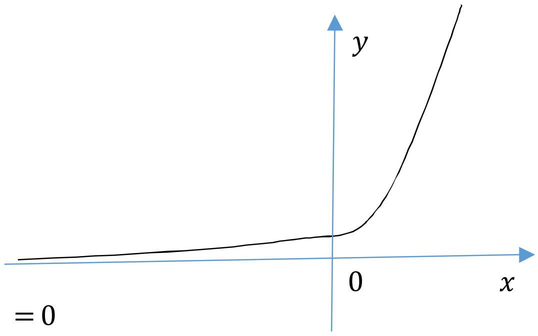
Lectura grafică și determinarea limitelor de funcții – 2 –

Funcția exponențială $f:\mathbb{R}\to(0,\infty)$, $f(x)=a^x$, $a\in(0,\infty)\setminus\{1\}$ $\quad a^x>0$

$a\in(1,\infty)$
f este crescătoare
f este convexă

$\lim\limits_{x\to\infty} a^x = +\infty$

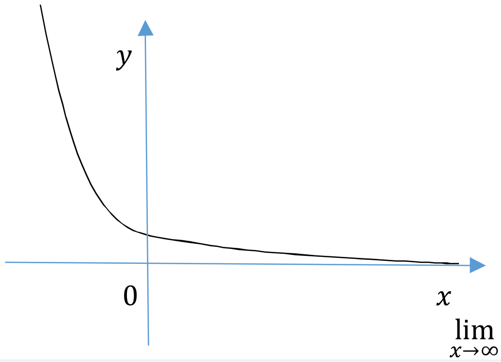


$\lim\limits_{x\to-\infty} a^x = 0$

$e^{-\infty}=0 \qquad e^{\infty}=\infty$

$a\in(0,1)$
f este descrescătoare
f este convexă

$\lim\limits_{x\to-\infty} a^x = +\infty$



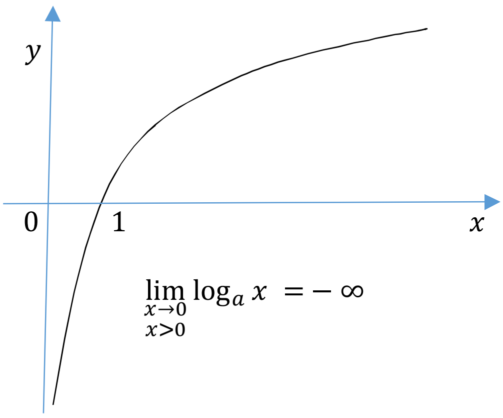
$\lim\limits_{x\to\infty} a^x = 0$

$\left(\frac{1}{2}\right)^{-\infty}=\infty \qquad \left(\frac{1}{2}\right)^{\infty}=0$

Funcția logaritmică $f:(0,\infty)\to\mathbb{R}$, $f(x)=\log_a x$, $a\in(0,\infty)\setminus\{1\}$

$a\in(1,\infty)$
f este crescătoare
f este concavă

$\lim\limits_{x\to\infty} \log_a x = +\infty$

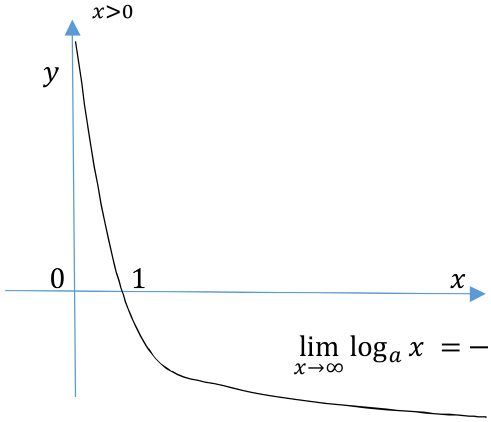


$\lim\limits_{\substack{x\to0\\x>0}} \log_a x = -\infty$

$ln\,0_+ = -\infty \qquad ln\,\infty=\infty$

$a\in(0,1)$
f este descrescătoare
f este convexă

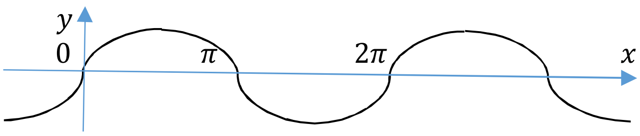
$\lim\limits_{\substack{x\to0\\x>0}} \log_a x = +\infty$



$\lim\limits_{x\to\infty} \log_a x = -\infty$

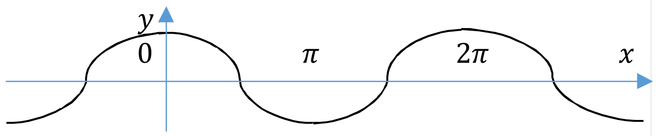
$\log_{\frac{1}{2}} 0_+ = \infty \qquad \log_{\frac{1}{2}} \infty = -\infty$

