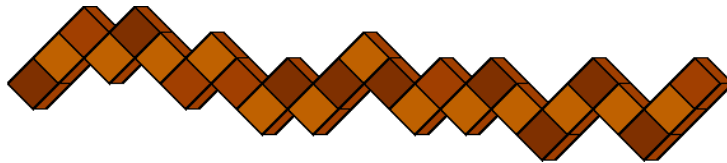


Funkcionálne programovanie 3

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- parametrický polymorfizmus na príkladoch funkcionálov (map, filter, foldl, foldr)
- backtracking (ako príklad na list-comprehension)



Cvičenie:

- funkcionály (map, filter, foldr, ...)
- backtracking v Haskellu,



Phil Wadler, λ -man



Funkcia je hodnotou - zatiaľ len argumentom

učíme sa z Prelude.hs:

štandardná knižnica haskellu obsahuje množstvo:

- užitočných funkcií,
- vzorových funkcií.

- zober zo zoznamu tie prvky, ktoré spĺňajú bool-podmienku (test)
booleovská podmienka príde ako argument funkcie a má typ (a -> Bool):

```
filter      :: (a -> Bool) -> [a] -> [a]      -- ( ) nie sú zbytočné  
filter p xs = [ x | x <- xs, p x ]
```

```
> filter even [1..10]  
[2,4,6,8,10]
```

- rozdel' zoznam na zoznam menších, rovných a väčších prvkov ako prvý:
riešenie menej efektívne ale v istom zmysle elegantnejšie

```
tripivot (x:xs) = ( filter (<x) xs, filter (==x) xs, filter (>x) xs )
```



Funkcia (predikát) argumentom

učíme sa z Prelude.hs:

- ber zo zoznamu prvky, kým platí logická podmienka (test):

```
takeWhile :: (a -> Bool) -> [a] -> [a]
takeWhile p [] = []
takeWhile p (x:xs) | p x = x : takeWhile p xs
                   | otherwise = []
```

```
> takeWhile (>0) [1,2,-1,3,4]
[1,2]
```

- vyhod' tie počiatkové prvky zoznamu, pre ktoré platí podmienka:

```
dropWhile :: (a -> Bool) -> [a] -> [a]
dropWhile p [] = []
dropWhile p xs@(x:xs') | p x = dropWhile p xs'
                       | otherwise = xs
```

```
> dropWhile (>0) [1,2,-1,3,4]
[-1,3,4]
```



Príklad (porozdel'uj)

Definujte `porozdeluj :: (a -> Bool) -> [a] -> [[a]]`, ktorá rozdelí zoznam na podzoznamy, v ktorých súvisle platí podmienka daná 1. argumentom

`porozdeluj (>0) [1,2,0,3,4,5,-1,6,7] = [[1,2],[3,4,5],[6,7]]`

`porozdeluj (\x -> x `mod` 3 > 0) [1..10] = [[1,2],[4,5],[7,8],[10]].`

`porozdeluj p [] = []`

`porozdeluj p xs =`

`(takeWhile p xs) :`

-- prefix, kým platí p je prvým prvkom

`porozdeluj p`

-- rekurzívne volanie na ďalšie prvky

`(dropWhile (\x -> (not (p x))))` -- odstráň, kým neplatí p

`(dropWhile p xs))`

-- odstráň, kým platí p

`Main> porozdeluj (>0) [1,2,0,0,3,4,-1,5]
[[1,2],[3,4],[5]]`



Funktor map

- funktor, ktorý aplikuje funkciu (1.argument) na všetky prvky zoznamu

map $:: (a \rightarrow b) \rightarrow [a] \rightarrow [b]$

map f [] = []

map f (x:xs) = f x : map f xs

-- alebo map f xs = [f x | x <-xs]

- Príklad použitia:

map (+1) [1,2,3,4,5] = [2,3,4,5,6]

map odd [1,2,3,4,5] = [True,False,True,False,True]

and (map odd [1,2,3,4,5]) = False

all p xs = and (map p xs) -- all p xs = p platí pre všetky prvky zoznamu xs

map head [[1,0,0], [2,1,0], [3,0,1]] = [1, 2, 3]

map tail [[1,0,0], [2,1,0], [3,0,1]] = [[0,0], [1,0], [0,1]]

map (0:) [[1],[2],[3]] = [[0,1],[0,2],[0,3]]

map (++[0]) [[1],[2],[3]] = [[1,0],[2,0],[3,0]]

← \x->x++[0]

Transponuj maticu

-- transponuj pomocou map, nie list-comprehension

transponuj :: Matica -> Matica

transponuj [] = []

transponuj ([]:xss) = transponuj xss

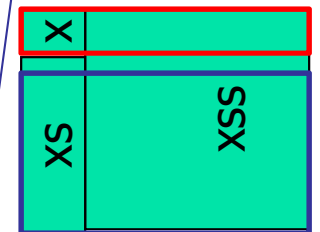
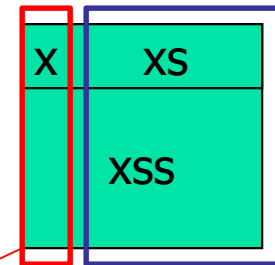
transponuj ((x:xs):xss) = (x:(**map head xss**)):
(transponuj (xs:(**map tail xss**)))

