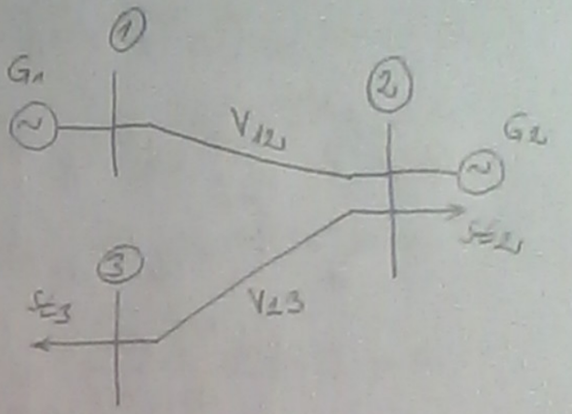
ZI 2014. - AES

① U k+1 iteraciji odredi napone u kV koristeći NR (nadopunu), $S_B = 100$ MVA

$$PQ \rightarrow S_{t2} = 80 + j30 \text{ MVA}$$
$$PQ \rightarrow S_{t3} = 50 + j10 \text{ MVA}$$
$$S_{g2} = 50 + j30 \text{ MVA}$$



$$U_1 = 230 \angle 0^\circ \text{ kV}$$
$$U_2^{(k)} = 221 \angle -3^\circ \text{ kV}$$
$$U_3^{(k)} = 215 \angle -5^\circ \text{ kV}$$

$$Y = \begin{bmatrix} 13{,}13 \angle -84{,}8^\circ & 18{,}26 \angle 95{,}2^\circ & 0 \\ 18{,}26 \angle 95{,}2^\circ & 32{,}80 \angle -84{,}6^\circ & 14{,}61 \angle 95{,}2^\circ \\ 0 & 14{,}61 \angle 95{,}2^\circ & 14{,}61 \angle -84{,}5^\circ \end{bmatrix} \text{ p.u.}$$

1)

$$U_1 = \frac{23}{22} \text{ p.u.}, \quad \delta_1 = 0^\circ = 0 \text{ rad}$$
$$U_2^{(k)} = \frac{221}{220} \text{ p.u.}, \quad \delta_2 = -3^\circ = -3^\circ \cdot \frac{\pi}{180^\circ} \text{ rad} = -\frac{\pi}{60} \text{ rad}$$
$$U_3^{(k)} = \frac{43}{44} \text{ p.u.}, \quad \delta_3 = -5^\circ = -5^\circ \cdot \frac{\pi}{180^\circ} \text{ rad} = -\frac{\pi}{36} \text{ rad}$$

2)

$$S_2 = S_{g2} - S_{t2} = -30 - j10 \text{ MVA} = -0{,}3 - j0{,}1 \text{ p.u.}$$
$$S_3 = S_{g3} - S_{t3} = -50 - j10 \text{ MVA} = -0{,}5 - j0{,}1 \text{ p.u.}$$

$$P_2 = -0{,}3 \text{ p.u.}, \quad Q_2 = -0{,}1 \text{ p.u.}$$
$$P_3 = -0{,}5 \text{ p.u.}, \quad Q_3 = -0{,}1 \text{ p.u.}$$

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3)

$$P_{2rač}^{(k)} = \sum_{j=1}^{3} U_2^{(k)} U_j^{(k)} Y_{2j} \cos\left(\delta_2^{(k)} - \delta_j^{(k)} - \Theta_{2j}\right) =$$
$$= U_2^{(k)} U_1^{(k)} Y_{21} \cos\left(\delta_2^{(k)} - \delta_1^{(k)} - \Theta_{21}\right) +$$
$$\left(U_2^{(k)}\right)^2 Y_{22} \cos\left(-\Theta_{22}\right) + U_2^{(k)} U_3^{(k)} Y_{23} \cos\left(\delta_2^{(k)} - \delta_3^{(k)} - \Theta_{23}\right) =$$
$$= -2{,}73516 + 3{,}11487 - 0{,}80063 = -0{,}42092 \text{ p.u.}$$

$$P_{3\,\text{rač}}^{(k)} = \sum_{j=1}^{3} U_3^{(k)} U_j^{(k)} Y_{3j} \cos\left(\delta_3^{(k)} - \delta_j^{(k)} - \Theta_{3j}\right) =$$

$$= \cancel{U_3^{(k)} U_1^{(k)} Y_{31} \cos\left(\delta_3^{(k)} - \delta_1^{(k)} - \Theta_{31}\right)} +$$

