

Development of a cardiac model for the study of sudden cardiac death in infants

Mateo Alonso García-Cabezón

September 2021

Ventricular hiPSC-CMs model formulation

1 Extracellular and intracellular ionic concentrations

$$\begin{aligned} \text{Na}_o &= 151 \text{ mM} \\ \text{Ca}_o &= 1.8 \text{ mM} \\ \text{K}_o &= 1.8 \text{ mM} \\ \text{Na}_i^0 &= 9.991 \text{ mM} \\ \text{K}_i^0 &= 150 \text{ mM} \\ \text{Ca}_i^0 &= 0.0002 \text{ mM} \\ \text{Ca}_{\text{SR}}^0 &= 0.12 \text{ mM} \end{aligned}$$

2 Cell size and dimensions

$$\begin{aligned} C &= 98.7109 \text{ pF} \\ V_c &= 8800 \mu\text{m}^3 \\ V_{\text{SR}} &= 583.73 \mu\text{m}^3 \end{aligned}$$

3 Maximum conductances and currents

$$\begin{aligned}
\sigma_{\text{Na}} &= 3.6712302e3 \text{ S/F} \\
\sigma_{\text{CaL}} &= 8.635702e - 5 \text{ m}^3/(\text{F} \cdot \text{s}) \\
\sigma_{\text{to}} &= 29.9038 \text{ S/F} \\
\sigma_{\text{Kr}} &= 29.8667 \text{ S/F} \\
\sigma_{\text{Ks}} &= 2.041 \text{ S/F} \\
\sigma_{\text{K1}} &= 28.1492 \text{ S/F} \\
\sigma_{\text{f}} &= 30.10312 \text{ S/F} \\
\sigma_{\text{pCa}} &= 0.4125 \text{ A/F} \\
\sigma_{\text{bCa}} &= 0.9 \text{ S/F} \\
\sigma_{\text{bCa}} &= 0.69264 \text{ S/F} \\
g_{\text{rel}} &= 17 \text{ s}^{-1} \\
P_{\text{NaK}} &= 1.841424 \text{ A/F} \\
K_{\text{NaCa}} &= 1633.33 \text{ A/F} \\
V_{\text{max,up}} &= 0.728832 \text{ mM/s} \\
V_{\text{leak}} &= 4.4444e - 4 \text{ s}^{-1}
\end{aligned}$$

4 Buffers

Sarcoplasmic reticulum

$$\text{CaSR}_{\text{buf}} = \frac{1}{1 + \frac{B_{\text{SR}} \cdot K_{\text{bufSR}}}{(\text{CaSR} + K_{\text{bufSR}})^2}}$$

Cytosol

$$\begin{aligned}
\frac{\partial \text{CaB}_{\text{TnC}}}{\partial t} &= k_{\text{TnC}}^{\text{on}} \text{Ca}_i \cdot (B_{\text{TnC}}^{\text{T}} - \text{CaB}_{\text{TnC}}) - k_{\text{TnC}}^{\text{off}} \text{CaB}_{\text{TnC}} \\
\frac{\partial \text{CaB}_{\text{CAM}}}{\partial t} &= k_{\text{CAM}}^{\text{on}} \text{Ca}_i \cdot (B_{\text{CAM}}^{\text{T}} - \text{CaB}_{\text{CAM}}) - k_{\text{CAM}}^{\text{off}} \text{CaB}_{\text{CAM}} \\
\frac{\partial \text{CaB}_{\text{SLH}}}{\partial t} &= k_{\text{SLH}}^{\text{on}} \text{Ca}_i \cdot (B_{\text{SLH}}^{\text{T}} - \text{CaB}_{\text{SLH}}) - k_{\text{SLH}}^{\text{off}} \text{CaB}_{\text{SLH}} \\
\frac{\partial \text{CaB}_{\text{SR}}}{\partial t} &= k_{\text{SR}}^{\text{on}} \text{Ca}_i \cdot (B_{\text{SR}}^{\text{T}} - \text{CaB}_{\text{SR}}) - k_{\text{SR}}^{\text{off}} \text{CaB}_{\text{SR}} \\
\frac{\partial \text{CaB}_{\text{Rhod-2}}}{\partial t} &= k_{\text{Rhod-2}}^{\text{on}} \text{Ca}_i \cdot (B_{\text{Rhod-2}}^{\text{T}} - \text{CaB}_{\text{Rhod-2}}) - k_{\text{Rhod-2}}^{\text{off}} \text{CaB}_{\text{Rhod-2}} \\
k_{\text{on}}^{\text{TnC}} &= 32.7 \mu\text{M}^{-1} \text{s}^{-1} \\
k_{\text{off}}^{\text{TnC}} &= 19.6 \text{s}^{-1} \\
B_{\text{TnC}} &= 70 \mu\text{M} \\
k_{\text{on}}^{\text{CAM}} &= 35 \mu\text{M}^{-1} \text{s}^{-1} \\
k_{\text{off}}^{\text{CAM}} &= 200 \text{s}^{-1} \\
B_{\text{CAM}} &= 24 \mu\text{M} \\
k_{\text{on}}^{\text{SLH}} &= 100 \mu\text{M}^{-1} \text{s}^{-1} \\
k_{\text{off}}^{\text{SLH}} &= 30 \text{s}^{-1}
\end{aligned}$$