-- riešenie z minulej prednášky

transpose [] = []

transpose ([] : xss) = transpose xss

transpose ((x:xs) : xss) = (x : [h | (h:t) <- xss]) :
transpose (xs : [t | (h:t) <- xss])



Ďalšie (známe) funkcionály –

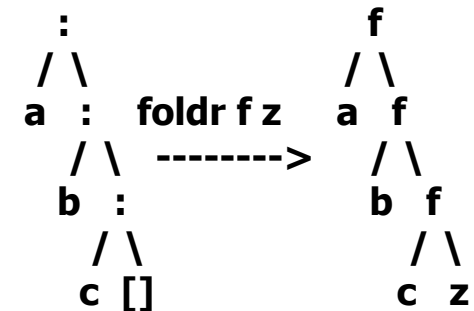
(foldr – schéma rekurzcie na zoznamoch)

foldr :: (a -> b -> b) -> b -> [a] -> b

foldr f z [] = z

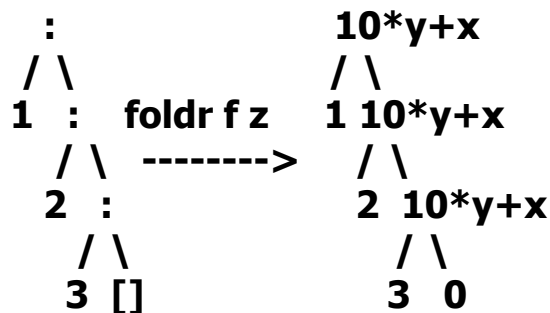
foldr f z (x:xs) = f x (foldr f z xs)

a : b : c : [] -> f a (f b (f c z))



Main> foldr (+) 0 [1..100]

5050



Main> foldr (\x y->10*y+x) 0 [1,2,3]

321

nepomenovaná analógia fcie: pom x y = 10*y+x

-- g je vnorená lokálna funkcia

foldr :: (a -> b -> b) -> b -> [a] -> b

foldr f z = g

where g [] = z

g (x:xs) = f x (g xs)

Ďalšie (známe) funkcionály -

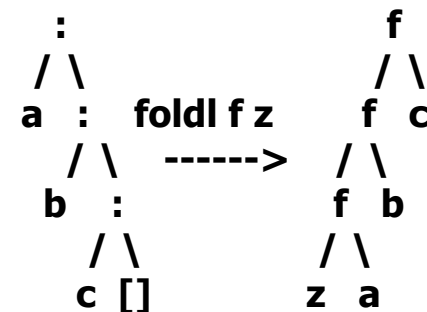
(foldl – schéma iterácie na zoznamoch)

foldl :: (a -> b -> a) -> a -> [b] -> a

foldl f z [] = z

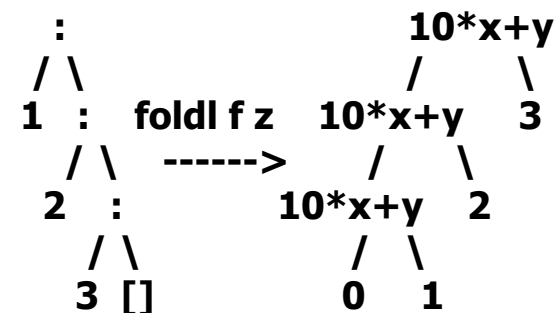
foldl f z (x:xs) = foldl f (f z x) xs

a : b : c : [] -> f (f (f z a) b) c



Main> foldl (+) 0 [1..100]
5050

Main> foldl (\x y->10*x+y) 0 [1,2,3]
123





Vypočítajte

- `foldr max (-999) [1,2,3,4]`
`foldl max (-999) [1,2,3,4]`
- `foldr (_ -> \y ->(y+1)) 0 [3,2,1,2,4]`
`foldl (\x -> _ ->(x+1)) 0 [3,2,1,2,4]`

- `foldr (-) 0 [1..100] =`

$$(1-(2-(3-(4-\dots-(100-0)))))) = 1-2 + 3-4 + 5-6 + \dots + (99-100) = -50$$

- `foldl (-) 0 [1..100] =`

$$(\dots(((0-1)-2)-3) \dots - 100) = -5050$$



foldr a foldl ešte raz

- `foldl (+) 0 [1,2,3,4]`
`foldr (+) 0 [1,2,3,4]`

10

- `foldr max (-999) [1,2,3,4]`
`foldl max (-999) [1,2,3,4]`

4

- `foldr (_ -> \y -> (y+1)) 0 [3,2,1,2,4]`
`foldl (\x -> _ -> (x+1)) 0 [3,2,1,2,4]`

5

- `rozpoj :: [(a,b)] -> ([a],[b])`
`rozpoj = foldr (\(x,y) -> \ (xs,ys) -> (x:xs, y:ys)) ([], [])`

