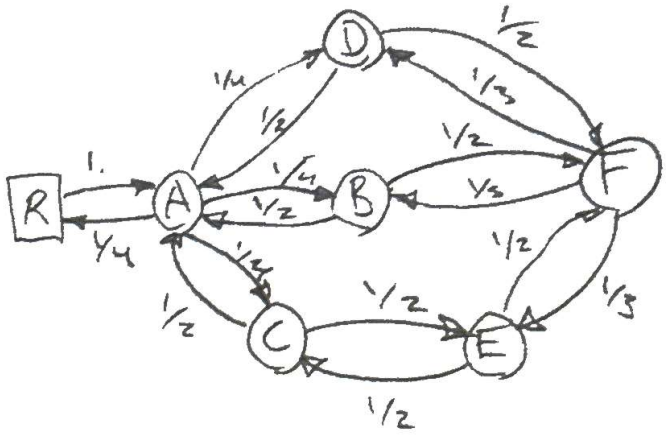
Homework 1 Planning, Learning and Decision Making

a) The markov process has 7 different states A-F and R (Recycling Station). The transitions between states, and their probabilities are given by the transition matrix:

	A	B	C	D	E	F	R
A	0	1/4	1/4	1/4	0	0	1/4
B	1/2	0	0	0	0	1/2	0
C	1/2	0	0	0	1/2	0	0
D	1/2	0	0	0	0	1/2	0
E	0	0	1/2	0	0	1/2	0
F	0	1/3	0	1/3	1/3	0	0
R	1	0	0	0	0	0	0



b) At $t=0$ the state is R, $P(S_0=R)=1$. In the next timestep, $t=1$, the state will be A. This is given by the transition matrix $P(S_1=A \mid S_0=R)=1$

In the next step, $t=2$, the possible states are E, C, D, R, with equal probability, of 0,25. The other states are not possible.

$P(S_2=B \mid S_1=A) = P(S_2=C \mid S_1=A) = P(S_2=D \mid S_1=A) = P(S_2=R \mid S_1=A) = 0,25$

$P(S_2 \in \{A,E,F\} \mid S_1=A) = 0$

c) The expected time to get from R to R is denoted by t_{RR} and given by:

$$t_{RR} = 30 + t_{AR}$$

where 30 minutes comes from going to A, and t_{AR} minutes denotes the expected to get from A to R.

Similarly the different expected return times (t_{XY} denotes the expected time to go from X to Y) are given by:

$$t_{AR} = \frac{1}{4} \cdot 30 + \frac{1}{4}(40 + t_{BR}) + \frac{1}{4}(55 + t_{CR}) + \frac{1}{4}(70 + t_{DR})$$

$$t_{BR} = \frac{1}{2}(40 + t_{AR}) + \frac{1}{2}(80 + t_{FR})$$

$$t_{CR} = \frac{1}{2}(55 + t_{AR}) + \frac{1}{2}(55 + t_{ER})$$

$$t_{DR} = \frac{1}{2}(70 + t_{AR}) + \frac{1}{2}(70 + t_{FR})$$

$$t_{ER} = \frac{1}{2}(55 + t_{CR}) + \frac{1}{2}(20 + t_{FR})$$

$$t_{FR} = \frac{1}{3}(20 + t_{ER}) + \frac{1}{3}(70 + t_{DR}) + \frac{1}{3}(80 + t_{BR})$$

By rearranging the numbers we see that the solution to the system of equations equals

$$\begin{bmatrix} -1 & 1/4 & 1/4 & 1/4 & 0 & 0 \\ 1/2 & -1 & 0 & 0 & 0 & 1/2 \\ 1/2 & 0 & -1 & 0 & 1/2 & 0 \\ 1/2 & 0 & 0 & -1 & 0 & 1/2 \\ 0 & 0 & 1/2 & 0 & -1 & 1/2 \\ 0 & 1/3 & 0 & 1/3 & 1/3 & -1 \end{bmatrix} \begin{bmatrix} -48{,}75 \\ -60 \\ -55 \\ -70 \\ -37.5 \\ -140/3 \end{bmatrix} = \begin{bmatrix} t_{AR} \\ t_{BR} \\ t_{CR} \\ t_{DR} \\ t_{ER} \\ t_{FR} \end{bmatrix}$$

Via gaussian elimination we. get

$t_{AR} = 810$ (minutes)

$= 13\frac{1}{1}\ 30$ min

This meahs that

$t_{RR} = 14h$

Given that the car left the Recycling Plant at 10:00, the car is expected to return at 00:00, midnight.