vertices - but could they have been drawn another way?

# Ambiguities on Faces

- Trouble occurs because:
  - trilinear interpolant is only linear along the edges
  - on a face, it becomes a bilinear function ... and for correct topology we must join the correct pair of intersections



Case 3 has two triangle pieces cutting off corners!

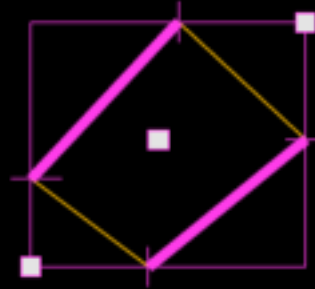


.. but here is another interpretation!

6 configurations include ambiguous faces

## Ambiguities on Faces

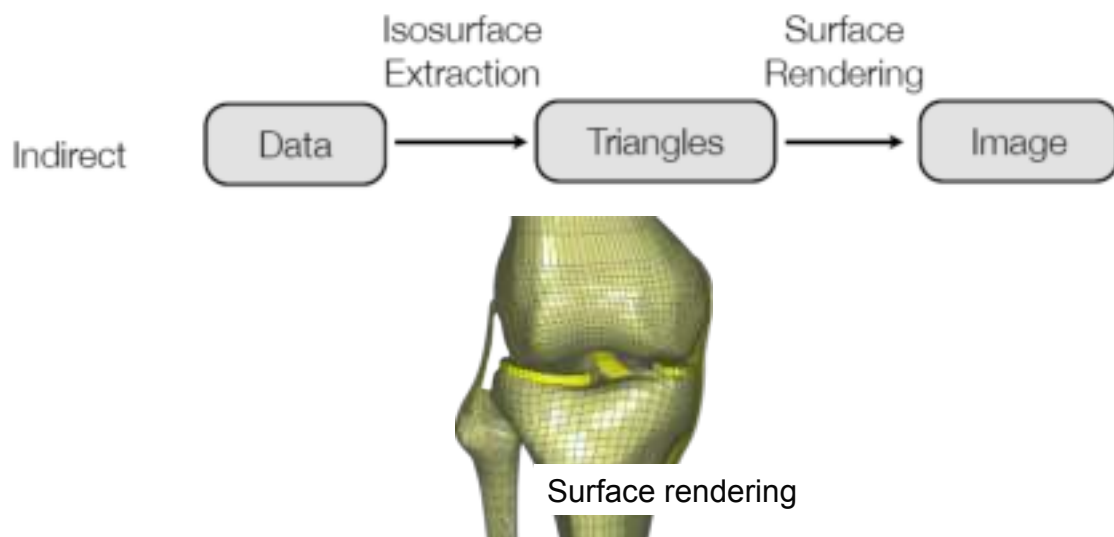
- On the front face, we have exactly the same ambiguity problem we had with contouring
- We can determine which pair of intersections to connect by looking at value at saddle point



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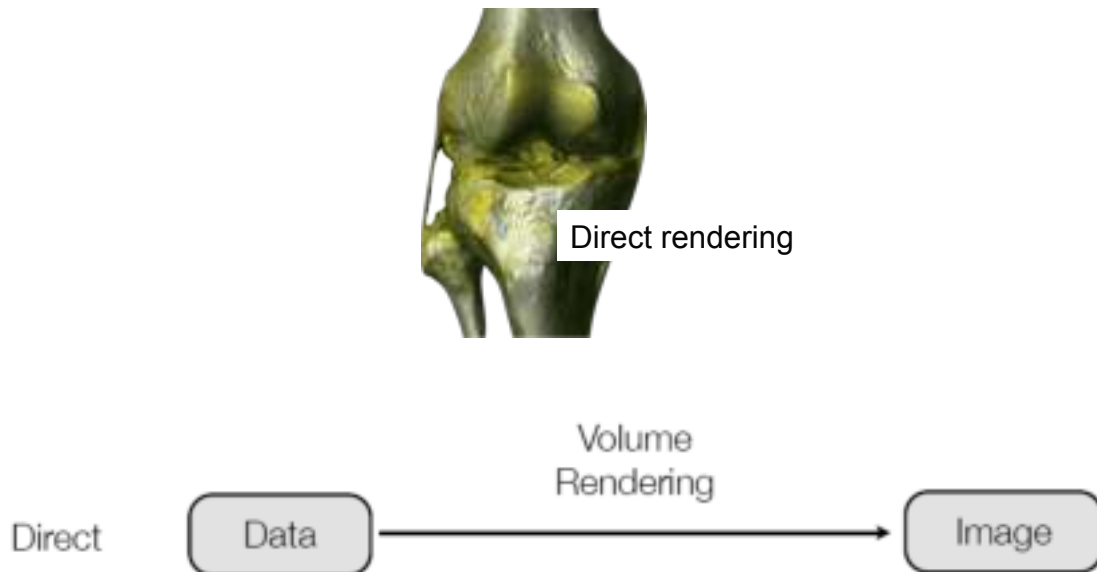
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## Volume visualization approaches