Funcția sinus $sin:\mathbb{R}\to[-1,1]$



$\nexists \lim\limits_{x\to-\infty} sinx \qquad \nexists \lim\limits_{x\to+\infty} sinx$

Funcția cosinus $cos:\mathbb{R}\to[-1,1]$



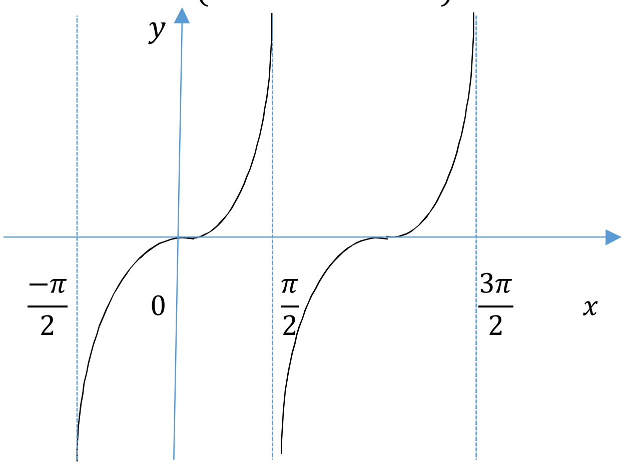
$\nexists \lim\limits_{x\to-\infty} cosx \qquad \nexists \lim\limits_{x\to+\infty} cosx$

$$\lim_{x\to+\infty}\frac{sinx}{x}=0$$

$$\lim_{x\to 0}\frac{sinx}{x}=1$$

Funcția tangentă

$$tg:\mathbb{R}\backslash\left\{\frac{(2k+1)\pi}{2},k\in\mathbb{Z}\right\}\to\mathbb{R}$$

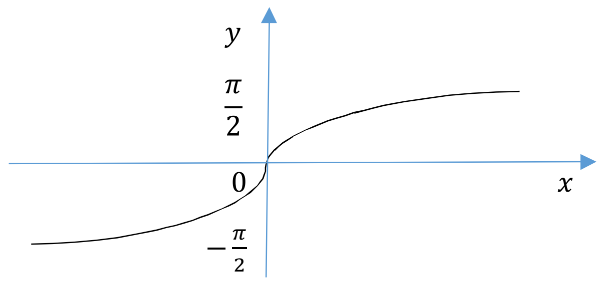


$$\lim_{\substack{x\to\frac{\pi}{2}\\ x<\frac{\pi}{2}}} tgx=+\infty$$

$$\lim_{\substack{x\to\frac{\pi}{2}\\ x>\frac{\pi}{2}}} tgx=-\infty$$

Funcția arctangentă

$$arctg:\mathbb{R}\ \to\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

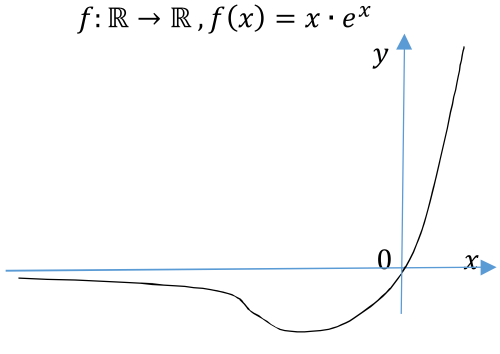


$$\lim_{x\to+\infty} arctg\ x=\frac{\pi}{2}$$

$$\lim_{x\to-\infty} arctg\ x=-\frac{\pi}{2}$$

$$arctg\ \infty=\frac{\pi}{2}$$

$$f:\mathbb{R}\to\mathbb{R},f(x)=x\cdot e^x$$

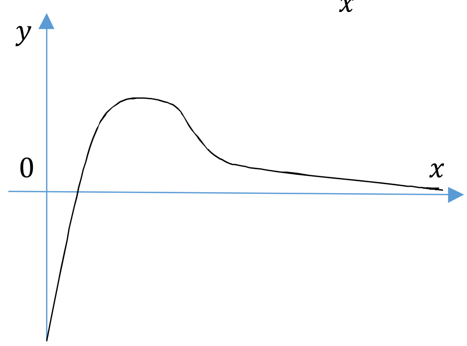


$$\lim_{x\to-\infty} x\cdot e^x=-\infty\cdot 0=\lim_{x\to-\infty}\frac{x}{e^{-x}}=\lim_{x\to+\infty}\frac{-x}{e^x}=0$$

Obs. La ∞ funcția exponențială crește mai repede decât funcția polinomială.

$$\lim_{x\to+\infty} x\cdot e^x=\infty\cdot\infty=\infty$$

$$f:(0,\infty)\to\mathbb{R},f(x)=\frac{ln\,x}{x}$$



$$\lim_{\substack{x\to 0\\ x>0}}\frac{ln\,x}{x}=\frac{ln0_+}{0_+}=-\infty\cdot\infty=-\infty$$

$$\lim_{x\to+\infty}\frac{ln\,x}{x}=0$$

Obs. La ∞ funcția polinomială crește mai repede decât funcția logaritmică.