## 2.3.2 Gauss-Newtonov postupak za sustav nelinearne jedbe.

- sustav $\underline{G}(\underline{x}) = \underline{\phi}$

$$\underline{G}(\underline{x}) = \begin{bmatrix} f_1(\underline{x}) \\ f_2(\underline{x}) \\ \vdots \\ f_n(\underline{x}) \end{bmatrix}$$

PRIM

nelinearne f-je

$$x_1^2 + x_2^2 - 1 = 0$$
$$x_1 - 2x_2 = 0$$

PRISTUPI – a) jedna f-ja cilja

$$F(\underline{x}) = \sum g_i^2(\underline{x})$$

b) sustav jbi $g_i(\underline{x}) = g_i(\underline{x}_0) + \nabla g_i(x_0)(\underline{x} - \underline{x}_0) + \dots$

$$\underline{G}(x) \approx \underline{G}(\underline{x}_0) + \underline{J}(\underline{x}_0)(\underline{x} - \underline{x}_0)$$

J – Jakobijan

$$\underline{J} = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \dots & \frac{\partial g_1}{\partial x_n} \\ \\ \frac{\partial g_n}{\partial x_1} & \partial g & \dots & \frac{\partial g_n}{\partial x_n} \end{bmatrix} \Rightarrow \underline{J}(\underline{x}_0)(\underline{x} - \underline{x}_0) = -\underline{G}(\underline{x}_0)$$

općenito $\underline{X}_{k+1} = \underline{X}_k - \underline{J}^{-1}(\underline{X}_k)\,\underline{G}(\underline{X}_k)$

rješava se kao:

$$\Delta\underline{x} = \underline{x}_{k+1} - \underline{x}_k = -\underline{J}(\underline{x}_k)\,G(\underline{x}_k)$$

$$\underline{J}(\underline{x}_k)\cdot\Delta\underline{x}_k = -\underline{G}(\underline{x}_k)$$

tu izračunam $\Delta x$ i znam slj. iteraciju x

algoritam:

$\underline{x}_k = \underline{x}_0$;

ponavljaj:

izračunaj $G(\underline{x}_k)$

— || — $\underline{J}(\underline{x}_k)$

riješi $\underline{J}\,\Delta\underline{x} = -\underline{G}$;

$\underline{x}_k \mathrel{+}= \Delta\underline{x}$

dok je $(\Delta\underline{x} > \varepsilon)$

modifikacija! $J^{-1}(x_0) \approx \text{konst}$

- Računati s konstantnom J matricom (izračunato u poč. točki)

problem – loša stabilnost (konvergencija!)

PRIM $x_1^2 + x_2^2 - 1 = 0 = g_1(\underline{x})$
$x_1 - 2x_2 = 0 = g_2(\underline{x})$ $\Big\} G(\underline{x})$

$$\underline{J} = \begin{bmatrix} 2x_1 & 2x_2 \\ 1 & -2 \end{bmatrix}$$

recimo $\underline{X}_0(1,1) \Rightarrow G(X_0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ ($g_1(x_0)$, $g_2$)

$\underline{J}\,\underline{\Delta x} = -\underline{G}$

$$\begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix} \Delta x = -\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$\Downarrow$

$\Delta x_1 = 0 \; ; \; \Delta x_2 = \frac{1}{2}$

$$\underline{x}_1 = \underline{x}_0 + \underline{\Delta x} = \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}$$

KRAJ 1. ITERACIJE

$$\underline{J}(\underline{x}_1) = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} \underline{\Delta x} = -\begin{bmatrix} 1/4 \\ 0 \end{bmatrix}$$

$\Delta x_1 = -\frac{1}{10} \; ; \; \Delta x_2 = -\frac{1}{20}$

$\underline{x}_2 = (0.9, 0.45)$

KRAJ 2. IT.

$\underline{x}_3 = (0.89\dot{4}, 0.447\dot{2})$

$\underline{G}(\underline{x}_3) = (3 \cdot 10^{-5}, 0)$

$G(X_2) =$