

# ANALIZA ELEKTROENERGETSKOG SUSTAVA

Predavanje br. 11.

- Proračun jednopolnog kratkog spoja

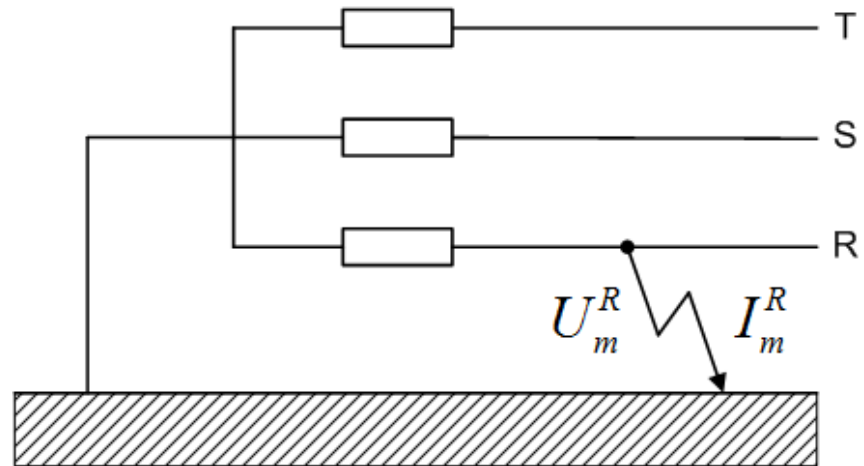
– U bolesnom čvorištu treba postaviti jednadžbe simetričnih komponenata:

$${}^R U_m = 0$$

$${}^R I_m = I_m^d + I_m^i + I_m^0$$

$${}^T I_m = {}^S I_m = 0$$

$$\begin{vmatrix} {}^R U_m \\ {}^S U_m \\ {}^T U_m \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a^2 & a^2 \end{vmatrix} \cdot \begin{vmatrix} U_m^0 \\ U_m^d \\ U_m^i \end{vmatrix}$$



$$U_m^0 + U_m^d + U_m^i = 0$$

$$^S I_m = 0$$

$$^T I_m = 0$$



$$I_m^0 + a^2 \cdot I_m^d + a \cdot I_m^i = 0$$

$$I_m^0 + a \cdot I_m^d + a^2 \cdot I_m^i = 0$$

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$$I_m^0 = I_m^d = I_m^i$$

$$U_m^d = \sum_{j=1}^n I_j \cdot Z_{m,j}^d + Z_{m,m}^d \cdot I_m^0$$

$$U_m^i = Z_{m,m}^i \cdot I_m^0$$

$$U_m^0 = Z_{m,m}^0 \cdot I_m^0$$

$$\sum_{j=1}^n I_j \cdot Z_{m,j}^d = U_m^Z \quad \longrightarrow \quad \text{prije kvara}$$

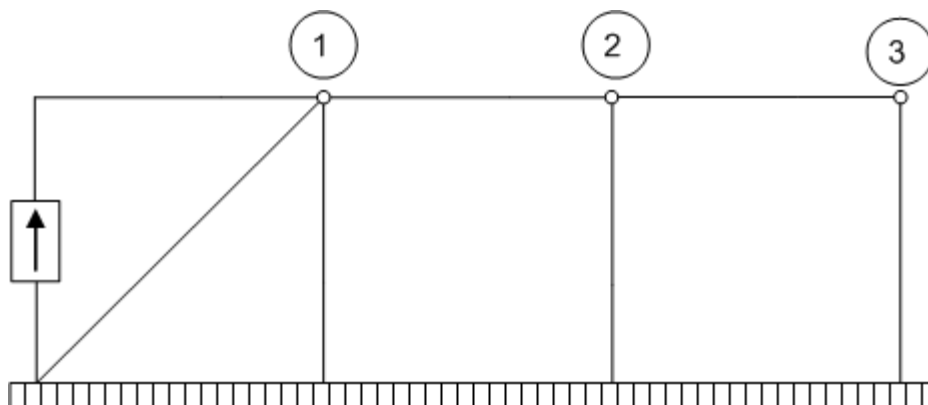
$$U_m^Z + Z_{mm}^d \cdot I_m^0 + Z_{mm}^i \cdot I_m^0 + Z_{mm}^0 \cdot I_m^0 = 0$$

$$I_m^0 = -\frac{U_m^Z}{Z_{mm}^d + Z_{mm}^i + Z_{mm}^0}$$

$$I_{KV} = -3 \cdot I_m^0 = \frac{3 \cdot U_m^Z}{Z_{mm}^d + Z_{mm}^i + Z_{mm}^0} = \frac{3 \cdot U_m^Z}{2 \cdot Z_{mm}^d + Z_{mm}^0}$$

$U_m^Z$  - fazni napon trofazne mreže

- Primjer:



$$Z^d = Z^i = \begin{vmatrix} 0.385 & 0.154 & 0.077 \\ 0.154 & 0.461 & 0.230 \\ 0.077 & 0.230 & 0.615 \end{vmatrix}$$

$$Z^0 = \begin{vmatrix} 0.770 & 0.308 & 0.154 \\ 0.308 & 0.922 & 0.460 \\ 0.154 & 0.460 & 1.230 \end{vmatrix}$$

$$\begin{bmatrix} U_1^d \\ U_2^d \\ U_3^d \end{bmatrix} = Z^d \cdot \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} I_1^d \\ 0 \\ 0 \end{bmatrix} \right\}$$

$$U_1^d = 0.385 \cdot 1 + 0.385 \cdot I_1^d$$

$$U_1^i = 0.385 \cdot I_1^i$$

$$U_1^0 = 0.77 \cdot I_1^0$$

$$I_1^0 = I_1^i = I_1^d = -\frac{0.385}{0.385 + 0.385 + 0.77} = -0.25 \text{ A}$$

$$\begin{bmatrix} U_1^d \\ U_2^d \\ U_3^d \end{bmatrix} = |Z^d| \cdot \begin{bmatrix} 1 - 0.25 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.288 \\ 0.115 \\ 0.058 \end{bmatrix}$$

