Guide de l'outil `Coq`
Passage de la déduction naturelle à `Coq`

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Tactiques existantes en `Coq`

Déduction Naturelle	*Nom*	*Équivalent* `Coq`	*Tactique*
$\dfrac{}{A \vdash A}$	Hyp	$\dfrac{}{\Gamma,\ H : A \vdash A}$	`exact H.`
$\dfrac{\Gamma \vdash G}{\Gamma, A \vdash G}$	Aff	$\dfrac{\Gamma \vdash G}{\Gamma, H : A \vdash G}$	`clear H.`
$\dfrac{\Gamma \vdash A \to G \quad \Gamma \vdash A}{\Gamma \vdash G}$	$E_{\to}$	$=$	`cut A.`
$\dfrac{\Gamma,\ A \vdash G}{\Gamma \vdash A \to G}$	$I_{\to}$	$\dfrac{\Gamma,\ H : A \vdash G}{\Gamma \vdash A \to G}$	`intro H.`
$\dfrac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B}$	$I_{\wedge}$	$=$	`split.`
$\dfrac{\Gamma \vdash A}{\Gamma \vdash A \vee B}$	$I_{\vee}^{1}$	$=$	`left.`
$\dfrac{\Gamma \vdash B}{\Gamma \vdash A \vee B}$	$I_{\vee}^{2}$	$=$	`right.`
$\dfrac{}{\Gamma \vdash A \vee \neg A}$	$TiersExclu$	$=$	`apply (classic A).`
$\dfrac{\Gamma, x : A \vdash (G\ x)}{\Gamma \vdash \forall x : A.(G\ x)}$	$I_{\forall}$	$=$	`intro x.`
$\dfrac{}{\Gamma \vdash x = x}$	$I_{=}$	$=$	`reflexivity.`
$\dfrac{\Gamma \vdash [b \mid a]G}{\Gamma,\ a = b \vdash G}$	$E_{=}$	$\dfrac{\Gamma,\ H : a = b \vdash [b \mid a]G}{\Gamma,\ H : a = b \vdash G}$	`rewrite -> H.`

Tactiques équivalentes en `Coq`

Déduction Naturelle	*Nom*	*Équivalent* `Coq`	*Tactique*

$\dfrac{\Gamma \vdash \bot}{\Gamma \vdash G}$	$E_\bot$	$\dfrac{\dfrac{\overline{\Gamma,\ H : \bot \vdash G}}{\Gamma \vdash \bot \to G} \quad \Gamma \vdash \bot}{\Gamma \vdash G}$	`cut False.`
			`intro H.`
			`contradiction.`

$\dfrac{\Gamma, \neg G \vdash \bot}{\Gamma \vdash G}$	$E_\bot$	$\dfrac{\dfrac{\overline{\Gamma,\ H : \bot \vdash G}}{\Gamma \vdash \bot \to G} \quad \Gamma \vdash \bot}{\Gamma \vdash G}$	`cut (G \/ ~G).`
			`intro Hgng.`
			`elim Hgng.`
			`intros Hg Hng.`
			`exact Hg.`
			`cut False.`
			`intro H.`
			`contradiction. ...`
			`apply (classic G).`

$\dfrac{\Gamma \vdash A \wedge B}{\Gamma \vdash A}$	$E^1_\wedge$	$\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma,\ H : A \wedge B,\ HA : A,\ HB : B \vdash A}}{\Gamma,\ H : A \wedge B \vdash A \to B \to A}}{\Gamma,\ H : A \wedge B \vdash A}}{\Gamma \vdash A \wedge B \to A} \quad \Gamma \vdash A \wedge B}{\Gamma \vdash A}$	`cut (A /\ B).`
			`intro H.`
			`elim H.`
			`intros HA HB.`
			`exact HA.`

$\dfrac{\Gamma \vdash A \wedge B}{\Gamma \vdash B}$	$E^2_\wedge$	idem	`cut (A /\ B).`
			`intro H.`
			`elim H.`
			`intros HA HB.`
			`exact HB.`

$\dfrac{\Gamma \vdash A \vee B \quad \Gamma, H1 : A \vdash G \quad \Gamma, H2 : B \vdash G}{\Gamma \vdash G}$	$E_\vee$	$\dfrac{\Gamma, H : A \vee B \vdash A \to G \quad \Gamma, H : A \vee B \vdash B \to G}{\Gamma, H : A \vee B \vdash G}$	`elim H.`
$\dfrac{\Gamma, H : A \vdash \neg B \quad \Gamma, H : A \vdash B}{\Gamma \vdash \neg A}$	$I_\neg$	$\dfrac{\Gamma, H : A \vdash \bot}{\Gamma \vdash \neg A}$	`unfold not.` `intro H.`
$\dfrac{\Gamma, H : A \to B \vdash A}{\Gamma, H : A \to B \vdash B}$	*Apply*	$=$	`apply H.`