`rozpoj [(1,11),(2,22),(3,33)]`
`([1,2,3],[11,22,33])`



Funkcia je hodnotou

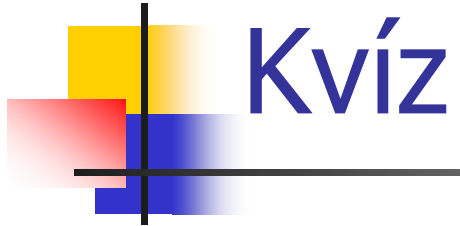
- $[a \rightarrow a]$ je zoznam funkcií typu $a \rightarrow a$
napríklad: $[(+1), (+2), (*3)]$ je $[\backslash x \rightarrow x+1, \backslash x \rightarrow x+2, \backslash x \rightarrow x*3]$
- čo je foldr $(.)$ id $[(+1), (+2), (*3)]$??
akého je typu $[a \rightarrow a]$
foldr $(.)$ id $[(+1), (+2), (*3)]$ 100 303
foldl $(.)$ id $[(+1), (+2), (*3)]$ 100 ???

lebo skladanie fcií je asociatívne:

- $((f . g) . h) x = (f . g) (h x) = f (g (h x)) = f ((g . h) x) = (f . (g . h)) x$
- $\text{mapf} :: [a \rightarrow b] \rightarrow [a] \rightarrow [b]$ -- rovnako dlhé zoznamy
 $\text{mapf } [] _ = []$
 $\text{mapf } _ [] = []$
 $\text{mapf } (f:fs) (x:xs) = (f x):\text{mapf } fs xs$

[11,22,33]

$\text{mapf } [(+1), (+2), (+3)] [10,20,30]$



Kvíz

$\text{foldr } (:) [] \text{ xs} = \text{xs}$

$\text{foldr } (:) \text{ ys xs} = \text{xs} ++ \text{ys}$

$\text{foldr } ? ? \text{ xs} = \text{reverse xs}$



Priemerný prvok

Ak chceme vypočítať aritmetický priemer (a-priemer) prvkov zoznamu, matice, ... potrebujeme poznať ich súčet a počet. Ako to urobíme na jeden prechod štruktúrou pomocou foldr/foldl ? ...počítame dvojicu hodnôt, súčet a počet:

- priemerný prvok zoznamu

priemer xs = sum/count where (sum, count) = sumCount xs

sumCount xs = foldr (\x -> \ (sum, count) -> (sum+x, count+1)) (0, 0) xs

- priemerný prvok matice

je a-priemer a-priemerov riadkov matice a-priemerom hodnôt matice ?

sumCount' :: [[Float]] -> (Float, Float)

sumCount' xs =

foldr (\x -> \ (sum, count) -> scitaj (sumCount x) (sum, count)) (0, 0) xs

where scitaj (a,b) (c,d) = (a+c, b+d)

priemer' :: [[Float]] -> Float

priemer' = uncurry (/) . sumCount'

uncurry :: (a->b->c) -> (a,b) -> c

uncurry f (a,b) = f a b

curry :: ((a,b) -> c) -> (a->b->c)

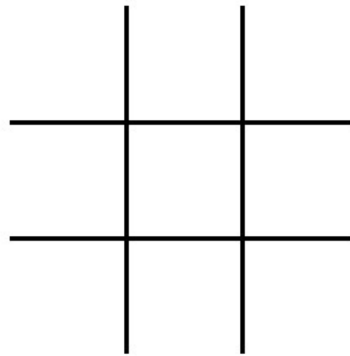
curry g a b = g (a,b)



Bactracking

(ľahký úvod)

- vložte 6 kameňov do mriežky 3x3, tak aby v žiadnom smere (riadok, stĺpec, uhlopriečka) neboli tri.



- pri najivnom prehľadávaní - všetkých možností je $2^9 = 512$
- ak poznáme kombinácie bez opakovania - možností je už len 9 nad 6, teda 9 nad 3, čo je 84



Riešenie „brute force“

(s ním prežijete len pri smiešne jednoduchých úlohách)

```
subset' ys xs = and (map (\x -> elem x xs) ys)
```

```
subset'' xs ys = all (`elem` ys) xs
```

```
isOk      :: [Int] -> Bool
```

```
isOk xs =
```

```
    not (subset' [0,1,2] xs) && not (subset' [3,4,5] xs) && not (subset' [6,7,8] xs) &&  
    not (subset' [0,3,6] xs) && not (subset' [1,4,7] xs) && not (subset' [2,5,8] xs) &&  
    not (subset' [0,4,8] xs) && not (subset' [2,4,6] xs)
```

vygenerujeme všetky podmnožiny [0..8] a vyberieme správne dĺžky 6

```
powerSet      :: [Int] -> [[Int]]
```

```
powerSet []   = [[]]
```

```
powerSet (x:xs) = map (x:) ps ++ ps where ps = powerSet xs
```

```
solve3x3 ' = filter (\x -> 6 == length x) (filter isOk (powerSet [0..8]))
```



