Computational Methods in Finance
Computational project 2

2 June 2023

Instructions

This project shall be solved in groups of three or four people. The solution of the project shall be sent by email to prsantunes@tecnico.ulisboa.pt by 1 July 2023. The submission must include a pdf file with a short report and a zip file contained all the Matlab files that were developed to solve the computational part of the project. Please include a Matlab script allowing to reproduce all the numerical results and figures that are presented in the report. In particular, it is important that the Matlab code concerning pseudo-random numbers include the seed number in order to allow to reproduce the results presented in the report.

1. Write Matlab routines for generating pseudo-random numbers from a standard normal distribution $\mathcal{N}(0,1)$ using the acceptance-rejection method and the linear congruential generator for obtaining realizations of a uniform distribution over $[0,1]$.

2. Write Matlab routines of Euler-Maruyama and Milstein methods for solving a general stochastic differential equation (SDE) of type

$$dX(t) = a(t, X(t))dt + b(t, X(t))dB(t),\ 0 < t \leq T \tag{1}$$

   with initial condition $X(0) = x_0 \in \mathbb{R}$ and determine the approximate solution at equally spaced points

$$t_0 = 0,\ t_1 = h,\ t_2 = 2h, ..., t_N = Nh = T, \text{ where } h = \frac{T}{N},$$

   for some $N \in \mathbb{N}$.

3. Consider the following SDE

$$dS(t) = \mu S(t)dt + \sigma S(t)dB(t),\ 0 < t \leq T \tag{2}$$

   whose solution is given by the geometric Brownian motion,

$$S(t) = S(0)\exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B(t)\right). \tag{3}$$

   Apply the routines developed in 2. to solve this SDE using Euler-Maruyama and Milstein methods for the following parameters $T = 1$, $\mu = 0.5$ and $\sigma = 0.3$ and present and discuss the following numerical simulations:

   (a) For a given realization of the Brownian motion, in the same figure plot the exact solution defined by (3) and the numerical solutions obtained by Euler-Maruyama and Milstein methods, taking $h = 0.001$.

   (b) Consider the following values of $h = 0.005 \times \left(\frac{1}{2}\right)^i,\ i = 0, 1, 2, 3$ and for each of the values of h, run 500 000 simulations of Euler-Maruyama and Milstein methods and use them to estimate the order of strong convergence and order of weak convergence of both methods.