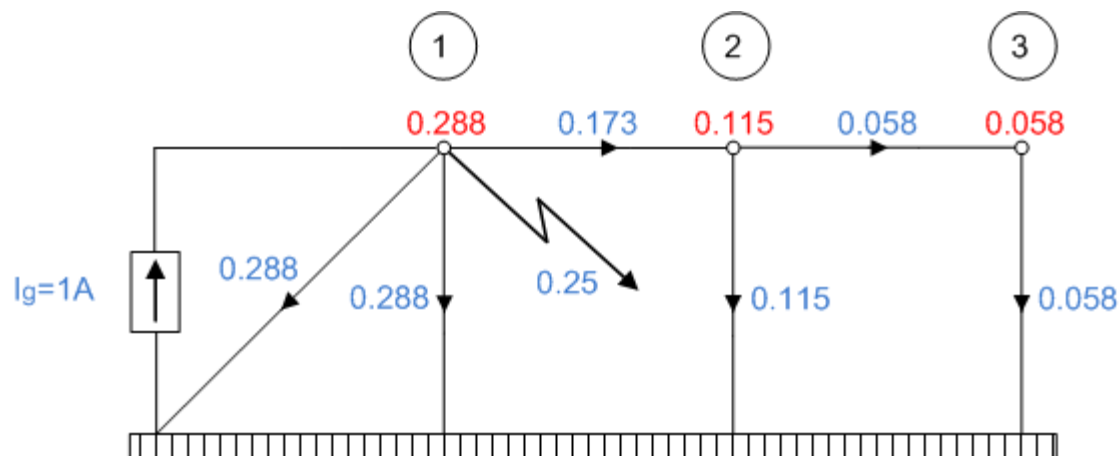
$$\begin{bmatrix} U_1^i \\ U_2^i \\ U_3^i \end{bmatrix} = |Z^i| \cdot \begin{bmatrix} -0.25 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.096 \\ -0.0385 \\ -0.0193 \end{bmatrix}$$

$$\begin{bmatrix} U_1^0 \\ U_2^0 \\ U_3^0 \end{bmatrix} = |Z^0| \cdot \begin{bmatrix} -0.25 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.192 \\ -0.077 \\ -0.039 \end{bmatrix}$$

Struja bolesnog čvorišta u fazi R:

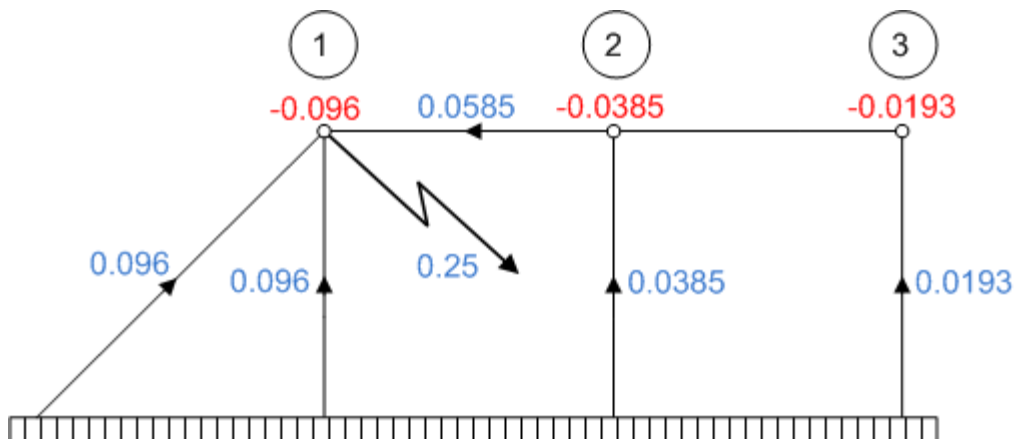
$$I_1^R = I^0 + I^d + I^i = 3 \cdot (-0.25) = -0.75$$

