Analiza elektroenergetskog sustava Uvod

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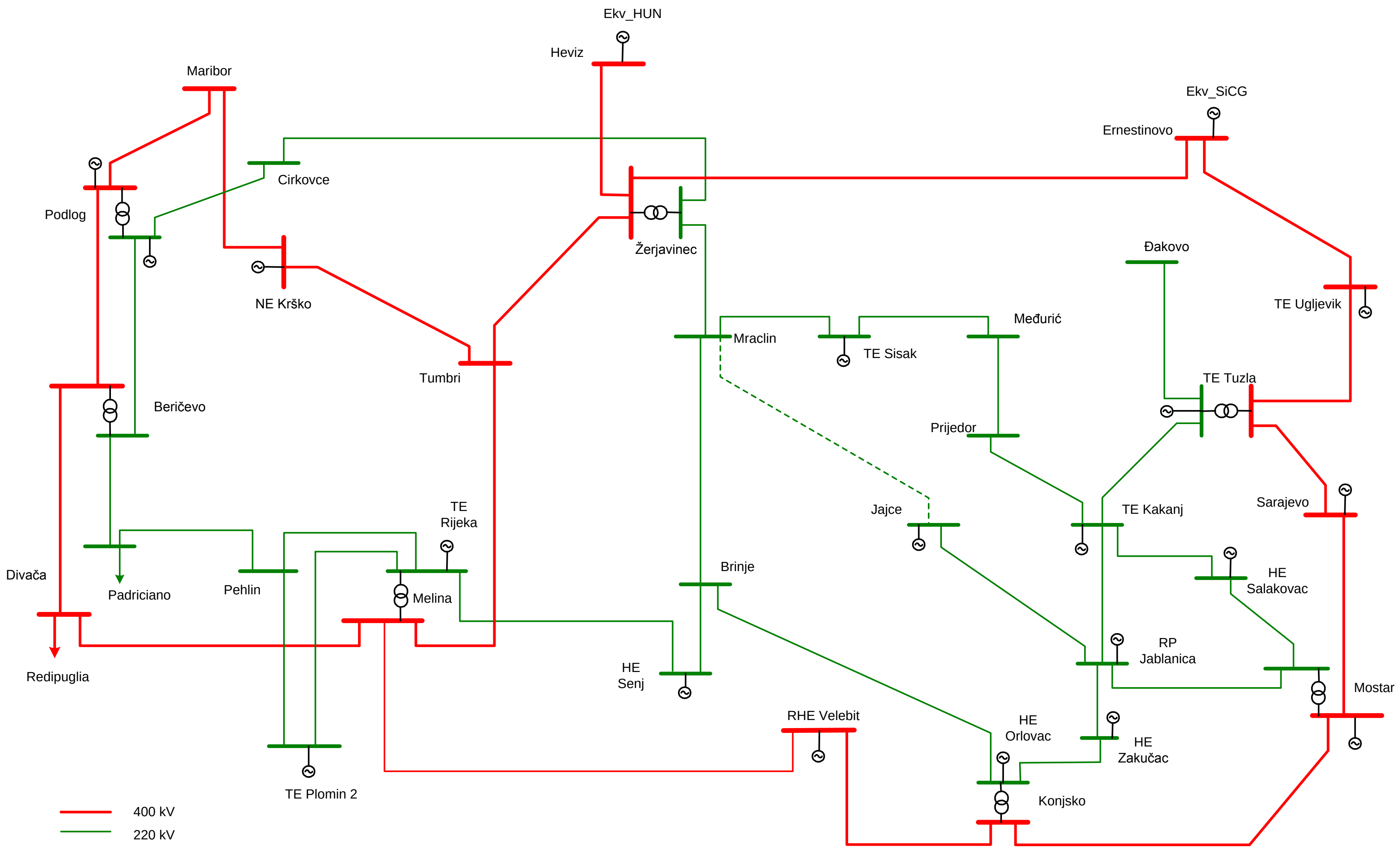
Osnovni pojmovi

- Elektroenergetski sustav
  - proizvodnja (izvori EE)
  - potrošnja
  - elektroenergetska mreža
- Elektroenergetska mreža
  - prijenosna
  - distribucijska (razdjelna)
  - industrijska
  - razvod (kućne instalacije)
- Kriteriji razgraničenja EEM
  - tehnički, funkcionalni
- Podjela EEM
  - nazivni napon, topologija, način napajanja, način uzemljenja, specifične karakteristike, ...

Primjer prijenosne mreže



Nazivne vrijednosti

- Definirane za sve elemente EES-a (generatori, transformatori, asinkroni motori, nadzemni vodovi, kabeli, prigušnice, kond. baterije, prekidači, rastavljači, sabirnice, ...)
- Nazivni napon

  - "ime" naponske razine

  - standardizirane vrijednosti (VVN, VN, SN, NN)

  - 765, 735, 500, 400, 220, 110, 35, 30, 20, 10, 6.3, 5.25, 3, 1, 0.4 kV
- Nazivna struja

  - definirana trajno dozvoljenim zagrijavanjem
- Nazivna snaga

  - najčešće izvedena iz nazivnog napona i struje

Karakteristike pogona EES-a

- Pogonska stanja
  - stacionarno stanje (normalni pogon)
  - kvazistacionarno stanje
  - poremećaj
  - havarijsko stanje (djelomični/potpuni raspad EES-a)
- Uzroci poremećaja:
  - ispad generatora
  - ispad potrošnje
  - ispad vodova, transformatora, ...
  - kratki spoj
- Jednakost proizvodnje i potrošnje (u svakom trenutku)

  proizvodnja = potrošnja + gubici
- Regulacija djelatne snage i frekvencije
- Regulacija napona i jalove snage

Važniji proračuni EES-a

- Proračun tokova snaga

  - osnovni proračun stacionarnog stanja EES-a

  - rješavanje sustava nelinearnih jednadžbi

  - cilj: određivanje napona (po iznosu i kutu) u svim čvorištima mreže

  - rezultat proračuna: proizvodnja generatora, tokovi snaga (struja) u granama, gubici snage

  - koristi se u vođenju pogona EES-a, studijskim analizama, planiranju razvoja, ...

  - metode za proračun zasnovane na simboličkom računu (primjena fazora) – vrijedi za sinusoidalne veličine

  - jednofazni i trofazni proračuni

Važniji proračuni EES-a

- Proračun kratkog spoja

  - proračun za stanje kvara u EES-u (poprečni i uzdužni kvarovi, simetrični i nesimetrični kvarovi)

  - primjena simboličkog računa za određivanje početne struje kratkog spoja

  - klasična i matrična metoda proračuna

  - primjena metode simetričnih komponenata (direktni, inverzni, nulti sustav)

  - rješavanje sustava linearnih jednadžbi

  - rezultat proračuna: struje i naponi kratkog spoja na mjestu kvara, ali i u ostalim dijelovima mreže

  - osnova za proračun udarne, rasklopne i trajne struje kratkog spoja

  - važan za dimenzioniranje opreme, proračun podešenja relejne zaštite, proračun uzemljenja i dr.)

Važniji proračuni EES-a

- Proračun stabilnosti EES-a

  - bitan za procjenu stabilnosti rada EES-a u normalnim pogonskim stanjima i u uvjetima poremećaja)

  - stabilnost kuta, napona i frekvencije

  <u>Kutna stabilnost (statička i prijelazna)</u>

  - statička (procjena stabilnosti stacionarne radne točke EES-a)

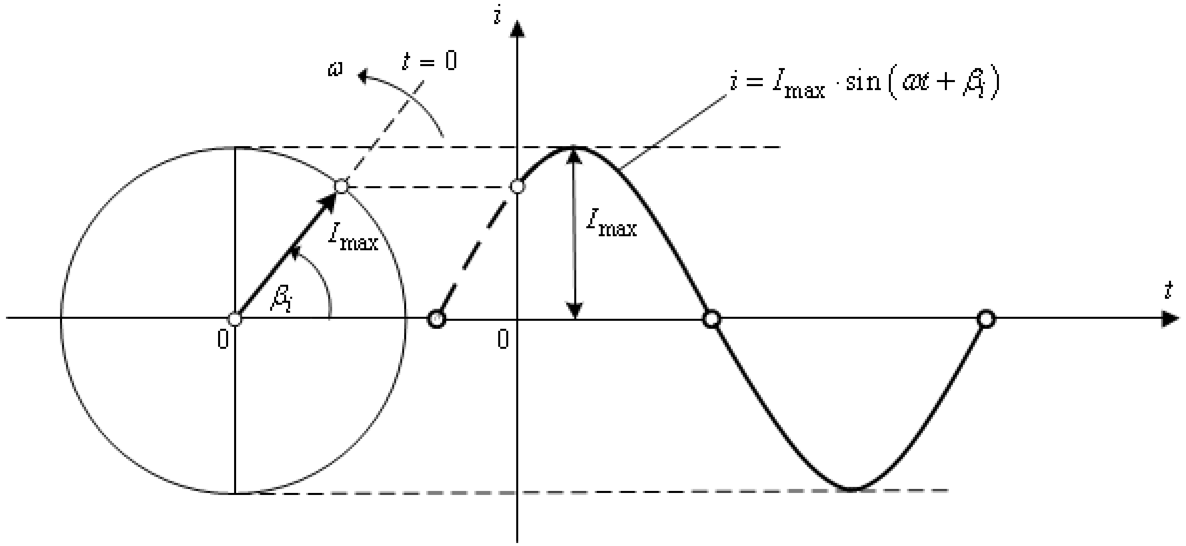
  - prijelazna (procjena stabilnosti EES-a nakon poremećaja)

  - složeniji modeli generatora (reaktancije, moment tromosti, prigušenja, ...)

  - rješavanje sustava diferencijalnih jednadžbi

  - rezultat prijelazne stabilnosti: njihanje kuteva rotora generatora, njihanje djelatnih i jalovih snaga, naponske i srujne prilike generatora, ...

Izraz za snagu u kompleksnom području



- Struja u realnom području: $i = I_{\max} \cdot \sin(\omega t + \beta_i)$
- Struja u kompleksnom području: $\overline{i} = I_{\max} \cdot e^{j(\omega t + \beta_i)}$
- Primjenom Moivre-ovog stavka:

$$\overline{i} = I_{\max} \cdot e^{j(\omega t + \beta_i)} = I_{\max} \cdot \cos(\omega t + \beta_i) + j \cdot I_{\max} \cdot \sin(\omega t + \beta_i)$$

Izraz za snagu u kompleksnom području

- Za napon i struju elementa strujnog kruga u trenutku *t=0* vrijedi:

$$\bar{i} = I \cdot e^{j\beta_i}$$

$$\bar{u} = U \cdot e^{j\beta_u}$$

$$\varphi = \beta_u - \beta_i$$

- Za srednje vrijednosti djelatne i jalove snage vrijedi:

$$P = U \cdot I \cdot \cos\varphi$$

$$Q = U \cdot I \cdot \sin\varphi$$

- Ukoliko bi se snaga elementa, za kojeg su poznate vrijednosti napona i struje u ***kompleksnom*** području, računala po istom pravilu kao za ***realno*** područje vrijedilo bi:

$$\bar{u} \cdot \bar{i} = U \cdot e^{j\beta_u} \cdot I \cdot e^{j\beta_i} = U \cdot I \cdot e^{j(\beta_u + \beta_i)}$$

$$\bar{u} \cdot \bar{i} = U \cdot I \cdot \cos(\beta_u + \beta_i) + j \cdot U \cdot I \cdot \sin(\beta_u + \beta_i)$$

Izraz za snagu u kompleksnom području

- Navedenim izrazom snaga nije prikazana kao funkcija karakterističnog kuta φ koji predočuje fazni pomak između struje i napona.
- Zbog toga se snaga u kompleksnom području uobičajeno računa kao:

$$\bar{u} \cdot \bar{i}^* = U \cdot e^{j\beta_u} \cdot I \cdot e^{-j\beta_i} = U \cdot I \cdot e^{j(\beta_u - \beta_i)} = U \cdot I \cdot e^{j\varphi}$$

$$\bar{u} \cdot \bar{i}^* = U \cdot I \cdot \cos\varphi + j \cdot U \cdot I \cdot \sin\varphi$$

$$\bar{u} \cdot \bar{i}^* = P + jQ$$

- Množenjem kompleksne vrijednosti napona s konjugirano kompleksnom vrijednosti struje dobije se tzv. ***kompleksna snaga***:

$$S = P + jQ$$

Analiza elektroenergetskog sustava
Osnovni zakoni i teoremi

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Osnovni zakoni i teoremi za proračun mreža

- Ohmov zakon
- Kirchoffovi zakoni (za struje i napone)
- Teoremi za aktivne mreže

  - superpozicije

  - kompenzacije

  - Theveninov

  - Nortonov

  - Millmanov
- Teoremi za pasivne mreže

  - transfiguracija trokut - zvijezda

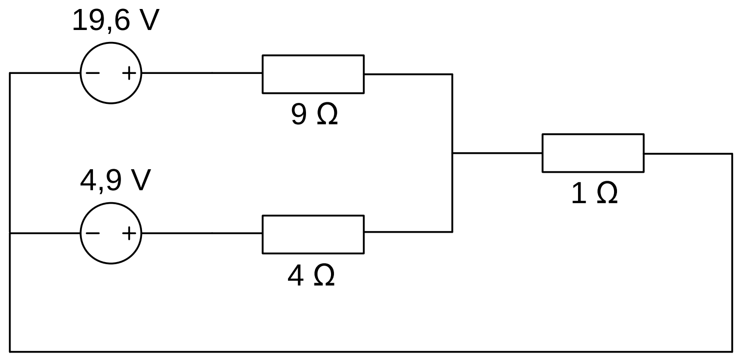
  - eliminacija čvorišta (opća zvijezda u opći poligon)

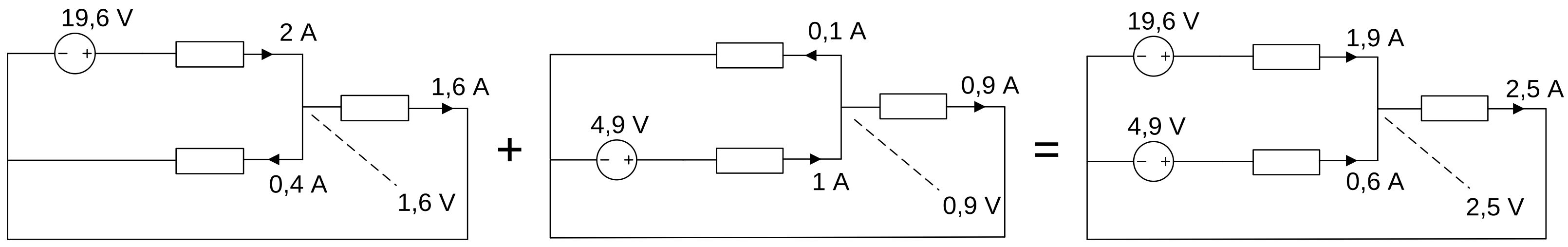
  - reciprociteta

Teorem superpozicije

- Primjena:
  - Samo za linearne mreže (linearan odnos struja-napon)
  - Vrijedi i za heterogene struje (npr. istosmjerne i izmjenične struje)

- Primjer:





Teorem kompenzacije

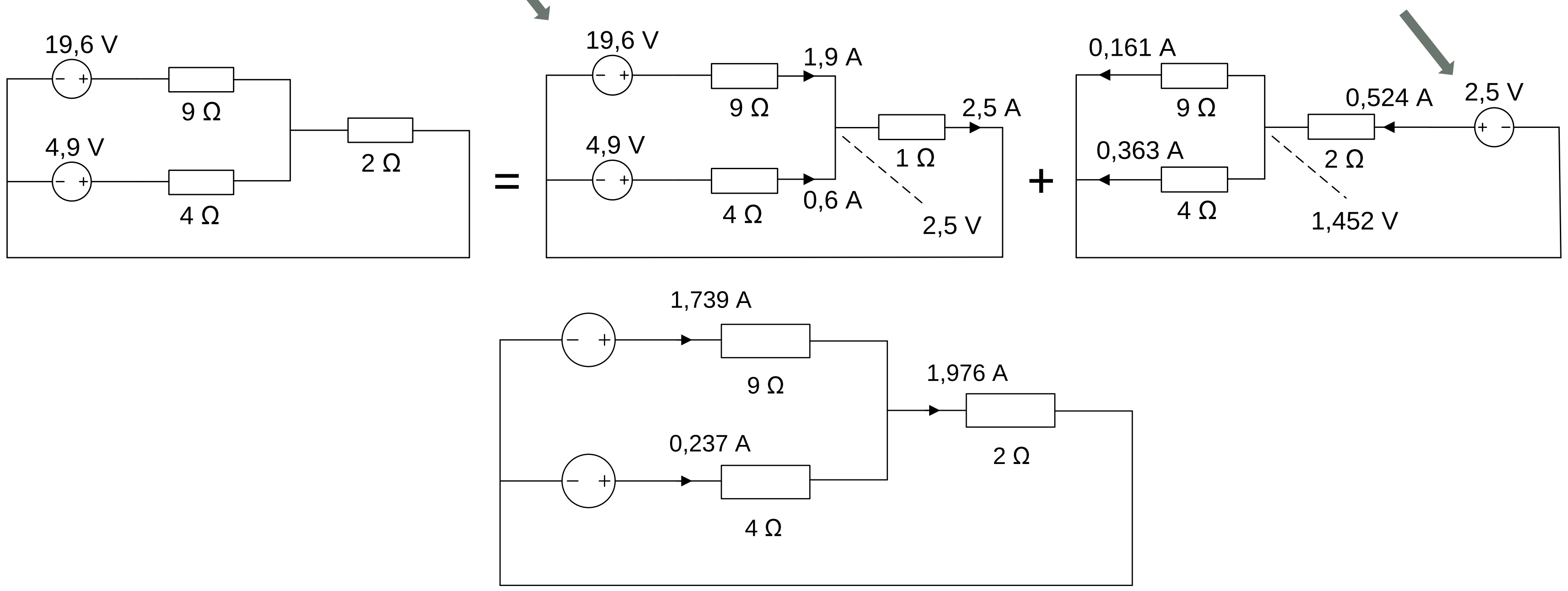
- Primjena:
  - Za već analizirane mreže kod kojih se jedna impedancija promijenila
  - Za aktivne i linearne mreže
- Primjer:

Mreža analizirana u prethodnom primjeru!

Nova EMS:

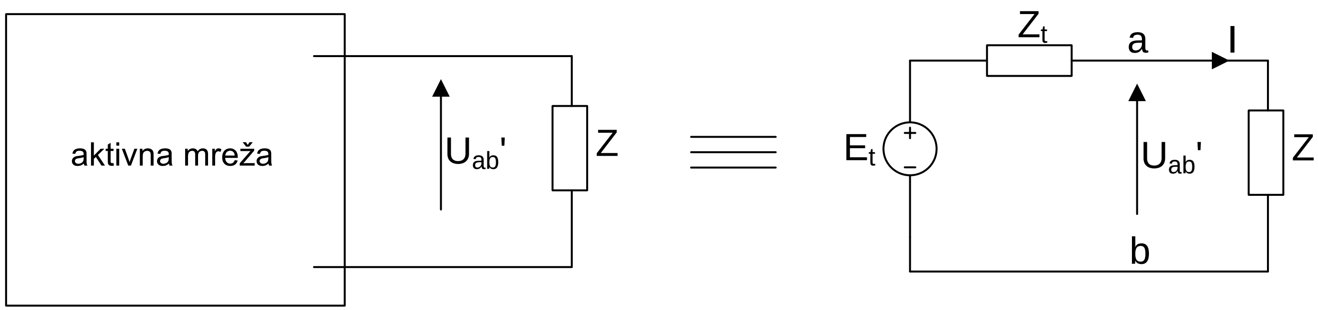
$$\Delta Z \cdot I = (Z_{poč} - Z_{kon}) \cdot I$$

$$\Delta Z \cdot I = (1\,\Omega - 2\,\Omega) \cdot 2.5\,A = -2.5\,V$$



Theveninov teorem

- Svaka stvarna aktivna mreža promatrana iz 2 čvorišta može se nadomjestiti fiktivnom mrežom koja je serijski spoj impedancije Z_T i EMS-e E_T, a naziva se Theveninov ekvivalent.



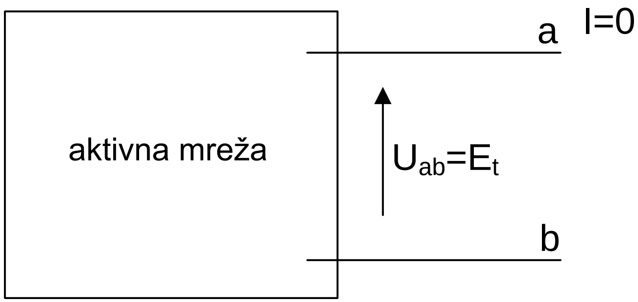
$$\vec{I} = \frac{\vec{E}_T}{\vec{Z}_T + \vec{Z}}$$

$$\vec{U}'_{ab} = \vec{E}_T - \vec{I} \cdot \vec{Z}_T = \vec{E}_T - \frac{\vec{E}_T}{\vec{Z}_T + \vec{Z}} \cdot \vec{Z}_T = \vec{E}_T \cdot \left(1 - \frac{\vec{Z}_T}{\vec{Z}_T + \vec{Z}}\right)$$

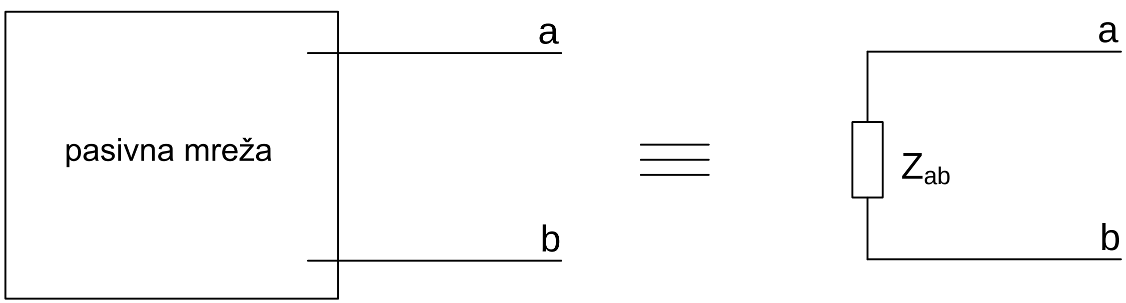
$$\vec{U}'_{ab} = \vec{E}_T \cdot \frac{\vec{Z}}{\vec{Z}_T + \vec{Z}}$$

Theveninov teorem

- Postupak određivanja Theveninovog ekvivalenta:
  - Određivanje E_T:

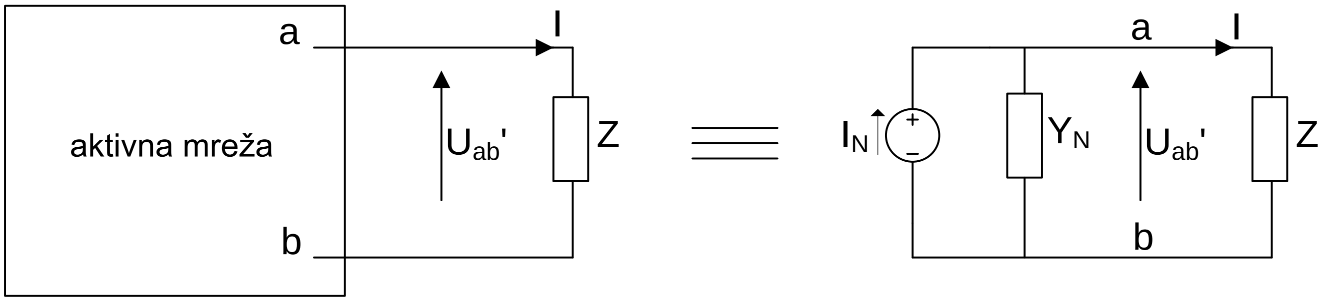


  - Određivanje Z_T:



Nortonov teorem

- Svaka stvarna aktivna mreža promatrana iz 2 čvorišta može se nadomjestiti fiktivnom mrežom koja je paralelni spoj admitancije Y_N i strujnog izvora I_N:

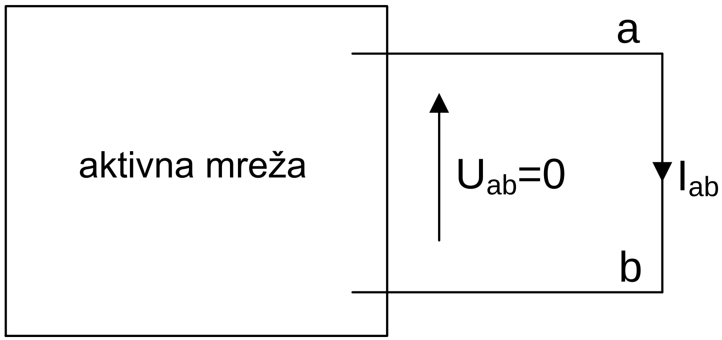


- Izvod iz Theveninovog teorema (prazni hod, kratki spoj)

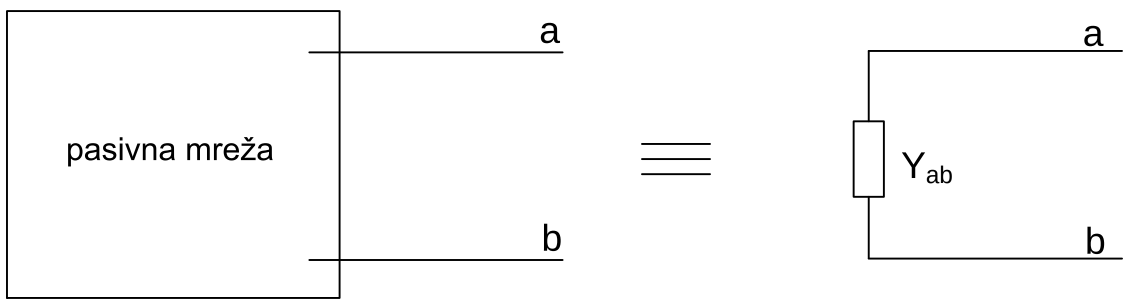
$$\vec{\mathrm{I}}_{\mathrm{N}} = \frac{\vec{\mathrm{E}}_{\mathrm{T}}}{\vec{\mathrm{Z}}_{\mathrm{T}}} \quad ; \; \vec{Y}_N = \frac{1}{\vec{\mathrm{Z}}_{\mathrm{T}}}$$

Nortonov teorem

- Postupak određivanja Nortonovog ekvivalenta korištenjem Theveninovog ekvivalenta:
  - Određivanje I_N:

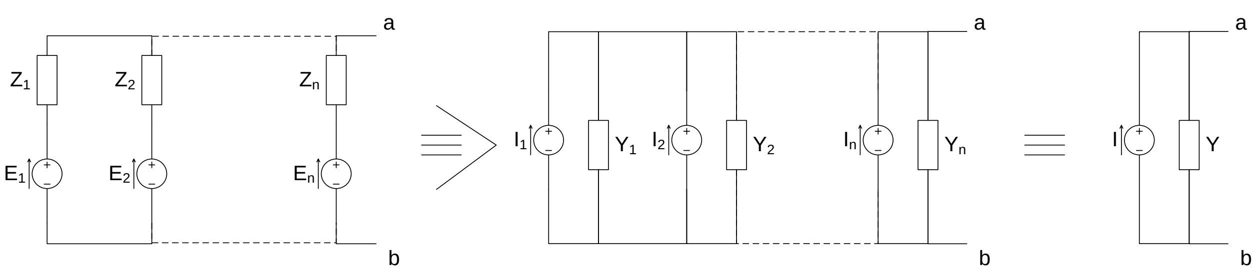


  - Određivanje Y_N:



Millmanov teorem

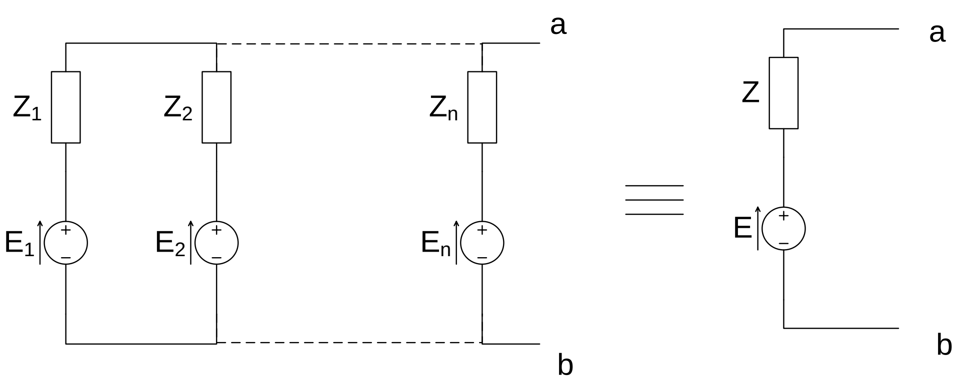
- Kompletna aktivna mreža može se ekvivalentirati pomoću Nortonovog i Theveninovog teorema u impedantni ili admitantni oblik.
  - Admitantni oblik



$$\vec{I} = \sum_{i=1}^{n} \vec{I}_i = \sum_{i=1}^{n} \frac{\vec{E}_i}{\vec{Z}_i}$$

$$\vec{Y} = \sum_{i=1}^{n} \vec{Y}_i$$

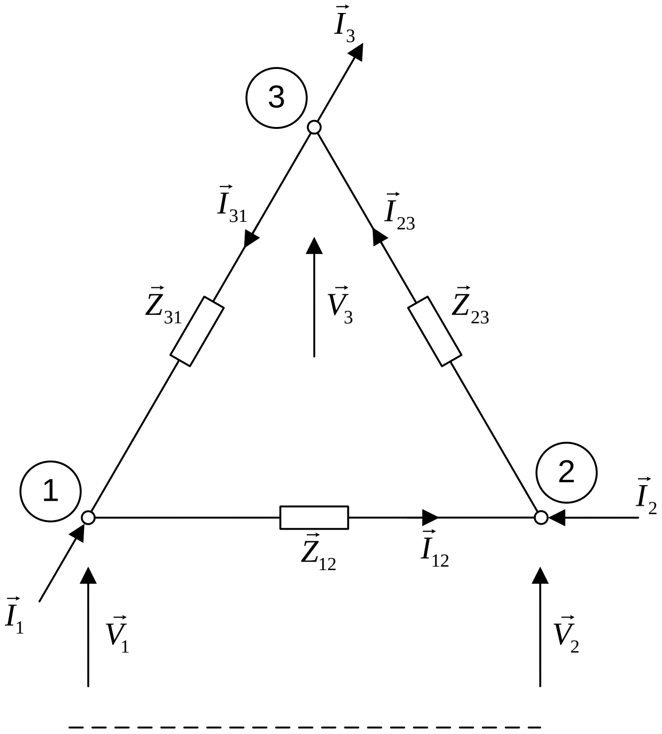
  - Impedantni oblik

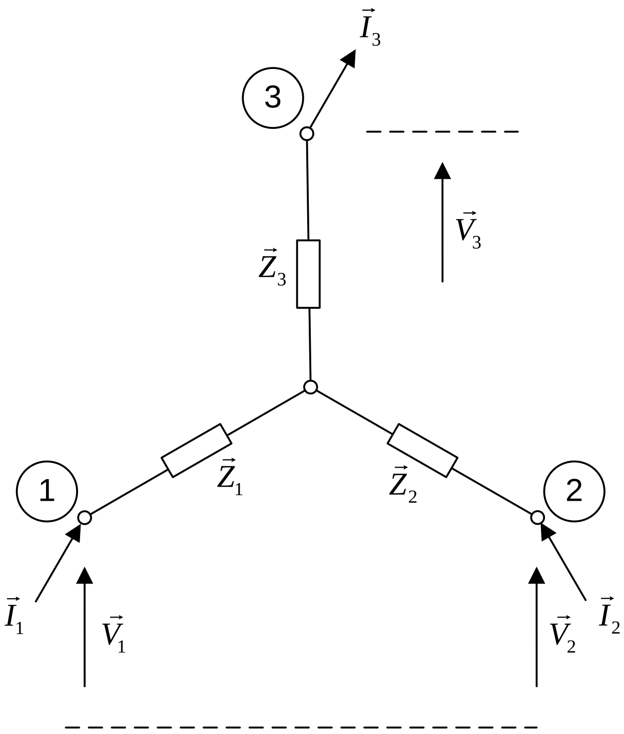


$$\vec{Z} = \frac{1}{\vec{Y}} = \frac{1}{\sum_{i=1}^{n} \frac{1}{\vec{Z}_i}}$$

$$\vec{E} = \frac{\vec{I}}{\vec{Y}} = \frac{\sum_{i=1}^{n} \frac{\vec{E}_i}{\vec{Z}_i}}{\sum_{i=1}^{n} \frac{1}{\vec{Z}_i}}$$

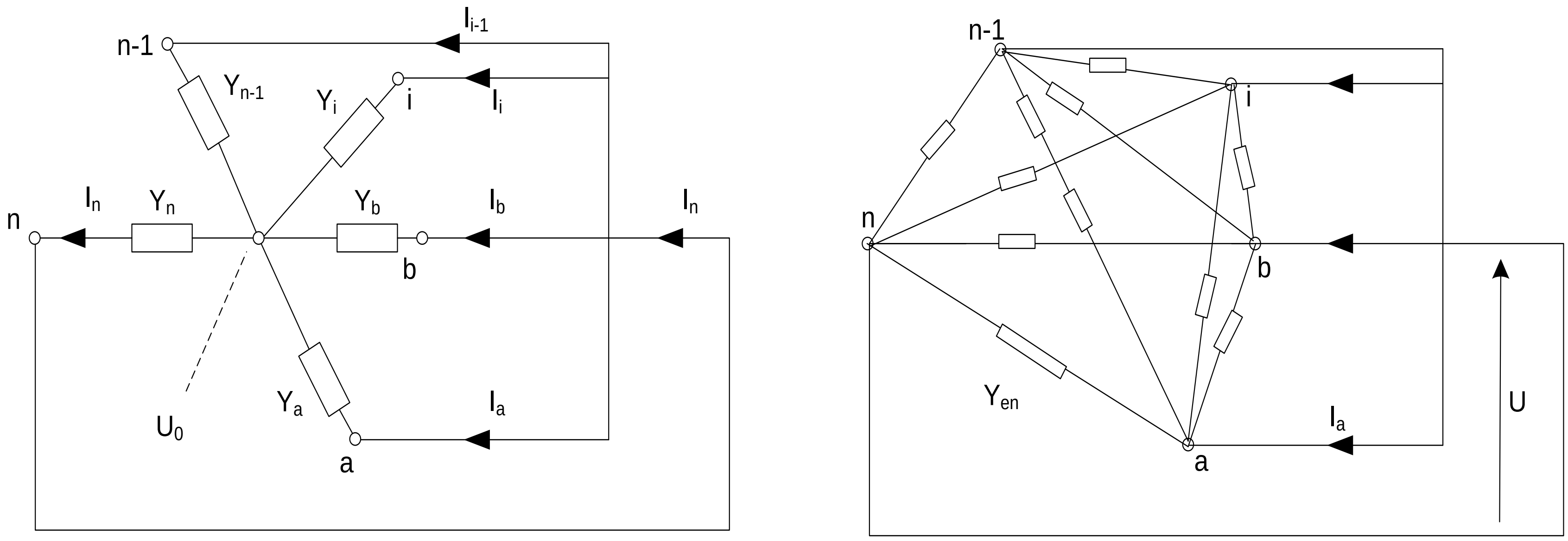
Transfiguracija trokut-zvijezda





$$\vec{Z}_1 = \frac{\vec{Z}_{13} \cdot \vec{Z}_{12}}{\vec{Z}_{12} + \vec{Z}_{13} + \vec{Z}_{23}} \quad ; \quad \vec{Z}_2 = \frac{\vec{Z}_{23} \cdot \vec{Z}_{12}}{\vec{Z}_{12} + \vec{Z}_{13} + \vec{Z}_{23}} \quad ; \quad \vec{Z}_3 = \frac{\vec{Z}_{13} \cdot \vec{Z}_{23}}{\vec{Z}_{12} + \vec{Z}_{13} + \vec{Z}_{23}}$$

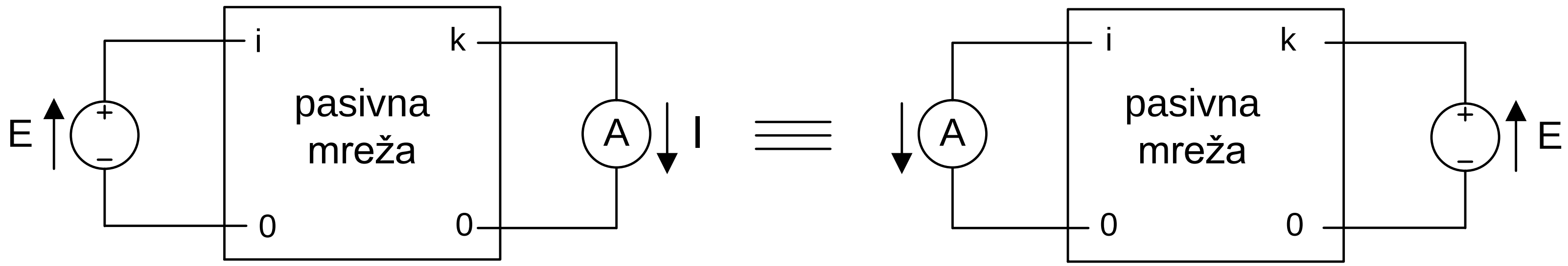
Eliminacija čvorišta (opća zvijezda u opći poligon)



$$Y_{jk} = \frac{Y_j \cdot Y_k}{\sum_{i=a}^{n} Y_i}$$

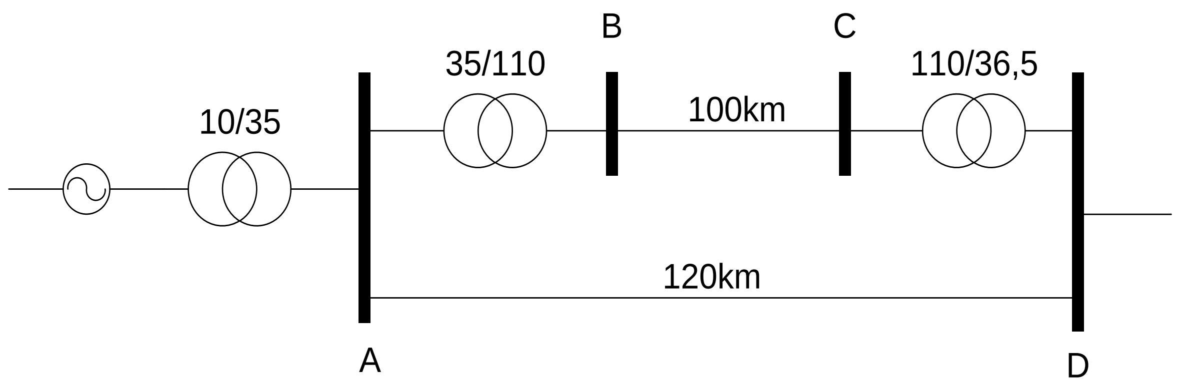
Teorem reciprociteta

- Neka u pasivnoj mreži djeluje samo jedna EMS na odabranom paru čvorišta. Ampermetar priključen na neki drugi par čvorišta mjeri određenu struju.
- Ukoliko se zamijene mjesta spoja EMS-e i ampermetra, u recipročnoj mreži ampermetar će ponovno pokazivati istu vrijednost.



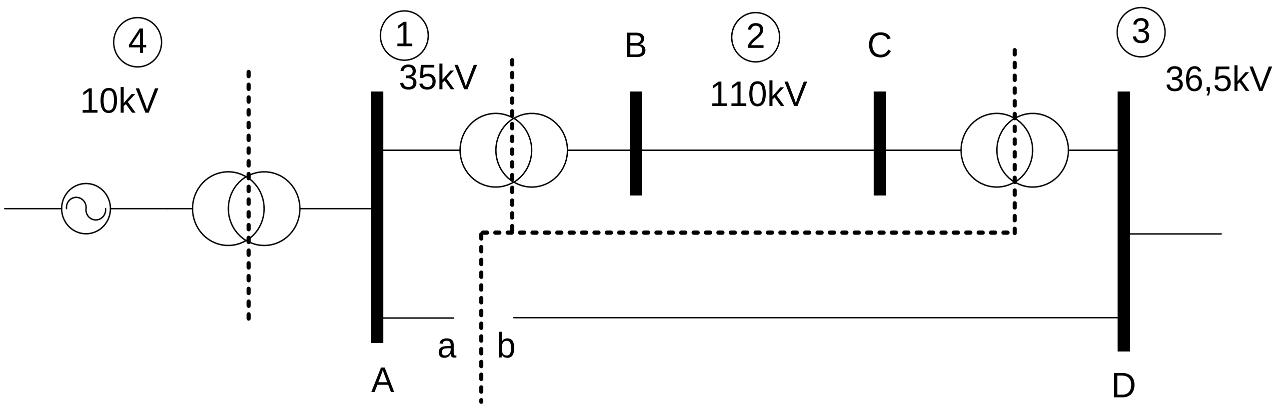
Problem različitih naponskih razina

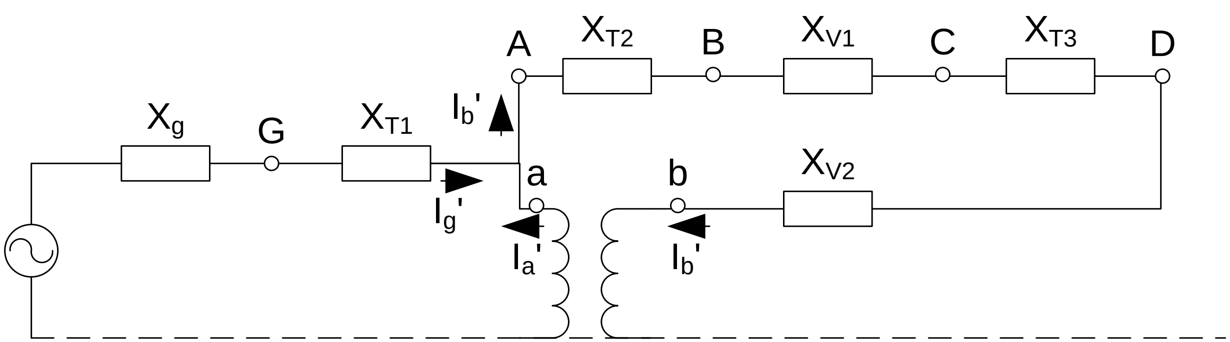
- Energetski transformatori – veza između mreža različitih naponskih razina (svrha transformatora, vrste, ...)
- Nazivni naponi mreže i transformatora
- Radijalne (zrakaste), zamkaste (petljaste) mreže
- Struje izjednačenja (nejednaki prij. odnosi)
- Primjer: U mreži na slici odredi prilike u praznom hodu za U_A=34 kV. Za sve transformatore u_k=10%, a nazivna snaga je 20 MVA. (Stvarni prijenosni omjer transformatora jednak je nazivnom.). Podatci generatora su: U_n=10,5 kV; S_n=20 MVA; X_d=115%. Reaktancija voda je X_V=0.4 Ω/km.



