**Computational Methods in Finance**
**Computational project 1**

**16 March 2023**

**Instructions**

This project shall be solved in groups of three or four people. The solution of the project shall be sent by email to prsantunes@tecnico.ulisboa.pt by 16 April 2023. The submission must include a pdf file with a short report and a zip file contained all the Matlab files that were developed to solve the computational part of the project. Please include a Matlab script allowing to reproduce all the numerical results and figures that are presented in the report.

1. Consider the region

$$\mathcal{R}_V^T = \{(S,t),\ 0 < S < S^*,\ 0 \le t \le T\}\,, \tag{1}$$

for a sufficiently large $S^*$ and the terminal/boundary value problem for Black-Scholes equation defining $V(S,t)$ to be the value of an option at the point $(S,t)$. In order to replace the terminal value problem associated with Black-Scholes equation by an initial value problem we perform a change of variables $U(S,t) := V(S,T-t)$ and consider the problem

$$\begin{cases} \frac{\partial U}{\partial t} = \frac{\sigma^2}{2}S^2\frac{\partial^2 U}{\partial S^2} + rS\frac{\partial U}{\partial S} - rU & \text{in } \mathcal{R}_V^T \\ U(S,0) = u_0(S) & S \in [0,S^*] \\ U(0,t) = u_a(t) & t \in [0,T] \\ U(S^*,t) = u_b(t) & t \in [0,T] \end{cases} \tag{2}$$

for some functions $u_0$, $u_a$ and $u_b$ that depend on the type of option and are assumed to be known.

   (a) Write Matlab routines for solving problem (2) using the explicit method and also using Crank-Nicolson method.

   (b) Calculate and plot the solutions of (2) obtained by both numerical methods in the case of a European call option, for the parameters $r = 0.06$, $\sigma = 0.3$, $T = 1$ and $K = 10$ and taking $S^* = 15$.

   (c) Consider the case of a European put option with parameters $r = 0.08$, $\sigma = 0.4$, $T = 4$ and $K = 5$. Imagine you would be asked to calculate the value of the option at time instants $t = 1$, $t = 2$ and $t = 3$ assuming that for all those instants of time the market price of the asset is $S = 5$. Run the routine with different discretization parameters $h_t$ and $h_S$ in order to have an intuition about the error of the approximations for values for $V(5,1)$, $V(5,2)$ and $V(5,3)$ in such a way to show approximations for those values with two decimal digits.