

Calculul limitelor de funcții

$$\infty - \infty$$

Exerciții rezolvate

$$1. \lim_{x \rightarrow \infty} \left(\frac{x^2 - 2x - 3}{x + 1} - \frac{x^2 - 2}{x} \right) = \infty - \infty = \lim_{x \rightarrow \infty} \frac{-3x^2 - x + 2}{x^2 + x} = \lim_{x \rightarrow \infty} \frac{-3x^2}{x^2} = -3$$

$$2. \lim_{x \rightarrow \infty} \left(\sqrt{4x^2 - x + 1} - 2x \right) = \infty - \infty = \lim_{x \rightarrow \infty} \frac{4x^2 - x + 1 - 4x^2}{\sqrt{4x^2 - x + 1} + 2x} = \lim_{x \rightarrow \infty} \frac{-x}{4x} = -\frac{1}{4}$$

$$3. \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 + x + 1} + x \right) = \infty - \infty = \lim_{x \rightarrow -\infty} \frac{x^2 + x + 1 - x^2}{\sqrt{x^2 + x + 1} - x} = \lim_{x \rightarrow -\infty} \frac{x}{|x| - x} = \lim_{x \rightarrow -\infty} \frac{x}{-2x} = -\frac{1}{2}$$

$$4. \lim_{x \rightarrow \infty} \left(x - \sqrt[3]{x^3 + x^2 + x + 1} \right) = \infty - \infty = \lim_{x \rightarrow \infty} \frac{x^3 - (x^3 + x^2 + x + 1)}{x^2 + x\sqrt[3]{x^3 + x^2 + x + 1} + \sqrt[3]{x^3 + x^2 + x + 1}^2} =$$

$$= \lim_{x \rightarrow \infty} \frac{-x^2}{3x^2} = -\frac{1}{3}$$

$$5. \lim_{x \rightarrow \infty} \left(\ln(x^3 + x^2 + 1) - \ln(x^2 + x + 1) \right) = \infty - \infty = \lim_{x \rightarrow \infty} \ln \frac{x^3 + x^2 + 1}{x^2 + x + 1} = \lim_{x \rightarrow \infty} \ln x = \infty$$

Exerciții propuse

$$1. \lim_{x \rightarrow -\infty} \left(\sqrt{9x^2 - 3x + 1} + 3x \right)$$

$$6. \lim_{x \rightarrow \infty} \left(\ln(4x + 1) - \ln(2x + 1) \right)$$

$$2. \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + x + 1} - x \right)$$

$$7. \lim_{x \rightarrow -\infty} \left(x - 2 + \sqrt{x^2 + 4x + 1} \right)$$

$$3. \lim_{x \rightarrow -\infty} \left(\sqrt[3]{x^3 + x^2 + x + 1} - x \right)$$

$$8. \lim_{\substack{x \rightarrow -2 \\ x > -2}} \left(\frac{x}{x + 2} - \frac{1}{x^2 - 4} \right)$$

$$4. \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 3x + 1} - \sqrt{x^2 + 1} \right)$$

$$9. \lim_{x \rightarrow \infty} \left(x - \ln(e^x + 1) \right)$$

$$5. \lim_{x \rightarrow \infty} \left(\sqrt{x^2 - x + 1} - x + 1 \right)$$

$$10. \lim_{x \rightarrow \infty} \left(\ln(e^x + 1) - \ln(e^{2x} + 1) \right)$$

$$11. \text{Determinați } a, b \in \mathbb{R} \text{ astfel încât } \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + x + 1} - ax + b \right) = \frac{3}{2}.$$

$$1) \frac{1}{2}$$

$$6) \ln 2$$

$$2) \frac{1}{2}$$

$$7) -4$$

$$3) \frac{1}{3}$$

$$8) -\infty$$

$$4) \frac{3}{2}$$

$$9) 0$$

$$5) \frac{1}{2}$$

$$10) -\infty$$

$$11) a = 1, b = 1$$