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# Volume visualization approaches

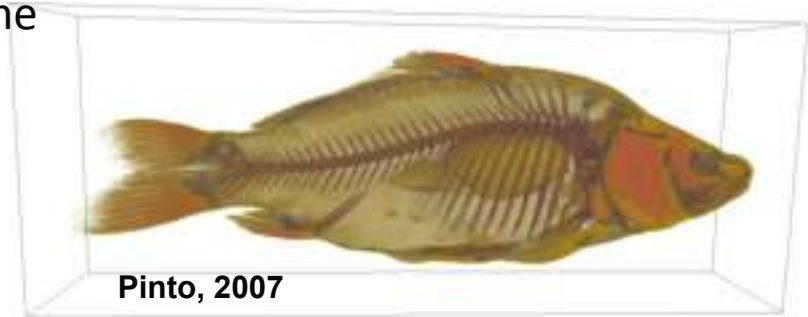


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## Direct volume rendering

- The data is considered to represent a semi-transparent, light-emitting medium (which absorbs light too)
  - Based on laws of physics
- Volume data is used as a whole
- Color and opacity are used to distinguish materials within the volume



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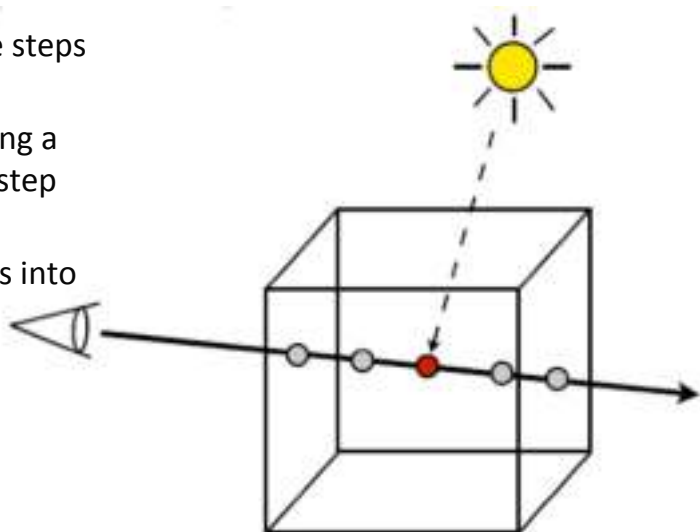
UFRGS

# Direct volume rendering

- Three stages of volume rendering
  - **Sampling:** Selecting the steps through the volume
  - **Classification:** Computing a color and opacity for a step
  - **Compositing:** Blending together classified steps into a final image

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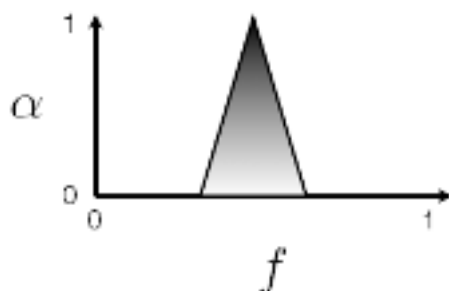
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Silva, 2000-2001