# ANALIZA ELEKTROENERGETSKOG SUSTAVA – predavanje br. 11



Direktna mreža

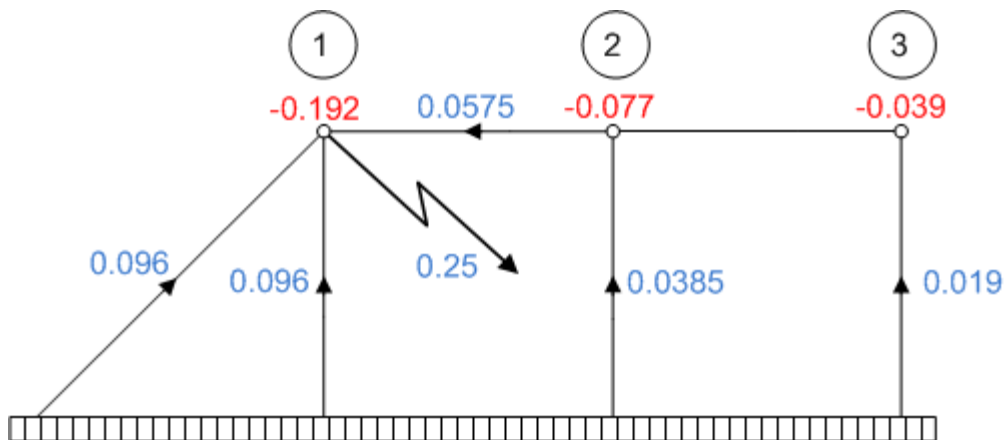
$$I_g^d = 0.712 A$$



Inverzna mreža



# ANALIZA ELEKTROENERGETSKOG SUSTAVA – predavanje br. 11

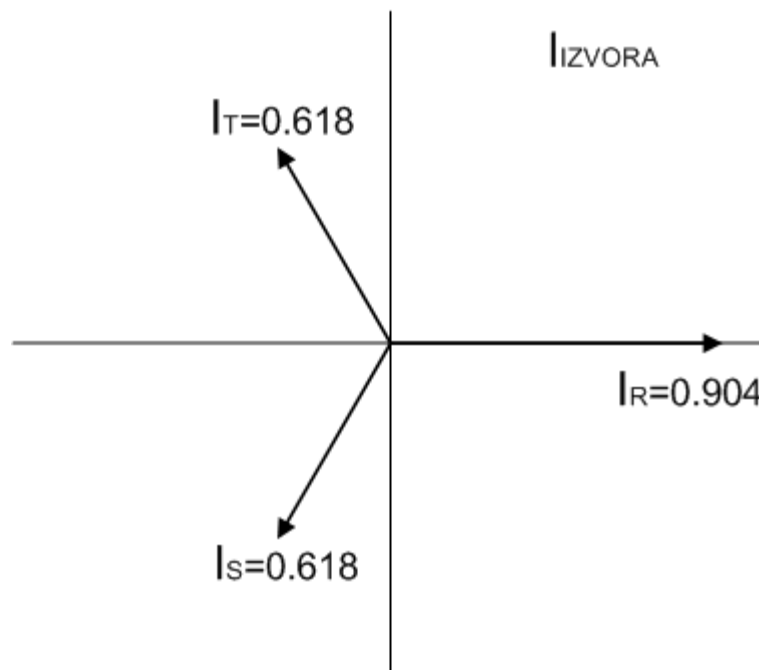


nulta mreža

$$\begin{bmatrix} U_1^R \\ U_1^S \\ U_1^T \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} -0.192 \\ 0.288 \\ -0.096 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.298 - j0.316 \\ -0.298 + j0.316 \end{bmatrix}$$

$$\begin{bmatrix} I_1^R \\ I_1^S \\ I_1^T \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} -0.25 \\ -0.25 \\ -0.25 \end{bmatrix} = \begin{bmatrix} -0.75 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I_{IZV}^R \\ I_{IZV}^S \\ I_{IZV}^T \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} 0.096 \\ 0.712 \\ 0.096 \end{bmatrix} = \begin{bmatrix} 0.904 \\ -0.312 - j0.534 \\ -0.312 + j0.534 \end{bmatrix}$$



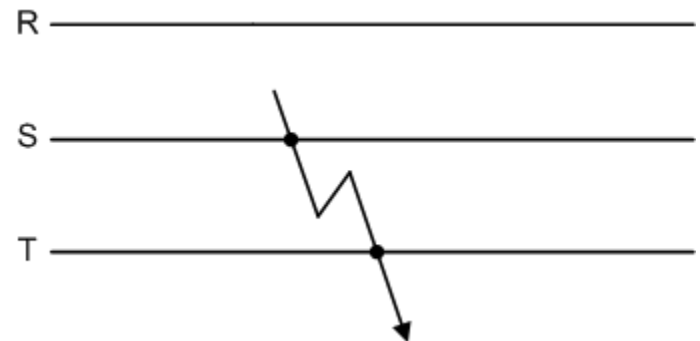
- Proračun dvopolnog kratkog spoja

$$\begin{bmatrix} U_1 \\ \vdots \\ U_m \\ \vdots \\ U_n \end{bmatrix}^d = |Z^d| \cdot \left\{ \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}^d + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_m^d \\ \vdots \\ 0 \end{bmatrix} \right\} \quad ; \quad \begin{bmatrix} U_1 \\ \vdots \\ U_m \\ \vdots \\ U_n \end{bmatrix}^i = |Z^i| \cdot \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_m^i \\ \vdots \\ 0 \end{bmatrix}$$

– Na mjestu kvara:

$$U_S = U_T$$

$$I_S = -I_T$$



$$U_S = U^0 + a^2 \cdot U^d + a \cdot U^i$$

$$U_T = U^0 + a \cdot U^d + a^2 \cdot U^i$$

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$$U^0 + a^2 \cdot U^d + a \cdot U^i = U^0 + a \cdot U^d + a^2 \cdot U^i$$

$$(a^2 - a) \cdot U^d = (a^2 - a) \cdot U^i$$

$$U^d = U^i$$

- Analogno:  $I^d = -I^i$
- Uvrstimo sada iz matričnih jednačbi u ove izraze:

$$U_m^d = Z U_m^d + Z_{mm}^d \cdot I_m^d$$

$$U_m^i = Z_{mm}^i \cdot I_m^i$$

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$$Z U_m^d + Z_{mm}^d \cdot I_m^d = Z_{mm}^i \cdot I_m^i = -Z_{mm} \cdot I_m^d$$

$$I_m^d = -\frac{Z U_m^d}{Z_{mm}^d + Z_{mm}^i}$$

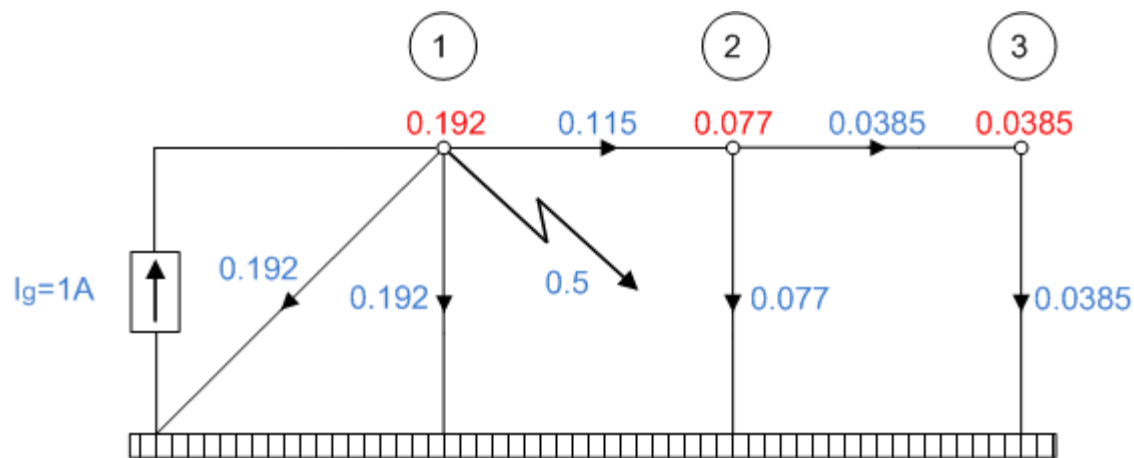
$$I_m^d = -\frac{0.385}{0.385 + 0.385} = -0.5 \text{ A}$$

$$I_m^i = 0.5 \text{ A}$$

$$\begin{vmatrix} U_1^d \\ U_2^d \\ U_3^d \end{vmatrix}^B = \begin{vmatrix} 0.385 \\ 0.154 \\ 0.077 \end{vmatrix}^Z - \begin{vmatrix} 0.192 \\ 0.077 \\ 0.0385 \end{vmatrix} = \begin{vmatrix} 0.192 \\ 0.077 \\ 0.0385 \end{vmatrix}$$