Problem različitih naponskih razina

- Primjer (nastavak):





$36,5/35 = U_{B3}/U_{B1}$

Relativne veličine u proračunima mreža

- Rješenje problema – svođenje cijele mreže na jednu naponsku razinu
- Stvarne (fizikalne) veličine
- Bazne veličine
- Relativne veličine
- Slična preslikavanja (Laplaceova transformacija, simbolički račun, metoda simetričnih komponenti, ...)
- Linearno preslikavanje (linearan odnos napon-struja)
- 3 osnovne metode

  - metoda otpora

  - metoda jediničnih vrijednosti (per unit)

  - metoda reduciranih admitancija

Metoda otpora

- Osnovne pretpostavke:
  1. stvarni sustav – trofazni, preslikani (reducirani) sustav – jednofazni
  2. bazni napon izabire se proizvoljno

$$S = S' \ , \quad S\,(MVA) - \text{snaga stvarnog (trofaznog) sustava} ,$$

$$S'(MVA) - \text{snaga reduciranog (jednofaznog) sustava}$$

$$U' = U \cdot \frac{U_B}{U_n} \ , \ U\,(kV) - \text{stvarni napon} ,$$

$$U_B\,(kV) - \text{bazni napon (proizvoljno izabran)}$$

$$U_n\,(kV) - \text{nazivni napon određenog dijela mreže}$$

Metoda otpora

$$\sqrt{3} \cdot U \cdot I^{*} = U' \cdot (I')^{*} \Rightarrow I' = \sqrt{3} \cdot I \cdot \frac{U_n}{U_B} \quad (A)$$

$$Z' = \frac{U'}{I'} = \frac{U_B^2}{U_n^2} \cdot \frac{U}{\sqrt{3} \cdot I} = \frac{U_B^2}{U_n^2} \cdot Z \quad (\Omega)$$

$$Y' = \frac{I'}{U'} = \frac{U_n^2}{U_B^2} \cdot \frac{\sqrt{3} \cdot I}{U} = \frac{U_n^2}{U_B^2} \cdot Y \quad (S)$$

- Izbor baznog napona – najveći dio mreže ili npr. 100 kV
- Svi elementi u reduciranom sustavu izraženi stvarnim jedinicama
- Svi naponi u reduciranom sustavu – oko baznog napona

Metoda jediničnih vrijednosti

- Osnovne pretpostavke:
  1. stvarni sustav – trofazni, preslikani sustav – trofazni
  2. bazna snaga izabire se proizvoljno

$$S_{p.u.} = \frac{S}{S_B} \;, \quad S\,(MVA) - \text{snaga stvarnog (trofaznog) sustava},$$

$$S_B\,(MVA) - \text{bazna snaga (trofazna)}$$

$$U_{p.u.} = \frac{U}{U_B} = \frac{U}{U_n}, \quad U_B\,(kV) - \text{bazni napon određenog dijela mreže}$$

$$I_{p.u.} = \frac{I}{I_B} = \frac{\dfrac{S}{\sqrt{3}\cdot U}}{\dfrac{S_B}{\sqrt{3}\cdot U_n}} = \frac{S}{S_B}\cdot\frac{U_n}{U}, \quad U\,(kV) - \text{stvarni napon}$$

Metoda jediničnih vrijednosti

$$Z_{p.u.} = \frac{Z}{Z_B} = Z \cdot \frac{S_B}{U_n^2} \;\; (p.u.)$$

$$Y_{p.u.} = \frac{Y}{Y_B} = Y \cdot \frac{U_n^2}{S_B} \;\; (p.u.)$$

- Izbor bazne snage – proizvoljan, npr. 100 MVA
- Svi elementi u p.u. sustavu izraženi jediničnim vrijednostima
- Svi naponi u p.u. – oko vrijednosti 1 (0,9 – 1,1)
- Metoda postotnih vrijednosti (p.c.) – sve vrijednosti izražene u postotnim vrijednostima u odnosu na bazne vrijednosti

  (p.u. vrijednosti pomnožene sa 100 %)

Relativne veličine u proračunima mreža

- Metoda jediničnih vrijednosti (p.u.)
- Metoda otpora
- Metoda reduciranih admitancija

METODA OTPORA (relativne vrijednosti)	**METODA REDUCIRANIH ADMITANCIJA**	**JEDINIČNE VRIJEDNOSTI (per unit)**
$U' = U \cdot \frac{U_B}{U_n}$	$U_B = 1\ \text{kV}$ ili $S_B = 1\text{MVA}$ $U_r = \frac{U}{U_n}$	$U_{p.u.} = \frac{U}{U_{Bi}} = \frac{U}{U_{ni}}$
$I' = \frac{\sqrt{3} \cdot U_n \cdot I}{U_B}$	$I' = \sqrt{3} \cdot U_n \cdot I$	$I_{p.u.} = \frac{S}{S_B} = \frac{\sqrt{3} \cdot U \cdot I}{S_B}$
$Z' = \left(\frac{U_B}{U_n}\right)^2 \cdot Z$	$Z_r = \frac{Z}{U_n^{\ 2}}$	$Z_{p.u.} = Z \cdot \frac{S_B}{U_n^2}$
$Y' = \left(\frac{U_n}{U_B}\right)^2 \cdot Y$	$Y_r = \frac{U_n^{\ 2}}{Z} = U_n^{\ 2} \cdot Y$	$Y_{p.u.} = Y \cdot \frac{U_n^2}{S_B}$

Modeli elemenata EES-a

- Nadzemni vodovi i kabeli
- Energetski transformatori (2-namotni, 3-namotni)
- Sinkroni generatori

<u>Nadzemni vodovi</u>

- Modeli s raspodijeljenim parametrima (prijenosne jednadžbe)
- Modeli s koncentriranim parametrima (približni, točni)
- Trofazni model
- Jednofazni model
- Modeli u simetričnim komponentama (direktni, inverzni, nulti)

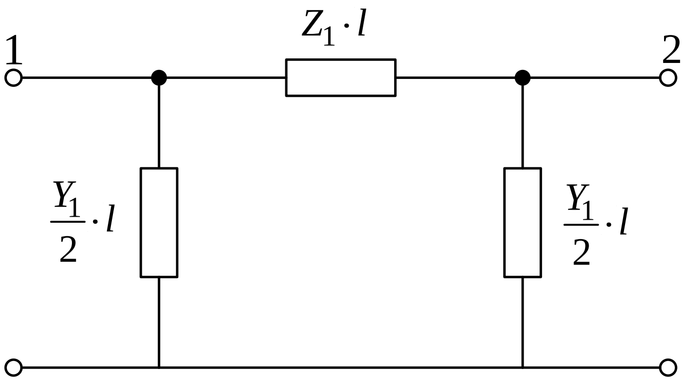
<u>Energetski kabeli</u>

- Osnovna razlika (izolacija)
- Modeli vrlo slični

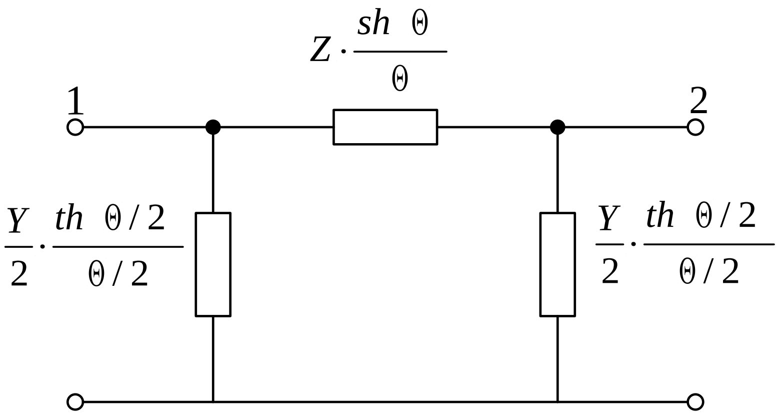
Nadzemni vodovi

- **Primjer:**

  Zadani su parametri 110 kV voda duljine 140 km: R_1=0.12 Ω/km, X_1=0.41 Ω/km, B_1=2.8 μS/km. Izračunajte parametre približne i točne π sheme voda.



Približna π shema voda



Točna π shema voda

Nadzemni vodovi

- **Rješenje:**
  - Približna π shema:

$$Z_\pi = Z_1 \cdot l = 16.8 + j57.4\ \Omega$$

$$\frac{Y_\pi}{2} = \frac{Y_1}{2} \cdot l = j1.96 \cdot 10^{-4}\ S$$

  - Točna π shema:

$$\Theta = \gamma \cdot l = \sqrt{Z_1 \cdot Y_1} \cdot l = 0.022 + j0.152$$

$$Z_\pi = Z_1 \cdot l \cdot \frac{sh\ \Theta}{\Theta} = 16.674 + j57.203\ \Omega$$

$$\frac{Y_\pi}{2} = \frac{Y_1}{2} \cdot l \cdot \frac{th\ \Theta/2}{\Theta/2} = j1.964 \cdot 10^{-4}\ S$$

Analiza elektroenergetskog sustava
Modeli elemenata mreže

Prof. dr. sc. Ivica Pavić

Prof. dr. sc. Marko Delimar

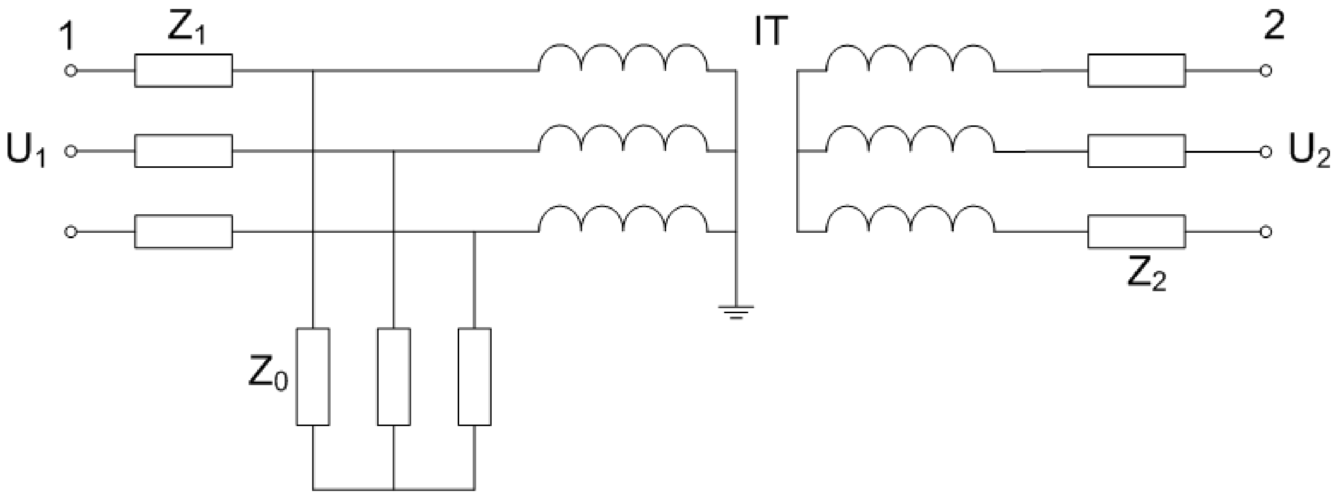
Energetski transformator

- Energetski transformatori
- Mjerni transformatori (naponski, strujni)

Energetski transformator

- Trofazni, jednofazni
- Broj namota (dvonamotni, tronamotni, autotransformatori)
- Grupe spoja (YNd5, YNyn0, Yz5, Dyn5...)
- Načini regulacije:
  - bez regulacije (fiksni prijenosni omjer)
  - s otcjepima (beznaponska pauza)
  - regulacijski (pod naponom)
- Uzdužna, poprečna (kosa) regulacija
- Pasivni element, privid aktivnog elementa (struje izjednačenja)

Model dvonamotnog transformatora



$Z_1 = R_1+jX_1$, R_1 – otpor primarnog namota, X_1 – reaktancija primarnog namota

$Z_2 = R_2+jX_2$, R_2 – otpor sekundarnog namota, X_2 – reaktancija sekundarnog namota

$Z_0 = R_0+jX_0$, R_0 – otpor poprečne impedancije (gubici u željezu)

X_0 – reaktancija poprečne impedancije (struja magnetiziranja)

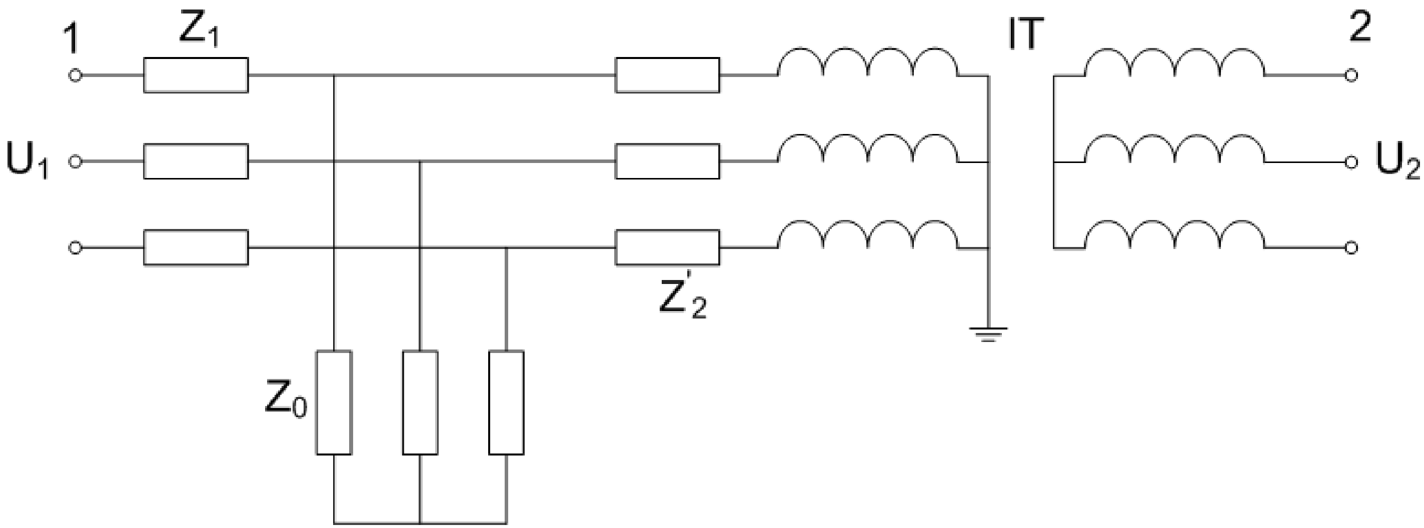
IT – idealni transfomator

Preračunavanje impedancija:

$$S = \frac{U_1^2}{Z_1} = \frac{U_2^2}{Z_1^{''}} \; , \; Z_1^{''} = Z_1 \left(\frac{U_2}{U_1} \right)^2$$

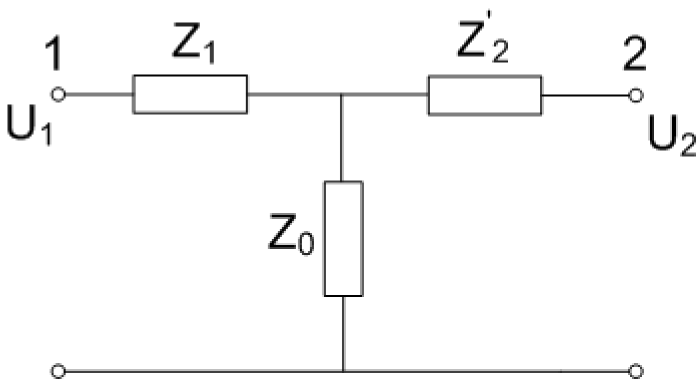
$$S = \frac{U_2^2}{Z_2} = \frac{U_1^2}{Z_2^{'}} \; , \; Z_2^{'} = Z_2 \left(\frac{U_1}{U_2} \right)^2$$

Preračunavanje na primarnu stranu

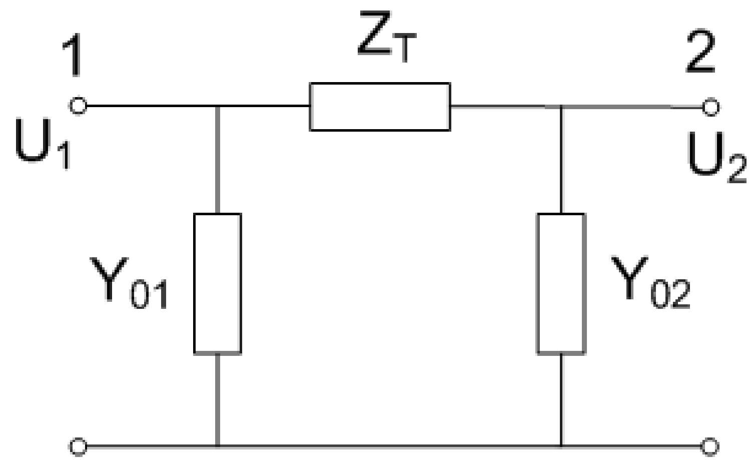


Pretpostavka: transformator je simetričan element – jednofazni model

T – shema transformatora preračunata na primarnu stranu



π-model transformatora



Pretvorba zvijezda – trokut

$$Y_T = \frac{Y_1 + Y_2^{'}}{Y_1 + Y_2^{'} + Y_0} = \frac{\frac{1}{Z_1} + \frac{1}{Z_2^{'}}}{\frac{1}{Z_1} + \frac{1}{Z_2^{'}} + \frac{1}{Z_0}} = \frac{1}{Z_1 + Z_2^{'} + \frac{Z_1 \cdot Z_2^{'}}{Z_0}} \approx \frac{1}{Z_1 + Z_2^{'}} = \frac{1}{Z_T} \quad , \quad Z_0 >> Z_1 , Z_2^{'}$$

$$Y_{01} = \frac{\frac{1}{Z_1} + \frac{1}{Z_0}}{\frac{1}{Z_1} + \frac{1}{Z_2^{'}} + \frac{1}{Z_0}} = \frac{1}{Z_1 + Z_0 + \frac{Z_1 \cdot Z_0}{Z_2^{'}}} = \frac{1}{Z_0\left(1 + \frac{Z_1}{Z_0} + \frac{Z_1}{Z_2^{'}}\right)} \approx \frac{1}{2Z_0} \quad , \quad Z_0 >> Z_1 , \; Z_1 \approx Z_2^{'}$$

$$Y_{02} \approx \frac{1}{2Z_0}$$

Određivanje uzdužnih impedancije transformatora

Nazivni podaci transformatora:

U_{n1} – nazivni linijski napon VN strane transformatora

U_{n2} – nazivni linijski napon NN strane transformatora

S_n – nazivna snaga transformatora

u_k (%) – napon kratkog spoja transformatora

P_k – gubici kratkog spoja (gubici u bakru)

i_0 (%) – struja magnetiziranja transformatora

P_0 – gubici praznog hoda (gubici u željezu)

Pokus kratkog spoja

$$u_k(\%) = \frac{I_n \cdot |Z_T|}{\frac{U_n}{\sqrt{3}}} \cdot 100$$

$$P_k = 3 \cdot I_n^2 \cdot R_T = \left(\frac{S_n}{U_n}\right)^2 \cdot R_T \Rightarrow R_T = \frac{P_k}{S_n^2} \cdot U_n^2$$

$$|Z_T| = \frac{u_k(\%)}{100} \cdot \frac{U_n}{I_n \cdot \sqrt{3}} = \frac{u_k(\%)}{100} \cdot \frac{U_n^2}{S_n} = u_k \cdot \frac{U_n^2}{S_n}$$

$$X_T = \sqrt{|Z_T|^2 - R_T^2} = \frac{U_n^2}{S_n} \sqrt{u_k^2 - \left(\frac{P_k}{S_n}\right)^2}$$

$$Z_T = \frac{U_n^2}{S_n} \left[\frac{P_k}{S_n} + j\sqrt{u_k^2 - \left(\frac{P_k}{S_n}\right)^2} \right]$$

Određivanje uzdužnih i poprečnih elemenata

Pokus praznog hoda

$$I_0 = i_0 \cdot I_n = \frac{U_n}{\sqrt{3}} \cdot \left|Y_0\right| \quad \Rightarrow \quad \left|Y\right|_0 = i_0 \cdot \frac{S_n}{U_n^2}$$

$$P_0 = 3\left(\frac{U_n}{\sqrt{3}}\right)^2 \cdot G_0 \quad \Rightarrow \quad G_0 = \frac{P_0}{U_n^2}$$

$$B_0 = \sqrt{\left|Y_0\right|^2 - G_0^2} = \frac{S_n}{U_n^2}\sqrt{i_0^2 - \left(\frac{P_0}{S_n}\right)^2}$$

$$Y_0 = G_0 - jB_0 = \frac{S_n}{U_n^2}\left[\frac{P_0}{S_n} - j\sqrt{i_0^2 - \left(\frac{P_0}{S_n}\right)^2}\right]$$

Okvirne vrijednosti parametara

u_k = 4 – 12 %

i_0 = 1 – 2,5 % I_n

P_k = 0,5 – 2,5 % S_n

P_0 = 15 – 40 % P_k

Transformator kao element mreže

- Prijenosni omjeri svih transformatora jednaki nazivnim naponima mreže (npr. 220/110, 400/110, 400/220)
  - svi elementi mreže se preračunaju na jedan naponski nivo
  - idealni transformatori mogu se zanemariti
- Prijenosni omjeri transformatora nisu jednaki nazivnim naponima mreže (npr. 220/115, 400/115, 400/231)
  - idealni transformatori se ne mogu zanemariti
  - primjena metode otpora ili p.u.

<u>Metoda otpora</u>

$$Z_T' = \left(\frac{U_B}{U_n}\right)^2 \cdot Z_T = \frac{U_B^2}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{u_k^2 - \left(\frac{P_k}{S_n}\right)^2}\right] \; [\Omega]$$

$$Y_0' = \left(\frac{U_n}{U_B}\right)^2 \cdot Y_0 = \frac{S_n}{U_B^2}\left[\frac{P_0}{S_n} - j\sqrt{i_0^2 - \left(\frac{P_0}{S_n}\right)^2}\right] \; [S]$$

<u>Metoda p.u.</u>

$$Z_{Tp.u.} = \frac{S_B}{U_B^2} \cdot Z_T = \frac{S_B}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{u_k^2 - \left(\frac{P_k}{S_n}\right)^2}\right] \; [p.u.]$$

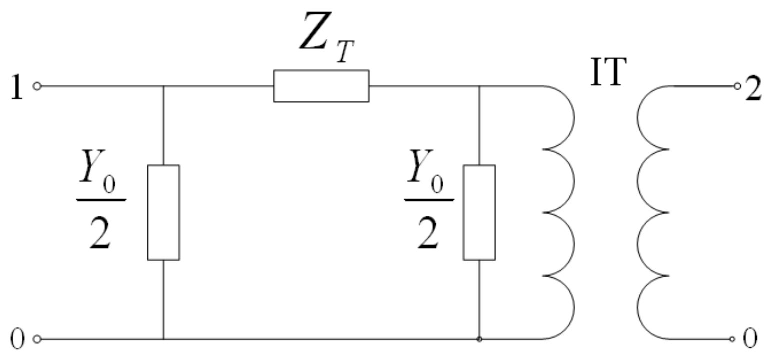
$$Y_{0p.u.} = \frac{U_B^2}{S_B} \cdot Y_0 = \frac{S_n}{S_B}\left[\frac{P_0}{S_n} - j\sqrt{i_0^2 - \left(\frac{P_0}{S_n}\right)^2}\right] \; [p.u.]$$

Primjer modeliranja transformatora

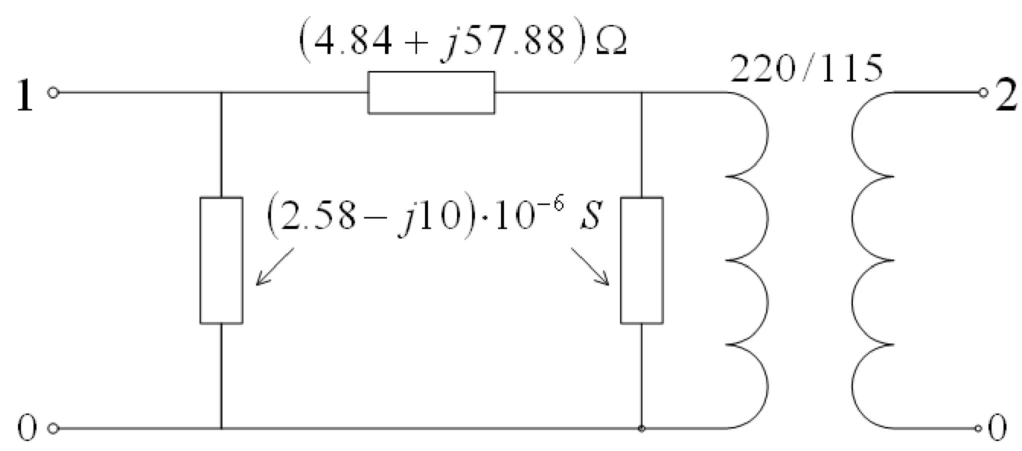
$S_n = 100\ MVA\ ;\ u_k\ \% = 12\%\ ;\ i_0 = 1\%$

$P_k = 1\%\ od\ S_n\ ;\ P_0 = 25\%\ P_k = 0{,}25\%\ od\ S_n$

$Prijenosni\ odnos\ 220/115\ kV$



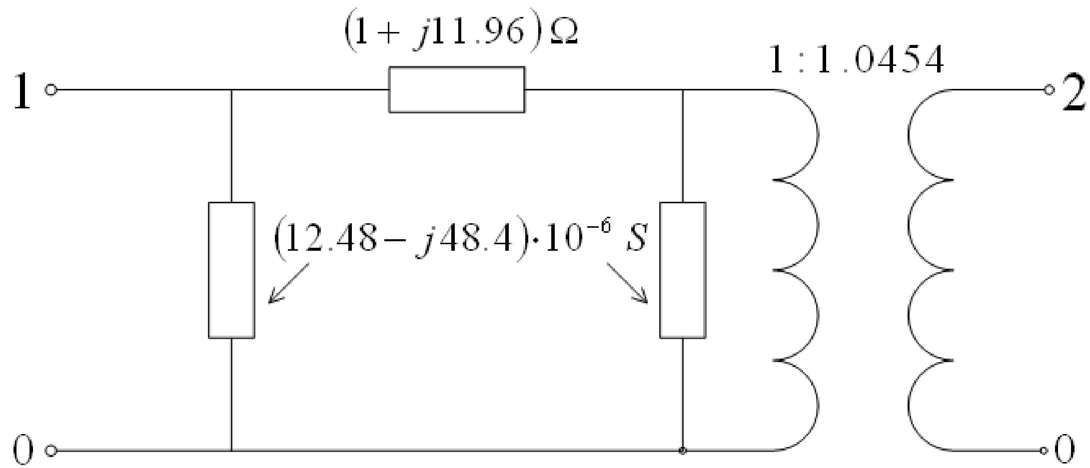
Stvarni model transformatora



Modeli transformatora

Metoda otpora

$$U_B = 100\ kV$$



Elementi preračunati na 220 kV stranu (stvarni napon transformatora jednak nazivnom naponu mreže)

Prijenosni omjer transformatora:

$$\frac{\frac{220}{115}}{\frac{220}{110}} = \frac{110}{115} = 1:\frac{115}{110} = 1:1{,}04545$$

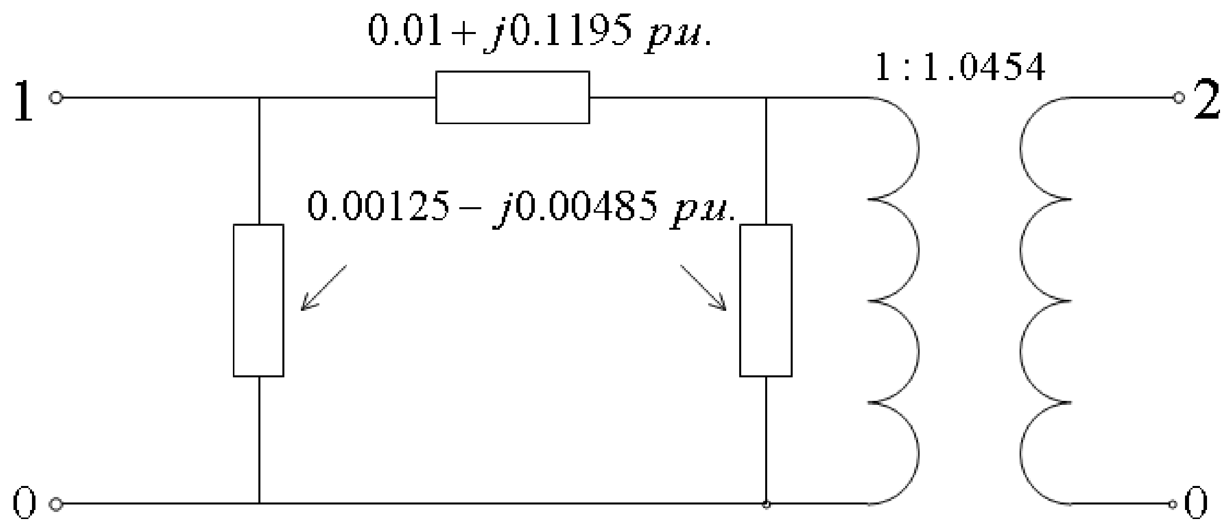
stvarni prijenosni omjer 220/115

nazivni prijenosni omjer 220/110

Modeli transformatora

Metoda p.u.

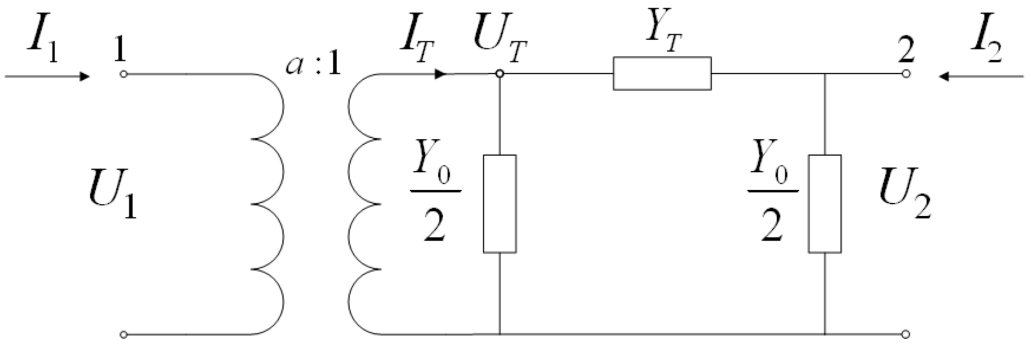
$S_B = 100\ MVA$



- Bitno: nazivni prijenosni omjer (1) s one strane na koju se preračunavaju elementi
- Poprečna admitancija se često zanemaruje
- Ponekad se može zanemariti i djelatni otpor uzdužne impedancije

Model bez idealnog transformatora

- Naponski i strujni odnosi u model s idealnim transformatorom



$$\frac{U_1}{U_T} = a = \frac{I_T}{I_1}$$

$$U_1 = a \cdot U_T \qquad I_T = a \cdot I_1$$

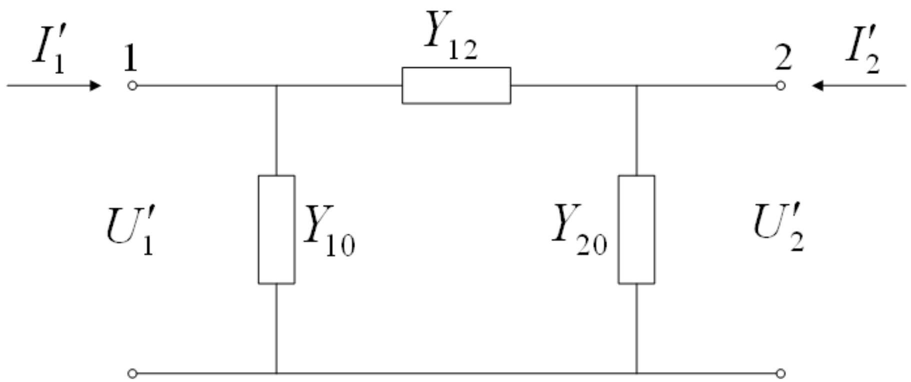
$$I_T = \left(U_T - U_2\right) \cdot Y_T + \frac{Y_0}{2} \cdot U_T$$

$$I_1 = \left(\frac{U_T}{a} - \frac{U_2}{a}\right) \cdot Y_T + \frac{Y_0}{2} \cdot \frac{U_T}{a} = \left(\frac{U_1}{a^2} - \frac{U_2}{a}\right) \cdot Y_T + \frac{Y_0}{2} \cdot \frac{U_1}{a^2} = \left(\frac{U_1}{a} - U_2\right) \cdot \frac{Y_T}{a} + \frac{Y_0}{2} \cdot \frac{U_1}{a^2}$$

$$I_2 = \left(U_2 - U_T\right) \cdot Y_T + \frac{Y_0}{2} \cdot U_2 = \left(U_2 - \frac{U_1}{a}\right) \cdot Y_T + \frac{Y_0}{2} \cdot U_2 = \left(U_2 \cdot a - U_1\right) \cdot \frac{Y_T}{a} + \frac{Y_0}{2} \cdot U_2$$

Model bez idealnog transformatora

- Pretvorba modela s idealnim transformatorom u model bez idealnog transformatora (pasivni četveropol)



$$I_1' = (U_1' - U_2') \cdot Y_{12} + U_1' \cdot Y_{10}$$

$$I_2' = (U_2' - U_1') \cdot Y_{12} + U_2' \cdot Y_{20}$$

- Parametri se određuju usporedbom struja pri pokusima kratkog spoja

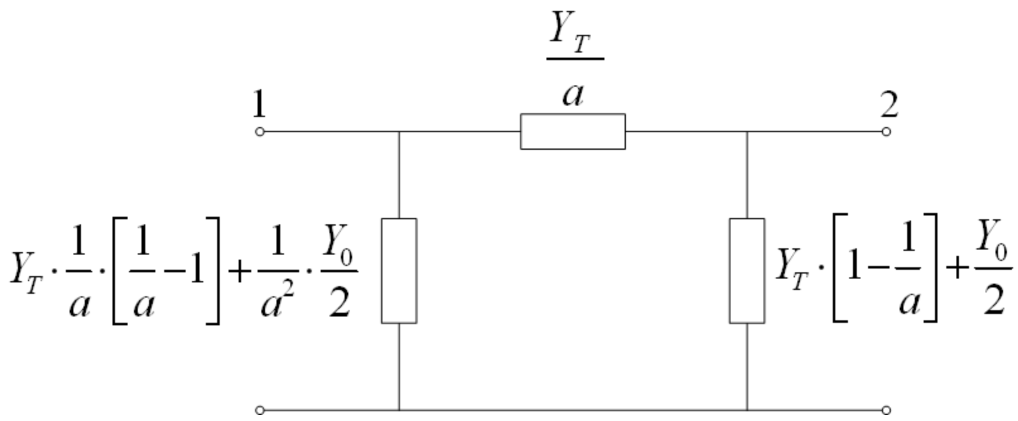
k.s u točki 2: $U_1 = 1$, $U_2 = 0$

k.s u točki 1: $U_2 = 1$, $U_1 = 0$

$$Y_{12} = \frac{Y_T}{a} \qquad Y_{01} = Y_T \cdot \frac{1}{a} \cdot \left[\frac{1}{a} - 1\right] + \frac{1}{a^2} \cdot \frac{Y_0}{2}$$

$$Y_{02} = Y_T \cdot \left[1 - \frac{1}{a}\right] + \frac{Y_0}{2}$$

Model bez idealnog transformatora



$$a = \frac{a_s}{a_n} = \frac{\frac{U_1}{U_2}}{\frac{U_{n1}}{U_{n2}}}$$

- Elementi se preračunavaju na čvorište 2 (nazivni napon transformatora odgovara nazivnom naponu mreže)
- Prijenosni omjer (*a*) računa se na onu stranu koja se regulira (napon te strane transformatora nije jednak nazivnom naponu mreže)

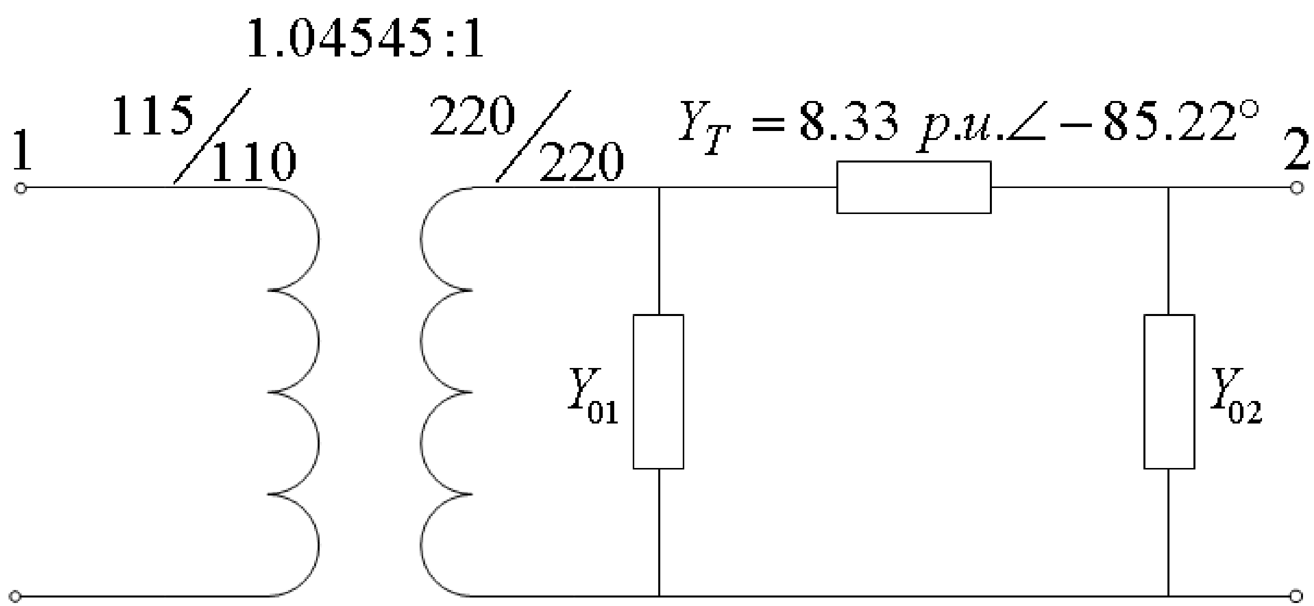
Model bez idealnog transformatora

Primjer

$$S_n = 100\ MVA\ ;\ u_k\ \% = 12\%\ ;\ i_0 = 1\%$$

$$P_k = 1\%\ od\ S_n\ ;\ P_0 = 25\%\ P_k = 0{,}25\%\ od\ S_n$$

$$Prijenosni\ odnos\ 220/115\ kV$$



Model bez idealnog transformatora

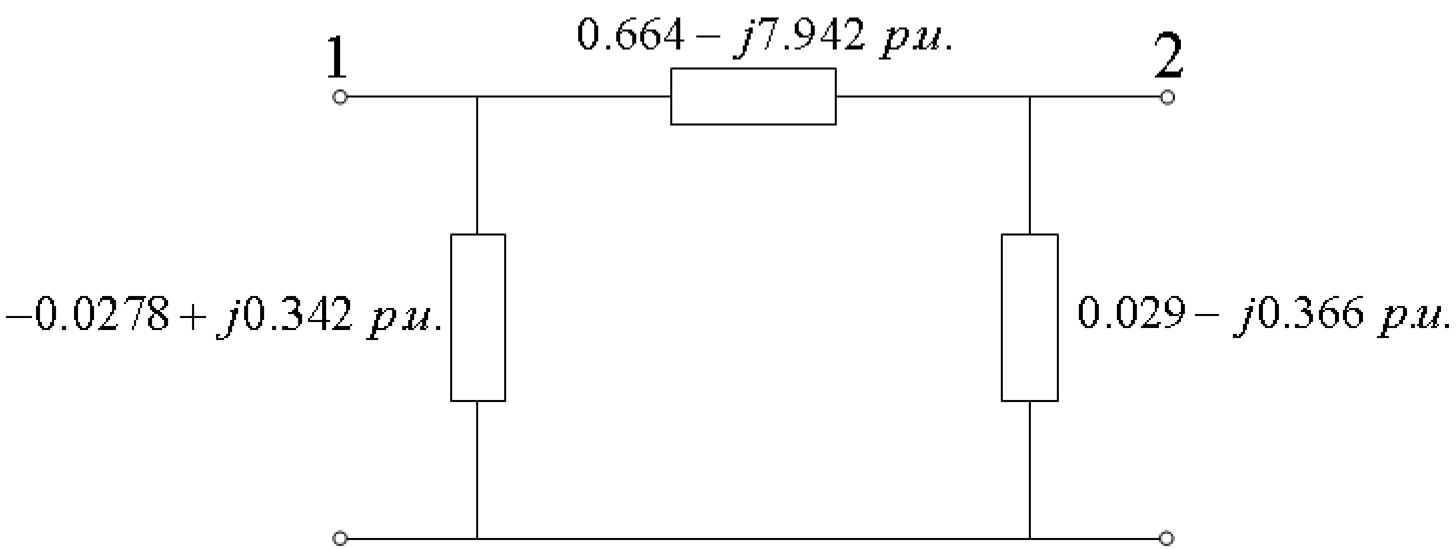
Primjer:

$$Y_{12} = \frac{Y_T}{a} = \frac{8.33\ p.u.\angle - 85.22°}{1.0454} = 7.97\ p.u.\angle - 85.22°$$

$$Y_{12} = \frac{Y_T}{a} = 0.664 - j7.942\ p.u.$$

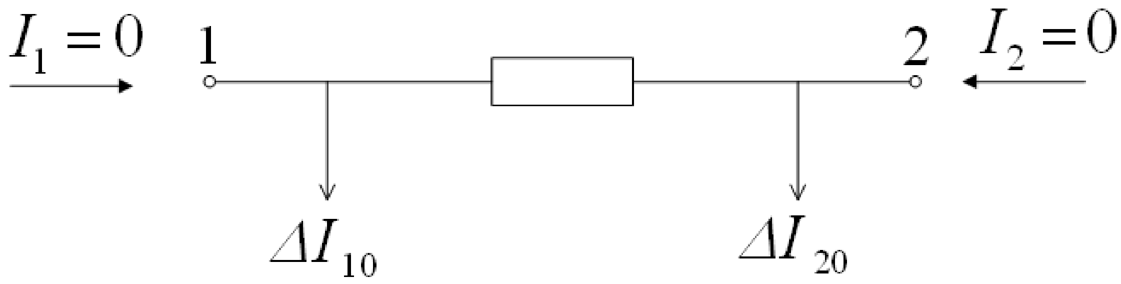
$$Y_{01} = Y_T \cdot \frac{1}{a} \cdot \left[\frac{1}{a} - 1\right] + \frac{1}{a^2} \cdot \frac{Y_0}{2} = -0.0278 + j0.342\ p.u.$$

$$Y_{02} = Y_T \cdot \left[1 - \frac{1}{a}\right] + \frac{Y_0}{2} = 0.029 - j0.366\ p.u.$$



Model bez idealnog transformatora

Prazni hod (zanemaren Y_0)



$$\Delta I_{10} = U_1 \cdot (-0.0289 + j0.346)$$

$$\Delta I_{20} = U_2 \cdot (0.0302 - j0.361)$$

$$\sum I = 0$$

$$\Delta I_{10} = -\Delta I_{20}$$

Model bez idealnog transformatora

Prazni hod (zanemaren Y_0)

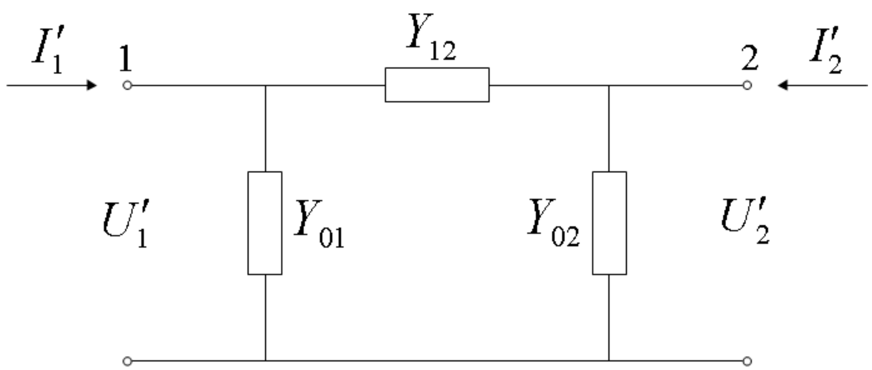
$$\frac{U_1}{U_2} = \frac{0.0302 - j0.361}{0.0289 - j0.34} = \frac{0.3628\angle 85.22°}{0.3471\angle 85.22°}$$

$$\Delta I_2 = U_2 \cdot Y_{02} = U_2 \cdot Y_T \cdot \left(1 - \frac{1}{a}\right)$$

$$\Delta I_1 = U_1 \cdot Y_{01} = U_1 \cdot Y_T \cdot \frac{1}{a} \cdot \left(\frac{1}{a} - 1\right)$$

$$\frac{U_1}{U_2} = a$$

Matrični oblik modela transformatora



Opći oblik:

$$\begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} Y_{12} + Y_{01} & -Y_{12} \\ -Y_{12} & Y_{12} + Y_{02} \end{vmatrix} \cdot \begin{vmatrix} U_1 \\ U_2 \end{vmatrix}$$

$$\begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} \frac{Y_T}{a} + Y_T \cdot \frac{1}{a} \cdot \left[\frac{1}{a} - 1\right] & -\frac{Y_T}{a} \\ -\frac{Y_T}{a} & \frac{Y_T}{a} + Y_T \cdot \left[1 - \frac{1}{a}\right] \end{vmatrix} \cdot \begin{vmatrix} U_1 \\ U_2 \end{vmatrix}$$

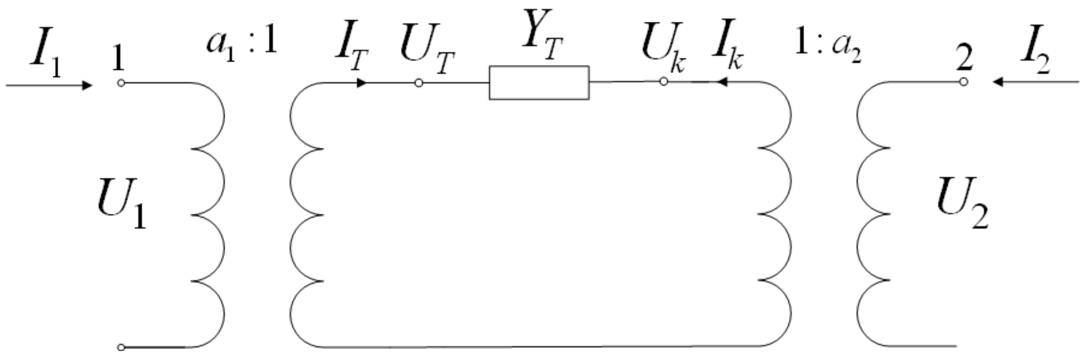
$$\begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} \frac{Y_T}{a^2} & -\frac{Y_T}{a} \\ -\frac{Y_T}{a} & Y_T \end{vmatrix} \cdot \begin{vmatrix} U_1 \\ U_2 \end{vmatrix}$$

matrica admitancija čvorišta

Opći model transformatora

- Nenazivni prijenosni omjeri na obje strane

- Model s idealnim transformatorima



$$\frac{U_1}{U_T} = a_1 = \frac{I_T}{I_1} \quad , \quad \frac{U_2}{U_k} = a_2 = \frac{I_k}{I_2}$$

$$I_1 \cdot a_1 = \left(\frac{U_1}{a_1} - \frac{U_2}{a_2} \right) \cdot Y_T = (U_1 \cdot a_2 - U_2 \cdot a_1) \cdot \frac{Y_T}{a_1^2 \cdot a_2}$$

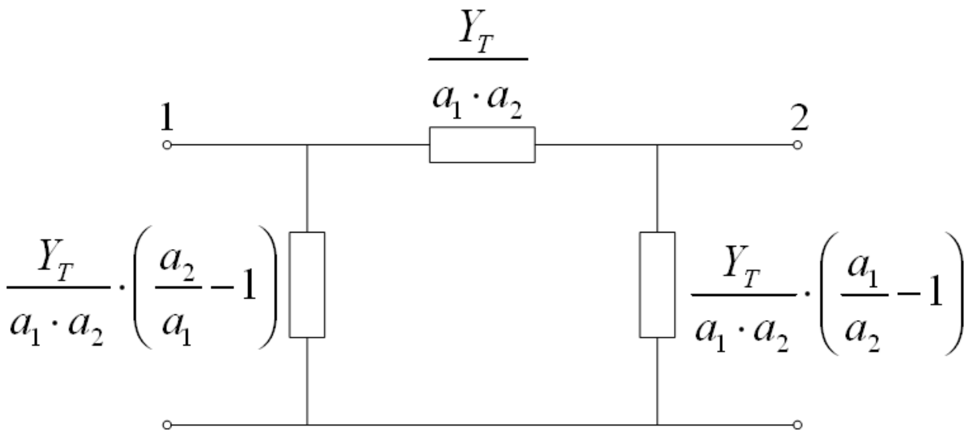
$$I_1 = \left(U_1 \cdot \frac{a_2}{a_1} - U_2 \right) \cdot \frac{Y_T}{a_1 \cdot a_2}$$

$$\vdots$$

$$I_2 = \left(U_2 \cdot \frac{a_1}{a_2} - U_1 \right) \cdot \frac{Y_T}{a_1 \cdot a_2}$$