Riešenie „kombinatorické“

(s ním prežijete len pri smiešne jednoduchých úlohách)

generujeme len 6 prvkové kombinácie [0..8] a z nich vyberieme správne

```
kbo          :: [Int] -> Int -> [[Int]]
kbo _ 0      = [[]]
kbo [] _     = []
kbo (x:xs) k = [x:ys | ys <- kbo xs (k-1)] ++ kbo xs k
```

```
solve3x3"    = filter isOk (kbo [0..8] 6)
```

generujeme len správne 6 prvkové kombinácie [0..8]

Backtracking = testujeme správnosť riešenia už počas jeho tvorby

```
kbo'         :: [Int] -> Int -> [[Int]]
kbo' _ 0      = [[]]
kbo' [] _     = []
kbo' (x:xs) k = [x:ys | ys <- kbo' xs (k-1), isOk (x:ys)] ++ kbo' xs k
```

```
solve3x3     = kbo' [0..8] 6
```




Backtracking

magické číslo

?	381654729
2	38
3	381
4	3816
5	38165
6	381654
7	3816547
8	38165472
9	381654729

číslo s neopakujúcimi sa ciframi 1..9, ktorého prvých i-cifier je deliteľných i

-- (čiastočným) riešením je zoznam cifier od konca, t.j. [3,2,1] je číslo 123
type Riesenie = [Int] -- typ riešenia

-- nájdi všetky magické riešenia, ktoré obsahujú predpísané cifry
magicBacktrack :: [Int] -> [Riesenie] -- hľadáme zoznam
magicBacktrack [] = [[]] -- riešení
magicBacktrack cifry = [c:ciastocneRiesenie |
c<-cifry,
ciastocneRiesenie<-magicBacktrack (diff cifry [c]),
jeMagicke (c:ciastocneRiesenie)]

-- diff (rozdiel zoznamov) poznáme z minulej prednášky, alebo cifry \ [c]

-- jeMagicke je test, ktorý overí konzistentnosť nového čiastočného riešenia

Backtracking

jeMagické

```
jeMagicke :: Riesenie -> Bool
jeMagicke cs = jeMagicke' 1 0 (reverse cs)

jeMagicke' :: Int->Int->Riesenie -> Bool
jeMagicke' _ _ [] = True
jeMagicke' i stareCislo (cifra:cs)
    = noveCislo `mod` i == 0
      &&
        jeMagicke' (i+1) noveCislo cs
where
    noveCislo = 10*stareCislo+cifra
```

```
Main> magicBacktrack [1,2,3,4,5,6,7,8,9]
[[9,2,7,4,5,6,1,8,3]]
```

t.j. jediné riešenie je
381654729



opäť jeMagicke

Definujme pomocou schém rekurzie foldr a iterácie foldl

foldr:

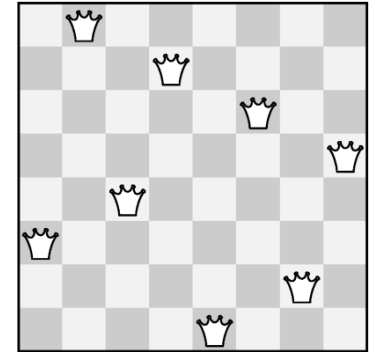
```
jeMagicke"      :: Riesenie -> Bool
jeMagicke" xs   = res
  where (_,_,res) =
    foldr (\cifra -> \(count, cislo, result) ->
      (count+1, 10*cislo+cifra,
       result && ( (10*cislo+cifra) `mod` (count+1) == 0)))
      (0, 0, True)
    xs
```

foldl: na cvičení...

```
Main> jeMagicke" [9,2,7,4,5,6,1,8,3]
True
```

Backtracking

8 dám na šachovnici



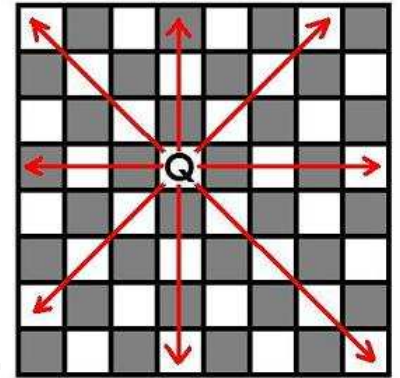
```
-- definujeme typ popisujúci riešenie problému
type RiesenieDam = [Int]
-- pozície dám v stĺpcoch 1,2,3, ..., n
-- príklad riešenia [3,1,6,2,5,7,4,0]
-- hľadáme permutáciu [0..N-1], [0..7]

-- definujeme rekurzívnu funkciu, ktorá vráti zoznam všetkých riešení
damyBacktrack :: Int -> [RiesenieDam] -- všetky riešenia,
-- argument Int určuje, ako ďaleko v "riešení" sme

damyBacktrack 0 = [[]] -- jedno triviálne riešenie pre 0x0
damyBacktrack (n+1) = [
  dama:ciastocneRiesenie |
    ciastocneRiesenie <- damyBacktrack n, -- rekurzívne volanie nájde
-- čiastočné riešenie
    dama <- [0..7], -- rozšírime čiastočné riešenie
-- o ďalšiu dámu na nové r.
    damyOk n dama ciastocneRiesenie] -- otestuj, či je to ešte riešenie
```