$$\begin{aligned}
B_{\text{SLH}} &= 50 \mu\text{M} \\
k_{\text{on}}^{\text{SR}} &= 100 \mu\text{M}^{-1} \text{s}^{-1} \\
k_{\text{off}}^{\text{SR}} &= 60 \text{s}^{-1} \\
B_{\text{SR}} &= 20 \mu\text{M} \\
k_{\text{on}}^{\text{Rhod-2}} &= 100 \mu\text{M}^{-1} \text{s}^{-1} \\
k_{\text{off}}^{\text{Rhod-2}} &= 150 \text{s}^{-1} \\
B_{\text{Rhod-2}} &= 20 \mu\text{M}
\end{aligned}$$

5 Other constants

$$\begin{aligned}
K_{\text{up}} &= 0.00025 \text{ mM} \\
K_{\text{pCa}} &= 0.0005 \text{ mM} \\
F &= 96485.3415 \text{ C/M} \\
R &= 8.314472 \text{ J/(M} \cdot \text{K)} \\
T &= 310 \text{ K} \\
L_0 &= 0.025 \text{ dimensionless}
\end{aligned}$$

6 Initial conditions

$$\begin{aligned}
h_0 &= 0.75 \text{ dimensionless} \\
j_0 &= 0.75 \text{ dimensionless} \\
m_0 &= 0 \text{ dimensionless} \\
d_0 &= 0 \text{ dimensionless} \\
f_{\text{Ca}_0} &= 1 \text{ dimensionless} \\
f_{1,0} &= 1 \text{ dimensionless} \\
f_{2,0} &= 1 \text{ dimensionless} \\
r_0 &= 0 \text{ dimensionless} \\
q_0 &= 1 \text{ dimensionless} \\
X_{r_1,0} &= 0 \text{ dimensionless} \\
X_{r_2,0} &= 1 \text{ dimensionless} \\
X_{s_0} &= 0 \text{ dimensionless} \\
X_{f_0} &= 0.1 \text{ dimensionless} \\
V_0 &= -70e - 3 \text{ V} \\
O_0 &= 0 \text{ dimensionless} \\
R_{1,0} &= 0 \text{ dimensionless} \\
R_{2,0} &= 0 \text{ dimensionless} \\
C_0 &= 1 \text{ dimensionless}
\end{aligned}$$

7 Membrane potential

$$\frac{dV}{dt} = -\frac{1}{C} (I_{\text{Na}} + I_{\text{CaL}} + I_{\text{f}} + I_{\text{Kr}} + I_{\text{Ks}} + I_{\text{to}} + I_{\text{NaCa}} + I_{\text{NaK}} + I_{\text{pCa}} + I_{\text{bNa}} + I_{\text{bCa}} - I_{\text{stim}})$$