Opći model transformatora

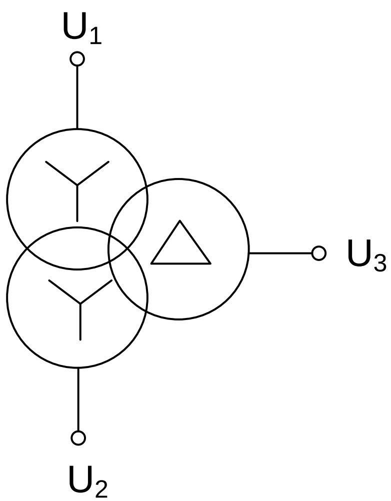
- Pretvorba modela s idealnim transformatorom u model bez idealnog transformatora (pasivni četveropol)

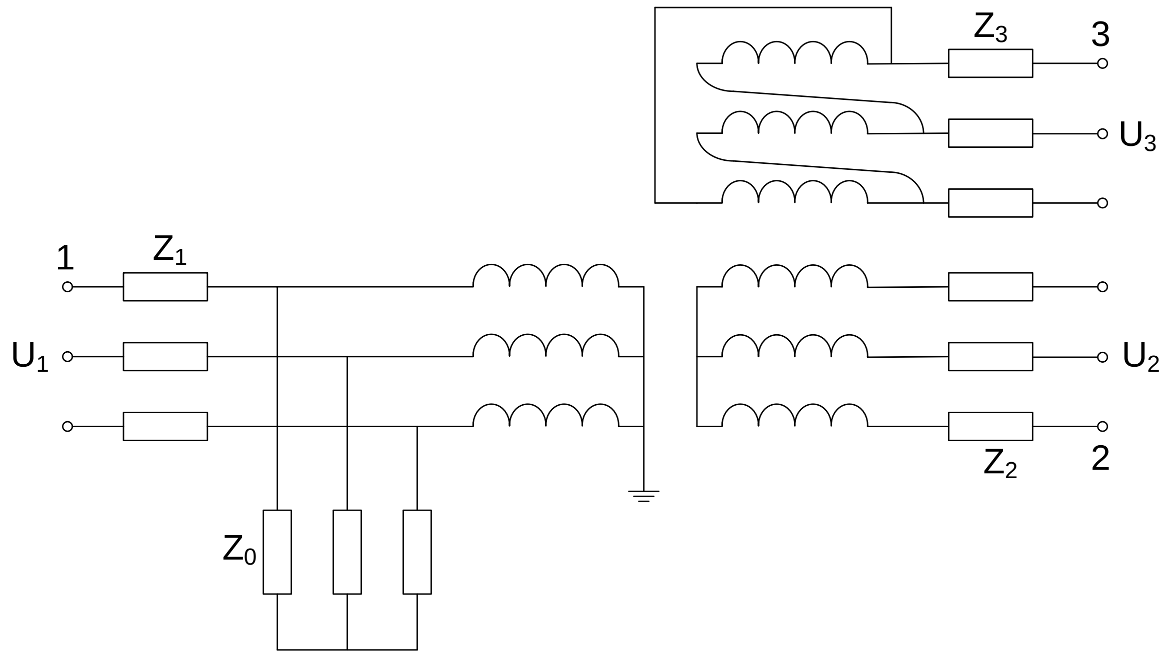


$$\begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} \frac{Y_T}{a_1^2} & -\frac{Y_T}{a_1 \cdot a_2} \\ -\frac{Y_T}{a_1 \cdot a_2} & \frac{Y_T}{a_2^2} \end{vmatrix} \cdot \begin{vmatrix} U_1 \\ U_2 \end{vmatrix}$$

Tronamotni transformator

- Primjena i uloga tronamotnih transformatora
- Dimenzioniranje tercijarnog namota





- Pokusi kratkog spoja i praznog hoda
- Određivanje u_{k12}, u_{k13}, u_{k23} i P_{k12}, P_{k13}, P_{k23}
- Prolazna snaga (nazivna snaga slabijeg namota)

Tronamotni transformator

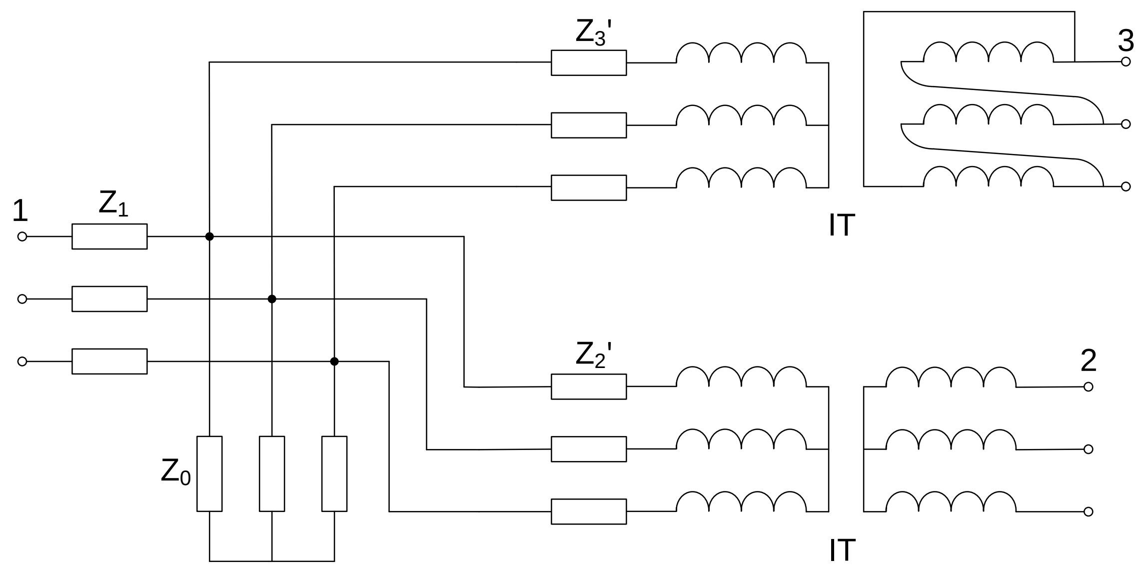
$$Z_{12} = \frac{U_{n1}^2}{S_{n12}}\left[\frac{P_{k12}}{S_{n12}} + j\sqrt{u_{k12}^2 - \left(\frac{P_{k12}}{S_{n12}}\right)^2}\right]\ [\Omega] \quad Z_{13} = \frac{U_{n1}^2}{S_{n13}}\left[\frac{P_{k13}}{S_{n13}} + j\sqrt{u_{k13}^2 - \left(\frac{P_{k13}}{S_{n13}}\right)^2}\right]\ [\Omega]$$

$$Z_{23} = \frac{U_{n1}^2}{S_{n23}}\left[\frac{P_{k23}}{S_{n23}} + j\sqrt{u_{k23}^2 - \left(\frac{P_{k23}}{S_{n23}}\right)^2}\right]\ [\Omega]$$

- Preračunavanje elemenata na primarnu stranu

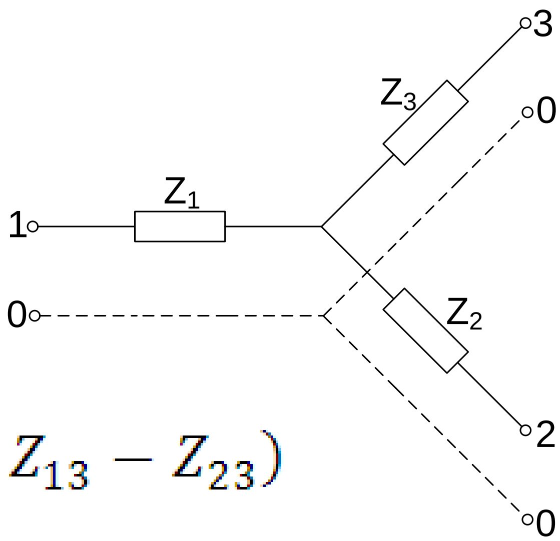
$$Z_2' = \left(\frac{U_1}{U_2}\right)^2 \cdot Z_2$$

$$Z_3' = \left(\frac{U_1}{U_3}\right)^2 \cdot Z_3$$



Tronamotni transformator

- Jednofazni model uz zanemarenu poprečnu granu



$$Z_{12} = Z_1 + Z_2$$

$$Z_{13} = Z_1 + Z_3$$

$$Z_{23} = Z_2 + Z_3$$

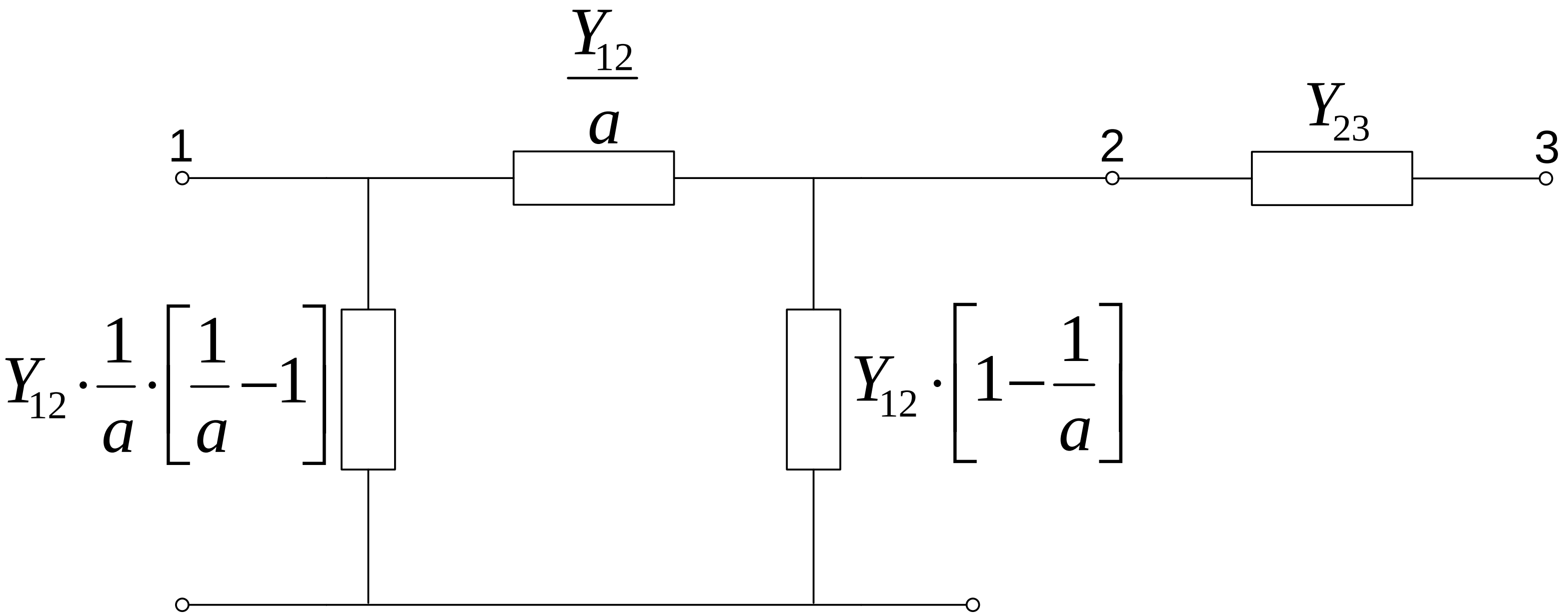
$$Z_1 = \frac{1}{2}(Z_{12} + Z_{13} - Z_{23})$$

$$Z_2 = \frac{1}{2}(Z_{12} + Z_{23} - Z_{13})$$

$$Z_3 = \frac{1}{2}(Z_{13} + Z_{23} - Z_{12})$$

Tronamotni transformator

- U praksi je Z_2 vrlo blizu vrijednosti 0 ili čak i negativan
- Z_2 se često zanemaruje (koristi se pojednostavljena shema)



Sinkroni strojevi

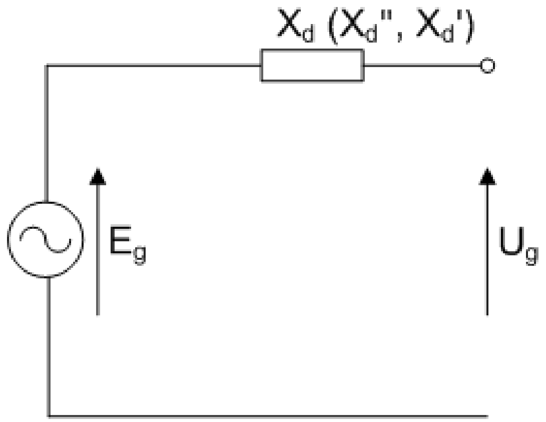
- Sinkroni generator
- Sinkroni motor
- Sinkroni kompenzator

Model generatora

- Za tokove snaga vrlo jednostavan (izvor djelatne i jalove snage)
- Za kratki spoj nešto složeniji (idealni izvor i reaktancija)

$$X_d = \frac{X_d\%}{100} \cdot \frac{U_n^2}{S_n} \; [\Omega]$$

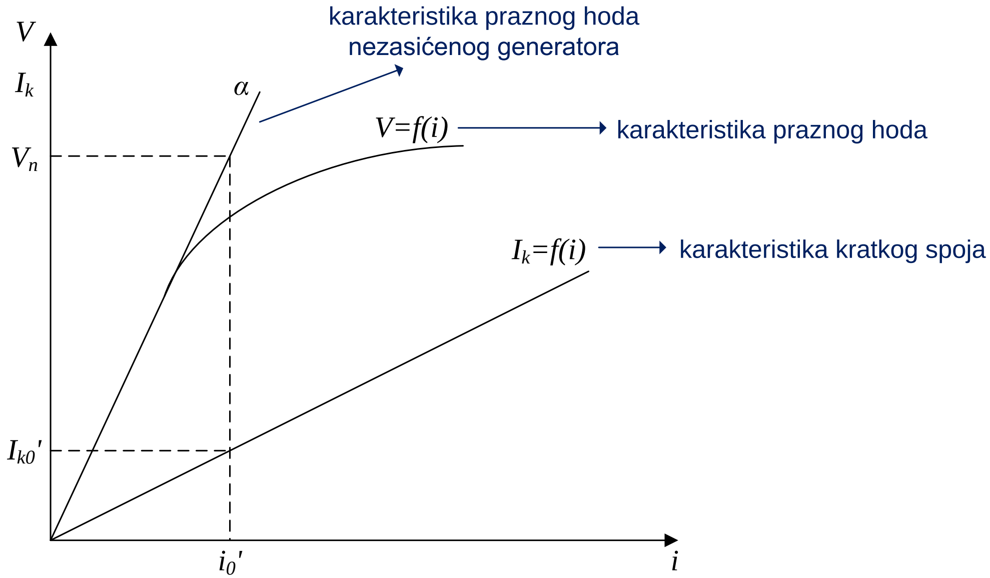
$$X_d'' = \frac{X_d''\%}{100} \cdot \frac{U_n^2}{S_n} \; [\Omega]$$



$$X_d''[p.u.] = \frac{X_d''}{Z_B} = \frac{X_d''\%}{100} \cdot \frac{U_n^2}{S_n} \cdot \frac{S_B}{U_n^2} = \frac{X_d''\%}{100} \cdot \frac{S_B}{S_n} \; [\mathrm{p.u.}]$$

Sinkroni strojevi

- *R* generatora je zanemaren
- X_d je sinkrona reaktancija, a X_d'' početna ili subtranzijentna reaktancija



V_n – nazivni fazni napon
i – struja uzbude generatora
i_0' – nazivna struja uzbude
i_{k0}' – struja kratkog spoja kod nazivne uzbude

- Sinkrona reaktancija se definira kao omjer nazivnog napona i struje generatora u kratkom spoju:

$$X_d = \frac{V_n}{I_{k0}'} \ [\Omega]$$

Sinkroni strojevi

- Sinkrona reaktancija izražena u postotnoj vrijednosti:

$$X_d[\%] = \frac{X_d}{X_n} \cdot 100\,\%$$

- Pri tome je:

$$X_n = \frac{V_n}{I_n}\ [\Omega]$$

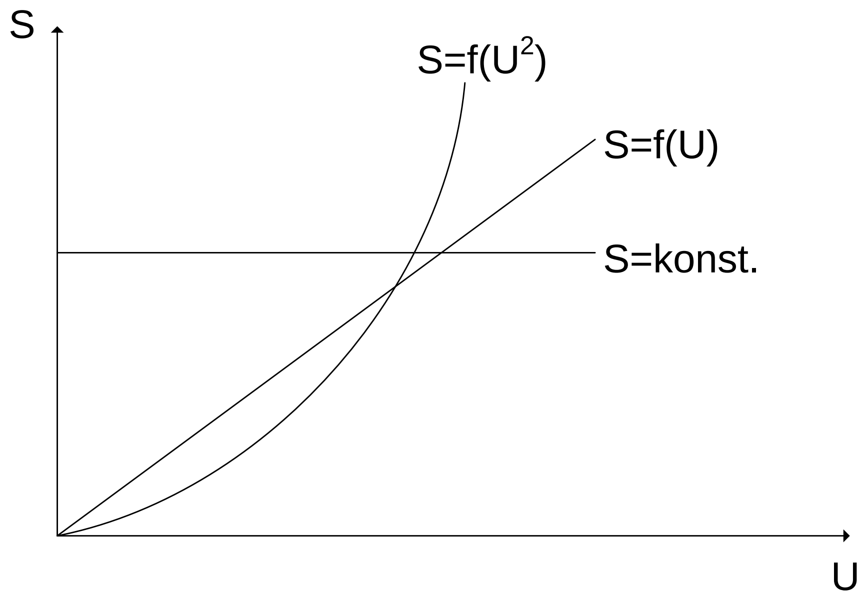
- Iz navedenoga slijedi:

$$X_d[\%] = X_d \cdot \frac{I_n}{V_n} \cdot 100\% = X_d \cdot \frac{3 \cdot V_n \cdot I_n}{3 \cdot V_n^{\,2}} \cdot 100\% = X_d \cdot \frac{S_n}{U_n^{\,2}} \cdot 100\%$$

$$\boxed{X_d = \frac{X_d[\%]}{100} \cdot \frac{U_n^{\,2}}{S_n}\ [\Omega]}$$

Modeliranje opterećenja

- Tri osnovna modela opterećenja (tereta) s obzirom na ovisnost o naponu:
  1. Konstantna impedancija - najjednostavniji model tereta $S=f(U^2)$
  2. Konstantna struja - $S=f(U)$
  3. Konstantna snaga - karakteristično za visokonaponske mreže



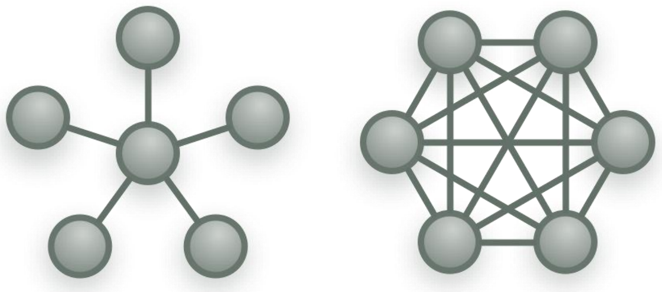
Analiza elektroenergetskog sustava Predavanje 4: Matrične metode

Prof. dr. sc. Ivica Pavić

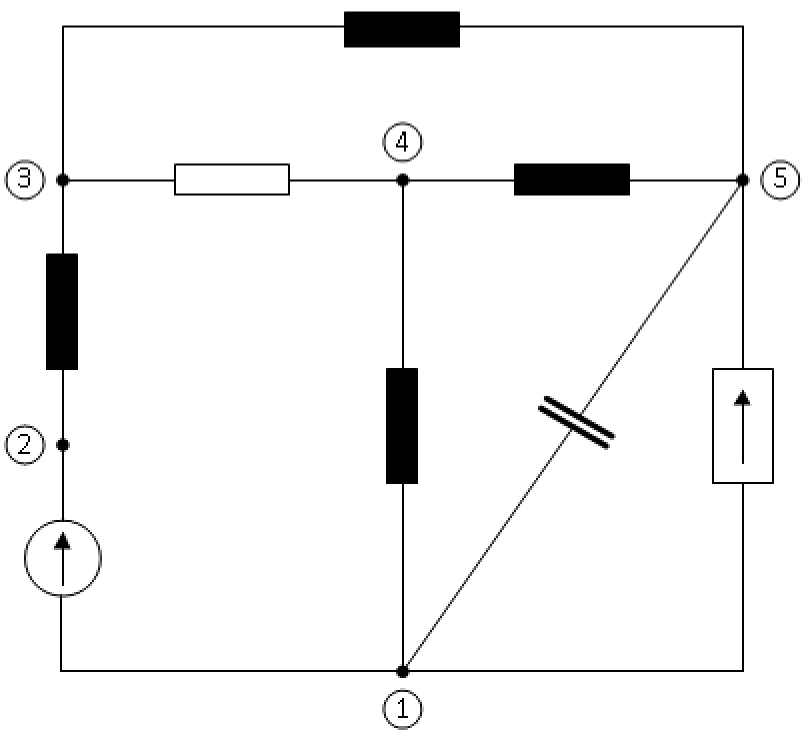
Izv. prof. dr. sc. Marko Delimar

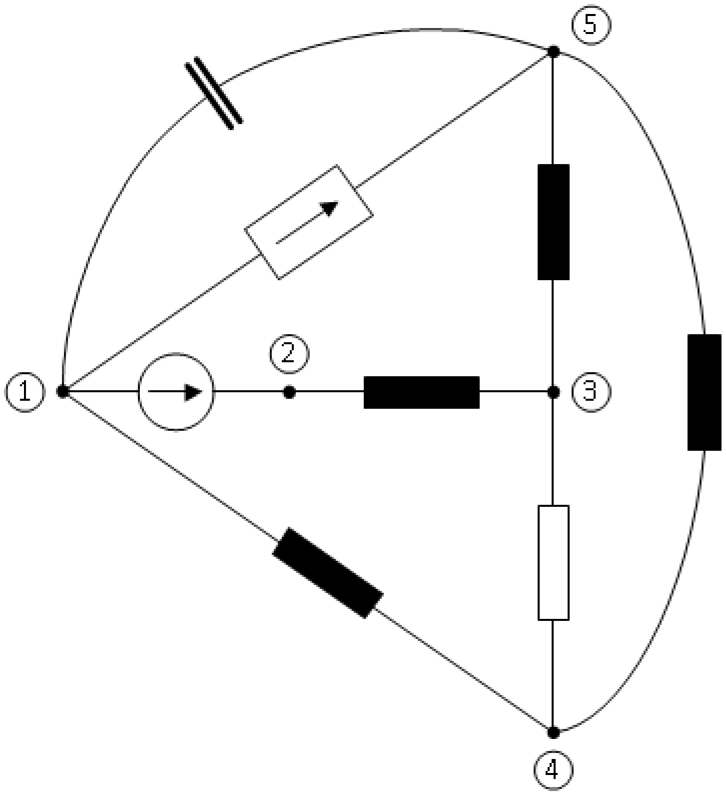
Topologija

- Topologija ili teorija grafova - disciplina matematike
- Geometrijska struktura mreža
  - Čvorište
  - Grana
  - Graf
  - Stablo
  - Put
  - Petlja



Primjer - dva modela iste mreže

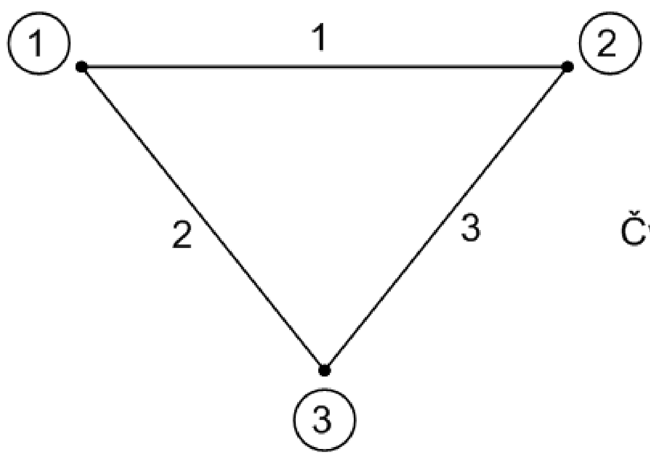




Čvorišta i grane

- Grana (element) se prikazuje dužinom

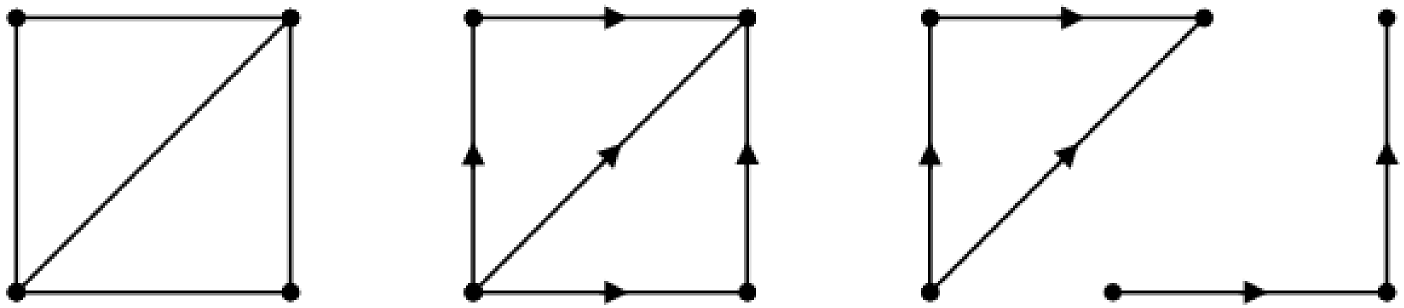
- Grana započinje i završava čvorištem
- Čvorište je krajnja točka grane
- Iz nekog čvorišta može izlaziti više grana
- Ako se čvorište i grana međusobno dodiruju, kažemo da su incidentni



Čvorište 1 je incidentno s granama 1 i 2

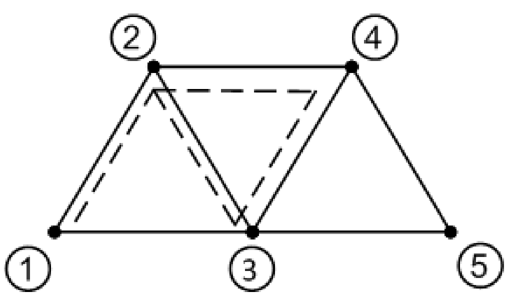
Graf

- Graf pokazuje geometrijsku vezu između elemenata mreže
- Ako svakom elementu grafa (granama) dodijelimo smjer, graf je **orijentiran**
- Dio grafa (podniz elemenata grafa) nazivamo **podgraf** (subgraf)

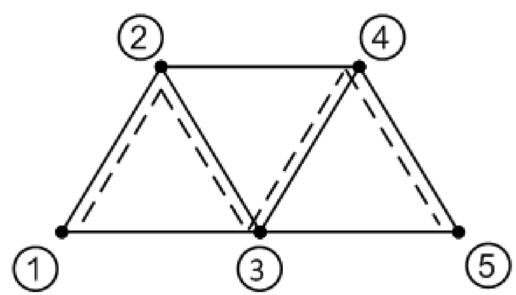


Put i petlja

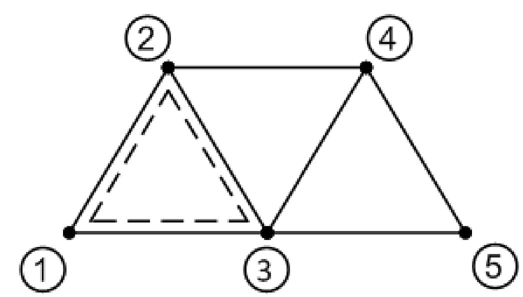
- Kad u mreži pođemo od nekog čvorišta i putujemo po granama dodirujući pri tome nova ili već dodirnuta čvorišta i konačno se zaustavimo kod nekog čvorišta kažemo da smo prešli **put**
- Ako je početno čvorište različito od završnog kažemo da je put **otvoren**, a ako je isto kažemo da je put **zatvoren**
- Put je **jednostavan** ako se idući njime nijedno čvorište ne prođe dva puta
- Zatvoreni jednostavni put naziva se **petlja**



otvoreni put



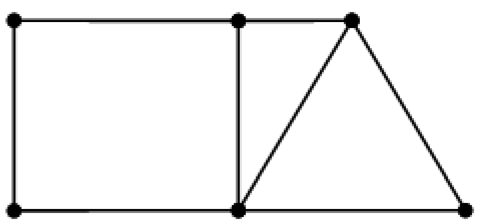
jednostavni otvoreni put



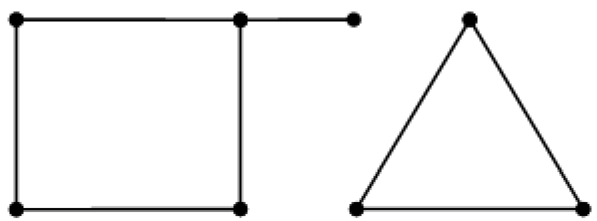
jednostavni zatvoreni put

Suvisla i nesuvisla mreža

- Ako su svaka dva različita čvorišta međusobno povezana putem mreža je **suvisla** (povezana)
- Ako se u nekoj petlji suvisle mreže ukloni bilo koja grana, mreža ostaje suvisla



suvisla mreža



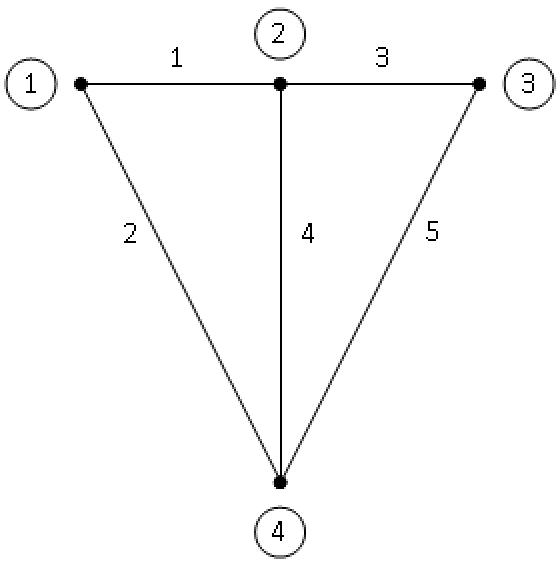
nesuvisla mreža

Stablo (1)

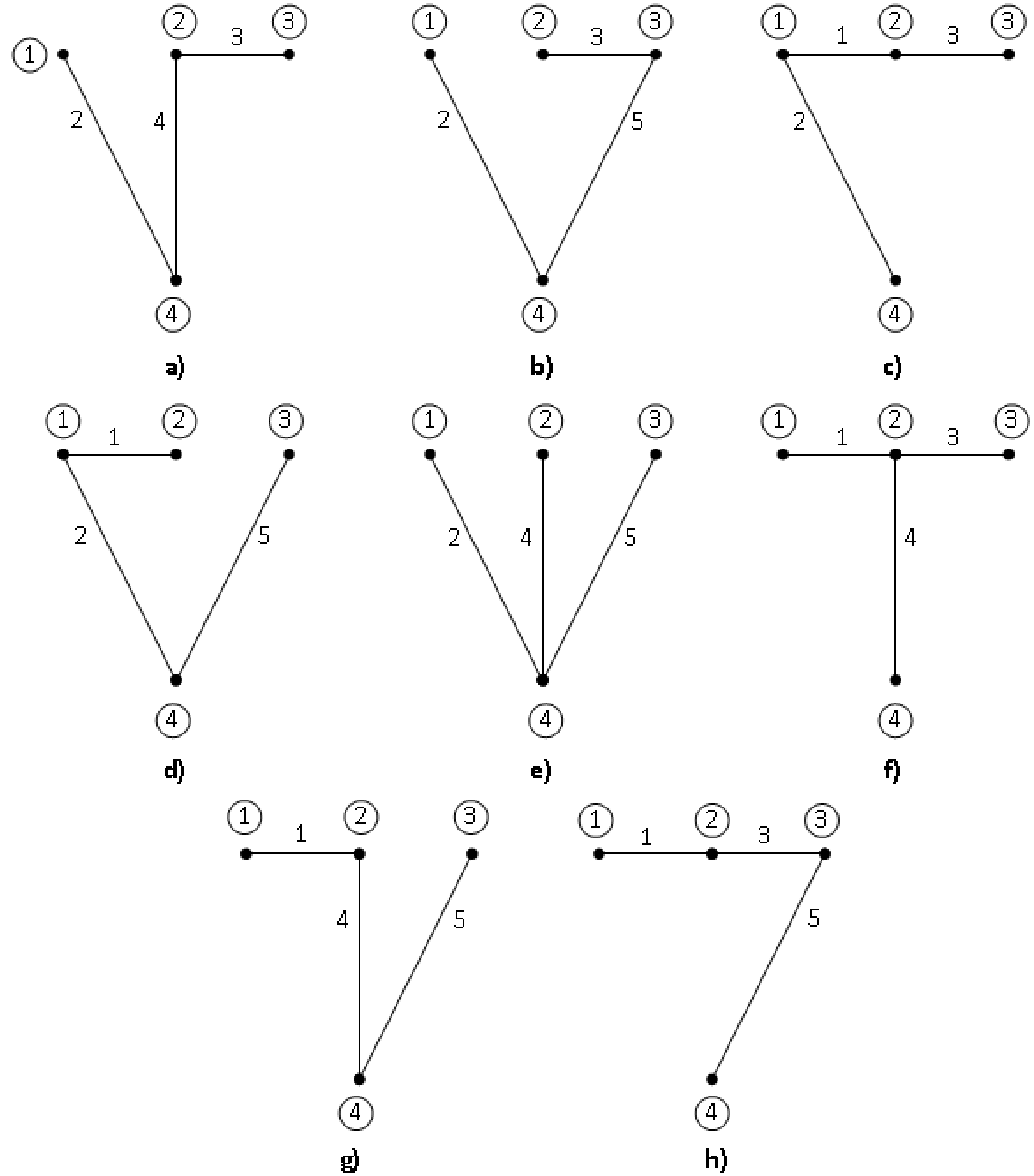
- Suvisla mreža bez petlji je **stablo**
- Iz svake suvisle mreže koja ima petlje možemo postepenim uklanjanjem grana načiniti stablo (s istim brojem čvorišta)
- Stablo je, dakle, podgraf koji sadrži sva čvorišta grafa, a nema niti jedan zatvoren put

- Grane koje tvore stablo su **zavisne grane**
- Nazivamo ih zavisne zbog toga što čine mrežu suvislom
- Ostale grane su nezavisne i čine petlje

Stablo (2)



Zadani graf

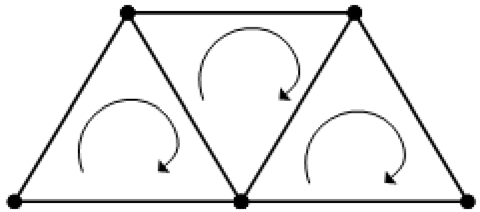


Sva stabla zadanog grafa

Oznake i odnosi

- n – broj čvorišta
- g – ukupni broj grana mreže
- p – broj temeljnih petlji (jednak je broju nezavisnih grana)
- g_{min} – broj grana stabla
- g_{max} – broj grana potreban da bi se svi parovi čvorišta međusobno povezali (s po jednom granom)
- $g_{min} = n-1$
- $g_{max} = n(n-1)/2$
- $p = g - g_{min} = g-n+1$

Primjer



$$n = 5$$

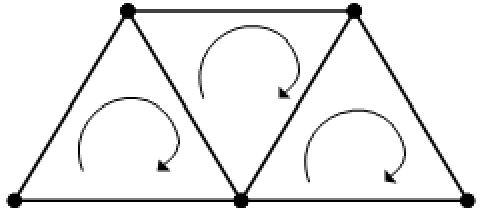
$$g_{min} = 4$$

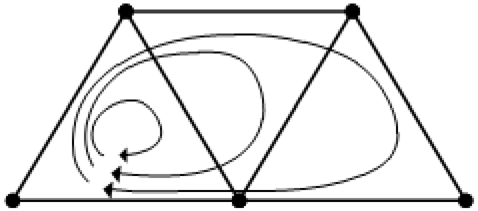
$$g_{max} = \frac{5 \cdot 4}{2} = 10$$

$$g = 7$$

$$p = g - n + 1 = 3$$

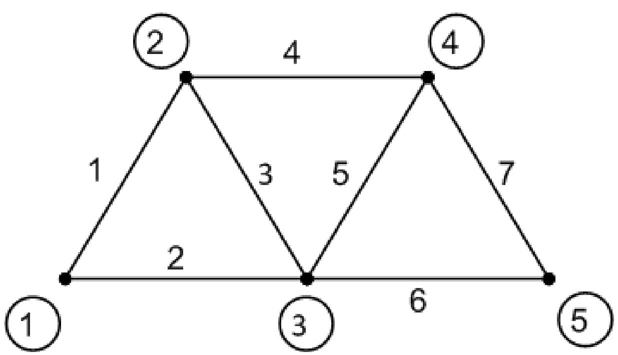
Napomena: koje petlje odabrati nije jednoznačno, ali je ukupni broj temeljnih petlji jednoznačno određen





Spojna matrica - M

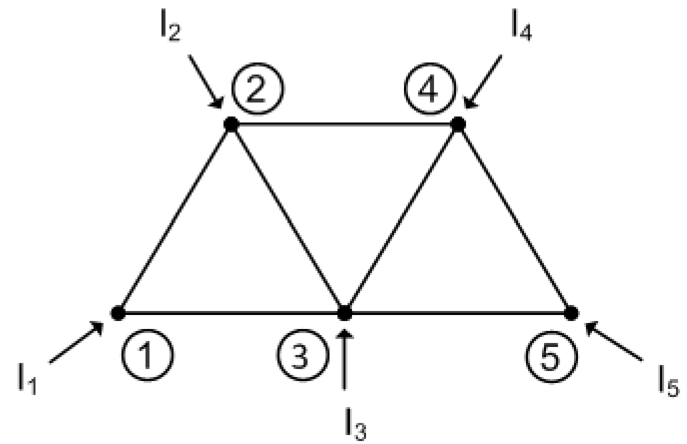
- Matrica koja daje vezu između čvorišta i grana
- Svaki stupac sadrži jednu pozitivnu i jednu negativnu jedinicu, jer smaka grana počinje i završava u nekom čvorištu
- Svaki redak ima barem jedan element različit od nule (ako je mreža suvisla)



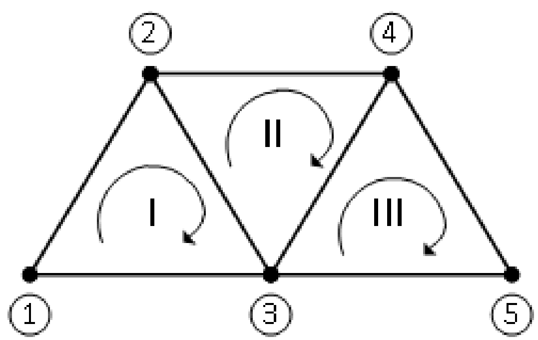
$$
\begin{array}{cc}
 & g \longrightarrow \\
n & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\downarrow & \\
M = \begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \\ (5) \end{array} &
\begin{array}{|c|c|c|c|c|c|c|}
\hline
1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hline
-1 & 0 & 1 & 1 & 0 & 0 & 0 \\ \hline
0 & -1 & -1 & 0 & 1 & 1 & 0 \\ \hline
0 & 0 & 0 & -1 & -1 & 0 & 1 \\ \hline
0 & 0 & 0 & 0 & 0 & -1 & -1 \\ \hline
\end{array}
\end{array}
$$

Jednadžbe mreže

- **Metoda čvorišta** zasnovana na prvom Kirchoffovom* zakonu (K. z. za struje)



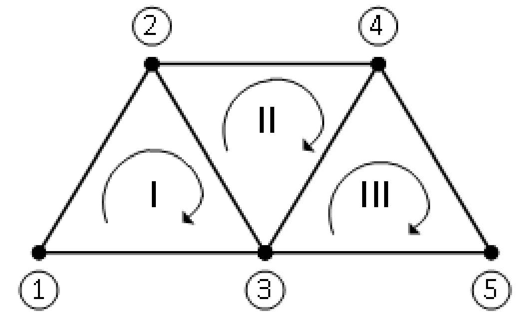
- **Metoda petlji** zasnovana na drugom Kirchoffovom zakonu (K. z. za napone)



***Gustav Robert Kirchhoff** (1824 – 1887), njemački fizičar

Metoda petlji (samo na informaciju)

- Poznajemo sve impedancije grana ($z_{1\text{-}2}$, $z_{1\text{-}3}$, …, $z_{i\text{-}j}$)



$$E_I = I_I \cdot z_{12} + (I_I - I_{II}) \cdot z_{23} + I_I \cdot z_{13}$$
$$E_{II} = I_{II} \cdot z_{24} + (I_{II} - I_{III}) \cdot z_{34} + (I_{II} - I_I) \cdot z_{23}$$
$$E_{III} = (I_{III} - I_{II}) \cdot z_{34} + I_{III} \cdot z_{45} + I_{III} \cdot z_{35}$$

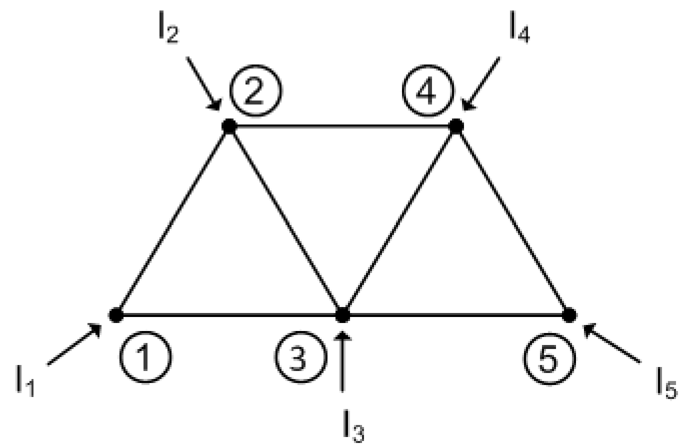
$$E_I = I_I \cdot (z_{12} + z_{13} + z_{23}) + I_{II} \cdot (-z_{23})$$
$$E_{II} = I_I \cdot (-z_{23}) + I_{II} \cdot (z_{23} + z_{24} + z_{34}) + I_{III} \cdot (-z_{34})$$
$$E_{III} = I_{II} \cdot (-z_{34}) + I_{III} \cdot (z_{34} + z_{35} + z_{45})$$

$$\begin{bmatrix} E_I \\ E_{II} \\ E_{III} \end{bmatrix} = \begin{bmatrix} z_{12} + z_{13} + z_{23} & -z_{23} & 0 \\ -z_{23} & z_{23} + z_{24} + z_{34} & -z_{34} \\ 0 & -z_{34} & z_{34} + z_{35} + z_{45} \end{bmatrix} \cdot \begin{bmatrix} I_I \\ I_{II} \\ I_{III} \end{bmatrix}$$

Matrica impedancija grana

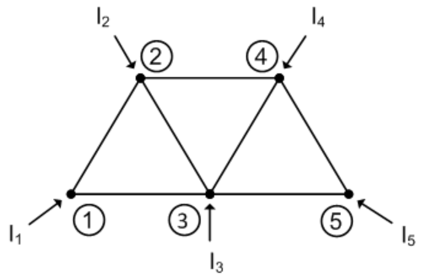
Metoda čvorišta (1)

- Kod analize elektroenergetskih mreža dominantno se koristi ova metoda



- Naponske izvore pretvorimo u strujne izvore
- Poznajemo sve admitancije grana ($y_{1\text{-}2}$, $y_{1\text{-}3}$, ..., $y_{i\text{-}j}$)
- Struju koja ulazi u čvorište (mrežu) uzimamo s pozitivnim predznakom

Metoda čvorišta (2)



- Napišemo prvi Kirchoffov zakon za sva čvorišta:

$$I_1 = (U_1 - U_2)\cdot y_{1-2} + (U_1 - U_3)\cdot y_{1-3} + \dots + (U_1 - U_n)\cdot y_{1-n}$$
$$I_2 = (U_2 - U_1)\cdot y_{2-1} + (U_2 - U_3)\cdot y_{2-3} + \dots + (U_2 - U_n)\cdot y_{2-n}$$
$$\vdots$$
$$I_n = (U_n - U_1)\cdot y_{n-1} + (U_n - U_2)\cdot y_{n-2} + \dots + (U_n - U_{n-1})\cdot y_{n-(n-1)}$$

- Izlučimo napone i jednadžbe uredimo ovako:

$$I_1 = U_1\cdot(y_{1-2} + y_{1-3} + \cdots + y_{1-n}) + U_2\cdot(-y_{1-2}) + \cdots + U_n\cdot(-y_{1-n})$$
$$I_2 = U_1\cdot(-y_{2-1}) + U_2\cdot(y_{2-1} + y_{2-3} + \cdots + y_{2-n}) + \cdots + U_n\cdot(-y_{2-n})$$
$$\vdots$$
$$I_n = U_1\cdot(-y_{1-n}) + U_2\cdot(-y_{2-n}) + \cdots + U_n\cdot\left(y_{1-n} + y_{2-n} + \cdots + y_{(n-1)-n}\right)$$

Metoda čvorišta (3)

$$\begin{vmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{vmatrix} = \begin{vmatrix} \sum_{i=2}^{n} y_{1-i} & -y_{1-2} & \cdots & -y_{1-n} \\ -y_{2-1} & \sum_{\substack{i=1 \\ i\neq 2}}^{n} y_{2-i} & \cdots & -y_{2-n} \\ \vdots & \vdots & \ddots & \vdots \\ -y_{n-1} & -y_{n-2} & \cdots & \sum_{i=1}^{n-1} y_{n-i} \end{vmatrix} \cdot \begin{vmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{vmatrix}$$

Metoda čvorišta (4)

- Kraće: $\boldsymbol{I} = \boldsymbol{Y} \cdot \boldsymbol{U}$

- Gdje je $\boldsymbol{Y}$ - matrica admitancije čvorišta (simetrična)

- Elementi matrice:
  - dijagonalni $Y_{i,i}$ (vlastita admitancija čvorišta)

$$Y_{i,i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} y_{i-j}$$

  - vandijagonalni $Y_{i,j}$ (međusobna admitancija čvorišta)

$$Y_{i,j} = -y_{i-j}$$

Metoda čvorišta (5)

- Vektor napona možemo izračunati:

$$\boldsymbol{U} = \boldsymbol{Z} \cdot \boldsymbol{I} \quad \text{gdje je} \quad \boldsymbol{Z} = \boldsymbol{Y}^{-1}$$

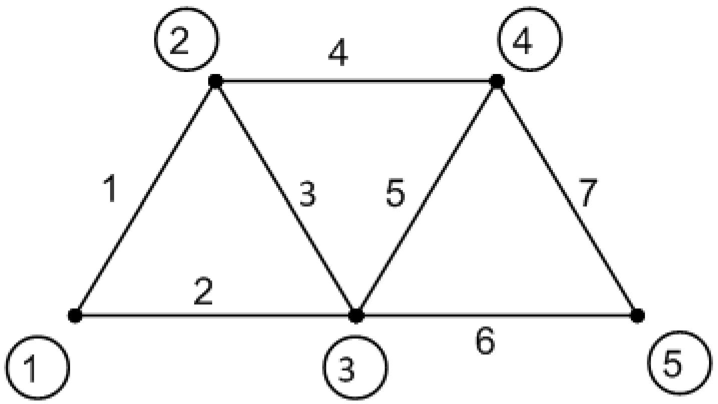
Y matricu nije uvijek moguće invertirati

  - Z - matrica impedancija čvorišta
  - $z_{i,i}$ - vlastita impedancija čvorišta
  - $z_{i,j}$ - međusobna impedancija čvorišta

$$\boldsymbol{Y} = \boldsymbol{M} \cdot \boldsymbol{y} \cdot \boldsymbol{M}^T$$

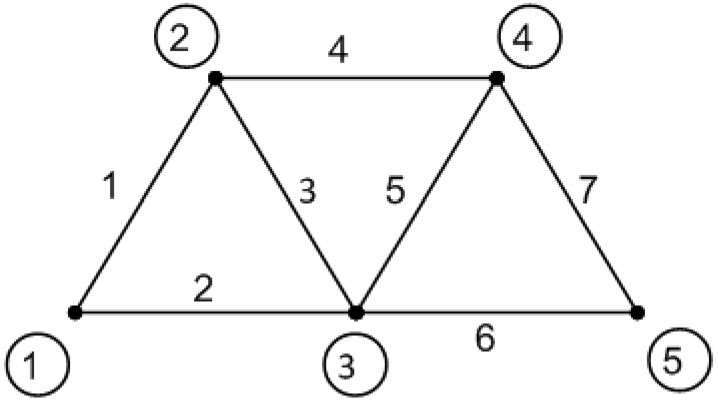
  - $\boldsymbol{M}$ - spojna matrica
  - $\boldsymbol{M}$ je (***n***x***g***) matrica koja daje vezu između čvorišta i grana

Primjer 1 (1)



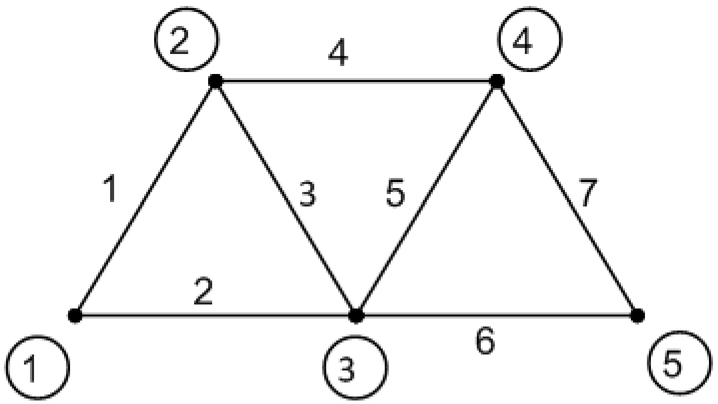
$$M = \begin{array}{c|ccccccc} n \downarrow \; g \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline (1) & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ (2) & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ (3) & 0 & -1 & -1 & 0 & 1 & 1 & 0 \\ (4) & 0 & 0 & 0 & -1 & -1 & 0 & 1 \\ (5) & 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{array}$$

Primjer 1 (2)



$$Y = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} y_{1\text{-}2} & & & & & & \\ & y_{1\text{-}3} & & & & & \\ & & y_{2\text{-}3} & & & & \\ & & & y_{2\text{-}4} & & & \\ & & & & y_{3\text{-}4} & & \\ & & & & & y_{3\text{-}5} & \\ & & & & & & y_{4\text{-}5} \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Primjer 1 (3)

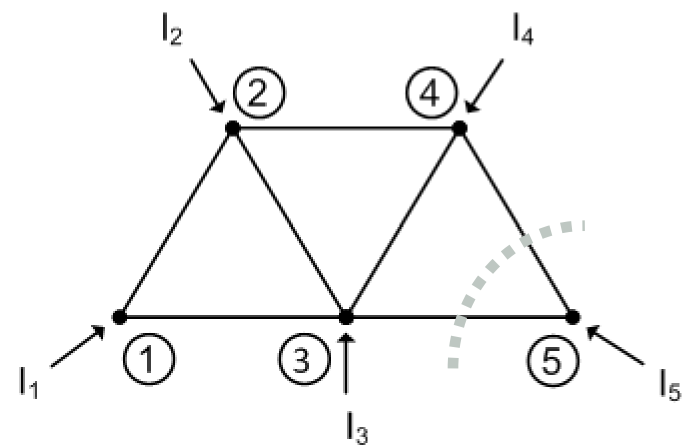


$$Y = \begin{vmatrix} y_{1-2}+y_{1-3} & -y_{1-2} & -y_{1-3} & 0 & 0 \\ -y_{1-2} & y_{1-2}+y_{2-3}+y_{2-4} & -y_{2-3} & -y_{2-4} & 0 \\ -y_{1-3} & -y_{2-3} & y_{1-3}+y_{2-3}+y_{3-4}+y_{3-5} & -y_{3-4} & -y_{3-5} \\ 0 & -y_{2-4} & -y_{3-4} & y_{2-4}+y_{3-4}+y_{4-5} & -y_{4-5} \\ 0 & 0 & -y_{3-5} & -y_{4-5} & y_{3-5}+y_{4-5} \end{vmatrix}$$

Ovu matricu <u>nije</u> moguće invertirati.

