Analiza elektroenergetskog sustava

Auditorne vježbe 01

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2013./2014.

Numeričke veličine u proračunima mreža

- Kako bi elektroenergetski sustav mogao biti analiziran potrebno ga je modelirati, odnosno predstaviti nekim od matematičkih modela.
- Za određivanje nadomjesne sheme mreže potrebno je prvotno svaki element te mreže predstaviti nekim od modela (npr. za prijenosne vodove se uobičajeno koriste π modeli).
- Prisutnost transformatora kao elementa mreže, odnosno postojanje različitih naponskih razina u sustavu, značajno povećava složenost proračuna.
- Problem se riješava na način da se proračun izvodi na nadomjesnoj shemi u kojoj su svi parametri mreže svedeni na jednu naponsku razinu.
- Postupak kojim se mreža svodi na jednu naponsku razinu naziva se metodom otpora ili metodom apsolutnih vrijednosti.
- Ovaj postupak ima nekoliko varijanata u kojima se računa s relativnim vrijednostima kao što su metoda jediničnih vrijednosti (*per unit*) ili metoda reduciranih admitancija.

Numeričke veličine u proračunima mreža

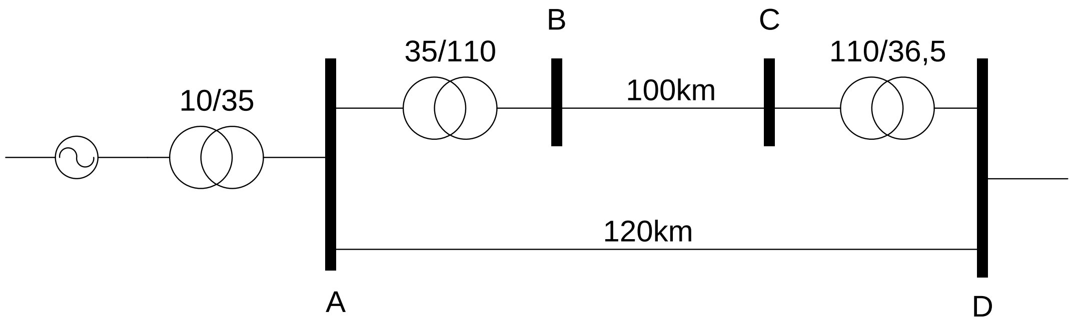
- Bazni napon na koji se u metodi otpora preračunavaju veličine elemenata mreže može biti napon neke od postojećih naponskih razina mreže ili neki proizvoljno odabrani napon (npr. U_B=100 kV)
- U metodi otpora sve veličine zadržavaju svoju prvobitnu fizikalnu dimenziju jer se reduciranje izvodi korištenjem bezdimenzionalnog faktora (U_B/U_{ni}).
- Metoda jediničnih vrijednosti (*per unit*) se ubraja u metode relativnih vrijednosti jer se sve veličine promatraju u odnosu na neku osnovnu, baznu veličinu čime one postaju bezdimenzionalne veličine.
- U metodi jediničnih vrijednosti sve bazne veličine se definiraju korištenjem baznih napona, koji se uobičajeno odabiru kao nazivni naponi određenog dijela mreže, te bazne snage koja se odabire proizvoljno (npr. S_B=100 MVA) i vrijedi za sve elemente mreže.

Numeričke veličine u proračunima mreža

Metoda otpora	Metoda reduciranih admitancija	Metoda jediničnih vrijednosti
$U' = U \cdot \frac{U_B}{U_n}$	$U_r = \frac{U}{U_n}$	$U_{p.u.} = \frac{U}{U_{Bi}} = \frac{U}{U_{ni}}$
$I' = \frac{\sqrt{3} \cdot U_n \cdot I}{U_B}$	$I' = \sqrt{3} \cdot U_n \cdot I$	$I_{p.u.} = \frac{S}{S_B} = \frac{\sqrt{3} \cdot U \cdot I}{S_B}$
$Z' = \left(\frac{U_B}{U_n}\right)^2 \cdot Z$	$Z_r = \frac{Z}{U_n^{\ 2}}$	$Z_{p.u.} = Z \cdot \frac{S_B}{U_n^2}$
$Y' = \left(\frac{U_n}{U_B}\right)^2 \cdot Y$	$Y_r = \frac{U_n^{\ 2}}{Z} = U_n^{\ 2} \cdot Y$	$Y_{p.u.} = Y \cdot \frac{U_n^2}{S_B}$

Zadatci

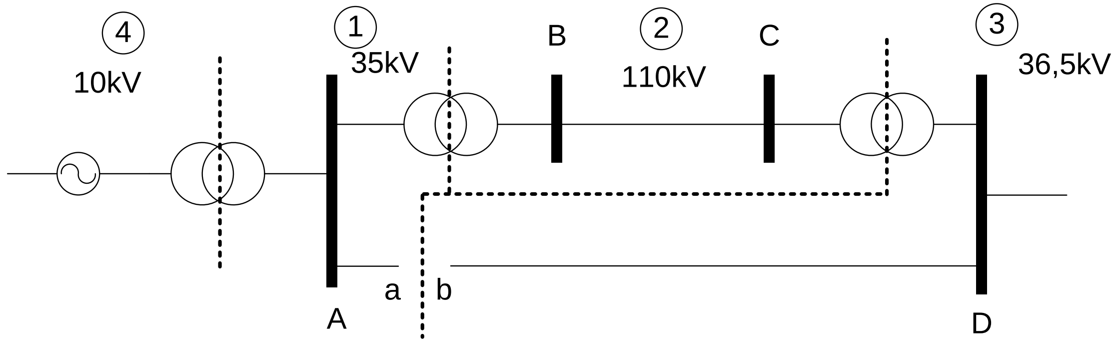
- **ZADATAK 1.** U mreži na slici odredite prilike u praznom hodu uz poznat napon U_A=34 kV. Za sve transformatore je poznat napon kratkog spoja u_k=10%, a nazivna snaga je 20 MVA (stvarni prijenosni omjer transformatora jednak je nazivnom). Podatci generatora su: U_n=10,5 kV; S_n=20 MVA; X_d=115%. Jedinična reaktancija vodova je X_V=0.4 Ω/km.



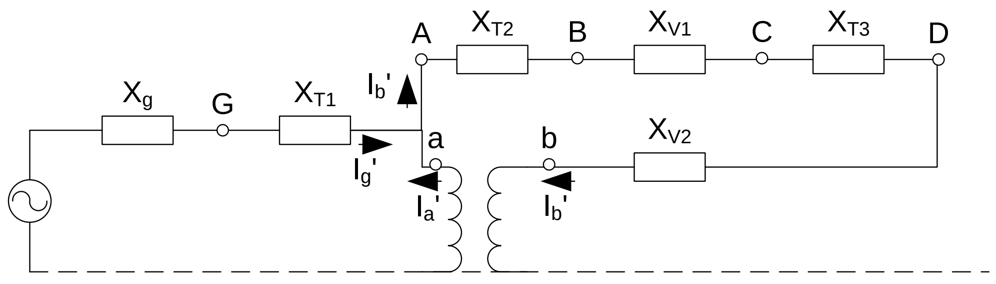
- **RJEŠENJE:**
  - Problem petlji koje se ne zaključuju – umnožak stvarnih prijenosnih omjera transformatora u petlji različit od jedan.
  - Ukoliko je umnožak prijenosnih omjera transformatora različit od jedan u petlji se javlja struja izjednačenja.
  - Petlja **A-B-C-D-A**:

$$\left(U_{nT2-1}/U_{nT2-2}\right)\cdot\left(U_{nT3-1}/U_{nT3-2}\right)=\left(110/35\right)\cdot\left(36.5/110\right)\neq 1$$

  - Za primjenu metode otpora potrebno je petlju otvoriti na jednom mjestu kako bi se mogli odrediti bazni naponi:



- U nadomjesnoj shemi mreže između točaka *a* i *b* umetnuti idealni transformator preuzima ulogu struje izjednačenja:



- Stvarne vrijednosti reaktancija elemenata nadomjesne sheme:

$$x_g = \frac{x_{d\%}}{100} \cdot \frac{U_{ng}^{\;2}}{S_{ng}} = \frac{115}{100} \cdot \frac{10,5^2}{20} = 6,339\ \Omega$$

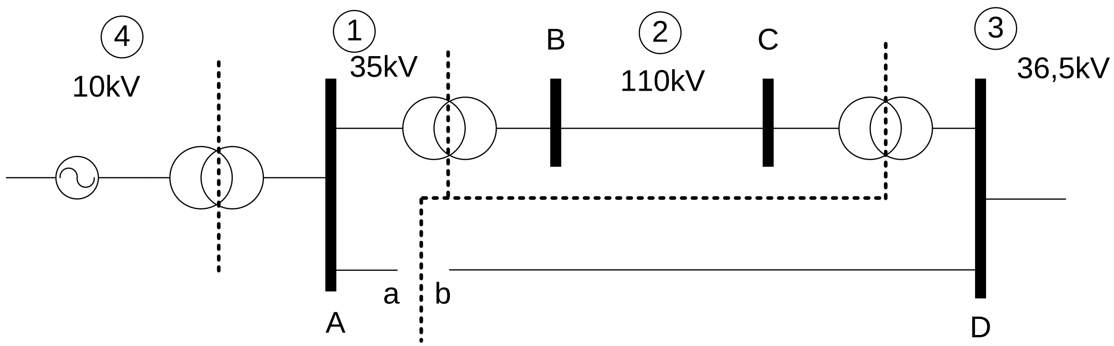
$$x_{T1} = x_{T2} = \frac{u_{k\%}}{100} \cdot \frac{U_n^{\;2}}{S_{nT}} = \frac{10}{100} \cdot \frac{35^2}{20} = 6,125\ \Omega$$

$$x_{V1} = 0,4 \cdot 100 = 40\ \Omega$$

$$x_{T3} = \frac{10}{100} \cdot \frac{110^2}{20} = 60,5\ \Omega$$

$$x_{V2} = 0,4 \cdot 120 = 48\ \Omega$$

- Za bazne napone naponskih razina *1-4* je uzeto:



$$U_{B1} = 35\ kV$$
$$U_{B2} = 110\ kV$$
$$U_{B3} = 36.5\ kV$$
$$U_{B4} = 10\ kV$$

- Korištenjem metode otpora preračunavanjem na bazni napon U_B=U_{B1}:

$$x_g' = x_g \cdot \left(\frac{U_{B1}}{U_{B4}}\right)^2 = 6,339 \cdot 3,5^2 = 77,653\Omega$$

$$x_{T1}' = x_{T2}' = 6,125 \cdot \left(\frac{U_{B1}}{U_{B1}}\right)^2 = 6,125\Omega$$

$$x_{V1}' = 40 \cdot \left(\frac{U_{B1}}{U_{B2}}\right)^2 = 40 \cdot 0,318^2 = 4,045\Omega$$

$$x_{T3}' = 60,5 \cdot 0,318^2 = 6,118\Omega$$

$$x_{V2}' = 48 \cdot 0,959^2 = 44,145\Omega$$

- Proračun u reduciranoj mreži (nadomjesnoj shemi):

$$U_a' = U_A = 34\ kV$$

$$U_b' = 34\ kV \cdot \frac{35}{36,5} = 32,6\ kV$$

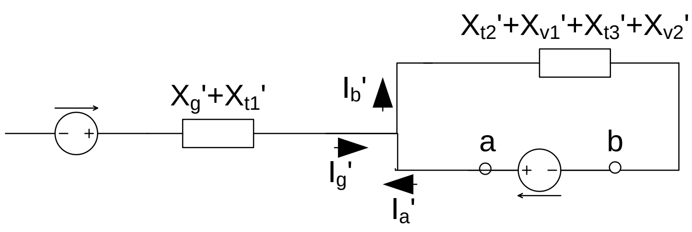
$$U_a' - U_b' = 34\ kV - 32,6\ kV = 1,4\ kV$$

$$\sum X' = 60,41\,\Omega$$

$$I_b' = \frac{1,4\ kV}{\sqrt{3} \cdot j60,41\,\Omega} = -j13,4A$$

$$I_a' = I_b' \cdot \frac{35}{36,5} = -j12,85\ A$$

$$I_g' = I_b' - I_a' = -j0,55\ A$$



- Za napone u mreži vrijedi:

$$U_D' = 32{,}6 - j13{,}4 \cdot j44{,}1 \cdot \sqrt{3} \cdot 10^{-3} = 33{,}62\ kV$$

$$U_C' = 33{,}62 - j13{,}4 \cdot j6{,}13 \cdot \sqrt{3} \cdot 10^{-3} = 33{,}76\ kV$$

$$U_B' = 33{,}76 - j13{,}4 \cdot j4{,}05 \cdot \sqrt{3} \cdot 10^{-3} = 33{,}85\ kV$$

$$U_A' = 33{,}85 - j13{,}4 \cdot j6{,}13 \cdot \sqrt{3} \cdot 10^{-3} = 34\ kV$$

$$U_G' = 34 - j0{,}55 \cdot j6{,}1 \cdot \sqrt{3} \cdot 10^{-3} = 34{,}006\ kV$$

- Stvarne vrijednosti:

$$U_A = 34\ kV$$

$$I_a = I_a' = -j12{,}85\ A$$

$$I_b = I_b' \cdot \frac{U_{B1}}{U_{B3}} = -j12{,}85\ A$$

$$I_g = I_g' \cdot \frac{U_{B1}}{U_{B4}} = -j1{,}925\ A$$

$$I_{V1} = -j4{,}26\ A$$

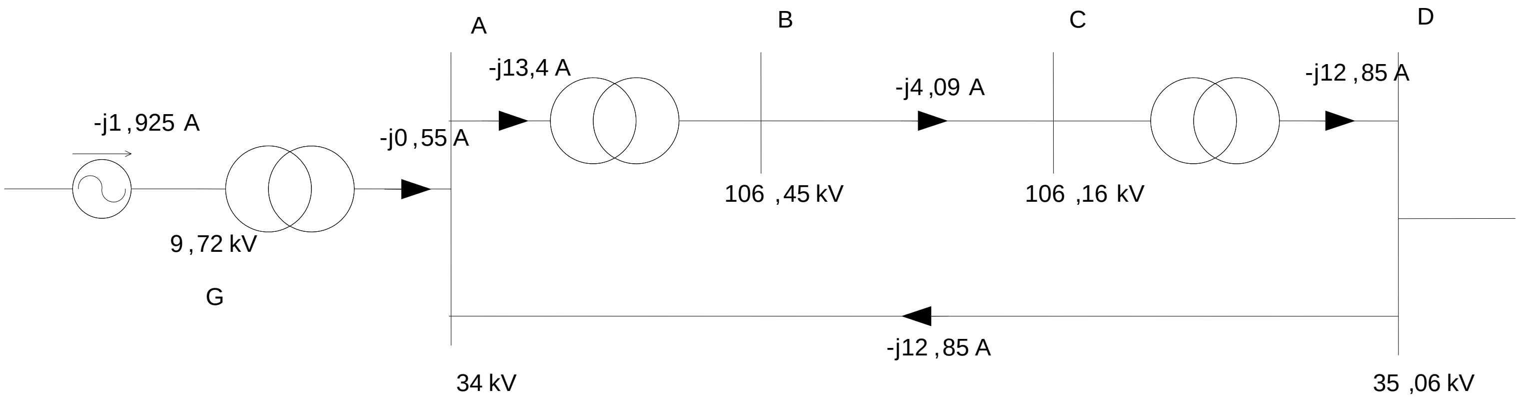
$$U_B = U_B' \cdot \frac{U_{B2}}{U_{B1}} = 106{,}45\ kV$$

$$U_C = U_C' \cdot \frac{U_{B2}}{U_{B1}} = 106{,}16\ kV$$

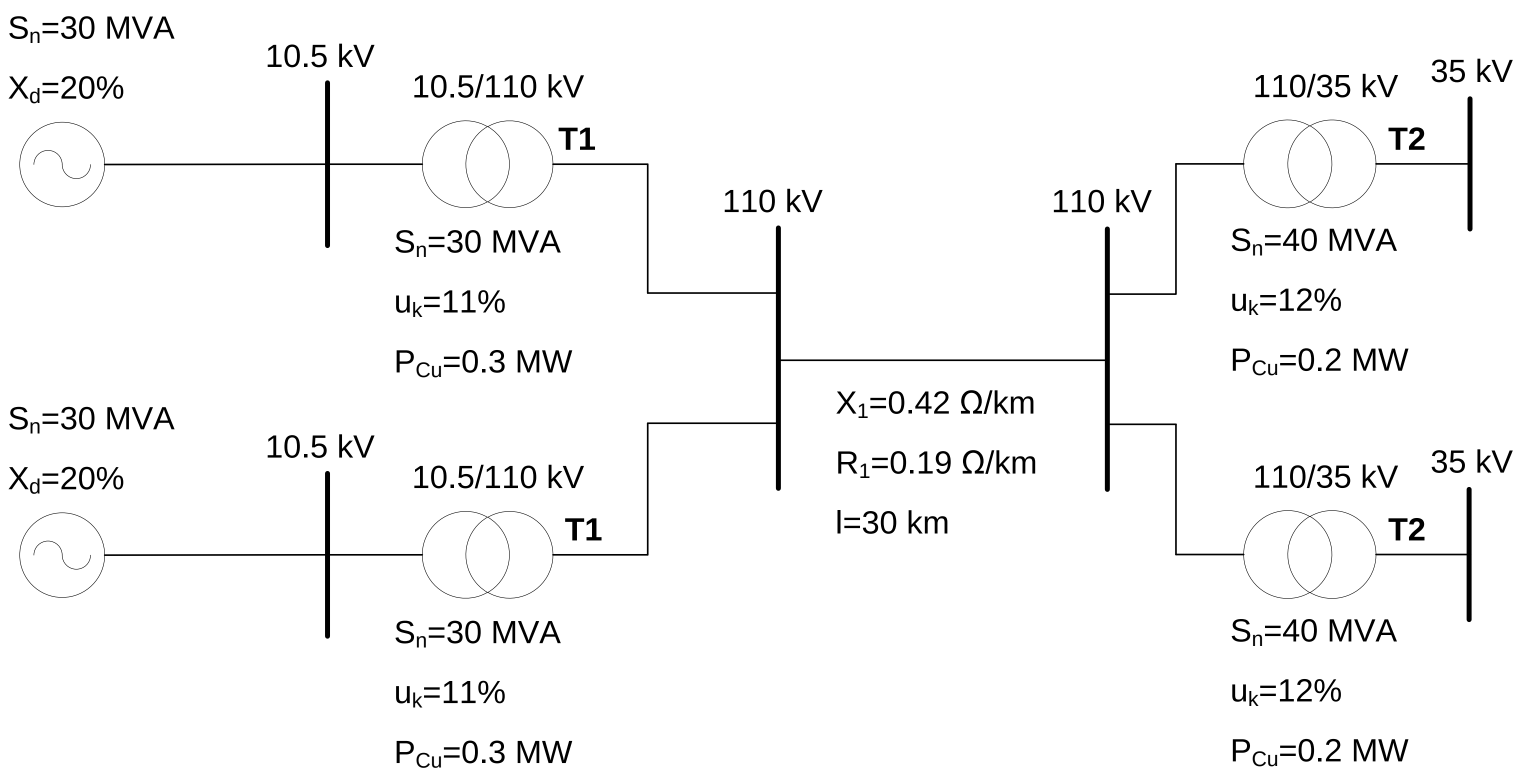
$$U_D = U_D' \cdot \frac{U_{B3}}{U_{B1}} = 35{,}06\ kV$$

$$U_G = U_G' \cdot \frac{U_{B4}}{U_{B1}} = 9{,}72\ kV$$

- Rješenje:


A
B
C
D
-j13,4 A
-j4 ,09 A
-j12 ,85 A
-j1 ,925 A
-j0 ,55 A
106 ,45 kV
106 ,16 kV
9 ,72 kV
G
-j12 ,85 A
34 kV
35 ,06 kV

- **ZADATAK 2.** Za mrežu prikazanu slikom odredite parametre svih elemenata :
  a) korištenjem metode otpora za bazni napon U_B= 100 kV
  b) korištenjem metode jediničnih vrijednosti (*per unit*) za baznu snagu S_B= 100MVA.



