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Distributions of city sizes in Mexico during the 20th century

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ABSTRACT

We present a study of the distribution of cities in Mexico along the 20th century, based on information collected in censuses every ten years. The size-rank and survival cumulative distributions are constructed to evaluate the presence of scaling, its deviation from the Zipf's law and their evolution along the period of observation. We find that the size of cities S, approximately follow a power-law with the rank $r, S(r) \sim r^{-\alpha}$, where the exponents take values between $\alpha \approx 0.7$ to $\alpha \approx 1.1$ for years 1900 and 2000, respectively. The local fluctuations in the scaling behavior are evaluated by means of a local exponent, and the deviation of the size predicted by the Zipf's law $(\alpha = 1)$ and the real size of each city is analyzed. Our calculations show that local exponents follow transitions between values above and below of the Zipfian regime and the deviations are more remarkable at the beginning and at the end of the 20th century. Besides, the cumulative distributions confirm the presence of scaling for the same records with a reasonable agreement with the scaling exponents observed in the size-rank distributions. Moreover, we examine the role of a recent introduced property named coherence. Finally, we explain our findings in terms of the socio-demographic evolution of Mexico along the 20th century.

1. Introduction

Since Pareto [1] in the 19th century up to the present day, many complex collective phenomena have been statistically described by scaling laws. This approach has been used in a variety of research fields ranging from social to natural sciences [2]. For example, from economics [3–5], linguistics [6,7] and sociology [8] to physics [9–11] and biology [12]. The phenomena described by scaling laws have no characteristic scales and usually are described by power laws of the fractal type. Scaling relationships were first found on empirical grounds. A clear case of this is that of the empirical laws of seismology, such as the so-called Omori law for aftershocks temporal distribution [13] or the Gutenberg–Richter law for the magnitude distribution of seisms [14]. Several decades after these laws were established empirically, Bak et al. [15,16] proposed a more fundamental explanation for them based on the concept of self-organized critical systems. The same happened with the scaling laws of the Zipf type, followed by the distribution of cities by size [17], which were first empirically established and until recent times more detailed mechanisms for their explanation have been proposed [18,19]. On the other hand, as it is well known, only the self-similar fractals, as the Koch's curve for instance, have scaling laws over an arbitrary number of scales. However, the scaling properties in real world objects and phenomena are incomplete in the sense that the corresponding power laws typically have one or more crossovers in their scaling exponents [20]. For example, the Gutenberg–Richter law for earthquakes extends along many decades from millimeters up to thousands of kilometers, however, it has a crossover

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around of $M = 7.5$, which has to do with a break in self-similarity, from small to large earthquakes occurring at a point where the dimensions of the event equals the down-dip width of the seismological layer (see Fig. 1 of Ref. [21]). Small earthquakes grow in both length and width; their rupture dimensions have no bounds. On the other hand, large earthquakes have no bounds in rupture length, but their down-dip width is limited by thickness of the region capable of generating earthquakes [21].

Another case where a crossover plays a very important role is that corresponding to empirical data of distribution of the money and wealth in many countries as in UK for example [22,23]. When a log–log plot of cumulative probability distribution against total of capital (wealth) is made, clearly two well defined regions are found, one following a Pareto-type behavior for rich people and other following an experimental distribution of the Boltzmann-type for the lower part of the distribution for the great majority (about 90%) of the population. Some well-founded models explain these different behaviors [22,24–26]. For the case of the city size distribution, deviations from Zipf's law and crossovers have been reported [27–30]. As in the case of the money and wealth distributions, crossovers between big ($S > 10^5$ inhabitants) and small scales, and between developed and developing countries have been identified. For the case of Japan for instance, Sasaki et al. reported that the rank-size distributions of towns and villages can be well approximated by log normal distributions while for cities a power-law distribution is observed. On the other hand, Malacarne et al. [31] also have reported that for the cases of Brazil and USA a notorious deviation from an asymptotic power-law of the Zipf type is found when cities of all sizes are considered. Until the present day, there is no consensus about the best fit of city size distributions. For example, Bee et al. [32] question the claim that largest US cities are Pareto distributed and based on multiple tests on real data they assert that the distribution is lognormal, and largely depends on sample sizes. On the other hand, Giesen et al. [33] by using untruncated settlement size data from eight countries show that the "double Pareto lognormal" distribution provides a better fit to actual city sizes than the simple lognormal distribution. More recently, Cristelli et al. [27] have reported a very important property called "coherence" of the sample of objects to be studied, which drives the resulting power-law towards a perfect Zipf's law ($\alpha = 1$) or towards marked deviations from it. Another important feature of city size distributions (CSD), is the time evolution of the power-law exponents depending of many social, economic and political phenomena [27,31]. The CSD has been studied for many countries as USA ([34]), Spain [35], France [28], Japan [28], India [36], China [36,37], Brazil [38] among others. The study of particular cases is important because they help to enrich the phenomenology of the CSD problem. In the present article we study the case of Mexico by analyzing each decade censuses from 1900 up to 2000. The main goal of this work is to evaluate presence and departures of the scaling behavior in CSD, and deviations with respect to the Zipf's regime. Our results clearly reflect some of the main social and economic transitions of Mexico along the 20th century under a macroscopic overview through the evolution of the CSD scaling exponents. Our paper is organized as follows. In Section 2, we describe the main aspects of the data of cities under study and some sociodemographic aspects which are important for the discussion of the results. In Section 3, the results are presented together with a discussion, and finally in Section 4 some concluding remarks are given.