- Ne možemo dobiti: $\boldsymbol{Z} = \boldsymbol{Y}^{-1}$

Metoda čvorišta, drugi put (1)



- Jedno čvorište (npr. 5), proglasimo referentnim
- Napone čvorišta određujemo relativno u odnosu na to čvorište
- Poznajemo struje koje ulaze u ostala čvorišta

Metoda čvorišta, drugi put (2)

- Ponovo prvi Kirchoffov zakon za sva čvorišta, osim za referentno

$$
\begin{aligned}
I_1 &= (U_1 - U_2) \cdot y_{1-2} + (U_1 - U_3) \cdot y_{1-3} + \cdots + (U_1 - U_n) \cdot y_{1-n} \\
I_2 &= (U_2 - U_1) \cdot y_{2-1} + (U_2 - U_3) \cdot y_{2-3} + \cdots + (U_2 - U_n) \cdot y_{2-n} \\
&\vdots \\
I_{n-1} &= (U_{n-1} - U_1) \cdot y_{(n-1)-1} + \cdots + (U_{n-1} - U_n) \cdot y_{(n-1)-n}
\end{aligned}
$$

  - (***n-1***) jednadžbi s ***n*** nepoznanica
  - n-ta jednadžba je : $\sum_{i=1}^{n} I_i = 0$

  - Prema tome moramo znati još jedan napon U_n, a ostalih (***n-1***) ćemo izračunati

Metoda čvorišta, drugi put (3)

- Umjesto da izlučimo napone U_1, U_2, U_3, … kao u prethodnom slučaju (slajd 16), izlučimo razlike napona $(U_1 - U_n)$, $(U_2 - U_n)$, $(U_3 - U_n)$, … i dobijemo ove jednadžbe:

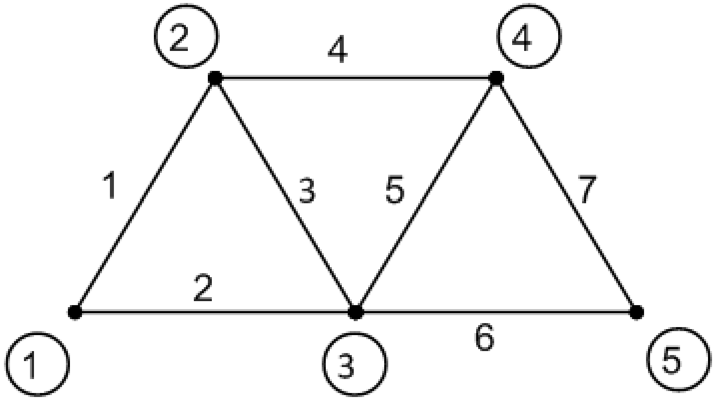
$$I_1 = (U_1 - U_n) \cdot (y_{1-2} + y_{1-3} + \cdots + y_{1-n}) + (U_2 - U_n) \cdot (-y_{1-2}) + \cdots$$
$$+ \left(U_{(n-1)} - U_n\right) \cdot \left(-y_{1-(n-1)}\right)$$
$$I_2 = (U_1 - U_n) \cdot (-y_{2-1}) + (U_2 - U_n) \cdot (y_{2-1} + y_{2-3} + \cdots + y_{2-n}) + \cdots$$
$$+ \left(U_{(n-1)} - U_n\right) \cdot \left(-y_{2-(n-1)}\right)$$
$$\vdots$$
$$I_{n-1} = (U_1 - U_n) \cdot \left(-y_{(n-1)-1}\right) + (U_2 - U_n) \cdot \left(-y_{(n-1)-2}\right) + \cdots$$
$$+ \left(U_{(n-1)} - U_n\right) \cdot \left(y_{(n-1)-1} + y_{(n-1)-2} + \cdots + y_{(n-1)-n}\right)$$

Metoda čvorišta, drugi put (4)

$$\begin{vmatrix} I_1 \\ I_2 \\ \vdots \\ I_{n-1} \end{vmatrix} = \begin{vmatrix} \sum\limits_{i=2}^{n} y_{1-i} & -y_{1-2} & \cdots & -y_{1-(n-1)} \\ -y_{2-1} & \sum\limits_{\substack{i=1 \\ i\neq 2}}^{n} y_{2-i} & \cdots & -y_{2-(n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ -y_{(n-1)-1} & -y_{(n-1)-2} & \cdots & \sum\limits_{\substack{i=1 \\ i\neq n-1}}^{n} y_{(n-1)-i} \end{vmatrix} \cdot \begin{vmatrix} U_1 - U_n \\ U_2 - U_n \\ \vdots \\ U_{(n-1)} - U_n \end{vmatrix}$$

- Odnosno: $\boldsymbol{I} = \boldsymbol{Y} \cdot \Delta \boldsymbol{U}$

Primjer 1 (4)



$$Y = \begin{bmatrix} y_{1-2}+y_{1-3} & -y_{1-2} & -y_{1-3} & 0 & 0 \\ -y_{1-2} & y_{1-2}+y_{2-3}+y_{2-4} & -y_{2-3} & -y_{2-4} & 0 \\ -y_{1-3} & -y_{2-3} & y_{1-3}+y_{2-3}+y_{3-4}+y_{3-5} & -y_{3-4} & -y_{3-5} \\ 0 & -y_{2-4} & -y_{3-4} & y_{2-4}+y_{3-4}+y_{4-5} & -y_{4-5} \\ 0 & 0 & -y_{3-5} & -y_{4-5} & y_{3-5}+y_{4-5} \end{bmatrix}$$

Diskusija o Y i Z matricama

- Y – matrica admitancija čvorišta
- Matrica admitancija čvorišta je kvadratna n×n matrica
- Z – matrica impedancija čvorišta:

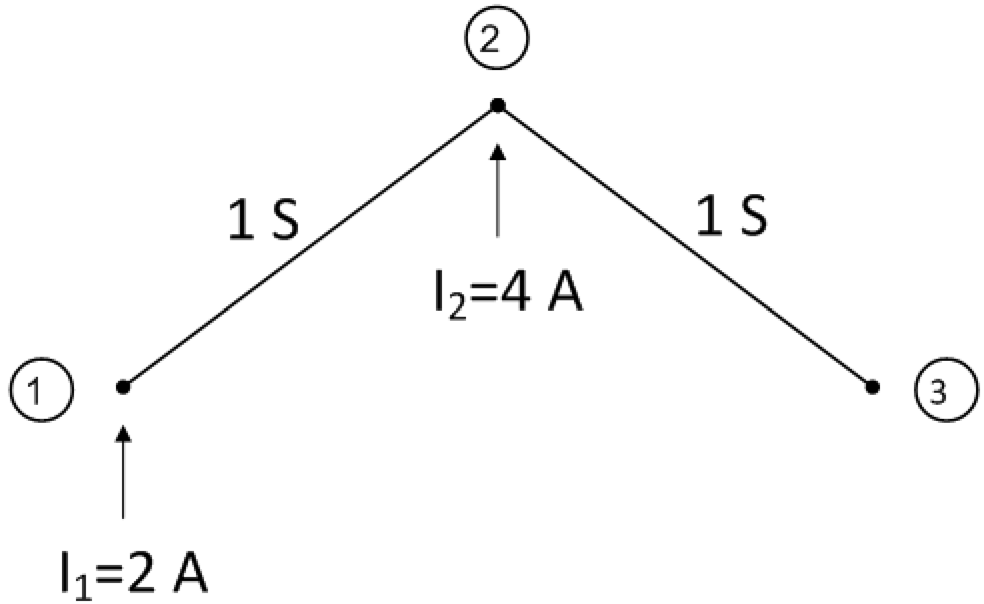
$$\boldsymbol{Z} = \boldsymbol{Y}^{-1}$$

Pitanje: Kada je Y matricu moguće invertirati?

Napomena:

- Drugim metodama dolazimo od <u>drugih</u> matrica
- Metodom petlji dolazimo do matrice impedancije petlji
- Matrica impedancije petlji je kvadratna p×p matrica
- Njoj inverzna matrica je matrica admitancija petlji

Primjer 2 (1)



$$Y_S = \left[\begin{array}{cc|c} 1 & -1 & 0 \\ -1 & 2 & -1 \\ \hline 0 & -1 & 1 \end{array}\right]$$

Y= singularna matrica

Čvorište 3 je referentno

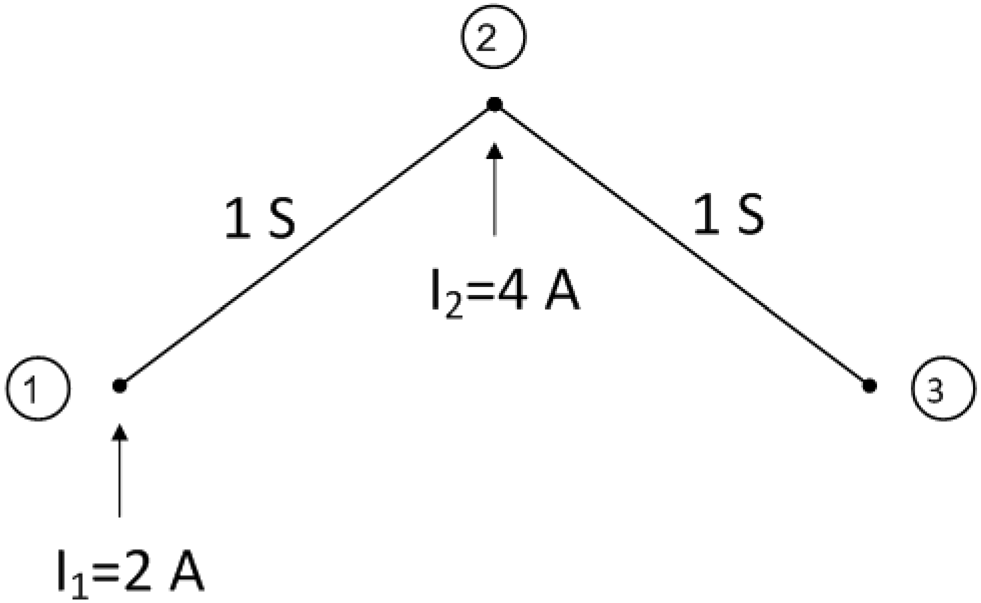
Primjer 2 (2)

$$Y = \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}$$

$$Y^{(1)} = \begin{vmatrix} \frac{1}{1} & \frac{-1}{1} \\ -\frac{-1}{1} & 2 - \frac{1}{1} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$Y^{(2)} = \begin{vmatrix} 1 + \frac{1}{1} & -\frac{-1}{1} \\ \frac{1}{1} & \frac{1}{1} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = Y^{-1}$$

Primjer 2 (3)

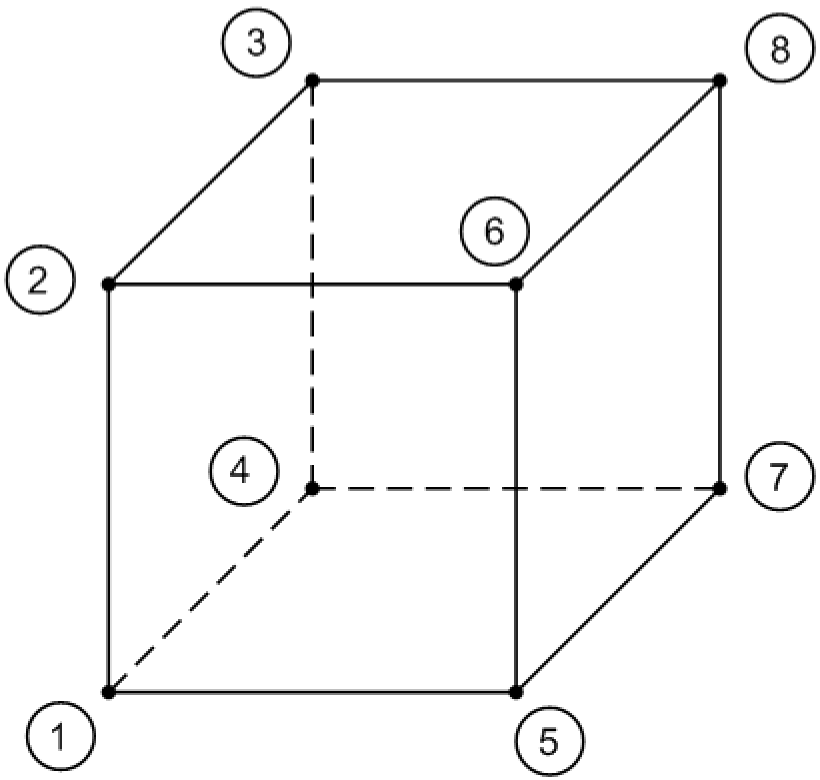


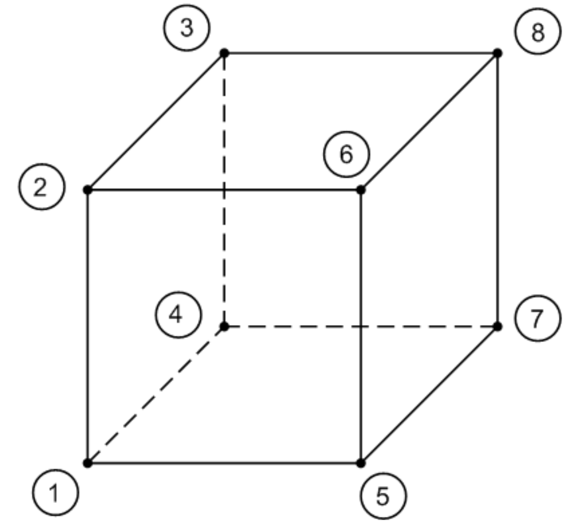
$$Z = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} \Delta U_1 \\ \Delta U_2 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} \cdot \begin{vmatrix} 2 \\ 4 \end{vmatrix} = \begin{vmatrix} 8 \\ 6 \end{vmatrix} \; V$$

$$(M \cdot y)^T \cdot \Delta U = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} 8 \\ 6 \\ 0 \end{vmatrix} = \begin{vmatrix} 2 \\ 6 \end{vmatrix} \; A$$

Primjer 3: Kocka





$$Y = \begin{vmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & 0 & 0 & -1 \\ -1 & 0 & -1 & 3 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 3 & -1 & -1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 3 & 0 & -1 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 & -1 \\ 0 & 0 & -1 & 0 & 0 & -1 & -1 & 3 \end{vmatrix}$$

- Ukoliko je pivot Y_{ii} :
  - Pivot:

$$Y_{ii}^{(m+1)} = \frac{1}{Y_{ii}^{(m)}}$$

  - Elementi Y_{ij} (u istom retku kao i pivot, j=1,2,...,n; j≠i)

$$Y_{ij}^{(m+1)} = \frac{Y_{ij}^{(m)}}{Y_{ii}^{(m)}}$$

  - Elementi Y_{ji} (u istom stupcu kao i pivot, j=1,2,...,n; j≠i)

$$Y_{ji}^{(m+1)} = -\frac{Y_{ji}^{(m)}}{Y_{ii}^{(m)}}$$

  - Ostali elementi Y_{kl} (k=1,2,...,n; k≠i, l=1,2,...,n; l≠i)

$$Y_{kl}^{(m+1)} = Y_{kl}^{(m)} - \frac{Y_{ki}^{(m)} \cdot Y_{il}^{(m)}}{Y_{ii}^{(m)}}$$

- m=1 , i=3

$$Y = \begin{vmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & \boxed{3} & \boxed{-1} & \boxed{0} & 0 & -1 & 0 \\ 0 & \boxed{-1} & \boxed{3} & \boxed{-1} & 0 & 0 & 0 \\ -1 & \boxed{0} & \boxed{-1} & \boxed{3} & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 3 & -1 & -1 \\ 0 & -1 & 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{vmatrix}$$

- m=2 , i=6

$$Y^{(1)} = \left|\begin{array}{ccccc|c|c}
3 & -1 & 0 & -1 & -1 & 0 & 0 \\
-1 & \boxed{\frac{8}{3}} & \frac{1}{3} & -\frac{1}{3} & \boxed{0} & \boxed{-1} & 0 \\
0 & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 \\
-1 & -\frac{1}{3} & \frac{1}{3} & \frac{8}{3} & 0 & 0 & -1 \\
-1 & \boxed{0} & 0 & 0 & \boxed{3} & \boxed{-1} & -1 \\
\hline
0 & \boxed{-1} & 0 & 0 & \boxed{-1} & \boxed{3} & 0 \\
\hline
0 & 0 & 0 & -1 & -1 & 0 & 3
\end{array}\right|$$

- m=3 , i=7

$$Y^{(2)} = \left|\begin{array}{cccccc|c}
3 & -1 & 0 & -1 & -1 & 0 & 0 \\
-1 & \frac{7}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & 0 \\
0 & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 \\
-1 & -\frac{1}{3} & \frac{1}{3} & \boxed{\frac{8}{3}} & \boxed{0} & 0 & \boxed{-1} \\
-1 & -\frac{1}{3} & 0 & \boxed{0} & \boxed{\frac{8}{3}} & \frac{1}{3} & \boxed{-1} \\
0 & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 \\
\hline
0 & 0 & 0 & \boxed{-1} & \boxed{-1} & 0 & \boxed{3}
\end{array}\right|$$

- m=4 , i=1

$$Y^{(3)} = \left|\begin{array}{c|cccccc}
3 & \boxed{-1} & 0 & \boxed{-1} & \boxed{-1} & 0 & 0 \\
\hline
\boxed{-1} & \boxed{\frac{7}{3}} & \frac{1}{3} & \boxed{-\frac{1}{3}} & \boxed{-\frac{1}{3}} & \frac{1}{3} & 0 \\
0 & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 \\
\boxed{-1} & \boxed{-\frac{1}{3}} & \frac{1}{3} & \boxed{\frac{7}{3}} & \boxed{-\frac{1}{3}} & 0 & \frac{1}{3} \\
\boxed{-1} & \boxed{-\frac{1}{3}} & 0 & \boxed{-\frac{1}{3}} & \boxed{\frac{7}{3}} & \frac{1}{3} & \frac{1}{3} \\
0 & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 \\
0 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & \frac{1}{3}
\end{array}\right|$$

- m=5 , i=2

$$Y^{(4)} = \left|\begin{array}{c|c|ccccc}
1/3 & -1/3 & 0 & -1/3 & -1/3 & 0 & 0 \\
\hline
-1 & 6/3 & 1/3 & -2/3 & -2/3 & 1/3 & 0 \\
\hline
0 & -1/3 & 1/3 & -1/3 & 0 & 0 & 0 \\
1/3 & -2/3 & 1/3 & 6/3 & -2/3 & 0 & 1/3 \\
1/3 & -2/3 & 0 & -2/3 & 6/3 & 1/3 & 1/3 \\
0 & -1/3 & 0 & 0 & -1/3 & 1/3 & 0 \\
0 & 0 & 0 & -1/3 & -1/3 & 0 & 1/3
\end{array}\right|$$

- m=6 , i=4

$$Y^{(5)} = \begin{vmatrix} 7/18 & 1/6 & 1/18 & -4/9 & -4/9 & 1/18 & 0 \\ 1/6 & 1/2 & 1/6 & -1/3 & -1/3 & 1/6 & 0 \\ 1/18 & 1/6 & 7/18 & -4/9 & -1/9 & 1/18 & 0 \\ 4/9 & 1/3 & 4/9 & 16/9 & -8/9 & 1/9 & 1/3 \\ 4/9 & 1/3 & 1/9 & -8/9 & 16/9 & 4/9 & 1/3 \\ 1/18 & 1/6 & 1/18 & -1/9 & -4/9 & 7/18 & 0 \\ 0 & 0 & 0 & -1/3 & -1/3 & 0 & 1/3 \end{vmatrix}$$

- m=7 , i=5

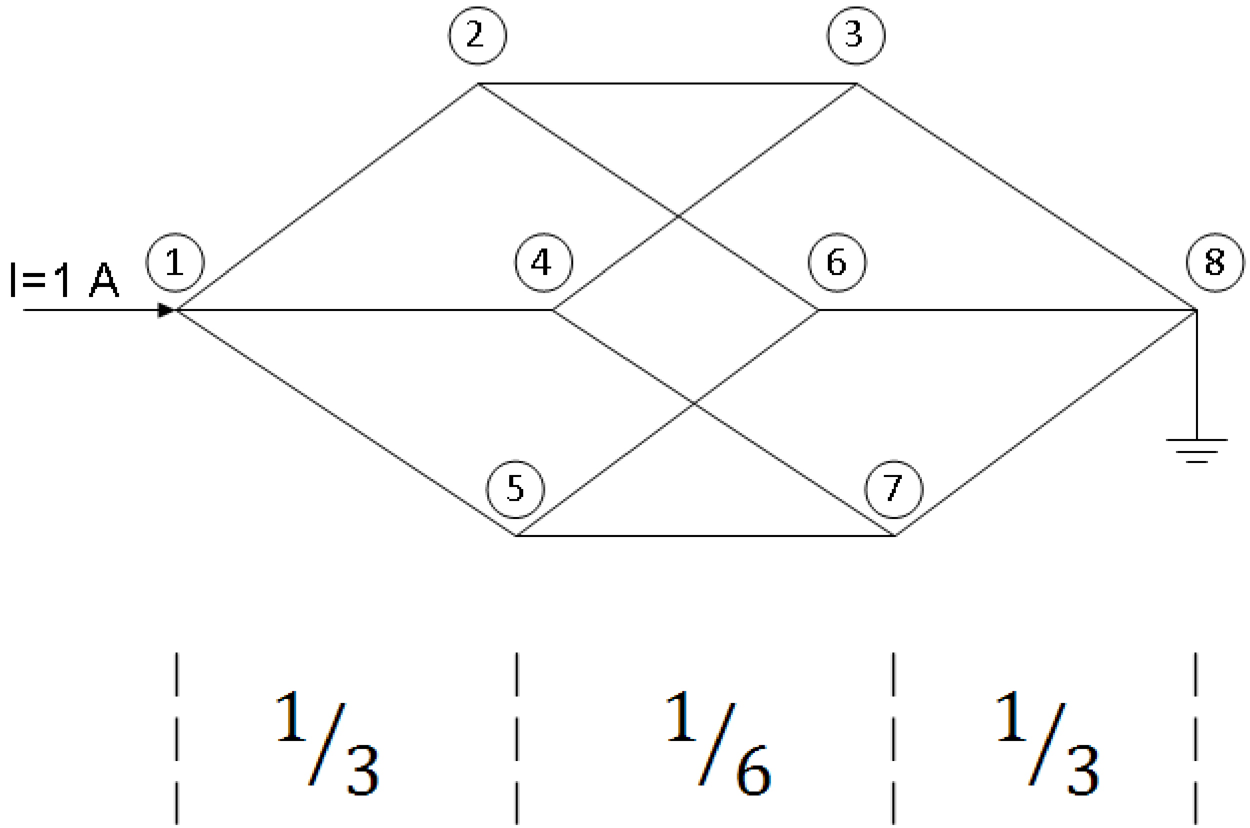
$$Y^{(6)} = \left[\begin{array}{cccc|c|cc}
1/2 & 3/12 & 1/6 & 1/4 & -2/3 & 1/12 & 1/12 \\
1/4 & 9/16 & 1/4 & 3/16 & -1/2 & 3/16 & 1/16 \\
1/6 & 1/4 & 1/2 & 1/4 & -1/3 & 1/12 & 1/12 \\
1/4 & 3/16 & 1/4 & 9/16 & -1/2 & 1/16 & 3/16 \\
\hline
2/3 & 1/2 & 1/3 & 1/2 & 4/3 & 1/2 & 1/2 \\
\hline
1/12 & 3/16 & 1/12 & 1/16 & -1/2 & 57/144 & 1/48 \\
1/12 & 1/16 & 1/12 & 3/16 & -1/2 & 1/48 & 19/48
\end{array}\right]$$

$$Z = Y^{-1} = Y^{(7)} = \begin{vmatrix} 5/6 & 1/2 & 1/3 & 1/2 & 1/2 & 1/3 & 1/3 \\ 1/2 & 3/4 & 3/8 & 3/8 & 3/8 & 3/8 & 1/4 \\ 1/3 & 3/8 & 7/12 & 3/8 & 1/4 & 5/24 & 5/24 \\ 1/2 & 3/8 & 3/8 & 3/4 & 3/8 & 1/4 & 3/8 \\ 1/2 & 3/8 & 1/4 & 3/8 & 3/4 & 3/8 & 3/8 \\ 1/3 & 3/8 & 5/24 & 1/4 & 3/8 & 7/12 & 5/24 \\ 1/3 & 1/4 & 5/24 & 3/8 & 3/8 & 5/24 & 7/12 \end{vmatrix}$$

- Y matrica je relativno prazna
- Z matrica je puna

- z_{11} - je nadomjesna impedancija između čvorišta 1-8
- z_{22} - je nadomjesna impedancija između čvorišta 2-8

- $z_{1,i}$ - je napon u čvorištu “i” ako je struja u čvorištu 1 = 1A


I=1 A
1
2
3
4
5
6
7
8
1/3
1/6
1/3

Pitanja i komentari

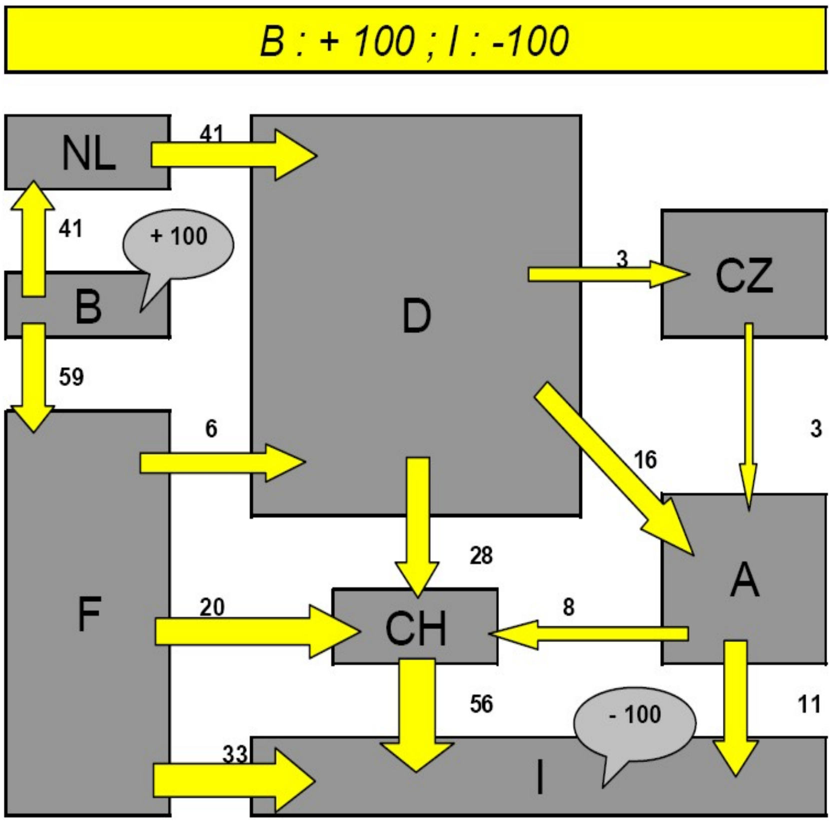
Analiza elektroenergetskog sustava
Predavanje 5: Proračun tokova snaga

Prof. dr. sc. Ivica Pavić

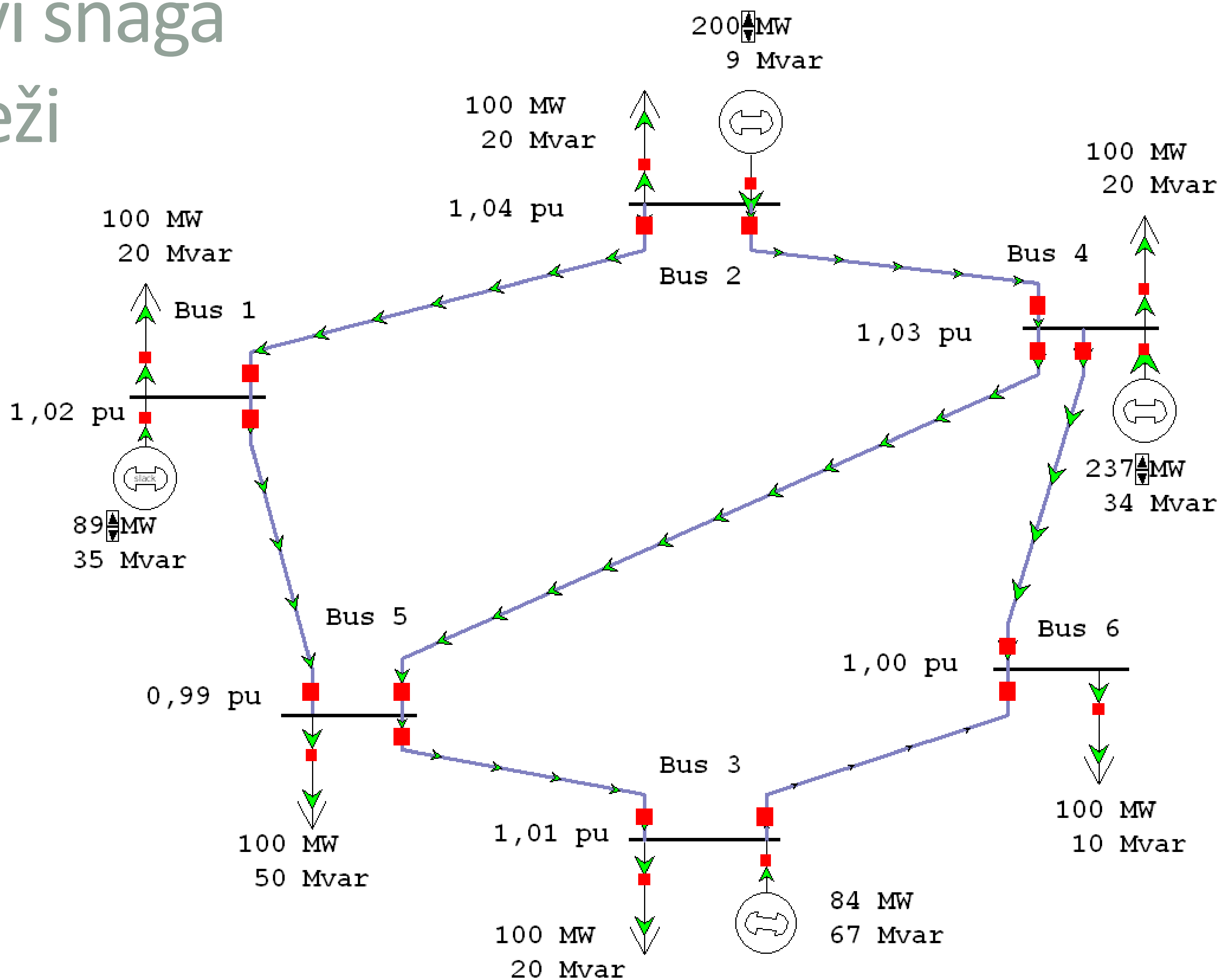
Izv. prof. dr. sc. Marko Delimar

Proračun tokova snaga

- Osnovni proračun u EE mrežama
- Potreban za...
  - Vođenje pogona EE sustava
  - Planiranje razvoja EE sustava
  - Projektiranje dijelova EE mreže
  - ...
- Pojednostavljeni primjer: prijenos 100 MW iz Belgije u Italiju



Tokovi snaga u mreži



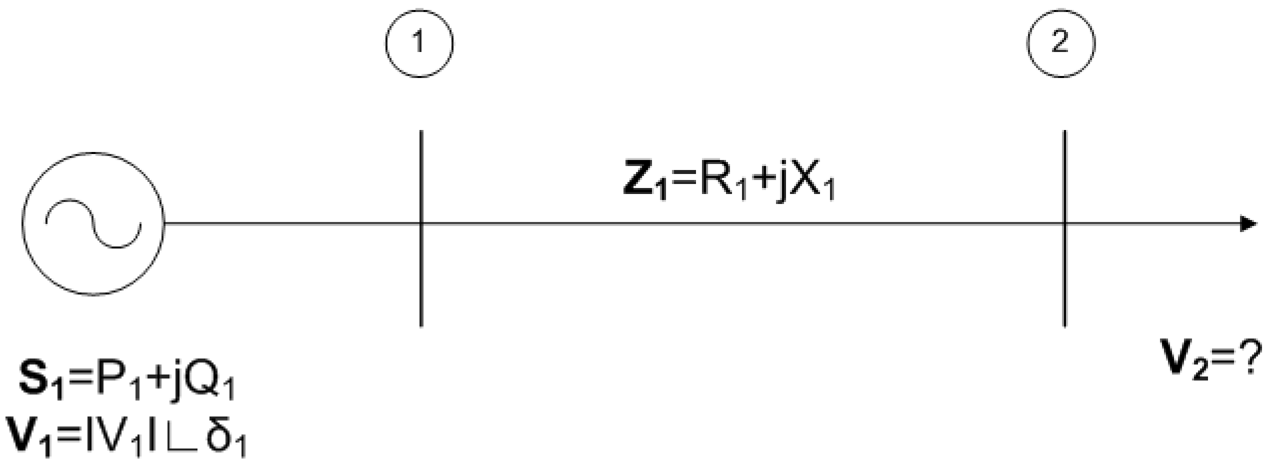
Power World Simulator
www.powerworld.com

Proračun tokova snaga

- Provodi se u EE mrežama u stacionarnom pogonskom stanju
- Cilj proračuna je odrediti napone po iznosu i kutu u svim čvorištima mreže, odnosno odrediti vektor stanja (iz kojeg se lako mogu izračunati preostale tražene vrijednosti)
- Rezultati proračuna:
  - Vektor napona svih čvorišta (iznos i kut)
- Iz dobivenih rezultata je moguće izračunati:
  - Iznose radne i jalove snage kroz grane mreže
  - Injekcije snage u čvorištima (raspodjela opterećenja izvora)
  - Gubitke snage u mreži

Primjer 1

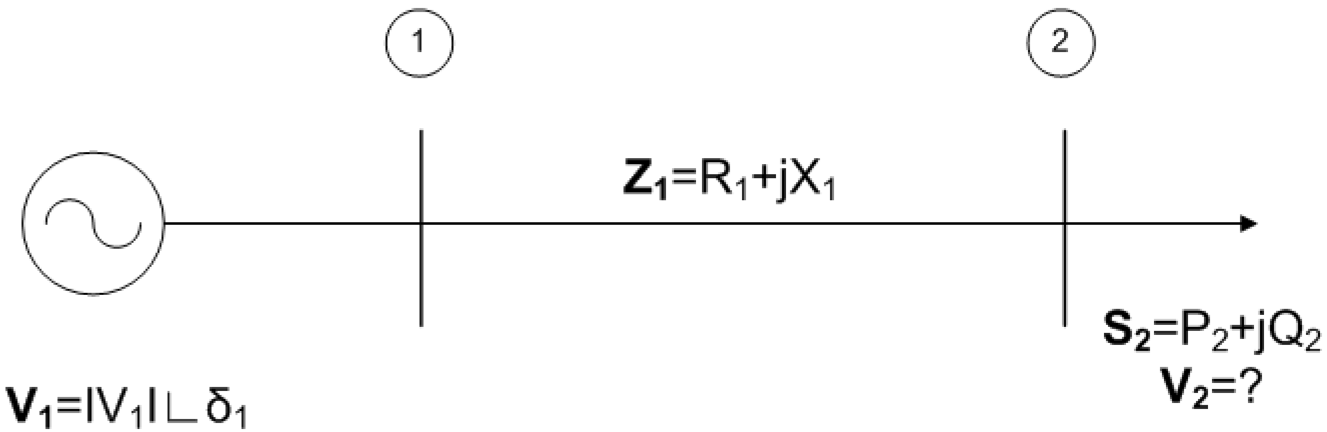
- Ukoliko je zadano:



- Direktnim proračunom je moguće izračunati napon $\mathbf{V_2}$ i snagu $\mathbf{S_2}$ u čvorištu

Primjer 2

- Ukoliko je pak zadano:



- Zbog nelinearnosti problema više nije moguće izravno izračunati $\mathbf{V_2}$ i $\mathbf{S_1}$
- Problem je potrebno rješavati primjenom iteracijskog postupka

Klasifikacija čvorišta

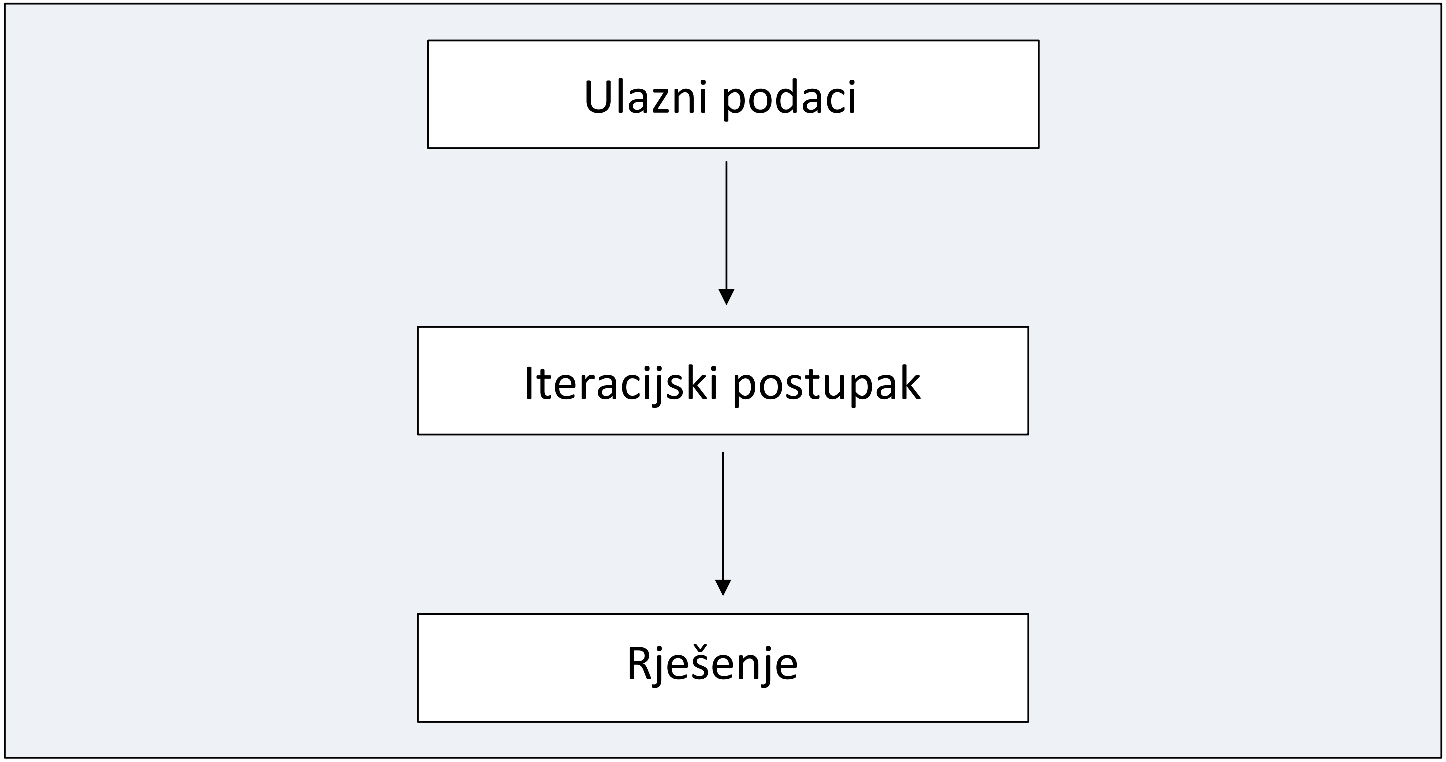
- Svako čvorište mreže (sabirnica) je definirano sa četiri električne vrijednosti:
  - Injektirana djelatna snaga Pi (MW)
  - Injektirana jalova snaga Qi (MVAr)
  - Iznos napona Vi
  - Fazni kut napona δi

- U proračunu tokova snage za svako čvorište (sabirnicu) vrijedi:
  - Dvije električne veličine su zadane
  - Dvije električne veličine su nepoznanice

Klasifikacija čvorišta

- Na osnovu toga koje su dvije vrijednosti poznate, a koje je potrebno izračunati, čvorišta se dijele na:
  - **Čvorišta tereta** (poznato P, Q)
    - najveći broj čvorišta
  - **Generatorska čvorišta** (poznato |V|, P)
    - ona čvorišta u kojima postoji regulacija napona
    - potrebna i ograničenja u proizvodnji jalove snage Q_{min}, Q_{max}
  - **Referentno čvorište** (poznato |V|, δ)
    - koristi se i naziv čvorište regulacijske elektrane

Struktura proračuna



Ulazni podatci

- Ulazni podatci koji moraju biti zadani (poznati):
  - Konfiguracija mreže
  - Parametri elemenata mreže (impedancije vodova)
  - Djelatna i jalova snaga u potrošačkim čvorištima
  - Iznos (modul) napona i djelatne snage u generatorskim čvorištima
  - Napon u referentnom čvorištu (kut se obično uzima za nulu)

- Kako bi iteracijski postupak mogao započeti potrebno je pretpostaviti početne vrijednosti napona čvorišta