- **RJEŠENJE:**
  - Stvarne vrijednosti parametara:

$$X_g = \frac{X_d[\%]}{100} \cdot \frac{U_g^{\ 2}}{S_{ng}}$$

$$X_g = \frac{20}{100} \cdot \frac{(10.5\,kV)^2}{30\,MVA} = 0.735\,\Omega$$

$$Z_{T1} = \frac{U_{nT1-2}^{\ \ 2}}{S_{nT1}} \cdot \left(\left(\frac{P_{CuT1}}{S_{nT1-2}} \right) + j\sqrt{(u_{kT1})^2 - \left(\frac{P_{CuT1}}{S_{nT1}} \right)^2} \right)$$

$$Z_{T1} = \frac{(110\,kV)^2}{30\,MVA} \cdot \left(\left(\frac{0.3\,MW}{30\,MVA} \right) + j\sqrt{(0.11)^2 - \left(\frac{0.3\,MW}{30\,MVA} \right)^2} \right) = 4.033 + j44.183\,\Omega$$

$$Z_V = (R_1 + jX_1) \cdot l_V$$

$$Z_V = (0.19 + j0.42\,\Omega/km) \cdot 30\,km = 5.7 + j12.6\,\Omega$$

$$Z_{T2} = \frac{U_{nT1-1}^{\ \ 2}}{S_{nT2}} \cdot \left(\left(\frac{P_{CuT2}}{S_{nT2-1}} \right) + j\sqrt{(u_{kT2})^2 - \left(\frac{P_{CuT2}}{S_{nT2}} \right)^2} \right)$$

$$Z_{T2} = \frac{(110\,kV)^2}{40\,MVA} \cdot \left(\left(\frac{0.2\,MW}{40\,MVA} \right) + j\sqrt{(0.12)^2 - \left(\frac{0.2\,MW}{40\,MVA} \right)^2} \right) = 1.513 + j36.268\,\Omega$$

a) Metoda otpora:

$$X_g' = \left(\frac{U_B}{U_{ng}}\right)^2 \cdot X_g$$

$$X_g' = \left(\frac{100\ kV}{10.5\ kV}\right)^2 \cdot 0.735\ \Omega = 66.667\ \Omega$$

$$Z_{T1}' = \left(\frac{U_B}{U_{nT1-2}}\right)^2 \cdot Z_{T1}$$

$$Z_{T1}' = \left(\frac{100\ kV}{110\ kV}\right)^2 \cdot \left(4.033 + j44.183\ \Omega\right) = 3.333 + j36.515\ \Omega$$

$$Z_V' = \left(\frac{U_B}{U_{nV}}\right)^2 \cdot Z_V$$

$$Z_V' = \left(\frac{100\ kV}{110\ kV}\right)^2 \cdot \left(5.7 + j12.6\ \Omega\right) = 4.711 + j10.413\ \Omega$$

$$Z_{T2}' = \left(\frac{U_B}{U_{nT2-1}}\right)^2 \cdot Z_{T2}$$

$$Z_{T2}' = \left(\frac{100\ kV}{110\ kV}\right)^2 \cdot \left(1.513 + j36.268\ \Omega\right) = 1.25 + j29.974\ \Omega$$

b) Metoda jediničnih vrijednosti (*per unit*):

$$X_g\left[p.u.\right]=\left(\frac{S_B}{U_{ng}^{\ 2}}\right)\cdot X_g$$

$$X_g\left[p.u.\right]=\left(\frac{100\ MVA}{\left(10.5\ kV\right)^2}\right)\cdot 0.735\,\Omega=0.667\ p.u.$$

$$Z_{T1}\left[p.u.\right]=\left(\frac{S_B}{U_{nT1-2}^{\ 2}}\right)\cdot Z_{T1}$$

$$Z_{T1}\left[p.u.\right]=\left(\frac{100\ MVA}{\left(110\ kV\right)^2}\right)\cdot\left(4.033+j44.183\,\Omega\right)=0.033+j0.365\ p.u.$$

$$Z_V\left[p.u.\right]=\left(\frac{S_B}{U_{nV}^{\ 2}}\right)\cdot Z_V$$

$$Z_V\left[p.u.\right]=\left(\frac{100\ MVA}{\left(110\ kV\right)^2}\right)\cdot\left(5.7+j12.6\,\Omega\right)=0.047+j0.104\ p.u.$$

$$Z_{T2}\left[p.u.\right]=\left(\frac{S_B}{\left(U_{nT2-1}\right)^2}\right)\cdot Z_{T2}$$

$$Z_{T2}\left[p.u.\right]=\left(\frac{100\ MVA}{\left(110\ kV\right)^2}\right)\cdot\left(1.513+j36.268\,\Omega\right)=0.013+j0.3\ p.u$$

Analiza elektroenergetskog sustava

Auditorne vježbe 02

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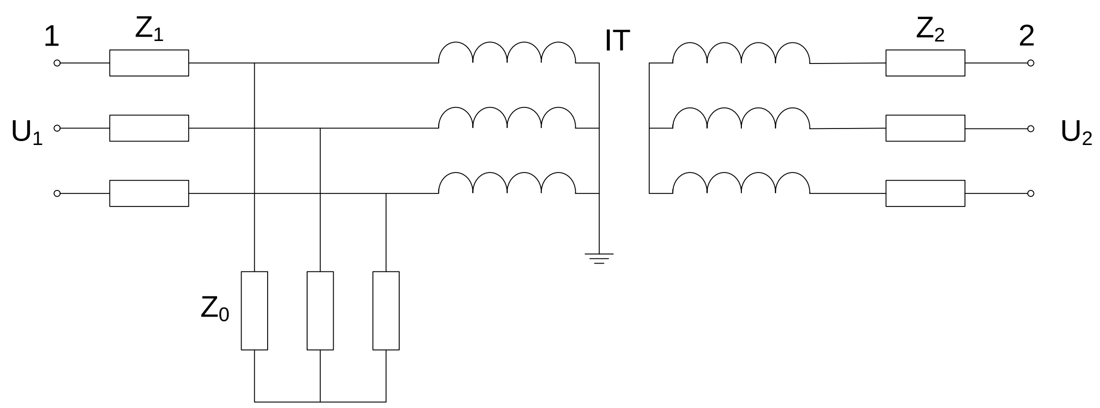
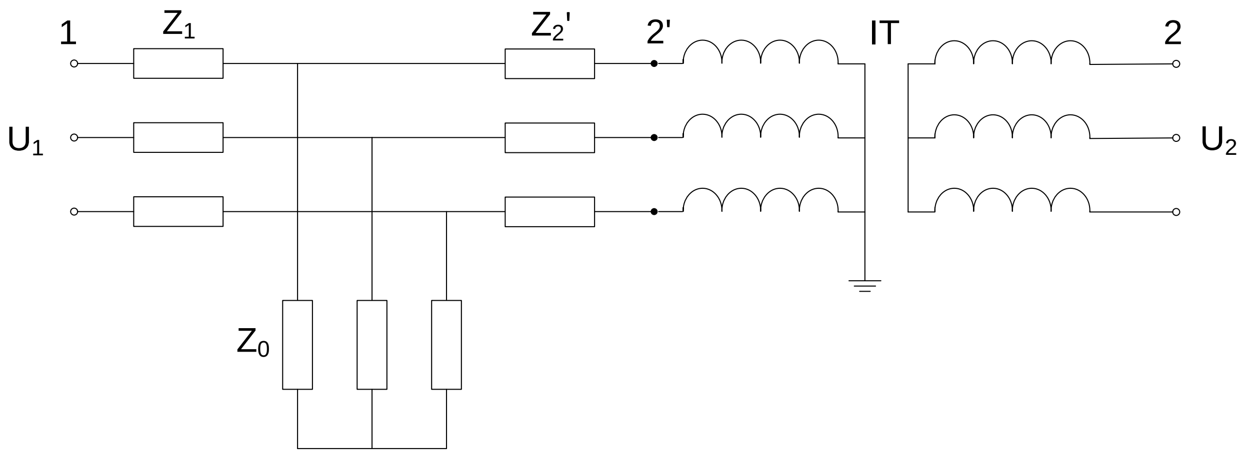
Nadomjesni modeli transformatora

- Transformator je pasivni element mreže, a u topološkom smislu se uzima kao grana mreže.
- Ekvivalentna shema transformatora je određena sa tri veličine: uzdužnom impedancijom primarne strane, uzdužnom impedancijom sekundarne strane, te impedancijom (admitancijom) poprečne grane.
- Trofazna nadomjesna shema dvonamotnog energetskog transformatora:



Nadomjesni modeli transformatora

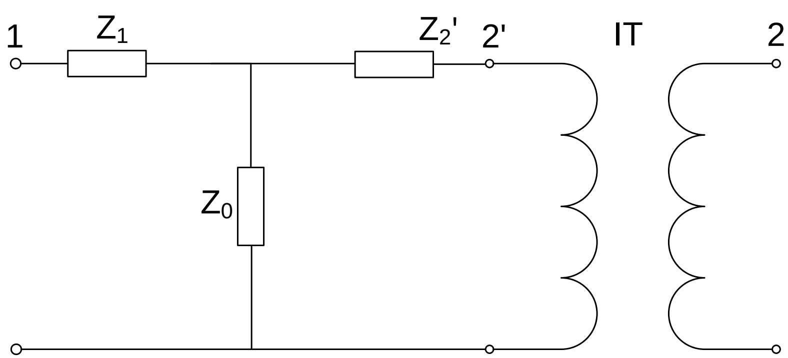
- Trofazna nadomjesna shema (T-shema) dvonamotnog energetskog trafo-a s parametrima preračunatim na primarnu naponsku stranu:



$$S = \frac{U_2^2}{Z_2} = \frac{U_1^2}{Z_2'}$$

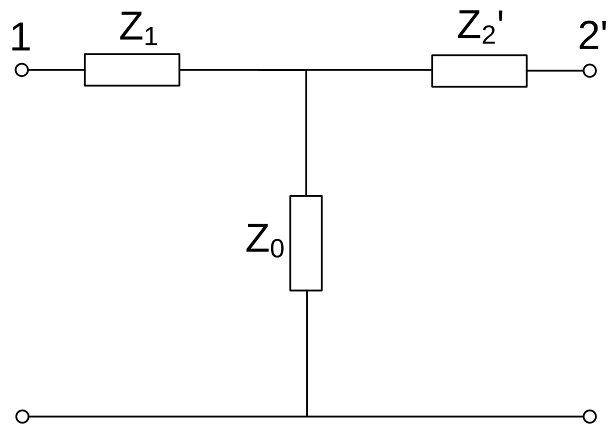
$$Z_2' = Z_2 \cdot \left(\frac{U_1^2}{U_2^2}\right)$$

- U simetričnom slučaju je navedenu nadomjesnu shemu moguće predstaviti jednofaznim modelom:



Nadomjesni modeli transformatora

- Ukoliko navedeni transformator ima prijenosni omjer jednak nazivnom prijenosnom omjeru, te ukoliko se parametri transformatora preračunaju u jedinične vrijednosti u nadomjesnoj shemi se izostavlja idealni transformator:



Za nazivni prijenosni omjer idealnog transformatora:

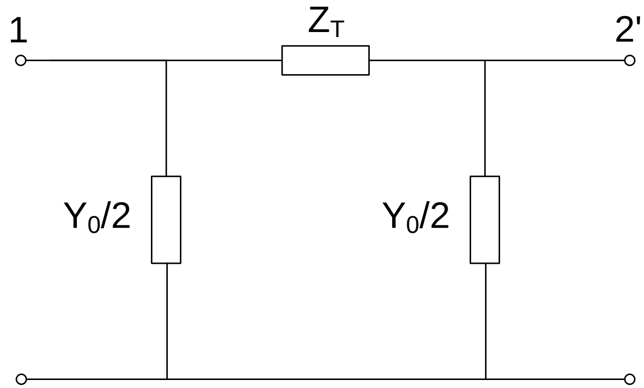
$$U_1/U_2 = U_{n1}/U_{n2}$$

$$U_1(p.u.) = U_2(p.u.) = 1\ p.u.$$

$$U_1(p.u.)/U_2(p.u.) = 1$$

Nadomjesni modeli transformatora

- Za potrebe analize mreža se uobičajeno koristi π shema :



$$\vec{Z}_T = \frac{U_n^2}{S_n} \cdot \left[\frac{P_k}{S_n} + j\sqrt{u_k^2 - \left(\frac{P_k}{S_n}\right)^2}\right] \ [\Omega]$$

$$\vec{Y}_0 = \frac{S_n}{U_n^2} \cdot \left[\frac{P_0}{S_n} - j\sqrt{i_0^2 - \left(\frac{P_0}{S_n}\right)^2}\right] \ [S]$$

$$\vec{Z}_T = \frac{S_B}{S_n} \cdot \left[\frac{P_k}{S_n} + j\sqrt{u_k^2 - \left(\frac{P_k}{S_n}\right)^2}\right] \ [p.u.]$$

$$\vec{Y}_0 = \frac{S_n}{S_B} \cdot \left[\frac{P_0}{S_n} - j\sqrt{i_0^2 - \left(\frac{P_0}{S_n}\right)^2}\right] \ [p.u.]$$

Uobičajena zanemarenja:

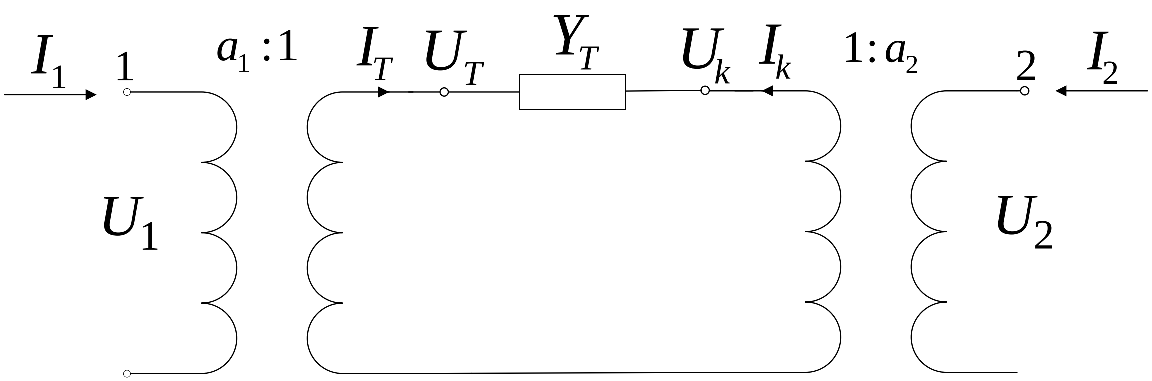
- Y_0, odnosno i_0 i P_0
- P_k

Okvirne vrijednosti parametara:

- u_k = 4 – 12 %
- i_0 = 1 – 2,5 % I_n
- P_k = 0,5 – 2,5 % S_n
- P_0 = 15 – 40 % P_k

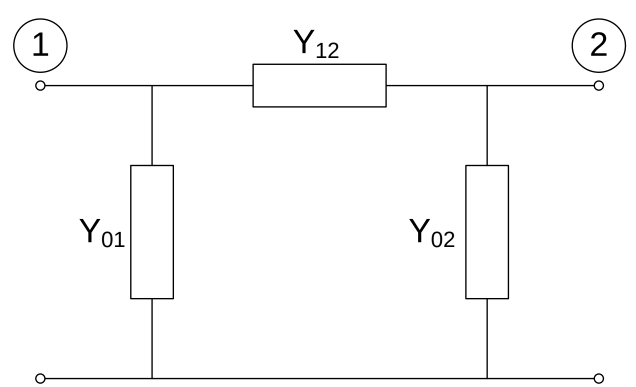
Nadomjesni modeli transformatora

- Transformator s nenazivnim prijenosnim omjerom, te zanemarenom poprečnom admitancijom (Y_0) :



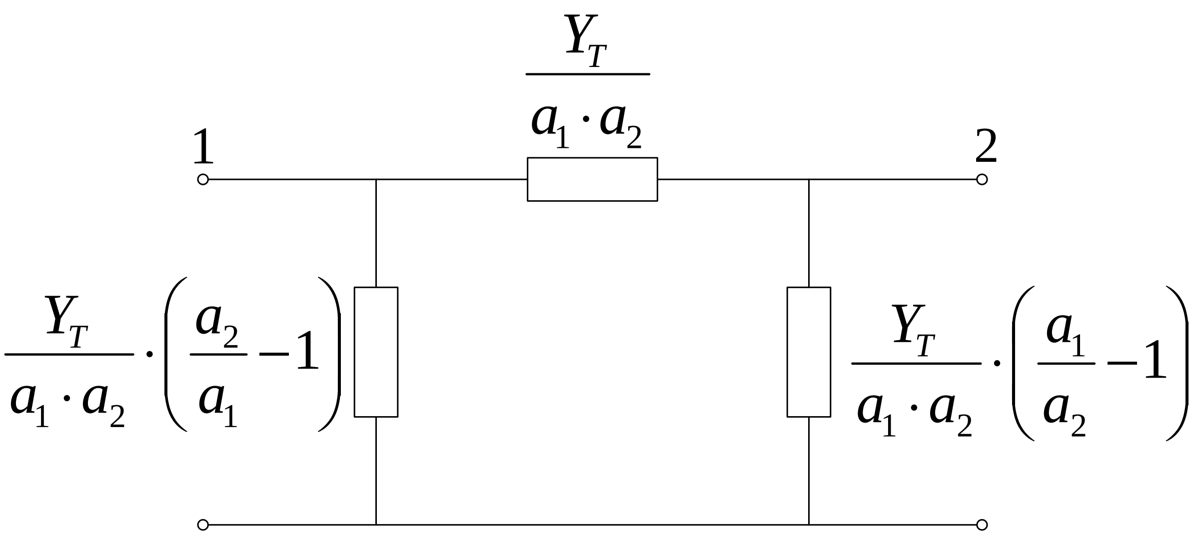
Nenazivni napon i na primarnoj i na sekundarnoj strani – Najopćenitiji slučaj

- Potrebno je dobiti model bez idealnih transformatora u sljedećem obliku (odnosno pasivni element):



Nadomjesni modeli transformatora

- Za parametre navedenog modela bez idealnih trafo-a vrijedi:



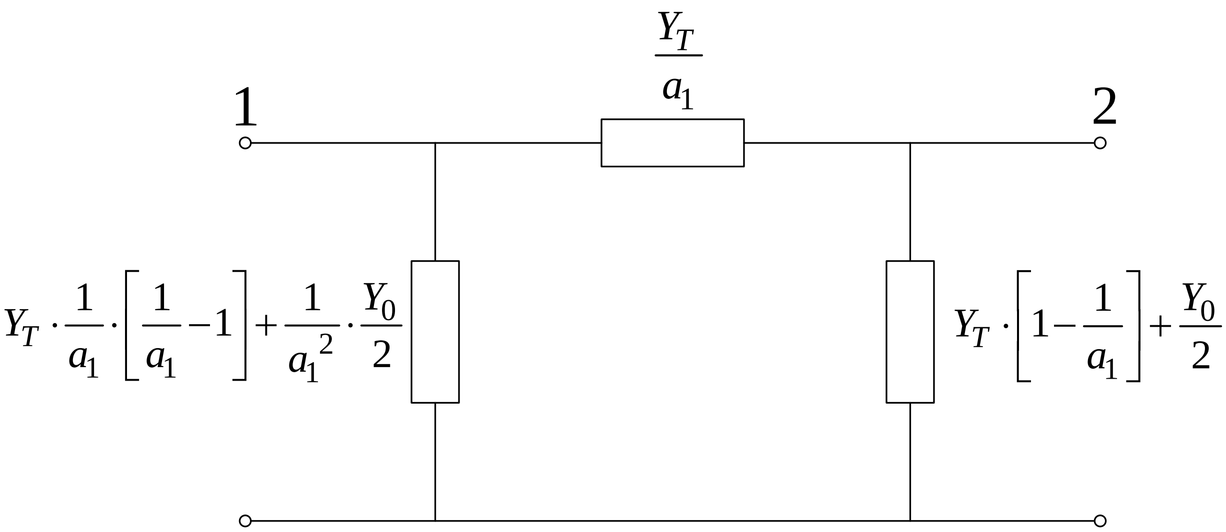
$$Y_{12} = \frac{Y_T}{a_1 \cdot a_2}$$

$$Y_{01} = \frac{Y_T}{a_1 \cdot a_2} \left(\frac{a_2}{a_1} - 1 \right)$$

$$Y_{02} = \frac{Y_T}{a_1 \cdot a_2} \left(\frac{a_1}{a_2} - 1 \right)$$

Nadomjesni modeli transformatora

- Za slučaj kada se u prijenosnom omjeru transformatora nenazivni napon javlja samo na primarnoj strani (i bez zanemarenja Y_0):



$$a_1 = U_{t1}/U_{n1}$$

$$a_2 = U_{n1}/U_{n1} = 1$$

$$Y_{12} = \frac{Y_T}{a_1}$$

$$Y_{01} = \frac{Y_T}{a_1}\left(\frac{1}{a_1} - 1\right) + \frac{1}{a_1^2}\frac{Y_0}{2}$$

$$Y_{02} = Y_T\left(1 - \frac{1}{a_1}\right) + \frac{Y_0}{2}$$

Zadatci

- **ZADATAK 1.** Zadan je transformator sa sljedećim podatcima:

Transformator T1	
S_n= 300 MVA	P_k= 700 kW
u_k= 12 %	P_0= 200 kW
i_0= 1 %	U_{n1}/U_{n2}= 400/110 kV

Ukoliko je stvarni prijenosni omjer transformatora 420/110 kV odredite parametre njegove π -sheme koristeći model:

a) s idealnim transformatorom

b) bez idealnog transformatora.