2. Data and sociodemographic aspects

One of the big issues in the construction of the relationship between population and rank of cities is the definition of a city [36]. Some authors use a spatial definition for cities, particularly based on the amount of building area. Other studies [39] use an administrative definition, i.e. government of specific states or cities. In the case of this study, we combine both perspectives. In first place, metropolitan zones are defined as municipalities agglomerations; that is, data corresponding to the total of the population of the municipalities and they were taken from different sources [38,40,41]. We follow the common practice in Mexico to assume the size of 15 000 inhabitants as the lower threshold to define a city [38]. Although there are some criticisms to this threshold, it remains as the most extended definition used in Mexico (our dataset can be freely accessed on the website *http://www.cslupiita.com.mx/citiesmx/*).

Along the 20th century, as many countries in the world, Mexico passed through important transformations, including its transition from a rural to an urban country [42]. As can be seen in Fig. 1, the annual growth rate (AGR) of the urban population was higher than the AGR of total population for the considered period. In the 1950's the largest difference between both AGR's is observed due to the industrialization policy, the promotion of the urbanization, and rural to urban migration. Thanks to the population policies since the 1970's, it seems that nowadays both AGR's are close each other. In 1900, just the 10.6% of the population lived in one of the 33 Mexican cities. By 2000, 364 cities formed the Mexican urban system with more than 60 million inhabitants (63% of the total).

Some authors such as Unikel [38,43], Garza [40] and Aguilar and Graizbord [42] have pointed out the importance of dividing the recent urban history of Mexico in three periods: I. 1900–1939, II. 1940–1979 and III. 1980–2000 (see Table 1). In 1900, Mexico was a rural country. The urban population (10.6%) was distributed in 33 cities and six of them concentrated the one half of the total. During the first period the percentage of urban population grew around 7% meanwhile the total of cities reached 43 (i.e. 30%).

In the second period (1940–1979), the process of urbanization took place in a faster rate due to a state policy of industrialization and urbanization. By the beginning of this period, around 20% of the total of population lived in 55 cities, meanwhile by the end the urban population reached 46% and the total of cities was 167. In other words, urban population doubled meanwhile the total of cities tripled.

The last period (named the neoliberal one), witnessed notable changes. In the first place the economic openness