# Direct volume rendering

- **Classification:** Use of a transfer function

$$f(x) = \mathbf{R} \rightarrow \mathbf{R}^4, s \rightarrow (r, g, b, \alpha)$$



Look up table

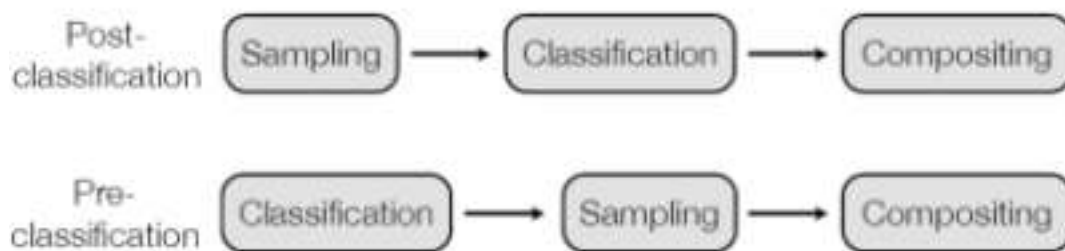


Silva, 2000-2001

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# Direct volume rendering

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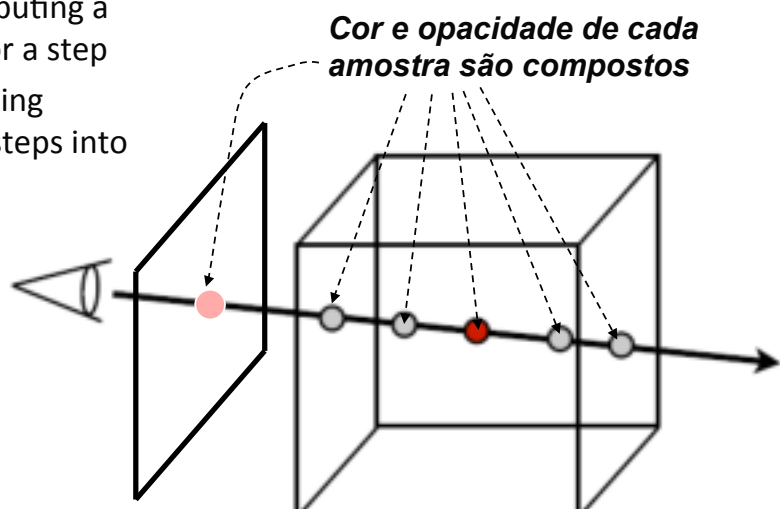


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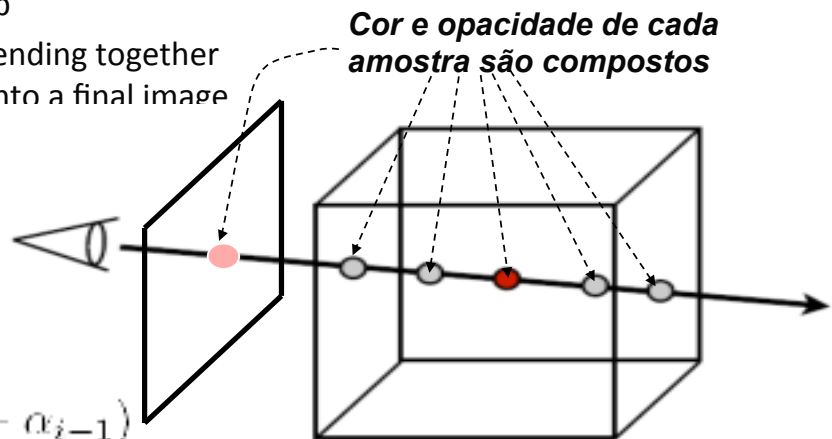


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# Direct volume rendering

- Three stages of volume rendering
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Ray casting: front to back

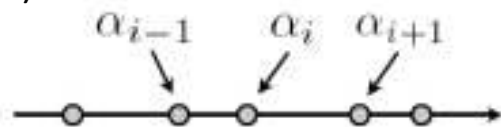
$$c_i = c_{i-1} + c_i \alpha_i (1 - \alpha_{i-1})$$

$$\alpha_i = \alpha_{i-1} + \alpha_i (1 - \alpha_{i-1})$$



# Direct volume rendering

- Image-based (backward projection)
  - Ray-casting (front to back)



