

Calculul limitelor de funcții

$$\frac{\infty}{\infty}$$

Exerciții rezolvate

$$1. \lim_{x \rightarrow \infty} \frac{x^2 - 2x - 3}{x^3 + x^2 + 1} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{x^2}{x^3} = \lim_{x \rightarrow \infty} \frac{1}{x} = \frac{1}{\infty} = 0$$

$$2. \lim_{x \rightarrow \infty} \frac{2x + 1}{\sqrt{4x^2 - 1}} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{2x}{2x} = 1$$

$$3. \lim_{x \rightarrow -\infty} \frac{2x + 1}{\sqrt{4x^2 - 1}} = \frac{\infty}{\infty} = \lim_{x \rightarrow -\infty} \frac{2x}{|2x|} = \lim_{x \rightarrow -\infty} \frac{2x}{-2x} = -1$$

$$4. \lim_{x \rightarrow \infty} \frac{2^x + 3^x}{3^x - 4^x} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{3^x}{-4^x} = \lim_{x \rightarrow \infty} \left[ -\left(\frac{3}{4}\right)^x \right] = 0, \left\{ \begin{array}{l} 3 > 2 > 1 \\ 4 > 3 > 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} 3^x > 2^x \\ 4^x > 3^x \end{array} \right\}, \frac{3}{4} \in (0,1) \Rightarrow \left(\frac{3}{4}\right)^x \xrightarrow{x \rightarrow \infty} 0$$

$$5. \lim_{x \rightarrow \infty} \frac{\ln(x^3 + x^2 + 1)}{\ln(x^2 + x + 1)} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{\ln x^3}{\ln x^2} = \lim_{x \rightarrow \infty} \frac{3 \ln x}{2 \ln x} = \frac{3}{2}$$

Exerciții propuse

$$1. \lim_{x \rightarrow -\infty} \frac{x^3 + x^2 + 1}{x^2 - 2x - 3}$$

$$2. \lim_{x \rightarrow \infty} \frac{5x + 1}{\sqrt{5x^2 + 1}}$$

$$3. \lim_{x \rightarrow -\infty} \frac{3x + 1}{\sqrt{16x^2 + 1}}$$

$$4. \lim_{x \rightarrow \infty} \frac{2^x + 3^{x+1}}{3^x - 2^x}$$

$$5. \lim_{x \rightarrow \infty} \frac{\ln(2x^2 + 1)}{\ln(x^2 + 1)}$$

$$6. \lim_{x \rightarrow \infty} \frac{\ln(e^x + 1)}{\ln(e^{2x} + 1)}$$

$$7. \lim_{x \rightarrow -\infty} \frac{7x + 1}{|x - 1|}$$

$$8. \lim_{x \rightarrow -\infty} \sqrt[3]{\frac{x^2}{7x + 1}}$$

$$9. \lim_{x \rightarrow \infty} \frac{\ln(e^x + 1)}{3x + 1}$$

$$10. \lim_{x \rightarrow -\infty} \frac{x + 1}{\sqrt{2x^2 + 1}}$$

$$11. \text{Determinați } a, b, c \in \mathbb{R} \text{ astfel încât } \lim_{x \rightarrow \infty} \frac{ax^2 + bx + c}{x + 1} = 1.$$

$$1) -\infty$$

$$2) \sqrt{5}$$

$$3) -\frac{3}{4}$$

$$4) 3$$

$$5) 1$$

$$11) a = 0, b = 1, c \in \mathbb{R}$$

$$6) \frac{1}{2}$$

$$7) -7$$

$$8) -\infty$$

$$9) \frac{1}{3}$$

$$10) 0$$