ANALIZA ELEKTROENERGETSKOG SUSTAVA

Predavanje br. 8.

- METODA GAUSS-SEIDEL POMOĆU Y MATRICE
  - Mreža od ***n*** čvorišta – jedno čvorište referentno

$$\left|\vec{I}\right| = \left|\vec{Y}\right| \cdot \left|\vec{U}\right|$$

$$\vec{I}_1 = \vec{Y}_{11} \cdot \vec{U}_1 + \vec{Y}_{12} \cdot \vec{U}_2 + \ldots + \vec{Y}_{1n} \cdot \vec{U}_n$$

$$\vec{I}_2 = \vec{Y}_{21} \cdot \vec{U}_1 + \vec{Y}_{22} \cdot \vec{U}_2 + \ldots + \vec{Y}_{2n} \cdot \vec{U}_n$$

$$\vdots$$

$$\vec{I}_{(n-1)} = \vec{Y}_{(n-1)1} \cdot \vec{U}_1 + \vec{Y}_{(n-1)2} \cdot \vec{U}_2 + \ldots + \vec{Y}_{(n-1)n} \cdot \vec{U}_n$$

$$\vec{U}_1 = \frac{1}{\vec{Y}_{11}} \cdot \left[\vec{I}_1 - \vec{Y}_{12} \cdot \vec{U}_2 - \vec{Y}_{13} \cdot \vec{U}_3 - \ldots - \vec{Y}_{1n} \cdot \vec{U}_n \right]$$

$$\vec{U}_2 = \frac{1}{\vec{Y}_{22}} \cdot \left[\vec{I}_2 - \vec{Y}_{21} \cdot \vec{U}_1 - \vec{Y}_{23} \cdot \vec{U}_3 - \ldots - \vec{Y}_{2n} \cdot \vec{U}_n \right]$$

$$\vdots$$

$$\vec{U}_{n-1} = \frac{1}{\vec{Y}_{(n-1)(n-1)}} \cdot \left[\vec{I}_{n-1} - \vec{Y}_{(n-1)1} \cdot \vec{U}_1 - \vec{Y}_{(n-1)3} \cdot \vec{U}_3 - \ldots - \vec{Y}_{(n-1)n} \cdot \vec{U}_n \right]$$

- Za čvorište **i**:

$$\vec{I}_i = \frac{\vec{S}_i^*}{\vec{U}_i^*}$$

$$\vec{U}_i^{(1)} = \frac{1}{\vec{Y}_{ii}} \cdot \left[\frac{\vec{S}_i^*}{\vec{U}_i^{*(0)}} - \vec{Y}_{i1} \cdot \vec{U}_1^{(0)} - \vec{Y}_{i2} \cdot \vec{U}_2^{(0)} - \ldots - \vec{Y}_{in} \cdot \vec{U}_n^{(0)} \right]$$

- Za neku iteraciju **k+1** i čvorište **i**:

$$\vec{U}_i^{(k+1)} = \frac{1}{\vec{Y}_{ii}} \cdot \left[\frac{\vec{S}_i^*}{\vec{U}_i^{*(k)}} - \vec{Y}_{i1} \cdot \vec{U}_1^{(k)} - \vec{Y}_{i2} \cdot \vec{U}_2^{(k)} - \ldots - \vec{Y}_{in} \cdot \vec{U}_n^{(k)} \right]$$

$$\vec{U}_i^{(k+1)} = \frac{\vec{S}_i^*}{\vec{Y}_{ii} \cdot \vec{U}_i^{*(k)}} - \frac{\vec{Y}_{i1}}{\vec{Y}_{ii}} \cdot \vec{U}_1^{(k)} - \ldots - \frac{\vec{Y}_{in}}{\vec{Y}_{ii}} \cdot \vec{U}_n^{(k)}$$

$$\vec{U}_i^{(k+1)} = \frac{\vec{S}_i^*}{\vec{Y}_{ii} \cdot \vec{U}_i^{*(k)}} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\vec{Y}_{ij}}{\vec{Y}_{ii}} \cdot \vec{U}_j^{(k)}$$

