TS6-b

lunes, mayo 26, 2025 12:06 AM

b) $y(n) = x(n-4) + x(n-3) + x(n-2) + x(n-1) + x(n)$

$y(z) = X(z).z^{-4} + X(z).z^{-3} + X(z).z^{-2} + X(z).z^{-1} + X(z)$

$y(z) = X(z).(z^{-4} + z^{-3} + z^{-2} + z^{-1} + 1)$

$$T(z) = \frac{Y(z)}{X(z)} = z^{-4} + z^{-3} + z^{-2} + z^{-1} + z^{0} = \frac{1 + z^{1} + z^{2} + z^{3} + z^{4}}{z^{4}}$$

$$T(j\Omega) = \frac{e^{0j\Omega} + e^{j\Omega} + e^{2j\Omega} + e^{3j\Omega} + e^{4j\Omega}}{e^{4j\Omega}}$$

$$= e^{2j\Omega}\left(e^{-2j\Omega} + e^{-j\Omega} + \frac{e^{0}}{e^{4j\Omega}} + e^{j\Omega} + e^{2j\Omega}\right)$$

$$= \underbrace{e^{-2j\Omega}}_{\text{fase}}\left(2\cos(2\Omega) + 2\cos(\Omega) + 1\right) \longrightarrow$$

$$|T(\Omega)| = 2\cos(2\Omega) + 2\cos(\Omega) + 1$$

$$\measuredangle T(\Omega) = -2\Omega$$

$1 + 2\cos(2\Omega) + 2\cos(\Omega) = 0$

$1 + 2\left(2\cos^2(\Omega) - 1\right) + 2\cos(\Omega) = 0$

$1 + 4\cos^2(\Omega) - 2 + 2\cos(\Omega) = 0$

$4\cos^2(\Omega) + 2\cos(\Omega) - 1$

$$\cos(\Omega) = \frac{-2 + \sqrt{4 - 4.4.(-1)}}{8} = \frac{-2 + \sqrt{20}}{8} \approx 0{,}31 \rightarrow \Omega \approx 1{,}257$$

$$\cos(\Omega) = \frac{-2 - \sqrt{20}}{8} \approx -0{,}81 \rightarrow \Omega \approx 2{,}51$$