Convergence of the maximum of a sample from a Uniform distribution

Objective: The primary purpose is to explore the convergence of $X_{(n)}$ for the Uniform distribution as the sample size (n) increases.

Task: Follow the subsequent steps to examine the convergence of $X_{(n)}$:

1. Open the Shiny app given in the URL: https://mateo.shinyapps.io/Conv-Prob/
2. Using the Shiny app, select the Uniform distribution, the sample size (n), the population maximum (γ) and ϵ based on the information given in Table 1.
3. In Table 1, fill the gaps by using the results from the Shiny app in order to infer curves on Figure 1 (convergence quickness of $X_{(n)}$) with their respective colors, which are associated to each distinct value of ϵ.

Table 1. Assessment of the empirical $F_{X_{(n)}}(\gamma - \epsilon)$ as n increases.

			$n = 3$	$n = 44$	$n = 101$	$n = 197$
$\gamma = 8.6$	$\epsilon = 0.1$	$F_{X_{(n)}}(\gamma - \epsilon) = P(\lvert X_{(n)} - \gamma\rvert \geq \epsilon)$	0.96			
	$\epsilon = 0.3$	$F_{X_{(n)}}(\gamma - \epsilon) = P(\lvert X_{(n)} - \gamma\rvert \geq \epsilon)$				0
	$\epsilon = 0.5$	$F_{X_{(n)}}(\gamma - \epsilon) = P(\lvert X_{(n)} - \gamma\rvert \geq \epsilon)$		0.07		

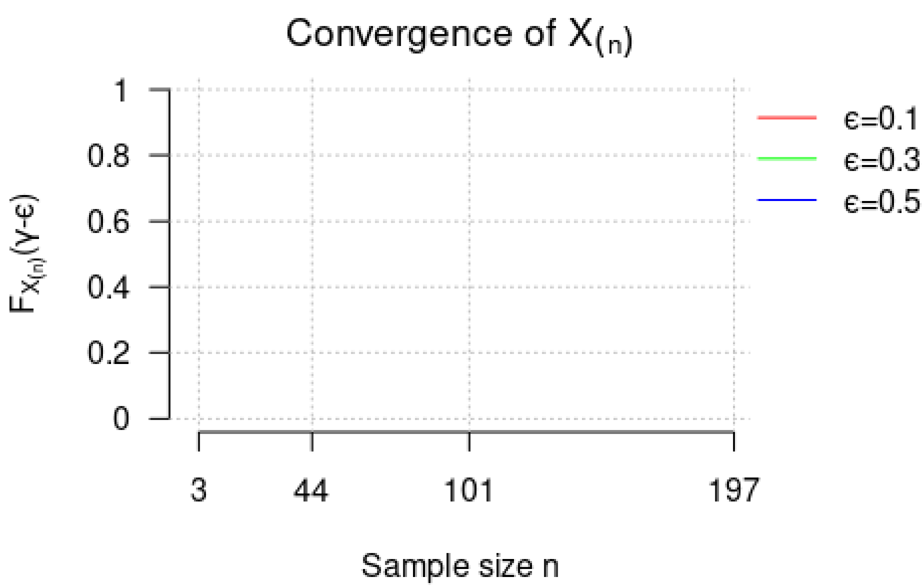

Figure 1. Template to illustrate the convergence swiftness of $X_{(n)}$ for each distinct value of ϵ.

4. In accordance with Table 1 and Figure 1:

- What can be inferred with regard to the pattern observed?

- It can be affirmed that $X_{(n)}$ is close to $\gamma = 8.6$ with high probability (when n is large)?

- What can be concluded about the convergence quickness of $X_{(n)}$ as ϵ rises?

Convergence of the minimum of a sample from a Shifted Exponential distribution

Objective: The primary purpose is to explore the convergence of $X_{(1)}$ for the Shifted Exponential distribution as the sample size (n) increases.

Task: Follow the subsequent steps to examine the convergence of $X_{(1)}$:

1. Open the Shiny app given in the URL: https://mateo.shinyapps.io/Conv-Prob/
2. Using the Shiny app, select the Shifted Exponential distribution, the sample size (n), the population minimum (γ) and ϵ based on the information given in Table 2.
3. In Table 2, fill the gaps by using the results from the Shiny app in order to infer curves on Figure 2 (convergence quickness of $X_{(1)}$) with their respective colors, which are associated to each distinct value of ϵ.

Table 2. Assessment of the empirical $F_{X_{(1)}}(\gamma+\epsilon)$ as n increases.

			$n=2$	$n=25$	$n=115$	$n=185$
$\gamma=4.2$	$\epsilon=0.1$	$F_{X_{(1)}}(\gamma+\epsilon)=P(\lvert X_{(1)}-\gamma\rvert<\epsilon)$			0.99	
	$\epsilon=0.3$	$F_{X_{(1)}}(\gamma+\epsilon)=P(\lvert X_{(1)}-\gamma\rvert<\epsilon)$	0.44			
	$\epsilon=0.5$	$F_{X_{(1)}}(\gamma+\epsilon)=P(\lvert X_{(1)}-\gamma\rvert<\epsilon)$				0.99

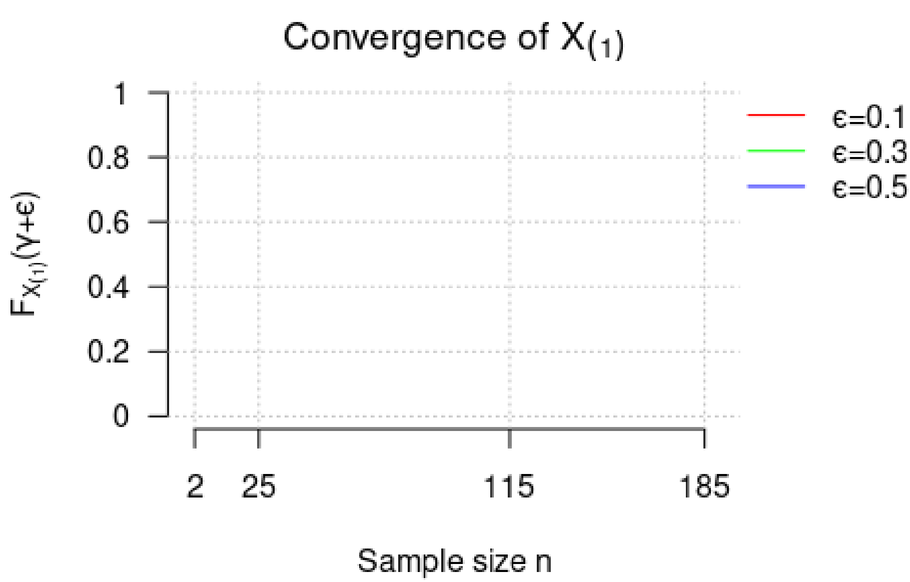

Figure 2. Template to illustrate the convergence swiftness of $X_{(1)}$ for each distinct value of ϵ.

4. In accordance with Table 2 and Figure 2:

- What can be inferred with regard to the pattern observed?

- It can be affirmed that $X_{(1)}$ is close to $\gamma = 4.2$ with high probability (when n is large)?

- What can be concluded about the convergence quickness of $X_{(1)}$ as ϵ rises?