$$KL_i = \frac{\vec{S}_i^*}{\vec{Y}_{ii}}$$

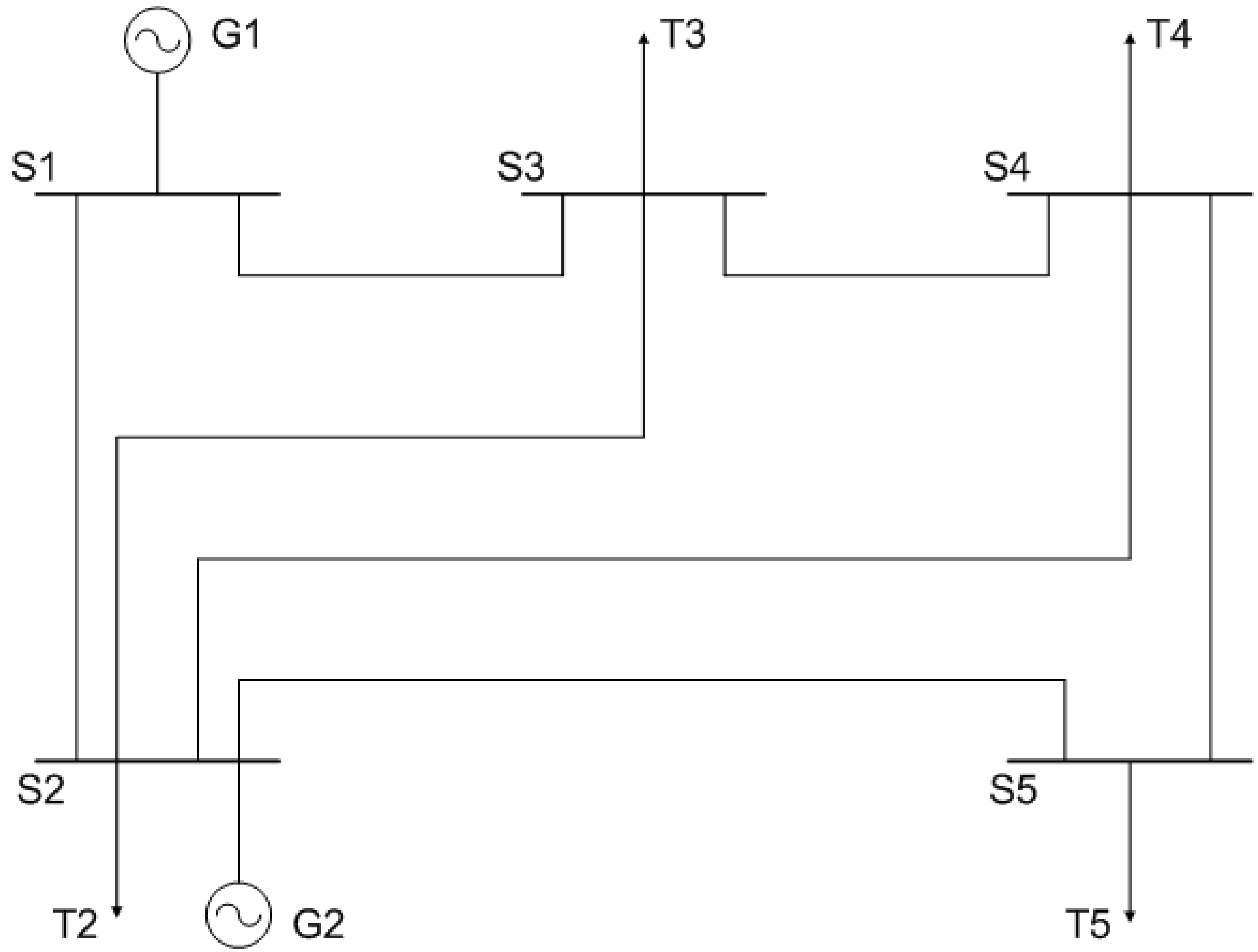
$$YL_{i,j} = \frac{\vec{Y}_{ij}}{\vec{Y}_{ii}}$$

$$\vec{U}_i^{(k+1)} = KL_i \cdot \frac{1}{\vec{U}_i^{*(k)}} - \sum_{j=1}^{i-1} YL_{i,j} \cdot \vec{U}_j^{(k+1)} - \sum_{j=i+1}^{n} YL_{i,j} \cdot \vec{U}_j^{(k)}$$

- Uvjet točnosti:

$$\left| \vec{U}_i^{(k+1)} - \vec{U}_i^{(k)} \right| < \varepsilon$$

- PRIMJER:



- Zadano:

i	j	$\vec{Z}_{i-j}\ (p.u.)$	$y'_{i-j}/2\ (p.u.)$	$\vec{y}_{i-j}$
1	2	0.02+j0.06	j0.03	5-j15
1	3	0.08+j0.24	j0.025	1.25-j3.75
2	3	0.06+j0.18	j0.02	1.66-j5
2	4	0.06+j0.18	j0.02	1.66-j5
2	5	0.04+j0.12	j0.015	2.5-j4.5
3	4	0.01+j0.03	j0.01	10-j30
4	5	0.08+j0.24	j0.025	1.25-j3.75

- Čvorište 1 je referentno

- Zadano:

Čv.	Generator			Teret		$\vec{Y}_i'$
	U	MW	Mvar	MW	Mvar	
1.	1.06+j0	/	/	/	/	j0.055
2.		40	30	20	10	j0.085
3.				45	15	j0.055
4.				40	5	j0.055
5.				60	10	j0.04

- Bazna snaga: $S_B = 100\ MVA$
- Tražena točnost: $\varepsilon = 0.001$

- Matrica **Y:**

$\vec{Y} =$

6.25-j18.695	-5+j15	-1.25+j3.75	0	0
-5+j15	10.833-j32.415	-1.66+j5	-1.66+j5	-2.5+j7.5
-1.25+j3.75	-1.66+j5	12.916-j38.695	-10+j30	0
0	-1.66+j5	-10+j30	12.916-j38.695	-1.25+j3.75
0	-2.5+j7.5	0	-1.25+j3.75	3.75-j11.21

$$\vec{Y}_{ij} = -\vec{y}_{i-j}$$

$$\vec{Y}_{ii} = \sum_{j=1}^{n} \vec{y}_{i-j} + \vec{Y}_i{}'$$

- Nulta iteracija: $\vec{U}_i^{(0)} = 1 + j0 \qquad i = 2, 3, 4, 5$

$$\vec{U}_2^{(k+1)} = \frac{KL_2}{\vec{U}_2^{*(k)}} - YL_{2,1} \cdot \vec{U}_1 - YL_{2,3} \cdot \vec{U}_3^{(k)} - YL_{2,4} \cdot \vec{U}_4^{(k)} - YL_{2,5} \cdot \vec{U}_5^{(k)}$$

$$\vec{U}_3^{(k+1)} = \frac{KL_3}{\vec{U}_3^{*(k)}} - YL_{3,1} \cdot \vec{U}_1 - YL_{3,2} \cdot \vec{U}_2^{(k+1)} - YL_{3,4} \cdot \vec{U}_4^{(k)}$$