8 Ionic currents

Na⁺ current

$$I_{\text{Na}} = \sigma_{\text{Na}} m^3 \cdot h \cdot j \cdot (V - E_{\text{Na}})$$

h gate

$$\frac{dh}{dt} = \frac{h_{\infty} - h}{\tau_h}$$

$$h_{\infty} = \frac{1}{\sqrt{1 + e^{\frac{V+72.1}{5.7}}}}$$

$$\alpha_h = \begin{cases} 0.057 \cdot e^{-\frac{V+80}{6.8}}, & \text{if } V < -40 \text{ mV} \\ 0, & \text{otherwise} \end{cases}$$

$$\beta_h = \begin{cases} 2.7 \cdot e^{0.079V} + 3.1 \cdot 10^5 \cdot e^{0.3485V}, & \text{if } V < -40 \text{ mV} \\ \frac{0.77}{0.13 \cdot \left(1 + e^{\frac{V+10.66}{-11.1}}\right)}, & \text{otherwise} \end{cases}$$

$$\tau_h = \begin{cases} \frac{1.5}{\alpha_h + \beta_h}, & \text{if } V < -40 \text{ mV} \\ 2.542, & \text{if } V \geq -40 \text{ mV} \end{cases}$$

j gate

$$\frac{dj}{dt} = \frac{j_{\infty} - j}{\tau_j}$$

$$j_{\infty} = h_{\infty}$$

$$\alpha_j = \begin{cases} \frac{(-25428e^{0.2444V} - 6.948 \cdot 10^{-6} \cdot e^{-0.04391V})(V + 37.78)}{1 + e^{0.311(V+79.23)}}, & \text{if } V < -40 \text{ mV} \\ 0, & \text{otherwise} \end{cases}$$

$$\beta_j = \begin{cases} \frac{0.02424e^{-0.01052V}}{1 + e^{-0.1378(V+40.14)}}, & \text{if } V < -40 \text{ mV} \\ \frac{0.6e^{0.057V}}{1 + e^{-0.1(V+32)}}, & \text{otherwise} \end{cases}$$

$$\tau_j = \frac{7}{\alpha_j + \beta_j}$$

m gate

$$\frac{dm}{dt} = \frac{m_{\infty} - m}{\tau_m}$$

$$m_{\infty} = \frac{1}{\left(1 + e^{\frac{-34.1-V}{5.9}}\right)^{1/3}}$$

$$\alpha_m = \frac{1}{1 + e^{\frac{-60-V}{5}}}$$

$$\beta_m = \frac{0.1}{1 + e^{\frac{V+35}{5}}} + \frac{0.1}{1 + e^{\frac{V-50}{200}}}$$

$$\tau_m = \alpha_m \cdot \beta_m$$

L-type Ca^{2+} current

$$I_{\text{CaL}} = \sigma_{\text{CaL}} \frac{4VF^2}{RT} \frac{\text{Ca}_i \cdot e^{\frac{2VF}{RT}} - 0.341 \cdot \text{Ca}_o}{e^{\frac{2VF}{RT}} - 1} \cdot d \cdot f_1 \cdot f_2 \cdot f_{\text{Ca}}$$

d gate

$$\frac{dd}{dt} = \frac{d_\infty - d}{\tau_d}$$

$$d_\infty = \frac{1}{1 + e^{-\frac{V+9.1}{7}}}$$

$$\alpha_d = \frac{1}{1 + e^{-\frac{-60-V}{5}}}$$

$$\beta_d = \frac{1.4}{1 + e^{\frac{V+5}{5}}}$$

$$\gamma_d = \frac{1}{1 + e^{\frac{50-V}{20}}}$$

$$\tau_d = \alpha_d \cdot \beta_d \cdot \gamma_d$$

f_{Ca} gate

$$\frac{df_{\text{Ca}}}{dt} = C_{f_{\text{Ca}}} \frac{f_{\text{Ca},\infty} - f_{\text{Ca}}}{\tau_{f_{\text{Ca}}}}$$

$$\alpha_{f_{\text{Ca}}} = \frac{1}{1 + \left(\frac{\text{Ca}_i}{0.0006}\right)^8}$$

$$\beta_{f_{\text{Ca}}} = \frac{0.1}{1 + e^{\frac{\text{Ca}_i - 0.0009}{0.0001}}}$$