$$U_3^{(k)} U_2^{(k)} Y_{32} \cos\left(\delta_3^{(k)} - \delta_2^{(k)} - \Theta_{32}\right) +$$

$$\left(U_3^{(k)}\right)^2 Y_{33} \cos\left(-\Theta_{33}\right) = -0{,}46025 \text{ p.u.}$$

$$Q_{2\,\text{rač}}^{(k)} = \sum_{j=1}^{3} U_2^{(k)} U_j^{(k)} Y_{2j} \sin\left(\delta_2^{(k)} - \delta_j^{(k)} - \Theta_{2j}\right) =$$

$$= U_2^{(k)} U_1^{(k)} Y_{21} \sin\left(\delta_2^{(k)} - \delta_1^{(k)} - \Theta_{21}\right) +$$

$$\left(U_2^{(k)}\right)^2 Y_{22} \sin\left(\cancel{\delta_2^{(k)} - \delta_2^{(k)}} - \Theta_{22}\right) +$$

$$U_2^{(k)} U_3^{(k)} Y_{23} \sin\left(\delta_2^{(k)} - \delta_3^{(k)} - \Theta_{23}\right) = -0{,}34928 \text{ p.u.}$$

$$Q_{3\,\text{rač}}^{(k)} = \sum_{j=1}^{3} U_3^{(k)} U_j^{(k)} Y_{3j} \sin\left(\delta_3^{(k)} - \delta_j^{(k)} - \Theta_{3j}\right) =$$

$$= \cancel{U_3^{(k)} U_1^{(k)} Y_{31} \sin\left(\delta_3^{(k)} - \delta_1^{(k)} - \Theta_{31}\right)} +$$

$$U_3^{(k)} U_2^{(k)} Y_{32} \sin\left(\delta_3^{(k)} - \delta_2^{(k)} - \Theta_{32}\right) +$$

$$\left(U_3^{(k)}\right)^2 Y_{33} \sin\left(-\Theta_{33}\right) = -0{,}34054 \text{ p.u.}$$

4)

$$J_{1\,2\times2} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} \end{bmatrix}, \qquad J_{4\,2\times2} = \begin{bmatrix} \frac{\partial Q_2}{\partial U_2} & \frac{\partial Q_2}{\partial U_3} \\ \frac{\partial Q_3}{\partial U_2} & \frac{\partial Q_3}{\partial U_3} \end{bmatrix}$$

$$\left(\frac{\partial P_2}{\partial \delta_2}\right)^{(k)} = -U_2^{(k)} U_1^{(k)} Y_{21} \sin\left(\delta_2^{(k)} - \delta_1^{(k)} - \Theta_{21}\right) - U_2^{(k)} U_3^{(k)} Y_{23} \sin\left(\delta_2^{(k)} - \delta_3^{(k)} - \Theta_{23}\right) = 33{,}30120 \text{ p.u.}$$

$$\left(\frac{\partial P_2}{\partial \delta_3}\right)^{(k)} = U_2^{(k)} U_3^{(k)} Y_{23} \sin\left(\delta_2^{(k)} - \delta_3^{(k)} - \Theta_{23}\right) = -14{,}32049 \text{ p.u.}$$

$$\left(\frac{\partial P_3}{\partial \delta_2}\right)^{(k)} = U_3^{(k)} U_2^{(k)} Y_{32} \sin\left(\delta_3^{(k)} - \delta_2^{(k)} - \Theta_{32}\right) = -14{,}22975 \text{ p.u.}$$

$$\left(\frac{\partial P_3}{\partial \delta_3}\right)^{(k)} = -U_3^{(k)} U_2^{(k)} Y_{32} \sin\left(\delta_3^{(k)} - \delta_2^{(k)} - \Theta_{32}\right) = 14{,}22975 \text{ p.u.}$$

$$J_1^{(k)} = \begin{bmatrix} 33,30120 & -14,32049 \\ -14,32975 & 14,32975 \end{bmatrix} \text{ p.u}$$

$$\left(\frac{\partial Q_2}{\partial U_2}\right)^{(k)} = U_1^{(k)} Y_{21} \sin\left(\delta_2^{(k)} - \delta_1^{(k)} - \Theta_{21}\right) + 2U_2^{(k)} Y_{22} \sin\left(-\Theta_{22}\right)$$
$$+ U_3^{(k)} Y_{23} \sin\left(\delta_2^{(k)} - \delta_3^{(k)} - \Theta_{23}\right) = 32,45520 \text{ p.u.}$$