$$\vec{U}_4^{(k+1)} = \frac{KL_4}{\vec{U}_4^{*(k)}} - YL_{4,2} \cdot \vec{U}_2^{(k+1)} - YL_{4,3} \cdot \vec{U}_3^{(k+1)} - YL_{4,5} \cdot \vec{U}_5^{(k)}$$

$$\vec{U}_5^{(k+1)} = \frac{KL_5}{\vec{U}_5^{*(k)}} - YL_{5,2} \cdot \vec{U}_2^{(k+1)} - YL_{5,4} \cdot \vec{U}_4^{(k+1)}$$

$$KL_2 = \frac{P_2 - jQ_2}{\vec{Y}_{22}} = \frac{0.2 - j0.2}{10.833 - j32.415} = 0.0074 + j0.0037$$

$$KL_3 = \frac{P_3 - jQ_3}{\vec{Y}_{33}} = -0.00698 - j0.0093$$

$$KL_4 = \frac{P_4 - jQ_4}{\vec{Y}_{44}} = -0.00427 - j0.00891$$

$$KL_5 = \frac{P_5 - jQ_5}{\vec{Y}_{55}} = -0.002413 - j0.004545$$

$$YL_{2,1} = \frac{\vec{Y}_{21}}{\vec{Y}_{22}} = \frac{-5 + j15}{10.833 - j32.415} = -0.46263 + j0.000177$$

$$YL_{2,3} = \frac{\vec{Y}_{23}}{\vec{Y}_{22}} = \frac{-1.66 + j5}{10.833 - j32.415} = -0.15421 + j0.00012$$

$$YL_{2,4} = \frac{\vec{Y}_{24}}{\vec{Y}_{22}} = -0.15421 + j0.00012$$

$$YL_{2,5} = \frac{\vec{Y}_{25}}{\vec{Y}_{22}} = -0.23131 + j0.00018$$

$$YL_{3,1} = -0.0969 + j0.00004$$

$$YL_{3,2} = -0.12920 + j0.00006$$

$$YL_{3,4} = -0.77518 + j0.00033$$

$$\mathrm{YL}_{4,2} = -0.1292 + \mathrm{j}0.00006$$

$$\mathrm{YL}_{4,3} = -0.77518 + \mathrm{j}0.00033$$

$$\mathrm{YL}_{4,5} = -0.0969 + \mathrm{j}0.00004$$

$$\mathrm{YL}_{5,2} = -0.66881 + \mathrm{j}0.00072$$

$$\mathrm{YL}_{5,4} = -0.3344 + \mathrm{j}0.00036$$

$$\vec{U}_2^{(1)} = 1.03752 + j0.0029$$

- Postupak se ubrzava uvođenjem faktora ubrzanja α :

$$\alpha = 1.4$$

$$\Delta\vec{\mathrm{U}}_2^{(1)} = \vec{\mathrm{U}}_2^{(1)} - \vec{\mathrm{U}}_2^{(0)} = 0.03752 + \mathrm{j}0.0029$$

$$\vec{\mathrm{U}}_{2\mathrm{ubrzani}}^{(1)} = \vec{\mathrm{U}}_2^{(0)} + \alpha \cdot \Delta\vec{\mathrm{U}}_2^{(1)} = 1 + \mathrm{j}0 + 0.05253 + \mathrm{j}0.00406$$

$$\vec{\bar{U}}^{(1)}_{2\text{ubrzani}} = \vec{\bar{U}}^{(0)}_{2} + \alpha \cdot \Delta\vec{\bar{U}}^{(1)}_{2} = 1 + j0 + 0.05253 + j0.00406$$

$$\vec{\bar{U}}^{(1)}_{2\text{ubrzani}} = 1.05253 + j0.00406$$

$$\vec{\bar{U}}^{(1)}_{3} = 1.00690 - j0.00921$$

$$\Delta\vec{\bar{U}}^{(1)}_{3} = 0.069 - j0.00021$$

$$\vec{\bar{U}}^{(1)}_{3\text{ubrz}} = \vec{\bar{U}}^{(0)}_{3} + \alpha \cdot \Delta\vec{\bar{U}}^{(1)}_{3} = 1.00966 - j0.01289$$

$$\vec{\bar{U}}^{(1)}_{4\text{ubrz}} = \vec{\bar{U}}^{(0)}_{4} + \alpha \cdot \Delta\vec{\bar{U}}^{(1)}_{4} = 1.01579 - j0.02635$$

$$\vec{\bar{U}}^{(1)}_{5\text{ubrz}} = \vec{\bar{U}}^{(0)}_{5} + \alpha \cdot \Delta\vec{\bar{U}}^{(1)}_{5} = 1.02728 - j0.07374$$

- 2. iteracija

$$\vec{U}_2^{(2)} = \frac{KL_2}{\vec{U}_{2ubrz}^{*(1)}} - YL_{2,1} \cdot \vec{U}_1 - YL_{2,3} \cdot \vec{U}_{3ubrz}^{(1)} - YL_{2,4} \cdot \vec{U}_{4ubrz}^{(1)} - YL_{2,5} \cdot \vec{U}_{5ubrz}^{(1)}$$

$$\Delta\vec{U}_2^{(2)} = \vec{U}_2^{(2)} - \vec{U}_{2ubrzani}^{(1)}$$

$$\vec{U}_{2ubrz}^{(2)} = \vec{U}_{2ubrzani}^{(1)} + \alpha \cdot \Delta\vec{U}_2^{(2)}$$

$$\vec{U}_3^{(2)} = \frac{KL_3}{\vec{U}_{3ubrz}^{*(1)}} - YL_{3,1} \cdot \vec{U}_1 - YL_{3,2} \cdot \vec{U}_{2ubrz}^{(2)} - YL_{3,4} \cdot \vec{U}_{4ubrz}^{(1)}$$

$$\vdots$$

- Rješenje:

Iteracija	Čv. 2	Čv. 3	Čv. 4	Čv. 5
1.	1.05253+j 0.00406	1.00966-j0.01289	1.01579-j0.02635	1.02727-j0.07374
2.	1.04528-j0.03015	1.02154-j0.04227	1.02451-j0.06353	1.01025-j0.08032
⋮	⋮	⋮	⋮	⋮
10.	1.04623-j0.05126	1.02036-j0.08917	1.0192-j0.09504	1.01211-j0.10904

- Tokovi snaga:

$$P_{12} + jQ_{12} = \vec{U}_1 \cdot (\vec{U}_1^* - \vec{U}_2^*) \cdot y_{1-2}^* + \vec{U}_1 \cdot \vec{U}_1^* \cdot \frac{{y'}_{1-2}^*}{2}$$

$$= 88.8 - j8.6\ MVA$$

$$P_{21} + jQ_{21} = \vec{U}_2 \cdot (\vec{U}_2^* - \vec{U}_1^*) \cdot y_{1-2}^* + \vec{U}_2 \cdot \vec{U}_2^* \cdot \frac{{y'}_{1-2}^*}{2}$$

$$= -87.4 + j6.2\ MVA$$

$$\Delta P_{1-2} + j\Delta Q_{1-2} = P_{12} + P_{21} + j(Q_{12} + Q_{21})$$

$$= 1.4\ MW - j2.4\ Mvar$$

- Tokovi snaga:

Vod	P [MW]	Q [Mvar]	ΔP [MW]	ΔQ [Mvar]
1-2	88.8	-8.6	1.4	-2.4
2-1	-87.4	6.2		
1-3	40.7	1.1	1.2	-1.9
3-1	-39.5	-3		
2-3	24.7	3.5	0.4	-3.3
3-2	-24.3	-6.8		
2-4	27.9	3.0	0.4	-2.9
4-2	-27.5	-5.9		
2-5	54.8	7.4	1.1	0.2
5-2	-53.7	-7.2		
3-4	18.9	-5.1	≈ 0	-1.9
4-3	-18.9	3.2		
4-5	6.3	-2.3	≈ 0	-5.1
5-4	-6.3	-2.8		

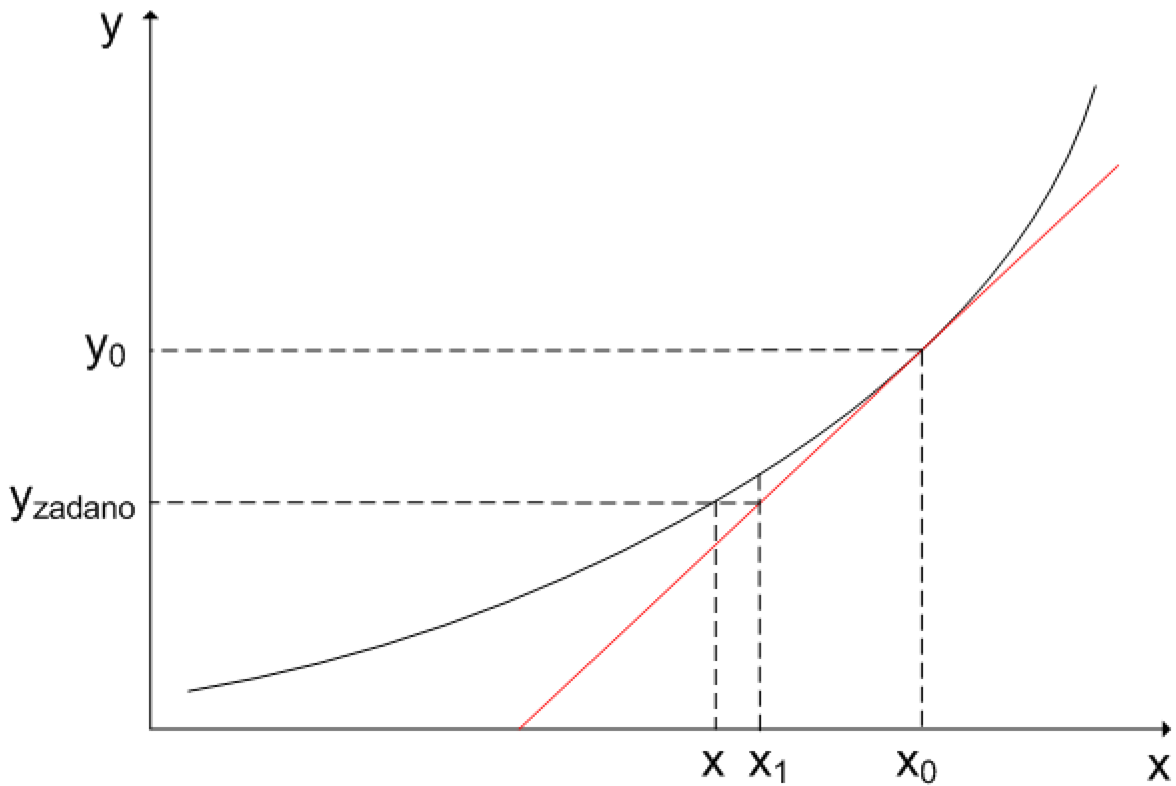
- **METODA NEWTON-RAPHSON**

$$f'(x_0) = \frac{\Delta y}{\Delta x}$$

$$f'(x_0) = \frac{f(x_0) - f(x_1)}{x_0 - x_1}$$

$$f'(x_0) = \frac{y_0 - y_{zadano}}{x_0 - x_1}$$

$$x_1 = x_0 - \frac{y_0 - y_{zadano}}{f'(x_0)}$$



- Približavanje rješenju (x) od nekog početnog, pretpostavljenog rješenja (x_0) pomoću tangenti