- Standardno se za početne iznose napona uzimaju nazivne vrijednosti ($\vec{V}_i = V_n \angle 0°, \vec{V}_i = 1.0 \angle 0°\ \mathrm{p.u.}$)

Matematički model proračuna

- Proračun tokova snage se temelji na metodi čvorišta:
  - pomoću Y matrice: $[\vec{\mathrm{I}}] = [\vec{\mathrm{Y}}] \cdot [\vec{\mathrm{V}}]$
  - pomoću Z matrice: $[\vec{\mathrm{V}}] = [\vec{\mathrm{Z}}] \cdot [\vec{\mathrm{I}}]$
- Pri čemu je:
  - $[\vec{I}]$ vektor vanjskih struja narinutih u čvorišta mreže
  - $[\vec{V}]$ vektor napona čvorišta
  - $[\vec{Y}]$ matrica admitancija mreže
  - $[\vec{Z}]$ matrica impedancija mreže

Matematički model proračuna

- Umjesto narinutih struja u čvorištima su zadane snage (pogledati ulazne podatke). Navedene jednadžbe je stoga potrebno pisati u sljedećem obliku:
  - pomoću Y matrice: $$\left[\left(\frac{\vec{S}}{\vec{V}}\right)^*\right] = [\vec{Y}] \cdot [\vec{V}]$$
  - pomoću Z matrice: $$[\vec{V}] = [\vec{Z}] \cdot \left[\left(\frac{\vec{S}}{\vec{V}}\right)^*\right]$$
- Zbog ovakve ovisnosti napon-snaga jednadžbe tokova snage se ubrajaju u nelinearne (kvadratne) algebarske jednadžbe

Matematički model proračuna

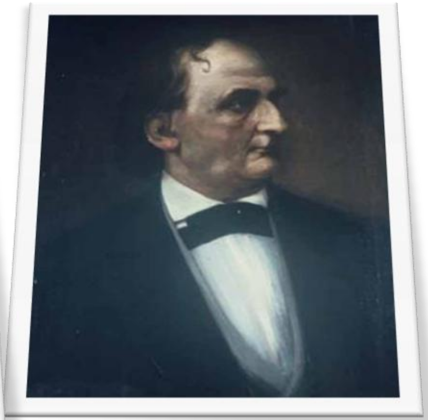
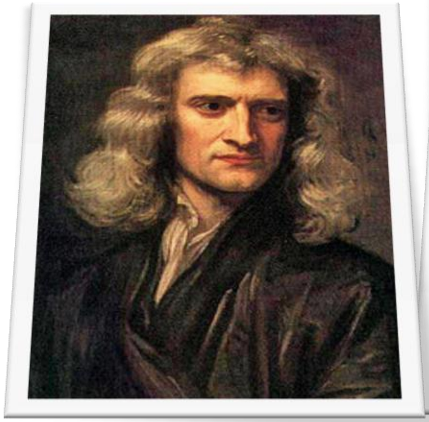
- Rješavanje nelinearnih (kvadratnih) jednadžbi zahtijeva primjenu iteracijskih postupaka
- Iteracijski proračun:
  - Odvija se kroz više iteracija (koraka)
  - U svakoj iteraciji se provjerava da li dobiveni rezultati zadovoljavaju postavljeni kriterij točnost
  - Rezultati koji ne zadovoljavaju postavljeni kriterij točnosti služe kao ulazni podaci za sljedeću iteraciju
- Osnovne metode proračuna tokova snaga temeljene na iteracijskim numeričkim postupcima su:
  - Gauss-Seidel metoda pomoću Z-matrice
  - Gauss-Seidel metoda pomoću Y-matrice
  - Newton-Raphson metoda

Gauss-Seidel

- Johann Carl Friedrich Gauss
(1777 – 1855)
njemački matematičar i fizičar
- Philipp Ludwig von Seidel
(1821 – 1896)
njemački matematičar

Newton-Raphson

- Sir Isaac Newton PRS MP
(1642 –1727)
engleski fizičar i matematičar
- Joseph Raphson
(1648 – 1715)
engleski matematičar


Snage u čvorištima

$$\vec{S}_i = \vec{U}_i \cdot \vec{I}_i^* = P_i + jQ_i$$

$$\vec{U}_i = \left|\vec{U}_i\right| \cdot e^{j\delta_i}$$

$$\vec{I}_i = \sum_{j=1}^{n} \vec{Y}_{ij} \cdot \vec{U}_j \quad \Rightarrow \quad \vec{I}_i^* = \sum_{j=1}^{n} \vec{Y}_{ij}^* \cdot \vec{U}_j^*$$

$$\vec{U}_j^* = \left|\vec{U}_j\right| \cdot e^{-j\delta_j}$$

$$\vec{Y}_{ij} = \left|\vec{Y}_{ij}\right| \cdot e^{j\Theta_{ij}} \quad ; \quad \vec{Y}_{ij}^* = \left|\vec{Y}_{ij}\right| \cdot e^{-j\Theta_{ij}}$$

$$\vec{S}_i = \vec{U}_i \cdot \vec{I}_i^* = \vec{U}_i \cdot \sum_{j=1}^{n} \vec{Y}_{ij}^* \cdot \vec{U}_j^* = \left|\vec{U}_i\right| \cdot e^{j\delta_i} \cdot \sum_{j=1}^{n} \left|\vec{Y}_{ij}\right| \cdot e^{-j\Theta_{ij}} \cdot \left|\vec{U}_j\right| \cdot e^{-j\delta_j}$$

$$\vec{S}_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{Y}_{ij}\right| \cdot \left|\vec{U}_j\right| \cdot e^{j(\delta_i - \delta_j - \Theta_{ij})}$$

Snage u čvorištima

$$\vec{S}_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \left[\cos\left(\delta_i - \delta_j - \Theta_{ij}\right) + j\sin(\delta_i - \delta_j - \Theta_{ij})\right]$$

- Djelatna snaga u čvorištu ***i*** (P_i)

$$P_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \cos\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

- Jalova snaga u čvorištu ***i*** (Q_i)

$$Q_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \sin\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

Snage u čvorištima

- Odnosno:

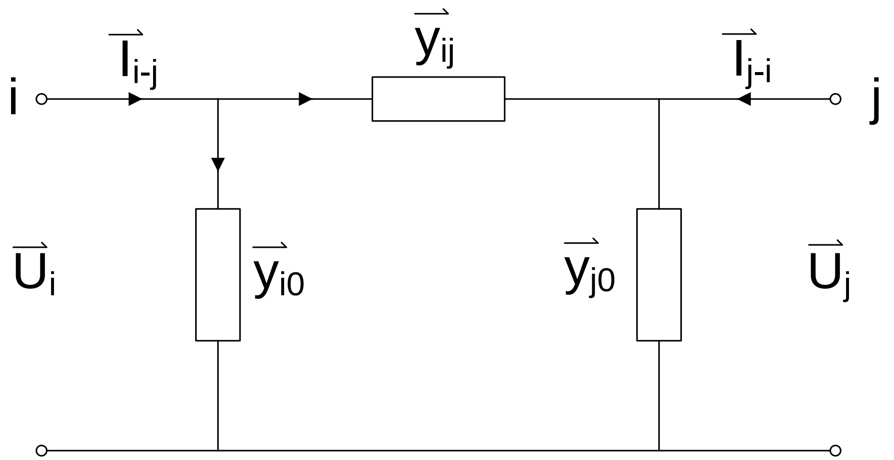
$$P_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \cos\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

$$P_i = \left|\vec{U}_i\right|^2 \cdot \left|\vec{Y}_{ii}\right| \cdot \cos\left(\Theta_{ii}\right) + \left|\vec{U}_i\right| \cdot \sum_{\substack{j=1 \\ j\neq i}}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \cos\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

$$Q_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \sin\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

$$Q_i = -\left|\vec{U}_i\right|^2 \cdot \left|\vec{Y}_{ii}\right| \cdot \sin\left(\Theta_{ii}\right) + \left|\vec{U}_i\right| \cdot \sum_{\substack{j=1 \\ j\neq i}}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \sin\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

Snage u granama



$$\vec{I}_{i-j} = \left(\vec{U}_i - \vec{U}_j\right)\cdot \vec{y}_{i-j} + \vec{U}_i \cdot \vec{y}_{i0}$$

$$\vec{S}_{i-j} = \vec{U}_i \cdot \vec{I}^*_{i-j} = \vec{U}_i\left[\left(\vec{U}^*_i - \vec{U}^*_j\right)\cdot \vec{y}^*_{i-j} + \vec{U}^*_i \cdot \vec{y}^*_{i0}\right]$$

$$\vec{I}_{j-i} = \left(\vec{U}_j - \vec{U}_i\right)\cdot \vec{y}_{i-j} + \vec{U}_j \cdot \vec{y}_{j0}$$

$$\vec{S}_{j-i} = \vec{U}_j \cdot \vec{I}^*_{j-i} = \vec{U}_j\left[\left(\vec{U}^*_j - \vec{U}^*_i\right)\cdot \vec{y}^*_{i-j} + \vec{U}^*_j \cdot \vec{y}^*_{j0}\right]$$

Snage u granama

- Gubitci snaga u granama:

$$\Delta\vec{S} = \vec{S}_{i-j} + \vec{S}_{j-i}$$

$$\Delta\vec{S} = \left(\vec{U}_i^* - \vec{U}_j^*\right) \cdot \vec{y}_{i-j}^* \cdot \left(\vec{U}_i - \vec{U}_j\right) + \left|\vec{U}_i\right|^2 \cdot \vec{y}_{i0}^* + \left|\vec{U}_j\right|^2 \cdot \vec{y}_{j0}^*$$

- Napomena: $\vec{y}_{i-j}$ - uzdužna admitancija grane

  $\vec{Y}_{ij}$ - element matrice admitancija čvorišta, međusobna admitancija čvorišta (između čvorišta **i** i **j**)

$$\vec{Y}_{ij} \neq \vec{y}_{i-j}$$

Gauss-Seidel pomoću Z matrice

Metoda Gauss-Seidel pomoću Z-matrice

- Mreža od ***n*** čvorišta – jedno čvorište referentno:

$$\vec{U}_i - \vec{U}_{ref.} = \sum_{\substack{j=1 \\ j \neq ref}}^{n} \vec{Z}_{ij} \cdot \vec{I}_j$$

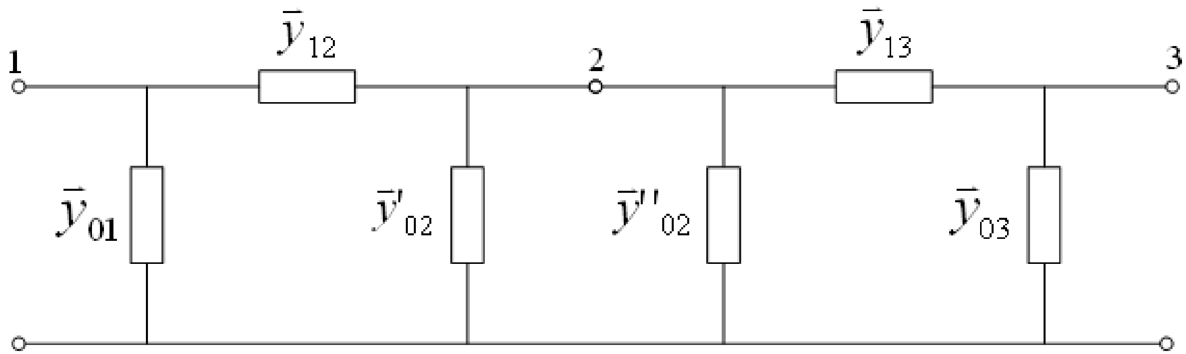
$$\left|\Delta\vec{U}\right| = \left|\vec{Z}\right| \cdot \left|\vec{I}\right|$$

$$\left|\vec{Z}\right| = \left|\vec{Y}\right|^{-1}$$

- Matrica **Y** se dobije uzimajući u obzir **samo uzdužne** parametre grana
- Poprečne admitancije grana sačinjavaju novu matricu **Y´**

Metoda Gauss-Seidel pomoću Z-matrice

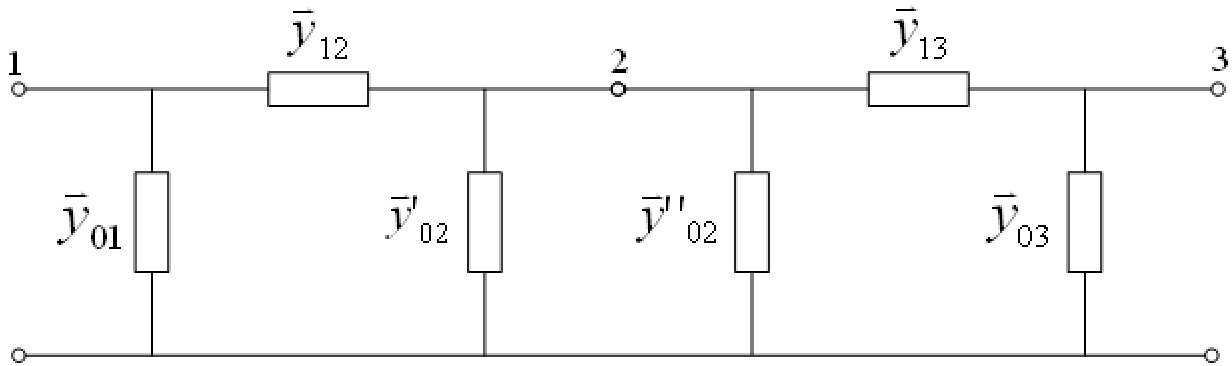
- Matrica **Y**:
  - Kod ove metode u matricu **Y** ulaze samo uzdužne admitancije
  - Poprečne admitancije se koriste za formiranje matrice **Y´**
  - Primjer:



$$\vec{Y} = \begin{bmatrix} \vec{y}_{12} & -y_{12} & 0 \\ -y_{12} & \vec{y}_{12} + \vec{y}_{13} & -\vec{y}_{13} \\ 0 & -\vec{y}_{13} & \vec{y}_{13} \end{bmatrix}$$

Metoda Gauss-Seidel pomoću Z-matrice

- Matrica **Y'**:
  - Primjer:



$$\vec{Y}' = \begin{bmatrix} \vec{y}_{01} \\ \vec{y}'_{02} + \vec{y}''_{02} \\ \vec{y}_{03} \end{bmatrix}$$

Metoda Gauss-Seidel pomoću Z-matrice

- Uz $\vec{U}_n = \vec{U}_{ref}$ vrijedi:

$$\vec{U}_1 - \vec{U}_n = \vec{Z}_{11} \cdot \vec{I}_1 + \vec{Z}_{12} \cdot \vec{I}_2 + \ldots + \vec{Z}_{1(n-1)} \cdot \vec{I}_{n-1}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\vec{U}_i - \vec{U}_n = \vec{Z}_{i1} \cdot \vec{I}_1 + \vec{Z}_{i2} \cdot \vec{I}_2 + \ldots + \vec{Z}_{i(n-1)} \cdot \vec{I}_{n-1}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\vec{U}_{n-1} - \vec{U}_n = \vec{Z}_{(n-1)1} \cdot \vec{I}_1 + \vec{Z}_{(n-1)2} \cdot \vec{I}_2 + \ldots + \vec{Z}_{(n-1)(n-1)} \cdot \vec{I}_{n-1}$$

- Struje u čvorištima je potrebno odrediti pomoću injekcija snaga u čvorištima

Metoda Gauss-Seidel pomoću Z-matrice

- Postupak proračuna:
  1. korak
     - Učitavanje podataka o mreži (konfiguracija, admitancije grana)
     - Učitavanje podataka o injekcijama snage u čvorištima
  2. korak
     - Formiranje matrice **Y´** (samo poprečne admitancije grana)
     - Formiranje matrice **Y** (samo uzdužne admitancije grana)
  3. korak
     - Računanje matrice **Z** $\left(\left|\vec{Z}\right| = \left|\vec{Y}\right|^{-1}\right)$
  4. korak
     - Početne vrijednosti napona čvorišta: $\vec{U}_i^{(0)} = 1 + j0 \text{ p.u.} = 1\angle 0^\circ \text{ p.u.}$
  5. korak
     - Računanje struja u čvorištima (nulta iteracija, k=0):

$$\vec{I}_i^{(0)} = \frac{\vec{S}_i^*}{\vec{U}_i^{*(0)}} - Y_i^{'} \cdot \vec{U}_i^{(0)} \qquad i = 1,2,\ldots,n-1$$

Metoda Gauss-Seidel pomoću Z-matrice

6. korak
   - Računanje napona $\vec{U}_i^{(1)}$ i struja $\vec{I}_i^{(1)}$ u čvorištima (k=1):

$$\vec{U}_1^{(1)} = \vec{U}_{ref} + \vec{Z}_{11} \cdot \vec{I}_1^{(0)} + \vec{Z}_{12} \cdot \vec{I}_2^{(0)} + \ldots + \vec{Z}_{1(n-1)} \cdot \vec{I}_{(n-1)}^{(0)}$$

$$\vec{I}_1^{(1)} = \frac{\vec{S}_1^*}{\vec{U}_1^{*(1)}} - \vec{U}_1^{(1)} \cdot Y_1'$$

$$\vec{U}_2^{(1)} = \vec{U}_{ref} + \vec{Z}_{21} \cdot \vec{I}_1^{(1)} + \vec{Z}_{22} \cdot \vec{I}_2^{(0)} + \ldots + \vec{Z}_{2(n-1)} \cdot \vec{I}_{(n-1)}^{(0)}$$

$$\vec{I}_2^{(1)} = \frac{\vec{S}_2^*}{\vec{U}_2^{*(1)}} - \vec{U}_2^{(1)} \cdot Y_2'$$

$$\vdots$$

$$\vec{U}_i^{(1)} = \vec{U}_{ref} + \sum_{j=1}^{i-1} \vec{Z}_{ij} \cdot \vec{I}_j^{(1)} + \sum_{j=i}^{n-1} \vec{Z}_{ij} \cdot \vec{I}_j^{(0)}$$

$$\vec{I}_i^{(1)} = \frac{\vec{S}_i^*}{\vec{U}_i^{*(1)}} - \vec{U}_i^{(1)} \cdot Y_i' \qquad za\ i = 1, 2, \ldots, n-1$$

Metoda Gauss-Seidel pomoću Z-matrice

7. korak
   - Provjera da li izračunate vrijednosti napona $\vec{U}_i^{(1)}$ zadovoljavaju unaprijed postavljeni uvjet točnosti:

$$\left|(\vec{U}_i^{(1)} - \vec{U}_i^{(0)})\right| < \varepsilon$$

$$\varepsilon = 0.001 \div 0.0001 \quad \text{(najčešće)}$$

   - Ako je postavljeni uvjet zadovoljen za svaki **i –** KRAJ PRORAČUNA, konačno rješenje je vektor stanja $\vec{U}_i^{(1)}$ (i=1,2,…, n-1)
   - U suprotnom prelazak u sljedeću iteraciju (k=2) korištenjem izračunatog vektora struja u čvorištima $\vec{I}_i^{(1)}$, te ponovnim izvršavanjem koraka 6 (dakle računaju se $\vec{U}_i^{(2)}$ i $\vec{I}_i^{(2)}$) i koraka 7
   - Ako postavljeni uvjet nije zadovoljen ni u drugoj iteraciji prelazi se u treću (k=3) korištenjem rezultata iz druge itd. dok se ne ostvari tražena točnost

Metoda Gauss-Seidel pomoću Z-matrice

- Općenito za neku iteraciju **k+1** vrijede sljedeći izrazi:

$$\vec{U}_i^{(k+1)} = \sum_{j=1}^{i-1} \vec{Z}_{ij} \cdot \vec{I}_j^{(k+1)} + \sum_{j=i}^{n-1} \vec{Z}_{ij} \cdot \vec{I}_j^{(k)} + \vec{U}_{ref}$$

$$\vec{I}_i^{(k+1)} = \frac{\vec{S}_i^*}{\vec{U}_i^{*(k+1)}} - \vec{U}_i^{(k+1)} \cdot Y_i' \qquad \text{za } i = 1,2,\ldots,n-1$$

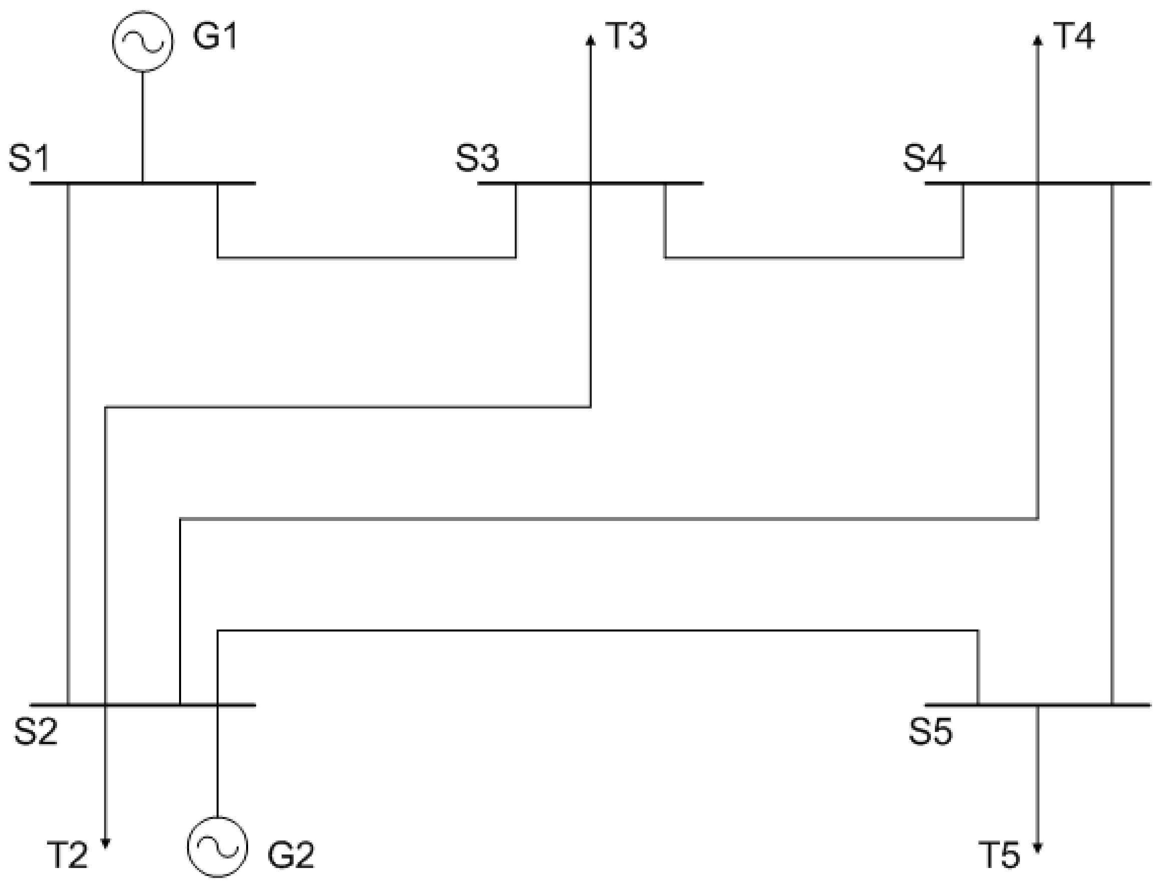
- Proračun se zaustavlja kada je ispunjeno:

$$\left| \left(\vec{U}_i^{(k+1)} - \vec{U}_i^{(k)} \right) \right| < \varepsilon \qquad \text{za } i = 1,2,\ldots,n-1$$

- Traženo rješenje (vektor stanja) je:

$$\vec{U}_i^{(k+1)} = \left| \vec{U}_i^{(k+1)} \right| \angle \delta_i^{k+1}$$

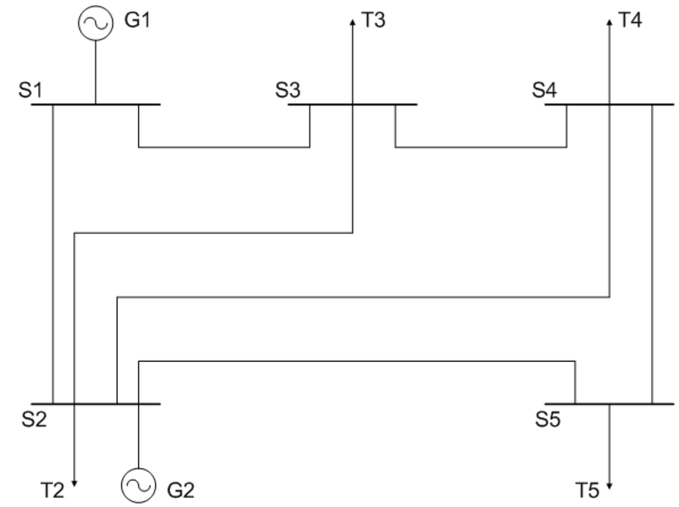
Primjer: Gauss-Seidel pomoću Z-matrice



Primjer: Gauss-Seidel pomoću Z-matrice

- Zadano:

i	j	$z_{i\text{-}j}$ (p.u.)	$y'_{i\text{-}j}/2$ (p.u.)
1	2	0.02+j0.06	j0.03
1	3	0.08+j0.24	j0.025
2	3	0.06+j0.18	j0.02
2	4	0.06+j0.18	j0.02
2	5	0.04+j0.12	j0.015
3	4	0.01+j0.03	j0.01
4	5	0.08+j0.24	j0.025



Čvorište 1 proglašavamo referentnim

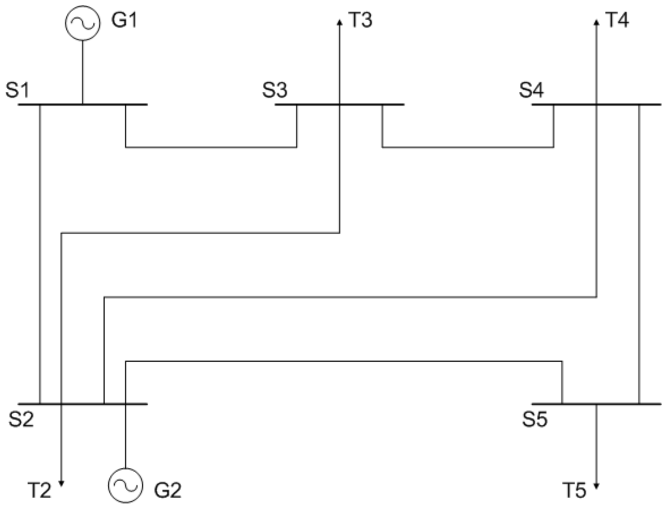
uzdužni parametri R+jX

poprečni parametri G+jB (već prepolovljeni)

Primjer: Gauss-Seidel pomoću Z-matrice

- Zadano:

Čv.	Generator			Teret	
	U	MW	Mvar		
1.	1.06+j0	/	/	/	/
2.		40	30	20	10
3.				45	15
4.				40	5
5.				60	10



- Bazna snaga: $S_B = 100\ MVA$
- Tražena točnost: $\varepsilon = 0.001$

Primjer: Gauss-Seidel pomoću Z-matrice

- Izračunamo uzdužne admitancije $$y_{i-j} = \frac{1}{z_{i-j}}$$

i	j	z_{i-j} (p.u.)
1	2	0.02+j0.06
1	3	0.08+j0.24
2	3	0.06+j0.18
2	4	0.06+j0.18
2	5	0.04+j0.12
3	4	0.01+j0.03
4	5	0.08+j0.24

→

y_{i-j} (p.u.)
5-j15
1.25-j3.75
1.66-j5
1.66-j5
2.5-j7.5
10-j30
1.25-j3.75

Primjer: Gauss-Seidel pomoću Z-matrice

- Matrica **Y** (sadrži samo uzdužne impedancije):

$\vec{Y}_{(5)} =$

6.25-j18.75	-5+j15	-1.25+j3.75	0	0
-5+j15	10.833-j32.5	-1.66+j5	-1.66+j5	-2.5+j7.5
-1.25+j3.75	-1.66+j5	12.916-j38.75	-10+j30	0
0	-1.66+j5	-10+j30	12.916-j38.75	-1.25+j3.75
0	-2.5+j7.5	0	-1.25+j3.75	3.75-j11.25

- Čvorište 1 je referentno pa odbacujemo 1. redak i 1. stupac

$\vec{Y} =$

10.833-j32.5	-1.66+j5	-1.66+j5	-2.5+j7.5
-1.66+j5	12.916-j38.75	-10+j30	0
-1.66+j5	-10+j30	12.916-j38.75	-1.25+j3.75
-2.5+j7.5	0	-1.25+j3.75	3.75-j11.25

Primjer: Gauss-Seidel pomoću Z-matrice

- Matrica impedancija čvorišta:

$$\vec{Z} = \vec{Y}^{-1} =$$

	(2)	(3)	(4)	(5)
(2)	0.016857+ j0.050571	0.012571+ j0.03771	0.013428+ j0.0402857	0.0151743+ j0.047143
(3)	0.012571+ j0.03771	0.0297143+ j0.089143	0.0262857+ j0.0788571	0.017143+ j0.0514286
(4)	0.013428+ j0.0402857	0.0262857+ j0.0788571	0.0317143+ j0.09514	0.0185238+ j0.0585714
(5)	0.0151743+ j0.047143	0.017143+ j0.0514286	0.0185238+ j0.0585714	0.043651+ j0.1309524

Primjer: Gauss-Seidel pomoću Z-matrice

- Matrica poprečnih admitancija Y':

$$Y' = \begin{array}{c} (2) \\ (3) \\ (4) \\ (5) \end{array} \begin{bmatrix} Y'_2 \\ Y'_3 \\ Y'_4 \\ Y'_5 \end{bmatrix} = \begin{bmatrix} j0.03 + j0.02 + j0.02 + j0.015 \\ j0.025 + j0.02 + j0.01 \\ j0.02 + j0.01 + j0.025 \\ j0.015 + j0.025 \end{bmatrix} = \begin{bmatrix} j0.085 \\ j0.055 \\ j0.055 \\ j0.04 \end{bmatrix}$$

Primjer: Gauss-Seidel pomoću Z-matrice

- k=0

$$\vec{I}_2^{(0)} = \frac{\vec{S}_2^*}{\vec{U}_2^{*(0)}} - Y_2^{'} \cdot \vec{U}_2^{(0)} = \frac{0.2 - j0.2}{1 - j0} - j0.085 \cdot 1 = 0.2 - j0.285$$

$$\vec{I}_3^{(0)} = \frac{\vec{S}_3^*}{\vec{U}_3^{*(0)}} - Y_3^{'} \cdot \vec{U}_3^{(0)} = \frac{-0.45 + j0.15}{1 - j0} - j0.055 \cdot 1 = -0.045 + j0.095$$

$$\vec{I}_4^{(0)} = \frac{\vec{S}_4^*}{\vec{U}_4^{*(0)}} - Y_4^{'} \cdot \vec{U}_4^{(0)} = \frac{-0.4 + j0.05}{1 - j0} - j0.055 \cdot 1 = -0.4 - j0.005$$

$$\vec{I}_5^{(0)} = \frac{\vec{S}_5^*}{\vec{U}_5^{*(0)}} - Y_5^{'} \cdot \vec{U}_5^{(0)} = \frac{-0.6 + j0.1}{1 - j0} - j0.04 \cdot 1 = -0.6 + j0.06$$

Primjer: Gauss-Seidel pomoću Z-matrice

- k=1

$$\vec{U}_2^{(1)} = \vec{U}_1 + \vec{Z}_{22} \cdot \vec{I}_2^{(0)} + \vec{Z}_{23} \cdot \vec{I}_3^{(0)} + \vec{Z}_{24} \cdot \vec{I}_4^{(0)} + \vec{Z}_{25} \cdot \vec{I}_5^{(0)} = 1.05122 - j0.05399$$

$$\vec{I}_2^{(1)} = \frac{\vec{S}_2^*}{\vec{U}_2^{*(1)}} - Y_2^{'} \cdot \vec{U}_2^{(1)} = 0.17544 - j0.208887$$

$$\vec{U}_3^{(1)} = \vec{U}_1 + \vec{Z}_{32} \cdot \vec{I}_2^{(1)} + \vec{Z}_{33} \cdot \vec{I}_3^{(0)} + \vec{Z}_{34} \cdot \vec{I}_4^{(0)} + \vec{Z}_{35} \cdot \vec{I}_5^{(0)} = 1.02777 - j0.09581$$

$$\vec{I}_3^{(1)} = \frac{\vec{S}_3^*}{\vec{U}_3^{*(1)}} - Y_3^{'} \cdot \vec{U}_3^{(1)} = -0.42585 - j0.12813$$

$$\vec{U}_4^{(1)} = \vec{U}_1 + \vec{Z}_{42} \cdot \vec{I}_2^{(1)} + \vec{Z}_{43} \cdot \vec{I}_3^{(1)} + \vec{Z}_{44} \cdot \vec{I}_4^{(0)} + \vec{Z}_{45} \cdot \vec{I}_5^{(0)} = 1.02521 - j0.0992$$

$$\vec{I}_4^{(1)} = \frac{\vec{S}_4^*}{\vec{U}_4^{*(1)}} - Y_4^{'} \cdot \vec{U}_4^{(1)} = -0.38782 + j0.02933$$

$$\vec{U}_5^{(1)} = \vec{U}_1 + \vec{Z}_{52} \cdot \vec{I}_2^{(1)} + \vec{Z}_{53} \cdot \vec{I}_3^{(1)} + \vec{Z}_{54} \cdot \vec{I}_4^{(1)} + \vec{Z}_{55} \cdot \vec{I}_5^{(0)} = 1.01913 - j0.11403$$

$$\vec{I}_5^{(1)} = \frac{\vec{S}_5^*}{\vec{U}_5^{*(1)}} - Y_5^{'} \cdot \vec{U}_5^{(1)} = -0.57518 + j0.1212$$

Primjer: Gauss-Seidel pomoću Z-matrice

- Provjera:

$$\left|\vec{U}_2^{(1)} - \vec{U}_2^{(0)}\right| = \left|(1.05112 - j0.05399) - (1 + j0)\right|$$

$$= \left|0.05112 - j0.05399\right|$$

$$\left|0.05112 - j0.05399\right| > 0.001$$

- Uvjet nije ispunjen (ostale napone nije ni potrebno dalje provjeravati) → prelazak u sljedeću iteraciju

- k=2

$$\vec{U}_2^{(2)} = \vec{U}_1 + \vec{Z}_{22} \cdot \vec{I}_2^{(1)} + \vec{Z}_{23} \cdot \vec{I}_3^{(1)} + \vec{Z}_{24} \cdot \vec{I}_4^{(1)} + \vec{Z}_{25} \cdot \vec{I}_5^{(1)}$$

$$\vec{I}_2^{(2)} = \frac{\vec{S}_2^*}{\vec{U}_2^{*(2)}} - Y_2' \cdot \vec{U}_2^{(2)}$$

$$\vdots$$

Primjer: Gauss-Seidel pomoću Z-matrice

Iteracija	Čv. 2	Čv. 3	Čv. 4	Čv. 5
0.	1+j0	1+j0	1+j0	1+j0
1.	1.05112-j0.05399	1.02777-j0.09581	1.02529-j0.0992	1.01913-j0.11403
2.	1.04622-j0.05286	1.02041-j0.08837	1.01924-j0.09454	1.0122-j0.10841
3.	1.04622-j0.05129	1.02035-j0.08924	1.01918-j0.09502	1.01212-j0.10908
	1.04748 $\angle$-28°	1.02425 $\angle$ -5°	1.02361 $\angle$-5.33°	1.018 $\angle$-6.15°

- U trećoj iteraciji je zadovoljen postavljeni kriterij točnosti

Gauss-Seidel pomoću Y matrice

Metoda Gauss-Seidel pomoću Y-matrice

- Mreža od ***n*** čvorišta – jedno čvorište referentno:

$$\left|\vec{I}\right| = \left|\vec{Y}\right| \cdot \left|\vec{U}\right|$$

$$\vec{I}_1 = \vec{Y}_{11} \cdot \vec{U}_1 + \vec{Y}_{12} \cdot \vec{U}_2 + \ldots + \vec{Y}_{1n} \cdot \vec{U}_n$$
$$\vec{I}_2 = \vec{Y}_{21} \cdot \vec{U}_1 + \vec{Y}_{22} \cdot \vec{U}_2 + \ldots + \vec{Y}_{2n} \cdot \vec{U}_n$$
$$\vdots$$
$$\vec{I}_{(n-1)} = \vec{Y}_{(n-1)1} \cdot \vec{U}_1 + \vec{Y}_{(n-1)2} \cdot \vec{U}_2 + \ldots + \vec{Y}_{(n-1)n} \cdot \vec{U}_n$$

Metoda Gauss-Seidel pomoću Y-matrice

$$\vec{U}_1 = \frac{1}{\vec{Y}_{11}} \cdot \left[\vec{I}_1 - \vec{Y}_{12} \cdot \vec{U}_2 - \vec{Y}_{13} \cdot \vec{U}_3 - \ldots - \vec{Y}_{1n} \cdot \vec{U}_n \right]$$

$$\vec{U}_2 = \frac{1}{\vec{Y}_{22}} \cdot \left[\vec{I}_2 - \vec{Y}_{21} \cdot \vec{U}_1 - \vec{Y}_{23} \cdot \vec{U}_3 - \ldots - \vec{Y}_{2n} \cdot \vec{U}_n \right]$$

$$\vdots$$

$$\vec{U}_{n-1} = \frac{1}{\vec{Y}_{(n-1)(n-1)}} \cdot \left[\vec{I}_{n-1} - \vec{Y}_{(n-1)1} \cdot \vec{U}_1 - \vec{Y}_{(n-1)3} \cdot \vec{U}_3 - \ldots - \vec{Y}_{(n-1)n} \cdot \vec{U}_n \right]$$

- Za čvorište **i**:

$$\vec{I}_i = \frac{\vec{S}_i^*}{\vec{U}_i^*}$$

$$\vec{U}_i^{(1)} = \frac{1}{\vec{Y}_{ii}} \cdot \left[\frac{\vec{S}_i^*}{\vec{U}_i^{*(0)}} - \vec{Y}_{i1} \cdot \vec{U}_1^{(0)} - \vec{Y}_{i2} \cdot \vec{U}_2^{(0)} - \ldots - \vec{Y}_{in} \cdot \vec{U}_n^{(0)} \right]$$

Metoda Gauss-Seidel pomoću Y-matrice

- Za neku iteraciju **k+1** i čvorište **i**:

$$\vec{U}_i^{(k+1)} = \frac{1}{\vec{Y}_{ii}} \cdot \left[\frac{\vec{S}_i^*}{\vec{U}_i^{*(k)}} - \vec{Y}_{i1} \cdot \vec{U}_1^{(k)} - \vec{Y}_{i2} \cdot \vec{U}_2^{(k)} - \ldots - \vec{Y}_{in} \cdot \vec{U}_n^{(k)} \right]$$

$$\vec{U}_i^{(k+1)} = \frac{\vec{S}_i^*}{\vec{Y}_{ii} \cdot \vec{U}_i^{*(k)}} - \frac{\vec{Y}_{i1}}{\vec{Y}_{ii}} \cdot \vec{U}_1^{(k)} - \ldots - \frac{\vec{Y}_{in}}{\vec{Y}_{ii}} \cdot \vec{U}_n^{(k)}$$

$$\vec{U}_i^{(k+1)} = \frac{\vec{S}_i^*}{\vec{Y}_{ii} \cdot \vec{U}_i^{*(k)}} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\vec{Y}_{ij}}{\vec{Y}_{ii}} \cdot \vec{U}_j^{(k)}$$

Metoda Gauss-Seidel pomoću Y-matrice

$$KL_i = \frac{\vec{S}_i^*}{\vec{Y}_{ii}}$$

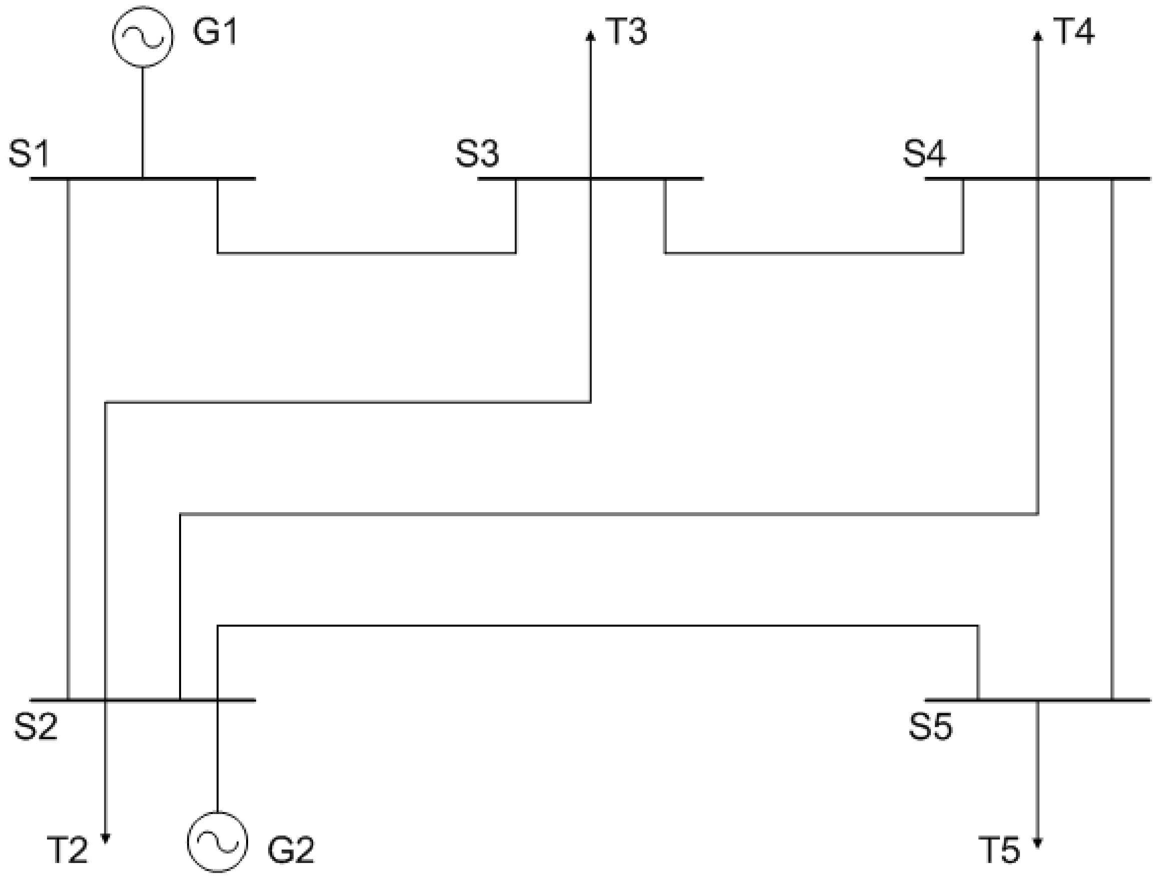
$$YL_{i,j} = \frac{\vec{Y}_{ij}}{\vec{Y}_{ii}}$$

$$\vec{U}_i^{(k+1)} = KL_i \cdot \frac{1}{\vec{U}_i^{*(k)}} - \sum_{j=1}^{i-1} YL_{i,j} \cdot \vec{U}_j^{(k+1)} - \sum_{j=i+1}^{n} YL_{i,j} \cdot \vec{U}_j^{(k)}$$

- Uvjet točnosti:

$$\left| \vec{U}_i^{(k+1)} - \vec{U}_i^{(k)} \right| < \varepsilon$$

Primjer: Gauss-Seidel pomoću Y-matrice

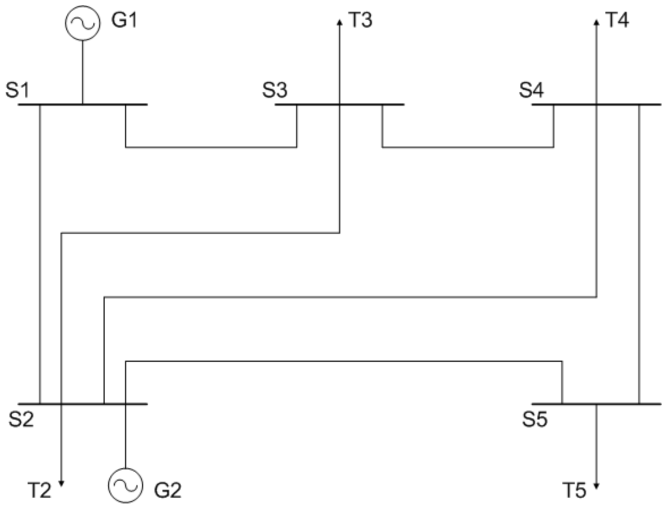


Primjer je isti kao i kod GS metode za Z-matricom

Primjer: Gauss-Seidel pomoću Y-matrice

- Zadano:

i	j	$z_{i\text{-}j}$ (p.u.)	$y'_{i\text{-}j}/2$ (p.u.)
1	2	0.02+j0.06	j0.03
1	3	0.08+j0.24	j0.025
2	3	0.06+j0.18	j0.02
2	4	0.06+j0.18	j0.02
2	5	0.04+j0.12	j0.015
3	4	0.01+j0.03	j0.01
4	5	0.08+j0.24	j0.025

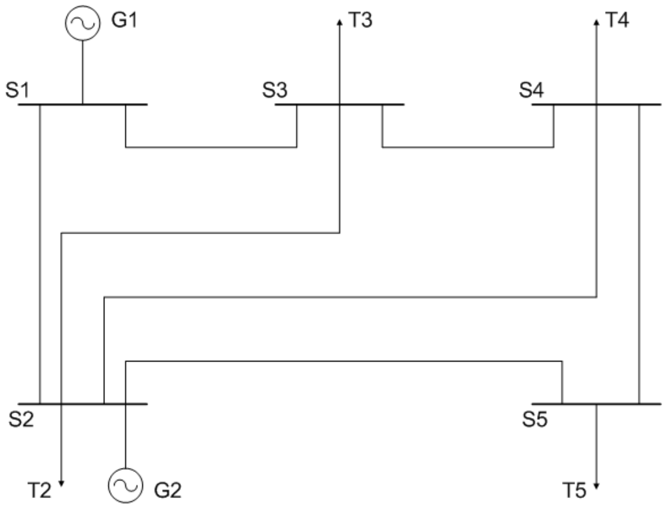


Čvorište 1 proglašavamo referentnim

Primjer: Gauss-Seidel pomoću Y-matrice

- Zadano:

Čv.	Generator			Teret	
	U	MW	Mvar		
1.	1.06+j0	/	/	/	/
2.		40	30	20	10
3.				45	15
4.				40	5
5.				60	10



