Generalized-Cost Delta Map (GCDM)

Indices, Sets, and Core Symbols

Origins i; airports $a \in \{\mathrm{HVN}, \mathrm{JFK}, \mathrm{LGA}, \mathrm{EWR}...\}$; modes $m \in \mathcal{M} = \{\mathrm{drive}, \mathrm{ride}, \mathrm{rail}\}$; trip legs $r \in \{1, 2\}$. Party sizes (N_A, N_C); values of time (v_A, v_C) in USD/hr; distance $d_{i,a}$ (mi). Desired curb-time window $[t_a^{\min}, t_a^{\max}]$; planned origin departure t^{depart}.

$$\mathrm{VOT}_{\mathrm{ppm}} = \frac{N_A v_A + N_C v_C}{60} \quad \text{(party value-of-time per minute).}$$

Access-time RV by mode m: $T_{i,a,m}$ with mean $\mu_{i,a,m}$, s.d. $\sigma_{i,a,m}$, quantile $Q_p(\cdot)$. Airport-process RVs: security S_a, bag/drop $C_a(\ell)$ (depends on luggage ℓ), curb→gate walk W_a with means $(\bar{S}_a, \bar{C}_a(\ell), \bar{W}_a)$. We compute an expected, risk-adjusted disutility (USD).

1) Cash Term $\mathrm{Cash}_{i,a}(m)$

A) Drive + park.

$$\mathrm{Park}_a(D) = \min\left(\lceil D \rceil\, R_a^{\mathrm{daily}},\ \lceil D/7 \rceil\, R_a^{\mathrm{weekly}}\right), \tag{1}$$

$$\mathrm{VehOp}_{i,a} = c_{\mathrm{mile}} \cdot 2d_{i,a}, \tag{2}$$

$$\mathrm{Cash}_{i,a}(\mathrm{drive}) = \mathrm{Park}_a(D) + \mathrm{Tolls}_{i,a} + \mathrm{VehOp}_{i,a} + \mathrm{RemLotFee}_a + \mathrm{ShuttleFee}_a + \mathrm{FareRisk}_a. \tag{3}$$

B) Rideshare.

$$\mathrm{Cash}_{i,a}(\mathrm{ride}) = s \cdot \left(F_0 + \alpha \cdot 2d_{i,a} + \beta \cdot 2\mu_{i,a,\mathrm{ride}} + \gamma\right) + \mathrm{Tip} + \mathrm{FareRisk}_a.$$

C) Rail + subway + AirTrain.

$$\mathrm{Cash}_{i,a}(\mathrm{rail}) = N_A f_{i,a}^{\mathrm{rail}} + N_C f_{i,a}^{\mathrm{rail,child}} + (N_A + N_C)\left(f_{i,a}^{\mathrm{xfer}} + f_a^{\mathrm{AirTrain}}\right) + \mathrm{StationPark}_i + \mathrm{FareRisk}_a.$$

2) Time Valuation $\mathrm{TimeVal}_{i,a}(m)$

Time-of-day aware. Let k index time bands with per-minute v_k, and let $\lambda \in [0, 1]$ weight child time.

$$\mathrm{TimeVal}_{i,a}(m) = r \cdot \sum_k \Big(\mathbb{E}[\Delta t_k]\ v_k\ (N_A + \lambda N_C)\Big).$$

Shortcut (constant VOT).

$$\mu_a^{\mathrm{proc}}(\ell) = \bar{S}_a + \bar{C}_a(\ell) + \bar{W}_a, \qquad \mathrm{TimeVal}_{i,a}(m) = \mathrm{VOT}_{\mathrm{ppm}} \cdot r \cdot \left(\mu_{i,a,m} + \mu_a^{\mathrm{proc}}(\ell)\right).$$

3) Reliability / Risk $\mathrm{Risk}_{i,a}(m)$ (CVaR with Shocks)

Let $A_{i,a,m} = T_{i,a,m}$ and $P_a = S_a + C_a + W_a$. Context multiplier $\kappa = \kappa_{\mathrm{weather}} \kappa_{\mathrm{event}} \kappa_{\mathrm{construction}}$; risk aversion $\rho \geq 0$.

$$\mathrm{Risk}_{i,a}(m) = \kappa \cdot \rho \cdot \mathrm{VOT}_{\mathrm{ppm}} \cdot r \cdot \Big(\mathrm{CVaR}_\alpha[A_{i,a,m}] - \mathbb{E}[A_{i,a,m}] + \mathrm{CVaR}_\alpha[P_a] - \mathbb{E}[P_a]\Big).$$

4) Schedule Alignment $\mathrm{Sched}_{i,a}(m)$

Curb-arrival RV $U = t^{\mathrm{depart}} - T_{i,a,m}$; earliness/lateness penalty with $\theta_L > \theta_E$:

$$\phi(u) = \begin{cases} \theta_E \; (t_a^{\min} - u)_+ \,, & u < t_a^{\min}, \\ 0, & u \in [t_a^{\min}, t_a^{\max}], \\ \theta_L \; (u - t_a^{\max})_+ \,, & u > t_a^{\max}. \end{cases} \qquad \mathrm{Sched}_{i,a}(m) = \mathrm{VOT}_{\mathrm{ppm}} \cdot \mathbb{E}[\phi(U)].$$