Backtracking

(ťažké: orezávanie stromu možností)



damyOk

:: Int -> Int -> RiesenieDam -> Bool

```
damyOk n ndama ciastocneRiesenie = and [ -- dámy sú ok, ak žiadna
  not (damaVOhrozeni ndama stlpec ciastocneRiesenie) | -- z nich nie je
  stlpec <- [0..(n-1)] ] -- v ohrození, pre všetky už položené dámy
```

damaVOhrozeni

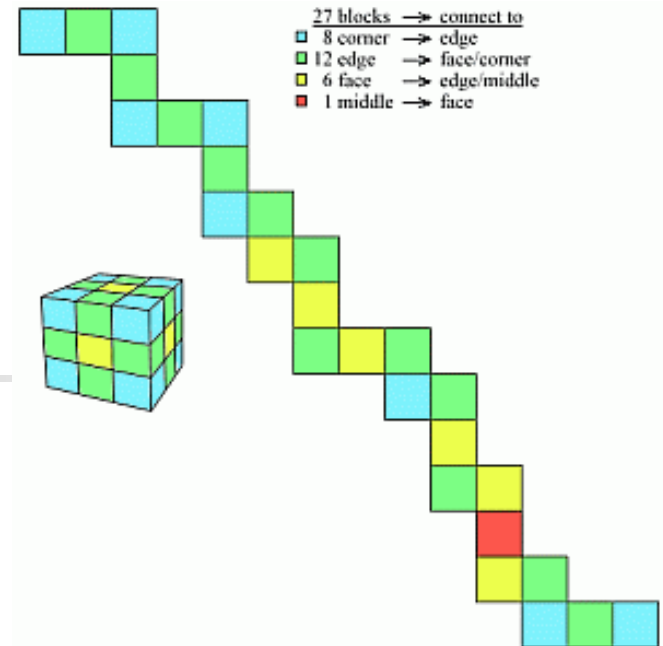
:: Int-> Int-> RiesenieDam->Bool

```
damaVOhrozeni ndama stlpec ciastocneRiesenie = -- dáma je v ohrození
  (ndama==ciastocneRiesenie!!stlpec) || -- od dámy v stlpec
  (abs(ndama - ciastocneRiesenie!!stlpec)==stlpec+1) -- ak sú v rovnakom riadku
  -- alebo diagonále
```

Main> damy

[[3,1,6,2,5,7,4,0],[4,1,3,6,2,7,5,0],[2,4,1,7,5,3,6,0],[2,5,3,1,7,4,6,0]]

Drevený had



- ako na to ?
- popíšeme hada dĺžkami jednotlivých „rebier“

```
type Had = [Int]
```

```
had1 :: Had
```

```
had1 = [3,3,3,3,2,2,2,3,3,2,2,3,2,3,2,2,3]
```

- počet „kĺbov“ = počet čiarok v zozname = $\text{length}(\text{mysnake}) - 1 = 16$
- každý kĺb je započítaný 2x, preto
 $(\text{sum } \text{mysnake}) - ((\text{length } \text{mysnake}) - 1) == 27$
- každý kĺb má 4 možné polohy, takže naivne $4^{16} = 4.294.967.296$ možností
- ak si kocku predstavíme v 3-rozmernom priestore, každé „rebro“ má smer niektorého z vektorov $[\pm 1, 0, 0]$, $[0, \pm 1, 0]$, $[0, 0, \pm 1]$
- v kĺbe sa had môže=musí zohnúť o 90° , ako vyzerá smer ďalšieho rebra ?
- kolmé vektory k vektoru $[\pm 1, 0, 0]$ sú $[0, \pm 1, 0]$ $[0, 0, \pm 1,]$
 k vektoru $[0, \pm 1, 0]$ sú $[\pm 1, 0, 0]$ $[0, 0, \pm 1,]$
 k vektoru $[0, 0, \pm 1]$ sú $[\pm 1, 0, 0]$ $[0, \pm 1, 0]$



Let's code



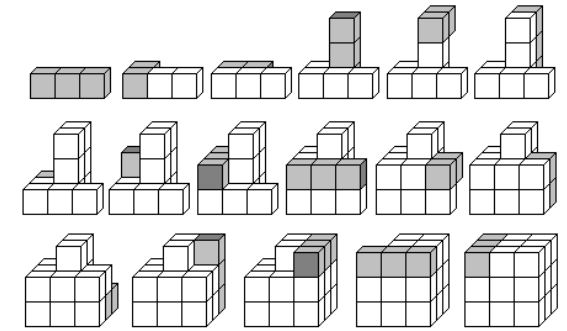
- kolmé vektory k vektoru

```
type SVektor = (Int, Int, Int)      -- dx, dy, dz
kolme        :: SVektor -> [SVektor]
kolme (_, 0, 0) = [(0, 1, 0), (0, -1, 0), (0, 0, 1), (0, 0, -1)]
kolme (0, _, 0) = [(1, 0, 0), (-1, 0, 0), (0, 0, 1), (0, 0, -1)]
kolme (0, 0, _) = [(0, 1, 0), (0, -1, 0), (1, 0, 0), (-1, 0, 0)]
```