- <u>METODA NEWTON-RAPHSON</u>
  - Mreža od ***n*** čvorišta – jedno čvorište referentno (čvorište **n**)
  - Poznate snage u čvorištima:
    - Generatorska i čvorišta tereta

$$P_i = U_i \cdot \sum_{j=1}^{n} U_j \cdot Y_{ij} \cdot cos\left(\delta_i - \delta_j - \Theta_{ij}\right) \qquad i = 1,2,\ldots,n-1$$

    - Čvorišta tereta

$$Q_i = U_i \cdot \sum_{j=1}^{n} U_j \cdot Y_{ij} \cdot sin\left(\delta_i - \delta_j - \Theta_{ij}\right) \qquad i = 1,2,\ldots,n-g-1$$

  - Potrebno izračunati: $U_i \qquad i = 1,2,\ldots,n-g-1$

$$\delta_i \qquad i = 1,2,\ldots,n-1$$

- POSTUPAK PRORAČUNA

1. korak
   - Učitavanje podataka o mreži (konfiguracija, admitancije grana)
   - Učitavanje podataka o injekcijama snage u čvorištima
2. korak
   - Početne vrijednosti napona čvorišta (pretpostavljeno rješenje)

$$\vec{\mathrm{U}}_{\mathrm{i}}^{(0)} = 1 + \mathrm{j}0\ \mathrm{p.u.} = 1\angle 0^\circ\ \mathrm{p.u.}$$

3. korak
   - Formiranje matrice **Y**
4. korak
   - Računanje snaga u čvorištima:

$$P_{ira\check{c}}^{(0)} = \sum_{j=1}^{n} U_i^{(0)} \cdot U_j^{(0)} \cdot Y_{ij} \cdot cos\left(\delta_i^{(0)} - \delta_j^{(0)} - \Theta_{ij}\right) \qquad i = 1, 2, \ldots, n-1$$

$$Q_{ira\check{c}}^{(0)} = \sum_{j=1}^{n} U_i^{(0)} \cdot U_j^{(0)} \cdot Y_{ij} \cdot sin\left(\delta_i^{(0)} - \delta_j^{(0)} - \Theta_{ij}\right) \qquad i = 1, 2, \ldots, n-1-g$$

5. korak

$$\Delta P_i^{(0)} = P_{izad} - P_{irač}^{(0)} \qquad i = 1, 2, \ldots, n-1$$

$$\Delta Q_i^{(0)} = Q_{izad} - Q_{irač}^{(0)} \qquad i = 1, 2, \ldots, n-1-g$$

6. korak
   - Provjera kriterija točnosti:

     $$\Delta P_i^{(0)} < \varepsilon$$

     $$\Delta Q_i^{(0)} < \varepsilon$$

   - Uvjet ispunjen – KRAJ PRORAČUNA
   - Uvjet nije ispunjen – računanje Jakobijeve matrice **J**

7. korak
   - Računanje $\Delta\delta_i^{(0)}, \Delta U_i^{(0)}$ pomoću $\Delta P_i^{(0)}, \Delta Q_i^{(0)}$ i Jakobijeve matrice

Jakobijeva matrica:

$$\begin{vmatrix}\Delta P\\ \Delta Q\end{vmatrix} = |J| \cdot \begin{vmatrix}\Delta\delta\\ \Delta U\end{vmatrix} = \begin{vmatrix}J_1 & J_2\\ J_3 & J_4\end{vmatrix} \cdot \begin{vmatrix}\Delta\delta\\ \Delta U\end{vmatrix} = \begin{vmatrix}\left(\dfrac{\partial P}{\partial \delta}\right) & \left(\dfrac{\partial P}{\partial U}\right)\\ \left(\dfrac{\partial Q}{\partial \delta}\right) & \left(\dfrac{\partial Q}{\partial U}\right)\end{vmatrix} \cdot \begin{vmatrix}\Delta\delta\\ \Delta U\end{vmatrix}$$

$$\begin{vmatrix}\Delta P_1\\ \vdots\\ \Delta P_{n-1}\\ \Delta Q_1\\ \vdots\\ \Delta Q_{n-1-g}\end{vmatrix} = \begin{vmatrix}\dfrac{\partial P_1}{\partial \delta_1} & \cdots & \dfrac{\partial P_1}{\partial \delta_{n-1}} & \dfrac{\partial P_1}{\partial U_1} & \cdots & \dfrac{\partial P_1}{\partial U_{n-1-g}}\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ \dfrac{\partial P_{n-1}}{\partial \delta_1} & \cdots & \dfrac{\partial P_{n-1}}{\partial \delta_{n-1}} & \dfrac{\partial P_{n-1}}{\partial U_1} & \cdots & \dfrac{\partial P_{n-1}}{\partial U_{n-1-g}}\\ \dfrac{\partial Q_1}{\partial \delta_1} & \cdots & \dfrac{\partial Q_1}{\partial \delta_{n-1}} & \dfrac{\partial Q_1}{\partial U_1} & \cdots & \dfrac{\partial Q_1}{\partial U_{n-1-g}}\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ \dfrac{\partial Q_{n-1-g}}{\partial \delta_1} & \cdots & \dfrac{\partial Q_{n-1-g}}{\partial \delta_{n-1}} & \dfrac{\partial Q_{n-1-g}}{\partial U_1} & \cdots & \dfrac{\partial Q_{n-1-g}}{\partial U_{n-1-g}}\end{vmatrix} \cdot \begin{vmatrix}\Delta \delta_1\\ \vdots\\ \Delta \delta_{n-1}\\ \Delta U_1\\ \vdots\\ \Delta U_{n-1-g}\end{vmatrix}$$