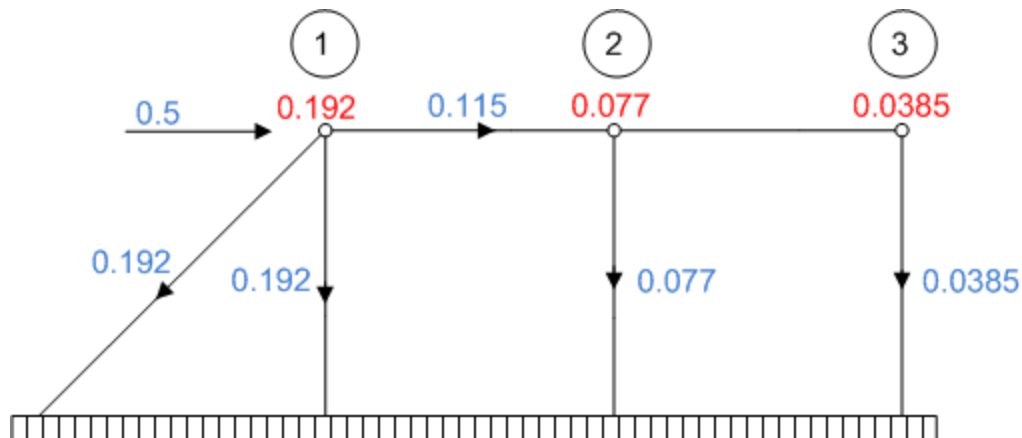
$$\begin{vmatrix} U_1^i \\ U_2^i \\ U_3^i \end{vmatrix} = |Z^i| \cdot \begin{vmatrix} 0.5 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0.192 \\ 0.077 \\ 0.0385 \end{vmatrix}$$

# ANALIZA ELEKTROENERGETSKOG SUSTAVA – predavanje br. 11



Direktna mreža

$$I_g^d = 1 A - 0.192 A = 0.808 A$$



Inverzna mreža

– Iz izvora po fazama R, S i T:

$$^R I_{IZV} = I_{IZV}^d + I_{IZV}^i = 0.808 - 0.192 = 0.616 \text{ A}$$

$$\begin{aligned} ^S I_{IZV} &= a^2 \cdot I_{IZV}^d + a \cdot I_{IZV}^i = \\ &= (-0.5 - j0.866) \cdot 0.808 - (-0.5 + j0.866) \cdot 0.192 = \\ &= -0.308 - j0.866 \text{ A} \end{aligned}$$

$$\begin{aligned} ^T I_{IZV} &= (-0.5 + j0.866) \cdot 0.808 - (-0.5 - j0.866) \cdot 0.192 = \\ &= -0.308 + j0.866 \text{ A} \end{aligned}$$

– Na mjestu kvara (poprečne struje):

$$^R I_m = I_{IZV}^d + I_{IZV}^i = 0 \text{ A}$$

$$^S I_m = a^2 \cdot I_m^d + a \cdot I_m^i = (a^2 - a) \cdot I_m^d = (-j\sqrt{3}) \cdot (-0.5) = j0.866 \text{ A}$$

$$^T I_m = a \cdot I_m^d + a^2 \cdot I_m^i = (a - a^2) \cdot I_m^d = -j0.866 \text{ A}$$

– U grani 1-2:

$$I_{1-2}^d = 0.115 = I_{1-2}^i$$

$$^R I_{1-2} = 0.23 \text{ A}$$

$$^S I_{1-2} = a^2 \cdot I_{1-2}^d + a \cdot I_{1-2}^i = (a^2 + a) \cdot I_{1-2}^d = -I_{1-2}^d = -0.115 \text{ A}$$

$$^T I_{1-2} = a \cdot I_{1-2}^d + a^2 \cdot I_{1-2}^i = (a + a^2) \cdot I_{1-2}^d = -0.115 \text{ A}$$

– U grani 1-0:

$$^R I_{1-0} = I_{1-0}^d + I_{1-0}^i = 0.192 + 0.192 = 0.385 \text{ A}$$

$$^S I_{1-0} = a^2 \cdot I_{1-0}^d + a \cdot I_{1-0}^i = (a^2 + a) \cdot I_{1-0}^d = -0.192 \text{ A}$$

$$^T I_{1-0} = a \cdot I_{1-0}^d + a^2 \cdot I_{1-0}^i = (a + a^2) \cdot I_{1-0}^d = -0.192 \text{ A}$$

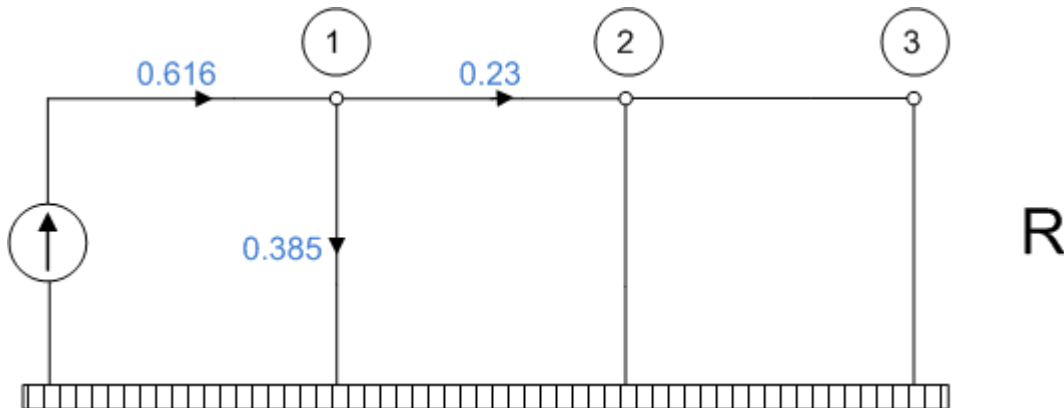


– U čvorištu 1:

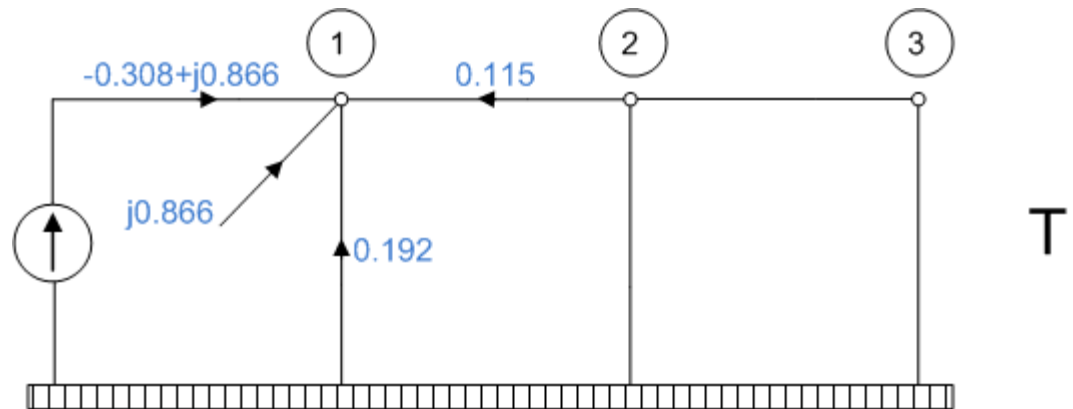
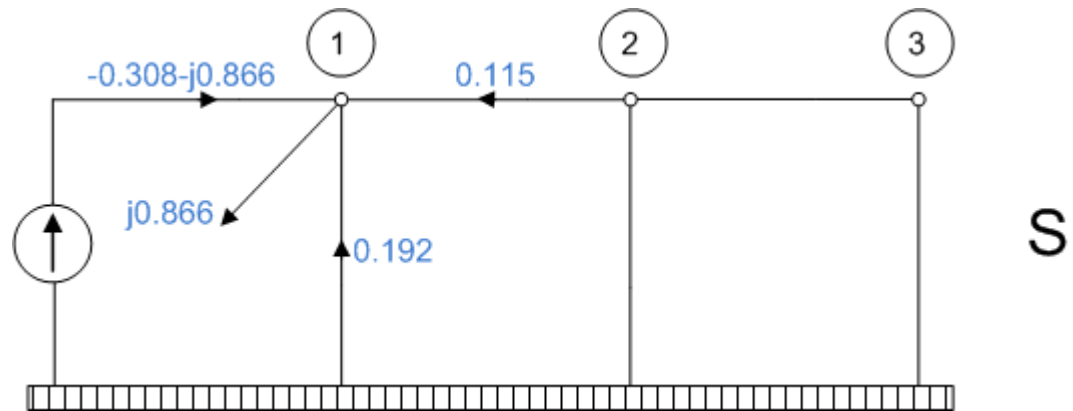
$$^R U_1 = U_1^d + U_1^i = 0.192 + 0.192 = 0.385 \text{ V}$$

$$^S U_1 = a^2 \cdot U_1^d + a \cdot U_1^i = -0.192 \text{ V}$$

$$^T U_1 = a \cdot U_1^d + a^2 \cdot U_1^i = -0.192 \text{ V}$$



# ANALIZA ELEKTROENERGETSKOG SUSTAVA – predavanje br. 11



- Proračun dvopolnog kratkog spoja s zemljom

$$U^d = Z U^d + Z^d \cdot I_m^d$$

$$U^i = Z^i \cdot I_m^i$$

$$U^0 = Z^0 \cdot I_m^0$$

$${}^s U = {}^T U = 0$$

$${}^R I = 0$$

$$U^0 + a^2 \cdot U^d + a \cdot U^i = U^0 + a \cdot U^d + a^2 \cdot U^i$$

$$U^0 = U^d = U^i$$

$${}^R I = I^0 + I^d + I^i = 0$$

$$I^i = -I^d - I^0$$

$$^ZU_m^d + Z_{mm}^d \cdot I_m^d = Z_{mm}^i \cdot I_m^i = Z_{mm}^0 \cdot I_m^0$$

$$^ZU_m^d + Z_{mm}^d \cdot I_m^d = Z_{mm}^i \cdot I_m^i \longrightarrow I_m^i = \frac{1}{Z_{mm}^i} \left( ^ZU_m^d + Z_{mm}^d \cdot I_m^d \right)$$

$$^ZU_m^d + Z_{mm}^d \cdot I_m^d = Z_{mm}^0 \cdot I_m^0$$

$$Z_{mm}^i \cdot I_m^i = Z_{mm}^0 \cdot I_m^0 \longrightarrow I_m^0 = \frac{Z_{mm}^i}{Z_{mm}^0} \cdot I_m^i$$

$$^ZU_m^d + Z_{mm}^d \cdot I_m^d = Z_{mm}^i \cdot (-I_m^d - I_m^0)$$

$$^ZU_m^d + Z_{mm}^d \cdot I_m^d + Z_{mm}^i \cdot I_m^d + \frac{Z_{mm}^i}{Z_{mm}^0} \cdot \left( ^ZU_m^d + Z_{mm}^d \cdot I_m^d \right) = 0$$