Skicirajte π-sheme za oba slučaja te označite odgovarajuće parametre. Parametre izrazite u admitantnom obliku u *per unit* vrijednostima koristeći S_B=100 MVA.

- **RJEŠENJE:**

$$Z_T = \frac{S_B}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{u_k^{\,2} - \left(\frac{P_k}{S_n}\right)^2}\right] = 7.778\cdot 10^{-4} + j0.04\ p.u.$$

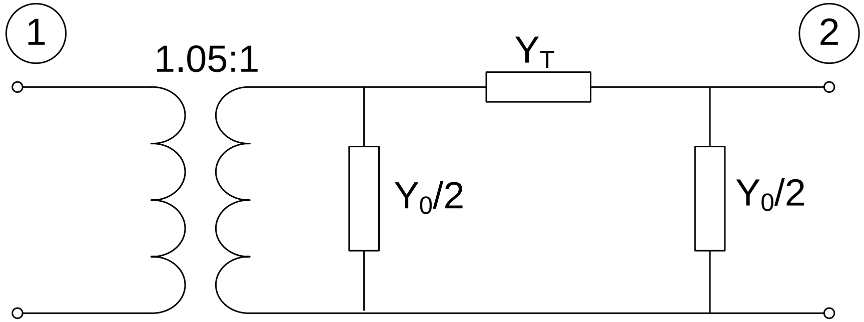
$$Y_T = \frac{1}{Z_T} = 0.486 - j24.995\ p.u.$$

$$Y_0 = \frac{S_n}{S_B}\left[\frac{P_0}{S_n} - j\sqrt{i_0^{\,2} - \left(\frac{P_0}{S_n}\right)^2}\right] = 2\cdot 10^{-3} - j0.03\ p.u.$$

$$\frac{Y_0}{2} = 1\cdot 10^{-3} - j0.015\ p.u.$$

$$a_1 = \frac{420}{400} = 1.05$$

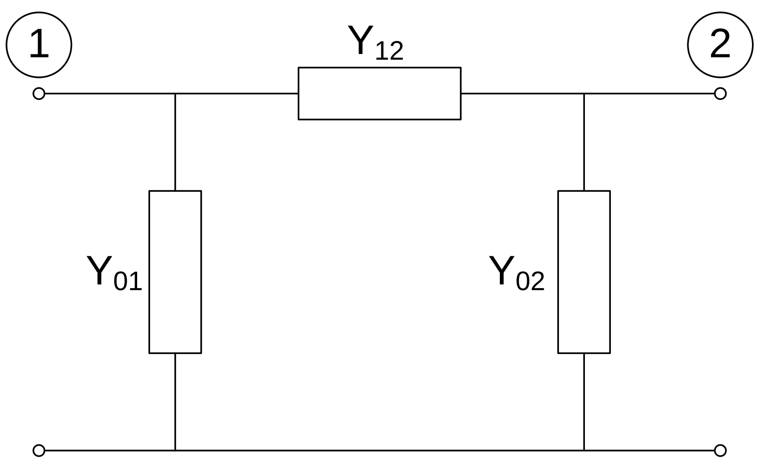
$$a_2 = \frac{110}{110} = 1$$



$$Y_{12} = \frac{Y_T}{a_1} = 0.463 - j23.805\ p.u.$$

$$Y_{01} = \frac{Y_T}{a_1}\left(\frac{1}{a_1} - 1\right) + \frac{1}{a_1^{\ 2}}\frac{Y_0}{2} = -0.02 + j1.106\ p.u.$$

$$Y_{02} = Y_T\left(1 - \frac{1}{a_1}\right) + \frac{Y_0}{2} = 0.025 - j1.22\ p.u.$$



- **ZADATAK 2:** U transformatorskoj stanici su paralelno spojena dva transformatora sa sljedećim podatcima:

T1	T2
S_n= 400 MVA	S_n= 400 MVA
u_k= 12 %	u_k= 12 %
420/220 kV	400/220 kV

Odredite napon na primaru (u kV) ukoliko je napon sekundara U_2=220 kV, a snaga kojom je transformatorska stanica opterećena iznosi S_2=200 MW uz $\cos(\varphi_2)$=0.95 *ind*.. Koristiti S_B=100 MVA.

- **RJEŠENJE:**

$$Z_T = \frac{S_B}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{{u_k}^2 - \left(\frac{P_k}{S_n}\right)^2}\right] = ju_k \cdot \frac{S_B}{S_n}\,[p.u.]$$

$$Y_T = \frac{1}{Z_T} = \frac{S_n}{ju_k \cdot S_B}\,[p.u.]$$

$$Y_T = -j33.333\ p.u.$$

$$Y_{T1} = Y_{T2} = Y_T$$

$$a_{1-T1} = \frac{420}{400} = 1.05 \qquad a_{2-T1} = \frac{220}{220} = 1$$

$$a_{1-T2} = \frac{400}{110} = 1 \qquad a_{2-T2} = \frac{220}{220} = 1$$

→ Transformator T1 ima nenazivni prijenosni omjer

- Transformator **T1**:

$$Y_{12-T1} = \frac{Y_T}{a_{1-T1}} = -j31.746\ p.u.$$

$$Y_{01-T1} = \frac{Y_T}{a_{1-T1}} \cdot \left(\frac{1}{a_{1-T1}} - 1 \right) = j1.512\ p.u.$$

$$Y_{02-T1} = Y_T \cdot \left(1 - \frac{1}{a_{1-T1}} \right) = -j1.587\ p.u.$$

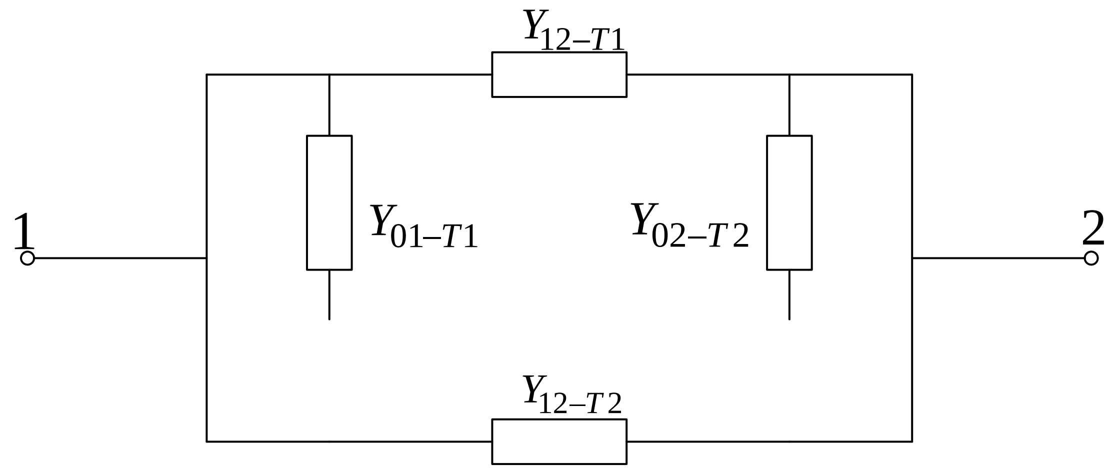
- Transformator **T2**:

$$Y_{12-T2} = Y_T = -j33.333\ p.u.$$

$$Y_{01-T2} = 0\ p.u.$$

$$Y_{02-T2} = 0\ p.u.$$

- Nadomjesna shema:



- Y-matrica (matrica admitancija čvorišta):

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Y] \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$\longrightarrow$ Pogledati predavanje 3., slajd 19.

$$[Y] = \begin{bmatrix} Y_{12-T1} + Y_{01-T1} + Y_{12-T2} + Y_{01-T2} & -\left(Y_{12-T1} + Y_{12-T2}\right) \\ -\left(Y_{12-T1} + Y_{12-T2}\right) & Y_{12-T1} + Y_{02-T1} + Y_{12-T2} + Y_{02-T2} \end{bmatrix}$$

- Zadano je:

$$S_2 = -200\ MVA$$

$$S_2 = \left(\frac{S_2}{S_B}\right)\left[\cos(\varphi_2) + j\sin(\arccos(\varphi_2))\right][p.u.]$$

$$S_2 = -1.9 - j0.624\ p.u.$$

$$U_2 = 1\ p.u.$$

$$I_2 = \left(\frac{S_2^*}{U_2^*}\right)$$

$$I_2 = -1.9 + j0.624\ p.u.$$

- Korištenjem Y-matrice:

$$I_2 = Y_{21} \cdot U_1 + Y_{22} \cdot U_2$$

$$U_1 = \frac{I_2 - Y_{22} \cdot U_2}{Y_{21}}$$

$$U_1 = 1.034 + j0.029\ p.u.$$

$$U_1 = 413.594 + j11.678\ kV = 413.759\angle 1.617^\circ\ kV$$

Analiza elektroenergetskog sustava

Auditorne vježbe 03

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2013./2014.

Matrične metode proračuna – metoda čvorišta

- Metoda grana (Ohmov i Kirchoffovi zakoni)

- Metoda petlji (2. Kirchoffov zakon: $\sum V_p = 0$, p – nezavisne petlje)

- Metoda čvorišta (1. Kirchoffov zakon: $\sum I_{cv} = 0$, cv – čvorišta mreže)
  - Struje čvorišta – injekcije struja
  - Naponi čvorišta – naponi prema jednom referentnom čvorištu

$$\begin{vmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{vmatrix} = \begin{vmatrix} \sum_{i=2}^{n} y_{1-i} & -y_{1-2} & \cdots & -y_{1-n} \\ -y_{2-1} & \sum_{\substack{i=1 \\ i\neq 2}}^{n} y_{2-i} & \cdots & -y_{2-n} \\ \vdots & \vdots & \ddots & \vdots \\ -y_{n-1} & -y_{n-2} & \cdots & \sum_{i=1}^{n-1} y_{n-i} \end{vmatrix} \cdot \begin{vmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{vmatrix}$$

Matrične metode proračuna – metoda čvorišta

- Matrica admitancija čvorišta (**Y**, [Y]):

$$[I] = [Y] \cdot [U]$$

$$[Y] = \begin{vmatrix} \sum_{i=2}^{n} y_{1-i} & -y_{1-2} & \cdots & -y_{1-n} \\ -y_{2-1} & \sum_{\substack{i=1 \\ i\neq 2}}^{n} y_{2-i} & \cdots & -y_{2-n} \\ \vdots & \vdots & \ddots & \vdots \\ -y_{n-1} & -y_{n-2} & \cdots & \sum_{i=1}^{n-1} y_{n-i} \end{vmatrix}$$

- Matrica [Y] je dimenzija *nxn*, pri čemu je *n* broj čvorišta mreže
- Ovako definirana matrica [Y] je singularna, a njen rang je *n-1*
- Matrica [Y] postaje regularna ukoliko se eliminira redak i stupac nekog čvorišta (referentnog): $[\boldsymbol{Z}] = [\boldsymbol{Y}]^{-1}$

$$[\boldsymbol{U}] = [\boldsymbol{Z}] \cdot [\boldsymbol{I}]$$

- Matrica [Z] je matrica impedancija čvorišta ([Y], [Z] →(*n-1xn-1)*)

Zadatci

- **ZADATAK 1.** Za regulacijski transformator prijenosnog omjera 220±10x1.5%/110 kV su zadani sljedeći podatci:

  S_n=150 MVA, u_k=11 %, P_k= 0.5 MW, i_0= 1%, P_0= 0.1 MW

  Regulacijska preklopka se nalazi na položaju n=+5.

  Odredite model transformatora korištenjem matrice admitancija čvorišta u *per unit* vrijednostima uz S_B=100 MVA.

- **RJEŠENJE:**

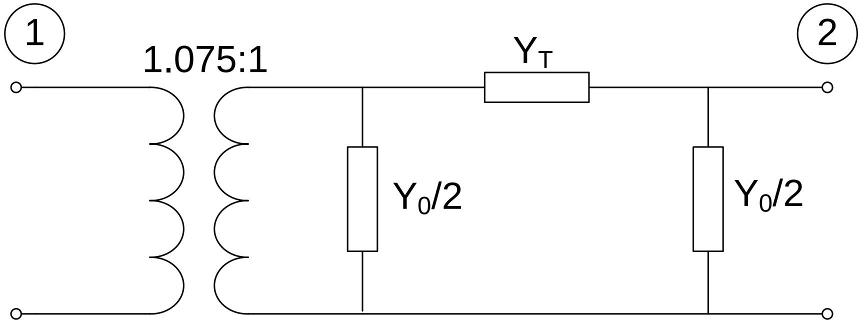
$$Z_T = \frac{S_B}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{{u_k}^2 - \left(\frac{P_k}{S_n}\right)^2}\right] = 2.222\cdot 10^{-3} + j0.073\ p.u.$$

$$Y_T = \frac{1}{Z_T} = 0.413 - j13.63\ p.u.$$

$$Y_0 = \frac{S_n}{S_B}\left[\frac{P_0}{S_n} - j\sqrt{{i_0}^2 - \left(\frac{P_0}{S_n}\right)^2}\right] = 1\cdot 10^{-3} - j0.015\ p.u.$$

$$a_1 = \frac{(220 + 5\cdot 0.015\cdot 220)}{220} = 1.075$$

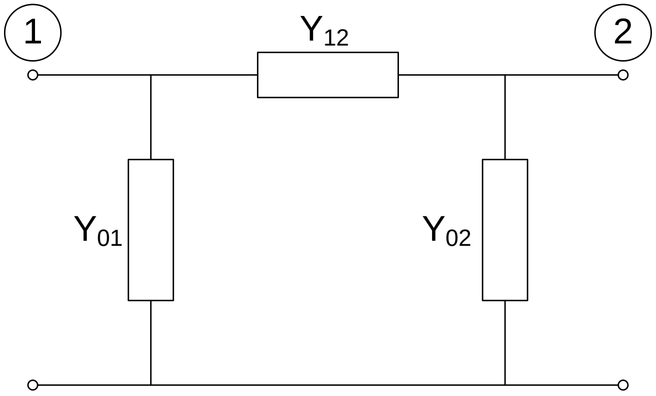
$$a_2 = \frac{110}{110} = 1$$



$$Y_{12} = \frac{Y_T}{a_1} = 0.384 - j12.679 \ p.u.$$

$$Y_{01} = \frac{Y_T}{a_1}\left(\frac{1}{a_1} - 1\right) + \frac{1}{{a_1}^2}\frac{Y_0}{2} = -0.026 + j0.878 \ p.u.$$

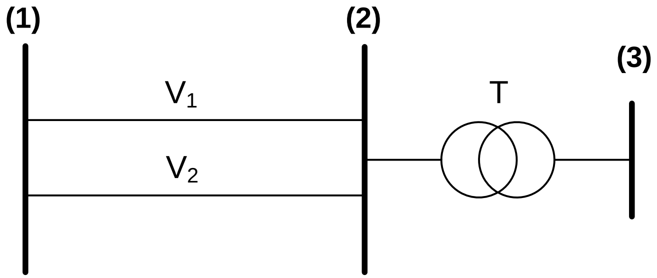
$$Y_{02} = Y_T\left(1 - \frac{1}{a_1}\right) + \frac{Y_0}{2} = 0.029 - j0.958 \ p.u.$$



$$\left[Y\right] = \begin{bmatrix} Y_{12} + Y_{01} & -Y_{12} \\ -Y_{12} & Y_{12} + Y_{02} \end{bmatrix}$$

$$\left[Y\right] = \begin{bmatrix} 0.358 - j11.801 & -0.384 + j12.679 \\ -0.384 + j12.679 & 0.414 - j13.638 \end{bmatrix} p.u.$$

- **ZADATAK 2.** Za mrežu zadanu slikom odredite matricu admitancija čvorišta:



Vodovi su jednaki i imaju iste parametre. Elemente matrice admitancija čvorišta je potrebno izračunati u *per unit* vrijednostima uz S_B=100 MVA. Podatci o elementima mreže su zadani u tablici:

Vodovi	**Transformator**
U_n= 220 kV	S_n=150 MVA
R_1= 0.08 Ω/km	P_k= 1.5 MW
X_1= 0.41 Ω/km	u_k= 10.5 %
B_1= 2.8 μS/km	231/110 kV
l= 25 km	