- test, či kocička je v kocke 3x3x3 (ľahko parametrizovateľné pre 4x4x4):

```
type Pozicia      = (Int, Int, Int)      -- x, y, z
vKocke3          :: Pozicia -> Bool
vKocke3 (x, y, z) = x `elem` [1..3] &&
                    y `elem` [1..3] &&
                    z `elem` [1..3]
```

Let's code



- ďalšia kocička v danom smere má súradnice

```
nPoz :: Pozicia -> SVektor -> Pozicia
```

```
nPoz (x,y,z) (dx,dy,dz) = (x+dx,y+dy,z+dz)
```

- kocičky celého rebra danej dĺžky (2, 3, event. 4)

```
type Rebro = [Pozicia]
```

```
rebro :: Pozicia -> SVektor -> Int -> Rebro
```

```
rebro start smer 2 = [nPoz start smer]
```

```
rebro start smer 3 = [nPoz (nPoz start smer) smer,  
                      nPoz start smer]
```

-- pre 4x4x4 ešte pridáme:

```
rebro start smer 4 = [nPoz (nPoz (nPoz start smer) smer) smer,  
                      nPoz (nPoz start smer) smer,  
                      nPoz start smer]
```


Zlož hada

<http://www.jaapsch.net/puzzles/javascript/snakecubej.htm>

```
zloz [[(1,1,1)]] (0,0,1) had1
```

```
zloz :: Solution -> SVektor -> Had -> [Solution]
```

```
zloz ciastocne smer [] = [ciastocne]
```

```
zloz ciastocne smer (len:rebra) =
```

```
  [ riesenie |
```

```
    kolmySmer <- kolmo smer,
```

```
    koniecHada <- [head (head ciastocne)],
```

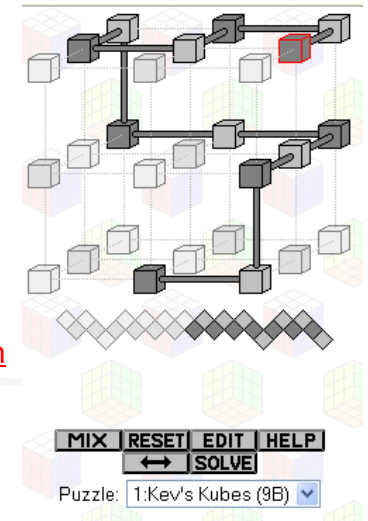
```
    noveRebro <- [rebro koniecHada kolmySmer len],
```

```
    all vKocke3 noveRebro,
```

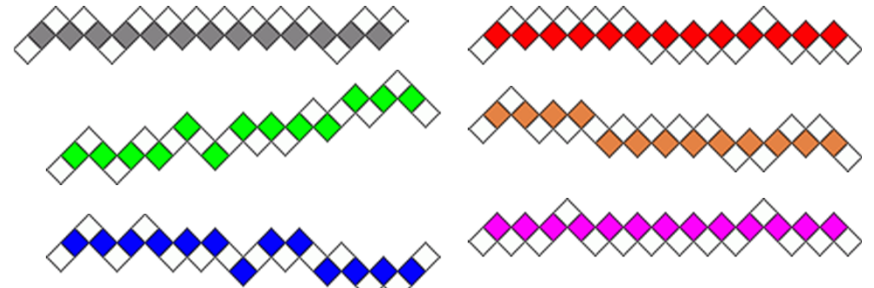
```
    all (`notElem` concat ciastocne) noveRebro,
```

```
    riesenie <- zloz (noveRebro:ciastocne) kolmySmer rebra
```

```
  ]
```