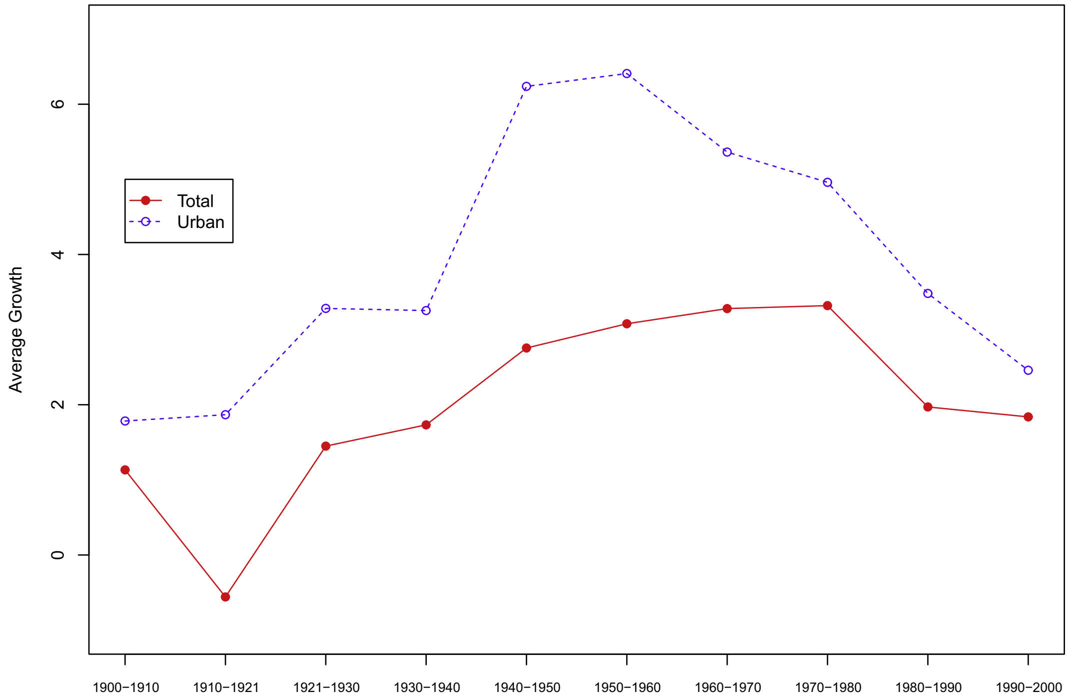

Fig. 1. Annual growth rates for the period 1900–2000. We show the cases of the urban (open circles) and the total (closed circles) population. It is noteworthy that the urban growth is higher that the total one.

Table 1
Main aspects of the three periods along the 20th century (see text for details). The number of cities, the increment of the total number of cities (ITC) and the urban population percentage (UPP) are listed.

Period	Year	Cities	ITC	% UPP
I. Post revolution	1900	33	–	10.6
	1910	33	0	11.3
	1921	37	4	14.4
	1939	43	6	17.2
II. Endogenous growth	1940	55	12	20.0
	1950	83	28	27.9
	1960	120	37	38.3
	1979	167	47	46.8
III. Neoliberal	1980	220	53	54.8
	1990	306	86	63.5
	2000	350	44	67.4

brought important transformations in both policy terms and spatial distribution of people. The most important is that urban population surpassed the 50% of the total. The second one is the increment in the total number of cities. By the beginning of the period, there were 220 but by the end the total reached 350. The social and economic impacts of the neoliberal policies during this period are evident. For example, the so-called macroeconomic indicators showed a seemingly general advancement, however, the microeconomic ones; that is, those that measure the actual economic status of the majority of population did not exhibited remarkable improvements, indeed, some of these indicators worsened.

The CSD in Mexico along the 20th century is a very interesting problem because of during this period it evolved from a mainly rural country towards a society that behaves like a mixture of a developed and a developing country.

3. Results and discussion

Prior to describe our results, we provide a brief description of some mathematical aspects of Zipf's and Pareto's distributions. Zipf's plots or size-rank distributions have the form $S(r) = ar^{-\alpha}$, where $S(r)$ and r are the size in terms of population of the cities and the rank, respectively; a, α are fitting parameters [2,44]. Commonly, when $\alpha = 1$ this relationship is referred as the Zipf's law or Zipfian regime; and when the statistics is based on the probability of having cities with sizes bigger than S, with a functional form given by, $P(>S) \sim S^{-\gamma}$, it is called Pareto's law and γ is an exponent related to the size-rank exponent through, $\gamma \approx 1/\alpha$ [44]. For simplicity, in what follows, we will refer to $P(>S)$ as the cumulative distribution function (CDF).