- J – Jakobijeva matrica
- J1, J2, J3, J4 – Jakobijeve podmatrice

$$J_1 = \frac{\partial P}{\partial \delta}$$

$$\frac{\partial P_i}{\partial \delta_i} = -U_i \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} U_j \cdot Y_{ij} \cdot sin\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

$$\frac{\partial P_i}{\partial \delta_j} = U_i \cdot U_j \cdot Y_{ij} \cdot sin\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

$$J_2 = \frac{\partial P}{\partial U}$$

$$\frac{\partial P_i}{\partial U_i} = 2 \cdot U_i \cdot Y_{ii} \cdot \cos(-\Theta_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^{n} U_j \cdot Y_{ij} \cdot \cos(\delta_i - \delta_j - \Theta_{ij})$$

$$\frac{\partial P_i}{\partial U_j} = U_i \cdot Y_{ij} \cdot \cos(\delta_i - \delta_j - \Theta_{ij})$$

$$J_3 = \frac{\partial Q}{\partial \delta}$$

$$\frac{\partial Q_i}{\partial \delta_i} = U_i \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} U_j \cdot Y_{ij} \cdot \cos(\delta_i - \delta_j - \Theta_{ij})$$

$$\frac{\partial Q_i}{\partial \delta_j} = -U_i \cdot U_j \cdot Y_{ij} \cdot \cos(\delta_i - \delta_j - \Theta_{ij})$$

$$J_4 = \frac{\partial Q}{\partial U}$$

$$\frac{\partial Q_i}{\partial U_i} = 2 \cdot U_i \cdot Y_{ii} \cdot sin\left(-\Theta_{ii}\right) + \sum_{\substack{j=1 \\ j \neq i}}^{n} U_j \cdot Y_{ij} \cdot sin\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

$$\frac{\partial Q_i}{\partial U_j} = U_i \cdot Y_{ij} \cdot sin\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

$$\begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix} = \begin{vmatrix} J_1 & J_2 \\ J_3 & J_4 \end{vmatrix} \cdot \begin{vmatrix} \Delta \delta \\ \Delta U \end{vmatrix}$$

$$\begin{vmatrix} \Delta \delta \\ \Delta U \end{vmatrix} = \begin{vmatrix} J_1 & J_2 \\ J_3 & J_4 \end{vmatrix}^{-1} \cdot \begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix}$$

- Uz zanemarenje J2 i J3 vrijedi:

$$\begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix} = \begin{vmatrix} J_1 & 0 \\ 0 & J_4 \end{vmatrix} \cdot \begin{vmatrix} \Delta \delta \\ \Delta U \end{vmatrix}$$

$$|\Delta \delta| = |J_1|^{-1} \cdot |\Delta P|$$

$$|\Delta U| = |J_4|^{-1} \cdot |\Delta Q|$$

- Općenito za **k**-tu iteraciju vrijedi:

$$P_{irač}^{(k)} = \sum_{j=1}^{n} U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{ij} \cdot cos\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, ..., n-1$$

$$Q_{irač}^{(k)} = \sum_{j=1}^{n} U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{ij} \cdot sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, ..., n-1-g$$

$$\Delta P_i^{(k)} = P_{izad} - P_{irač}^{(k)} \quad i = 1, 2, ..., n-1$$

$$\Delta Q_i^{(k)} = Q_{izad} - Q_{irač}^{(k)} \quad i = 1, 2, ..., n-1-g$$

- Jakobijeva podmatrica **J1**

$$\left(\frac{\partial P_i}{\partial \delta_i}\right)^{(k)} = -U_i^{(k)} \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} U_j^{(k)} \cdot Y_{ij} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, ..., n-1$$

$$\left(\frac{\partial P_i}{\partial \delta_j}\right)^{(k)} = U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{ij} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, ..., n-1; \; j = 1, 2, ..., n-1$$

- Jakobijeva podmatrica **J4**

$$\left(\frac{\partial Q_i}{\partial U_i}\right)^{(k)} = 2 \cdot U_i^{(k)} \cdot Y_{ii} \cdot \sin\left(-\Theta_{ii}\right) + \sum_{\substack{j=1 \\ j \neq i}}^{n} U_j^{(k)} \cdot Y_{ij} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, \dots, n-1-g$$

$$\left(\frac{\partial Q_i}{\partial U_j}\right)^{(k)} = U_i^{(k)} \cdot Y_{ij} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, \dots, n-1-g \; ; \; j = 1, 2, \dots, n-1-g$$

$$\left|\Delta\delta\right|^{(k)} = \left|J_1^{(k)}\right|^{-1} \cdot \left|\Delta P\right|^{(k)}$$

$$\left|\Delta U\right|^{(k)} = \left|J_4^{(k)}\right|^{-1} \cdot \left|\Delta Q\right|^{(k)}$$