5) Transfer & Handling Frictions $\mathrm{Xfer}_{i,a}(m)$

Rail transfers with miss probability $\pi_{\mathrm{miss}}(\Delta_k)$ and headway H_k:

$$\mathrm{XferRail}_{i,a} = (N_A + \lambda N_C) \cdot \mathrm{VOT}_{\mathrm{ppm}} \cdot \sum_k \pi_{\mathrm{miss}}(\Delta_k)\, H_k.$$

Checked-bag penalty and terminal geometry:

$$\mathrm{BagPen}_a(\ell) = \mathrm{VOT}_{\mathrm{ppm}} \cdot \Big(\Delta \bar{C}_a(\ell) + \rho\big(\,\mathrm{CVaR}_\alpha[C_a] - \mathbb{E}[C_a]\big)\Big), \qquad \mathrm{GeomPen}_a = \mathrm{VOT}_{\mathrm{ppm}} \cdot r \cdot (\mathrm{IntWalk}_a + \mathrm{Shuttle}_a).$$

$$\mathrm{Xfer}_{i,a}(m) = \mathrm{XferRail}_{i,a} + \mathrm{BagPen}_a(\ell) + \mathrm{GeomPen}_a.$$

6) Comfort / Disutility $\mathrm{Comfort}_{i,a}(m)$

Let $\omega_m(t) \geq 1$ inflate perceived minutes in uncomfortable periods (crowding, night driving):

$$\mathrm{Comfort}_{i,a}(m) = \mathrm{VOT}_{\mathrm{ppm}} \cdot r \cdot \mathbb{E}\left[\int (\omega_m(t) - 1)\, dt\right].$$

7) Environment / Carbon (Optional) $\mathrm{Carbon}_{i,a}(m)$

With social cost of carbon SC (USD/metric-ton) and emission rate e_m (kg CO_2/mi):

$$\mathrm{Carbon}_{i,a}(m) = \frac{SC}{1000} \cdot e_m \cdot 2 d_{i,a}.$$

8) Per-Mode GC and Soft-Min Across Modes

$\mathrm{GC}_{i,a}(m) = \mathrm{Cash}_{i,a}(m) + \mathrm{TimeVal}_{i,a}(m) + \mathrm{Risk}_{i,a}(m) + \mathrm{Sched}_{i,a}(m) + \mathrm{Xfer}_{i,a}(m) + \mathrm{Comfort}_{i,a}(m) + \mathrm{Carbon}_{i,a}(m).$

Inclusive-value (soft-min) with scale $\mu_{\mathrm{sm}} > 0$:

$$\widetilde{\mathrm{GC}}_{i,a} = -\frac{1}{\mu_{\mathrm{sm}}} \log \sum_{m \in \mathcal{M}} \exp\big(-\mu_{\mathrm{sm}}\, \mathrm{GC}_{i,a}(m)\big), \qquad \mathrm{GC}_{i,a} = \widetilde{\mathrm{GC}}_{i,a}.$$

9) Delta for the Map

$$\Delta \mathrm{GC}_i = \mathrm{GC}_{i,\mathrm{HVN}} - \min\left(\mathrm{GC}_{i,\mathrm{JFK}}, \mathrm{GC}_{i,\mathrm{LGA}}, \mathrm{GC}_{i,\mathrm{EWR}}\right).$$

10) Door-to-Destination (d)

Misconnect at hub h with layover L:

$$\mathrm{MCpen}(h,L) = \pi_{\mathrm{mis}}(h,L) \cdot \big(\mathrm{VOT}_{\mathrm{ppm}} \cdot \mathbb{E}[\text{reprotect delay}] + \text{Hotel/meal}\big).$$

Destination schedule penalty for arrive-by $T^\star$ with arrival-time RV $\mathsf{Arr}_{a\to d}$:

$$\mathrm{DestSched}_{a\to d} = \mathrm{VOT}_{\mathrm{ppm}} \cdot \mathbb{E}\big[\Theta_E\ (T^\star - \mathsf{Arr}_{a\to d})_+ + \Theta_L\ (\mathsf{Arr}_{a\to d} - T^\star)_+\,\big].$$

Air-side delay/cancel risk and in-flight time value:

$$\mathrm{AirRisk}_{a\to d} = \rho_{\mathrm{air}}\,\mathrm{VOT}_{\mathrm{ppm}}\big(\,\mathrm{CVaR}_\alpha[\mathrm{Delay}_{a\to d}] - \mathbb{E}[\mathrm{Delay}_{a\to d}]\big) + \mathrm{CancelProb}_{a\to d} \cdot \mathrm{VOT}_{\mathrm{ppm}} \cdot \mathbb{E}[\text{reprotect delay}],$$

$$\mathrm{AirTimeVal}_{a\to d} = \mathrm{VOT}_{\mathrm{ppm}} \cdot \mathbb{E}[\mathrm{AirTime}_{a\to d}].$$

Door-to-destination generalized cost:

$$\mathrm{GC}^{\mathrm{D2D}}_{i,a\to d} = \mathrm{GC}_{i,a} + \mathrm{AirTimeVal}_{a\to d} + \mathrm{AirRisk}_{a\to d} + \mathrm{MCpen}(h,L) + \mathrm{DestSched}_{a\to d}.$$