$$\gamma_{f_{\text{Ca}}} = \frac{0.1}{1 + e^{\frac{\text{Ca}_i - 0.00075}{0.0008}}}$$

$$f_{\text{Ca},\infty} = \frac{\alpha_{f_{\text{Ca}}} + \beta_{f_{\text{Ca}}} + \gamma_{f_{\text{Ca}}}}{1.3156}$$

$$\tau_{f_{\text{Ca}}} = 2 \text{ ms}$$

$$C_{f_{\text{Ca}}} = \begin{cases} 0, & \text{if } (f_{\text{Ca},\infty} > f_{\text{Ca}}) \text{ and } (V > -60 \text{ mV}) \\ 1, & \text{otherwise} \end{cases}$$

f_1 gate

$$\frac{df_1}{dt} = \frac{f_{1,\infty} - f_1}{\tau_{f_1}}$$

$$f_{1,\infty} = \frac{1}{1 + e^{\frac{V+26}{3}}}$$

$$\tau_{f_1} = \left(1102.5e^{-\left[\frac{(V+27)^2}{15}\right]^2} + \frac{200}{1 + e^{\frac{13-V}{10}}} + \frac{180}{1 + e^{\frac{V+30}{10}}} + 20 \right) \cdot \begin{cases} 1 + 1433 \cdot (\text{Ca}_i - 50 \cdot 10^{-6}), & \frac{df_1}{dt} > 0 \\ 1, & \text{otherwise} \end{cases}$$

f_2 gate

$$\frac{df_2}{dt} = \frac{f_{2,\infty} - f_2}{\tau_{f_2}}$$

$$f_{2,\infty} = \frac{0.67}{1 + e^{\frac{V+35}{4}}} + 0.33$$

$$\tau_{f_2} = \left(600e^{-\frac{(V+25)^2}{170}} + \frac{31}{1 + e^{\frac{25-V}{10}}} + \frac{16}{1 + e^{\frac{V+30}{10}}} \right)$$

Funny current

$$I_f = \sigma_f \cdot X_f \cdot (V - E_f)$$

X_f gate

$$\frac{dX_f}{dt} = \frac{X_{f\infty} - X_f}{\tau_{X_f}}$$

$$X_{r_1\infty} = \frac{1}{1 + e^{\frac{V+77.85}{5}}}$$

$$\tau_{X_f} = \frac{1900}{1 + e^{\frac{V+15}{10}}}$$

Transient outward current

$$I_{to} = \sigma_{to} \cdot r \cdot q \cdot (V - E_K)$$

r gate

$$\frac{dr}{dt} = \frac{r_\infty - r}{\tau_r}$$

$$r_\infty = \frac{1}{1 + e^{\frac{22.3-V}{18.75}}}$$

$$\tau_r = \frac{14.40516}{1.037e^{0.09(V+30.61)} + 0.369e^{-0.12(V+23.84)}} + 2.75352$$

q gate

$$\frac{dq}{dt} = \frac{q_\infty - q}{\tau_q}$$

$$q_\infty = \frac{1}{1 + e^{\frac{V+53}{13}}}$$

$$\tau_q = \frac{39.102}{0.57e^{0.08(V+44)} + 0.065e^{0.1(V+45.93)}} + 6.06$$

Rapid delayed rectifier K^+ current

$$I_{K_r} = \sigma_{K_r} \sqrt{\frac{K_0}{5.4}} \cdot X_{r_1} \cdot X_{r_2} \cdot (V - E_K)$$

X_{r_1} gate

$$\begin{aligned}
\frac{dX_{r_1}}{dt} &= \frac{X_{r_1\infty} - X_{r_1}}{\tau_{X_{r_1}}} \\
X_{r_1\infty} &= \frac{1}{1 + e^{\frac{V_{1/2}-V}{4.9}}} \\
V_{1/2} &= -1000 \cdot \left[\frac{RT}{FQ} \ln \frac{\left(\frac{1+Ca_o}{2.6}\right)^4}{L_0 \left(1 + \frac{Ca_o}{0.58}\right)^4} - 0.019 \right] \\
\alpha_{X_{r_1}} &= \frac{450}{1 + e^{\frac{-45-V}{10}}} \\
\beta_{X_{r_1}} &= \frac{6}{1 + e^{\frac{V+30}{11.5}}} \\
\tau_{X_{r_1}} &= \alpha_{X_{r_1}} \cdot \beta_{X_{r_1}}
\end{aligned}$$