- **RJEŠENJE:**

$$Z_V = \left(R_1 + jX_1\right)\cdot l \cdot \frac{S_B}{U_n^2} = 4.132\cdot 10^{-3} + j0.021\ p.u.$$

$$Y_V = \frac{1}{Z_V} = 8.876 - j45.488\ p.u.$$

$$Y_{0V} = \frac{\left(j\cdot B_1 \cdot l\right)}{2}\cdot\frac{U_n^2}{S_B} = j0.017\ p.u.$$

$$Z_T = \frac{S_B}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{u_k^{\ 2} - \left(\frac{P_k}{S_n}\right)^2}\right] = 6.667\cdot 10^{-3} + j0.07\ p.u.$$

$$a_1 = \frac{231}{220} = 1.05$$

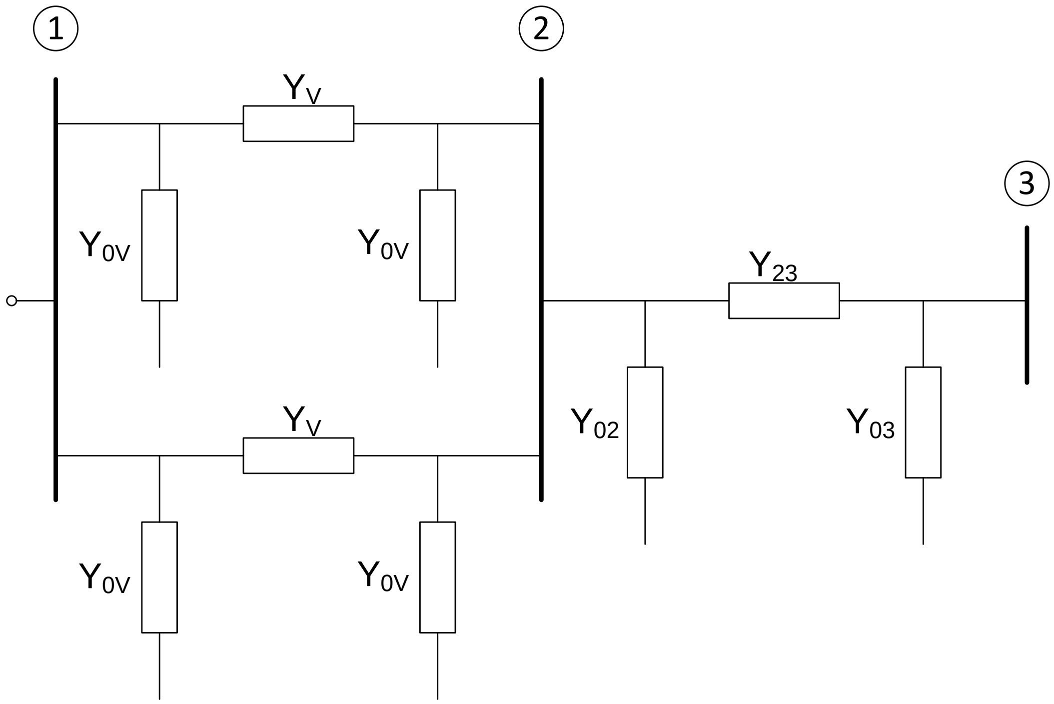
$$a_2 = \frac{110}{110} = 1$$

$$Y_T = \frac{1}{Z_T} = 1.361 - j14.221\ p.u.$$

$$Y_{23} = \frac{Y_T}{a_1} = 1.296 - j13.544\ p.u.$$

$$Y_{02} = \frac{Y_T}{a_1}\left(\frac{1}{a_1} - 1\right) = -0.062 + j0.645\ p.u.$$

$$Y_{03} = Y_T\left(1 - \frac{1}{a_1}\right) = 0.065 - j0.677\ p.u.$$



$$[Y]=\begin{bmatrix} 2\cdot Y_V+2\cdot Y_{0V} & -2\cdot Y_V & 0 \\ -2\cdot Y & 2\cdot Y_V+2\cdot Y_{0V}+Y_{23}+Y_{02} & -Y_{23} \\ 0 & -Y_{23} & Y_{23}+Y_{03} \end{bmatrix}$$

$$[Y]=\begin{bmatrix} 17.751-j90.941 & -17.751+j90.975 & 0 \\ -17.751+j90.975 & 18.985-j103.84 & -1.296+j13.544 \\ 0 & -1.296+j13.544 & 1.361-j14.221 \end{bmatrix} p.u.$$

Analiza elektroenergetskog sustava

Auditorne vježbe 04

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2013./2014.

Proračun tokova snaga

- Za stacionarno pogonsko stanje EES-a
- Cilj proračuna je određivanje svih napona u EES-u (vektor stanja)
- Pomoću izračunatih napona je moguće odrediti:
  - Tokove snaga u svim granama
  - Injekcije snaga u svim čvorištima
  - Gubitke snage u mreži
- Osnovne metode proračuna tokova snaga:
  - Metoda Gauss-Seidel pomoću Z matrice
  - Metoda Gauss-Seidel pomoću Y matrice
  - Newton-Raphson metoda
- Klasifikacija čvorišta (čvorište je definirano sa 4 podatka: $|V_i|$, δ_i, P_i, Q_i)
  - Čvorišta tereta (zadano P_i i Q_i, P-Q čvorišta, čvorišta snage)
  - Generatorska čvorišta (zadano P_i i $|V_i|$, P-V čvorišta, čvorišta s kontrolom napona)
  - Referentno čvorište (zadano $|V_i|$ i δ_i, čvorište regulacijske elektrane, bilančno čvorište)

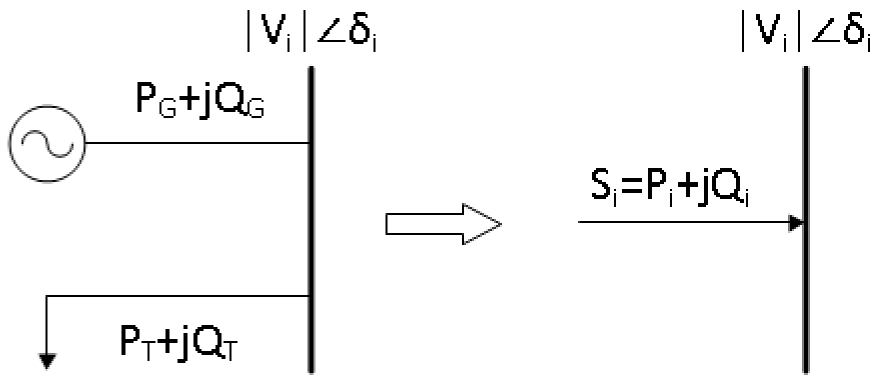
Zašto iterativne metode?
•Sustav nelinearnih jednadžbi:

$$\left[\left(\frac{S}{V}\right)^*\right] = [Y]\cdot[U]$$

$$[U] = [Z]\cdot\left[\left(\frac{S}{V}\right)^*\right]$$

Proračun tokova snaga

- Klasifikacija čvorišta
  - Injekcije snage u čvorištu:

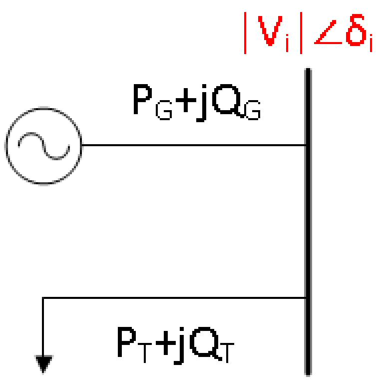


Vrijedi:

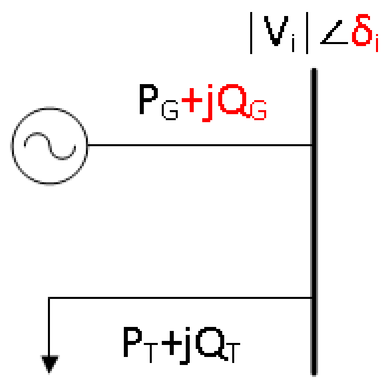
$P_i = P_G - P_T$

$Q_i = Q_G - Q_T$

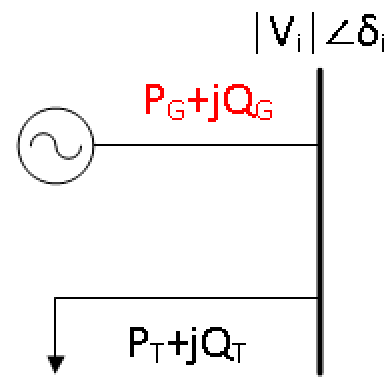
  - Crvenom bojom su označene nepoznate vrijednosti, a crnom poznate



Čvorište tereta



Generatorsko čvorište



Referentno čvorište

Proračun tokova snaga

- Metoda Gauss-Seidel pomoću Z-matrice u iteraciji ***k+1***:

$$\vec{I}_i^{(k+1)} = \frac{\vec{S}_i^*}{\vec{U}_i^{*(k+1)}} - \vec{U}_i^{(k+1)} \cdot Y_i' \qquad i = 1, 2, \ldots, n; \quad i \neq ref.$$

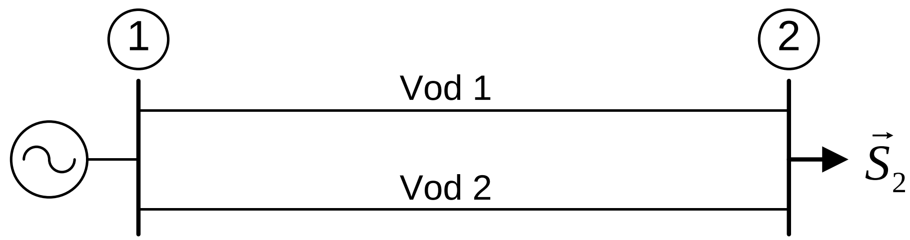
$$\vec{U}_i^{(k+1)} = \vec{U}_{ref} + \sum_{\substack{j=1 \\ j\neq ref}}^{i-1} \vec{Z}_{ij} \cdot \vec{I}_j^{(k+1)} + \sum_{\substack{j=i \\ j\neq ref}}^{n} \vec{Z}_{ij} \cdot \vec{I}_j^{(k)} \qquad i = 1, 2, \ldots, n; \quad i \neq ref.$$

- Uvjet točnosti:

$$\left| \left(\vec{U}_i^{(k+1)} - \vec{U}_i^{(k)} \right) \right| < \varepsilon \qquad za\ i = 1, 2, \ldots, n; \quad i \neq ref.$$

Zadatci

- **ZADATAK 1.** Za mrežu prikazanu slikom su poznati sljedeći podatci:



	R [Ω]	X [Ω]	B [mS]
Vod 1	0	32	0.22
Vod 2	0	30	0.24

Napon u čvorištu 1 je poznat i iznosi U_1=220 kV, a snaga u čvorištu 2 iznosi S_2=-120-j30 MVA. Korištenjem metode Gauss-Seidel pomoću Z matrice izračunajte napon u čvorištu 2 uz traženu točnost ε=0.01. Nazivni napon mreže je 220 kV. Koristiti baznu snagu S_B=100 MVA te napon čvorišta 2 u nultoj iteraciji $U_2^{(0)}$=220 kV.

- **RJEŠENJE:**

$$Z_{12-V1} = jX_{V1} \cdot \frac{S_B}{U_n^2} = j0.066\ p.u.$$

$$Z_{12-V2} = jX_{V2} \cdot \frac{S_B}{U_n^2} = j0.062\ p.u.$$

$$Y_{12-V1} = \frac{1}{Z_{12-V1}} = -j15.125\ p.u.$$

$$Y_{12-V2} = \frac{1}{Z_{12-V2}} = -j16.133\ p.u.$$

$$Y_{0-V1}/2 = \frac{jB_{V1}}{2} \cdot \frac{U_n^2}{S_B} = j0.053\ p.u.$$

$$Y_{0-V2}/2 = \frac{jB_{V2}}{2} \cdot \frac{U_n^2}{S_B} = j0.058\ p.u.$$

$$Y_{(2)} = \begin{bmatrix} Y_{12-V1} + Y_{12-V2} & -\left(Y_{12-V1} + Y_{12-V2}\right) \\ -\left(Y_{12-V1} + Y_{12-V2}\right) & Y_{12-V1} + Y_{12-V2} \end{bmatrix} = \begin{bmatrix} -j31.258 & j31.258 \\ j31.258 & -j31.258 \end{bmatrix} p.u.$$

$$Y = \left[Y_{12-V1} + Y_{12-V2}\right] = \left[-j31.258\right]$$

$$Z = Y^{-1} = j0.032\ p.u.$$

$$Y' = \left[\left(Y_{0-V1}/2\right) + \left(Y_{0-V2}/2\right)\right] = \left[\,j0.111\right] p.u.$$

$$U_1 = 1 + j0 \ p.u.$$

$$S_2 = -1.2 - j0.3 \ p.u.$$

$$U_2^{(0)} = 1 + j0 \ p.u.$$

$$I_2^{(0)} = \frac{S_2^*}{\left(U_2^{(0)}\right)^*} - Y'_2 \cdot U_2^{(0)} = -1.2 + j0.189 \ p.u.$$

$$U_2^{(1)} = U_1 + Z_{22} \cdot I_2^{(0)} = 0.994 - j0.038 \ p.u.$$

$$I_2^{(1)} = \frac{S_2^*}{\left(U_2^{(1)}\right)^*} - Y'_2 \cdot U_2^{(1)} = -1.198 + j0.237 \ p.u.$$

$$\left| U_2^{(1)} - U_2^{(0)} \right| = 0.039$$

$$U_2^{(2)} = 0.992 - j0.038 \ p.u.$$

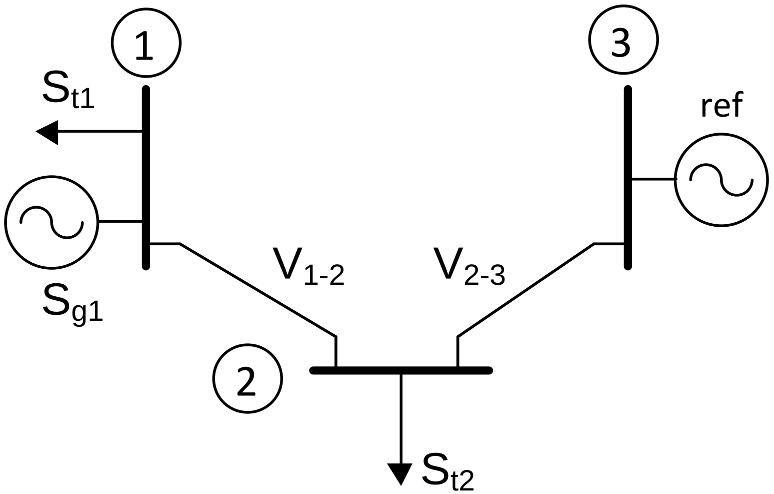
$$I_2^{(2)} = -1.2 + j0.238 \ p.u$$

$$\left| U_2^{(2)} - U_2^{(1)} \right| = 1.556 \cdot 10^{-3}$$

$$U_2^{(2)} = 218.493 \angle -2.2^\circ$$

- **ZADATAK 2.** Za mrežu zadanu slikom odredite napone u prvoj iteraciji koristeći metodu Gauss-Seidel pomoću Z matrice (S_B=100 MVA). Vodovi V_{1-2} i V_{2-3} su jednaki te imaju sljedeće parametre:

Vodovi V_{1-2} i V_{2-3}
U_n= 110 kV
R_1= 0 Ω/km
X_1= 0.40 Ω/km
B_1= 0 μS/km
l= 100 km



Napon u čvorištu 3 je poznat te iznosi U_3=110∠0° kV. Za napone u čvorištima 1 i 2 koristite početne vrijednosti $U_1^{(0)}=U_2^{(0)}=110\angle 0°$ kV.

Snage trošila i generatora u čvorištima 1 i 2 su poznate i iznose:

S_{1t}= 30+j10 MVA

S_{2t}= 50+j15 MVA

S_{1g}= 20+j6 MVA

- **RJEŠENJE:**

$$Z_{12} = Z_{23} = jX_1 \cdot l \cdot \frac{S_B}{U_n^2} = j0.331\ p.u.$$

$$Y_{12} = Y_{23} = \frac{1}{Z_{12}} = -j3.025\ p.u.$$

$$Y_{(3)} = \begin{bmatrix} Y_{12} & -Y_{12} & 0 \\ -Y_{12} & Y_{12}+Y_{23} & -Y_{23} \\ 0 & -Y_{23} & Y_{23} \end{bmatrix} = \begin{bmatrix} -j3.025 & j3.025 & 0 \\ j3.025 & -j6.05 & j3.025 \\ 0 & j3.025 & -j3.025 \end{bmatrix} p.u.$$

$$Y = \begin{bmatrix} -j3.025 & j3.025 \\ j3.025 & -j6.05 \end{bmatrix} p.u.$$

$$Z = Y^{-1} = \begin{bmatrix} j0.661 & j0.331 \\ j0.331 & j0.331 \end{bmatrix} p.u.$$

$$Y' = \begin{bmatrix} 0 \\ 0 \end{bmatrix} p.u.$$

$$U_3 = 1 + j0 \ p.u.$$

$$S_1 = -0.3 - j0.1 + 0.2 + j0.06 = -0.1 + j0.04 \ p.u.$$

$$S_2 = -0.5 - j0.15 \ p.u.$$

$$U_1^{(0)} = U_2^{(0)} = 1 + j0 \ p.u.$$

$$I_1^{(0)} = \frac{S_1^*}{\left(U_1^{(0)}\right)^*} - Y'_1 \cdot U_1^{(0)} = -0.1 + j0.04 \ p.u.$$

$$I_2^{(0)} = \frac{S_2^*}{\left(U_2^{(0)}\right)^*} - Y'_2 \cdot U_2^{(0)} = -0.5 + j0.15 \ p.u.$$

$$U_1^{(1)} = U_3 + Z_{11} \cdot I_1^{(0)} + Z_{12} \cdot I_2^{(0)} = 0.924 - j0.231 \ p.u.$$

$$I_1^{(1)} = \frac{S_1^*}{\left(U_1^{(1)}\right)^*} - Y'_1 \cdot U_1^{(1)} = -0.092 + j0.066 \ p.u.$$

$$U_2^{(1)} = U_3 + Z_{21} \cdot I_1^{(1)} + Z_{22} \cdot I_2^{(0)} = 0.929 - j0.196 \ p.u.$$

$$I_2^{(1)} = \frac{S_2^*}{\left(U_2^{(1)}\right)^*} - Y'_2 \cdot U_2^{(1)} = -0.483 + j0.263 \ p.u.$$

$$U_1^{(1)} = 104.775 \angle -14.1^\circ \ kV$$

$$U_2^{(1)} = 104.378 \angle -11.9^\circ \ kV$$

Analiza elektroenergetskog sustava

Auditorne vježbe 05

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2013./2014.

Proračun tokova snaga

- Metoda Gauss-Seidel pomoću Y-matrice u iteraciji ***k+1***:

$$KL_i = \frac{\vec{S}_i^*}{\vec{Y}_{ii}} \qquad i = 1, 2, \ldots, n; \quad i \neq ref.$$

$$YL_{i,j} = \frac{\vec{Y}_{ij}}{\vec{Y}_{ii}} \qquad i = 1, 2, \ldots, n; \quad i \neq ref.; \quad j = 1, 2, \ldots, n; \quad j \neq i$$

$$\vec{U}_i^{(k+1)} = KL_i \cdot \frac{1}{\vec{U}_i^{*(k)}} - \sum_{j=1}^{i-1} YL_{i,j} \cdot \vec{U}_j^{(k+1)} - \sum_{j=i+1}^{n} YL_{i,j} \cdot \vec{U}_j^{(k)}$$

- Korištenjem faktora ubrzanja ***α***:

$$\vec{U}_{i-ubr}^{(k+1)} = \vec{U}_i^{(k)} + \alpha \cdot \left(\vec{U}_i^{(k+1)} - \vec{U}_i^{(k)} \right)$$

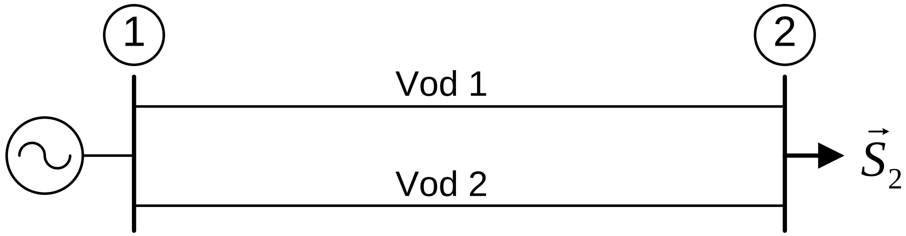
- Uvjet točnosti:

$$\left| \left(\vec{U}_i^{(k+1)} - \vec{U}_i^{(k)} \right) \right| < \varepsilon \qquad za\ i = 1, 2, \ldots, n; \quad i \neq ref.$$

$$\left| \left(\vec{U}_i^{(k+1)} - \vec{U}_{i-ubr}^{(k)} \right) \right| < \varepsilon \quad za\ \alpha \neq 1$$

Zadatci

- **ZADATAK 1.** Za mrežu prikazanu slikom su poznati sljedeći podatci:



	R [Ω]	X [Ω]	B [mS]
Vod 1	0	32	0.22
Vod 2	0	30	0.24

Napon u čvorištu 1 je poznat i iznosi U_1=220 kV, a snaga u čvorištu 2 iznosi S_2=-120-j30 MVA. Korištenjem metode Gauss-Seidel pomoću Y matrice izračunajte napon u čvorištu 2 uz traženu točnost ε=0.01. Nazivni napon mreže je 220 kV. Koristiti baznu snagu S_B=100 MVA te napon čvorišta 2 u nultoj iteraciji $U_2^{(0)}$=220 kV.