- Bazna snaga: $S_B = 100\ MVA$
- Tražena točnost: $\varepsilon = 0.001$

Primjer: Gauss-Seidel pomoću Y-matrice

- Matrica **Y**:

$\vec{Y} =$

6.25-j18.695	-5+j15	-1.25+j3.75	0	0
-5+j15	10.833-j32.415	-1.66+j5	-1.66+j5	-2.5+j7.5
-1.25+j3.75	-1.66+j5	12.916-j38.695	-10+j30	0
0	-1.66+j5	-10+j30	12.916-j38.695	-1.25+j3.75
0	-2.5+j7.5	0	-1.25+j3.75	3.75-j11.21

Puna prava Y matrica sa uzdužnim i poprečnim elementima

$$\vec{Y}_{ij} = -\vec{y}_{i-j}$$

$$\vec{Y}_{ii} = \sum_{j=1}^{n} \vec{y}_{i-j} + \vec{Y}_i'$$

- Početne vrijednosti napona: $\vec{U}_i^{(0)} = 1 + j0 \quad i = 2, 3, 4, 5$

Primjer: Gauss-Seidel pomoću Y-matrice

$$\vec{U}_2^{(k+1)} = \frac{KL_2}{\vec{U}_2^{*(k)}} - YL_{2,1} \cdot \vec{U}_1 - YL_{2,3} \cdot \vec{U}_3^{(k)} - YL_{2,4} \cdot \vec{U}_4^{(k)} - YL_{2,5} \cdot \vec{U}_5^{(k)}$$

$$\vec{U}_3^{(k+1)} = \frac{KL_3}{\vec{U}_3^{*(k)}} - YL_{3,1} \cdot \vec{U}_1 - YL_{3,2} \cdot \vec{U}_2^{(k+1)} - YL_{3,4} \cdot \vec{U}_4^{(k)}$$

$$\vec{U}_4^{(k+1)} = \frac{KL_4}{\vec{U}_4^{*(k)}} - YL_{4,2} \cdot \vec{U}_2^{(k+1)} - YL_{4,3} \cdot \vec{U}_3^{(k+1)} - YL_{4,5} \cdot \vec{U}_5^{(k)}$$

$$\vec{U}_5^{(k+1)} = \frac{KL_5}{\vec{U}_5^{*(k)}} - YL_{5,2} \cdot \vec{U}_2^{(k+1)} - YL_{5,4} \cdot \vec{U}_4^{(k+1)}$$

Primjer: Gauss-Seidel pomoću Y-matrice

$$KL_2 = \frac{P_2 - jQ_2}{\vec{Y}_{22}} = \frac{0.2 - j0.2}{10.833 - j32.415} = 0.0074 + j0.0037$$

$$KL_3 = \frac{P_3 - jQ_3}{\vec{Y}_{33}} = -0.00698 - j0.0093$$

$$KL_4 = \frac{P_4 - jQ_4}{\vec{Y}_{44}} = -0.00427 - j0.00891$$

$$KL_5 = \frac{P_5 - jQ_5}{\vec{Y}_{55}} = -0.002413 - j0.004545$$

Primjer: Gauss-Seidel pomoću Y-matrice

$$YL_{2,1} = \frac{\vec{Y}_{21}}{\vec{Y}_{22}} = \frac{-5 + j15}{10.833 - j32.415} = -0.46263 + j0.000177$$

$$YL_{2,3} = \frac{\vec{Y}_{23}}{\vec{Y}_{22}} = \frac{-1.66 + j5}{10.833 - j32.415} = -0.15421 + j0.00012$$

$$YL_{2,4} = \frac{\vec{Y}_{24}}{\vec{Y}_{22}} = -0.15421 + j0.00012$$

$$YL_{2,5} = \frac{\vec{Y}_{25}}{\vec{Y}_{22}} = -0.23131 + j0.00018$$

$$YL_{3,1} = -0.0969 + j0.00004$$

$$YL_{3,2} = -0.12920 + j0.00006$$

$$YL_{3,4} = -0.77518 + j0.00033$$

Primjer: Gauss-Seidel pomoću Y-matrice

$$YL_{4,2} = -0.1292 + j0.00006$$

$$YL_{4,3} = -0.77518 + j0.00033$$

$$YL_{4,5} = -0.0969 + j0.00004$$

$$YL_{5,2} = -0.66881 + j0.00072$$

$$YL_{5,4} = -0.3344 + j0.00036$$

$$\vec{U}_2^{(1)} = 1.03752 + j0.0029$$

- Postupak se ubrzava uvođenjem faktora ubrzanja **α**:

$$\alpha = 1.4$$

$$\Delta\vec{U}_2^{(1)} = \vec{U}_2^{(1)} - \vec{U}_2^{(0)} = 0.03752 + j0.0029$$

$$\vec{U}_{2ubrzani}^{(1)} = \vec{U}_2^{(0)} + \alpha \cdot \Delta\vec{U}_2^{(1)} = 1 + j0 + 0.05253 + j0.00406$$

Primjer: Gauss-Seidel pomoću Y-matrice

$$\vec{U}_{2ubrzani}^{(1)} = \vec{U}_2^{(0)} + \alpha \cdot \Delta\vec{U}_2^{(1)} = 1 + j0 + 0.05253 + j0.00406$$

$$\vec{U}_{2ubrzani}^{(1)} = 1.05253 + j0.00406$$

$$\vec{U}_3^{(1)} = 1.00690 - j0.00921$$

$$\Delta\vec{U}_3^{(1)} = 0.069 - j0.00021$$

$$\vec{U}_{3ubrz}^{(1)} = \vec{U}_3^{(0)} + \alpha \cdot \Delta\vec{U}_3^{(1)} = 1.00966 - j0.01289$$

$$\vec{U}_{4ubrz}^{(1)} = \vec{U}_4^{(0)} + \alpha \cdot \Delta\vec{U}_4^{(1)} = 1.01579 - j0.02635$$

$$\vec{U}_{5ubrz}^{(1)} = \vec{U}_5^{(0)} + \alpha \cdot \Delta\vec{U}_5^{(1)} = 1.02728 - j0.07374$$

Primjer: Gauss-Seidel pomoću Y-matrice

- 2. iteracija

$$\vec{U}_2^{(2)} = \frac{KL_2}{\vec{U}_{2ubrz}^{*(1)}} - YL_{2,1} \cdot \vec{U}_1 - YL_{2,3} \cdot \vec{U}_{3ubrz}^{(1)} - YL_{2,4} \cdot \vec{U}_{4ubrz}^{(1)} - YL_{2,5} \cdot \vec{U}_{5ubrz}^{(1)}$$

$$\Delta\vec{U}_2^{(2)} = \vec{U}_2^{(2)} - \vec{U}_{2ubrzani}^{(1)}$$

$$\vec{U}_{2ubrz}^{(2)} = \vec{U}_{2ubrzani}^{(1)} + \alpha \cdot \Delta\vec{U}_2^{(2)}$$

$$\vec{U}_3^{(2)} = \frac{KL_3}{\vec{U}_{3ubrz}^{*(1)}} - YL_{3,1} \cdot \vec{U}_1 - YL_{3,2} \cdot \vec{U}_{2ubrz}^{(2)} - YL_{3,4} \cdot \vec{U}_{4ubrz}^{(1)}$$

$$\vdots$$

Primjer: Gauss-Seidel pomoću Y-matrice

- Rješenje:

Iteracija	U_2	U_3	U_4	U_5
1.	1.05253+ j0.00406	1.00966- j0.01289	1.01579- j0.02635	1.02727- j0.07374
2.	1.04528- j0.03015	1.02154- j0.04227	1.02451- j0.06353	1.01025- j0.08032
⋮	⋮	⋮	⋮	⋮
10.	1.04623- j0.05126	1.02036- j0.08917	1.0192- j0.09504	1.01211- j0.10904

Primjer: Gauss-Seidel pomoću Y-matrice

- Tokovi snaga:

$$P_{12} + jQ_{12} = \vec{U}_1 \cdot (\vec{U}_1^* - \vec{U}_2^*) \cdot y_{1-2}^* + \vec{U}_1 \cdot \vec{U}_1^* \cdot \frac{{y'}_{1-2}^*}{2}$$

$$= 88.8 - j8.6 \ MVA$$

$$P_{21} + jQ_{21} = \vec{U}_2 \cdot (\vec{U}_2^* - \vec{U}_1^*) \cdot y_{1-2}^* + \vec{U}_2 \cdot \vec{U}_2^* \cdot \frac{{y'}_{1-2}^*}{2}$$

$$= -87.4 + j6.2 \ MVA$$

$$\Delta P_{1-2} + j\Delta Q_{1-2} = P_{12} + P_{21} + j(Q_{12} + Q_{21})$$

$$= 1.4 \ MW - j2.4 \ Mvar$$

Primjer: Gauss-Seidel pomoću Y-matrice

- Tokovi snaga:

Vod	P [MW]	Q [Mvar]	ΔP [MW]	ΔQ [Mvar]
1-2 2-1	88.8 -87.4	-8.6 6.2	1.4	-2.4
1-3 3-1	40.7 -39.5	1.1 -3	1.2	-1.9
2-3 3-2	24.7 -24.3	3.5 -6.8	0.4	-3.3
2-4 4-2	27.9 -27.5	3.0 -5.9	0.4	-2.9
2-5 5-2	54.8 -53.7	7.4 -7.2	1.1	0.2
3-4 4-3	18.9 -18.9	-5.1 3.2	0	-1.9
4-5 5-4	6.3 -6.3	-2.3 -2.8	0	-5.1

Pitanja i komentari

Analiza elektroenergetskog sustava Predavanje 6: Proračun tokova snaga

Prof. dr. sc. Ivica Pavić

Izv. prof. dr. sc. Marko Delimar

Newton-Raphson metoda

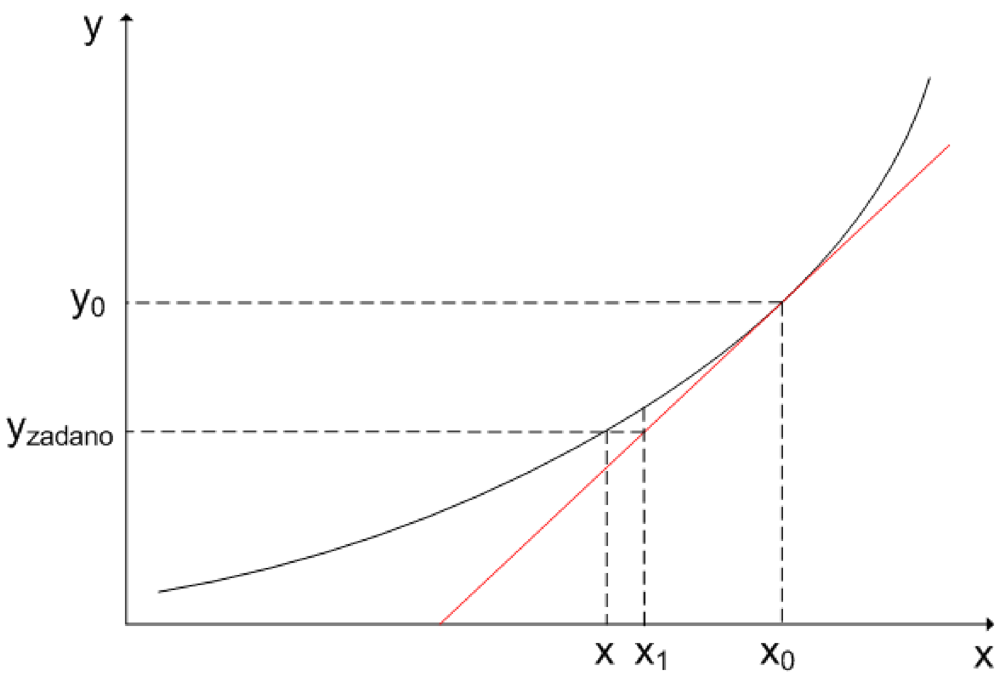
Metoda Newton-Raphson

$$f'(x_0) = \frac{\Delta y}{\Delta x}$$

$$f'(x_0) = \frac{f(x_0) - f(x_1)}{x_0 - x_1}$$

$$f'(x_0) = \frac{y_0 - y_{zadano}}{x_0 - x_1}$$

$$x_1 = x_0 - \frac{y_0 - y_{zadano}}{f'(x_0)}$$



- Približavanje rješenju (x) od nekog početnog, pretpostavljenog rješenja (x_0) pomoću tangenti

Metoda Newton-Raphson

- Mreža od **n** čvorišta – jedno čvorište referentno (čvorište ***n***)
- Poznate snage u čvorištima:
  - Generatorska i čvorišta tereta

$$P_i = U_i \cdot \sum_{j=1}^{n} U_j \cdot Y_{ij} \cdot cos\left(\delta_i - \delta_j - \Theta_{ij}\right) \qquad i = 1,2,\ldots,n-1$$

  - Čvorišta tereta

$$Q_i = U_i \cdot \sum_{j=1}^{n} U_j \cdot Y_{ij} \cdot sin\left(\delta_i - \delta_j - \Theta_{ij}\right) \qquad i = 1,2,\ldots,n-g-1$$

- Potrebno izračunati: $U_i \qquad i = 1, 2, \ldots, n-g-1$

$$\delta_i \qquad i = 1, 2, \ldots, n-1$$

Metoda Newton-Raphson

- Postupak proračuna:
  1. korak
     - Učitavanje podataka o mreži (konfiguracija, admitancije grana)
     - Učitavanje podataka o injekcijama snage u čvorištima
  2. korak
     - Formiranje matrice **Y** (pune prave **Y** matrice)
  3. korak
     - Početne vrijednosti napona čvorišta: $\vec{\mathrm{U}}_{\mathrm{i}}^{(0)} = 1 + \mathrm{j}0\ \mathrm{p.u.} = 1\angle 0^\circ\ \mathrm{p.u.}$
  4. korak
     - Računanje snaga u čvorištima:

$$P_{ira\check{c}}^{(0)} = \sum_{j=1}^{n} U_i^{(0)} \cdot U_j^{(0)} \cdot Y_{ij} \cdot cos\left(\delta_i^{(0)} - \delta_j^{(0)} - \Theta_{ij}\right) \qquad i = 1, 2, ..., n-1$$

$$Q_{ira\check{c}}^{(0)} = \sum_{j=1}^{n} U_i^{(0)} \cdot U_j^{(0)} \cdot Y_{ij} \cdot sin\left(\delta_i^{(0)} - \delta_j^{(0)} - \Theta_{ij}\right) \qquad i = 1, 2, ..., n-1-g$$

Metoda Newton-Raphson

5. korak

$$\Delta P_i^{(0)} = P_{izad} - P_{irač}^{(0)} \qquad i = 1, 2, \ldots, n-1$$

$$\Delta Q_i^{(0)} = Q_{izad} - Q_{irač}^{(0)} \qquad i = 1, 2, \ldots, n-1-g$$

6. korak
   - Provjera kriterija točnosti:

     $$\Delta P_i^{(0)} < \varepsilon$$

     $$\Delta Q_i^{(0)} < \varepsilon$$
   - Uvjet ispunjen – KRAJ PRORAČUNA
   - Uvjet nije ispunjen – računanje Jakobijeve matrice **J**
7. korak
   - Računanje $\Delta\delta_i^{(0)}, \Delta U_i^{(0)}$ pomoću $\Delta P_i^{(0)}, \Delta Q_i^{(0)}$ i Jakobijeve matrice

Metoda Newton-Raphson

8. korak

$$U_i^{(1)} = U_i^{(0)} + \Delta U_i^{(0)}$$

$$\delta_i^{(1)} = \delta_i^{(0)} + \Delta\delta_i^{(0)}$$

9. korak
   - Obavljanje iteracijskog postupka ponavljanjem koraka 4, 5, 6, 7 i 8 (i korištenjem rezultata iz prethodne iteracije) dok nije ispunjen postavljeni kriterij točnosti

Metoda Newton-Raphson

- Jakobijeva matrica

$$\begin{vmatrix}\Delta P\\ \Delta Q\end{vmatrix}=|J|\cdot\begin{vmatrix}\Delta\delta\\ \Delta U\end{vmatrix}=\begin{vmatrix}J_1 & J_2\\ J_3 & J_4\end{vmatrix}\cdot\begin{vmatrix}\Delta\delta\\ \Delta U\end{vmatrix}=\begin{vmatrix}\left(\dfrac{\partial P}{\partial\delta}\right) & \left(\dfrac{\partial P}{\partial U}\right)\\ \left(\dfrac{\partial Q}{\partial\delta}\right) & \left(\dfrac{\partial Q}{\partial U}\right)\end{vmatrix}\cdot\begin{vmatrix}\Delta\delta\\ \Delta U\end{vmatrix}$$

$$\begin{vmatrix}\Delta P_1\\ \vdots\\ \Delta P_{n-1}\\ \Delta Q_1\\ \vdots\\ \Delta Q_{n-1-g}\end{vmatrix}=\begin{vmatrix}\dfrac{\partial P_1}{\partial\delta_1} & \cdots & \dfrac{\partial P_1}{\partial\delta_{n-1}} & \dfrac{\partial P_1}{\partial U_1} & \cdots & \dfrac{\partial P_1}{\partial U_{n-1-g}}\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ \dfrac{\partial P_{n-1}}{\partial\delta_1} & \cdots & \dfrac{\partial P_{n-1}}{\partial\delta_{n-1}} & \dfrac{\partial P_{n-1}}{\partial U_1} & \cdots & \dfrac{\partial P_{n-1}}{\partial U_{n-1-g}}\\ \dfrac{\partial Q_1}{\partial\delta_1} & \cdots & \dfrac{\partial Q_1}{\partial\delta_{n-1}} & \dfrac{\partial Q_1}{\partial U_1} & \cdots & \dfrac{\partial Q_1}{\partial U_{n-1-g}}\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ \dfrac{\partial Q_{n-1-g}}{\partial\delta_1} & \cdots & \dfrac{\partial Q_{n-1-g}}{\partial\delta_{n-1}} & \dfrac{\partial Q_{n-1-g}}{\partial U_1} & \cdots & \dfrac{\partial Q_{n-1-g}}{\partial U_{n-1-g}}\end{vmatrix}\cdot\begin{vmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_{n-1}\\ \Delta U_1\\ \vdots\\ \Delta U_{n-1-g}\end{vmatrix}$$

Metoda Newton-Raphson

- Jakobijeva matrica – **J**
- Jakobijeve podmatrice – **J1**, **J2**, **J3**, **J4**

- **J1**:

$$J_1 = \frac{\partial P}{\partial \delta}$$

$$\frac{\partial P_i}{\partial \delta_i} = -U_i \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} U_j \cdot Y_{ij} \cdot sin\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

$$\frac{\partial P_i}{\partial \delta_j} = U_i \cdot U_j \cdot Y_{ij} \cdot sin\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

Metoda Newton-Raphson

- **J2**: $$J_2 = \frac{\partial P}{\partial U}$$

$$\frac{\partial P_i}{\partial U_i} = 2 \cdot U_i \cdot Y_{ii} \cdot \cos(-\Theta_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^{n} U_j \cdot Y_{ij} \cdot \cos(\delta_i - \delta_j - \Theta_{ij})$$

$$\frac{\partial P_i}{\partial U_j} = U_i \cdot Y_{ij} \cdot \cos(\delta_i - \delta_j - \Theta_{ij})$$

- **J3**: $$J_3 = \frac{\partial Q}{\partial \delta}$$

$$\frac{\partial Q_i}{\partial \delta_i} = U_i \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} U_j \cdot Y_{ij} \cdot \cos(\delta_i - \delta_j - \Theta_{ij})$$

$$\frac{\partial Q_i}{\partial \delta_j} = -U_i \cdot U_j \cdot Y_{ij} \cdot \cos(\delta_i - \delta_j - \Theta_{ij})$$

Metoda Newton-Raphson

- **J4**:

$$J_4 = \frac{\partial Q}{\partial U}$$

$$\frac{\partial Q_i}{\partial U_i} = 2 \cdot U_i \cdot Y_{ii} \cdot sin(-\Theta_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^{n} U_j \cdot Y_{ij} \cdot sin(\delta_i - \delta_j - \Theta_{ij})$$

$$\frac{\partial Q_i}{\partial U_j} = U_i \cdot Y_{ij} \cdot sin(\delta_i - \delta_j - \Theta_{ij})$$

- Vrijedi:

$$\begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix} = \begin{vmatrix} J_1 & J_2 \\ J_3 & J_4 \end{vmatrix} \cdot \begin{vmatrix} \Delta \delta \\ \Delta U \end{vmatrix}$$

$$\begin{vmatrix} \Delta \delta \\ \Delta U \end{vmatrix} = \begin{vmatrix} J_1 & J_2 \\ J_3 & J_4 \end{vmatrix}^{-1} \cdot \begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix}$$

Metoda Newton-Raphson

- Općenito za **k**-tu iteraciju vrijedi:

$$P_{irač}^{(k)} = \sum_{j=1}^{n} U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{ij} \cdot cos\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \qquad i = 1, 2, \dots, n-1$$

$$Q_{irač}^{(k)} = \sum_{j=1}^{n} U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{ij} \cdot sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \qquad i = 1, 2, \dots, n-1-g$$

$$\Delta P_i^{(k)} = P_{izad} - P_{irač}^{(k)} \qquad i = 1, 2, \dots, n-1$$

$$\Delta Q_i^{(k)} = Q_{izad} - Q_{irač}^{(k)} \qquad i = 1, 2, \dots, n-1-g$$

Metoda Newton-Raphson

- Jakobijeva podmatrica **J1** (iteracija **k**)

$$\left(\frac{\partial P_i}{\partial \delta_i}\right)^{(k)} = -U_i^{(k)} \cdot \sum_{\substack{j=1 \\ j\neq i}}^{n} U_j^{(k)} \cdot Y_{ij} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, \ldots, n-1$$

$$\left(\frac{\partial P_i}{\partial \delta_j}\right)^{(k)} = U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{ij} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, \ldots, n-1;\ j = 1, 2, \ldots, n-1$$

- Jakobijeva podmatrica **J2** (iteracija **k**)

$$\left(\frac{\partial P_i}{\partial U_i}\right)^{(k)} = 2 \cdot U_i^{(k)} \cdot Y_{ii} \cdot \cos\left(-\Theta_{ii}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{n} U_j^{(k)} \cdot Y_{ij} \cdot \cos\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, \ldots, n-1$$

$$\left(\frac{\partial P_i}{\partial U_j}\right)^{(k)} = U_i^{(k)} \cdot Y_{ij} \cdot \cos\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, \ldots, n-1;\ j = 1, 2, \ldots, n-1-g$$

Metoda Newton-Raphson

- Jakobijeva podmatrica **J3** (iteracija **k**)

$$\left(\frac{\partial Q_i}{\partial \delta_i}\right)^{(k)} = U_i^{(k)} \cdot \sum_{\substack{j=1 \\ j\neq i}}^{n} U_j^{(k)} \cdot Y_{ij} \cdot \cos\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, \dots, n-1-g$$

$$\left(\frac{\partial Q_i}{\partial \delta_j}\right)^{(k)} = -U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{ij} \cdot \cos\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, \dots, n-1-g;\ j = 1, 2, \dots, n-1$$

- Jakobijeva podmatrica **J4** (iteracija **k**)

$$\left(\frac{\partial Q_i}{\partial U_i}\right)^{(k)} = 2 \cdot U_i^{(k)} \cdot Y_{ii} \cdot \sin\left(-\Theta_{ii}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{n} U_j^{(k)} \cdot Y_{ij} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, \dots, n-1-g$$

$$\left(\frac{\partial Q_i}{\partial U_j}\right)^{(k)} = U_i^{(k)} \cdot Y_{ij} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad i = 1, 2, \dots, n-1-g\ ;\ j = 1, 2, \dots, n-1-g$$

Metoda Newton-Raphson

- Naponi u iteraciji **k+1**:

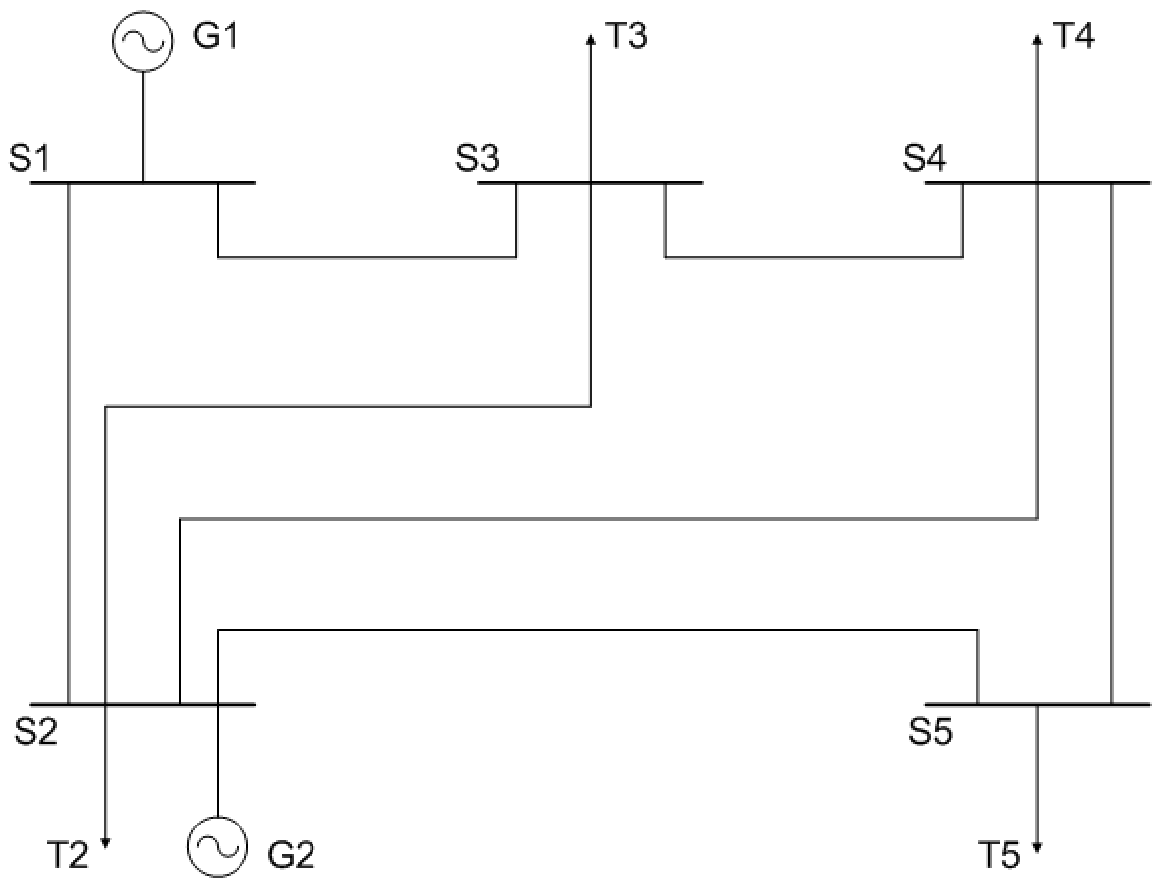
$$\left|\Delta\delta\right|^{(k)} = \left|J_1^{(k)}\right|^{-1} \cdot \left|\Delta P\right|^{(k)}$$

$$\left|\Delta U\right|^{(k)} = \left|J_4^{(k)}\right|^{-1} \cdot \left|\Delta Q\right|^{(k)}$$

$$U_i^{(k+1)} = U_i^{(k)} + \Delta U_i^{(k)}$$

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)}$$

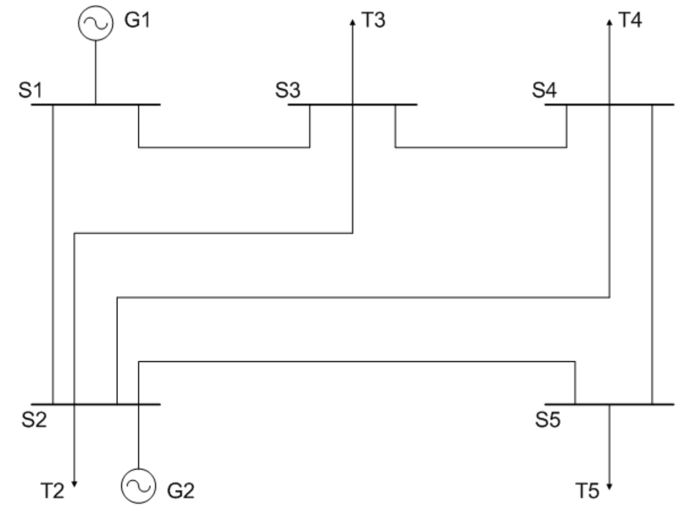
Primjer: Newton-Raphson metoda



Primjer: Newton-Raphson metoda

- Zadano:

i	j	$z_{i\text{-}j}$ (p.u.)	$y'_{i\text{-}j}/2$ (p.u.)
1	2	0.02+j0.06	j0.03
1	3	0.08+j0.24	j0.025
2	3	0.06+j0.18	j0.02
2	4	0.06+j0.18	j0.02
2	5	0.04+j0.12	j0.015
3	4	0.01+j0.03	j0.01
4	5	0.08+j0.24	j0.025

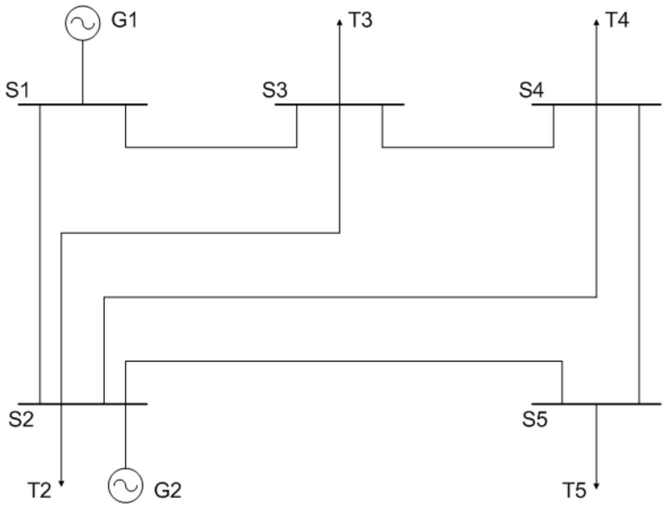


Čvorište 1 proglašavamo referentnim

Primjer: Newton-Raphson metoda

- Zadano:

Čv.	Generator			Teret	
	U	MW	Mvar		
1.	1.06+j0	/	/	/	/
2.		40	30	20	10
3.				45	15
4.				40	5
5.				60	10



- Bazna snaga: $S_B = 100\ MVA$
- Tražena točnost: $\varepsilon = 0.001$

Primjer: Newton-Raphson metoda

- Matrica **Y**:

$\vec{Y} =$

6.25-j18.695	-5+j15	-1.25+j3.75	0	0
-5+j15	10.833-j32.415	-1.66+j5	-1.66+j5	-2.5+j7.5
-1.25+j3.75	-1.66+j5	12.916-j38.695	-10+j30	0
0	-1.66+j5	-10+j30	12.916-j38.695	-1.25+j3.75
0	-2.5+j7.5	0	-1.25+j3.75	3.75-j11.21

$\vec{Y} =$

19.712 ∠-71.5145°	15.81 ∠108.435°	3.95 ∠108.435°	0	0
15.81 ∠108.435°	34.18 ∠-71.52°	5.27 ∠108.435°	5.27 ∠108.435°	7.9 ∠108.435°
3.95 ∠108.435°	5.27 ∠108.435°	40.79 ∠-71.54°	31.62 ∠108.435°	0
0	5.27 ∠108.435°	31.62 ∠108.435°	40.79 ∠-71.54°	3.95 ∠108.435°
0	7.9 ∠108.435°	0	3.95 ∠108.435°	11.82 ∠-71.52°

Primjer: Newton-Raphson metoda

- Početne vrijednosti napona: $\vec{U}_i^{(0)} = 1 + j0 \quad i = 2, 3, 4, 5$
- Snage u čvorištima:

$$P_{2izr}^{(0)} = -0.3 \qquad P_{4izr}^{(0)} = 0$$

$$Q_{2izr}^{(0)} = -0.985 \qquad Q_{4izr}^{(0)} = -0.055$$

$$P_{3izr}^{(0)} = -0.075 \qquad P_{5izr}^{(0)} = 0$$

$$Q_{3izr}^{(0)} = -0.28 \qquad Q_{5izr}^{(0)} = -0.04$$

Primjer: Newton-Raphson metoda

$$\Delta P_2^{(0)} = P_{2zad} - P_{2izr}^{(0)} = 0.5 \qquad \Delta Q_2^{(0)} = 1.185$$

$$\Delta P_3^{(0)} = -0.375 \qquad \Delta Q_3^{(0)} = 0.13$$

$$\Delta P_4^{(0)} = -0.4 \qquad \Delta Q_4^{(0)} = 0.005$$

$$\Delta P_5^{(0)} = -0.6 \qquad \Delta Q_5^{(0)} = -0.06$$

$$\begin{vmatrix} \Delta P_2^{(0)} \\ \vdots \\ \Delta P_5^{(0)} \\ \Delta Q_2^{(0)} \\ \vdots \\ \Delta Q_5^{(0)} \end{vmatrix} = \begin{vmatrix} J_1 & J_2 \\ J_3 & J_4 \end{vmatrix} \cdot \begin{vmatrix} \Delta \delta_2^{(0)} \\ \vdots \\ \Delta \delta_5^{(0)} \\ \Delta U_2^{(0)} \\ \vdots \\ \Delta U_5^{(0)} \end{vmatrix}$$

Primjer: Newton-Raphson metoda

$$|J| = \left|\begin{array}{cccc|cccc}
33.4 & -5 & -5 & -7.5 & 10.533 & -1.66 & -1.66 & -2.5 \\
-5 & 38.975 & -30 & 0 & -1.66 & 12.84 & -10 & 0 \\
-5 & -30 & 38.75 & -3.75 & -1.66 & -10 & 12.916 & -1.25 \\
-7.5 & 0 & -3.75 & 11.25 & -2.5 & 0 & -1.25 & 3.75 \\
\hline
-11.338 & 1.66 & 1.66 & 2.5 & 31.43 & -5 & -5 & -7.5 \\
1.66 & -12.991 & 10 & 0 & -5 & 38.415 & -30 & 0 \\
1.66 & 10 & -12.916 & 1.25 & -5 & -30 & 38.64 & -3.75 \\
2.5 & 0 & 1.25 & -3.75 & -7.5 & 0 & -3.75 & 11.17
\end{array}\right|$$

Primjer: Newton-Raphson metoda

$$\begin{vmatrix} \Delta\delta_2^{(0)} \\ \vdots \\ \Delta\delta_5^{(0)} \\ \Delta U_2^{(0)} \\ \vdots \\ \Delta U_5^{(0)} \end{vmatrix} = \begin{vmatrix} -0.05068 \\ -0.0911 \\ -0.09733 \\ -0.11268 \\ 0.05494 \\ 0.03134 \\ 0.03091 \\ 0.026 \end{vmatrix}$$

$$\delta_2^{(1)} = \delta_2^{(0)} + \Delta\delta_2^{(0)} = -0.05068 = -2.904°$$

$$\delta_3^{(1)} = -0.0911 = -3.22°$$

$$\delta_4^{(1)} = -0.09733 = -5.577°$$

$$\delta_5^{(1)} = -0.11288 = -6.455°$$

$$U_2^{(1)} = U_2^{(0)} + \Delta U_2^{(0)} = 1.05449$$

$$U_3^{(1)} = 1.03134$$

$$U_4^{(1)} = 1.03091$$

$$U_5^{(1)} = 1.026$$

Primjer: Newton-Raphson metoda

$$P_{2izr}^{(1)} = U_2^{(1)} \sum_{j=1}^{5} U_j^{(1)} \cdot Y_{ij} \cdot cos\left(\delta_i^{(1)} - \delta_j^{(1)} - \Theta_{ij}\right)$$

$$\vdots$$

$$P_{5izr}^{(1)} = U_5^{(1)} \sum_{j=1}^{5} U_j^{(1)} \cdot Y_{ij} \cdot cos\left(\delta_i^{(1)} - \delta_j^{(1)} - \Theta_{ij}\right)$$

$$Q_{2izr}^{(1)} = U_2^{(1)} \sum_{j=1}^{5} U_j^{(1)} \cdot Y_{ij} \cdot sin\left(\delta_i^{(1)} - \delta_j^{(1)} - \Theta_{ij}\right)$$

$$\vdots$$

$$Q_{5izr}^{(1)} = U_5^{(1)} \sum_{j=1}^{5} U_j^{(1)} \cdot Y_{ij} \cdot sin\left(\delta_i^{(1)} - \delta_j^{(1)} - \Theta_{ij}\right)$$

$$\Delta P_2^{(1)} = P_{2zad} - P_{2izr}^{(1)}$$

$$\vdots$$

$$\Delta P_5^{(1)} = P_{5zad} - P_{5izr}^{(1)}$$

$$\vdots$$

$$\Delta Q_2^{(1)} = Q_{2zad} - Q_{2izr}^{(1)}$$

$$\vdots$$

$$\Delta Q_5^{(1)} = Q_{5zad} - Q_{5izr}^{(1)}$$

Newton-Raphson metoda

Neke varijante Newton-Raphson metode

Newton-Raphson u pravokutnim koordinatama

- Umjesto

$$\begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix} = \begin{vmatrix} \left(\dfrac{\partial P}{\partial \delta}\right) & \left(\dfrac{\partial P}{\partial U}\right) \\ \left(\dfrac{\partial Q}{\partial \delta}\right) & \left(\dfrac{\partial Q}{\partial U}\right) \end{vmatrix} \cdot \begin{vmatrix} \Delta \delta \\ \Delta U \end{vmatrix}$$

- Koristimo

$$\begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix} = \begin{vmatrix} \left(\dfrac{\partial P}{\partial e}\right) & \left(\dfrac{\partial P}{\partial f}\right) \\ \left(\dfrac{\partial Q}{\partial e}\right) & \left(\dfrac{\partial Q}{\partial f}\right) \end{vmatrix} \cdot \begin{vmatrix} \Delta e \\ \Delta f \end{vmatrix}$$

- Gdje je

$$\vec{U} = U \angle \delta = e + \mathrm{j}\, f$$

Newton-Raphson s korekcijom napona (1)

- Umjesto

$$\begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix} = \begin{vmatrix} \left(\dfrac{\partial P}{\partial \delta}\right) & \left(\dfrac{\partial P}{\partial U}\right) \\ \left(\dfrac{\partial Q}{\partial \delta}\right) & \left(\dfrac{\partial Q}{\partial U}\right) \end{vmatrix} \cdot \begin{vmatrix} \Delta \delta \\ \Delta U \end{vmatrix}$$

- Koristimo

$$\begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix} = \begin{vmatrix} \left(\dfrac{\partial P}{\partial \delta}\right) & \left(\dfrac{\partial P}{\partial U} \cdot U\right) \\ \left(\dfrac{\partial Q}{\partial \delta}\right) & \left(\dfrac{\partial Q}{\partial U} \cdot U\right) \end{vmatrix} \cdot \begin{vmatrix} \Delta \delta \\ \Delta U / U \end{vmatrix}$$

- Ovaj pristup se često koristi kod rješavanja NR metode kada se koristi potpuna J matrica, pogotovo kada se ne koristi [pu] metoda

Newton-Raphson s korekcijom napona (2)

- Tada jednadžbe izgledaju ovako

$$
\begin{bmatrix} \Delta P_1 \\ \vdots \\ \Delta P_{n-1} \\ \Delta Q_1 \\ \vdots \\ \Delta Q_{n-1-g} \end{bmatrix} = \begin{bmatrix}
\frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_{n-1}} & \frac{\partial P_1}{\partial U_1}\cdot U_1 & \cdots & \frac{\partial P_1}{\partial U_{n-1-g}}\cdot U_{n-1-g} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial P_{n-1}}{\partial \delta_1} & \cdots & \frac{\partial P_{n-1}}{\partial \delta_{n-1}} & \frac{\partial P_{n-1}}{\partial U_1}\cdot U_1 & \cdots & \frac{\partial P_{n-1}}{\partial U_{n-1-g}}\cdot U_{n-1-g} \\
\frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_{n-1}} & \frac{\partial Q_1}{\partial U_1}\cdot U_1 & \cdots & \frac{\partial Q_1}{\partial U_{n-1-g}}\cdot U_{n-1-g} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial Q_{n-1-g}}{\partial \delta_1} & \cdots & \frac{\partial Q_{n-1-g}}{\partial \delta_{n-1}} & \frac{\partial Q_{n-1-g}}{\partial U_1}\cdot U_1 & \cdots & \frac{\partial Q_{n-1-g}}{\partial U_{n-1-g}}\cdot U_{n-1-g}
\end{bmatrix} \cdot \begin{bmatrix} \Delta \delta_1 \\ \vdots \\ \Delta \delta_{n-1} \\ \Delta U_1 / U_1 \\ \vdots \\ \Delta U_{n-1-g} / U_{n-1-g} \end{bmatrix}
$$

„Brzi” Newton-Raphson postupak

- Jakobijeva matrica računa se samo u nultom koraku iteracije
- U svim daljnjim koracima, koristi se ista Jakobijeva matrica

Alternativno:

- Jakobijeva matrica računa se samo u prvih nekoliko koraka iteracije (npr. nultom i prvom)
- U svim daljnjim koracima, koristi se zadnja izračunata Jakobijeva matrica

Razdvojeni Newton-Raphson postupak

- Umjesto

$$\begin{vmatrix}\Delta P\\ \Delta Q\end{vmatrix} = \begin{vmatrix}\left(\dfrac{\partial P}{\partial \delta}\right) & \left(\dfrac{\partial P}{\partial U}\right)\\ \left(\dfrac{\partial Q}{\partial \delta}\right) & \left(\dfrac{\partial Q}{\partial U}\right)\end{vmatrix} \cdot \begin{vmatrix}\Delta \delta\\ \Delta U\end{vmatrix}$$

- Koristimo

$$\begin{vmatrix}\Delta P\\ \Delta Q\end{vmatrix} = \begin{vmatrix}\left(\dfrac{\partial P}{\partial \delta}\right) & (0)\\ (0) & \left(\dfrac{\partial Q}{\partial U}\right)\end{vmatrix} \cdot \begin{vmatrix}\Delta \delta\\ \Delta U\end{vmatrix}$$

- Pri čemu zanemarujemo $\mathbf{J_2}$ i $\mathbf{J_3}$
  i dobivamo dva potpuno odvojena sustava jednadžbi:

$$|\Delta\delta| = |\mathbf{J}_1|^{-1} \cdot |\Delta P|$$

$$|\Delta U| = |\mathbf{J}_4|^{-1} \cdot |\Delta Q|$$

Brzi razdvojeni Newton-Raphson postupak

- Kombinacija prethodna dva postupka
- Dakle računaju se dvije odvojene Jakobijeve matrice (J1 i J4) i to samo u prvom koraku iteracije

- Velik broj proračuna Newton-Raphson metodom radi se upravo brzim razdvojenim postupkom

Engleski:

- *Fast-decoupled Newton-Raphson Power-flow*

Pitanje…

- Koje su dimenzije Jakobijeve matrice za mrežu od 10 čvorišta od čega su elektrane (s regulacijom napona) u 5 čvorišta?