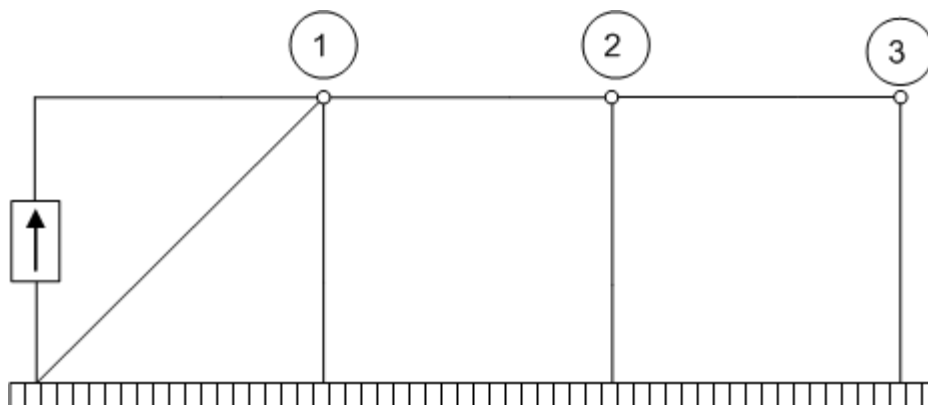
$$^ZU_m^d \cdot \left( 1 + \frac{Z_{mm}^i}{Z_{mm}^0} \right) + \left( Z_{mm}^d + Z_{mm}^i + \frac{Z_{mm}^d \cdot Z_{mm}^i}{Z_{mm}^0} \right) \cdot I_m^d = 0$$

$$I_m^d = -Z U_m^d \cdot \frac{Z_{mm}^0 + Z_{mm}^i}{Z_{mm}^d \cdot Z_{mm}^0 + Z_{mm}^i \cdot Z_{mm}^0 + Z_{mm}^d \cdot Z_{mm}^i}$$

$$\begin{aligned} I_m^i &= \frac{Z U_m^d}{Z_{mm}^i} - Z U_m^d \cdot \frac{Z_{mm}^d}{Z_{mm}^i} \cdot \frac{Z_{mm}^0 + Z_{mm}^i}{Z_{mm}^d \cdot Z_{mm}^0 + Z_{mm}^i \cdot Z_{mm}^0 + Z_{mm}^d \cdot Z_{mm}^i} \\ &= Z U_m^d \cdot \left( \frac{Z_{mm}^d \cdot Z_{mm}^0 + Z_{mm}^i \cdot Z_{mm}^0 + Z_{mm}^d \cdot Z_{mm}^i - Z_{mm}^d \cdot Z_{mm}^0 - Z_{mm}^d \cdot Z_{mm}^i}{Z_{mm}^i \cdot (Z_{mm}^d \cdot Z_{mm}^0 + Z_{mm}^i \cdot Z_{mm}^0 + Z_{mm}^d \cdot Z_{mm}^i)} \right) \\ &= Z U_m^d \cdot \left( \frac{Z_{mm}^0}{Z_{mm}^d \cdot Z_{mm}^0 + Z_{mm}^i \cdot Z_{mm}^0 + Z_{mm}^d \cdot Z_{mm}^i} \right) \end{aligned}$$

$$I_m^0 = Z U_m^d \cdot \left( \frac{Z_{mm}^i}{Z_{mm}^d \cdot Z_{mm}^0 + Z_{mm}^i \cdot Z_{mm}^0 + Z_{mm}^d \cdot Z_{mm}^i} \right)$$

- Primjer:



$$Z^d = Z^i = \begin{vmatrix} 0.385 & 0.154 & 0.077 \\ 0.154 & 0.461 & 0.230 \\ 0.077 & 0.230 & 0.615 \end{vmatrix}$$

$$Z^0 = \begin{vmatrix} 0.770 & 0.308 & 0.154 \\ 0.308 & 0.922 & 0.460 \\ 0.154 & 0.460 & 1.230 \end{vmatrix}$$

$$I_m^d = -0.385 \cdot \frac{0.77 + 0.385}{0.385 \cdot 0.77 + 0.385 \cdot 0.77 + 0.385 \cdot 0.385} = -0.60 \text{ A}$$

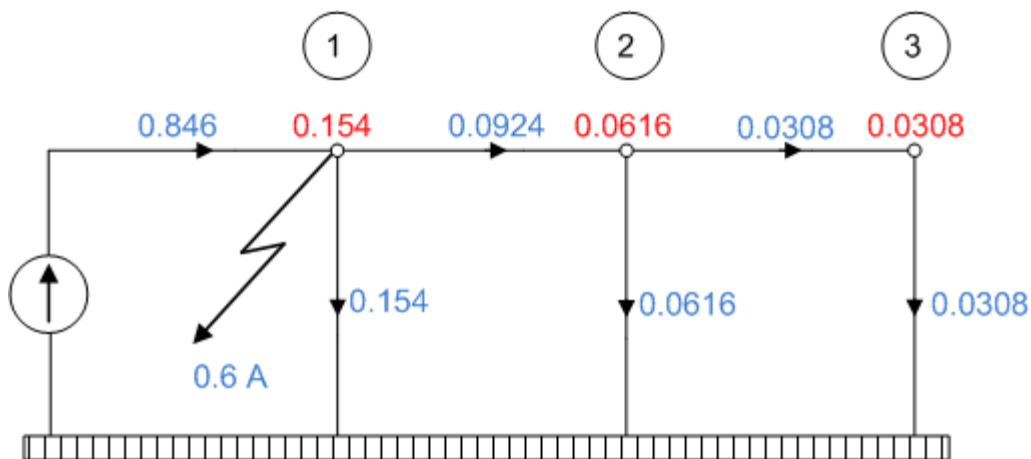
$$I_m^i = 0.385 \cdot \frac{0.77}{0.385 \cdot 1.925} = 0.4 \text{ A}$$

$$I_m^0 = 0.385 \cdot \frac{0.385}{0.385 \cdot 1.925} = 0.2 \text{ A}$$

$$\begin{bmatrix} U_1^d \\ U_2^d \\ U_3^d \end{bmatrix}^B = \begin{bmatrix} U_1^d \\ U_2^d \\ U_3^d \end{bmatrix}^Z + |Z^d| \cdot \begin{bmatrix} I_m^d \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.385 \\ 0.154 \\ 0.077 \end{bmatrix} - \begin{bmatrix} 0.385 \cdot 0.6 \\ 0.154 \cdot 0.6 \\ 0.077 \cdot 0.6 \end{bmatrix} = \begin{bmatrix} 0.154 \\ 0.0616 \\ 0.0308 \end{bmatrix}$$