$$U_i^{(k+1)} = U_i^{(k)} + \Delta U_i^{(k)}$$

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)}$$

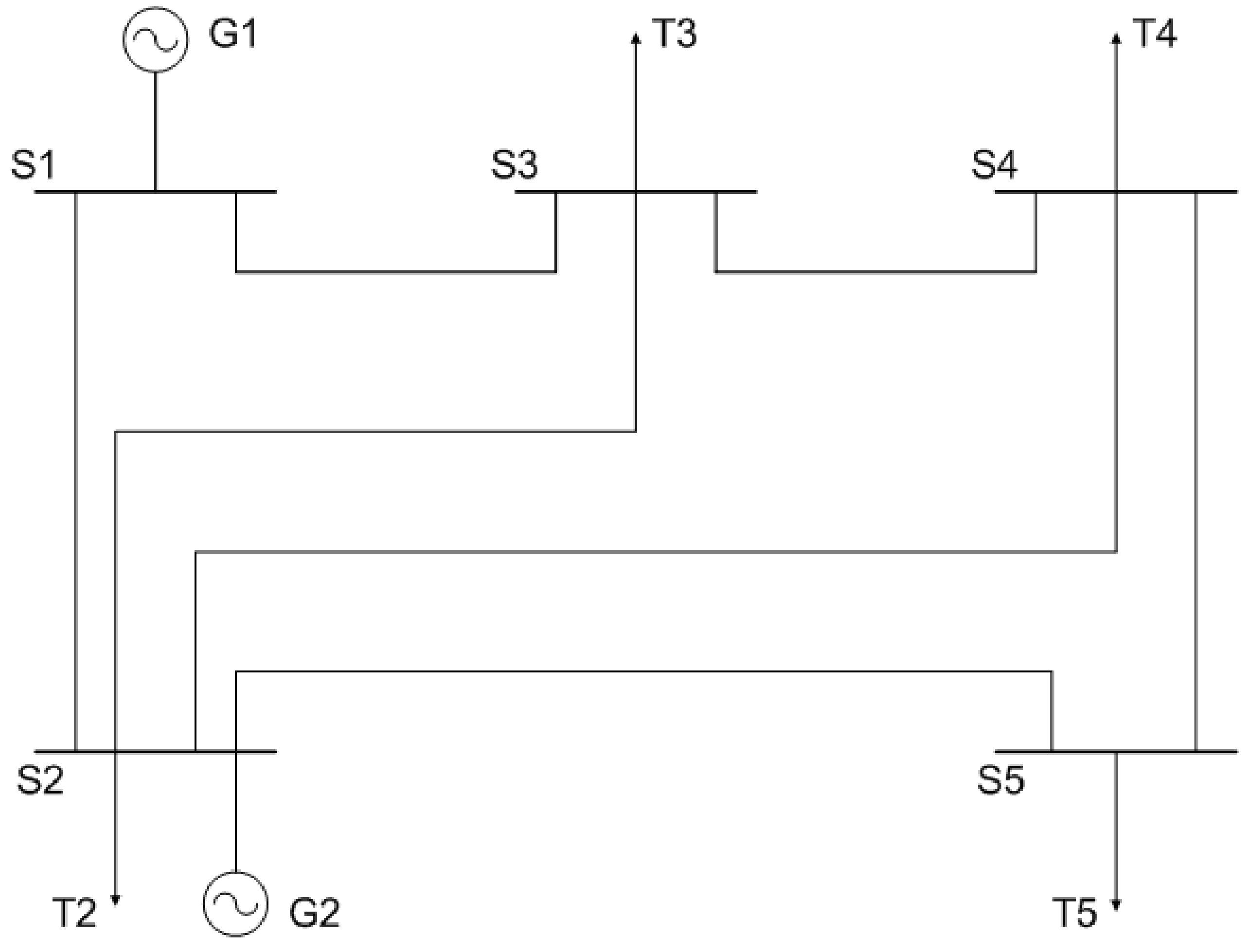
8. korak

$$U_i^{(1)} = U_i^{(0)} + \Delta U_i^{(0)}$$

$$\delta_i^{(1)} = \delta_i^{(0)} + \Delta \delta_i^{(0)}$$

9. korak
   - Obavljanje iteracijskog postupka ponavljanjem koraka 4, 5, 6, 7 i 8 (i korištenjem rezultata iz prethodne iteracije) dok nije ispunjen postavljeni kriterij točnosti

- PRIMJER (prethodni):



- Zadano:

i	j	$\vec{Z}_{i-j}\ (p.u.)$	$y'_{i-j}/2\,(p.u.)$	$\vec{y}_{i-j}$
1	2	0.02+j0.06	j0.03	5-j15
1	3	0.08+j0.24	j0.025	1.25-j3.75
2	3	0.06+j0.18	j0.02	1.66-j5
2	4	0.06+j0.18	j0.02	1.66-j5
2	5	0.04+j0.12	j0.015	2.5-j4.5
3	4	0.01+j0.03	j0.01	10-j30
4	5	0.08+j0.24	j0.025	1.25-j3.75

- Čvorište 1 je referentno

- Zadano:

Čv.	Generator			Teret		$\vec{Y}_i'$
	U	MW	Mvar	MW	Mvar	
1.	1.06+j0	/	/	/	/	j0.055
2.		40	30	20	10	j0.085
3.				45	15	j0.055
4.				40	5	j0.055
5.				60	10	j0.04

- Bazna snaga: $S_B = 100\ MVA$
- Tražena točnost: $\varepsilon = 0.001$

- Matrica **Y:**

$\vec{Y} =$

6.25-j18.695	-5+j15	-1.25+j3.75	0	0
-5+j15	10.833-j32.415	-1.66+j5	-1.66+j5	-2.5+j7.5
-1.25+j3.75	-1.66+j5	12.916-j38.695	-10+j30	0
0	-1.66+j5	-10+j30	12.916-j38.695	-1.25+j3.75
0	-2.5+j7.5	0	-1.25+j3.75	3.75-j11.21

$\vec{Y} =$

19.712 ∠ -71.5145°	15.81 ∠ 108.435	3.95 ∠ 108.435	0	0
15.81 ∠ 108.435	34.18 ∠ -71.52	5.27 ∠ 108.435	5.27 ∠ 108.435	7.9 ∠ 108.435
3.95 ∠ 108.435	5.27 ∠ 108.435	40.79 ∠ -71.54	31.62 ∠ 108.435	0
0	5.27 ∠ 108.435	31.62 ∠ 108.435	40.79 ∠ -71.54	3.95 ∠ 108.435
0	7.9 ∠ 108.435	0	3.95 ∠ 108.435	11.82 ∠ -71.52

