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Calculul limitelor de funcții

$$\frac{0}{0}$$

Exerciții rezolvate

1. $\lim\limits_{x\to 1}\dfrac{x^2-2x+1}{x^3-1}=\dfrac{0}{0}=\lim\limits_{x\to 1}\dfrac{(x-1)^2}{(x-1)(x^2+x+1)}=\lim\limits_{x\to 1}\dfrac{x-1}{x^2+x+1}=0$

2. $\lim\limits_{x\to 0}\dfrac{sin3x}{x}=\dfrac{0}{0}=\underbrace{\lim\limits_{x\to 0}\dfrac{sin3x}{3x}}_{1}\cdot 3=3$

3. $\lim\limits_{x\to 0}\dfrac{5^x-3^x}{x}=\dfrac{0}{0}=\lim\limits_{x\to 0}\dfrac{5^x-1+1-3^x}{x}=\lim\limits_{x\to 0}\left(\dfrac{5^x-1}{x}-\dfrac{3^x-1}{x}\right)=ln5-ln3=ln\dfrac{5}{3}$

4. $\lim\limits_{x\to 0}\dfrac{ln(x^2+x+1)}{x}=\dfrac{0}{0}=\lim\limits_{x\to 0}\dfrac{ln(x^2+x+1)}{x^2+x}\cdot\dfrac{x^2+x}{x}=$

$=\lim\limits_{x\to 0}\underbrace{\dfrac{ln(x^2+x+1)}{x^2+x}}_{1}\cdot\dfrac{x(x+1)}{x}=\lim\limits_{x\to 0}(x+1)=1$

5. $\lim\limits_{x\to 1}\dfrac{\sqrt[7]{2-x}-1}{x-1}=\dfrac{0}{0}=\lim\limits_{x\to 1}\dfrac{\big(1+(1-x)\big)^{\frac{1}{7}}-1}{x-1}=-\dfrac{1}{7}$

Exerciții propuse

1. $\lim\limits_{x\to 0}\dfrac{x+x^2}{arcsin\,x}$

2. $\lim\limits_{x\to 2}\dfrac{\sqrt{x^2+5}-3}{x-2}$

3. $\lim\limits_{x\to -1}\dfrac{x^2-8x-9}{3x^2+2x-1}$

4. $\lim\limits_{x\to 2}\dfrac{4-x^2}{3^x-9}$

5. $\lim\limits_{x\to 0}\dfrac{cosx-cos3x}{x^2}$

6. $\lim\limits_{x\to 0}\dfrac{ln(1+tg5x)}{x}$

7. $\lim\limits_{x\to 1}\dfrac{x^7-7x+6}{(x-1)^2}$

8. $\lim\limits_{x\to -1}\dfrac{\sqrt{3-x}-2}{x+\sqrt{x+2}}$

9. $\lim\limits_{\substack{x\to 1\\ x<1}}\dfrac{\sqrt[3]{x^3-3x+2}}{x-1}$

10. $\lim\limits_{x\to 1}\dfrac{x-1}{x+x^2+\cdots+x^n-n},n\in\mathbb{N}^*$

11. Determinați $a,b\in\mathbb{R}$ astfel încât $\lim\limits_{x\to 1}\dfrac{ax^2+bx+1}{x-1}=1.$

1) 1

2) $\dfrac{2}{3}$

3) $\dfrac{5}{2}$

4) $-\dfrac{4}{9ln3}$

5) 4

6) 5

7) 21

8) $-\dfrac{1}{6}$

9) $-\infty$

10) $\dfrac{2}{n(n+1)}$

11) $a=2,b=-3$