- **RJEŠENJE:**

$$Z_{12-V1} = jX_{V1} \cdot \frac{S_B}{U_n^2} = j0.066\ p.u.$$

$$Z_{12-V2} = jX_{V2} \cdot \frac{S_B}{U_n^2} = j0.062\ p.u.$$

$$Y_{12-V1} = \frac{1}{Z_{12-V1}} = -j15.125\ p.u.$$

$$Y_{12-V2} = \frac{1}{Z_{12-V2}} = -j16.133\ p.u.$$

$$Y_{0-V1}/2 = \frac{jB_{V1}}{2} \cdot \frac{U_n^2}{S_B} = j0.053\ p.u.$$

$$Y_{0-V2}/2 = \frac{jB_{V2}}{2} \cdot \frac{U_n^2}{S_B} = j0.058\ p.u.$$

$$Y = \begin{bmatrix} Y_{12-V1} + Y_{12-V2} + \left(Y_{0-V1}/2\right) + \left(Y_{0-V2}/2\right) & -\left(Y_{12-V1} + Y_{12-V2}\right) \\ -\left(Y_{12-V1} + Y_{12-V2}\right) & Y_{12-V1} + Y_{12-V2} + \left(Y_{0-V1}/2\right) + \left(Y_{0-V2}/2\right) \end{bmatrix}$$

$$Y = \begin{bmatrix} -j31.147 & j31.258 \\ j31.258 & -j31.147 \end{bmatrix} p.u.$$

$$U_1 = 1 + j0 \ p.u.$$

$$S_2 = -1.2 - j0.3 \ p.u.$$

$$U_2^{(0)} = 1 + j0 \ p.u.$$

$$KL_2 = \frac{S_2^*}{Y_{2,2}} = -9.632 \cdot 10^{-3} - j0.039$$

$$YL_{21} = \frac{Y_{2,1}}{Y_{2,2}} = -1.004$$

$$U_2^{(1)} = \frac{KL_2}{\left(U_2^{(0)}\right)^*} - YL_{21} \cdot U_1 = 0.994 - j0.039 \ p.u.$$

$$\left|U_2^{(1)} - U_2^{(0)}\right| = 0.039$$

$$U_2^{(2)} = \frac{KL_2}{\left(U_2^{(1)}\right)^*} - YL_{21} \cdot U_1 = 0.992 - j0.038 \ p.u.$$

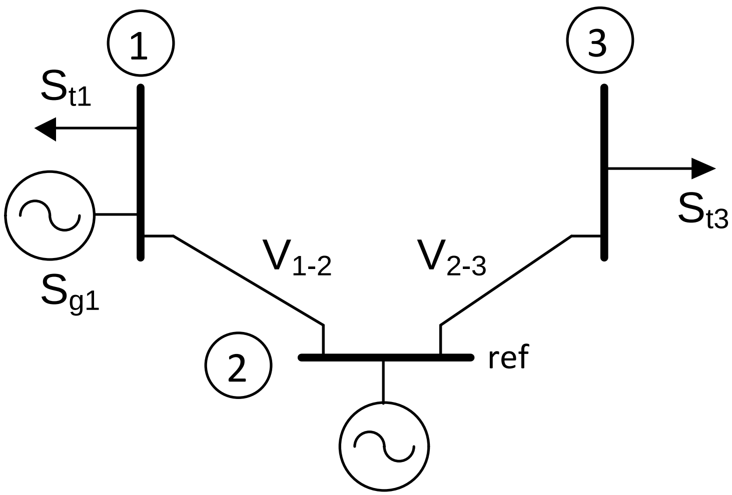
$$\left|U_2^{(2)} - U_2^{(1)}\right| = 1.557 \cdot 10^{-3}$$

$$U_2^{(2)} = 218.49\angle -2.2^\circ \ kV$$

- **ZADATAK 2.** Za mrežu zadanu slikom odredite napone u prvoj iteraciji koristeći metodu Gauss-Seidel pomoću Y matrice.

Vodovi V_{1-2} i V_{2-3} su jednaki te imaju sljedeće parametre:

Vodovi V_{1-2} i V_{2-3}
U_n= 110 kV
R_1= 0.15 Ω/km
X_1= 0.40 Ω/km
B_1= 2.7 μS/km
l= 50 km



Napon u čvorištu 2 je poznat te iznosi U_2=115 kV. Za napone u čvorištima 1 i 3 koristite početne vrijednosti $U_1^{(0)}=U_3^{(0)}=110\angle 0°$ kV.

Snage trošila i generatora u čvorištima 1 i 3 su poznate i iznose:

S_{1t}= 50+j15 MVA

S_{3t}= 40+j10 MVA

S_{1g}= 20+j5 MVA

Koristiti faktor ubrzanja α=1.2 i baznu snagu S_B=100 MVA.

- **RJEŠENJE:**

$$Z_{12} = Z_{23} = jX_1 \cdot l \cdot \frac{S_B}{U_n^2} = 0.062 + j0.165 \ p.u.$$

$$Y_{12} = Y_{23} = \frac{1}{Z_{12}} = 1.989 - j5.304 \ p.u.$$

$$Y_{0-12}/2 = Y_{0-23}/2 = \frac{jB_1 \cdot l}{2} \cdot \frac{U_n^2}{S_B} = j8.167 \cdot 10^{-3} \ p.u.$$

$$Y = \begin{bmatrix} 1.989 - j5.296 & -1.989 + j5.304 & 0 \\ -1.989 + j5.304 & 3.978 - j10.592 & -1.989 + j5.304 \\ 0 & -1.989 + j5.304 & 1.989 - j5.296 \end{bmatrix} p.u.$$

$$KL_1 = \frac{S_1^*}{Y_{11}} = -0.035 - j0.043$$

$$KL_3 = \frac{S_3^*}{Y_{33}} = -0.041 - j0.06$$

$$YL_{12} = \frac{Y_{12}}{Y_{11}} = -1.001 + j5.076 \cdot 10^{-3}$$

$$YL_{13} = \frac{Y_{13}}{Y_{11}} = 0$$

$$YL_{31} = \frac{Y_{31}}{Y_{33}} = 0$$

$$YL_{32} = \frac{Y_{32}}{Y_{33}} = -1.001 + j5.076 \cdot 10^{-3}$$

$$U_1^{(0)} = 1 + j0 \ p.u.$$

$$U_2 = \frac{115}{110} = 1.045 \ p.u.$$

$$U_3^{(0)} = 1 + j0 \ p.u.$$

$$S_1 = -0.3 - j0.1 \ p.u.$$

$$S_3 = -0.4 - j0.1 \ p.u.$$

$$U_1^{(1)} = \frac{KL_1}{\left(U_1^{(0)}\right)^*} - YL_{12} \cdot U_2 - YL_{13} \cdot U_3^{(0)} = 1.012 - j0.044 \ p.u.$$

$$U_{1-ubr}^{(1)} = U_1^{(0)} + \alpha \cdot \left(U_1^{(1)} - U_1^{(0)}\right) = 1.014 - j0.053 \ p.u.$$

$$U_3^{(1)} = \frac{KL_3}{\left(U_3^{(0)}\right)^*} - YL_{31} \cdot U_{1-ubr}^{(1)} - YL_{32} \cdot U_2 = 1.005 - j0.061 \ p.u.$$

$$U_1^{(1)} = 111.389 \angle -2.488^\circ \ kV$$

$$U_3^{(1)} = 110.801 \angle -3.444^\circ \ kV$$

Proračun tokova snaga

- Proračun injekcija snaga u čvorištima:

$$\vec{S}_i = \vec{U}_i \cdot \vec{I}_i^* = P_i + jQ_i$$

$$\vec{U}_i = \left|\vec{U}_i\right| \cdot e^{j\delta_i}$$

Pogledati predavanje 4., slajdovi 15.-18.

$$\vec{I}_i = \sum_{j=1}^{n} \vec{Y}_{ij} \cdot \vec{U}_j \quad \Rightarrow \quad \vec{I}_i^* = \sum_{j=1}^{n} \vec{Y}_{ij}^* \cdot \vec{U}_j^* \qquad \vec{Y}_{ij} = \left|\vec{Y}_{ij}\right| \cdot e^{j\Theta_{ij}} \quad ; \quad \vec{Y}_{ij}^* = \left|\vec{Y}_{ij}\right| \cdot e^{-j\Theta_{ij}}$$

$$\vec{S}_i = \vec{U}_i \cdot \vec{I}_i^* = \vec{U}_i \cdot \sum_{j=1}^{n} \vec{Y}_{ij}^* \cdot \vec{U}_j^* = \left|\vec{U}_i\right| \cdot e^{j\delta_i} \cdot \sum_{j=1}^{n} \left|\vec{Y}_{ij}\right| \cdot e^{-j\Theta_{ij}} \cdot \left|\vec{U}_j\right| \cdot e^{-j\delta_j}$$

$$\vec{S}_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{Y}_{ij}\right| \cdot \left|\vec{U}_j\right| \cdot e^{j(\delta_i - \delta_j - \Theta_{ij})}$$

$$\boxed{\vec{S}_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \left[\cos\left(\delta_i - \delta_j - \Theta_{ij}\right) + j\sin(\delta_i - \delta_j - \Theta_{ij})\right]}$$

Proračun tokova snaga

- Proračun injekcija snaga u čvorištima:
  - Djelatna snaga u čvorištu ***i***:

$$P_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \cos\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

  - Jalova snaga u čvorištu ***i***:

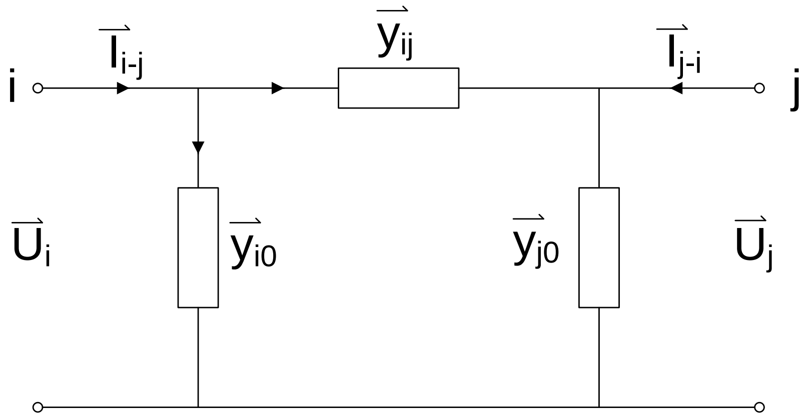
$$Q_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \sin\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

  - Gubitci u mreži:

$$P_g = \sum_{i=1}^{n} P_i$$

Proračun tokova snaga

- Proračun tokova snaga u granama:



$$\vec{S}_{i-j} = \vec{U}_i \cdot \vec{I}^*_{i-j} = \vec{U}_i \left[\left(\vec{U}^*_i - \vec{U}^*_j \right) \cdot \vec{y}^*_{i-j} + \vec{U}^*_i \cdot \vec{y}^*_{i0} \right]$$

$$\vec{S}_{j-i} = \vec{U}_j \cdot \vec{I}^*_{j-i} = \vec{U}_j \left[\left(\vec{U}^*_j - \vec{U}^*_i \right) \cdot \vec{y}^*_{i-j} + \vec{U}^*_j \cdot \vec{y}^*_{j0} \right]$$

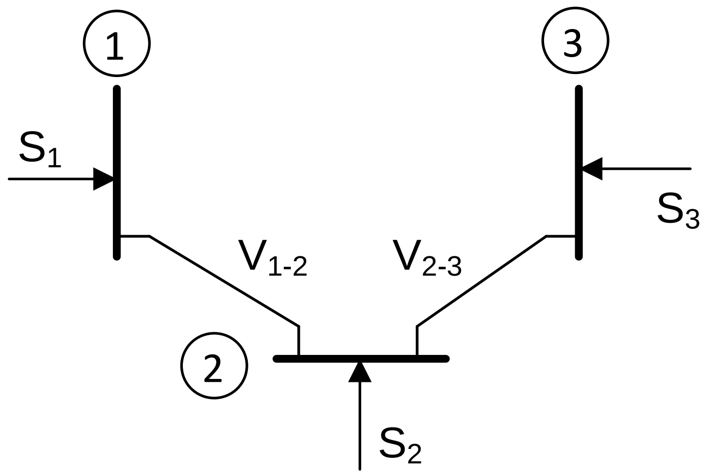
- Gubitci snage:

$$\Delta\vec{S} = \vec{S}_{i-j} + \vec{S}_{j-i}$$

$$\Delta\vec{S} = \left(\vec{U}^*_i - \vec{U}^*_j \right) \cdot \vec{y}^*_{i-j} \cdot \left(\vec{U}_i - \vec{U}_j \right) + \left| \vec{U}_i \right|^2 \cdot \vec{y}^*_{i0} + \left| \vec{U}_j \right|^2 \cdot \vec{y}^*_{j0}$$

- **ZADATAK 3.** Za mrežu zadanu slikom izračunajte:
  a) Injekcije djelatne i jalove snage u svim čvorištima (u MW)
  b) Ukupne gubitke djelatne snage u mreži (u MW).
  c) Tokove snage u granama mreže.

Vodovi V_{1-2} i V_{2-3}
U_n= 110 kV
R_1= 0.15 Ω/km
X_1= 0.40 Ω/km
B_1= 2.7 μS/km
l= 50 km



Naponi u čvorištima mreže su poznati i iznose:

U_1= 111.231 ∟-2.381° kV

U_2= 115 ∟0° kV

U_3= 110.432 ∟-3.302° kV

- **RJEŠENJE:**

a)

$$Z_{12}=Z_{23}=jX_1\cdot l\cdot\frac{S_B}{U_n^2}=0.062+j0.165\ p.u.$$

$$Y_{12}=Y_{23}=\frac{1}{Z_{12}}=1.989-j5.304\ p.u.$$

$$Y_{0-12}/2=Y_{0-23}/2=\frac{jB_1\cdot l}{2}\cdot\frac{U_n^2}{S_B}=j8.167\cdot10^{-3}\ p.u.$$

$$Y=\begin{bmatrix}1.989-j5.296 & -1.989+j5.304 & 0\\ -1.989+j5.304 & 3.978-j10.592 & -1.989+j5.304\\ 0 & -1.989+j5.304 & 1.989-j5.296\end{bmatrix}p.u.$$

$$Y=\begin{bmatrix}5.657\angle290.585° & 5.665\angle110.556° & 0\\ 5.665\angle110.556° & 11.314\angle290.585° & 5.665\angle110.556°\\ 0 & 5.665\angle110.556° & 5.657\angle290.585°\end{bmatrix}p.u.$$

$$U_1=1.011\angle-2.381°$$

$$U_2=1.045\angle0°$$

$$U_3=1.004\angle-3.302°$$

$$P_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \cos\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

$$P_1 = -0.30005\ p.u. = -30.005\ MW$$

$$P_2 = 0.71648\ p.u. = 71.648\ MW$$

$$P_3 = -0.4001\ p.u. = -40.010\ MW$$

$$Q_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \sin\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

$$Q_1 = -0.09995\ p.u. = -9.995\ M\text{var}$$

$$Q_2 = 0.20893\ p.u. = 20.893\ M\text{var}$$

$$Q_3 = -0.09988\ p.u. = -9.988\ M\text{var}$$

b)

$$P_g = \sum_{i=1}^{n} P_i = P_1 + P_2 + P_3$$

$$P_g = 0.01633\ p.u. = 1.633\ MW$$

c)

$$\vec{S}_{i-j} = \vec{U}_i \cdot \vec{I}^{\,*}_{i-j} = \vec{U}_i \left[\left(\vec{U}^{\,*}_i - \vec{U}^{\,*}_j \right) \cdot \vec{y}^{\,*}_{i-j} + \vec{U}^{\,*}_i \cdot \vec{y}^{\,*}_{i0} \right]$$

$$\vec{S}_{j-i} = \vec{U}_j \cdot \vec{I}^{\,*}_{j-i} = \vec{U}_j \left[\left(\vec{U}^{\,*}_j - \vec{U}^{\,*}_i \right) \cdot \vec{y}^{\,*}_{i-j} + \vec{U}^{\,*}_j \cdot \vec{y}^{\,*}_{j0} \right]$$

$$S_{12} = -0.3005 - j0.09995\ p.u. = -30.005 - j9.995\ MVA$$

$$S_{21} = 0.30601 + j0.09858\ p.u. = 30.601 + j9.858\ MVA$$

$$\Delta S_{12} = 0.597 - j0.137\ MVA$$

$$S_{23} = 0.41047 + j0.11035\ p.u. = 41.047 + j11.035\ MVA$$

$$S_{32} = -0.4001 - j0.09988\ p.u. = -40.01 - j9.988\ MVA$$

$$\Delta S_{23} = 1.036 + j1.047\ MVA$$

$$\Delta S = \Delta S_{12} + \Delta S_{23} = 1.633 + j0.91\ MVA$$

Analiza elektroenergetskog sustava

Auditorne vježbe 06

Prof. dr. sc. Ivica Pavić

Prof. dr. sc. Marko Delimar

2013./2014.