$$\left(\frac{\partial Q_3}{\partial U_2}\right)^{(k)} = U_3^{(k)} Y_{32} \sin\left(\delta_3^{(k)} - \delta_2^{(k)} - \Theta_{32}\right) = -14,16536 \text{ p.u.}$$

$$\left(\frac{\partial Q_2}{\partial U_3}\right)^{(k)} = U_2^{(k)} Y_{23} \sin\left(\delta_2^{(k)} - \delta_3^{(k)} - \Theta_{23}\right) = -14,65352 \text{ p.u.}$$

$$\left(\frac{\partial Q_3}{\partial U_3}\right)^{(k)} = U_2^{(k)} Y_{32} \sin\left(\delta_3^{(k)} - \delta_2^{(k)} - \Theta_{32}\right) + 2U_3^{(k)} Y_{33} \sin\left(-\Theta_{33}\right) =$$
$$= 13,86376 \text{ p.u.}$$

$$J_4^{(k)} = \begin{bmatrix} 32,45520 & -14,65352 \\ -14,16536 & 13,86376 \end{bmatrix} \text{ p.u.}$$

5)

$$\Delta P^{(k)} = \begin{bmatrix} P_2 - P_{2ra\check{c}}^{(k)} \\ P_3 - P_{3ra\check{c}}^{(k)} \end{bmatrix} = \begin{bmatrix} 0,12092 \\ -0,03975 \end{bmatrix} \text{ p.u.}$$

$$\Delta Q^{(k)} = \begin{bmatrix} Q_2 - Q_{2ra\check{c}}^{(k)} \\ Q_3 - Q_{3ra\check{c}}^{(k)} \end{bmatrix} = \begin{bmatrix} 0,24923 \\ 0,24054 \end{bmatrix} \text{ p.u.}$$

$$\Delta \delta^{(k)} = \left(J_1^{(k)}\right)^{-1} \Delta P^{(k)} = \begin{bmatrix} 0,053 & 0,053 \\ 0,053 & 0,123 \end{bmatrix} \begin{bmatrix} 0,121 \\ -0,0397 \end{bmatrix}$$
$$= \begin{bmatrix} 4,31 \cdot 10^{-3} \\ 1,53 \cdot 10^{-3} \end{bmatrix} \text{ rad}$$

$$\delta[rad] = \delta \cdot \frac{180^\circ}{\pi}$$

$$\Delta\delta^{(k)} = \begin{bmatrix} 0{,}246^\circ \\ 0{,}0876^\circ \end{bmatrix}$$

$$\delta[^\circ] = \delta \cdot \frac{\pi}{180^\circ}$$

$$\delta^{(k+1)} = \delta^{(k)} + \Delta\delta^{(k)} = \begin{bmatrix} -3^\circ \\ -5^\circ \end{bmatrix} + \begin{bmatrix} 0{,}246^\circ \\ 0{,}0876^\circ \end{bmatrix} = \begin{bmatrix} -2{,}754^\circ \\ -4{,}9124^\circ \end{bmatrix}$$

$$\Delta U^{(k)} = \left(J_4^{(k)}\right)^{-1} \Delta Q^{(k)} = \begin{bmatrix} 0{,}0571 & 0{,}0604 \\ 0{,}0584 & 0{,}1339 \end{bmatrix} \begin{bmatrix} 0{,}24923 \\ 0{,}24054 \end{bmatrix} =$$

$$= \begin{bmatrix} 0{,}0287 \\ 0{,}0467 \end{bmatrix} \text{ p.u.}$$

$$U^{(k+1)} = U^{(k)} + \Delta U^{(k)} = \begin{bmatrix} \frac{221}{220} \\ \frac{43}{44} \end{bmatrix} + \begin{bmatrix} 0{,}0287 \\ 0{,}0467 \end{bmatrix} = \begin{bmatrix} 1{,}0333 \\ 1{,}0239 \end{bmatrix} kV$$

$$U_2^{(k+1)} = 227{,}326 \text{ kV} = 227{,}326 \angle{-2{,}754^\circ} \text{ kV}$$

$$U_3^{(k+1)} = 225{,}258 \text{ kV} = 225{,}258 \angle{-4{,}9124^\circ} \text{ kV}$$

3.