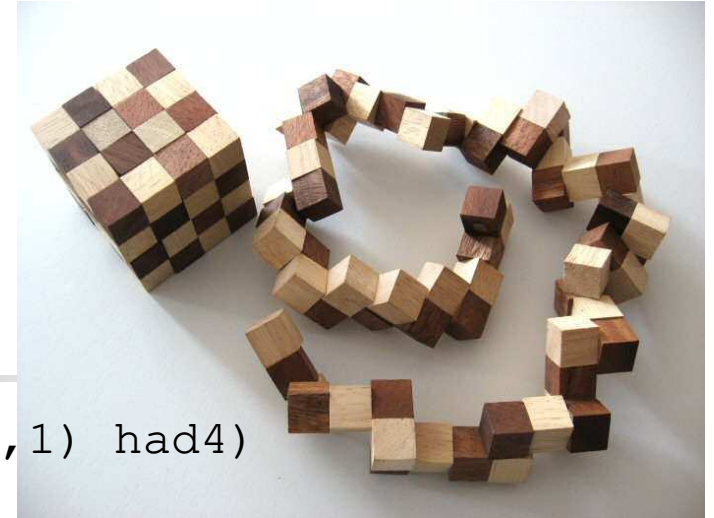
Výsledky 3x3x3



<http://www.jaapsch.net/puzzles/snakecube.htm>

```
Main> head(zloz3 [[(1,1,1)]] (0,0,1) had1)
[[ (3,3,3), (2,3,3)], [(1,3,3)], [(1,2,3)], [(2,2,3), (2,2,2)], [(2,2,1)
  ], [(2,1,1), (2,1,2)], [(2,1,3)], [(1,1,3)], [(1,1,2), (1,2,2)], [(1,3
    ,2), (2,3,2)], [(3,3,2)], [(3,2,2)], [(3,2,3)], [(3,1,3), (3,1,2)], [(
      3,1,1), (3,2,1)], [(3,3,1), (2,3,1)], [(1,3,1), (1,2,1)], [(1,1,1)]]
=== 4x4x4
*Main> :load "c:\\had.hs"
[1 of 1] Compiling Main                ( C:\\had.hs, interpreted )
Ok, modules loaded: Main.
*Main> head (zloz4 [[(2,1,1)]] (0,0,1) had4)
[[ (1,4,2)], [(1,4,1), (1,3,1), (1,2,1)], [(1,1,1)], [(1,1,2)], [(2,1,2)], [(2,
  ,2,2)], [(1,3,2)], [(2,3,2)], [(2,3,3)], [(2,2,3)], [(1,2,3), (1,3,3)], [(1,4,
    ,4), (1,3,4), (1,2,4)], [(1,1,4)], [(1,1,3)], [(2,1,3)], [(2,1,4), (2,2,4), (2,
      ,4,4), (3,4,4)], [(4,4,4)], [(4,3,4)], [(3,3,4)], [(3,3,3)], [(4,3,3)], [(4,4,
        )], [(2,4,3), (2,4,2)], [(2,4,1)], [(3,4,1)], [(3,4,2)], [(4,4,2)], [(4,4,1)],
          , [(4,3,2)], [(3,3,2)], [(3,2,2), (3,2,3)], [(3,2,4)], [(3,1,4), (3,1,3)], [(3,
            ,1,2), (4,1,3)], [(4,1,4)], [(4,2,4), (4,2,3), (4,2,2)], [(4,2,1)], [(4,1,1)],
              (3,2,1)], [(3,3,1)], [(2,3,1), (2,2,1)], [(2,1,1)]]
```

Výsledky 4x4x4



```
*Main> head (zloz4 [[(2,1,1)]] (0,0,1) had4)
```

```
*Main> head (zloz4 [[(2,1,1)]] (0,0,1) had4)
```