Proračun tokova snaga

- Metoda Newton-Raphson u iteraciji ***k+1***:

$$P_{ira\check{c}}^{(k)} = \sum_{j=1}^{n} U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{ij} \cdot cos\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \qquad i = 1, 2, ..., n-1$$

$$Q_{ira\check{c}}^{(k)} = \sum_{j=1}^{n} U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{ij} \cdot sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \qquad i = 1, 2, ..., n-1-g$$

$$\Delta P_i^{(k)} = P_{izad} - P_{ira\check{c}}^{(k)} \qquad i = 1, 2, ..., n-1$$

$$\Delta Q_i^{(k)} = Q_{izad} - Q_{ira\check{c}}^{(k)} \qquad i = 1, 2, ..., n-1-g$$

$$\begin{vmatrix} \Delta\delta^{(k)} \\ \Delta U^{(k)} \end{vmatrix} = \begin{vmatrix} J_1^{(k)} & J_2^{(k)} \\ J_3^{(k)} & J_4^{(k)} \end{vmatrix}^{-1} \cdot \begin{vmatrix} \Delta P_{ra\check{c}}^{(k)} \\ \Delta Q_{ra\check{c}}^{(k)} \end{vmatrix}$$

$$U_i^{(k+1)} = U_i^{(k)} + \Delta U_i^{(k)}$$

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)}$$

Proračun tokova snaga

- Uvjet točnosti:

$$\left|\Delta P_i^{(k)}\right| < \varepsilon \qquad i = 1, 2, \ldots, n-1$$

$$\left|\Delta Q_i^{(k)}\right| < \varepsilon \qquad i = 1, 2, \ldots, n-1-g$$

- Jakobijeva matrica:

$$\begin{vmatrix} \Delta P_{rač}^{(k)} \\ \Delta Q_{rač}^{(k)} \end{vmatrix} = \left| J^{(k)} \right| \cdot \begin{vmatrix} \Delta \delta^{(k)} \\ \Delta U^{(k)} \end{vmatrix} = \begin{vmatrix} J_1^{(k)} & J_2^{(k)} \\ J_3^{(k)} & J_4^{(k)} \end{vmatrix} \cdot \begin{vmatrix} \Delta \delta^{(k)} \\ \Delta U^{(k)} \end{vmatrix}$$

$$\begin{vmatrix} J_1^{(k)} & J_2^{(k)} \\ J_3^{(k)} & J_4^{(k)} \end{vmatrix} = \begin{vmatrix} \left(\dfrac{\partial P}{\partial \delta}\right)^{(k)} & \left(\dfrac{\partial P}{\partial U}\right)^{(k)} \\ \left(\dfrac{\partial Q}{\partial \delta}\right)^{(k)} & \left(\dfrac{\partial Q}{\partial U}\right)^{(k)} \end{vmatrix}$$

Proračun tokova snaga

- Računanje elemenata Jakobijeve matrice:
- Podmatrica $J_1^{(k)}$:

$$\left(\frac{\partial P_i}{\partial \delta_i}\right)^{(k)} = -U_i^{(k)} \cdot \sum_{\substack{j=1 \\ j\neq i}}^{n} U_j^{(k)} \cdot Y_{ij} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \qquad i = 1, 2, ..., n\, ;\, i \neq ref$$

$$\left(\frac{\partial P_i}{\partial \delta_j}\right)^{(k)} = U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{ij} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \qquad i = 1, 2, ..., n\, ;\, i \neq ref$$

$$j = 1, 2, ..., n\, ;\; j \neq ref$$

- Podmatrica $J_2^{(k)}$:

$$\left(\frac{\partial P_i}{\partial U_i}\right)^{(k)} = 2 \cdot U_i^{(k)} \cdot Y_{ii} \cdot \cos\left(-\Theta_{ii}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{n} U_j^{(k)} \cdot Y_{ij} \cdot \cos\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \qquad i = 1, 2, ..., n\, ;\, i \neq ref$$

$$\left(\frac{\partial P_i}{\partial U_j}\right)^{(k)} = U_i^{(k)} \cdot Y_{ij} \cdot \cos\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \qquad i = 1, 2, ..., n\, ;\, i \neq ref\, ;\; j = 1, 2, ..., n$$

$$j \neq ref\; ;\; j \neq gen$$

Proračun tokova snaga

- Računanje elemenata Jakobijeve matrice:
- Podmatrica $J_3^{(k)}$:

$$\left(\frac{\partial Q_i}{\partial \delta_i}\right)^{(k)} = U_i^{(k)} \cdot \sum_{\substack{j=1 \\ j\neq i}}^{n} U_j^{(k)} \cdot Y_{ij} \cdot \cos\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i=1,2,\dots,n;\ i\neq ref\ ;\ i\neq gen$$

$$\left(\frac{\partial Q_i}{\partial \delta_j}\right)^{(k)} = -U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{ij} \cdot \cos\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i=1,2,\dots,n;\ i\neq ref\ ;\ i\neq gen$$

$$j=1,2,\dots,n;\ j\neq ref$$

- Podmatrica $J_4^{(k)}$:

$$\left(\frac{\partial Q_i}{\partial U_i}\right)^{(k)} = 2\cdot U_i^{(k)} \cdot Y_{ii} \cdot \sin\left(-\Theta_{ii}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{n} U_j^{(k)} \cdot Y_{ij} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i=1,2,\dots,n;\ i\neq ref\ ;\ i\neq gen$$

$$\left(\frac{\partial Q_i}{\partial U_j}\right)^{(k)} = U_i^{(k)} \cdot Y_{ij} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i=1,2,\dots,n;\ i\neq ref\ ;\ i\neq gen$$

$$j=1,2,\dots,n;\ j\neq ref\ ;\ j\neq gen$$

Proračun tokova snaga

- <u>Uobičajena pojednostavljenja metode Newton-Raphson:</u>
- Ubrzana metoda Newton-Raphson – Jakobijeva matrica se računa samo u prvom koraku (ili u prvih nekoliko koraka)
- Razdvojena metoda Newton-Raphson – zanemaruju se podmatrice J_2 i J_3. U tom slučaju se sustav linearnih jednadžbi može razdvojiti u dva sustava:

$$\left|\Delta\delta^{(k)}\right| = \left|J_1^{(k)}\right|^{-1} \cdot \left|\Delta P_{rač}^{(k)}\right|$$

$$\left|\Delta U^{(k)}\right| = \left|J_4^{(k)}\right|^{-1} \cdot \left|\Delta Q_{rač}^{(k)}\right|$$

- Ubrzana razdvojena metoda Newton-Raphson – kombinacija gorenavedenih metoda. Koriste se samo Jakobijeve podmatrice J_1 i J_4 i to proračunate u prvom koraku.
- Metoda Newton-Raphson korištenjem pravokutnih koordinata

Zadatci

- **ZADATAK 1.** Za mrežu prikazanu slikom odredite numeričke vrijednosti Jakobijeve matrice u nultoj iteraciji:

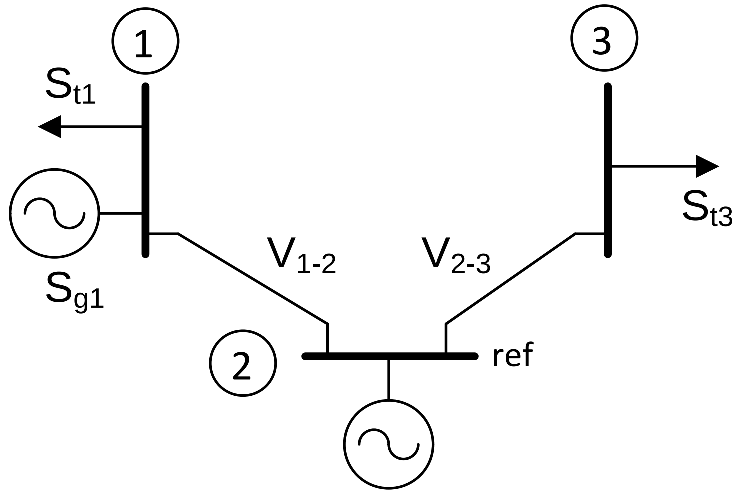
Napon u čvorištu 2 je poznat:

U_2=220 kV

Za napone u čvorištima 1 i 3 u nultoj iteraciji koristite vrijednosti:

$U_1^{(0)}$=218∠-3° kV

$U_3^{(0)}$=215∠-7° kV.



Matrica admitancija čvorišta za navedenu mrežu:

$$Y = \begin{bmatrix} 18.19\, p.u.\angle-84.8° & 18.26\, p.u.\angle95.2° & 0.0\, p.u.\angle0° \\ 18.26\, p.u.\angle95.2° & 32.80\, p.u.\angle-84.6° & 14.61\, p.u.\angle95.2° \\ 0.0\, p.u.\angle0° & 14.61\, p.u.\angle95.2° & 14.61\, p.u.\angle-84.5° \end{bmatrix}$$

- **RJEŠENJE:**

$$U_1^{(0)} = \frac{218}{220} = 0.991\ p.u. \qquad \delta_1 = -3^\circ$$

$$U_2 = \frac{220}{220} = 1.000\ p.u. \qquad \delta_2 = 0^\circ$$

$$U_3^{(0)} = \frac{215}{220} = 0.977\ p.u. \qquad \delta_3 = -7^\circ$$

- Elementi Jakobijeve podmatrice $J_1^{(0)}$:

$$\left(\frac{\partial P_1}{\partial \delta_1}\right)^{(0)} = -U_1^{(0)} \cdot \left(U_2 \cdot Y_{12} \cdot \sin\left(\delta_1^{(0)} - \delta_2 - \Theta_{12}\right) + U_3^{(0)} \cdot Y_{13} \cdot \sin\left(\delta_1^{(0)} - \delta_3^{(0)} - \Theta_{13}\right)\right) = 17.909$$

$$\left(\frac{\partial P_1}{\partial \delta_3}\right)^{(0)} = U_1^{(0)} \cdot U_3^{(0)} \cdot Y_{13} \cdot \sin\left(\delta_1^{(0)} - \delta_3^{(0)} - \Theta_{13}\right) = 0$$

$$\left(\frac{\partial P_3}{\partial \delta_1}\right)^{(0)} = 0$$

$$\left(\frac{\partial P_3}{\partial \delta_3}\right)^{(0)} = 13.955$$

- Elementi Jakobijeve podmatrice $J_2^{(0)}$: :

$$\left(\frac{\partial P_1}{\partial U_1}\right)^{(0)} = 2 \cdot U_1^{(0)} \cdot Y_{11} \cdot \cos(-\Theta_{11}) +$$

$$+ U_2 \cdot Y_{12} \cdot \cos\left(\delta_1^{(0)} - \delta_2 - \Theta_{12}\right) + U_3^{(0)} \cdot Y_{13} \cdot \cos\left(\delta_1^{(0)} - \delta_3^{(0)} - \Theta_{13}\right)$$

$$\left(\frac{\partial P_1}{\partial U_1}\right)^{(0)} = 0.663$$

$$\left(\frac{\partial P_1}{\partial U_3}\right)^{(0)} = U_1^{(0)} \cdot Y_{13} \cdot \cos\left(\delta_1^{(0)} - \delta_3^{(0)} - \Theta_{13}\right)$$

$$\left(\frac{\partial P_1}{\partial U_3}\right)^{(0)} = 0$$

$$\left(\frac{\partial P_3}{\partial U_1}\right)^{(0)} = 0$$

$$\left(\frac{\partial P_3}{\partial U_3}\right)^{(0)} = -0.35$$

- Elementi Jakobijeve podmatrice $J_3^{(0)}$: :

$$\left(\frac{\partial Q_1}{\partial \delta_1}\right)^{(0)} = U_1^{(0)} \cdot \left(U_2^{(0)} \cdot Y_{12} \cdot \cos\left(\delta_1^{(0)} - \delta_2^{(0)} - \Theta_{12}\right) + U_3^{(0)} \cdot Y_{13} \cdot \cos\left(\delta_1^{(0)} - \delta_3^{(0)} - \Theta_{13}\right)\right)$$

$$\left(\frac{\partial Q_1}{\partial \delta_1}\right)^{(0)} = -2.581$$

$$\left(\frac{\partial Q_1}{\partial \delta_3}\right)^{(0)} = -U_1^{(0)} \cdot U_3^{(0)} \cdot Y_{13} \cdot \cos\left(\delta_1^{(0)} - \delta_3^{(0)} - \Theta_{13}\right)$$

$$\left(\frac{\partial Q_1}{\partial \delta_3}\right)^{(0)} = 0$$

$$\left(\frac{\partial Q_3}{\partial \delta_1}\right)^{(0)} = 0$$

$$\left(\frac{\partial Q_3}{\partial \delta_3}\right)^{(0)} = -3.017$$

- Elementi Jakobijeve podmatrice $J_4^{(0)}$:

$$\left(\frac{\partial Q_1}{\partial U_1}\right)^{(0)} = 2\cdot U_1^{(0)}\cdot Y_{11}\cdot\sin\left(-\Theta_{11}\right)+U_2^{(0)}\cdot Y_{12}\cdot\sin\left(\delta_1^{(0)}-\delta_2^{(0)}-\Theta_{12}\right)+$$

$$+U_3^{(0)}\cdot Y_{13}\cdot\sin\left(\delta_1^{(0)}-\delta_3^{(0)}-\Theta_{13}\right)$$

$$\left(\frac{\partial Q_1}{\partial U_1}\right)^{(0)} = 17.828$$

$$\left(\frac{\partial Q_1}{\partial U_3}\right)^{(0)} = U_3^{(0)}\cdot Y_{13}\cdot\sin\left(\delta_1^{(0)}-\delta_3^{(0)}-\Theta_{13}\right)$$

$$\left(\frac{\partial Q_1}{\partial U_3}\right)^{(0)} = 0$$

$$\left(\frac{\partial Q_3}{\partial U_1}\right)^{(0)} = 0$$

$$\left(\frac{\partial Q_3}{\partial U_3}\right)^{(0)} = 14.144$$

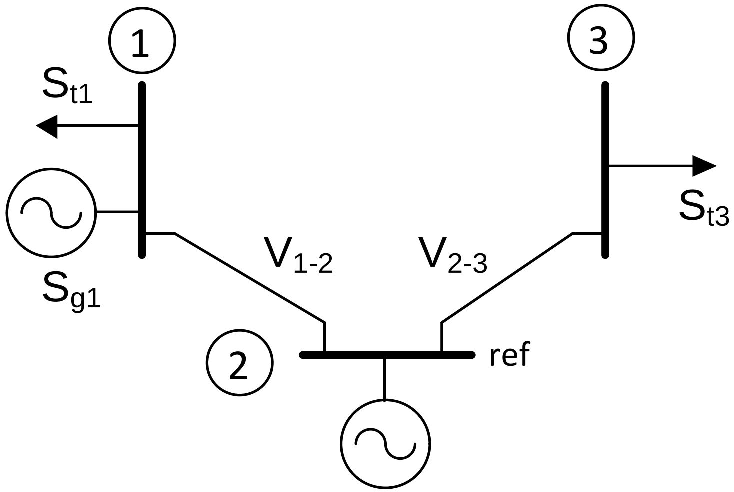
- **ZADATAK 2.** Za mrežu iz prethodnog zadatka odredite napone u prvoj iteraciji koristeći ubrzanu razdvojenu Newton-Raphson metodu.

S_{1t}= 40+j10 MVA

S_{3t}= 50+j20 MVA

S_{1g}= 50+j15 MVA

S_B=100 MVA



Napon u čvorištu 2 je poznat te iznosi U_2=220 kV. Za napone u čvorištima 1 i 3 koristite početne vrijednosti:

$U_1^{(0)}=218\angle -3°$ kV

$U_3^{(0)}=215\angle -7°$ kV.

Matrica admitancija čvorišta, te Jakobijeve podmatrice su zadane u prethodnom zadatku.

- **RJEŠENJE:**

$$U_1^{(0)} = \frac{218}{220} = 0.991\ p.u. \qquad \delta_1 = -3° = -0.052\ rad$$

$$U_2 = \frac{220}{220} = 1.000\ p.u. \qquad \delta_2 = 0° = 0\ rad$$

$$U_3^{(0)} = \frac{215}{220} = 0.977\ p.u. \qquad \delta_3 = -7° = -0.122\ rad$$

$$S_1 = 0.1 + j0.05\ p.u. \qquad P_1 = 0.1\ p.u. \qquad Q_1 = 0.05\ p.u.$$

$$S_3 = -0.5 - j0.2\ p.u. \qquad P_3 = -0.5\ p.u. \qquad Q_3 = -0.2\ p.u.$$

$$P_{1rač}^{(0)} = -0.962\ p.u. \qquad Q_{1rač}^{(0)} = -0.122\ p.u.$$

$$P_{3rač}^{(0)} = -1.680\ p.u. \qquad Q_{3rač}^{(0)} = -0.066\ p.u.$$

$$(\Delta P)^{(0)} = \begin{bmatrix} P_1 - P_{1rač}^{(0)} \\ P_3 - P_{3rač}^{(0)} \end{bmatrix} = \begin{bmatrix} 1.062 \\ 1.18 \end{bmatrix} p.u. \qquad (\Delta Q)^{(0)} = \begin{bmatrix} Q_1 - Q_{1rač}^{(0)} \\ Q_3 - Q_{3rač}^{(0)} \end{bmatrix} = \begin{bmatrix} 0.172 \\ -0.134 \end{bmatrix} p.u.$$

$$\left(J_1^{(0)}\right)^{-1} \approx \left(J_4^{(0)}\right)^{-1} = \begin{bmatrix} 0.056 & 0 \\ 0 & 0.072 \end{bmatrix}$$

$$\Delta\delta^{(0)} = \left(J_1^{(0)}\right)^{-1} \cdot \Delta P^{(0)}$$

$$\Delta\delta^{(0)} = \begin{bmatrix} 0.059 \\ 0.085 \end{bmatrix} rad$$

$$\Delta U^{(0)} = \left(J_4^{(0)}\right)^{-1} \cdot \Delta Q^{(0)}$$

$$\Delta U^{(0)} = \begin{bmatrix} 0.00962 \\ -0.00963 \end{bmatrix} p.u.$$

$$U^{(1)} = U^{(0)} + \Delta U^{(0)}$$

$$U^{(1)} = \begin{bmatrix} 1.001 \\ 0.968 \end{bmatrix} p.u. = \begin{bmatrix} 220.116 \\ 212.882 \end{bmatrix} kV$$

$$\delta^{(1)} = \delta^{(0)} + \Delta\delta^{(0)}$$

$$\delta^{(1)} = \begin{bmatrix} 0.007 \\ -0.037 \end{bmatrix} rad = \begin{bmatrix} 0.407^\circ \\ -2.133^\circ \end{bmatrix}$$

$$U_1^{(1)} = 220.116\angle 0.407^\circ\ kV$$

$$U_3^{(1)} = 212.882\angle -2.133^\circ\ kV$$

Analiza elektroenergetskog sustava

Auditorne vježbe 07

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2013./2014.