X_{r_2} gate

$$\begin{aligned}
\frac{dX_{r_2}}{dt} &= \frac{X_{r_2\infty} - X_{r_2}}{\tau_{X_{r_2}}} \\
X_{r_2\infty} &= \frac{1}{1 + e^{\frac{V+88}{50}}} \\
\alpha_{X_{r_2}} &= \frac{3}{1 + e^{\frac{-60-V}{20}}} \\
\beta_{X_{r_2}} &= \frac{1.12}{1 + e^{\frac{V-60}{20}}} \tau_{X_{r_2}} = \alpha_{X_{r_1}} \cdot \beta_{X_{r_1}}
\end{aligned}$$

Slow delayed rectifier K^+ current

$$I_{K_s} = \sigma_{K_s} \cdot X_s^2 \left[1 + \frac{0.6}{1 + \left(\frac{3.8 \cdot 10^{-5}}{Ca_i}\right)^2} \right] \cdot (V - E_{K_s})$$

X_s gate

$$\begin{aligned}
\frac{dX_s}{dt} &= \frac{X_{s\infty} - X_s}{\tau_{X_s}} \\
X_{s\infty} &= \frac{1}{1 + e^{\frac{-20-V}{16}}} \\
\alpha_{X_s} &= \frac{1100}{\sqrt{1 + e^{\frac{-10-V}{6}}}} \\
\beta_{X_s} &= \frac{1}{1 + e^{\frac{V-60}{20}}} \tau_{X_s} = \alpha_{X_s} \cdot \beta_{X_s}
\end{aligned}$$

Inward rectifier K^+ current

$$\begin{aligned}
I_{K_1} &= \sigma_{K_1} x_{K_1,\infty} \cdot \sqrt{\frac{K_o}{5.4}} (V - E_K) \\
X_{K_1\infty} &= \frac{\alpha_{K_1}}{\alpha_{K_1} + \beta_{K_1}} \\
\alpha_{X_{K_1}} &= \frac{3.91}{1 + e^{0.5942(V-E_K-200)}}
\end{aligned}$$

$$\beta_{X_{K_1}} = \frac{-1.509 \cdot e^{0.0002(V-E_K+100)} + e^{0.5886(V-E_K-10)}}{1 + e^{0.4547(V-E_K)}}$$

Na⁺/Ca²⁺ pump current

$$I_{\text{NaCa}} = K_{\text{NaCa}} \frac{e^{\frac{\gamma \cdot VF}{RT}} \cdot \text{Na}_i^3 \cdot \text{Ca}_o - e^{\frac{(\gamma-1)VF}{RT}} \cdot \text{Na}_o^3 \cdot \text{Ca}_i \cdot \alpha}{(K_{\text{mCa}} + \text{Ca}_o) \cdot (K_{\text{mNa}}^3 + \text{Na}_o^3) \cdot \left(1 + K_{\text{sat}} e^{\frac{(\gamma-1)VF}{RT}}\right)}$$

Na⁺/K⁺ pump current

$$I_{\text{NaK}} = \frac{\frac{\text{Na}_i \cdot P_{\text{NaK}} \cdot K_o}{K_o + K_{\text{mk}}}}{(\text{Na}_i + K_{\text{mNa}}) \cdot \left(1 + 0.1245 e^{\frac{-0.1VF}{RT}} + 0.0353 e^{\frac{-VF}{RT}}\right)}$$