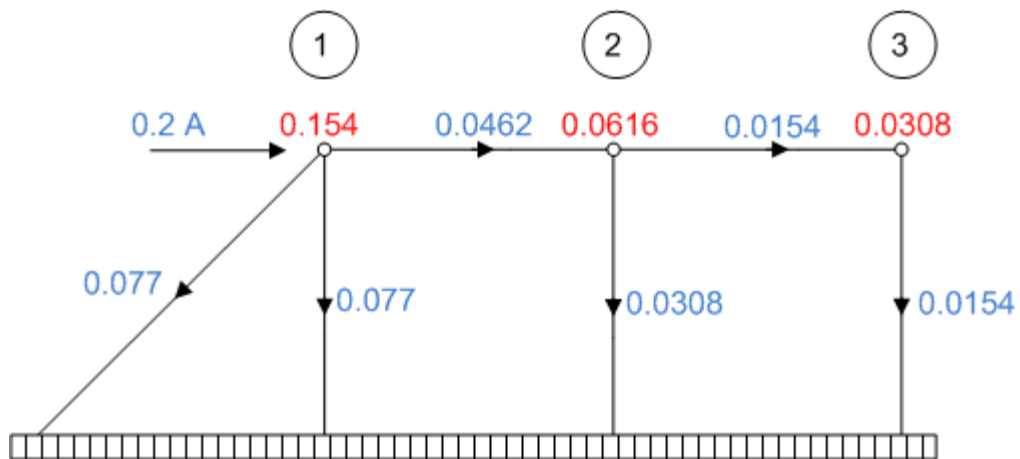
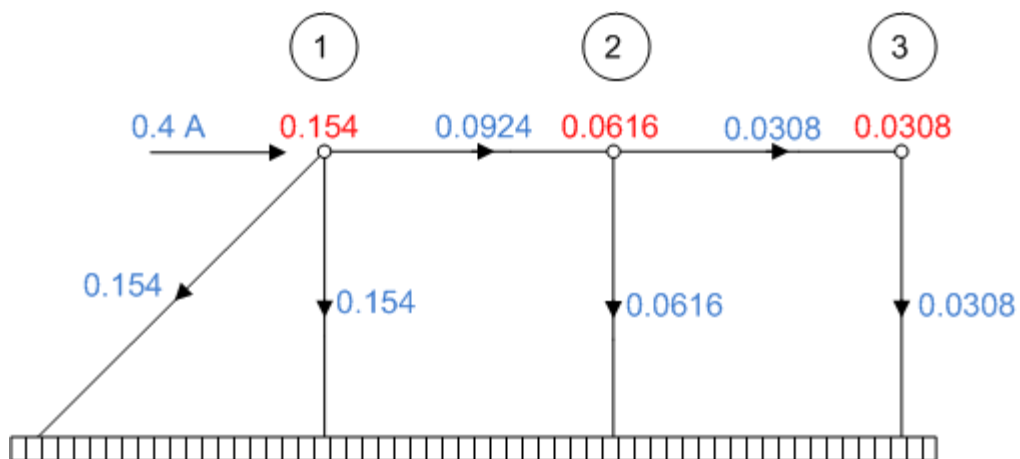
$$\begin{bmatrix} U_1^i \\ U_2^i \\ U_3^i \end{bmatrix}^B = |Z^i| \cdot \begin{bmatrix} I_m^i \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.385 \cdot 0.4 \\ 0.154 \cdot 0.4 \\ 0.077 \cdot 0.4 \end{bmatrix} = \begin{bmatrix} 0.154 \\ 0.0616 \\ 0.0308 \end{bmatrix}$$

$$\begin{bmatrix} U_1^0 \\ U_2^0 \\ U_3^0 \end{bmatrix}^B = \begin{bmatrix} Z^0 \end{bmatrix} \cdot \begin{bmatrix} I_m^0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.77 \cdot 0.2 \\ 0.308 \cdot 0.2 \\ 0.154 \cdot 0.2 \end{bmatrix} = \begin{bmatrix} 0.154 \\ 0.0616 \\ 0.0308 \end{bmatrix}$$





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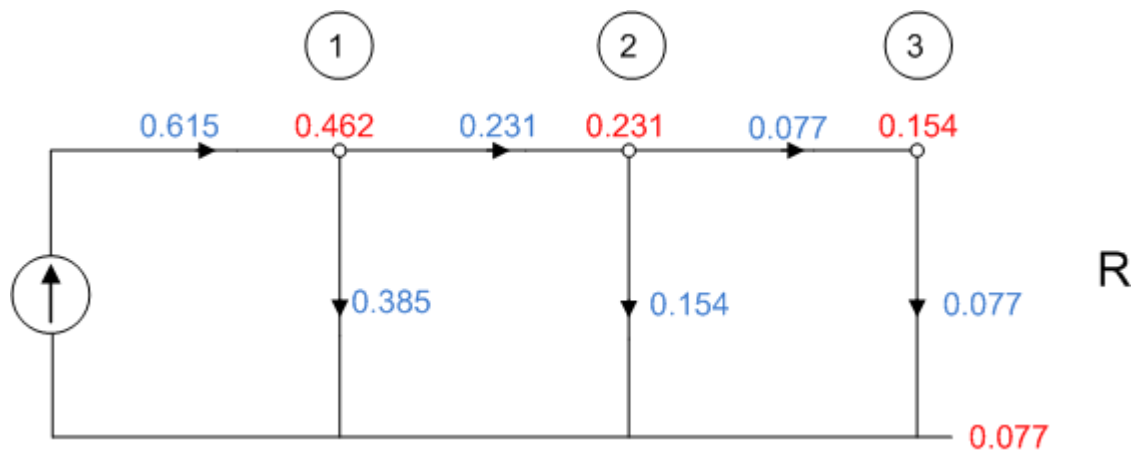
$$\begin{vmatrix} {}^R U_1 \\ {}^S U_1 \\ {}^T U_1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} U_1^0 \\ U_1^d \\ U_1^i \end{vmatrix} =$$

$$= \begin{vmatrix} 0.154 + 0.154 + 0.154 \\ 0.154 + (-0.77 - j0.133) + (-0.77 + j0.133) \\ 0.154 + (-0.77 + j0.133) + (-0.77 - j0.133) \end{vmatrix} = \begin{vmatrix} 0.462 \\ 0 \\ 0 \end{vmatrix}$$