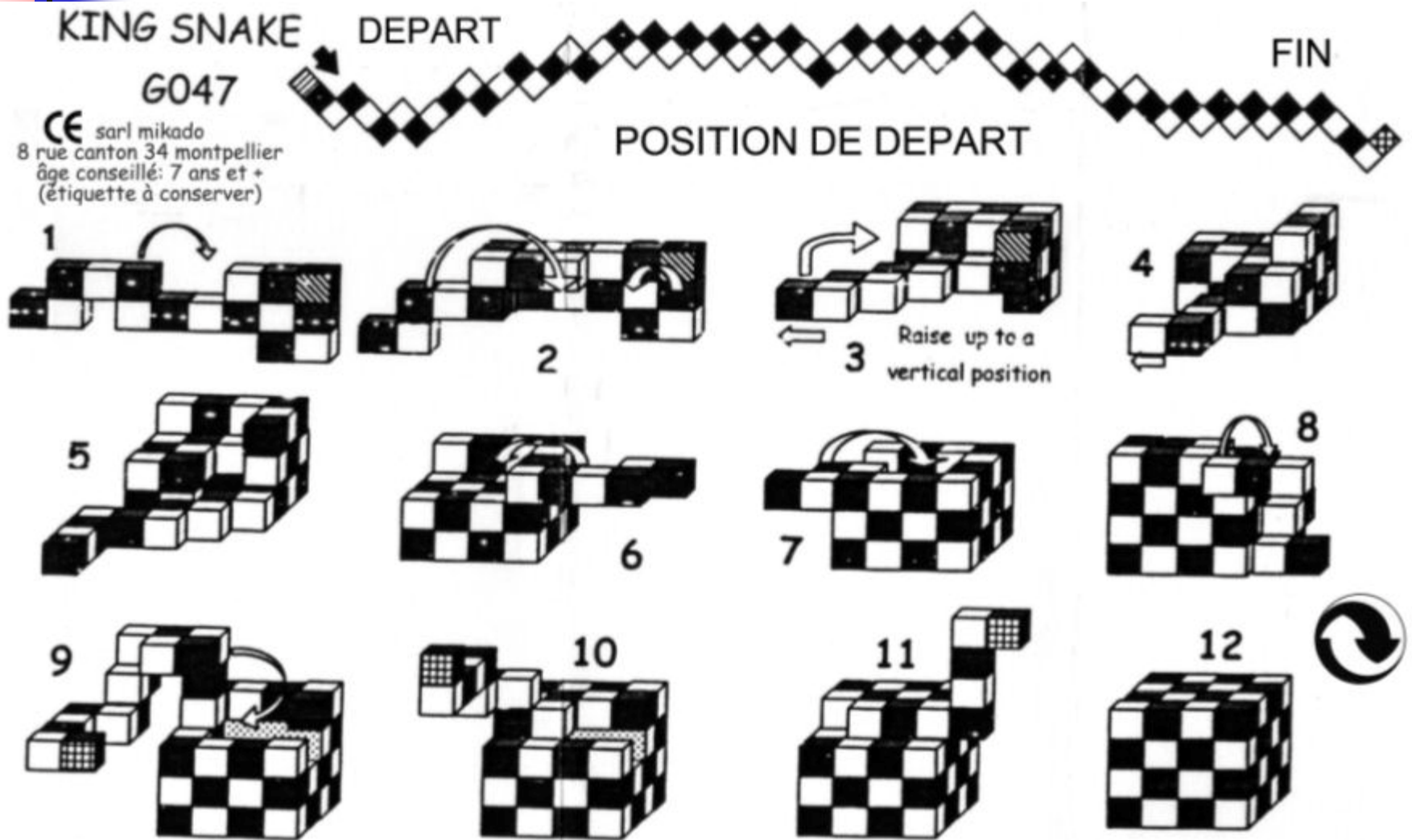
```
[[ (1,4,2) ], [ (1,4,1), (1,3,1), (1,2,1) ], [ (1,1,1) ], [ (1,1,2) ], [ (2,1,2) ], [ (2,2,2) ], [ (1,2,2) ], [ (1,3,2) ], [ (2,3,2) ], [ (2,3,3) ], [ (2,2,3) ], [ (1,2,3), (1,3,3) ], [ (1,4,3) ], [ (1,4,4), (1,3,4), (1,2,4) ], [ (1,1,4) ], [ (1,1,3) ], [ (2,1,3) ], [ (2,1,4), (2,2,4), (2,3,4) ], [ (2,4,4), (3,4,4) ], [ (4,4,4) ], [ (4,3,4) ], [ (3,3,4) ], [ (3,3,3) ], [ (4,3,3) ], [ (4,4,3), (3,4,3) ], [ (2,4,3), (2,4,2) ], [ (2,4,1) ], [ (3,4,1) ], [ (3,4,2) ], [ (4,4,2) ], [ (4,4,1) ], [ (4,3,1) ], [ (4,3,2) ], [ (3,3,2) ], [ (3,2,2), (3,2,3) ], [ (3,2,4) ], [ (3,1,4), (3,1,3) ], [ (3,1,2) ], [ (4,1,2), (4,1,3) ], [ (4,1,4) ], [ (4,2,4), (4,2,3), (4,2,2) ], [ (4,2,1) ], [ (4,1,1) ], [ (3,1,1), (3,2,1) ], [ (3,3,1) ], [ (2,3,1), (2,2,1) ], [ (2,1,1) ]]
```

```
*Main> head (zloz4 [[(1,1,1)]] (0,0,1) had4)
```

```
[[ (4,4,4) ], [ (4,3,4), (3,3,4), (2,3,4) ], [ (1,3,4) ], [ (1,4,4) ], [ (1,4,3) ], [ (1,3,3) ], [ (2,3,3) ], [ (2,4,3) ], [ (2,4,4) ], [ (3,4,4) ], [ (3,4,3) ], [ (4,4,3), (4,3,3) ], [ (4,2,3) ], [ (4,2,4), (3,2,4), (2,2,4) ], [ (1,2,4) ], [ (1,2,3) ], [ (1,1,3) ], [ (1,1,4), (2,1,4), (3,1,4) ], [ (4,1,4), (4,1,3) ], [ (4,1,2) ], [ (3,1,2) ], [ (3,1,3) ], [ (2,1,3) ], [ (2,2,3) ], [ (2,2,2), (2,3,2) ], [ (2,4,2), (3,4,2) ], [ (4,4,2) ], [ (4,4,1) ], [ (4,3,1) ], [ (4,3,2) ], [ (4,2,2) ], [ (4,2,1) ], [ (4,1,1) ], [ (3,1,1) ], [ (3,2,1), (3,2,2) ], [ (3,2,3) ], [ (3,3,3), (3,3,2) ], [ (3,3,1) ], [ (3,4,1), (2,4,1) ], [ (1,4,1) ], [ (1,4,2), (1,3,2), (1,2,2) ], [ (1,1,2) ], [ (2,1,2) ], [ (2,1,1), (2,2,1) ], [ (2,3,1) ], [ (1,3,1), (1,2,1) ], [ (1,1,1) ]]
```

http://collection.cassetete.free.fr/1_bois/cube_elastique3x3/cube_elastique_3x3.htm

http://www.jbd-jouetsenbois.com/king-snake-xml-380_385-1238.html



Prémia Japončici na plti

(<http://www.justonlinegames.com/games/river-iq-game.html>)