$U_n = 110\ kV$

$X_1 = 0{,}42\ \frac{\Omega}{km}$

$l_{12} = 50\ km$

$l_{13} = l_{23} = 100\ km = l$

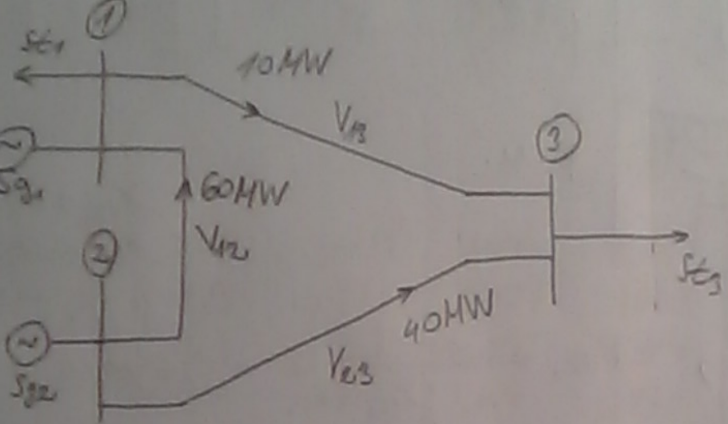
$S_{t1} = 80\ MW$

$S_{t3} = 50\ MW$

$S_{g2} = 100\ MW$

$\left.\begin{matrix} S_{t1} = 80\ MW \\ S_{t3} = 50\ MW \\ S_{g2} = 100\ MW \end{matrix}\right\} 130-100$

a) $S_B = 100\ MVA$ $\qquad S_{g1} = 30\ MW$



$Z_{12} = jX_1 l_{12} \frac{S_B}{U_n^2} = j0{,}17355\ p.u. \quad \Rightarrow \quad Y_{12} = -j5{,}7619\ p.u.$

$Z_{13} = Z_{23} = jX_1 l \frac{S_B}{U_n^2} = 0{,}34711\ p.u. \quad \Rightarrow \quad Y_{13} = Y_{23} = -j2{,}88095\ p.u.$

$$Y = \begin{bmatrix} \cancel{Y_{12}+Y_{13}} & \cancel{-Y_{12}} & \cancel{-Y_{13}} \\ \cancel{-Y_{12}} & Y_{12}+Y_{23} & -Y_{23} \\ \cancel{-Y_{13}} & -Y_{23} & Y_{13}+Y_{23} \end{bmatrix}$$

čv. 1 je ref.
$\delta_1 = 0$

$$Y = \begin{bmatrix} -j8{,}643 & j2{,}881 \\ j2{,}881 & -j5{,}762 \end{bmatrix} p.u.$$

$$Z = Y^{-1} = \begin{bmatrix} j0{,}138 & j0{,}069 \\ j0{,}069 & j0{,}208 \end{bmatrix} p.u.$$

TOKOVI SNAGE U ČVORIŠTIMA

$$P_1 = P_{g1} - P_{t1} = -50\ MW = -0{,}5\ p.u.$$
$$P_2 = P_{g2} - P_{t2} = 100\ MW = 1\ p.u.$$
$$P_3 = P_{g3} - P_{t3} = -50\ MW = -0{,}5\ p.u.$$

$$\sum_i P_i = 0$$

$$\begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = [Z] \begin{bmatrix} P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} j0{,}138 & j0{,}069 \\ j0{,}069 & j0{,}208 \end{bmatrix} \begin{bmatrix} 1 \\ -0{,}5 \end{bmatrix}$$

$$= \begin{bmatrix} +j0{,}1035 \\ -j0{,}035 \end{bmatrix} p.u.$$

$$X_{12} = Z_{12}$$
$$X_{13} = Z_{13} = X_{23}$$

$$P_{i-j} = \frac{\delta_i - \delta_j}{jX_{ij}}, \quad P_{12} = \frac{\delta_1 - \delta_2}{jX_{12}} = -0{,}6\ p.u. = -60\ MW$$

$$P_{13} = 0{,}1\ p.u. = 10\ MW$$

$$P_{2-3} = 0{,}4\ p.u. = 40\ MW$$

c) $S_{g1} = 30\ MW, \quad P_g = 0\ MW$

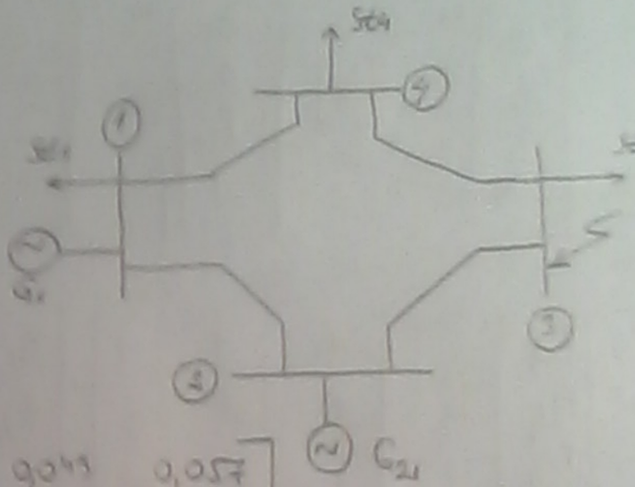
(2)

1KS u čvorištu 3, mreža u P.H.