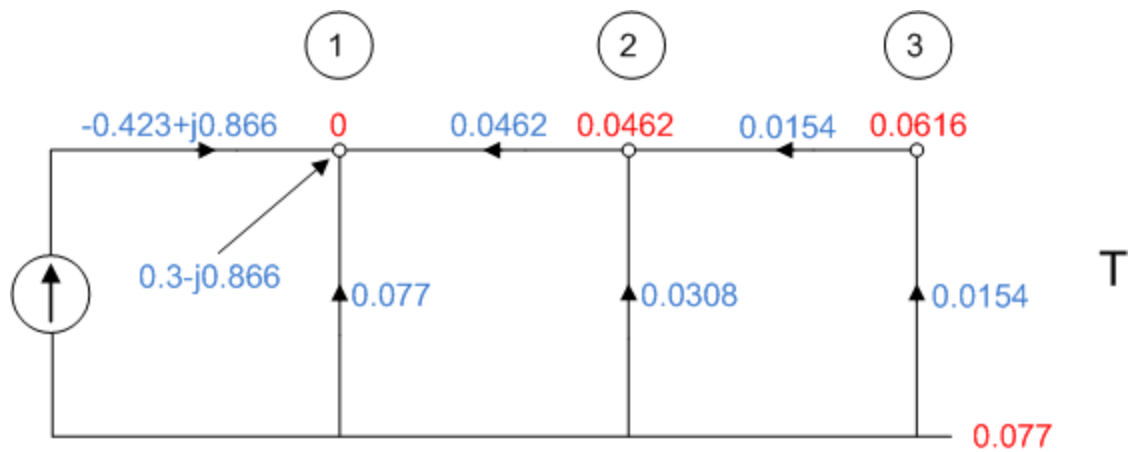
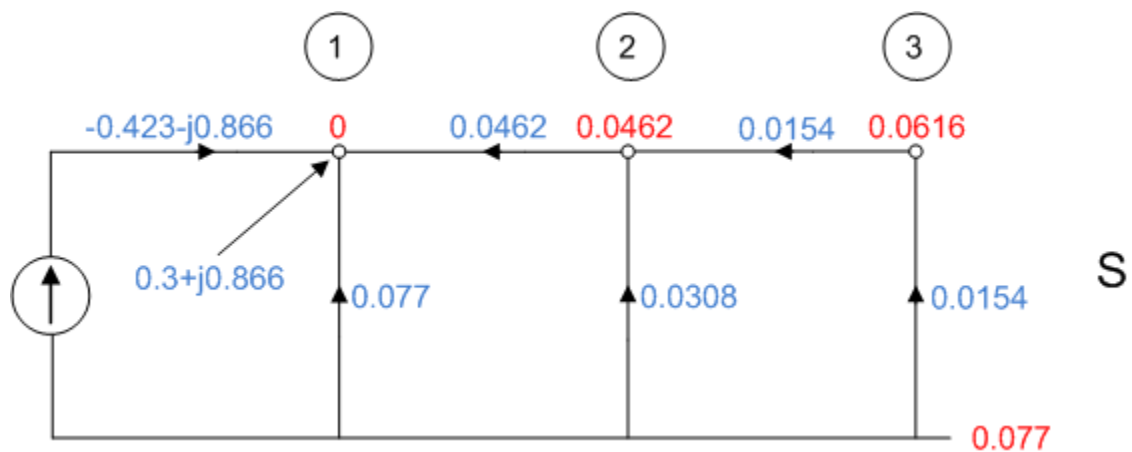
$$\begin{vmatrix} {}^R I_1 \\ {}^S I_1 \\ {}^T I_1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.2 \\ -0.6 \\ 0.4 \end{vmatrix}$$

$$= \begin{vmatrix} 0 \\ 0.2 + 0.3 + j0.5196 - 0.2 + j0.3464 \\ 0.2 + 0.3 - j0.3464 - 0.2 - j0.5196 \end{vmatrix} = \begin{vmatrix} 0 \\ 0.3 + j0.866 \\ 0.3 - j0.866 \end{vmatrix}$$

$$\begin{bmatrix} {}^R I_{IZV} \\ {}^S I_{IZV} \\ {}^T I_{IZV} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} -0.077 \\ 0.846 \\ -0.154 \end{bmatrix} = \begin{bmatrix} 0.615 \\ -0.423 - j0.846 \\ -0.423 + j0.866 \end{bmatrix}$$



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$$\begin{vmatrix} {}^R I_{1-2} \\ {}^S I_{1-2} \\ {}^T I_{1-2} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.0462 \\ 0.0924 \\ 0.0924 \end{vmatrix} = \begin{vmatrix} 0.231 \\ -0.0462 \\ -0.0462 \end{vmatrix}$$

$$\begin{vmatrix} {}^R I_{1-0} \\ {}^S I_{1-0} \\ {}^T I_{1-0} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.077 \\ 0.154 \\ 0.154 \end{vmatrix} = \begin{vmatrix} 0.385 \\ -0.077 \\ -0.077 \end{vmatrix}$$

$$\begin{vmatrix} {}^R I_{2-0} \\ {}^S I_{2-0} \\ {}^T I_{2-0} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.0308 \\ 0.0616 \\ 0.0616 \end{vmatrix} = \begin{vmatrix} 0.154 \\ -0.0308 \\ -0.0308 \end{vmatrix}$$

$$\begin{vmatrix} {}^R I_{2-3} \\ {}^S I_{2-3} \\ {}^T I_{2-3} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.0154 \\ 0.0308 \\ 0.0308 \end{vmatrix} = \begin{vmatrix} 0.077 \\ -0.0154 \\ -0.0154 \end{vmatrix}$$

$$\begin{vmatrix} {}^R I_{3-0} \\ {}^S I_{3-0} \\ {}^T I_{3-0} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.0154 \\ 0.0308 \\ 0.0308 \end{vmatrix} = \begin{vmatrix} 0.077 \\ -0.0154 \\ -0.0154 \end{vmatrix}$$