Cibernética III Taller 2 Corte 3 23.09.21

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1)

* $x[n] = \text{sen}[0{,}2n]\,\mu(n)$

$$Z\{\text{sen}[0{,}2n]\mu(n)\} = \frac{1}{2j} Z\{(e^{j0{,}2} - e^{-j0{,}2})\mu[n]\}$$

$$= \frac{1}{2j} Z\{\mu(n)e^{j0{,}2}\} - \frac{1}{2j} Z\{\mu(n)e^{-j0{,}2}\}$$

$$= \frac{1}{2j}\left(\frac{z}{z-e^{j0{,}2}} - \frac{z}{z-e^{-j0{,}2}}\right)$$

$$= \frac{1}{2j} \cdot \frac{ze^{j0{,}2} - ze^{-j0{,}2}}{z^2 - z(e^{j0{,}2}+e^{-j0{,}2})+1}$$

$$Z\{\text{sen}[0{,}2n]\mu(n)\} = \frac{z\,\text{sen}(0{,}2)}{z^2 - 2z\cos(0{,}2)+1} = X(z)$$

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$x[n] = n^2 (0,5)^n \mu[n] = (0,5)^n n^2 \mu[n]$

si $\quad y[n] = n^2 \mu(n)$

tenemos

$$Z\{n\mu(n)\} = W(z) = \frac{z}{(z-1)^2}$$

$$Z\{n^2\mu(n)\} = -z \frac{dW(z)}{dz}$$

$$Z\{n^2\mu(n)\} = -z \cdot \frac{(z-1)^2 - z\,2(z-1)}{(z-1)^4}$$

$$Z\{n^2\mu(n)\} = -z \cdot \frac{(z-1)^2 - 2z^2 + 2z}{(z-1)^4}$$

$$= -z \cdot \frac{z^2 - \cancel{2z} + 1 - 2z^2 \cancel{+2z}}{(z-1)^4}$$

$$= -z \cdot \frac{1 - z^2}{(z-1)^4} = z \cdot \frac{z^2 - 1}{(z-1)^4}$$

$$= z \cdot \frac{\cancel{(z-1)}(z+1)}{(z-1)^4}$$

$$Z\{n^2\mu(n)\} = \frac{z(z+1)}{(z-1)^3} = Y(z)$$

$$X(z) = Y(0,5^{-1}z) = Y(2z) = \frac{2z(2z+1)}{(2z-1)^3}$$

$$X(z) = \frac{2z(2z+1)}{(2z-1)^3}$$

$$* \quad Y(z) = \frac{2z}{z^2+2z+2}$$

$$\frac{1}{2} Y(z) = \frac{z}{z^2+2z+2} \cdot \frac{z^{-2}}{z^{-2}} = \frac{z^{-1}}{1+2z^{-1}+2z^{-2}}$$

$$Z\{a^n \operatorname{sen}(\Omega_0 n)\mu(n)\} = \frac{a z^{-1} \operatorname{sen}(\Omega_0)}{1-2az^{-1}\cos(\Omega_0)+a^2z^{-2}}$$

$$\cancel{1} + 2z^{-1} + 2z^{-2} = \cancel{1} - 2az^{-1}\cos(\Omega_0) + a^2 z^{-2}$$

$$2 = a^2 \rightarrow a = \sqrt{2}$$

$$2 = -2a \cdot \cos(\Omega_0)$$

$$2 = -2\sqrt{2} \cdot \cos(\Omega_0)$$

$$-\frac{1}{\sqrt{2}} = \cos(\Omega_0) \rightarrow \Omega_0 = \frac{3}{4}\pi$$

$$\frac{1}{2} y[n] = a^n \operatorname{sen}[\Omega_0 n]\mu(n) = (\sqrt{2})^n \operatorname{sen}\left[\frac{3}{4}\pi n\right]\mu(n)$$

$$y[n] = 2 \cdot (\sqrt{2})^n \operatorname{sen}\left[\frac{3}{4}\pi n\right]\mu(n)$$

