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Calculul limitelor de funcții

1^∞

Exerciții rezolvate

1. $\lim\limits_{x\to\infty}\left(1+\frac{1}{2x+1}\right)^x = 1^\infty = \lim\limits_{x\to\infty}\left[\left(1+\frac{1}{2x+1}\right)^{2x+1}\right]^{\frac{1}{2x+1}\cdot x} = e^{\lim\limits_{x\to\infty}\frac{x}{2x+1}} = e^{\frac{1}{2}}$

2. $\lim\limits_{x\to\infty}\left(\frac{x^2-2x-3}{x^2+x+1}\right)^{2x-1} = 1^\infty = \lim\limits_{x\to\infty}\left(1+\frac{x^2-2x-3}{x^2+x+1}-1\right)^{2x-1} = \lim\limits_{x\to\infty}\left(1+\frac{-3x-4}{x^2+x+1}\right)^{2x-1} =$

$= \lim\limits_{x\to\infty}\left[\left(1+\frac{-3x-4}{x^2+x+1}\right)^{\frac{x^2+x+1}{-3x-4}}\right]^{\frac{-3x-4}{x^2+x+1}\cdot(2x-1)} = e^{\lim\limits_{x\to\infty}\frac{(-3x-4)(2x-1)}{x^2+x+1}} = e^{\lim\limits_{x\to\infty}\frac{-6x^2}{x^2}} = e^{-6}$

3. $\lim\limits_{x\to\infty}\left(\frac{2x+1}{\sqrt{4x^2-1}}\right)^x = 1^\infty = \lim\limits_{x\to\infty}\left(1+\frac{2x+1}{\sqrt{4x^2-1}}-1\right)^x = \lim\limits_{x\to\infty}\left(1+\frac{2x+1-\sqrt{4x^2-1}}{\sqrt{4x^2-1}}\right)^x =$

$= \lim\limits_{x\to\infty}\left(1+\frac{4x^2+4x+1-4x^2+1}{\sqrt{4x^2-1}\left(2x+1+\sqrt{4x^2-1}\right)}\right)^x = \lim\limits_{x\to\infty}\left(1+\frac{4x+2}{\sqrt{4x^2-1}\left(2x+1+\sqrt{4x^2-1}\right)}\right)^x =$

$= \lim\limits_{x\to\infty}\left[\left(1+\frac{4x+2}{\sqrt{4x^2-1}\left(2x+1+\sqrt{4x^2-1}\right)}\right)^{\frac{\sqrt{4x^2-1}\left(2x+1+\sqrt{4x^2-1}\right)}{4x+2}}\right]^{\frac{4x+2}{\sqrt{4x^2-1}\left(2x+1+\sqrt{4x^2-1}\right)}\cdot x} =$

$= e^{\lim\limits_{x\to\infty}\frac{(4x+2)x}{\sqrt{4x^2-1}\left(2x+1+\sqrt{4x^2-1}\right)}} = e^{\lim\limits_{x\to\infty}\frac{4x^2}{2x\cdot 4x}} = e^{\frac{1}{2}}$

4. $\lim\limits_{x\to 0}(1+2x)^{\frac{1}{x}} = 1^\infty = \lim\limits_{x\to 0}\left[(1+2x)^{\frac{1}{2x}}\right]^{2x\cdot\frac{1}{x}} = e^2$

5. $\lim\limits_{x\to 1}x^{\frac{1}{x-1}} = 1^\infty = \lim\limits_{x\to 1}(1+x-1)^{\frac{1}{x-1}} = \mathrm{e}$

Exerciții propuse

1. $\lim\limits_{x\to-\infty}\left(\frac{x^2-x+1}{x^2-2x-3}\right)^x$
2. $\lim\limits_{x\to\infty}\left(\frac{5x+1}{5x-3}\right)^{\frac{x^2-1}{x}}$
3. $\lim\limits_{x\to\infty}\left(\frac{3x+1}{\sqrt{9x^2+1}}\right)^x$
4. $\lim\limits_{x\to\infty}\left(\frac{2^x+3^x}{3^x-2^x}\right)^x$
5. $\lim\limits_{x\to\infty}\left(\frac{x+\sqrt{x}}{x-\sqrt{x}}\right)^{5\sqrt{x}}$
6. $\lim\limits_{x\to 0}(1-6x)^{\frac{x+1}{x}}$
7. $\lim\limits_{x\to -1}(2x^2-1)^{\frac{1}{x+1}}$
8. $\lim\limits_{x\to 2}(x-1)^{\frac{1}{x^2-4}}$
9. $\lim\limits_{x\to 0}(1+sinx)^{\frac{1}{tgx}}$
10. $\lim\limits_{x\to -3}\left(\frac{x^2+2x-6}{x}\right)^{\frac{1}{3+x}}$

11. Determinați $a, b, c \in \mathbb{R}$ astfel încât $\lim_{x\to\infty}\left(\frac{ax^2+bx+c}{x+1}\right)^x = e$.

1) e

2) $e^{\frac{4}{5}}$

3) $e^{\frac{1}{3}}$

4) 1

5) e^{10}

6) e^{-6}

7) e^{-4}

8) $e^{\frac{1}{4}}$

9) e

10) $e^{\frac{5}{3}}$

11) $a = 0, b = 1, c = 2$