To start our analysis of the sizes of cities along the 20th century, we construct the size-rank plots for each decade. Fig. 2 shows the statistics of rank against the size of Mexican cities for ten censuses. The best fit, using the ordinary least square (OLS), leads to the exponent values showed in Table 2. We observe a clear increasing tendency in the exponents as the time evolves, with values above one for recent data. Interestingly, the CSD exponents since 1900 up to 1960 had a homogeneous behavior expressed by means of practically linear fittings (especially, if Mexico City is excluded). The α-exponents grew from 0.76 until 1.04 during this period; that is, the α's evolved from a value corresponding to an uneven distribution of cities up to a

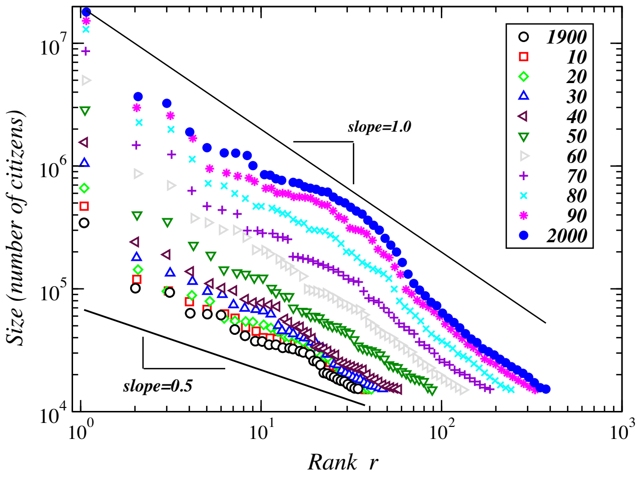

Fig. 2. Rank vs. size plot for cities in Mexico along the 20th century. We observe a Zipf's distribution of the form, $S(r) \sim r^{-\alpha}$. The value of the exponent α increased from 0.78 to 1.1 during the period under study, revealing changes in the city growth mechanisms. We also notice the deviations from the power-law behavior from year 1970 and up.

Table 2
Summary of scaling exponents observed in Zipf plots and cumulative distributions for cities in Mexico along the 20th century. The values of the exponents (α) for the Zipf's plots were estimated by means of the ordinary linear regression method, while the exponents for the CDF (γ) or Pareto distributions were obtained by means of the MLE method. A good agreement is observed between α and $\hat{\alpha}$ (where $\hat{\alpha}$ was obtained according to the relationship, $\hat{\alpha} \approx 1/\gamma$ [44]), except for the values corresponding to 1910, 1930 and 1940 (marked in boldface).

Year	Zipf's exponent (α)	CDF exponent (γ)	$\hat{\alpha} = 1/\gamma$
1900	0.761 ± 0.026	1.329 ± 0.231	0.752
1910	0.773 ± 0.031	1.187 ± 0.197	**0.842**
1920	0.846 ± 0.034	1.223 ± 0.195	0.817
1930	0.937 ± 0.031	1.172 ± 0.174	**0.853**
1940	0.976 ± 0.029	1.198 ± 0.161	**0.834**
1950	0.972 ± 0.025	1.075 ± 0.117	0.929
1960	1.045 ± 0.018	0.972 ± 0.087	1.028
1970	1.072 ± 0.020	0.944 ± 0.071	1.058
1980	1.124 ± 0.022	0.868 ± 0.057	1.151
1990	1.136 ± 0.025	0.868 ± 0.049	1.155
2000	1.159 ± 0.027	0.842 ± 0.044	1.186

more egalitarian distribution; that is, a more evenly spread of the population across the country. This global behavior from 1900 until 1960 can be interpreted within the context of the "coherence" concept coined by Cristelli et al. [27], which has to do with the internal consistency or completeness of the total sample under examination. To exemplify this concept these authors [27] show how the frequency of words in the Corpus of Contemporary American English displays a quasi-perfect Zipf's law and also how the rank of the Gross Domestic Product (GDP) of world countries has a Zipfian behavior for the 30th richest. Another case of a good Zipfian fitting is that corresponding to the CSD of Nigeria. For the GDP case, Cristelli et al. [27] interpret the Zipfian behavior in terms of globalization as a mechanism of internal coherence of this system. On the other hand, for Nigeria, despite being a developing country, it has an internal coherence stemming from a developing in a more uniform, isolated and self integrated fashion. Returning to the Mexico's case, Fig. 2 shows how from 1960 until 2000, despite of the α exponents are within the interval $[1.045, 1.115]$, the quality of the linear fitting is remarkable deteriorated, mainly due to the growth of cities with population between 10^5 and 10^6 inhabitants; that is an accelerated growing of intermediate city-sizes.