$$c_i = c_{i-1} + c_i \alpha_i (1 - \alpha_{i-1})$$

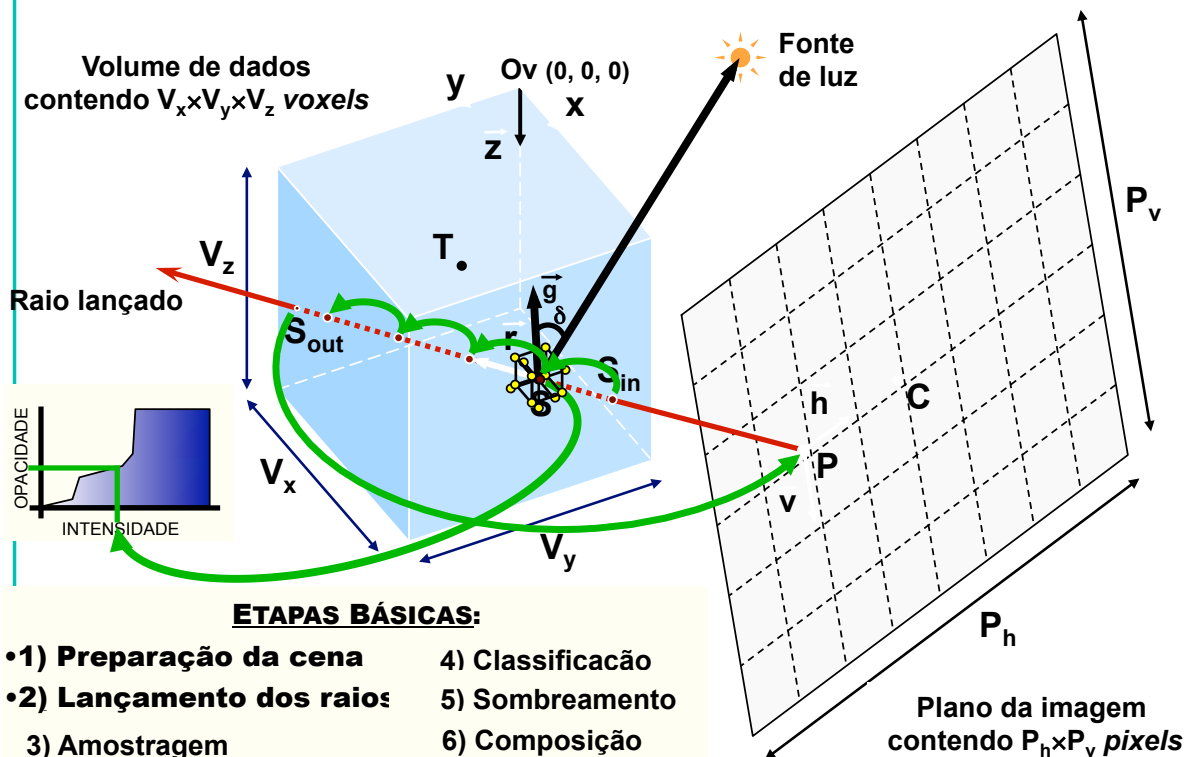
$$\alpha_i = \alpha_{i-1} + \alpha_i (1 - \alpha_{i-1})$$

- Object-based (forward projection, back to front)
  - Splatting
  - Texture mapping