Proračun tokova snaga-istosmjerni model

- Pojednostavljeni, približni proračun tokova djelatnih snaga
- Proračun nije iterativan
- Počiva na nekoliko osnovni pretpostavki (zanemarenja):
  - Zanemarene poprečne grane elemenata mreže
  - Zanemaren uzdužni otpor grana (R<<X, R≈0)
  - Razlike između vrijednosti kutova su jako male ($\sin(\delta_i-\delta_j)\approx(\delta_i-\delta_j)$)
  - Iznosi napona su približno isti ($U_i \approx U_j \approx 1.0$ p.u.)
- Uvrštavanjem navedenih pretpostavki u jednadžbe za proračun snaga u čvorištima:

$$P_i = \sum_{j=1}^{n} U_i \cdot U_j \cdot |Y_{ij}| \cdot \cos(-\Theta_{ij} + \delta_i - \delta_j) \quad \Rightarrow \quad P_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} |Y_{ij}| \cdot \sin(\delta_i - \delta_j) = \sum_{j=1}^{n} B_{ij} \cdot \delta_j$$

$$Q_i = \sum_{j=1}^{n} U_i \cdot U_j \cdot |Y_{ij}| \cdot \sin(-\Theta_{ij} + \delta_i - \delta_j) \quad \Rightarrow \quad Q_i = 0$$

$$B_{ij} \quad \rightarrow \quad \textit{element matrice}\, [B]$$

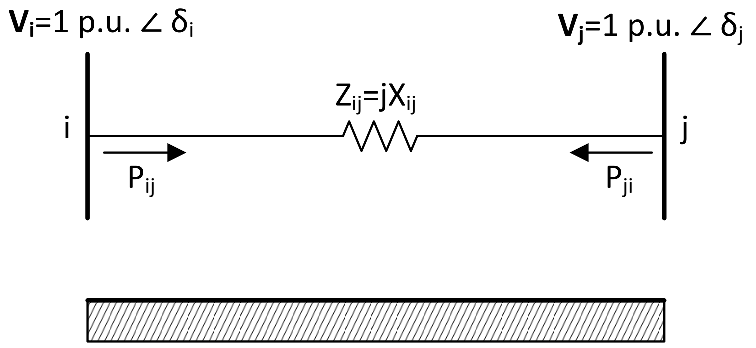
$$[Y] = -j[B] \quad \rightarrow \quad [B] = \left(\frac{1}{-j}\right) \cdot [Y]$$

Proračun tokova snaga-istosmjerni model

$$\begin{bmatrix} P_1 \\ \vdots \\ P_i \\ \vdots \\ P_{n-1} \end{bmatrix} = -j[B]\cdot\begin{bmatrix} \delta_1 \\ \vdots \\ \delta_i \\ \vdots \\ \delta_{n-1} \end{bmatrix} = [Y]\cdot\begin{bmatrix} \delta_1 \\ \vdots \\ \delta_i \\ \vdots \\ \delta_{n-1} \end{bmatrix} \Rightarrow [\delta] = [Y]^{-1}\cdot[P]$$

$$P_{i-j} = (\delta_i - \delta_j)\cdot(-jb_{ij}) = \frac{\delta_i - \delta_j}{jx_{i-j}}$$

$$P_{i-j} = -P_{j-i}$$

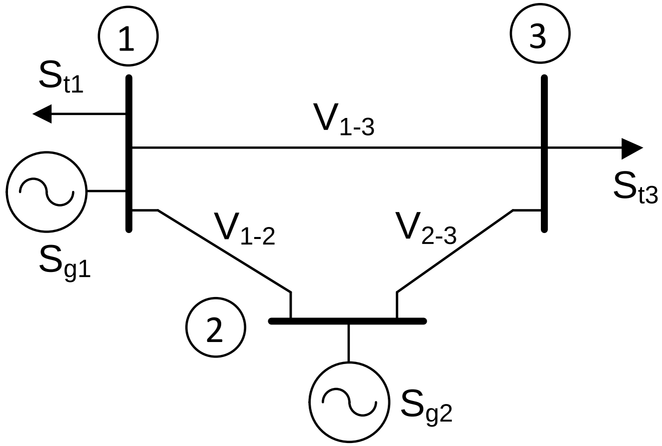


- Nema gubitaka djelatne snage (R=0)
- Nema prijenosa jalove snage

Zadatci

- **ZADATAK 1.** Mreža nazivnog napona 110kV je zadana slikom i sljedećim podatcima:

Vodovi V_{1-2},V_{1-3}, V_{2-3}
U_n= 110 kV
X_1= 0.42 Ω/km
l= 100 km



Potrošači: S_{1t}= 30 MW

S_{3t}= 50 MW

Generator: S_{2g}= 40 MW

Korištenjem istosmjernog modela odredite tokove snaga u granama mreže zadane slikom. Koristiti S_B=100 MVA.

- **RJEŠENJE:**

$$Z_{12} = Z_{13} = Z_{23} = jX_1 \cdot l \cdot \frac{S_B}{U_n^2} = j0.347\ p.u.$$

$$Y_{12} = Y_{13} = Y_{23} = \frac{1}{Z_{12}} = -j2.881\ p.u.$$

$$Y = \begin{bmatrix} Y_{12} + Y_{23} & -Y_{23} \\ -Y_{23} & Y_{13} + Y_{23} \end{bmatrix} p.u. = \begin{bmatrix} -j5.762 & j2.881 \\ j2.881 & -j5.762 \end{bmatrix} p.u. \qquad [Y] = -j[B]$$

$$Z = Y^{-1}$$

$$Z = \begin{bmatrix} j0.231 & j0.116 \\ j0.116 & j0.231 \end{bmatrix} p.u.$$

$$P_2 = 0.4\ p.u.$$

$$P_3 = -0.5\ p.u.$$

$$\begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = [Z] \cdot \begin{bmatrix} P_2 \\ P_3 \end{bmatrix}$$

$\delta_1 = 0$ - Čvorište 1 referentno.

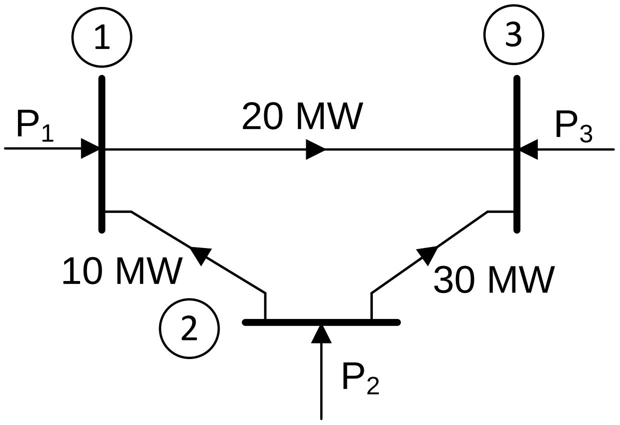
$$\begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} j0.035 \\ -j0.069 \end{bmatrix}$$

$$P_{i-j} = \left(\delta_i - \delta_j\right) \cdot \left(-jb_{ij}\right) = \frac{\delta_i - \delta_j}{jx_{i-j}}$$

$$P_{12} = \frac{\delta_1 - \delta_2}{jX_{12}} = -0.1\ p.u. = -10\ MW$$

$$P_{13} = \frac{\delta_1 - \delta_3}{jX_{13}} = 0.2\ p.u. = 20\ MW$$

$$P_{23} = \frac{\delta_2 - \delta_3}{jX_{23}} = 0.3\ p.u. = 30\ MW$$



Analiza elektroenergetskog sustava

Auditorne vježbe 08

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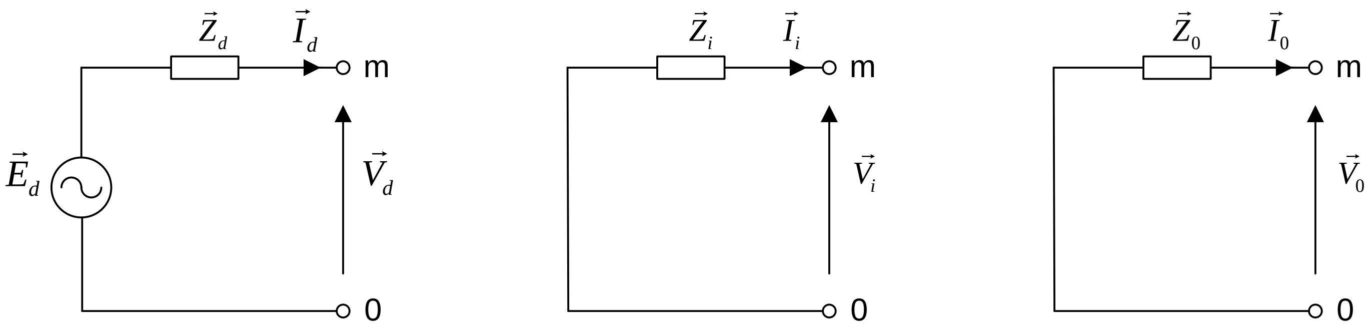
2013./2014.

Proračun kratkog spoja

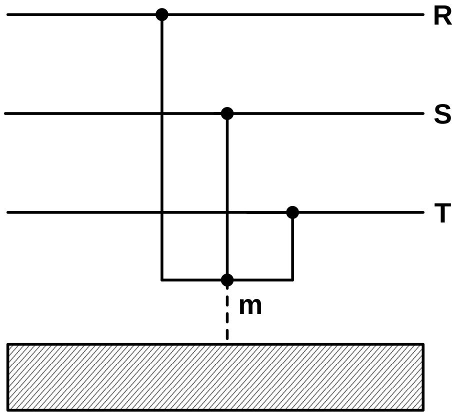
- Glavni cilj proračuna kratkog spoja je određivanje početne struje kratkog spoja I_k'' na samom mjesto kratkog spoja (iz nje je moguće odrediti udarnu, rasklopnu, prijelaznu i trajnu struju KS-a).
- Proračun kratkog spoja se uobičajeno obavlja reduciranjem EE mreže po Theveninovom teoremu u odnosu na čvorište u kojem je nastupio KS i neko referentno (neutralno) čvorište.
- Uz korištenje metode simetričnih komponenti redukcija mreže na Theveninov ekvivalent se provodi za direktnu, inverznu i nultu mrežu.
- Uz simetrično naponsko stanje stanje u izvorima:



$$\vec{V}_d = \vec{E}_d - \vec{I}_d \cdot \vec{Z}_d \qquad \vec{V}_i = -\vec{I}_i \cdot \vec{Z}_i \qquad \vec{V}_0 = -\vec{I}_0 \cdot \vec{Z}_0$$

Proračun kratkog spoja – trofazni kratki spoj

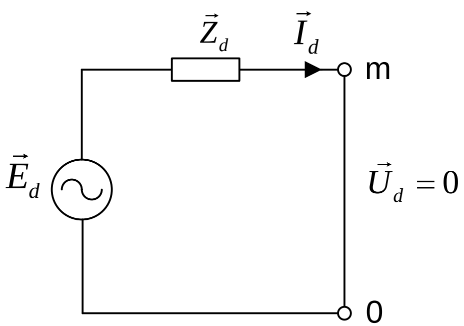
- Trofazni (tropolni) kratki spoj – simetrični kvar

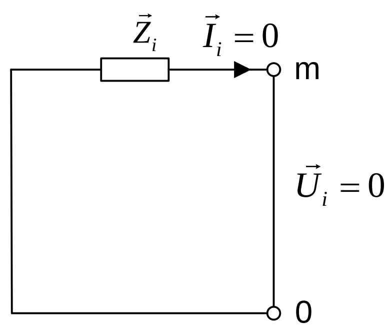


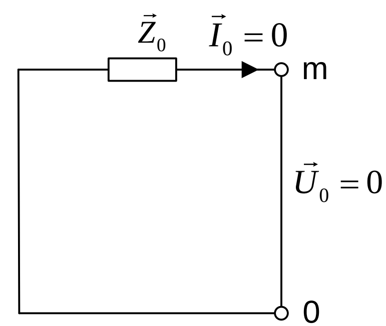
$$\vec{V}_{Rm} = \vec{V}_{Sm} = \vec{V}_{Tm} = 0 \qquad \vec{V}_R = \vec{V}_S = \vec{V}_T$$

$$\vec{I}_{Rm} + \vec{I}_{Sm} + \vec{I}_{Tm} = 0$$

- Izvođenjem iz navedenog slijedi:







$$\vec{I}_{dm} = \frac{\vec{E}_d}{\vec{Z}_{dmm}} = -\frac{{}^{Z}U_m^d}{\vec{Z}_{dmm}}$$

$$\vec{V}_{dm} = 0$$

$$\vec{I}_{im} = 0$$

$$\vec{V}_{im} = 0$$

$$\vec{I}_{0m} = 0$$

$$\vec{V}_{0m} = 0$$

Proračun kratkog spoja – trofazni kratki spoj

- Naponi mreže u kvaru se određuju korištenjem teorema superpozicije:

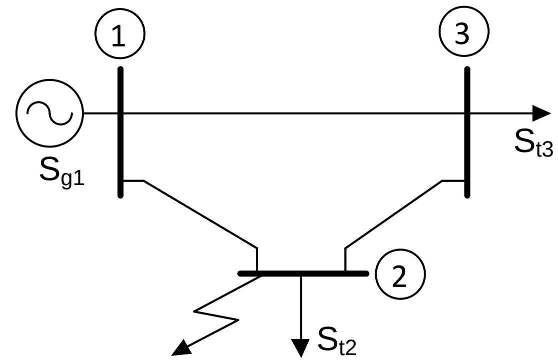
$$\begin{bmatrix} U_1 \\ \vdots \\ 0 \\ \vdots \\ U_n \end{bmatrix}^{kv} = \begin{bmatrix} U_1 \\ \vdots \\ U_m \\ \vdots \\ U_n \end{bmatrix}^{zdr} + \begin{bmatrix} Z_{11} & \cdots & Z_{1n} \\ \vdots & \ddots & \vdots \\ Z_{n1} & \cdots & Z_{nn} \end{bmatrix}_{KS} \cdot \begin{bmatrix} 0 \\ \vdots \\ I_m \\ \vdots \\ 0 \end{bmatrix}^{kv}$$

- Naponi zdrave mreže ($[U]^{zdr}$) su naponi čvorišta mreže u trenutku prije nastanka kvara.
- Ukoliko je mreže u trenutku prije nastanka kratkog spoja bila neopterećena (u praznom hodu) za sve napone zdrave mreže se uzima $U_i^{zdr}=1.0$ p.u. (uobičajeno).
- Matrica impedancija čvorišta [Z] je dimenzija *nxn*, i dobije se inverzijom matrice admitancija čvorišta [Y].

Zadatci

- **ZADATAK 1.** U mreži na slici je nastao tropolni kratki spoj u čvorištu 2. Nazivni napon mreže je 110 kV. Odredite struju kvara (u A) i napone u mreži.

Generator	Vodovi
U_n= 110 kV	U_n= 110 kV
S_n=120 MVA	X_1= 0.41 Ω/km
X_d''=19 %	l= 30 km



- Snage trošila u čvorištima su:

$$S_{2t} = 60+j30 \text{ MVA}$$

$$S_{3t} = 40+j10 \text{ MVA}$$

- Poznati su i naponi u čvorištima u trenutku prije nastanka kvara:

$$U_1 = 111.0 \angle 0° \text{ kV}$$

$$U_2 = 115.1 \angle -5.7° \text{ kV}$$

$$U_3 = 114.8 \angle -7.5° \text{ kV}$$

- **RJEŠENJE:**

$$x_{12} = x_{13} = x_{23} = x_1 \cdot l \cdot \frac{S_B}{U_n^2} = 0.102 \; p.u.$$

$$y_{12} = y_{13} = y_{23} = \frac{1}{j \cdot x_{12}} = -j9.837 \; p.u.$$

$$y_{Ti} = \frac{S_{Ti}^*}{\left|U_i\right|^2}$$

$$y_{T2} = \frac{S_{T2}^*}{\left|U_2\right|^2} = 0.548 - j0.274 \; p.u.$$

$$y_{T3} = \frac{S_{T3}^*}{\left|U_3\right|^2} = 0.367 - j0.092 \; p.u.$$

$$S_{T2} = 0.6 + j0.3 \; p.u.$$

$$S_{T3} = 0.4 + j0.1 \; p.u.$$

$$U_1 = 1.0091 \; p.u.$$

$$U_2 = 1.0464\angle -5.7° \; p.u. = 1.0412 - j0.1039 \; p.u.$$

$$U_3 = 1.0436\angle -7.5° \; p.u. = 1.0347 - j0.1362 \; p.u.$$

$$x_{d1}'' = x_d'' \cdot \frac{S_B}{S_n} = 0.158 \; p.u.$$

$$y_{d1}'' = \frac{1}{j \cdot x_{d1}''} = -j6.316 \; p.u.$$

$$Y=\begin{bmatrix} y_{12}+y_{13}+y_{d1}" & -y_{12} & -y_{13} \\ -y_{12} & y_{12}+y_{23}+y_{T2} & -y_{23} \\ -y_{13} & -y_{23} & y_{13}+y_{23}+y_{T3} \end{bmatrix}$$

$$Y=\begin{bmatrix} -j25.991 & j9.837 & j9.837 \\ j9.837 & 0.548-j19.949 & j9.837 \\ j9.837 & j9.837 & 0.367-j19.767 \end{bmatrix} p.u.$$

$$Z=Y^{-1}=\begin{bmatrix} 0.019+j0.146 & 0.026+j0.142 & 0.025+j0.143 \\ 0.026+j0.142 & 0.035+j0.204 & 0.033+j0.172 \\ 0.025+j0.143 & 0.033+j0.172 & 0.033+j0.207 \end{bmatrix} p.u.$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}^{kv}=\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}^{zdr}+Z\cdot\begin{bmatrix} 0 \\ I_2 \\ 0 \end{bmatrix}^{kv} \quad \Rightarrow U_2^{kv}=0=U_2^{zdr}+Z_{2,2}\cdot I_2 \quad \Rightarrow I_2=-\frac{U_2^{zdr}}{Z_{2,2}}$$

$$I_2^d = I_2 = -\frac{U_2^{zdr}}{Z_{2,2}} = \frac{-1.0412 + j0.1039}{0.035 + j0.204} = -0.348 + j5.049 \; p.u.$$

$$I_{KV} = -I_2 \cdot \frac{S_B}{\sqrt{3} \cdot U_n} = 0.183 - j2.65 \, kA = 2.657 \angle -86.057^\circ \, kA$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}^{kv} = \begin{bmatrix} 0.283 + j0.08 \\ 0 \\ 0.157 - j0.028 \end{bmatrix} p.u. = \begin{bmatrix} 32.337\angle 15.829^\circ \, kV \\ 0 \\ 17.558\angle -10.208^\circ \, kV \end{bmatrix}$$

Analiza elektroenergetskog sustava

Auditorne vježbe 09

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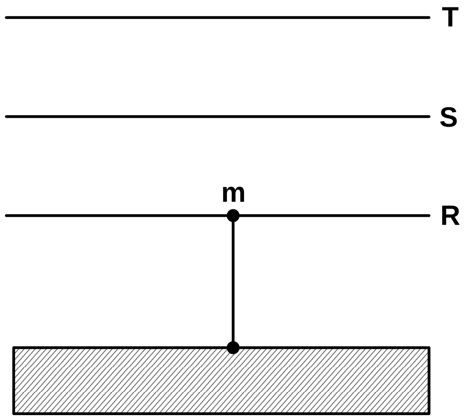
2013./2014.

Proračun kratkog spoja

- Za potrebe proračuna struja i napona kod nesimetričnog kratkog spoja u mreži potrebno je korištenjem metode simetričnih komponenti, izuzev direktnog modela mreže, odrediti i inverzni te nulti model mreže:
  - 3f KS : direktni model mreže
  - 2f KS : direktni i inverzni model mreže
  - 1f KS, 2f KS sa zemljom : direktni, inverzni i nulti model mreže
- Trofazni model mreže se nadomješta jednofaznim modelima u sustavu simetričnih komponenti (direktni, inverzni i nulti)
- Na temelju tih modela se određuju matrice admitancija čvorišta, odnosno matrice impedancija čvorišta za svaki sustav zasebno .