Pitanja i komentari

Analiza elektroenergetskog sustava
Predavanje 7: Proračun tokova snaga

Prof. dr. sc. Ivica Pavić

Izv. prof. dr. sc. Marko Delimar

Istosmjerni model tokova snaga

Istosmjerni model tokova snaga

- Pojednostavljena (aproksimativna) metoda
- Linearizacija problema tokova snaga
- Svrha: povećanje brzine proračuna (smanjenje točnosti)
- Primjena: dispečerski centri (veliki broj proračuna u kratkom vremenu)
- Istosmjerni model = proračun tokova djelatne snage
- Analize u kojima se koristi:

  - proračuni proširenog realnog vremena (PRV)
  - planiranje i praćenje razmjene djelatnih snaga između EES-a
  - procjena zagušenja mreže

Istosmjerni model tokova snaga

- Pretpostavke:
  - zanemarene poprečne grane
  - zanemarene konduktancije svih prijenosnih elemenata ($G_{ij}=0$)
  - razlike kuteva napona su male, tj. vrijedi: $\sin\left(\delta_i-\delta_j\right)\approx\delta_i-\delta_j$
  - iznosi napona čvorišta su približno isti (nazivne vrijednosti)

$$U_i \approx U_j \approx 1.0$$

$$\text{element } Y_{ij} = \left|Y_{ij}\right| / \ 90^\circ$$

$$\text{element } Y_{ii} = \left|Y_{ii}\right| / \ -90^\circ$$

$$P_i = \sum_{j=1}^{n} U_i \cdot U_j \cdot \left|Y_{ij}\right| \cdot \cos(-\Theta_{ij}+\delta_i-\delta_j)$$

$$Q_i = \sum_{j=1}^{n} U_i \cdot U_j \cdot \left|Y_{ij}\right| \cdot \sin(-\Theta_{ij}+\delta_i-\delta_j)$$

$$\cos\left[\left(\delta_i-\delta_j\right)-90^\circ\right]=\cos\left(\delta_i-\delta_j\right)\cdot\cos 90^\circ+\sin\left(\delta_i-\delta_j\right)\cdot\sin 90^\circ=\sin\left(\delta_i-\delta_j\right)$$

$$\sin\left[\left(\delta_i-\delta_j\right)-90^\circ\right]=\sin\left(\delta_i-\delta_j\right)\cdot\cos 90^\circ-\cos\left(\delta_i-\delta_j\right)\cdot\sin 90^\circ=-\cos\left(\delta_i-\delta_j\right)$$

Istosmjerni model tokova snaga

$$P_i = |Y_{ii}| \cdot \cos(90^\circ) + \sum_{\substack{j=1 \\ j \neq i}}^{n} |Y_{ij}| \cdot \sin(\delta_i - \delta_j) = \sum_{\substack{j=1 \\ j \neq i}}^{n} |Y_{ij}| \cdot \sin(\delta_i - \delta_j)$$

$$Q_i = |Y_{ii}| \cdot \sin(90^\circ) - \sum_{\substack{j=1 \\ j \neq i}}^{n} |Y_{ij}| \cdot \cos(\delta_i - \delta_j) = |Y_{ii}| - \sum_{\substack{j=1 \\ j \neq i}}^{n} |Y_{ij}| \cdot \cos(\delta_i - \delta_j) \approx 0$$

za male kuteve vrijedi: $\sin(\delta) \approx \delta$

Uvođenjem B_{ij} i B_{ii} umjesto $|Y_{ij}|$ i $|Y_{ii}|$ $\quad [Y] = -j[B]$

B_{ii} – dijagonalni element imaginarnog dijela matrice Y (pozitivna vrijednost)

B_{ij} – vandijagonalni element imaginarnog dijela matrice Y (negativna vrijednost)

$$P_i \approx -\sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} \cdot (\delta_i - \delta_j) = -\delta_i \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} + \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} \cdot \delta_j = \delta_i \cdot B_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} \cdot \delta_j = \sum_{j=1}^{n} B_{ij} \cdot \delta_j$$

$$P_i = \sum_{j=1}^{n-1} B_{ij} \cdot \delta_j \quad , \text{ za } \delta_n = \delta_{ref} = 0^\circ$$

Istosmjerni model tokova snaga

- Matrični zapis:

$$[P] = [B] \cdot [\Delta\delta]$$

$$[B] \text{ – matrica stupnja } (n-1)x(n-1)$$

$$[P] \text{ – vektor stupnja } (n-1)$$

$$[\Delta\delta] \text{ – vektor stupnja } (n-1)$$

$$P_i = P_g - P_l \quad , \quad P_i - \text{injekcija u čvorištu}$$

$$P_g - \text{proizvodnja u čvorištu}$$

$$P_l - \text{potrošnja u čvorištu}$$

- Rješenje: $[\Delta\delta] = [B]^{-1} \cdot [P]$

Istosmjerni model tokova snaga

- Zapis u pravokutnim koordinatama: $y_{ij} = g_{ij} - jb_{ij} = -jb_{ij} \ , \ g_{ij} = 0$

$$\begin{bmatrix} P_1 \\ \vdots \\ P_i \\ \vdots \\ P_{n-1} \end{bmatrix} = -j\left[B\right]\cdot\begin{bmatrix} \delta_1 \\ \vdots \\ \delta_i \\ \vdots \\ \delta_{n-1} \end{bmatrix} \quad , \quad B_{ii} = \sum_{j=1}^{n} b_{ij} \quad , \quad B_{ij} = -b_{ij}$$

$$\begin{bmatrix} \delta_1 \\ \vdots \\ \delta_i \\ \vdots \\ \delta_{n-1} \end{bmatrix} = j\left[B\right]^{-1}\cdot\begin{bmatrix} P_1 \\ \vdots \\ P_i \\ \vdots \\ P_{n-1} \end{bmatrix}$$

$$P_i = P_g - P_l \quad , \quad \begin{aligned} &P_i - \text{injekcija u čvorištu} \\ &P_g - \text{proizvodnja u čvorištu} \\ &P_l - \text{potrošnja u čvorištu} \end{aligned}$$

Istosmjerni model tokova snaga

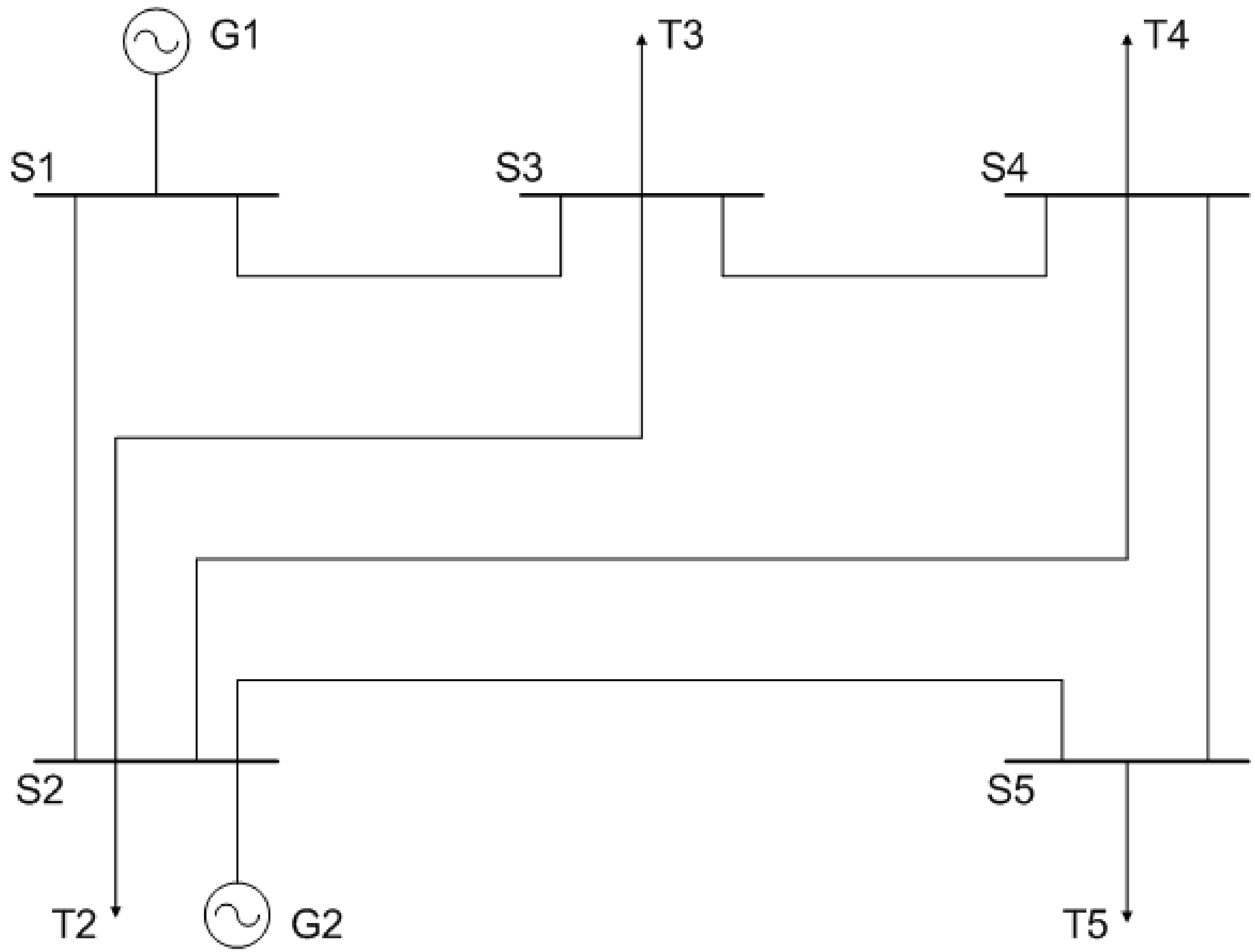
- Tokovi snaga u granama:

$$P_{i-j} = \left(\delta_i - \delta_j\right)\cdot\left(-jb_{ij}\right) = \frac{\delta_i - \delta_j}{jx_{i-j}}$$

$$P_{i-j} = -P_{j-i} \quad , \quad \text{nema gubitaka} \;-\; R_{ij} = 0$$

- Bilanca ostvarena u ref. čvorištu

Primjer: Istosmjerni model tokova snaga



Primjer: Istosmjerni model tokova snaga

- Zadano:

i	j	X	B
1	2	j0.06	-j16.667
1	3	j0.24	-j4.167
2	3	j0.18	-j5.556
2	4	j0.18	-j5.556
2	5	j0.12	-j8.333
3	4	j0.03	-j33.333
4	5	j0.24	-j4.167

Čv.	Pi (MW)	Pi (p.u.)
2.	20	0.2
3.	-45	-0.45
4.	-40	-0.4
5.	-60	-0.6

Primjer: Istosmjerni model tokova snaga

$$P_1 = -\sum_{i=2}^{5} P_i = -20 + 45 + 40 + 60 = 125\ MW$$

$$B = -j \left| \begin{array}{c|cccc} 20.833 & -16.667 & -4.167 & 0 & 0 \\ \hline -16.667 & 36.111 & -5.556 & -5.556 & -8.333 \\ -4.167 & -5.556 & 43.055 & -33.333 & 0 \\ 0 & -5.556 & -33.333 & 43.055 & -4.167 \\ 0 & -8.333 & 0 & -4.167 & 12.5 \end{array} \right|$$

Primjer: Istosmjerni model tokova snaga

$$B^{-1} = j\begin{vmatrix} 0.05057 & 0.03771 & 0.04029 & 0.04714 \\ 0.03771 & 0.08914 & 0.07886 & 0.05143 \\ 0.04029 & 0.07886 & 0.09514 & 0.05857 \\ 0.04714 & 0.05143 & 0.05857 & 0.13095 \end{vmatrix}$$

$$\begin{vmatrix} \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \end{vmatrix} = \left|B\right|^{-1} \cdot \begin{vmatrix} 0.2 \\ -0.45 \\ -0.4 \\ -0.6 \end{vmatrix} = j\begin{vmatrix} -0.05126 \\ -0.09497 \\ -0.10063 \\ -0.11571 \end{vmatrix}$$

Primjer: Istosmjerni model tokova snaga

$$P_{1-2} = \frac{j\left(\delta_1 - \delta_2\right)}{jX_{12}} = \frac{j\left[0-\left(-0.05126\right)\right]}{j0.06} = 0.85433 \quad \rightarrow \quad 85.4\ MW$$

$$P_{1-3} = \frac{j\left[0-\left(-0.09497\right)\right]}{j0.24} = 0.396 \quad \rightarrow \quad 39.6\ MW$$

$$P_{2-3} = \frac{j\left[-0.05126-\left(-0.09497\right)\right]}{j0.18} = 0.243 \quad \rightarrow \quad 24.3\ MW$$

$$P_{2-4} = 27.4\ MW$$

$$P_{2-5} = 53.7\ MW$$

$$P_{3-4} = 18.9\ MW$$

$$P_{4-5} = 6.3\ MW$$

Primjer: Istosmjerni model tokova snaga

- Isti primjer izračunat s Y_{ij}:

i	j	Z	Y
1	2	0.02+j0.06	5-j15
1	3	0.08+j0.24	1.25-j3.75
2	3	0.06+j0.18	1.66-j5
2	4	0.06+j0.18	1.66-j5
2	5	0.04+j0.12	2.5-j4.5
3	4	0.01+j0.03	10-j30
4	5	0.08+j0.24	1.25-j3.75

Čv.	Pi (MW)	Pi (p.u.)
2.	20	0.2
3.	-45	-0.45
4.	-40	-0.4
5.	-60	-0.6

Primjer: Istosmjerni model tokova snaga

$$Y = -j \cdot \begin{vmatrix} 19.76 & -15.81 & -3.95 & 0 & 0 \\ -15.81 & 34.25 & -5.27 & -5.27 & -7.9 \\ -3.95 & -5.27 & 40.84 & -31.62 & 0 \\ 0 & -5.27 & -31.62 & 40.84 & -3.95 \\ 0 & -7.9 & 0 & -3.95 & 11.85 \end{vmatrix}$$

$$Y^{-1} = j \cdot \begin{vmatrix} 0.05332 & 0.03977 & 0.04248 & 0.04970 \\ 0.03977 & 0.0940 & 0.08315 & 0.05423 \\ 0.04248 & 0.08315 & 0.10032 & 0.06176 \\ 0.04970 & 0.5423 & 0.06176 & 0.13811 \end{vmatrix}$$

Primjer: Istosmjerni model tokova snaga

$$\begin{vmatrix} \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \end{vmatrix} = |Y|^{-1} \cdot \begin{vmatrix} 0.2 \\ -0.45 \\ -0.4 \\ -0.6 \end{vmatrix} = j \cdot \begin{vmatrix} -0.054 \\ -0.10014 \\ -0.1061 \\ -0.122 \end{vmatrix}$$

$$P_{1-2} = (\delta_1 - \delta_2) \cdot y_{1-2} = j0.054 \cdot (-j15.81) = 0.85374 \rightarrow 85.4\ MW \qquad (\ 88.8\ MW\ )$$

$$P_{1-3} = (\delta_1 - \delta_3) \cdot y_{1-3} = j0.10014 \cdot (-j3.95) = 0.395553 \rightarrow 39.6\ MW \qquad (\ 40.7\ MW\ )$$

$$P_{2-3} = (\delta_2 - \delta_3) \cdot Y_{2-3} = j(-0.054 + 0.10014) \cdot (-j5.27) = 0.243 \rightarrow 24.3\ MW \qquad (\ 24.7\ MW\ )$$

$$P_{2-4} = (\delta_2 - \delta_4) \cdot Y_{2-4} = j(-0.054 + 0.10061) \cdot (-j5.27) = 0.275 \rightarrow 27.5\ MW \qquad (\ 27.9\ MW\ )$$

$$P_{2-5} = 53.7\ MW \qquad (\ 54.8\ MW\ )$$

$$P_{3-4} = 18.9\ MW \qquad (\ 18.9\ MW\ )$$

$$P_{4-5} = 6.3\ MW \qquad (\ 8.3\ MW)$$

Napomena : crvenom bojom su označeni rezultati dobiveni metodom Gauss-Seidel pomoću Y matrice

Pitanja i komentari

Analiza elektroenergetskog sustava Predavanje 8: Proračun kratkog spoja

Prof. dr. sc. Ivica Pavić

Izv. prof. dr. sc. Marko Delimar

Kratki spoj

Problematika kratkog spoja

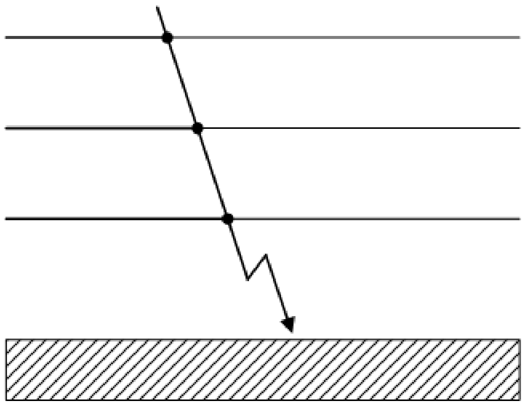
- Dvije vrste kvarova:
  - Uzdužni kvarovi – prekid vodiča
  - Poprečni kvarovi – proboj izolacije (kratki spojevi)
- Uzroci kratkih spojeva:
  - Slom izolacije:
    a) Zbog povećanja električnog naprezanja
    b) Zbog smanjenja dielektrične čvrstoće izolacije
    c) Zbog kombinacije uzroka pod a) i b)

Problematika kratkog spoja

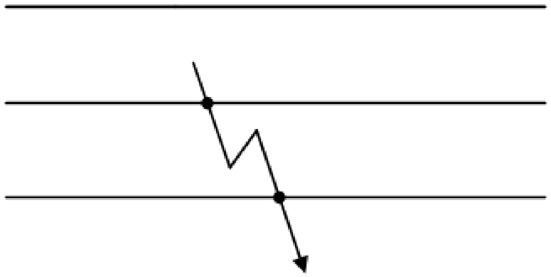
- Električna naprezanja izolacije:
  - Pogonski napon
  - Povišenje napona (Ferantijev efekt, zemljospoj)
  - Unutrašnji prenaponi (sklapanje, ferorezonancija)
  - Atmosferski prenaponi
  - Utjecaj mreže višeg napona (najgori: dodir)

Problematika kratkog spoja

- Trofazni kratki spoj:

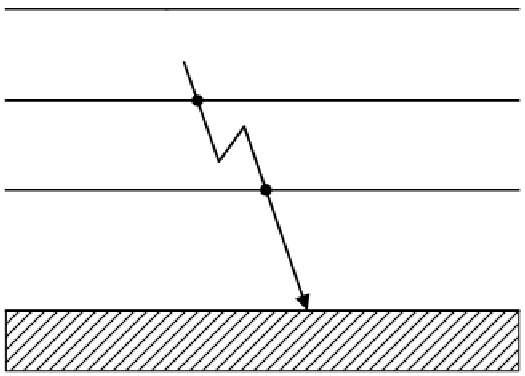


- Dvofazni kratki spoj:

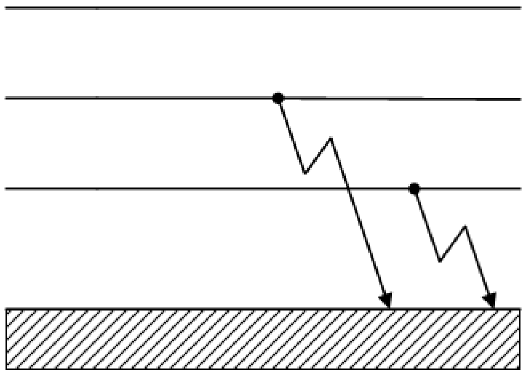


Problematika kratkog spoja

- Dvofazni kratki spoj sa zemljom:

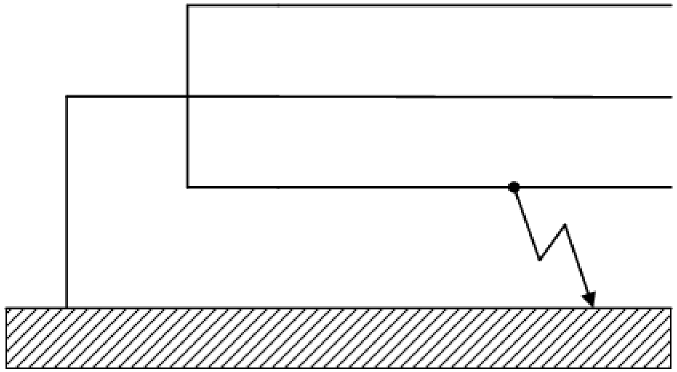


- Dvostruki zemljospoj neuzemljene mreže:

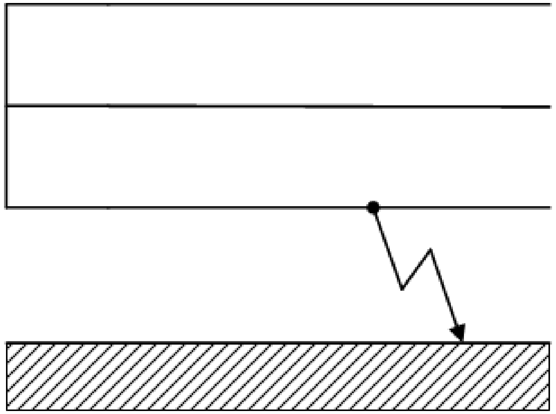


Problematika kratkog spoja

- Jednofazni kratki spoj:



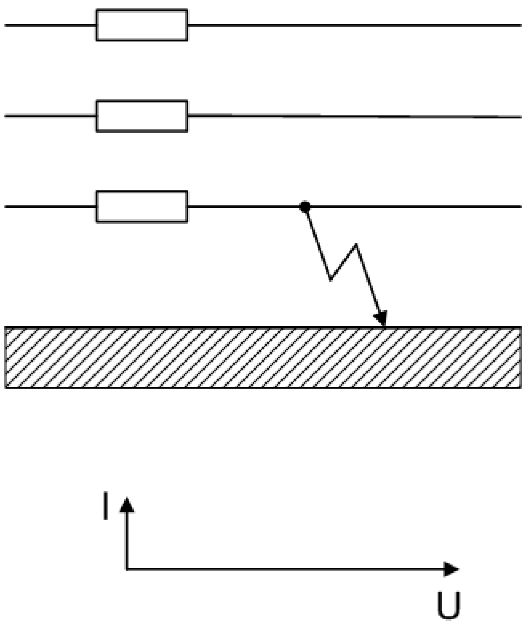
- Zemljospoj:



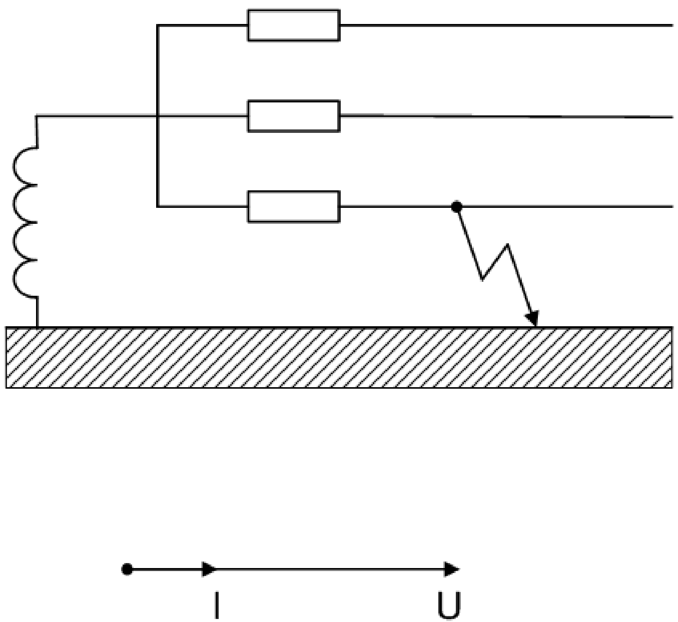
Problematika kratkog spoja

- Vektorska slika (kvalitativna):

Zemljospoj (izolirana mreža)



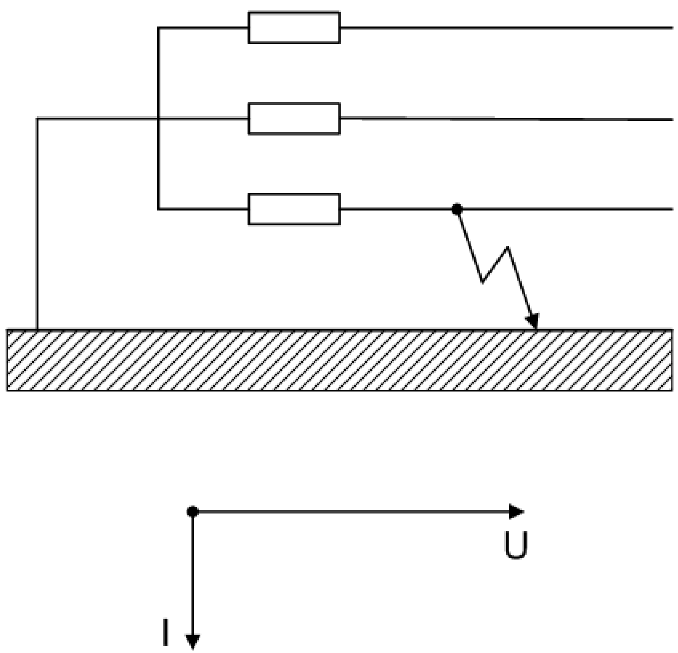
Zemljospoj (mreža uzemljena preko rezonantne prigušnice)



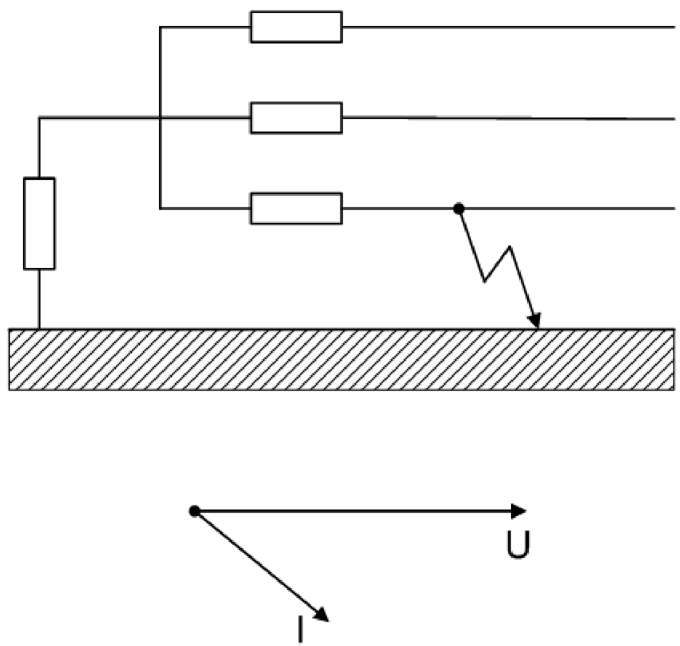
Problematika kratkog spoja

- Vektorska slika (kvalitativna):

Mreža uzemljena direktno



Mreža uzemljena preko otpora



Klasični proračun kratkog spoja

- Simetrične komponente:

$$^{R}U_i = {}^{0}U_i + {}^{d}U_i + {}^{i}U_i$$

$$^{S}U_i = {}^{0}U_i + {}^{d}U_i \cdot a^2 + {}^{i}U_i \cdot a$$

$$^{T}U_i = {}^{0}U_i + {}^{d}U_i \cdot a + {}^{i}U_i \cdot a^2$$

$$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$\begin{vmatrix} ^{R}U_i \\ ^{S}U_i \\ ^{T}U_i \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} ^{0}U_i \\ ^{d}U_i \\ ^{i}U_i \end{vmatrix}$$

$$\begin{vmatrix} ^{0}U_i \\ ^{d}U_i \\ ^{i}U_i \end{vmatrix} = \frac{1}{3} \cdot \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \cdot \begin{vmatrix} ^{R}U_i \\ ^{S}U_i \\ ^{T}U_i \end{vmatrix}$$

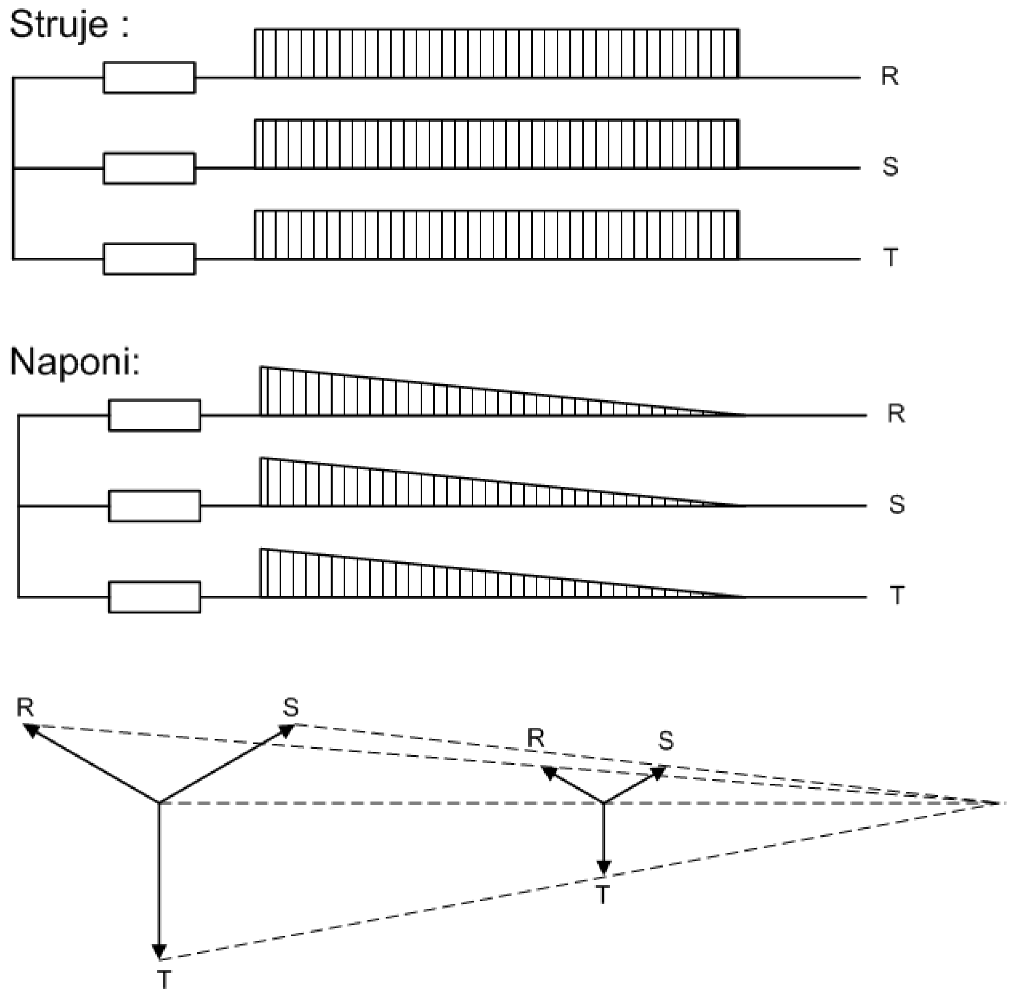
Problematika kratkog spoja

- Vektorske slike kratkog spoja:
  - Trofazni kratki spoj:

Prilike na mjestu kvara:

$$\begin{bmatrix} \overline{I}_R \\ \overline{I}_S \\ \overline{I}_T \end{bmatrix} = \frac{\overline{E}_1}{\overline{Z}_1} \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$$

$$\begin{bmatrix} \overline{V}_R \\ \overline{V}_S \\ \overline{V}_T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



Problematika kratkog spoja

- Vektorske slike kratkog spoja:
  - Dvofazni kratki spoj:

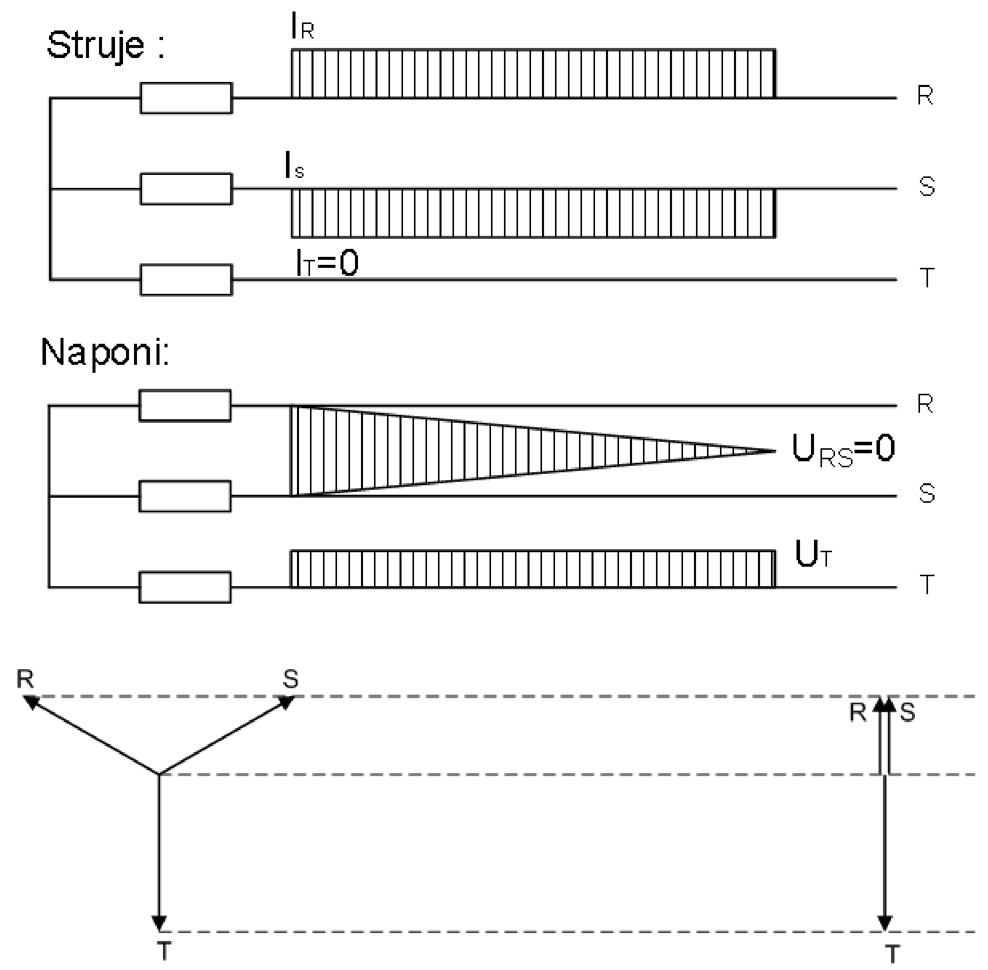
Prilike na mjestu kvara:

$$\begin{bmatrix} \overline{I}_R \\ \overline{I}_S \\ \overline{I}_T \end{bmatrix} = \frac{\overline{E}_1}{\overline{Z}_1 + \overline{Z}_2} \begin{bmatrix} a^2 - a \\ a - a^2 \\ 0 \end{bmatrix}$$

$$a^2 - a = -j\sqrt{3}$$

$$\begin{bmatrix} \overline{V_R} \\ \overline{V_S} \\ \overline{V_T} \end{bmatrix} = \frac{\overline{E_1}}{\overline{\overline{Z_1} + \overline{Z_2}}} \begin{bmatrix} (a^2 + a)\overline{Z_2} \\ (a + a^2)\overline{Z_2} \\ 2\overline{Z_2} \end{bmatrix}$$

$$a^2 + a = -1$$



Problematika kratkog spoja

- Vektorske slike kratkog spoja:
  - Jednofazni kratki spoj:

Prilike na mjestu kvara:

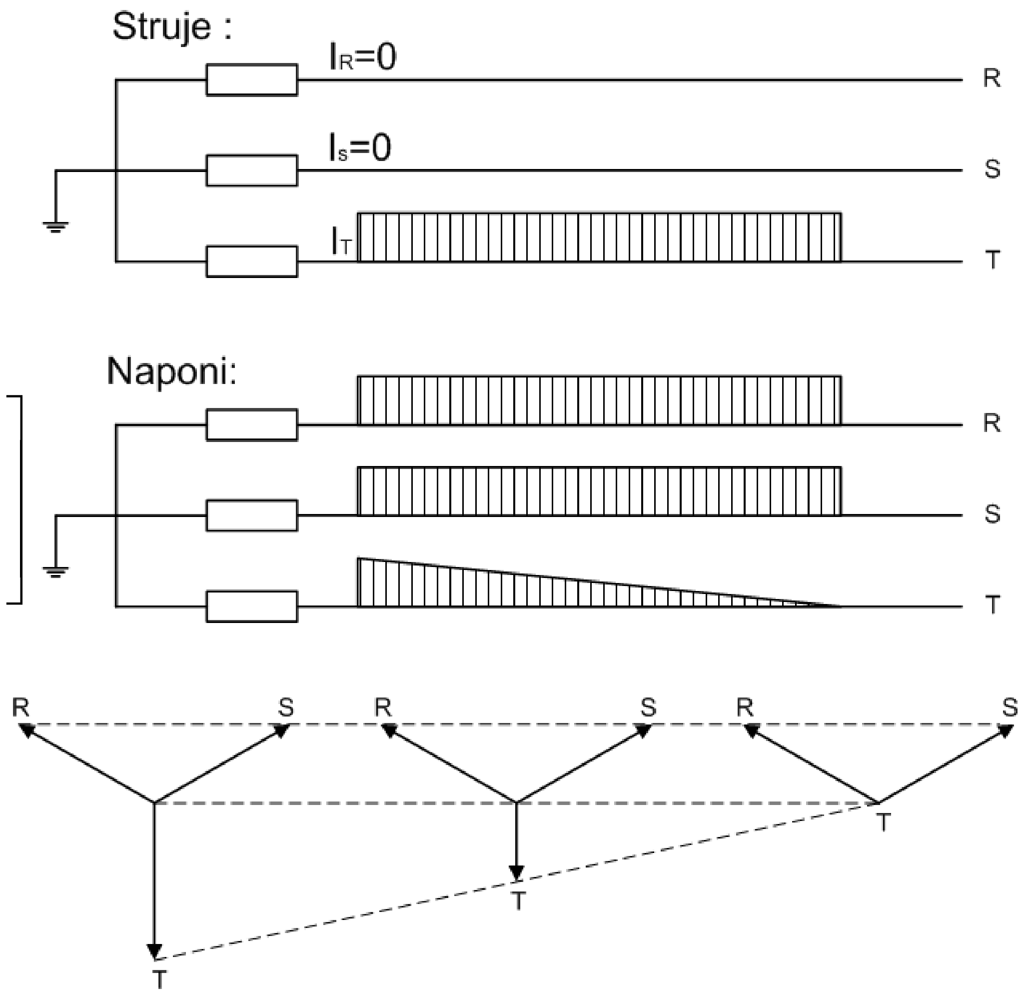
$$\begin{bmatrix} \overline{I}_R \\ \overline{I}_S \\ \overline{I}_T \end{bmatrix} = \frac{3\overline{E}_1}{\overline{Z}_1 + \overline{Z}_2 + \overline{Z}_0} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \overline{V}_R \\ \overline{V}_S \\ \overline{V}_T \end{bmatrix} = \frac{\overline{E}_1}{\overline{Z}_1 + \overline{Z}_2 + \overline{Z}_0} \begin{bmatrix} (a^2 - a)\overline{Z}_2 + (a^2 - 1)\overline{Z}_0 \\ (a - a^2)\overline{Z}_2 + (a - 1)\overline{Z}_0 \\ 0 \end{bmatrix}$$

$$a^2 - a = -j\sqrt{3}$$

$$a^2 - 1 = -1{,}5 - j\frac{\sqrt{3}}{2}$$

$$a - 1 = -1{,}5 + j\frac{\sqrt{3}}{2}$$



Problematika kratkog spoja

- Primjena

  - dimenzioniranje opreme
  - podešenje relejne zaštite
  - dimenzioniranje uzemljivača
  - određivanje elektromagnetskog utjecaja na ostale objekte
  - itd.

- Relejna zaštita:
  - Uređaji koji automatski upravljaju iskapčanjem u mreži ako nastane kratki spoj.
  - Zahtjevi:
    1. Osjetljivost
    2. Kritičnost
    3. Brzina
    4. Selektivnost
    5. Pouzdanost (rezerva)

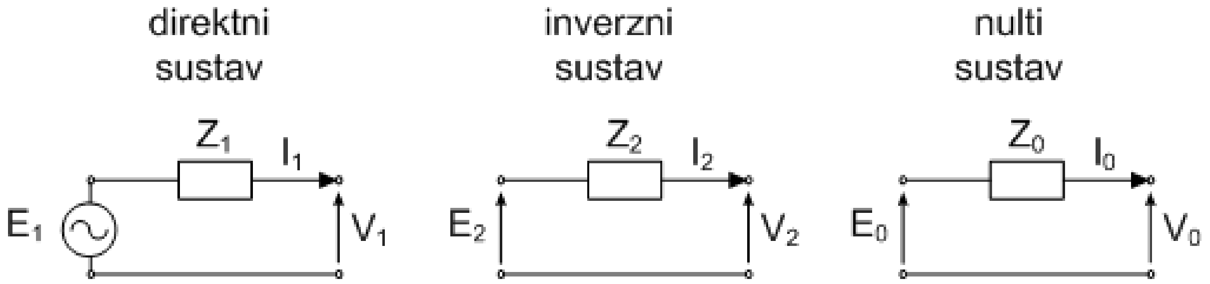
Problematika kratkog spoja

- Generacije uređaja za zaštitu od kratkog spoja:

Generacija	Uređaj	Način mjerenja	Vremensko podešavanje	Kriterij za djelovanje
1	Osigurači	Primarno	Donekle	Nadstrujno
2	Okidači	Primarno	Grubo	Nadstrujno ili podnaponski
3	Elektromehanički releji	Sekundarno	Dobro	Složen
4	Statički releji	Sekundarno	Fino	Složen
5	Numerička zaštita	Sekundarno (daljinsko)	Egzaktno (po potrebi)	Korisnički definiran

Klasični proračun kratkog spoja

- Primjena metode simetričnih komponenti
- Jednofazni model
- Redukcija mreže na mjesto kvara (direktni, inverzni, nulti sustav)
  a) nadomjesna impedancija na mjestu kratkog spoja – ukupna impedancija cijelog sustava gledana s mjesta kvara (primjena pravila za serijski i paralelni spoj, transfiguraciju zvijezda- trokut i dr.)
  b) određivanje struje na mjestu kvara (ovisno o vrsti)
  c) određivanje napona u čvorištima i struja u granama mreže (povratak na stvarnu mrežu)
  d) zakret napona i struja na transformatorima (ovisno o grupi spoja)



Matrični proračun kratkog spoja

- Klasični proračun – mreža u praznom hodu (dobro za dimenzioniranje opreme)
- Matrični proračun – općenitiji (prazni hod ili trenutno stanje, tj. stanje prije nastanka kratkog spoja – stanje u **zdravoj mreži**)
- Dodatni podaci za proračun (generator, teret)
- Model generatora:

$$E_i^{''} = U_i + I_i \cdot jX_{d,i}^{''} = U_i + \frac{S_i^*}{U_i^*} \cdot jX_{d,i}^{''}$$

$E_i^{''}$ – unutarnja elektromotorna sila generatora u čvorištu i (početna vrijednost)

S_i – snaga generatora u čvorištu i određena proračunom tokova snaga

- Model tereta:

$$I_i = \frac{S_i^*}{U_i^*} = U_i \cdot y_i \;\Rightarrow\; y_i = \frac{S_i^*}{\left|U_i\right|^2} \;,\; y_i \text{ – admitancija tereta u čvorištu } i$$

Matrični proračun kratkog spoja

a) <u>Stanje prije nastanka kratkog spoja – mreža u praznom hodu</u>

- Referentno čvorište: nul-točka sustava (zemlja)
- Primjena teorema superpozicije (nazivni naponi u mreži prije kvara, struja kratkog spoja za mrežu u praznom hodu u čvorištu *m*)

$$\begin{bmatrix} U_1 \\ \vdots \\ 0 \\ \vdots \\ U_n \end{bmatrix}^{kv} = \begin{bmatrix} U_1 \\ \vdots \\ U_m \\ \vdots \\ U_n \end{bmatrix}^{zdr} + \begin{bmatrix} Z_{11} & \cdots & Z_{1n} \\ \vdots & \ddots & \vdots \\ Z_{n1} & \cdots & Z_{nn} \end{bmatrix}_{KS} \cdot \begin{bmatrix} 0 \\ \vdots \\ I_m \\ \vdots \\ 0 \end{bmatrix}^{kv} , \quad \begin{bmatrix} U_1 \\ \vdots \\ U_m \\ \vdots \\ U_n \end{bmatrix}^{zdr} = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{p.u.}$$

- Sve struje su jednake nuli (0), osim struje na mjestu kvara
- Matrica impedancije čvorišta dobivena inverzijom matrice admitancije čvorišta u koju su ušle sve admitancije (vodovi, generatori, transformatori, tereti)
- Admitancije generatora i transformatora dodane u dijagonalu matrice admitancije čvorišta

Matrični proračun kratkog spoja

b) Stanje prije nastanka kratkog spoja – opterećena mreža

- Primjena teorema superpozicije (struje opterećene mreže prije kratkog spoja + struje kratkog spoja za mrežu u praznom hodu)
- Stanje "zdrave mreže" – određeno tokovima snaga

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_{n-1} \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1,n-1} \\ \vdots & \ddots & \vdots \\ Y_{n-1,1} & \cdots & Y_{n-1,n-1} \end{bmatrix}_{TS} \cdot \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_{n-1} \end{bmatrix} \Rightarrow \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_{n-1} \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1,n-1} \\ \vdots & \ddots & \vdots \\ Z_{n-1,1} & \cdots & Z_{n-1,n-1} \end{bmatrix}_{TS} \cdot \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_{n-1} \end{bmatrix}$$

- Matrice $[Y]_{KS}$ i $[Z]_{KS}$ razlikuju se od matrica $[Y]_{TS}$ i $[Z]_{TS}$
- Za primjenu superpozije treba koristiti istu matricu [Y] za tokove snaga i kratki spoj (vektor struja za tokove snaga sadrži samo struje generatora – idealni strujni izvori, struje opterećenja su uzete u obzir preko admitancija dodanih u dijagonalne članove matrice [Y])

Matrični proračun kratkog spoja

b) Stanje prije nastanka kratkog spoja – opterećena mreža

- Superpozicija

Kvar u čvorištu *m*

$$\begin{bmatrix} U_1 \\ \vdots \\ 0 \\ \vdots \\ U_n \end{bmatrix}^{kv} = [Z]_{KS} \cdot \left\{ \begin{bmatrix} I_1 \\ \vdots \\ 0 \\ \vdots \\ I_n \end{bmatrix}^{zdr} + \begin{bmatrix} 0 \\ \vdots \\ I_m \\ \vdots \\ 0 \end{bmatrix}^{kv} \right\} = \begin{bmatrix} U_1 \\ \vdots \\ U_m \\ \vdots \\ U_n \end{bmatrix}^{zdr} + [Z]_{KS} \cdot \begin{bmatrix} 0 \\ \vdots \\ I_m \\ \vdots \\ 0 \end{bmatrix}^{kv}$$

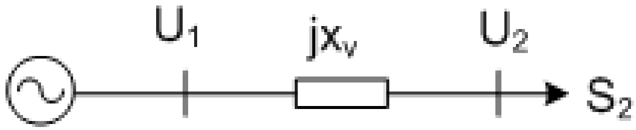
struje $I_1, I_2, \ldots, I_n = 0$, osim za generatorska čvorišta

$$I_i = \frac{E_i^{''}}{x_{d,i}^{''}} = E_i^{''} \cdot y_{d,i}^{''} \quad , \quad i - \text{generatorsko čvorište}$$

Matrični proračun kratkog spoja

b) Stanje prije nastanka kratkog spoja – opterećena mreža

- Primjer



- Zadano:

U_2=1+j0 p.u. , S_2=1 p.u. , jX_v = j1 p.u. , jX_d"= j0,2 p.u.