$$c_i = c_i \alpha_i + c_{i+1} (1 - \alpha_i)$$

# Ray casting

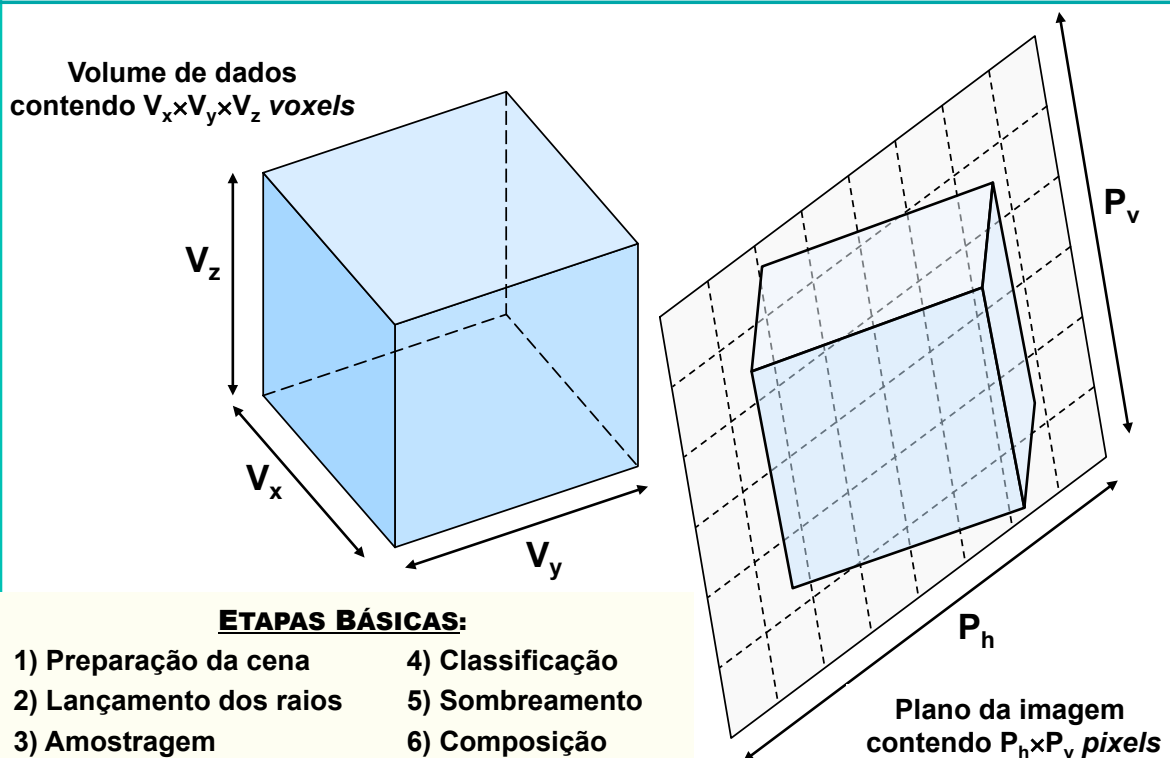
Silva, 2000-2001



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# Ray casting

Silva, 2000-2001

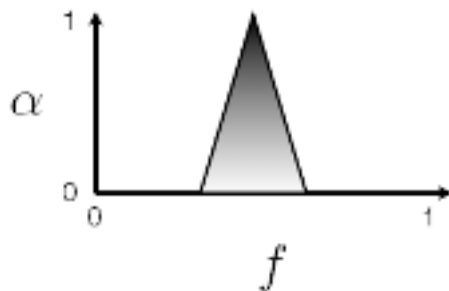


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# Transfer functions

- **Classification: color and opacity**

$$f(x) = \mathbf{R} \rightarrow \mathbf{R}^4, s \rightarrow (r, g, b, \alpha)$$



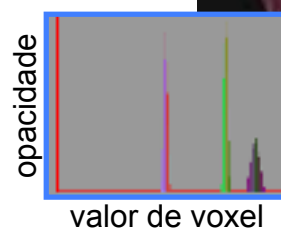
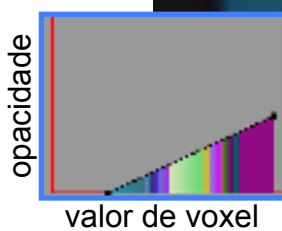
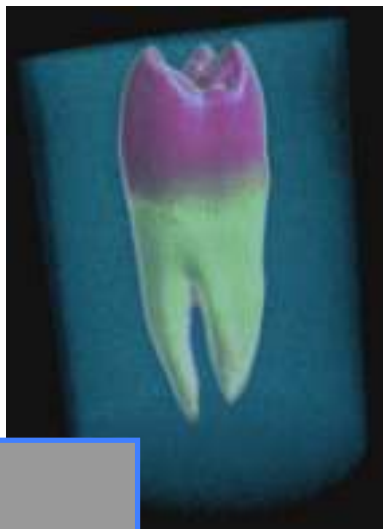
Look up table



Silva, 2000-2001

C. Freitas (UFRGS)

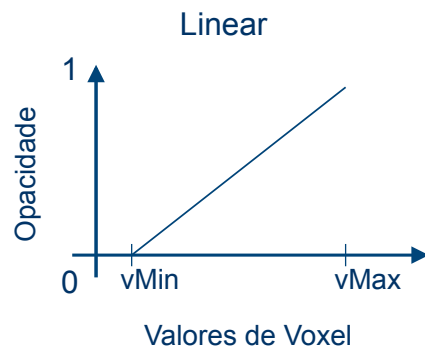
# Transfer functions



C. Freitas (UFRGS)

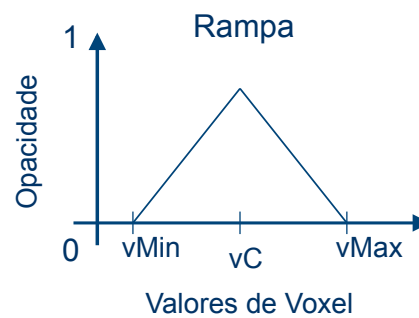
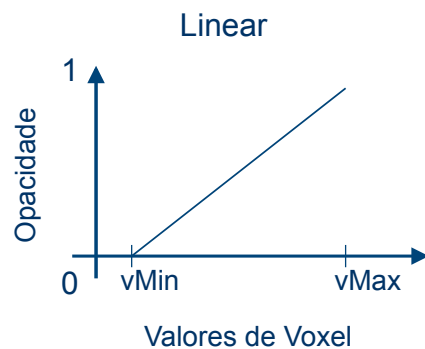
## ❖ FTs de opacidade 1-D