Proračun kratkog spoja – jednofazni kratki spoj

- Jednofazni (jednopolni) kratki spoj:



$$\vec{V}_{Rm} = 0$$

$$\vec{I}_{Sm} = \vec{I}_{Tm} = 0$$

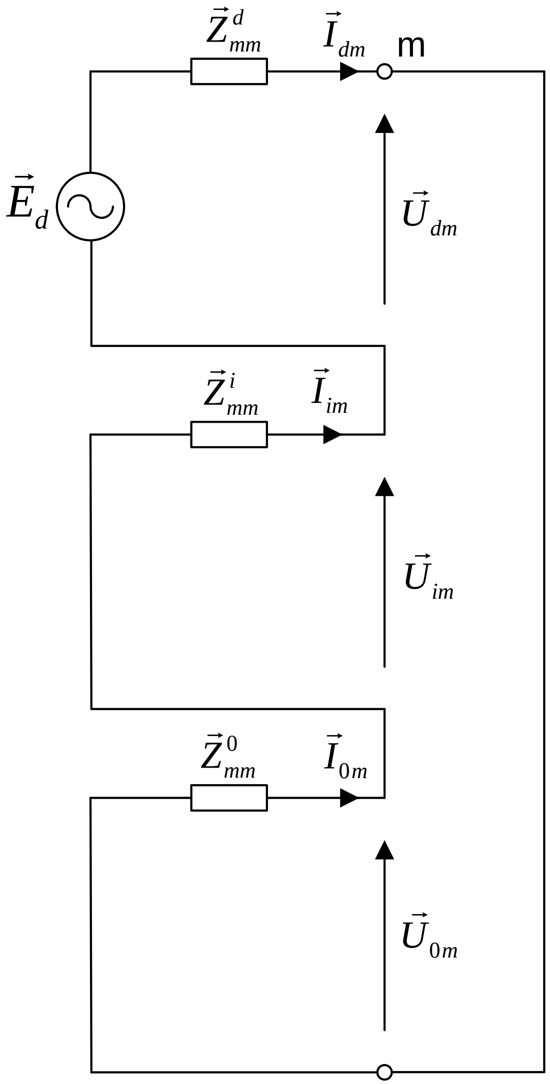
- Raspisivanjem navedenih uvjeta u sustavu simetričnih komponenti dobije se:

$$\vec{I}_{dm} = \vec{I}_{im} = \vec{I}_{0m} = \frac{\vec{E}_d}{\vec{Z}^0_{mm} + \vec{Z}^d_{mm} + \vec{Z}^i_{mm}} = -\frac{{}^{zdr}\vec{U}^d_m}{\vec{Z}^0_{mm} + \vec{Z}^d_{mm} + \vec{Z}^i_{mm}}$$

$$\vec{U}_{dm} + \vec{U}_{im} + \vec{U}_{0m} = 0$$

Proračun kratkog spoja – jednofazni kratki spoj

- Izvedenim izrazima odgovara nadomjesna shema fiktivnog sustava:



Proračun kratkog spoja – jednofazni kratki spoj

- Za napone u čvorištima mreže vrijedi:

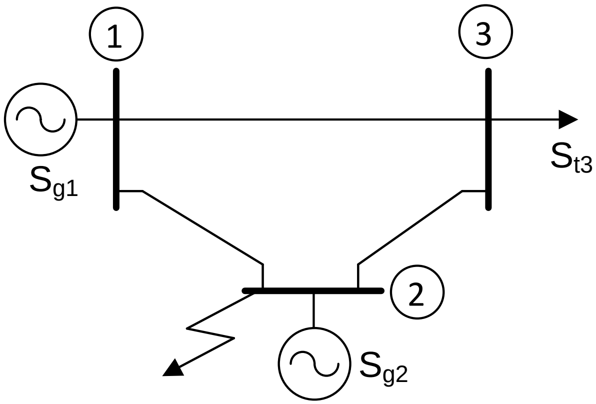
$$\begin{bmatrix} U_1^d \\ \vdots \\ U_m^d \\ \vdots \\ U_n^d \end{bmatrix}^{kv} = \begin{bmatrix} U_1^d \\ \vdots \\ U_m^d \\ \vdots \\ U_n^d \end{bmatrix}^{zdr} + \begin{bmatrix} Z_{11}^d & \cdots & Z_{1n}^d \\ \vdots & \ddots & \vdots \\ Z_{n1}^d & \cdots & Z_{nn}^d \end{bmatrix}_{KS} \cdot \begin{bmatrix} 0 \\ \vdots \\ I_m^d \\ \vdots \\ 0 \end{bmatrix}^{kv}$$

$$\begin{bmatrix} U_1^i \\ \vdots \\ U_m^i \\ \vdots \\ U_n^i \end{bmatrix}^{kv} = \begin{bmatrix} Z_{11}^i & \cdots & Z_{1n}^i \\ \vdots & \ddots & \vdots \\ Z_{n1}^i & \cdots & Z_{nn}^i \end{bmatrix}_{KS} \cdot \begin{bmatrix} 0 \\ \vdots \\ I_m^i \\ \vdots \\ 0 \end{bmatrix}^{kv}$$

$$\begin{bmatrix} U_1^0 \\ \vdots \\ U_m^0 \\ \vdots \\ U_n^0 \end{bmatrix}^{kv} = \begin{bmatrix} Z_{11}^0 & \cdots & Z_{1n}^0 \\ \vdots & \ddots & \vdots \\ Z_{n1}^0 & \cdots & Z_{nn}^0 \end{bmatrix}_{KS} \cdot \begin{bmatrix} 0 \\ \vdots \\ I_m^0 \\ \vdots \\ 0 \end{bmatrix}^{kv}$$

Zadatci

- **ZADATAK 1.** U mreži prikazanoj slikom je nastao jednofazni kratki spoj u čvorištu 2 u trenutku kada je mreža bila u praznom hodu. Odredite napone po fazama (u kV) u svim čvorištima mreže. Nazivni napon mreže je 110 kV:



Matrice impedancija čvorišta za direktni, inverzni i nulti sustav su zadane i iznose:

$$Z_d = Z_i = j\begin{bmatrix} 0.063 & 0.037 & 0.05 \\ 0.037 & 0.063 & 0.05 \\ 0.05 & 0.05 & 0.1 \end{bmatrix} [p.u.] \qquad Z_0 = j\begin{bmatrix} 0.125 & 0.075 & 0.1 \\ 0.075 & 0.125 & 0.1 \\ 0.1 & 0.1 & 0.2 \end{bmatrix} [p.u.]$$

RJEŠENJE:

$$I_{d2} = -\frac{{}^{zdr}U_2^d}{Z_{22}^d + Z_{22}^i + Z_{22}^0} = -\frac{1}{j0.063 + j0.063 + j0.125} = j3.984 \ p.u.$$

$$I_{02} = I_{d2} = j3.984 \ p.u.$$

$$I_{i2} = I_{d2} = j3.984 \ p.u.$$

$$\begin{bmatrix} U_1^d \\ U_2^d \\ U_3^d \end{bmatrix}^{kv} = \begin{bmatrix} U_1^d \\ U_2^d \\ U_3^d \end{bmatrix}^{zdr} + Z^d \cdot \begin{bmatrix} 0 \\ I_{d2} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + j \begin{bmatrix} 0.063 & 0.037 & 0.05 \\ 0.037 & 0.063 & 0.05 \\ 0.05 & 0.05 & 0.1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ j3.984 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.853 \\ 0.749 \\ 0.801 \end{bmatrix} p.u.$$

$$\begin{bmatrix} U_1^i \\ U_2^i \\ U_3^i \end{bmatrix}^{kv} = Z^i \cdot \begin{bmatrix} 0 \\ I_{i2} \\ 0 \end{bmatrix} = j \begin{bmatrix} 0.063 & 0.037 & 0.05 \\ 0.037 & 0.063 & 0.05 \\ 0.05 & 0.05 & 0.1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ j3.984 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.147 \\ -0.251 \\ -0.199 \end{bmatrix} p.u.$$

$$\begin{bmatrix} U_1^0 \\ U_2^0 \\ U_3^0 \end{bmatrix}^{kv} = Z^0 \cdot \begin{bmatrix} 0 \\ I_{02} \\ 0 \end{bmatrix} = j \begin{bmatrix} 0.125 & 0.075 & 0.1 \\ 0.075 & 0.125 & 0.1 \\ 0.1 & 0.1 & 0.2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ j3.984 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.299 \\ -0.498 \\ -0.398 \end{bmatrix} p.u.$$

$$\begin{bmatrix} U_1^R \\ U_1^S \\ U_1^T \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} U_1^0 \\ U_1^d \\ U_1^i \end{bmatrix} = \begin{bmatrix} 0.406 \\ -0.651 - j0.866 \\ -0.651 + j0.866 \end{bmatrix} p.u.$$

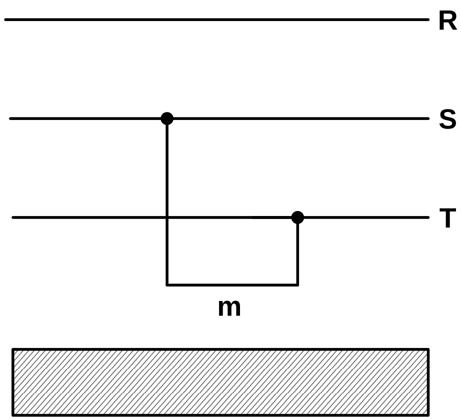
$$\begin{bmatrix} U_1^R \\ U_1^S \\ U_1^T \end{bmatrix} = \begin{bmatrix} 44.701 \\ -71.653 - j95.263 \\ -71.653 + j95.263 \end{bmatrix} kV = \begin{bmatrix} 44.701\angle 0° \\ 119.202\angle -126.949° \\ 119.202\angle 126.949° \end{bmatrix} kV$$

$$\begin{bmatrix} U_2^R \\ U_2^S \\ U_2^T \end{bmatrix} = \begin{bmatrix} 0\angle 0° \\ 125.806\angle -130.78° \\ 125.806\angle 130.78° \end{bmatrix} kV$$

$$\begin{bmatrix} U_3^R \\ U_3^S \\ U_3^T \end{bmatrix} = \begin{bmatrix} 22.351\angle 0° \\ 122.436\angle -128.916° \\ 122.436\angle 128.916° \end{bmatrix} kV$$

Proračun kratkog spoja – dvofazni kratki spoj

- Dvofazni (dvopolni) kratki spoj:



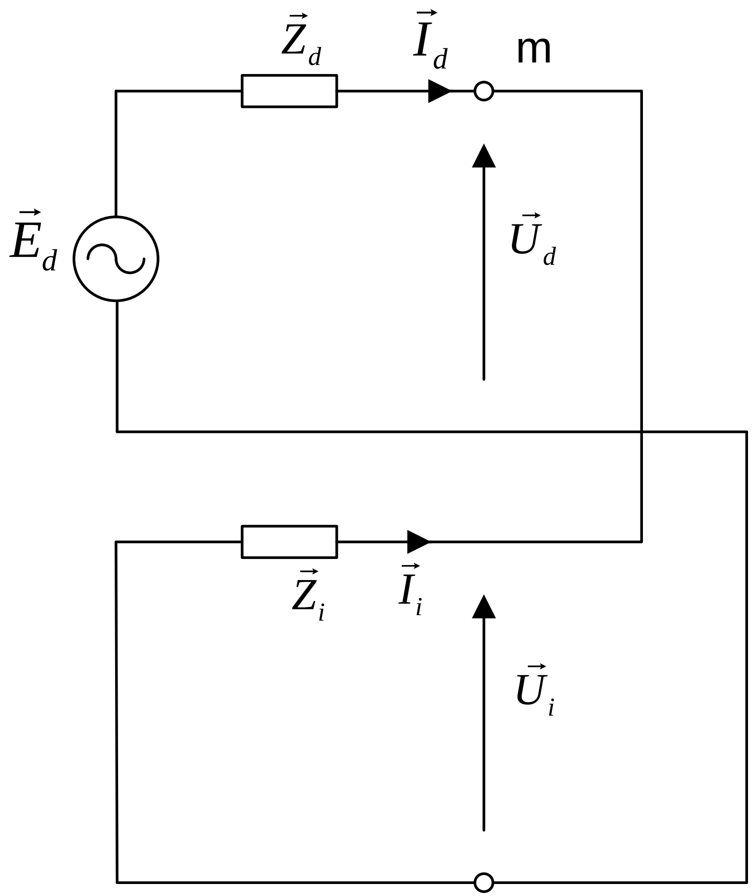
$$\vec{V}_{Sm} = \vec{V}_{Tm}$$
$$\vec{I}_{Sm} = -\vec{I}_{Tm}$$

- Raspisivanjem navedenih uvjeta u sustavu simetričnih komponenti dobije se:

$$\vec{I}_{dm} = -\vec{I}_{im} = \frac{\vec{E}_d}{\vec{Z}_{mm}^d + \vec{Z}_{mm}^i} = -\frac{{}^{zdr}\vec{U}_m^d}{\vec{Z}_{mm}^d + \vec{Z}_{mm}^i}$$
$$\vec{U}_{dm} = \vec{U}_{im}$$
$$\vec{I}_{0m} = 0$$
$$\vec{U}_{0m} = 0$$

Proračun kratkog spoja – dvofazni kratki spoj

- Izvedenim izrazima odgovara nadomjesna shema fiktivnog sustava:



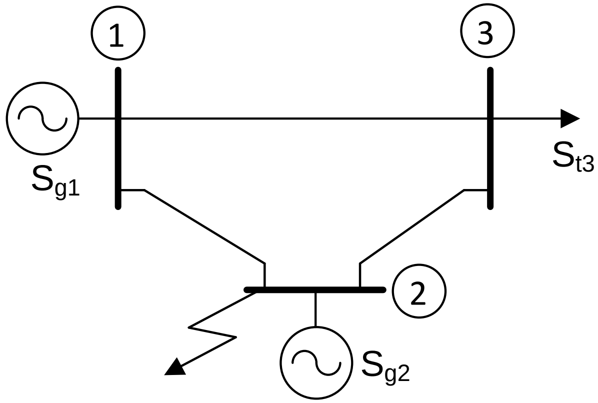
Proračun kratkog spoja – dvofazni kratki spoj

- Za napone u čvorištima mreže vrijedi:

$$\begin{bmatrix} U_1^d \\ \vdots \\ U_m^d \\ \vdots \\ U_n^d \end{bmatrix}^{kv} = \begin{bmatrix} U_1^d \\ \vdots \\ U_m^d \\ \vdots \\ U_n^d \end{bmatrix}^{zdr} + \begin{bmatrix} Z_{11}^d & \cdots & Z_{1n}^d \\ \vdots & \ddots & \vdots \\ Z_{n1}^d & \cdots & Z_{nn}^d \end{bmatrix}_{KS} \cdot \begin{bmatrix} 0 \\ \vdots \\ I_m^d \\ \vdots \\ 0 \end{bmatrix}^{kv}$$

$$\begin{bmatrix} U_1^i \\ \vdots \\ U_m^i \\ \vdots \\ U_n^i \end{bmatrix}^{kv} = \begin{bmatrix} Z_{11}^i & \cdots & Z_{1n}^i \\ \vdots & \ddots & \vdots \\ Z_{n1}^i & \cdots & Z_{nn}^i \end{bmatrix}_{KS} \cdot \begin{bmatrix} 0 \\ \vdots \\ I_m^i \\ \vdots \\ 0 \end{bmatrix}^{kv}$$

- **ZADATAK 2.** U mreži prikazanoj slikom je nastao dvofazni kratki spoj u čvorištu 2 u trenutku kada je mreža bila u praznom hodu. Odredite napone po fazama (u kV) u svim čvorištima mreže. Nazivni napon mreže je 110 kV:



Matrice impedancija čvorišta za direktni i inverzni sustav su zadane i iznose:

$$Z_d = Z_i = j\begin{bmatrix} 0.063 & 0.037 & 0.05 \\ 0.037 & 0.063 & 0.05 \\ 0.05 & 0.05 & 0.1 \end{bmatrix} [p.u.]$$

RJEŠENJE:

$$I_{d2} = -\frac{{}^{zdr}U_2^d}{Z_{22}^d + Z_{22}^i} = -\frac{1}{j0.063 + j0.063} = j7.937\ p.u.$$

$$I_{i2} = -I_{d2} = -j7.937\ p.u.$$

$$I_{02} = 0$$

$$\begin{bmatrix} U_1^d \\ U_2^d \\ U_3^d \end{bmatrix}^{kv} = \begin{bmatrix} U_1^d \\ U_2^d \\ U_3^d \end{bmatrix}^{zdr} + Z^d \cdot \begin{bmatrix} 0 \\ I_{d2} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + j\begin{bmatrix} 0.063 & 0.037 & 0.05 \\ 0.037 & 0.063 & 0.05 \\ 0.05 & 0.05 & 0.1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ j7.937 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.706 \\ 0.5 \\ 0.603 \end{bmatrix} p.u.$$

$$\begin{bmatrix} U_1^i \\ U_2^i \\ U_3^i \end{bmatrix}^{kv} = Z^i \cdot \begin{bmatrix} 0 \\ I_{i2} \\ 0 \end{bmatrix} = j\begin{bmatrix} 0.063 & 0.037 & 0.05 \\ 0.037 & 0.063 & 0.05 \\ 0.05 & 0.05 & 0.1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -j7.937 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.294 \\ 0.5 \\ 0.397 \end{bmatrix} p.u.$$

$$\begin{bmatrix} U_1^0 \\ U_2^0 \\ U_3^0 \end{bmatrix}^{kv} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} p.u.$$

$$\begin{bmatrix} U_1^R \\ U_1^S \\ U_1^T \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} U_1^0 \\ U_1^d \\ U_1^i \end{bmatrix} = \begin{bmatrix} 1 \\ -0.5 - j0.357 \\ -0.5 + j0.357 \end{bmatrix} p.u.$$

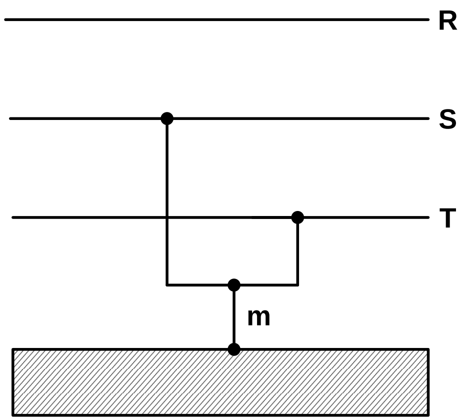
$$\begin{bmatrix} U_1^R \\ U_1^S \\ U_1^T \end{bmatrix} = \begin{bmatrix} 110 \\ -55 - j39.315 \\ -55 + j39.315 \end{bmatrix} kV = \begin{bmatrix} 110\angle 0^\circ \\ 67.607\angle -144.442^\circ \\ 67.607\angle 144.442^\circ \end{bmatrix} kV$$

$$\begin{bmatrix} U_2^R \\ U_2^S \\ U_2^T \end{bmatrix} = \begin{bmatrix} 110\angle 0^\circ \\ 55\angle 180^\circ \\ 55\angle 180^\circ \end{bmatrix} kV$$

$$\begin{bmatrix} U_3^R \\ U_3^S \\ U_3^T \end{bmatrix} = \begin{bmatrix} 110\angle 0^\circ \\ 58.407\angle -160.333^\circ \\ 58.407\angle 160.333^\circ \end{bmatrix} kV$$

Proračun kratkog spoja – dvofazni kratki spoj sa zemljom

- Dvofazni (dvopolni) kratki spoj sa zemljom:



$$\vec{V}_{Sm} = \vec{V}_{Tm} = 0$$

$$\vec{I}_{Rm} = 0$$

- Raspisivanjem navedenih uvjeta u sustavu simetričnih komponenti dobije se:

$$\vec{I}_{dm} = -\frac{{}^{zdr}\vec{U}_m^d \cdot \left(\vec{Z}_{mm}^i + \vec{Z}_{mm}^0\right)}{\vec{Z}_{mm}^d \cdot \vec{Z}_{mm}^i + \vec{Z}_{mm}^d \cdot \vec{Z}_{mm}^0 + \vec{Z}_{mm}^i \cdot \vec{Z}_{mm}^0}$$

$$\vec{I}_{im} = -\frac{{}^{zdr}\vec{U}_m^d \cdot \vec{Z}_{mm}^0}{\vec{Z}_{mm}^d \cdot \vec{Z}_{mm}^i + \vec{Z}_{mm}^d \cdot \vec{Z}_{mm}^0 + \vec{Z}_{mm}^i \cdot \vec{Z}_{mm}^0}$$

$$\vec{I}_{0m} = -\frac{{}^{zdr}\vec{U}_m^d \cdot \vec{Z}_{mm}^i}{\vec{Z}_{mm}^d \cdot \vec{Z}_{mm}^i + \vec{Z}_{mm}^d \cdot \vec{Z}_{mm}^0 + \vec{Z}_{mm}^i \cdot \vec{Z}_{mm}^0}$$

$$\vec{U}_{dm} = \vec{U}_{im} = \vec{U}_{0m}$$

Proračun kratkog spoja – dvofazni kratki spoj sa zemljom

- Temeljem izvedenih izraza je moguće nacrtati shemu fiktivnog sustava za simetrične komponente: