VaR on option portfolio

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Options

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- american options
- exotic options

European options

Value of european option at its expiration time:

- Call: $C_T = \max\{S_T - K, 0\}$

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- Call: $C_T = \max\{S_T - K, 0\}$
- Put: $P_T = \max\{K - S_T, 0\}$

Black-Scholes model

Price of european call option:

- $V_t = S_t \Phi(d_1) - Ke^{-R(T-t)}\Phi(d_2)$
  - K ... strike price
  - R ... risk-free interest rate
  - S_t ... the underlying asset's value at time t
- $d_1 = \frac{\ln(\frac{S_0}{K}e^{RT}) + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}}$
- $d_2 = d_1 - \sigma\sqrt{T}$

VaR

VaR definiton

Let X be a random variable on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and $\alpha \in (0,1)$. $\mathrm{VaR}_\alpha(X)$ is defined as the $(1-\alpha)$ quantile of -X. Then

$$\mathrm{VaR}_\alpha(X) := -\inf\{x \in \mathbb{R} \mid F_X(x) > \alpha\} = F_{-X}^{-1}(1-\alpha).$$

Non-linear VaR

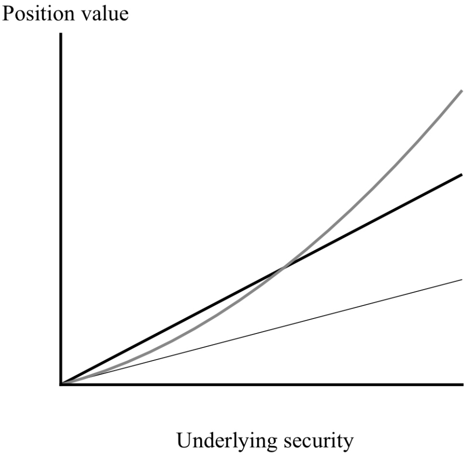

Figure: Linear and non-linear function of payoff.

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- european call option: $\Phi(d_1)$

The Gamma

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$$\Gamma = \frac{\partial^2 V}{\partial S^2}$$

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- european call option: $e^{-R(T-t)} \frac{\Phi(d1)}{S_t \sigma \sqrt{T-t}}$

The Theta

The Theta

$$\Theta = \frac{\partial V}{\partial t}$$

The Vega

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$$\mathcal{V} = \frac{\partial V}{\partial \sigma}$$

Delta-Gamma-Theta Approach

Importance of the three Greeks used for VaR calculation:

- Delta: the potential change in the option's value associated with a unit shift in the underlying asset's price;
- Gamma: important role in VaR calculations as the underlying asset price fluctuates more significantly;
- Theta: time decay is a major factor in the approximation of the overall risk of the portfolio.