ECON250: Intermediate Microeconomics

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1 Production

For production we assume you product only one good. Next, the firms goal is to minimize cost of making some fixed amount. Third, we can buy as many inputs as possible to achieve the optimal amount (Input markets are competitive). Firms do not have a budget constraint (Functioning credit markets).
A Production function is a relationship between quantity of outputs for quantity of inputs. More formally,

$$Q : K \times L \to Q$$

In the short run we have fixed capital (K)
<u>Defn</u>
(i) The Short run is an empirically a few months to a year.
(ii) Marginal product is additional unit of output from using one more unit of input. (Other inputs held constant)
(iii) The average product is the quantity of output per unit of input. We find from our production function the marginal product of labor is $MP_L := \frac{\partial Q}{\partial L}$ and $AP_L := \frac{Q}{L}$

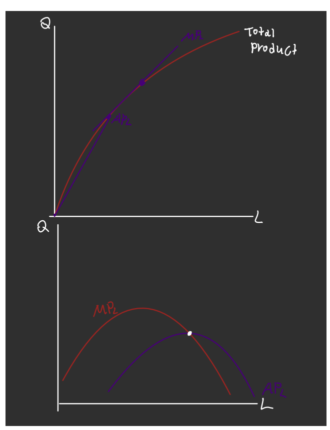

Figure 1: Total Product and APL and MPL curves

We find that there is diminishing MPL because we fix K, MPL is decreasing every additional input. We can also observe APL is the slope of the secant line from the origin to the point. When MPL is less than APL then APl is decreasing similarly, when MPL is more than APL then APL is increasing. APL moves more slowly and is generally more flat than marginal product. We also see that APL is maximized when it is equal to MPL hence when MPL is the secant line from the origin and the point of maximization.

1.1 Iso-Quants

Long run has both capital and Labor flexible hence we plot level curves.
Definition: An Iso quant is all combinations of labor and capital which yield the same output.
Some notes about Isoquants

- Iso-quants further from the origin have higher output
- They don't cross
- They are downward sloping
- They are concave up

Reasons:
(i) If you have more of K,L then obviously higher Q
(ii) If they did then the same K,L give a different Q
(iii) To stay on the curve you need to give some of the other one up
(iv) Balanced input are more productive than a lot of one and a little of the other.
Iso-quants have cardinal ranking
Curvature of Iso-quants
(i) Less curve $\implies$ More substitutable$\equiv$ More curved $\implies$ Less substitutable
(ii) Perfect Substitutes are lines
(iii) Perfect complements are complements
Definition: The marginal rate of technical substitution is how many K can give up using an additional L and still make the same output.
$MRTS_K^L$:Slope of the isoquant
$MRST_K^L = \frac{MP_L}{MP_K} = \frac{\frac{\partial Q}{\partial L}}{\frac{\partial Q}{\partial K}}$ Isocost Lines:
Definition: All curves where input combinations yeild the same cost.
$C = RK + WL$ where R is the rental rate of capital and W is the wage.
RK is the total cost of capital
WL is the total cost of labor
The slope if $\frac{W}{R}$

1.2 Cost Minimization:

To motivate, we want to find a way to minimize cost with a fixed quantity Q_0
Steps for Cost Minimization:
(i) find the $MRTS_K^L$ and $\frac{W}{R}$
(ii) from our tangency condition we set $MRTS_K^L = \frac{W}{R}$
(iii) find K as a function of L or vise versa (iii) Write the isoquant for the fixed quantity i.e. $Q_0 = f(K, L)$
(iv) using (iii) write $Q_0 = f(K \vee L)$
(v) Hence we can solve for L and K
The interpretation is that we increase inputs until you get the most quantity per dollar.
Corner Solutions:
Case 1: when we choose K=0 we must have $\frac{MP_L}{W} \geq \frac{MP_K}{R} \iff MRTS_K^L \geq \frac{W}{R}$
This can be interpreted as the Product per dollar of Capital is greater than the product per dollar of Labor
Case 2: when we choose L=0 we must have $\frac{MP_L}{W} \leq \frac{MP_K}{R} \iff MRTS_K^L \leq \frac{W}{R}$

1.3 Returns to Scale:

Definition:
Constant Return to Scale: Doubling inputs implies double outputs $Q(cK, cL) = cQ(K, L)$(Linearity?)
Increasing Returns to Scale: Doubling inputs results in more than double outputs $Q(cK, cL) > cQ(K, L)$
Decreasing Returns to Scale: Doubling inputs results in more than double outputs $Q(cK, cL) < cQ(K, L)$
Ex: Given $Q(K, L) = K^{0.9}L^{0.1} + 2KL$ do we see increasing, decreasing or constant returns?
Solution:
$Q(2K, 2L) = (2K)^{0.9}(2L)^{0.1} + 2(2K)(2L) = 2(K^{0.9}L^{0.1} + 2(2KL)$
$2Q(K, L) = 2K^{0.9}L^{0.1} + 4KL$
Hence $Q(2K, 2L) \geq 2Q(K, L)$ implies we have increasing returns to scale.
Example:
Learning effects are an example of increasing returns to scale
Agglomeration effects are another example.(More benefit when you bring more people together)

1.4 Cost Introduction

Types of Costs: What we care about
Accounting Costs are the direct cost of operation
Opportunity Costs are the forgone value of using an input
Sunk Costs: Already paid cost that cannot be recovered (Should not effect decisions)

Economic Costs = Accounting Cost + Opportunity Cost
Definition:
(i) Fixed Costs (Not depending on output level (Q)
(ii) Variable costs (Depending based on output level , VC=F(Q))
(iii) Total Costs = Fixed + Variable Costs
(iv) Marginal Cost: Additional Cost for 1 more Q, $MC := \frac{\partial(TC)}{\partial Q} =^{Why?} \frac{\partial VC}{\partial Q}$
(v) $AFC := \frac{FC}{Q}$
(vi) $AVC := \frac{VC}{Q}$
(vii) $ATC := \frac{TC}{Q} = AVC + AFC$ Marginal Cost eventually rises We see the average fixed cost $(\frac{FC}{Q})$ is always the same
Average variable cost starts near marginal cost and is slower than MC
Average total cost starts near AFC and decreases until it reaches Marginal cost
Propn: The marginal average relation is that when MC is increasing it is pulling the average up. And when it is decreasing it is dragging it down. When average is equal to marginal average is maximized.
Deriving Short Run total Cost
Goal: Cost as a function of output
Method: $LRTC(K, L) := RK + WL$ Since we are in the short run we have a fixed K. hence $SRTC(Q) = RK_0 + WL$ Given R, K_0 and W we can find L in terms of Q and then write SRTC as a function of Q
Deriving Long Run total cost
Method: Minimize $LRTC(L, K) = RK + WL, w.r.tQ(K, L) = f(K, L)$
Then we find K and L and write $LRTC(K, L)$ into $LRTC(Q)$
Example: Find LRTC for $Q(K, L) = K^2L, LRTC(L, K) = 50L + 100K$
Solution: $MRST^L_K = \frac{K}{2L}$ from tangency we set it equal to $\frac{1}{2} = \frac{K}{2L} \iff L = K$ then from $Q(K, L)$ we find that since K and L are equal we have $Q = K^3 = L^3 \implies L = K = Q^{\frac{1}{3}}$
hence, $LRTC(Q) = 150Q^{\frac{1}{3}}$
Example: Given $Q = min(2K, L), LRTC(L, K) = 50L + 100K$
Solution: We look that the points of tangency will always be along the line $2K = L$ hence we can find that $Q = 2K = L$ since they are equal they result in the same output quantity
Hence, LRTC(Q)=100Q

1.5 Profit Maximization

Definition:
Case 1: Economies of Scale
ATC Falls as output increases $Q \uparrow \implies ATC \downarrow$
Case 2: Constant economies of Scale
ATC is constant as output increases
$Q \uparrow \implies ATC \rightarrow$
Case 3: dis economies of Scale
ATC Increases as output increases

$Q \uparrow \implies ATC \downarrow$
<u>Short Run Shutdown</u>
In the short run, we do want to minimize the loss regarding fixed costs and fixed costs are sunk costs in the Shortrun(SR). It is possible that the short run curves may look different from the long run curves. To determine if we shut down, we look at producer surplus.
<u>Definition:</u> The producer surplus is how much the firm get after they subtract their variable costs.

$$P.S. := TR - VC$$

This is the rectangle below P^* and above $AVC(Q^*)$ from 0 to Q^*
From this we can see a firm will want to operate given their producer surplus is positive, otherwise they will be loosing more than if they where to just shutdown and take the fixed cost.
From this we can consider the following equalities

$$P.S. \geq 0 \iff Q^*(P^* - AVC(Q^*)) \geq 0 \iff P^* \geq AVC(Q^*)$$

We assume when $P.S. = 0$ you continue to operate.
From this we find that a firms short run supply curve is the marginal cost above the point where AVC is minimized.
<u>Example:</u> The Market for coconut water is perfectly competitive, The total costs for Fritzie's new firm are given by $TC(Q) = 400 + 2Q + 2Q^2$. The market price is 50.
(a) Determine whether the firm operates in the long run.
<u>Solution:</u> $MC(Q) = 2 + 4Q$
$\pi_{max} : 50 = 2 + 4Q \implies Q = 12$
$\implies TC(12) = 400 + 24 + 288, TR(12) = 50 \cdot 12 = 600$
$TC > TR \implies \pi < 0 \implies Shutdown$
(b) Determine whether the firm operates in the short run.
<u>Solution:</u> $AVC(Q) = 2 + 2Q \iff AVC(12) = 2 + 24 = 26$
$AVC(12) < P^*$ hence we continue to operate.
<u>Example:</u> Suppose that the market for rainbow loom bracelets is perfectly competitive. Each firm has a total cost curve given by:
$TC(Q) = 500 + Q + 0.4Q^2$
(a) Derive the short-run supply curve for a single firm. Give quantity supplied as a function of price.
<u>Solution:</u> $MC = 1 + \frac{8}{10}Q$
$\pi_{max} : P = MC \iff P = 1 + \frac{8}{10}Q \iff \boxed{\frac{10}{8}P - \frac{10}{8} = Q, P \geq 1.}$
(b) Suppose there are 60 identical producers with these cost curves. Find the formula for the market supply curve for these producers.
<u>Solution:</u> $Q_m = 60(\frac{5}{4}P - \frac{5}{4}) = 75P - 75$ Notice these set of people enter when

Price is greater than or equal to 1 hence $\boxed{Q_m = \begin{cases} 0 & P < 1, \\ 75P - 75 & P \geq 1. \end{cases}}$ (c) In addition to the identical producers above there is another producer with the following firm supply curve: $P = 10 + \frac{2}{10}Q$ find the formula for market supply when this firm is also in the market.
Notice, the only enter the market when Price is greater than or equal to 10 hence

$$\boxed{Q_m = \begin{cases} 0 & P < 1, \\ 75P - 75 & 1 \leq P \leq 10, \\ \\ 80P - 125 & P \geq 10 \end{cases}}$$

<u>LR: Supply</u>
Firms supply curve is just MC above ATC min since we wont operate if π is strictly less than 0
Note:MC_{SR} might not equal MC_{LR}
Market:
1) Starts out with high price and positive profits
2) Firms enter causing an expansion of market supply.
3) Market Supply expands
4) Equilibrium price falls until Profit is 0.
In the Long Run economic profit is 0.
<u>Definition:</u>
1) Decreasing cost Industries
End up with downward sloping market supply.
Demand is not high enough to cause many firms
e.g. Aircraft manufactures
2) Increasing cost industries
Upward sloping market Supply
Market costs rise when more inputs are purchased
e.g Rubber tyres
Horizontally add upward sloping Q_{firm}
3) Constant Cost Industry
Input prices are constant even as you purchase a lot
Unlimited number of identical firms
e.g. Commodities: Vegetable Oil.
In this case we see thagt since you keep adding firms, your supply curves gets shallower and shallower until it is a horizontal line tangent to the point where ATC is minimized.

Suppose we have some finite number of low cost firms and an infinite number of high cost firms. We see that the market supply will be a piecewise function in long run equilibrium. But we would see that after a certain point, then it would appear that the low cost firms are making profit. However that is not the case because in long run equilibrium $\pi_e = 0$

Definition:

We says the economic rent or $E_{rent} = TR - TC$ is the returns to specialized inputs above what you paid for them.

Example:

Suppose that the market for jet-packs is perfectly competitive. Each firm must higher a manager and all managers are 100 dollars. There is a finite number of managers with extraordinary talent. Firms with extraordinary managers' long run cost curve are given by $TC_E = 100 + 4Q^2$

There is a potentially unlimited supply of managers with average skills. A firm with an average skilled managers has a long run total cost firm given by $TC_A = 100 + 25Q^2$

At long run equilibrium both types of firms are in the market.

(a) What is the long run equilibrium price of Jet packs?

Solution:

$MC_E = 8Q, MC_A = 50Q$

$ATC_E = \frac{100}{Q} + 4Q \implies \frac{d}{dx}ATC_E = \frac{-100}{Q^2} + 4 = 0 \iff Q = 5$

$ATC_A = \frac{100}{Q} + 25Q \implies \frac{d}{dx}ATC_A = \frac{-100}{Q^2} + 25 = 0 \iff Q = 2$

$MC_A = 100 \ P^* = \begin{cases} 8Q & Q < 2, \\ 100 & Q \geq 2. \end{cases}$

Given that both types of firms enter the market we know that $Q \geq 2$ hence $P^* = 100$

(b) When market is in long run equilibrium how much economic profit does a firm with an average managers earn?

Solution:

$\pi_e = 0$

(c) How much economic profit does a firm with an extraordinary managers earn? How much economic rent does a firm with an extraordinary manager make?

Solution:

$\pi_e = 0$

$E_{rent} = TR - TC_E = \frac{25}{2}100 - 100 - 4(\frac{25}{2})^2 = 525$