2.3

* $G(z) = \dfrac{(2z-1)z}{2(z-1)(z+0{,}5)} = \dfrac{Y(z)}{X(z)}$

$$Y(z)\,2\,(z-1)(z+0{,}5) = X(z)\,z\,(2z-1)$$

$$Y(z)(2z-2)(z+0{,}5) = X(z)(2z^2 - z)$$

$$Y(z)(2z^2 + z - 2z - 1) = 2z^2X(z) - zX(z)$$

$$Y(z)2z^2 + Y(z)z - Y(z) = 2z^2X(z) - zX(z)$$

$$Y(z) - Y(z)z + Y(z)2z^2 = 2z^2X(z) - zX(z)$$

$$Y(z)z^{-2} - Y(z)z^{-1} + 2Y(z) = 2X(z) - X(z)z^{-1}$$

$$2y[n] - y[n-1] + y[n-2] = 2x[n] - x[n-1]$$

* $G(z) = \dfrac{z^2+3z+2}{z^3+4z^2+3z+2} = \dfrac{Y(z)}{X(z)}$

$$X(z)(z^2+3z+2) = Y(z)(z^3+4z^2+3z+2)$$

$$X(z)z^2 + X(z)3z + X(z)2 = Y(z)z^3 + Y(z)4z^2 + Y(z)3z + Y(z)2$$

$$X(z)z^{-1} + X(z)3z^{-2} + X(z)2z^{-3} = Y(z) + 4Y(z)z^{-1} + 3Y(z)z^{-2} + 2Y(z)z^{-3}$$

$$y[n] + 4y[n-1] + 3y[n-2] + 2y[n-3] = x[n-1] + 3x[n-2] + 2x[n-3]$$

* $y[n] - y[n-1] - y[n-2] = x[n] - x[n-1]$

$$Y(z) - Y(z)z^{-1} - Y(z)z^{-2} = X(z) - X(z)z^{-1}$$

$$Y(z)\left(1 - z^{-1} - z^{-2}\right) = X(z)\left(1 - z^{-1}\right)$$

$$\frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - z^{-1} - z^{-2}} \cdot \frac{z^2}{z^2} = \frac{z^2 - z}{z^2 - z - 1} = \frac{z(z-1)}{z^2 - z - 1} = G(z)$$

2.5

* $y[n-1] + y[n] = x[n]$

con $x[n] = (0{,}5)^n$, $y[-1] = 1$

$$\mathcal{Z}\{y[n-1]\} + \mathcal{Z}\{y[n]\} = \mathcal{Z}\{x[n]\}$$

$$z^{-1} Y(z) + y[-1] + Y(z) = X(z)$$

reemplazando la condición inicial:

$$X(z) = \mathcal{Z}\{0.5^n\} = \frac{z}{z-0{,}5}$$

$$1 + Y(z)(z^{-1}+1) = \frac{z}{z-0{,}5}$$

$$Y(z)(z^{-1}+1) = \frac{z - 1(z-0{,}5)}{z-0{,}5}$$

$$Y(z) = \frac{\cancel{z} - \cancel{z} + 0{,}5}{(z-0{,}5)(z^{-1}+1)} = \frac{0{,}5}{(z-0{,}5)(z^{-1}+1)}$$

$$z^{-1} \cdot Y(z) = \frac{0{,}5\,z}{(z-0{,}5)(1+z)} \cdot z^{-1}$$

$$\frac{Y(z)}{z} = \frac{0{,}5}{(1+z)(z-0{,}5)}$$

$$\frac{Y(z)}{z} = \frac{0{,}5}{(1+z)(z-0{,}5)}$$

$$\frac{0{,}5}{(1+z)(z-0{,}5)} = \frac{A}{1+z} + \frac{B}{z-0{,}5}$$

$$\frac{0{,}5}{(1+z)(z-0{,}5)} = \frac{A(z-0{,}5)+B(1+z)}{(1+z)(z-0{,}5)} = \frac{Az - A0{,}5 + B + Bz}{(1+z)(z-0{,}5)}$$

$$B - A0{,}5 = 0{,}5$$

$$A + B = 0$$

$$A = -B$$

$$\downarrow$$

$$B + B0{,}5 = 0{,}5$$

$$B = \frac{0{,}5}{1{,}5} = \frac{1}{3} \rightarrow A = -\frac{1}{3}$$

$$\frac{Y(z)}{z} = -\frac{1}{3} \cdot \frac{1}{1+z} + \frac{1}{3} \cdot \frac{1}{z-0{,}5}$$

$$Z^{-1}\{Y(z)\} = -\frac{1}{3} Z^{-1}\left\{\frac{z}{z-(-1)}\right\} + \frac{1}{3} Z^{-1}\left\{\frac{z}{z-0{,}5}\right\}$$

$$y[n] = -\frac{1}{3} \cdot -1^{n} \mu[n] + \frac{1}{3} 0{,}5^{n} \mu[n]$$

$$y[n] = \frac{1}{3} 0{,}5^{n} \mu[n] - \frac{1}{3} \cdot -1^{n} \mu[n]$$

* $y[n] + \frac{1}{2} y[n-1] - \frac{1}{2} y[n-2] = x[n]$

con $y[-1] = 1$, $y[-2] = 0$, $x[n] = \mu[n]$

$$\mathcal{Z}\{y[n]\} - \frac{1}{2}\mathcal{Z}\{y[n-1]\} - \frac{1}{2}\mathcal{Z}\{y[n-2]\} = \mathcal{Z}\{x[n]\}$$

$$Y(z) - \frac{1}{2}\left(z^{-1}Y(z) + y[-1]\right) - \frac{1}{2}\left(z^{-2}Y(z) + \cancel{y[-2]}\right) = X(z)$$

$$Y(z) - \frac{1}{2}z^{-1}Y(z) - \frac{1}{2} - \frac{1}{2}z^{-2}Y(z) = X(z)$$

Reemplazando la condición inicial

$$X(z) = \mathcal{Z}\{\mu[n]\} = \frac{z}{z-1}$$

$$Y(z)\left(1 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2}\right) = \frac{z}{z-1} + \frac{1}{2}$$

$$Y(z)\left(1 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2}\right) = \frac{z + \frac{1}{2}z - \frac{1}{2}}{z-1} = \frac{\frac{3}{2}z - \frac{1}{2}}{z-1}$$

$$Y(z)\left(1 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2}\right) = \frac{3z-1}{2z-2}$$

$$Y(z) = \frac{3z-1}{(2z-2)\left(1 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2}\right)} \cdot \frac{z^2}{z^2}$$

$$Y(z) = \frac{(3z-1)z^2}{(2z-2)\left(z^2 - \frac{1}{2}z - \frac{1}{2}\right)}$$

$$y_{(z)} = \frac{z^2(3z-1)}{(2z-2)(z^2 - \frac{1}{2}z - \frac{1}{2})}$$

$$\frac{y_{(z)}}{z} = \frac{3z^2 - z}{(2z-1)(z^2 - \frac{1}{2}z - \frac{1}{2})}$$

$$\frac{3z^2 - z}{(2z-1)(z^2 - \frac{1}{2}z - \frac{1}{2})} = \frac{A}{2z-1} + \frac{Bz + C}{z^2 - \frac{1}{2}z - \frac{1}{2}}$$

$$3z^2 - z = A(z^2 - \frac{1}{2}z - \frac{1}{2}) + (Bz + C)(2z - 1)$$

$$3z^2 - z = \underline{Az^2} - \underline{\frac{1}{2}Az} - \underline{\frac{1}{2}A} + \underline{2Bz^2} \underline{- Bz} + \underline{2Cz} \underline{- C}$$

$$3z^2 - z = (A + 2B)z^2 + \left(-\frac{1}{2}A - B + 2C\right)z - \frac{1}{2}A - C$$

$$A + 2B = 3$$

$$2B = 3 - A \rightarrow B = \frac{3}{2} - \frac{A}{2}$$

$$-\frac{1}{2}A - B + 2C = -1$$

$$-\frac{1}{2}A - C = 0$$

$$C = -\frac{1}{2}A$$

$$\frac{3}{2} + \frac{1}{4}$$

$$-\frac{1}{2}A - B + 2C = -1$$

$$\cancel{-\frac{1}{2}A} - \frac{3}{2} + \cancel{\frac{1}{2}A} - A = -1$$

$$A + \frac{3}{2} = 1 \rightarrow A = -\frac{1}{2}, \; B = \frac{7}{4}, \; C = \frac{1}{4}$$

$$\frac{Y(z)}{z} = -\frac{1}{2}\cdot\frac{1}{2z-1} + \frac{\frac{7}{4}z+\frac{1}{4}}{z^2-\frac{1}{2}z-\frac{1}{2}}$$

$$\frac{Y(z)}{z} = -\frac{1}{2}\cdot\frac{1}{2z-1} + \frac{7z+1}{4z^2-2z-2}$$

$$\frac{Y(z)}{z} = -\frac{1}{2}\cdot\frac{1}{2z-1} + \frac{1}{2}\cdot\frac{7z+1}{(2z+1)(z-1)}$$

$$\frac{7z+1}{(2z+1)(z-1)} = \frac{A}{2z+1} + \frac{B}{z-1}$$

$$7z+1 = A(z-1) + B(2z+1)$$

$$7z+1 = \underline{Az} - A + \underline{2Bz} + B = z(A+2B) - A + B$$

$$A+2B = 7$$

$$-A+B = 1$$

$$B = 1+A$$

$$A + 2(1+A) = 7$$

$$A+2+2A = 7$$

$$3A = 5 \rightarrow A = \frac{5}{3} \rightarrow B = \frac{8}{3}$$

$$\frac{Y(z)}{z} = -\frac{1}{2}\cdot\frac{1}{2z-1} + \frac{1}{2}\left(\frac{\frac{5}{3}}{2z+1} + \frac{\frac{8}{3}}{z-1}\right)$$

$$\frac{Y(z)}{z} = -\frac{1}{2}\cdot\frac{1}{2z-1} + \frac{5}{6}\cdot\frac{1}{2z+1} + \frac{4}{3}\cdot\frac{1}{z-1}$$

$$\frac{Y(z)}{z} = -\frac{1}{2} \cdot \frac{1}{2z-1} + \frac{5}{6} \cdot \frac{1}{2z+1} + \frac{4}{3} \cdot \frac{1}{z-1}$$

$$\frac{Y(z)}{z} = -\frac{1}{2} \cdot \frac{1}{2(z-\frac{1}{2})} + \frac{5}{6} \cdot \frac{1}{2(z+\frac{1}{2})} + \frac{4}{3} \cdot \frac{1}{z-1}$$

$$\frac{Y(z)}{z} = -\frac{1}{4} \cdot \frac{1}{z-\frac{1}{2}} + \frac{5}{12} \cdot \frac{1}{z-(-\frac{1}{2})} + \frac{4}{3} \cdot \frac{1}{z-1}$$

$$Y(z) = -\frac{1}{4} \cdot \frac{z}{z-\frac{1}{2}} + \frac{5}{12} \cdot \frac{z}{z-(-\frac{1}{2})} + \frac{4}{3} \cdot \frac{z}{z-1}$$

$$\mathcal{Z}^{-1}\{Y(z)\} = -\frac{1}{4} \mathcal{Z}^{-1}\left\{\frac{z}{z-\frac{1}{2}}\right\} + \frac{5}{12} \mathcal{Z}^{-1}\left\{\frac{z}{z-(-\frac{1}{2})}\right\} + \frac{4}{3} \mathcal{Z}^{-1}\left\{\frac{z}{z-1}\right\}$$

$$y[n] = -\frac{1}{4}\left(\frac{1}{2}\right)^n \mu[n] + \frac{5}{12}\left(-\frac{1}{2}\right)^n \mu[n] + \frac{4}{3}(1)^n \mu[n]$$