9 Ca²⁺ dynamics

$$\begin{aligned} \frac{d\text{Ca}_i}{dt} &= \left(j_{\text{leak}} - j_{\text{up}} + j_{\text{rel}} - \frac{I_{\text{CaL}} - 2I_{\text{NaCa}} + I_{\text{pCa}} + I_{\text{bCa}}}{2FV_c} \right) - \sum_j \frac{\partial \text{CaB}_j}{\partial t} \\ \frac{d\text{CaSR}}{dt} &= \text{CaSR}_{\text{buf}} \frac{V_c}{V_{\text{SR}}} (j_{\text{up}} - j_{\text{leak}} - j_{\text{rel}}) \\ j_{\text{up}} &= V_{\text{max,up}} \frac{1}{1 + \frac{K_{\text{up}}^2}{\text{Ca}_i^2}} \\ j_{\text{leak}} &= V_{\text{leak}} \cdot (\text{CaSR} - \text{Ca}_i) \\ j_{\text{rel}} &= g_{\text{rel}} \cdot \text{O} \cdot (\text{CaSR} - \text{Ca}_i) \end{aligned}$$

9.1 RyR scheme

$$\begin{aligned} \frac{d\text{O}}{dt} &= k_{\text{co}}\text{C} + k_{\text{iO}}\text{R}_1 - (k_{\text{oc}}\text{O} + k_{\text{oi}}\text{O}) \\ \frac{d\text{C}}{dt} &= k_{\text{oc}}\text{O} + k_{\text{ic}}\text{R}_2 - (k_{\text{co}}\text{C} + k_{\text{ci}}\text{C}) \\ \frac{d\text{R}_2}{dt} &= k_{\text{ci}}\text{C} + k_{\text{i}_1\text{i}_2}\text{R}_1 - (k_{\text{ic}}\text{R}_1 + k_{\text{i}_2\text{i}_1}\text{R}_1) \\ \frac{d\text{R}_1}{dt} &= k_{\text{ci}}\text{C} + k_{\text{i}_1\text{i}_2}\text{R}_1 - (k_{\text{ic}}\text{R}_1 + k_{\text{i}_2\text{i}_1}\text{R}_1) \\ k_{\text{CaSR}} &= \frac{1 + (150 \mu\text{M}/\text{CaSR})^{30}}{1 + (\text{CaSR}/300 \mu\text{M})^5} \\ k_{\text{co}} &= k_{\text{a}} \cdot \frac{(\text{Ca}_i/\text{Ca}_i^*)^3}{1 + (\text{Ca}_i/\text{Ca}_i^*)^3} \cdot \frac{1}{k_{\text{CaSR}}} \\ f_2 &= 1 + \left(\frac{200 \mu\text{M}}{\text{CaSR}} \right)^2 \\ k_{\text{ci}} &= 10^{-4} \cdot \text{Ca}_i \cdot f_2 \end{aligned}$$

$$\begin{aligned}
k_a &= 3 \text{ S/F} \\
Ca_i^* &= 1 \mu\text{M} \\
k_{oc} &= k_{i1i2} = 0.01 \text{ ms}^{-1} \\
k_{1d} &= 2 \text{ mM} \\
k_{ic} &= k_{io} \\
k_{ci} &= k_{oi}
\end{aligned}$$

Ca²⁺ pump current

$$I_{pCa} = \sigma_{pCa} \frac{Ca_i}{Ca_i + K_{pCa}}$$

10 Na⁺ dynamics

$$\frac{dNa}{dt} = -C \frac{I_{Na} + I_{bNa} + 3I_{NaK} + 3I_{NaCa}}{FV_c}$$

Background currents

$$\begin{aligned}
I_{bNa} &= \sigma_{bNa} \cdot (V - E_{Na}) \\
I_{bCa} &= \sigma_{bCa} \cdot (V - E_{Ca})
\end{aligned}$$

11 Reversal potentials

$$\begin{aligned}
E_{Na} &= \frac{RT}{F} \ln \frac{Na_o}{Na_i} \\
E_K &= \frac{RT}{F} \ln \frac{K_o}{K_i} \\
E_{K_s} &= \frac{RT}{F} \ln \frac{K_o + P_{K,Na} \cdot Na_o}{K_i + P_{K,Na} \cdot Na_i} \\
E_{Ca} &= \frac{RT}{2F} \ln \frac{Ca_o}{Ca_i} \\
E_f &= -17 \text{ mV}
\end{aligned}$$