# VaR on option portfolio

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- Put: $P_T = \max\{K - S_T, 0\}$

# Black-Scholes model

Price of european call option:

- $V_t = S_t\Phi(d_1) - Ke^{-R(T-t)}\Phi(d_2)$
  - $K$ ... strike price
  - $R$ ... risk-free interest rate
  - $S_t$ ... the underlying asset's value at time $t$
- $d_1 = \frac{\ln(\frac{S_0}{K}e^{RT}) + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}}$
- $d_2 = d_1 - \sigma\sqrt{T}$

# VaR

### VaR definiton

Let $X$ be a random variable on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and $\alpha \in (0,1)$. $\mathrm{VaR}_\alpha(X)$ is defined as the $(1-\alpha)$ quantile of -$X$. Then

$$\mathrm{VaR}_\alpha(X) := -\inf\{x \in \mathbb{R} \mid F_X(x) > \alpha\} = F_{-X}^{-1}(1-\alpha).$$

# Non-linear VaR

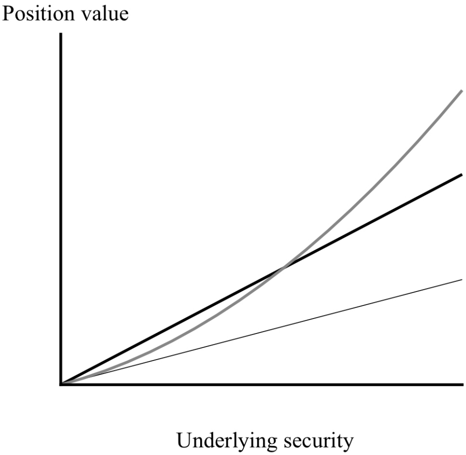


Figure: Linear and non-linear function of payoff.

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- European call option: $\Phi(d_1)$

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In general, the gamma is at its maximum point when the stock is near the strike of the option.

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Long positions generally have a negative theta and short positions a positive one.

# The Vega

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$$\mathcal{V} = \frac{\partial V}{\partial \sigma}$$

# Delta-Gamma-Theta Approach

Importance of the three Greeks used for VaR calculation:

- Delta: the potential change in the option's value associated with a unit shift in the underlying asset's price;
- Gamma: important role in VaR calculations as the underlying asset price fluctuates more significantly;
- Theta: time decay is a major factor in the approximation of the overall risk of the portfolio.