Tactiques propres à Coq

Déduction Naturelle	*Nom*	*Équivalent* Coq	*Tactique*
$\dfrac{\Gamma \vdash \neg A \quad \Gamma \vdash A}{\Gamma \vdash G}$	$E_{\neg}$	$=$	`absurd A.`
$\dfrac{}{\Gamma \vdash A \vee \neg A}$	$TiersExclu$	$=$	`apply (classic A).`
$\dfrac{\Gamma \vdash \neg\neg A}{\Gamma \vdash A}$	$Pierce$	$=$	`apply (NNPP A).`
$\dfrac{\Gamma, H1 : A, H2 : A \to B, H3 : B \vdash G}{\Gamma, H1 : A, H2 : A \to B \vdash G}$	$ModusPonens$	$\dfrac{\Gamma, H1 : A, H2 : A \to B \vdash B \to G}{\Gamma, H1 : A, H2 : A \to B \vdash G}$	`generalize (H2 H1).`
$\dfrac{\Gamma, x : A \vdash (G\ x)}{\Gamma \vdash \forall x : A.(G\ x)}$	$I_{\forall}$	$=$	`intro x.`
$\dfrac{\Gamma \vdash \forall x : A.(G\ x) \quad \Gamma \vdash y : A}{\Gamma \vdash (G\ y)}$	$E_{\forall}$	$\dfrac{\Gamma, y : A \vdash \forall x : A.(G\ x)}{\Gamma, y : A \vdash (G\ y)}$	`generalize y.`
$\dfrac{\Gamma \vdash (G\ y) \quad \Gamma \vdash y : A}{\Gamma \vdash \exists x : A.(G\ x)}$	$I_{\exists}$	$=$	`exists y.`
$\dfrac{\Gamma \vdash \exists x : A.(P\ x) \quad \Gamma, y : A, H : (P\ y) \vdash G}{\Gamma \vdash G}$	$E_{\exists}$	$\dfrac{\Gamma, H : \exists x : A.(P\ x) \vdash \forall y : A.(P\ y) \to G}{\Gamma, H : \exists x : A.(P\ x) \vdash G}$	`elim H.`
$\dfrac{\Gamma \vdash (G\ 0) \quad \Gamma \vdash \forall m : Nat.(G\ m) \to (G\ (S\ m))}{\Gamma \vdash \forall n : Nat.(G\ n)}$	E_{Nat}	$\approx$	`intro n ;elim n.`
$\dfrac{\Gamma \vdash (G\ 0) \quad \Gamma \vdash \forall m : Nat.(G\ (S\ m))}{\Gamma \vdash \forall n : Nat.(G\ n)}$	$Cas\ sur\ Nat$	$\approx$	`intro n ;case n.`
$\dfrac{\forall k \in [1, N] : \quad \Gamma, H : T = (C_k\ u_1 \ldots u_{n_k}) \vdash G}{\Gamma, T : (I\ v_1 \ldots v_n) \vdash G}$	$I\ inductif$	$\approx$	`inversion T.`
$\dfrac{}{\Gamma, H : t[\![(C\ u_1 \ldots u_n)]\!] = t[\![(C'\ v_1 \ldots v_p)]\!] \vdash G}$	$C \neq C'$	$=$	`discriminate H.`
$\dfrac{\Gamma \vdash u_1 = v_1 \to \ldots u_n = v_n \to G}{\Gamma, H : (C\ u_1 \ldots u_n) = (C\ v_1 \ldots v_n) \vdash G}$	$C\ injectif$	$\approx$	`injection H.`
$\dfrac{\Gamma \vdash G'}{\Gamma \vdash G}$	$G \rhd G'$	$=$	`simpl.`
$\dfrac{\Gamma, H : a = b \vdash [a \mid b]G}{\Gamma, H : a = b \vdash G}$	$b = a$	$=$	`rewrite <- H.`
$\dfrac{\Gamma, HA : A, HB : B \vdash G}{\Gamma, H : A \wedge B \vdash G}$	$E'_{\wedge}$	$=$	`destruct H as (HA,HB).`
$\dfrac{\Gamma, HA : A \vdash G \quad \Gamma, HB : B \vdash G}{\Gamma, H : A \vee B \vdash G}$	$E'_{\vee}$	$=$	`destruct H as [HA | HB].`