- odrediti struje kvara u kA i napone po fazama u kV u čvorištu 4

$U_n = 110$ kV

$S_B = 100$ MVA



$$Z_d = Z_i = j\begin{bmatrix} 0,071 & 0,029 & 0,043 & 0,057 \\ 0,029 & 0,071 & 0,057 & 0,043 \\ 0,043 & 0,057 & 0,186 & 0,114 \\ 0,057 & 0,043 & 0,114 & 0,186 \end{bmatrix} \text{ p.u.}$$

$$Z_0 = j\begin{bmatrix} 0,143 & 0,057 & 0,086 & 0,114 \\ 0,057 & 0,143 & 0,114 & 0,086 \\ 0,086 & 0,114 & 0,371 & 0,229 \\ 0,114 & 0,086 & 0,229 & \end{bmatrix} \text{ p.u.}$$

$$I_{3d} = I_{3i} = I_{30} = -\frac{{}^{d}U_3^{pkv}}{Z_{33}^{d} + Z_{33}^{i} + Z_{33}^{0}} = -\frac{1}{j0,186 + j0,186 + j0,371}$$

$$= j1,34589 \text{ p.u.} \quad \text{kapacitivna struja}$$

$$\begin{bmatrix} {}^{d}U_1 \\ {}^{d}U_2 \\ {}^{d}U_3 \\ {}^{d}U_4 \end{bmatrix}^{kv} = \begin{bmatrix} {}^{d}U_1 \\ {}^{d}U_2 \\ {}^{d}U_3 \\ {}^{d}U_4 \end{bmatrix}^{pkv} + \begin{bmatrix} Z_d \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ I_{3d} \\ 0 \end{bmatrix}^{kv} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + j\begin{bmatrix} 0,071 & 0,029 & 0,043 & 0,057 \\ 0,029 & 0,071 & 0,057 & 0,043 \\ 0,043 & 0,057 & 0,186 & 0,114 \\ 0,057 & 0,043 & 0,114 & 0,186 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0,94212 \\ 0,92328 \\ 0,74966 \\ 0,84656 \end{bmatrix} \text{ p.u.}$$

$$\begin{bmatrix} {}^{i}U_1 \\ {}^{i}U_2 \\ {}^{i}U_3 \\ {}^{i}U_4 \end{bmatrix}^{k\nu} = \left[\,{}^{i}\underline{Z}\,\right] \begin{bmatrix} 0 \\ 0 \\ I_{3i} \\ 0 \end{bmatrix}^{k\nu} = \begin{bmatrix} -0{,}05788 \\ -0{,}07672 \\ -0{,}25034 \\ -0{,}15344 \end{bmatrix} \ p.u.$$

$$\begin{bmatrix} {}^{o}U_1 \\ {}^{o}U_2 \\ {}^{o}U_3 \\ {}^{o}U_4 \end{bmatrix}^{k\nu} = \left[\,{}^{o}\underline{Z}\,\right] \begin{bmatrix} 0 \\ 0 \\ I_{3o} \\ 0 \end{bmatrix}^{k\nu} = \begin{bmatrix} -0{,}11574 \\ -0{,}15343 \\ -0{,}49932 \\ -0{,}30820 \end{bmatrix} \ p.u.$$

$$\begin{bmatrix} {}^{R}U_4 \\ {}^{S}U_4 \\ {}^{T}U_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} {}^{o}U_4 \\ {}^{d}U_4 \\ {}^{i}U_4 \end{bmatrix} = \begin{bmatrix} 0{,}38493 \\ -0{,}65476 - j0{,}86602 \\ -0{,}65476 + j0{,}86602 \end{bmatrix} p.u.$$

$$= \begin{bmatrix} 43{,}3412 \angle 0^\circ \\ 119{,}425 \angle -127{,}091^\circ \\ 119{,}425 \angle 127{,}091^\circ \end{bmatrix} kV$$

$$\text{ZA 3pKS} \begin{cases} I_{ks} = -I_3 = -j1{,}34589 \ p.u. \\ I_{ks} = I_{ks}[p.u.] \cdot \dfrac{S_B}{\sqrt{3}\,U_n} = -j0{,}7064 \ kA = 0{,}7064 \angle -90^\circ \ kA \end{cases}$$