- Nulta iteracija: $\vec{U}_i^{(0)} = 1 + j0 \qquad i = 2, 3, 4, 5$

$$P_{2izr}^{(0)} = -0.3 \qquad P_{4izr}^{(0)} = 0$$

$$Q_{2izr}^{(0)} = -0.985 \qquad Q_{4izr}^{(0)} = -0.055$$

$$P_{3izr}^{(0)} = -0.075 \qquad P_{5izr}^{(0)} = 0$$

$$Q_{3izr}^{(0)} = -0.28 \qquad Q_{5izr}^{(0)} = -0.04$$

$$\Delta P_2^{(0)} = P_{2zad} - P_{2izr}^{(0)} = 0.5$$

$$\Delta P_3^{(0)} = -0.375$$

$$\Delta P_4^{(0)} = -0.4$$

$$\Delta P_5^{(0)} = -0.6$$

$$\Delta Q_2^{(0)} = 1.185$$

$$\Delta Q_3^{(0)} = 0.13$$

$$\Delta Q_4^{(0)} = 0.005$$

$$\Delta Q_5^{(0)} = -0.06$$

$$\begin{vmatrix} \Delta P_2^{(0)} \\ \vdots \\ \Delta P_5^{(0)} \\ \Delta Q_2^{(0)} \\ \vdots \\ \Delta Q_5^{(0)} \end{vmatrix} = \begin{vmatrix} J1 & J2 \\ J3 & J4 \end{vmatrix} \cdot \begin{vmatrix} \Delta\delta_2^{(0)} \\ \vdots \\ \Delta\delta_5^{(0)} \\ \Delta U_2^{(0)} \\ \vdots \\ \Delta U_5^{(0)} \end{vmatrix}$$

$$|J| = \left| \begin{array}{cccc|cccc} 33.4 & -5 & -5 & -7.5 & 10.533 & -1.66 & -1.66 & -2.5 \\ -5 & 38.975 & -30 & 0 & -1.66 & 12.84 & -10 & 0 \\ -5 & -30 & 38.75 & -3.75 & -1.66 & -10 & 12.916 & -1.25 \\ -7.5 & 0 & -3.75 & 11.25 & -2.5 & 0 & -1.25 & 3.75 \\ \hline -11.338 & 1.66 & 1.66 & 2.5 & 31.43 & -5 & -5 & -7.5 \\ 1.66 & -12.991 & 10 & 0 & -5 & 38.415 & -30 & 0 \\ 1.66 & 10 & -12.916 & 1.25 & -5 & -30 & 38.64 & -3.75 \\ 2.5 & 0 & 1.25 & -3.75 & -7.5 & 0 & -3.75 & 11.17 \end{array} \right|$$

Napomena: najčešće se zbog lakšeg proračuna zanemaruju podmatrice J2 i J3, te se proračun obavlja samo pomoću podmatrica J1 i J4

$$\Delta\delta_2^{(0)} = -0.05068 \qquad \Delta U_2^{(0)} = 0.05494$$

$$\Delta\delta_3^{(0)} = -0.0911 \qquad \Delta U_3^{(0)} = 0.03134$$

$$\Delta\delta_4^{(0)} = -0.9733 \qquad \Delta U_4^{(0)} = 0.03091$$

$$\Delta\delta_5^{(0)} = -0.11268 \qquad \Delta U_5^{(0)} = 0.026$$

$$\delta_2^{(1)} = \delta_2^{(0)} + \Delta\delta_2^{(0)} = -0.05068\, rad$$

$$\delta_3^{(1)} = -0.0911\, rad$$

$$\delta_4^{(1)} = -0.09733\, rad$$

$$\delta_5^{(1)} = -0.11288\, rad$$

$$U_2^{(1)} = U_2^{(0)} + \Delta U_2^{(0)} = 1.05449$$

$$U_3^{(1)} = 1.03134$$

$$U_4^{(1)} = 1.03091$$

$$U_5^{(1)} = 1.026$$

$$P_{2izr}^{(1)} = U_2^{(1)} \sum_{j=1}^{5} U_j^{(1)} \cdot Y_{ij} \cdot cos\left(\delta_i^{(1)} - \delta_j^{(1)} - \Theta_{ij}\right)$$

$$\vdots$$

$$P_{5izr}^{(1)} = U_5^{(1)} \sum_{j=1}^{5} U_j^{(1)} \cdot Y_{ij} \cdot cos\left(\delta_i^{(1)} - \delta_j^{(1)} - \Theta_{ij}\right)$$

$$Q_{2izr}^{(1)} = U_2^{(1)} \sum_{j=1}^{5} U_j^{(1)} \cdot Y_{ij} \cdot sin\left(\delta_i^{(1)} - \delta_j^{(1)} - \Theta_{ij}\right)$$

$$\vdots$$

$$Q_{5izr}^{(1)} = U_5^{(1)} \sum_{j=1}^{5} U_j^{(1)} \cdot Y_{ij} \cdot sin\left(\delta_i^{(1)} - \delta_j^{(1)} - \Theta_{ij}\right)$$

$$\Delta P_2^{(1)} = P_{2zad} - P_{2izr}^{(1)}$$

$$\vdots$$

$$\Delta P_5^{(1)} = P_{5zad} - P_{5izr}^{(1)}$$

$$\vdots$$

$$\Delta Q_2^{(1)} = Q_{2zad} - Q_{2izr}^{(1)}$$

$$\vdots$$

$$\Delta Q_5^{(1)} = Q_{5zad} - Q_{5izr}^{(1)}$$