As it can be seen in Fig. 6, this notorious deviation is a contribution of the northern cities located near the border with USA [45]. Much of this phenomenon has to do with the great economic "gradient" exercised by a strong economy on a weak one, which is the case in the USA–Mexico interaction.

Next, we evaluate the properties of the scaling behavior observed in Fig. 2 from two perspectives: (i) The concordance of the global scaling exponent with the local one; and (ii) the deviation of the size predicted by the Zipf's law ($\alpha = 1$) and the real size of each city. Concerning (i), to asses the behavior of the local exponent, α_l, along different rank scales, we calculate this exponent in the following way,

$$\alpha_l = -\frac{d \log S(r)}{d \log r}. \quad (1)$$

The resulting values of α_l as a function of the rank r are showed in Fig. 3(a). We show that the local exponent is not stable, leading to different scaling behaviors, but the rounded crossover between different scales seems to be important when the global (over all scales) fit is performed, presenting a power law scaling of the form $1/r^{\alpha}$. A more detailed observation of the local exponents reveals the presence of crossings between values below and above one, indicating the scales where the rank statistics deviates from the Zipfian regime. Regarding (ii), we use the approach suggested by González [30] to evaluate the deviation of the real city-size S with respect to the one predicted by the Zipf's law (S^*). The deviation is captured by the statistics [30],

$$\ln(S^*/S) = (1/\alpha - 1)(\ln r - \ln N) - (1/\alpha)\hat{e}, \quad (2)$$

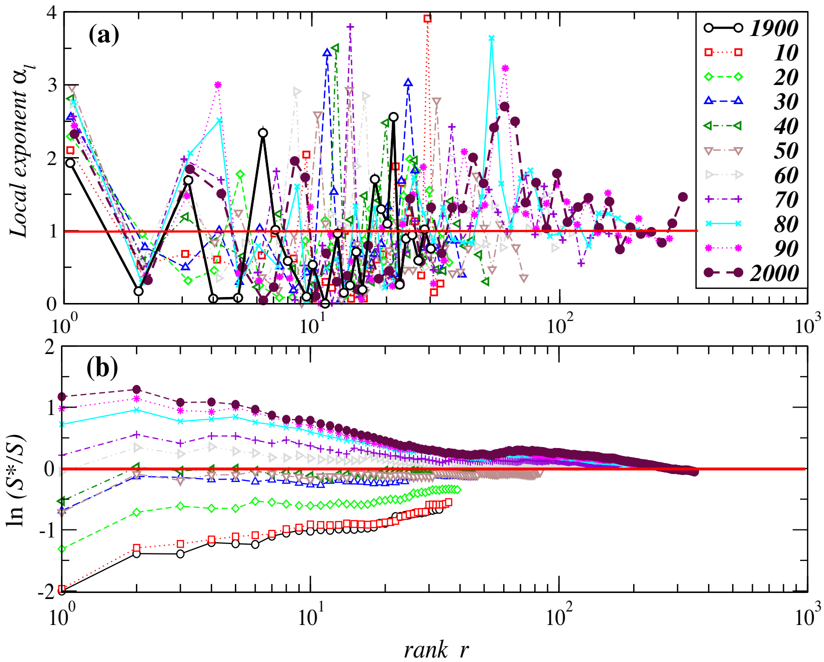

Fig. 3. (a) Dependence of local exponents α_l on the rank-scale for the Zipf's plots showed in Fig. 2. The variations around the global exponent estimated in Fig. 2, indicate that the global scaling behavior is not true locally. (b) $\ln(S^*/S)$ vs. rank-scale for the years showed in Fig. 2. In this plot, if the data is close to the zero value, it indicates that the distribution is close to the pure Zipf's law.

where N is the number of cities, α is the observed exponent and $\hat{e}$ is the error in the fit. Fig. 3(b) shows the results of the calculations from Eq. 2. As it is observed, the value of $\ln(S^*/S)$ is negative for censuses from 1900 up to 1950, and it becomes positive for 1960 and subsequent censuses for almost all rank scales. This behavior indicates in general that, for data within the period 1900–1950, the size predicted by the Zipf's law is smaller than the real one, whereas the opposite situation is presented for 1960–2000 censuses. We notice that for smaller cities (larger ranks), the deviations (both positive and negative) tend to be close to the zero value, indicating that for these sizes (ranks) both distributions (Zipf's law and size-rank) tend to be close each other.

In order to get additional information about the city-growth along the 20th century, we construct the scatter plot of $S(t)$ vs. $S(t+10)$ in a log–log plane. The resulting distributions of points are presented in Fig. 4. We observe a clear tendency of the points to follow a straight line along different scales, which confirms that the rate of growth is consistent with the Gibrat's law [29]. However, the northern "bump" present in Fig. 2, here it is observed as outlier-points. To robust the results obtained by means of the Zipf plots analysis, we also construct the cumulative distribution CDF, that is, the probability of having sizes larger than a given value S. Fig. 5 shows the CDF for the same data analyzed in Fig. 2. We also find that the distributions approximately follow a power law of the form $G(>S) \sim S^{-\gamma}$, where γ is the Pareto exponent. The maximum-likehood estimation method (MLE) [2] is used to calculate γ, which leads to exponents ranging from $\gamma = 1.32$ to $\gamma = 0.84$ for years 1900 and 2000, respectively (see Table 2). It is noteworthy

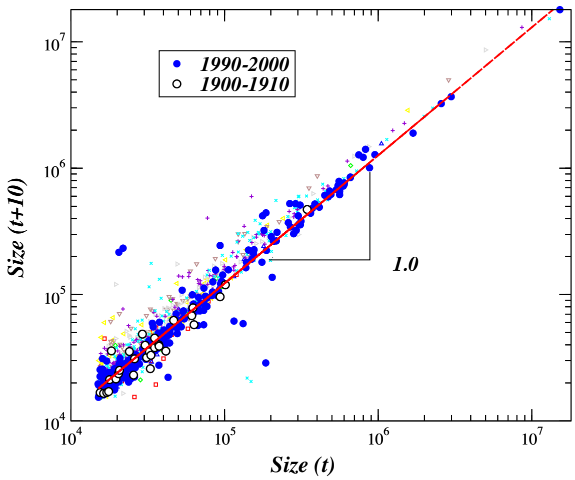

Fig. 4. Scatter plot of $S(t)$ vs. $S(t+10)$ for city sizes in Mexico along the 20th century. The distribution of points are clearly located around the straight line with slope one. We observe that for the year 2000, the data exhibits some outliers which are consistent with the irregular behavior of this dataset previously described in Figs. 2 and 3.

that there is a good concordance between the values estimated for γ and the size-rank exponents α, i. e., the relationship $\alpha \approx 1/\gamma$ is fulfilled [44].

Next, we explore the effect of the geographic distribution of cities over the whole statistics described above. To this end, we divide the country into three main regions namely, north, center and south [46]. In order to get clarity

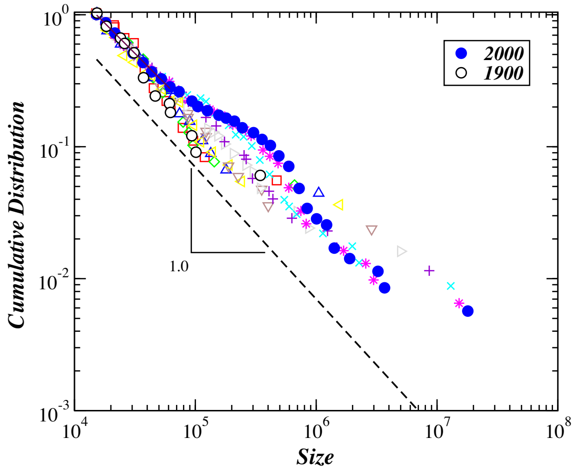

Fig. 5. Cumulative distribution of cities for ten censuses. For 1900 data the scaling exponent is close to one (see Table 2), and for 2000 data the value decreased to 0.79, indicating that the probability of sizes $S > 10^5$ is bigger than the other datasets from previous years.

in the statistics we only considered the data corresponding to the year 2000. The CDF of each region (subset) are showed in Fig. 6, the data indicate that the distributions for the center and south regions can be well described by a power law with exponents $\gamma_{center} = 0.681 \pm 0.002$ and $\gamma_{south} = 0.807 \pm 0.002$, whereas the north set leads to $\gamma_{north} = 0.6922 \pm 0.038$, but it is remarkable its deviation from the power law behavior. It is also noteworthy that this deviation observed in the data from the north region, indicates that the probability of cities $G(> S)$ is higher than the probabilities of the center and the south, for intermediate city sizes. This result confirms the fact that the north region contributes in large extent to the deviation in the power law behavior of the CSD reported in Fig. 2. In order to perform an equivalent analysis for subsets by using the Zipf's plot statistics, we take care of a recent property noticed by Cristelli et al. [27] about the coherence of the set under study. Within this context, the α's behavior in Fig. 2 suggests that despite its increase along 20th century the country has lost internal coherence due to uneven development between regions, such as the "bump" effect suggests (see Figs. 2, 5 and 6). Another peculiarity of Mexican CSD is that for the ten censuses (including the five with $\alpha > 1$) the size of the largest city (Mexico City) is greater than the intercept term of the corresponding fittings, unlike those reported by Soo [47], where shows otherwise for countries with $\alpha > 1$.

4. Concluding remarks

The present study shows how along the 20th century Mexico evolved from low values of α at the beginning of the century up to values above one, since the decade of the 1960's. These changes coincide with the Mexico's evolution from a rural society towards an urban country. However, nevertheless Mexico passed from an uneven distribution to a more spread population across its territory, the country did not reach the status of developed country; that is, $\alpha > 1$ does not imply economic development. Recently, a more suitable interpretation of the α behavior was proposed in terms of a concept called "coherence" [27]. Within this frame the Mexican CSD along the 20th century is better understood. The unequal development between regions has led to a noticeable deviation from an accurate Zipfian behavior even though the mean α values are slightly above one. Although some authors as Soo [47] suggest that political variables appear to matter more than economic geographic variables in determining the size distribution of cities, the case of Mexico reveals that the economic geography can play a very important role in CSD. In fact, the notorious "bump" observed in

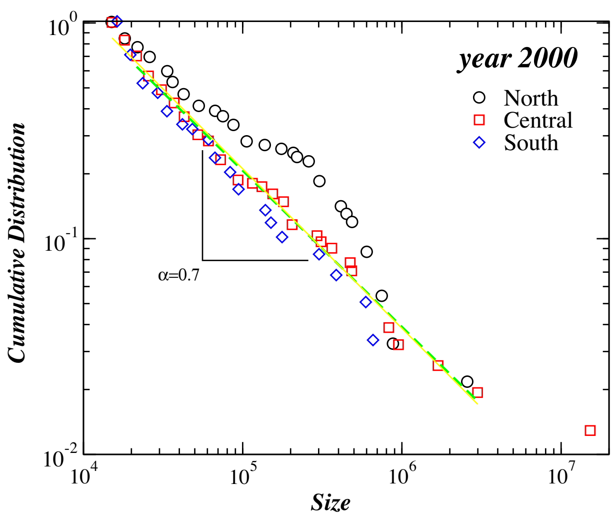

Fig. 6. Cumulative distribution of cities for year 2000 and from three regions. The country is divided into three regions: North, Central and South. The cities in each region are considered to construct the distributions in order to evaluate the contribution of each zone into the whole set distribution. It is noteworthy that the North one mainly contributes to the overall distribution, especially for intermediate city sizes.

Mexican data during the last decades of 20th century has a very clear geographic origin. When the Zipf's plots are observed at local scale, the scaling exponent exhibits variations which indicate that the scaling behavior is not true locally. Thus, only a mean global Zipfian analysis is suitable for urban systems of the Mexican-type. In summary, we believe that the CSD in Mexico along the 20th century is a very interesting problem due to peculiarities stemming from its geographic location an its internal social an economic heterogeneity.

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[46] The considered regions are formed by the following states: North (Baja California, Baja California Sur, Sonora, Durango, Sinaloa, Chihuahua, Coahuila, Nuevo León, Tamaulipas); Center (Nayarit, Zacatecas, San Luis Potosí, Aguascalientes, Jalisco, Guanajuato, Querétaro, Hidalgo, Estado de México, Michoacán, Tlaxcala, Distrito Federal, Puebla, Morelos, Veracruz) and South (Guerrero, Oaxaca, Tabasco, Chiapas, Campeche, Yucatán, Quintana Roo).
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