$$\alpha = f(v)$$



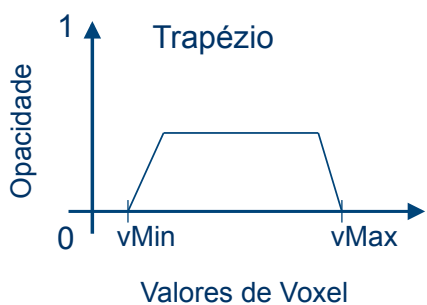
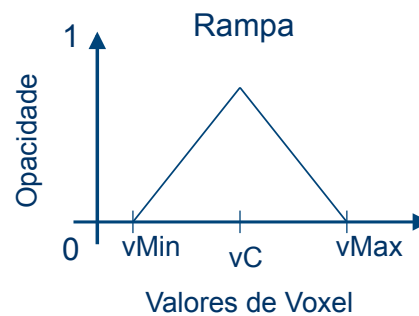
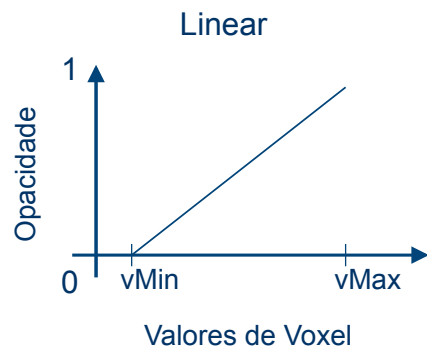
## ❖ FTs de opacidade 1-D

$$\alpha = f(v)$$



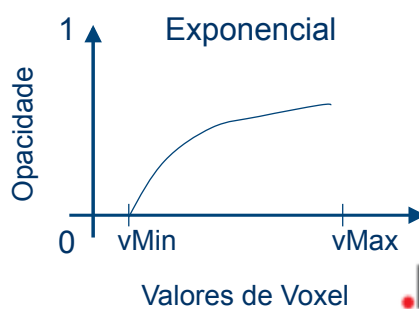
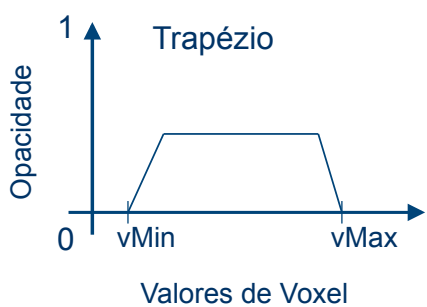
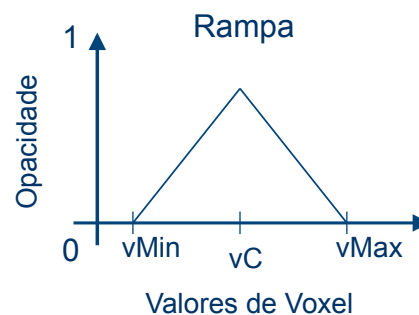
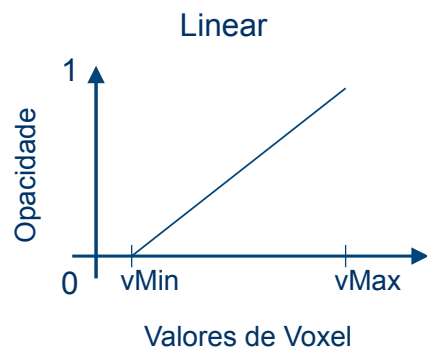
## ❖ FTs de opacidade 1-D

$$\alpha = f(v)$$

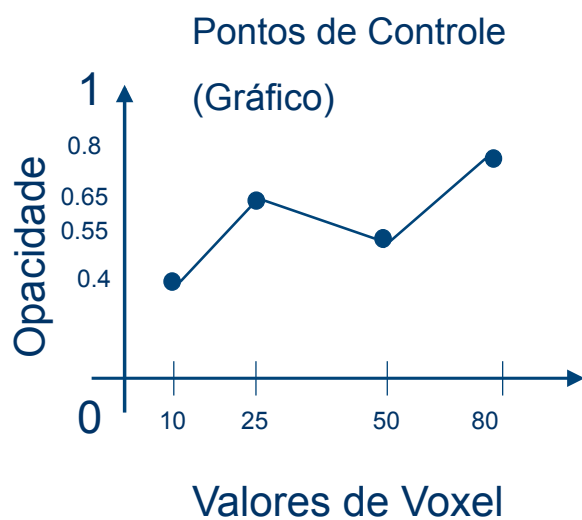


## ❖ FTs de opacidade 1-D

$$\alpha = f(v)$$

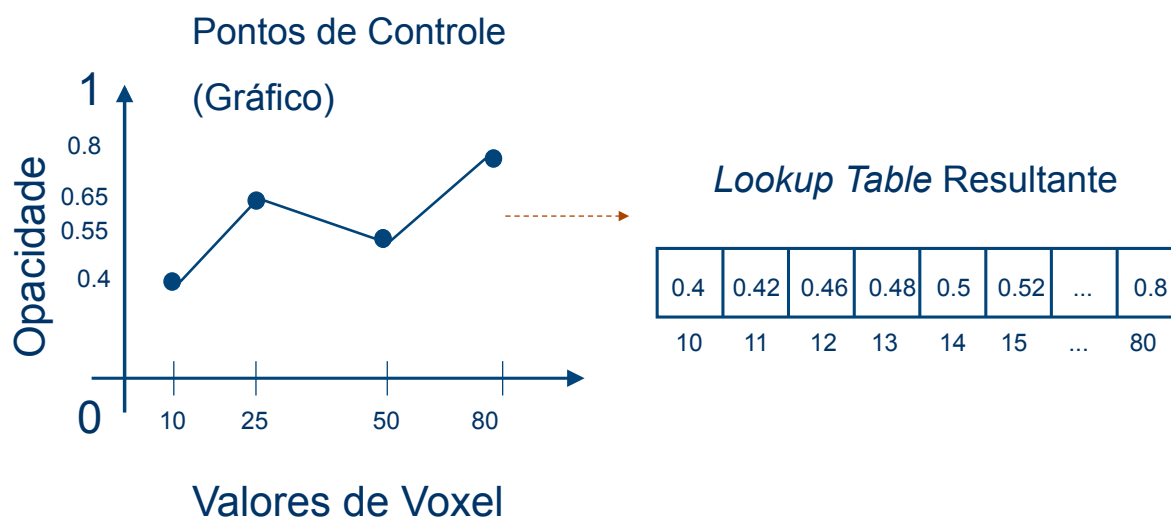


## ❖ Especificação Manual de FTs



C. Freitas (UFRGS)

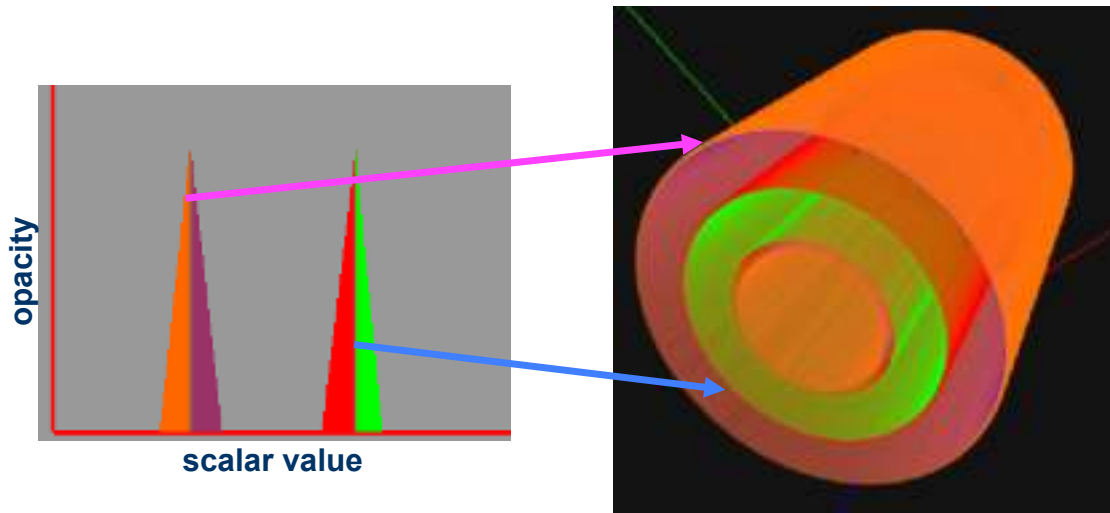
## ❖ Especificação Manual de FTs



C. Freitas (UFRGS)

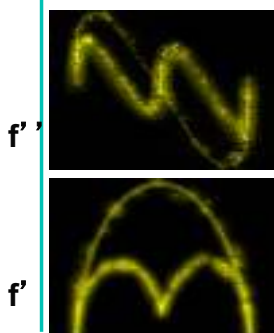
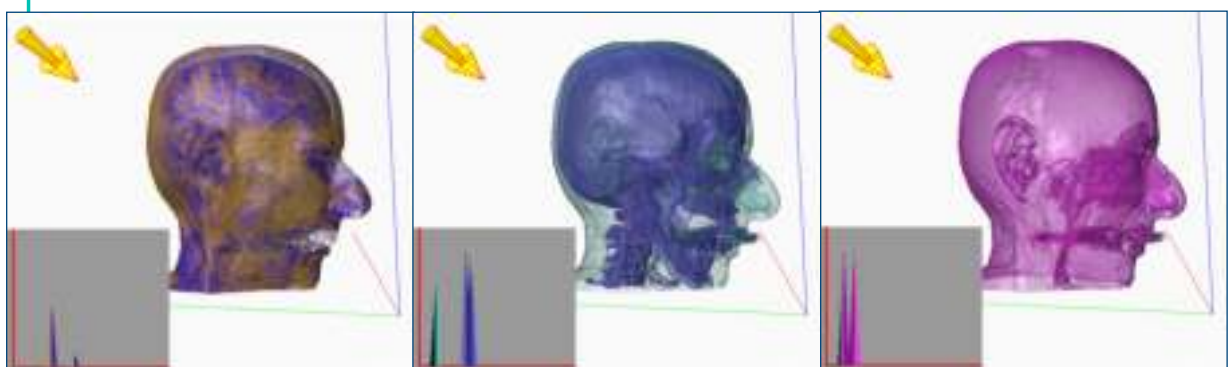


## Enhancing transition zones



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## Transfer functions

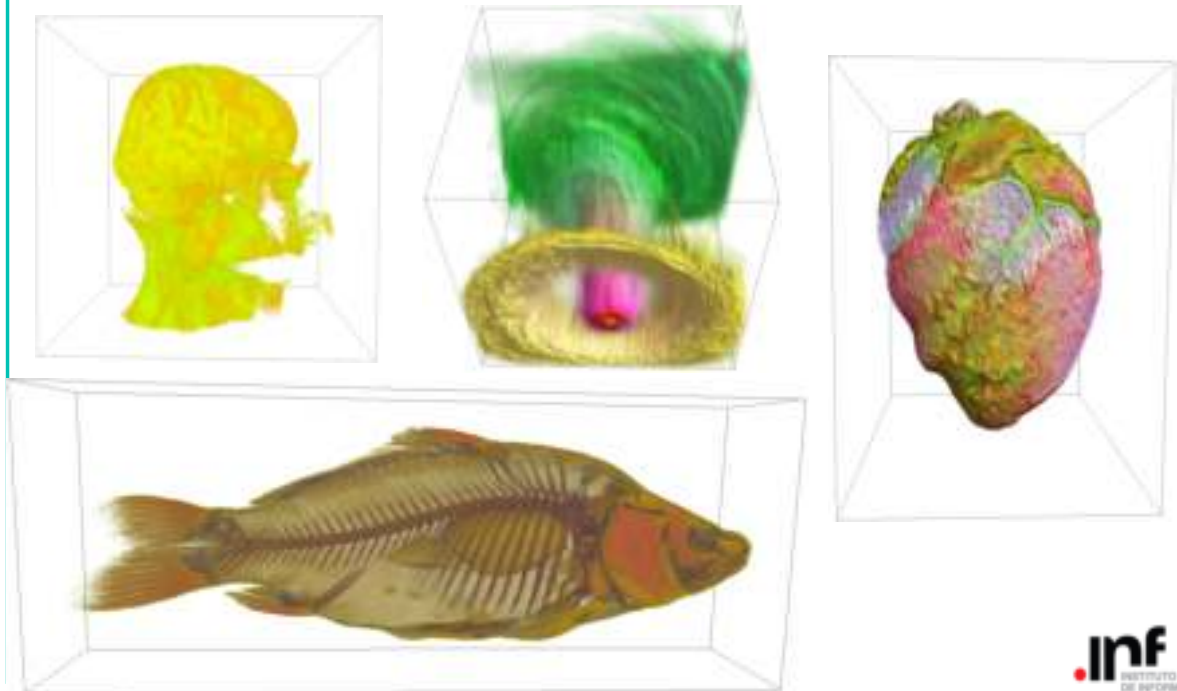


- Valores escalares de voxel são ordenados de acordo com a distância estimada até a borda mais próxima (Kindlmann e Durkin, 1998).

- Sorteia-se, entre os primeiros, um pequeno número de valores de voxel. Estes recebem opacidade elevada e descontinuidade de cor.

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# Results



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## Direct volume rendering

- Image-based (backward projection)
  - Ray-casting
- Object-based (forward projection)
  - Texture mapping



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