- Tokovi snaga (struja)

$$I_2 = \frac{S_2^*}{U_2^*} = \frac{-1}{1} = -1$$

$$\begin{bmatrix} I_1 \\ -1 \end{bmatrix} = \begin{bmatrix} -j1 & j1 \\ j1 & -j1 \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ 1 \end{bmatrix} \Rightarrow -1 = j1 \cdot U_1 - j1 \cdot 1 \Rightarrow U_1 = \frac{-1+j1}{j1} = 1 + j1$$

$$I_1 = -j1 \cdot (1 + j1) + j1 \cdot 1 = 1$$

Matrični proračun kratkog spoja

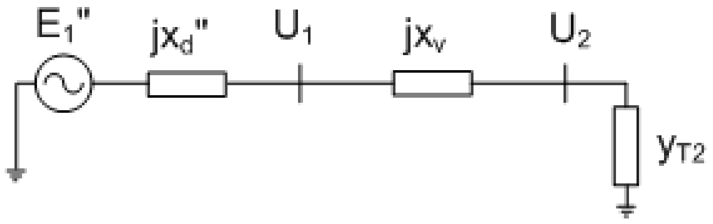
b) Stanje prije nastanka kratkog spoja – opterećena mreža

- Naponi zdrave mreže (tereti predočeni admitancijama)

$$I_2 = \frac{S_2^*}{U_2^*} = \frac{-1}{1} = -1$$

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}^{zdr} = [Z]_{KS} \cdot \begin{bmatrix} E_1'' \cdot y_{d1} \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} E_1'' \cdot y_{d1}'' \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_{KS} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}^{zdr}$$

$$y_{d1}'' = -j5 \ , \ \ y_{T2} = \frac{S_2^*}{\left|U_2\right|^2} = 1$$



$$E_1'' = U_1 + I_1 \cdot jx_{d1}'' = 1 + j1{,}2 \ \ , \ \ E_1'' \cdot y_{d1}'' = 6 - j5$$

$$\begin{bmatrix} 1 + j1 \\ 1 \end{bmatrix}^{zdr} = \begin{bmatrix} -j1 - j5 & j1 \\ j1 & -j1 + 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 6 - j5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 - j5 \\ 0 \end{bmatrix} = \begin{bmatrix} -j1 - j5 & j1 \\ j1 & -j1 + 1 \end{bmatrix} \cdot \begin{bmatrix} 1 + j1 \\ 1 \end{bmatrix}^{zdr}$$

$$-j6 \cdot (1 + j1) + j1 \cdot 1 = 6 - j5$$

$$j1 \cdot (1 + j1) + (-j1 + 1) \cdot 1 = 0$$

Matrični proračun kratkog spoja

b) Stanje prije nastanka kratkog spoja – opterećena mreža

- Određivanje struje kratkog spoja

$$\begin{bmatrix} U_1 \\ \vdots \\ 0 \\ \vdots \\ U_n \end{bmatrix}^{kv} = \begin{bmatrix} U_1 \\ \vdots \\ U_m \\ \vdots \\ U_n \end{bmatrix}^{zdr} + [Z]_{KS} \cdot \begin{bmatrix} 0 \\ \vdots \\ I_m \\ \vdots \\ 0 \end{bmatrix}^{kv}$$

$$U_m = 0$$

$$0 = U_m^{zdr} + Z_{mm} \cdot I_m^{kv}$$

$$I_m^{kv} = -\frac{U_m^{zdr}}{Z_{mm}}$$

$$I_{KS} = -I_m^{kv}$$

Pitanja i komentari

Analiza elektroenergetskog sustava
Predavanje 9: Proračun kratkog spoja

Prof. dr. sc. Ivica Pavić

Izv. prof. dr. sc. Marko Delimar

Kratki spoj

Problematika kratkog spoja

- Model električnih prilika u bolesnoj mreži:

$$\begin{vmatrix} U_1 \\ \vdots \\ U_m \\ \vdots \\ U_n \end{vmatrix}^B = |Z| \cdot \left\{ \begin{vmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{vmatrix} + \begin{vmatrix} 0 \\ \vdots \\ I_m \\ \vdots \\ 0 \end{vmatrix} \right\} \qquad I_k = -I_m$$

$$\begin{vmatrix} {}^dU_1 \\ \vdots \\ {}^dU_m \\ \vdots \\ {}^dU_n \end{vmatrix}^B = \begin{vmatrix} U_1 \\ \vdots \\ U_m \\ \vdots \\ U_n \end{vmatrix}^Z + \begin{vmatrix} {}^dZ_{1m} \cdot I_m \\ \vdots \\ {}^dZ_{mm} \cdot I_m \\ \vdots \\ {}^dZ_{nm} \cdot I_m \end{vmatrix}$$

Problematika kratkog spoja

- Ako postoji nesimetrični kratki spoj:

$$
{}^{d}U = \begin{vmatrix} {}^{d}U_1 \\ \vdots \\ {}^{d}U_m \\ \vdots \\ {}^{d}U_n \end{vmatrix} = \begin{vmatrix} {}^{d}Z_{11} & \cdots & {}^{d}Z_{1n} \\ \vdots & \ddots & \vdots \\ {}^{d}Z_{n1} & \cdots & {}^{d}Z_{nn} \end{vmatrix} \cdot \left\{ \begin{vmatrix} {}^{d}I_1 \\ {}^{d}I_2 \\ \vdots \\ {}^{d}I_n \end{vmatrix} + \begin{vmatrix} 0 \\ \vdots \\ {}^{d}I_m \\ \vdots \\ 0 \end{vmatrix} \right\}
$$

$$
{}^{i}U = \begin{vmatrix} {}^{i}U_1 \\ \vdots \\ {}^{i}U_m \\ \vdots \\ {}^{i}U_n \end{vmatrix} = \begin{vmatrix} {}^{i}Z_{11} & \cdots & {}^{i}Z_{1n} \\ \vdots & \ddots & \vdots \\ {}^{i}Z_{n1} & \cdots & {}^{i}Z_{nn} \end{vmatrix} \cdot \begin{vmatrix} 0 \\ 0 \\ \vdots \\ {}^{i}I_m \\ \vdots \\ 0 \end{vmatrix}
\qquad
{}^{0}U = \begin{vmatrix} {}^{0}U_1 \\ \vdots \\ {}^{0}U_m \\ \vdots \\ {}^{0}U_n \end{vmatrix} = \begin{vmatrix} {}^{0}Z_{11} & \cdots & {}^{0}Z_{1n} \\ \vdots & \ddots & \vdots \\ {}^{0}Z_{n1} & \cdots & {}^{0}Z_{nn} \end{vmatrix} \cdot \begin{vmatrix} 0 \\ 0 \\ \vdots \\ {}^{0}I_m \\ \vdots \\ 0 \end{vmatrix}
$$

Trofazni kratki spoj

$$^{d}U_m = 0$$

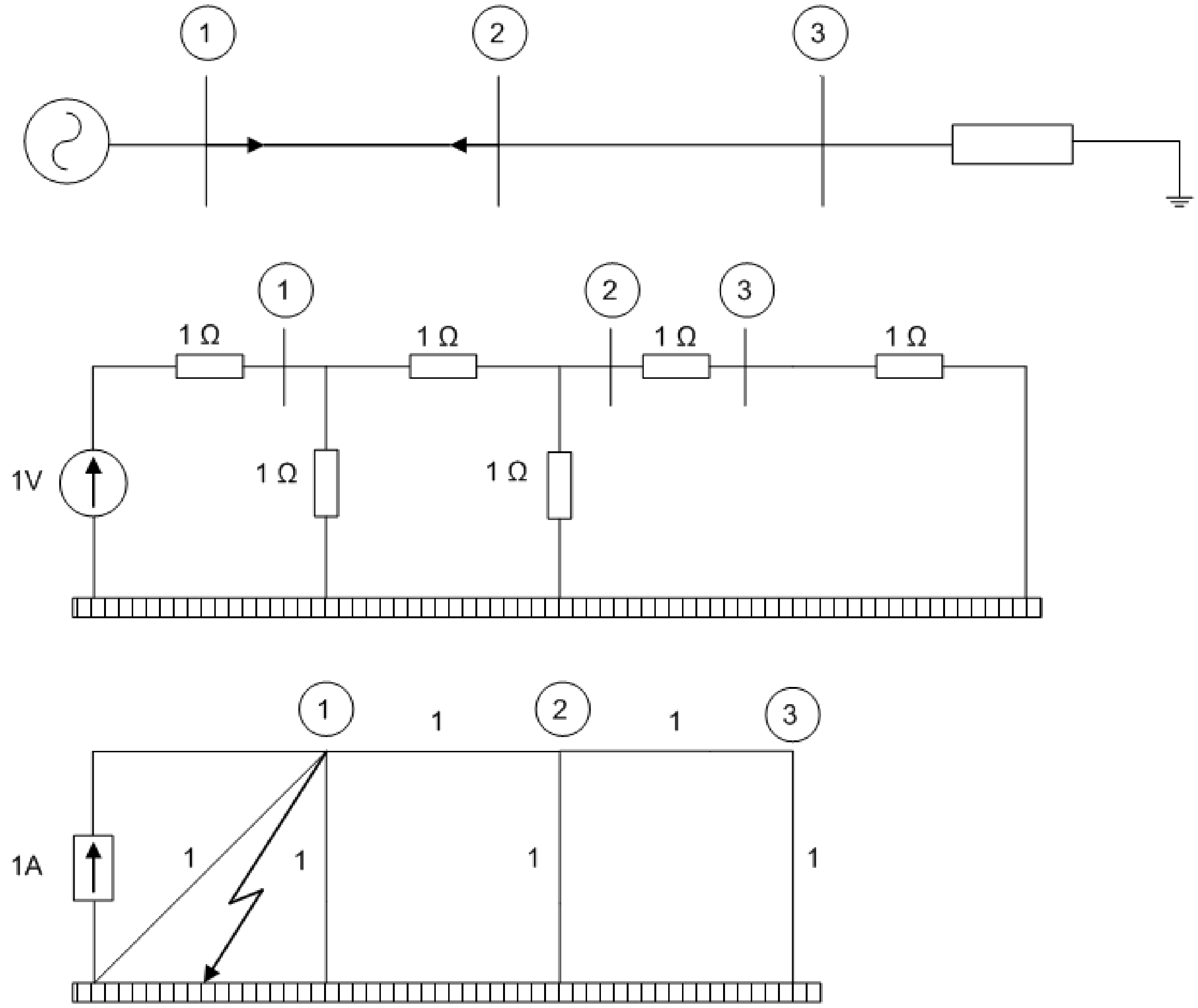
$$0 = U_m^z + {}^{d}Z_{mm} \cdot I_m$$

$$I_m = -\frac{U_m^z}{^{d}Z_{mm}}$$

$$\begin{vmatrix} ^{d}U_1 \\ \vdots \\ ^{d}U_m \\ \vdots \\ ^{d}U_n \end{vmatrix}^B = \begin{vmatrix} U_1^z - \frac{^{d}Z_{1m}}{^{d}Z_{mm}} \cdot U_m^Z \\ \vdots \\ 0 \\ \vdots \\ U_n^z - \frac{^{d}Z_{nm}}{^{d}Z_{mm}} \cdot U_m^Z \end{vmatrix}$$

Trofazni kratki spoj

- Primjer:



Trofazni kratki spoj

- Primjer:

$$Y = \begin{vmatrix} 3 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{vmatrix}$$

$$Y^{(1)} = \begin{vmatrix} 3 & -1 & 0 \\ -1 & 3-\frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \begin{vmatrix} 3 & -1 & 0 \\ -1 & \frac{5}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{vmatrix}$$

$$Y^{(2)} = \begin{vmatrix} \frac{1}{3} & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{5}{2}-\frac{1}{3} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \begin{vmatrix} \frac{1}{3} & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{13}{6} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{vmatrix}$$

$$Y^{(3)} = \begin{vmatrix} 0.385 & 0.154 & 0.077 \\ 0.154 & 0.461 & 0.23 \\ 0.077 & 0.23 & 0.615 \end{vmatrix} = Z$$

Trofazni kratki spoj

$$\begin{vmatrix} {}^dU_1 \\ {}^dU_2 \\ {}^dU_3 \end{vmatrix}^B = |Z| \cdot \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} + |Z| \cdot \begin{vmatrix} I_1 \\ 0 \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} {}^dU_1 \\ {}^dU_2 \\ {}^dU_3 \end{vmatrix}^B = \begin{vmatrix} 0.385 \\ 0.154 \\ 0.077 \end{vmatrix} + \begin{vmatrix} 0.385 \cdot {}^dI_1 \\ 0.154 \cdot {}^dI_1 \\ 0.077 \cdot {}^dI_1 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

$${}^dU_1 = 0.385 + 0.385 \cdot {}^dI_1 = 0$$

$${}^dI_1 = -1$$

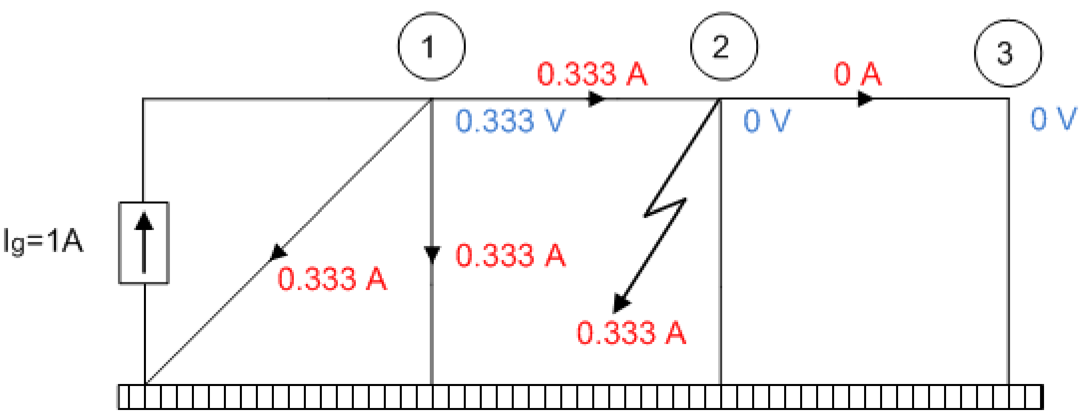
Struje generatora: 1A; Svi naponi: 0.

Trofazni kratki spoj

$$ {}^{d}U^{B} = 0 = 0.154 \cdot 1 + 0.461 \cdot {}^{d}I_2 $$

$$ {}^{d}I_2 = -\frac{0.154}{0.461}\,A = -0.334\,A $$

$$ \begin{vmatrix} {}^{d}U_1 \\ {}^{d}U_2 \\ {}^{d}U_3 \end{vmatrix}^{B} = \begin{vmatrix} 0.385 \\ 0.154 \\ 0.077 \end{vmatrix} - |Z| \cdot \begin{vmatrix} 0 \\ 0.334 \\ 0 \end{vmatrix} = \begin{vmatrix} 0.333 \\ 0 \\ 0 \end{vmatrix} $$



Izvor daje:
1 A-0.333 A= 0.667 A

Trofazni kratki spoj

$$\begin{vmatrix} {}^{d}U_1 \\ {}^{d}U_2 \\ {}^{d}U_3 \end{vmatrix}^B = \begin{vmatrix} {}^{d}U_1 \\ {}^{d}U_2 \\ {}^{d}U_3 \end{vmatrix}^Z + \left|Z\right| \cdot \begin{vmatrix} 0 \\ 0 \\ I_3 \end{vmatrix}$$

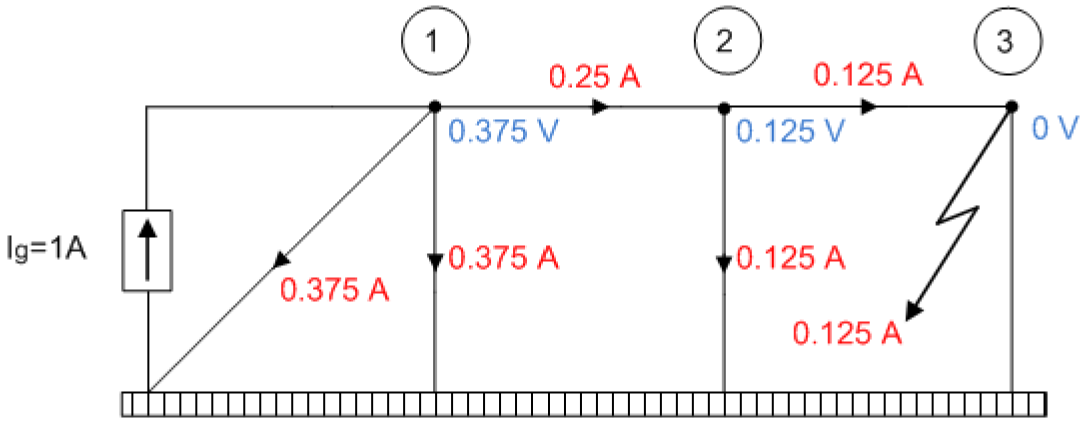
$$\begin{vmatrix} {}^{d}U_1 \\ {}^{d}U_2 \\ {}^{d}U_3 \end{vmatrix}^B = \begin{vmatrix} 0.385 \\ 0.154 \\ 0.077 \end{vmatrix} + \begin{vmatrix} Z_{13} \cdot {}^{d}I_3 \\ Z_{23} \cdot {}^{d}I_3 \\ Z_{33} \cdot {}^{d}I_3 \end{vmatrix}$$

$${}^{d}U_3^B = 0 = 0.077 + Z_{33} \cdot {}^{d}I_3$$

$${}^{d}I_3 = -\frac{0.077}{0.615}\ A = -0.125\ A$$

Trofazni kratki spoj

$$\begin{vmatrix} {}^{d}U_1 \\ {}^{d}U_2 \\ {}^{d}U_3 \end{vmatrix}^{B} = \begin{vmatrix} 0.385 \\ 0.154 \\ 0.077 \end{vmatrix} + \begin{vmatrix} 0.077\cdot(-0.125) \\ 0.23\cdot(-0.125) \\ 0.615\cdot(-0.125) \end{vmatrix} = \begin{vmatrix} 0.375 \\ 0.125 \\ 0 \end{vmatrix}$$



Jednofazni kratki spoj

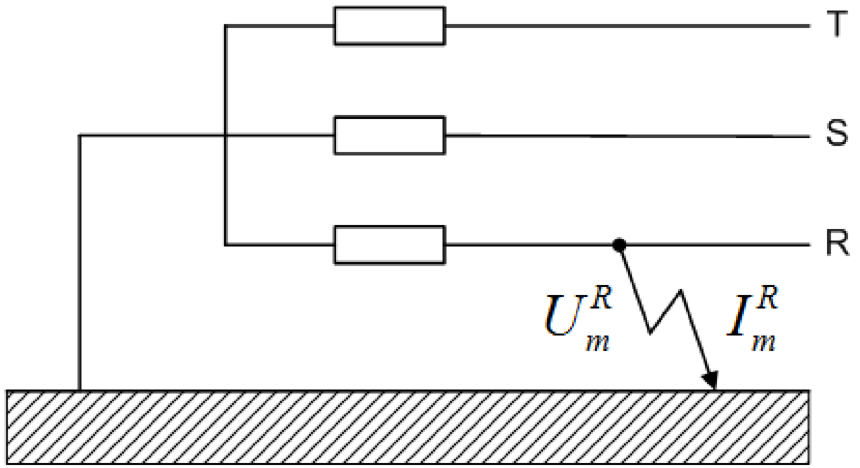
- U bolesnom čvorištu treba postaviti jednadžbe simetričnih komponenata:

$$ {}^{R}U_m = 0 $$

$$ {}^{R}I_m = I_m^d + I_m^i + I_m^0 $$

$$ {}^{T}I_m = {}^{S}I_m = 0 $$

$$ \begin{vmatrix} {}^{R}U_m \\ {}^{S}U_m \\ {}^{T}U_m \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a^2 & a^2 \end{vmatrix} \cdot \begin{vmatrix} U_m^0 \\ U_m^d \\ U_m^i \end{vmatrix} $$



Jednofazni kratki spoj

$$U_m^0 + U_m^d + U_m^i = 0$$
$${}^S I_m = 0$$
$${}^T I_m = 0$$

$\longrightarrow$

$$I_m^0 + a^2 \cdot I_m^d + a \cdot I_m^i = 0$$
$$I_m^0 + a \cdot I_m^d + a^2 \cdot I_m^i = 0$$
$$I_m^0 = I_m^d = I_m^i$$

$$U_m^d = \sum_{j=1}^{n} I_j \cdot Z_{m,j}^d + Z_{m,m}^d \cdot I_m^0$$
$$U_m^i = Z_{m,m}^i \cdot I_m^0$$
$$U_m^0 = Z_{m,m}^0 \cdot I_m^0$$

$$\sum_{j=1}^{n} I_j \cdot Z_{m,j}^d = U_m^Z$$ $\longrightarrow$ prije kvara

Jednofazni kratki spoj

$$U_m^Z + Z_{mm}^d \cdot I_m^0 + Z_{mm}^i \cdot I_m^0 + Z_{mm}^0 \cdot I_m^0 = 0$$

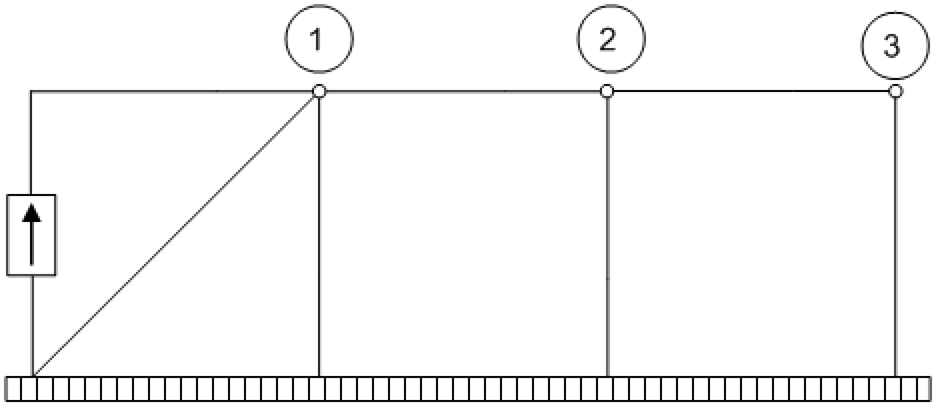
$$I_m^0 = -\frac{U_m^Z}{Z_{mm}^d + Z_{mm}^i + Z_{mm}^0}$$

$$I_{KV} = -3 \cdot I_m^0 = \frac{3 \cdot U_m^Z}{Z_{mm}^d + Z_{mm}^i + Z_{mm}^0} = \frac{3 \cdot U_m^Z}{2 \cdot Z_{mm}^d + Z_{mm}^0}$$

U_m^Z - fazni napon trofazne mreže

Jednofazni kratki spoj

- Primjer:



$$Z^d = Z^i = \begin{vmatrix} 0.385 & 0.154 & 0.077 \\ 0.154 & 0.461 & 0.230 \\ 0.077 & 0.230 & 0.615 \end{vmatrix}$$

$$Z^0 = \begin{vmatrix} 0.770 & 0.308 & 0.154 \\ 0.308 & 0.922 & 0.460 \\ 0.154 & 0.460 & 1.230 \end{vmatrix}$$

$$\begin{vmatrix} U_1^d \\ U_2^d \\ U_3^d \end{vmatrix} = Z^d \cdot \left\{ \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} + \begin{vmatrix} I_1^d \\ 0 \\ 0 \end{vmatrix} \right\}$$

$$U_1^d = 0.385 \cdot 1 + 0.385 \cdot I_1^d$$

$$U_1^i = 0.385 \cdot I_1^i$$

$$U_1^0 = 0.77 \cdot I_1^0$$

$$I_1^0 = I_1^i = I_1^d = -\frac{0.385}{0.385 + 0.385 + 0.77} = -0.25\ A$$

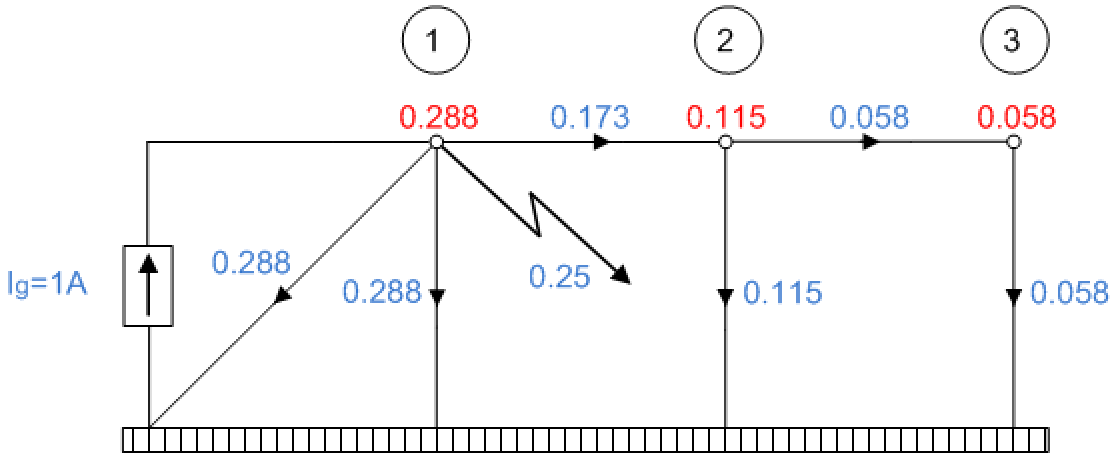
$$\begin{vmatrix} U_1^d \\ U_2^d \\ U_3^d \end{vmatrix} = \left| Z^d \right| \cdot \begin{vmatrix} 1 - 0.25 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0.288 \\ 0.115 \\ 0.058 \end{vmatrix}$$

$$\begin{vmatrix} U_1^i \\ U_2^i \\ U_3^i \end{vmatrix} = \left|Z^i\right| \cdot \begin{vmatrix} -0.25 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} -0.096 \\ -0.0385 \\ -0.0193 \end{vmatrix}$$

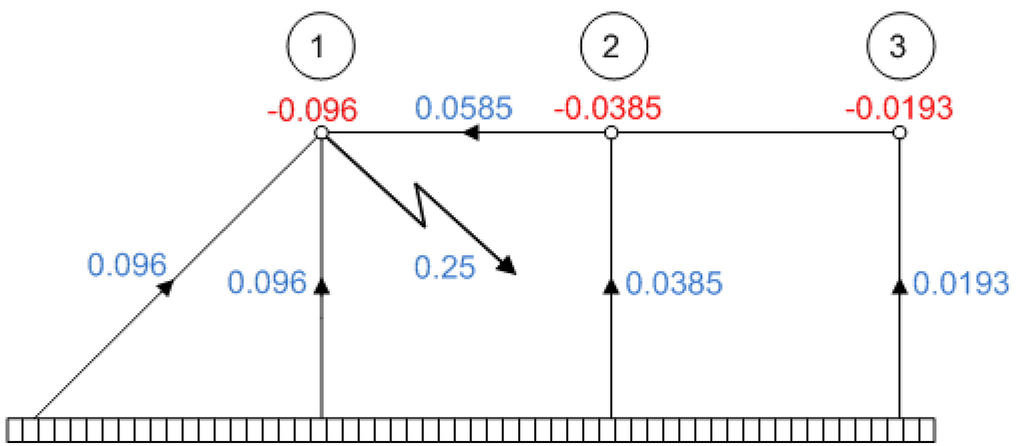
$$\begin{vmatrix} U_1^0 \\ U_2^0 \\ U_3^0 \end{vmatrix} = \left|Z^0\right| \cdot \begin{vmatrix} -0.25 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} -0.192 \\ -0.077 \\ -0.039 \end{vmatrix}$$

Struja bolesnog čvorišta u fazi R:

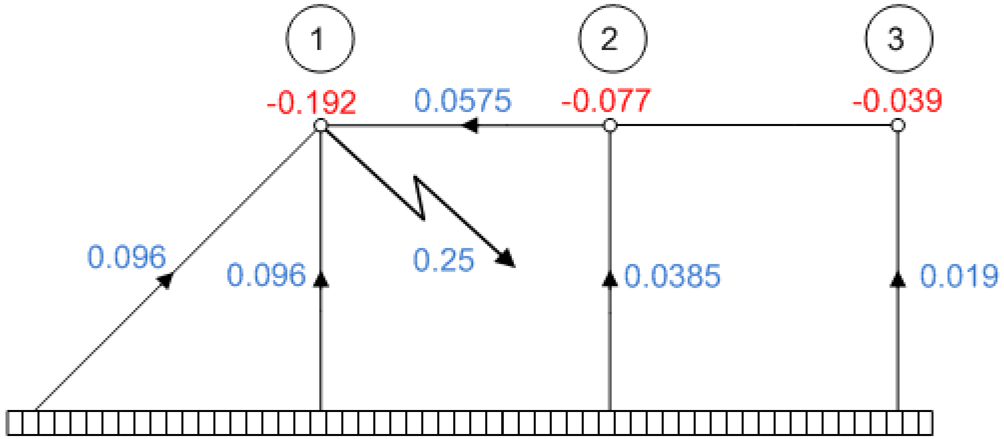
$$I_1^R = I^0 + I^d + I^i = 3 \cdot \left(-0.25\right) = -0.75$$



Direktna mreža



Inverzna mreža

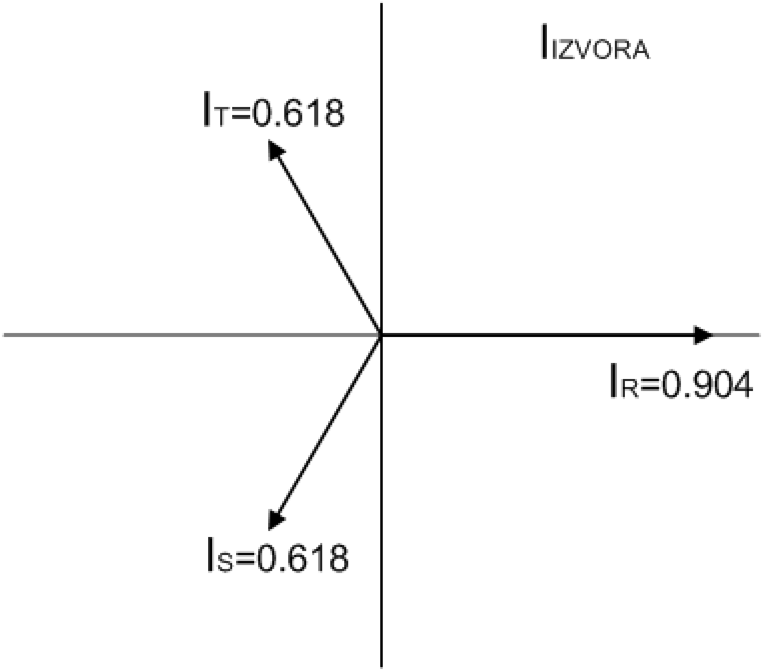


nulta mreža

$$\begin{vmatrix} U_1^R \\ U_1^S \\ U_1^T \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} -0.192 \\ 0.288 \\ -0.096 \end{vmatrix} = \begin{vmatrix} 0 \\ 0.298 - j0.316 \\ -0.298 + j0.316 \end{vmatrix}$$

$$\begin{vmatrix} I_1^R \\ I_1^S \\ I_1^T \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} -0.25 \\ -0.25 \\ -0.25 \end{vmatrix} = \begin{vmatrix} -0.75 \\ 0 \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} I_{IZV}^{R} \\ I_{IZV}^{S} \\ I_{IZV}^{T} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.096 \\ 0.712 \\ 0.096 \end{vmatrix} = \begin{vmatrix} 0.904 \\ -0.312 - j0.534 \\ -0.312 + j0.534 \end{vmatrix}$$



Pitanja i komentari

Analiza elektroenergetskog sustava
Predavanje 10: Proračun kratkog spoja

Prof. dr. sc. Ivica Pavić

Izv. prof. dr. sc. Marko Delimar

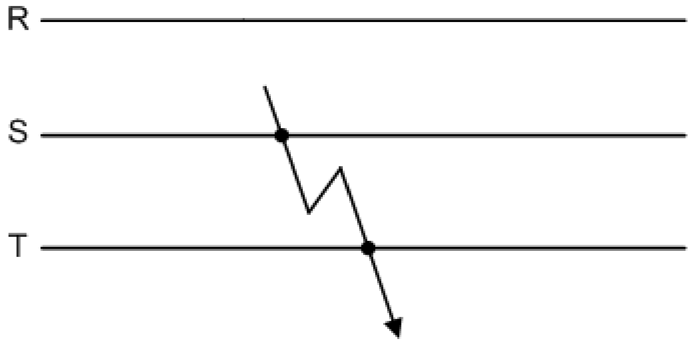
Kratki spoj

Dvofazni kratki spoj

$$\begin{vmatrix} U_1 \\ \vdots \\ U_m \\ \vdots \\ U_n \end{vmatrix}^d = \left|Z^d\right| \cdot \left\{ \begin{vmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{vmatrix}^d + \begin{vmatrix} 0 \\ 0 \\ \vdots \\ I_m^d \\ \vdots \\ 0 \end{vmatrix} \right\} \quad ; \quad \begin{vmatrix} U_1 \\ \vdots \\ U_m \\ \vdots \\ U_n \end{vmatrix}^i = \left|Z^i\right| \cdot \begin{vmatrix} 0 \\ 0 \\ \vdots \\ I_m^i \\ \vdots \\ 0 \end{vmatrix}$$

- Na mjestu kvara:

$$U_S = U_T$$
$$I_S = -I_T$$



Dvofazni kratki spoj

$$U_S = U^0 + a^2 \cdot U^d + a \cdot U^i$$
$$U_T = U^0 + a \cdot U^d + a^2 \cdot U^i$$

$$U^0 + a^2 \cdot U^d + a \cdot U^i = U^0 + a \cdot U^d + a^2 \cdot U^i$$
$$\left(a^2 - a\right) \cdot U^d = \left(a^2 - a\right) \cdot U^i$$
$$U^d = U^i$$

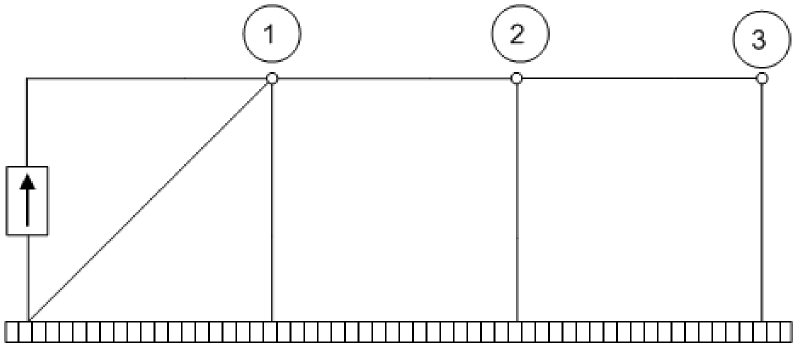
- Analogno iz drugog uvjeta: $I^d = -I^i$
- Uvrštavanjem matričnih jednadžbi u dobivene jednakosti:

$$U_m^d = {}^Z U_m^d + Z_{mm}^d \cdot I_m^d$$
$$U_m^i = Z_{mm}^i \cdot I_m^i$$

$${}^Z U_m^d + Z_{mm}^d \cdot I_m^d = Z_{mm}^i \cdot I_m^i = -Z_{mm} \cdot I_m^d \longrightarrow I_m^d = -\frac{{}^Z U_m^d}{Z_{mm}^d + Z_{mm}^i}$$

Dvofazni kratki spoj

- Primjer:



$$Z^d = Z^i = \begin{vmatrix} 0.385 & 0.154 & 0.077 \\ 0.154 & 0.461 & 0.230 \\ 0.077 & 0.230 & 0.615 \end{vmatrix}$$

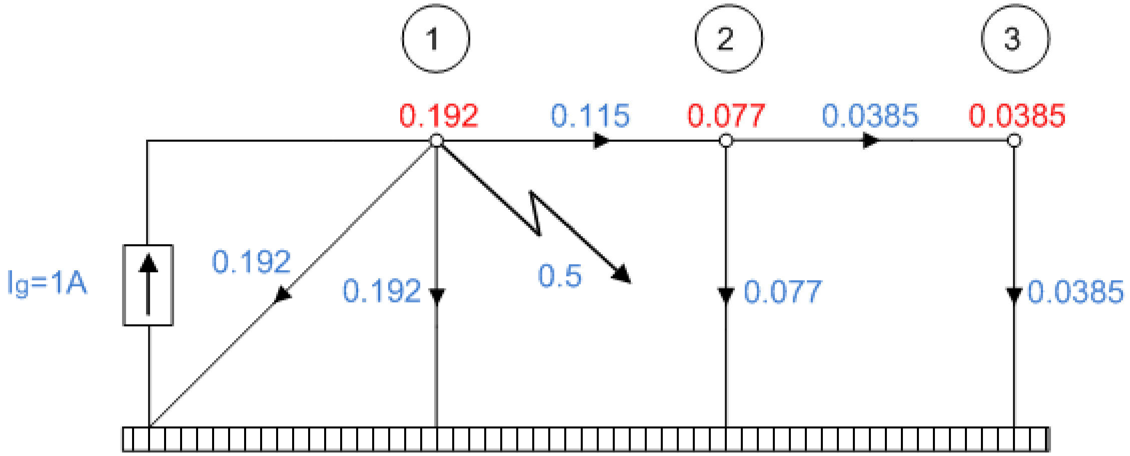
$$I_m^d = -\frac{0.385}{0.385+0.385} = -0.5\ A$$

$$I_m^i = 0.5\ A$$

$$\begin{vmatrix} U_1^d \\ U_2^d \\ U_3^d \end{vmatrix}^B = \begin{vmatrix} 0.385 \\ 0.154 \\ 0.077 \end{vmatrix}^Z - \begin{vmatrix} 0.192 \\ 0.077 \\ 0.0385 \end{vmatrix} = \begin{vmatrix} 0.192 \\ 0.077 \\ 0.0385 \end{vmatrix}$$

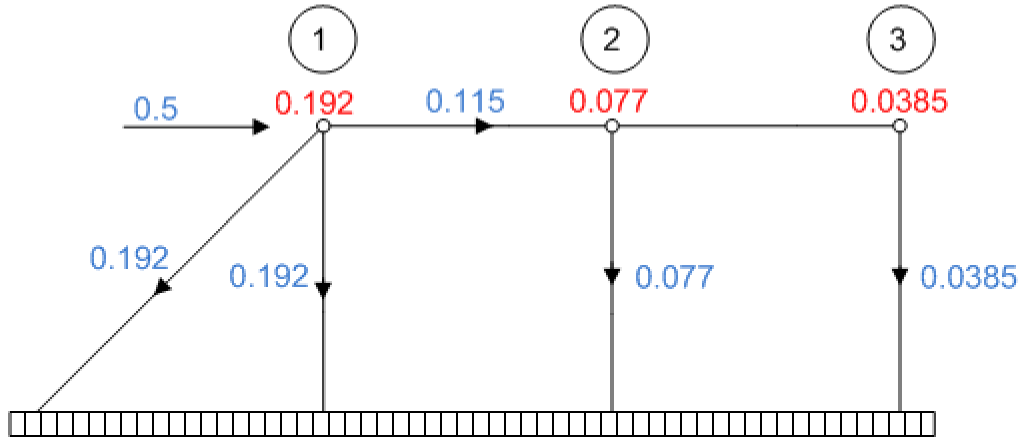
$$\begin{vmatrix} U_1^i \\ U_2^i \\ U_3^i \end{vmatrix} = \left| Z^i \right| \cdot \begin{vmatrix} 0.5 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0.192 \\ 0.077 \\ 0.0385 \end{vmatrix}$$

Dvofazni kratki spoj



Direktna mreža

$$I_g^d = 1\,A - 0.192\,A = 0.808\,A$$



Inverzna mreža

Dvofazni kratki spoj

- Iz izvora po fazama R, S i T:

$$^{R}I_{IZV} = I^{d}_{IZV} + I^{i}_{IZV} = 0.808 - 0.192 = 0.616\ A$$

$$^{S}I_{IZV} = a^2 \cdot I^{d}_{IZV} + a \cdot I^{i}_{IZV} =$$

$$= \left(-0.5 - j0.866\right) \cdot 0.808 - \left(-0.5 + j0.866\right) \cdot 0.192 =$$

$$= -0.308 - j0.866\ A$$

$$^{T}I_{IZV} = \left(-0.5 + j0.866\right) \cdot 0.808 - \left(-0.5 - j0.866\right) \cdot 0.192 =$$

$$= -0.308 + j0.866\ A$$

- Na mjestu kvara (poprečne struje):

$$^{R}I_{m} = I^{d}_{IZV} + I^{i}_{IZV} = 0\ A$$

$$^{S}I_{m} = a^2 \cdot I^{d}_{m} + a \cdot I^{i}_{m} = \left(a^2 - a\right) \cdot I^{d}_{m} = \left(-j\sqrt{3}\right) \cdot \left(-0.5\right) = j0.866\ A$$

$$^{T}I_{m} = a \cdot I^{d}_{m} + a^2 \cdot I^{i}_{m} = \left(a - a^2\right) \cdot I^{d}_{m} = -j0.866\ A$$

Dvofazni kratki spoj

- U grani 1-2 :

$$I_{1-2}^{d} = 0.115 = I_{1-2}^{i}$$

$$^{R}I_{1-2} = 0.23\ A$$

$$^{S}I_{1-2} = a^2 \cdot I_{1-2}^{d} + a \cdot I_{1-2}^{i} = \left(a^2 + a\right) \cdot I_{1-2}^{d} = -I_{1-2}^{d} = -0.115\ A$$

$$^{T}I_{1-2} = a \cdot I_{1-2}^{d} + a^2 \cdot I_{1-2}^{i} = \left(a + a^2\right) \cdot I_{1-2}^{d} = -0.115\ A$$

- U grani 1-0 :

$$^{R}I_{1-0} = I_{1-0}^{d} + I_{1-0}^{i} = 0.192 + 0.192 = 0.385\ A$$

$$^{S}I_{1-0} = a^2 \cdot I_{1-0}^{d} + a \cdot I_{1-0}^{i} = \left(a^2 + a\right) \cdot I_{1-0}^{d} = -0.192\ A$$

$$^{T}I_{1-0} = a \cdot I_{1-0}^{d} + a^2 \cdot I_{1-0}^{i} = \left(a + a^2\right) \cdot I_{1-0}^{d} = -0.192\ A$$

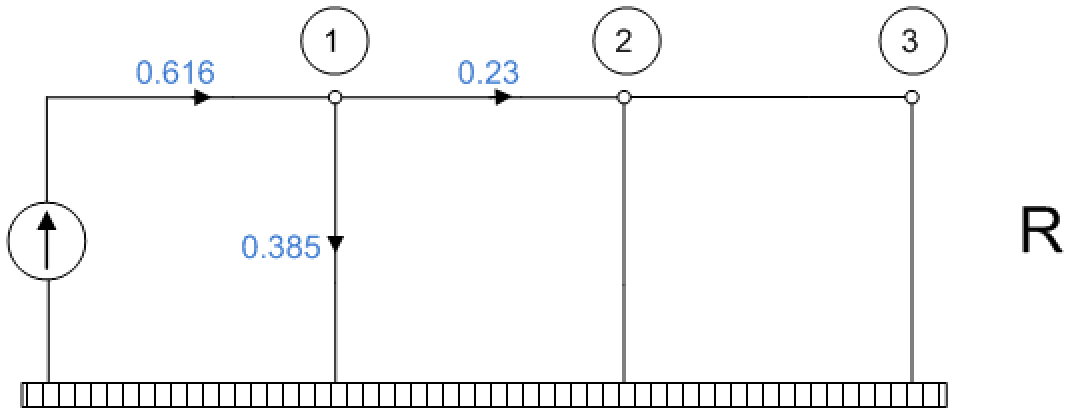
Dvofazni kratki spoj

- U čvorištu 1 :

$$^{R}U_1 = U_1^d + U_1^i = 0.192 + 0.192 = 0.385\,V$$

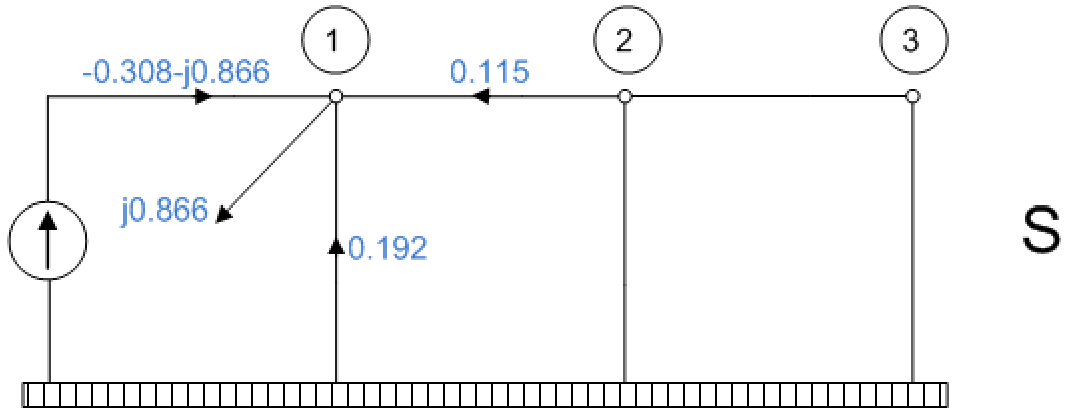
$$^{S}U_1 = a^2 \cdot U_1^d + a \cdot U_1^i = -0.192\,V$$

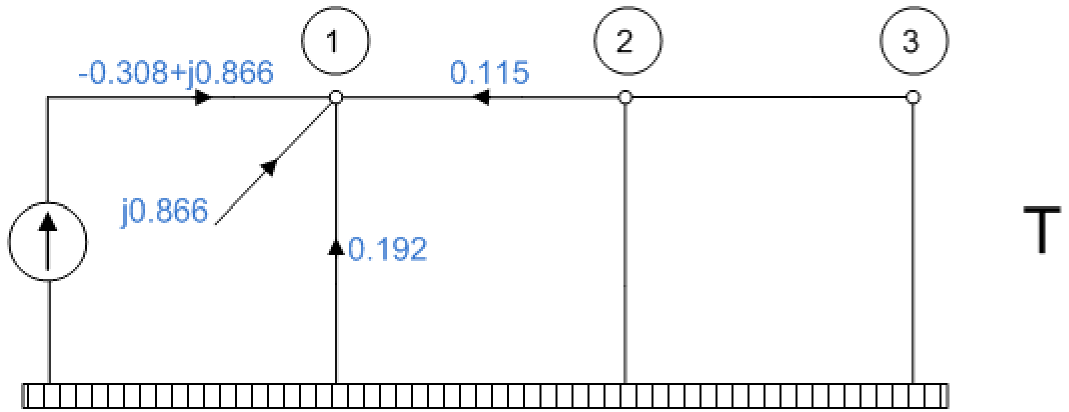
$$^{T}U_1 = a \cdot U_1^d + a^2 \cdot U_1^i = -0.192\,V$$



Dvofazni kratki spoj

- U čvorištu 1 :





Dvofazni kratki spoj sa zemljom

$$U^d = {}^Z U^d + Z^d \cdot I_m^d$$

$$U^i = Z^i \cdot I_m^i$$

$$U^0 = Z^0 \cdot I_m^0$$

$${}^S U = {}^T U = 0$$

$${}^R I = 0$$

$$U^0 + a^2 \cdot U^d + a \cdot U^i = U^0 + a \cdot U^d + a^2 \cdot U^i$$

$$U^0 = U^d = U^i$$

$${}^R I = I^0 + I^d + I^i = 0$$

$$I^i = -I^d - I^0$$

Dvofazni kratki spoj sa zemljom

$$^{Z}U_m^d + Z_{mm}^d \cdot I_m^d = Z_{mm}^i \cdot I_m^i = Z_{mm}^0 \cdot I_m^0$$

$$^{Z}U_m^d + Z_{mm}^d \cdot I_m^d = Z_{mm}^i \cdot I_m^i \longrightarrow I_m^i = \frac{1}{Z_{mm}^i}\left({}^{Z}U_m^d + Z_{mm}^d \cdot I_m^d \right)$$

$$^{Z}U_m^d + Z_{mm}^d \cdot I_m^d = Z_{mm}^0 \cdot I_m^0$$

$$Z_{mm}^i \cdot I_m^i = Z_{mm}^0 \cdot I_m^0 \longrightarrow I_m^0 = \frac{Z_{mm}^i}{Z_{mm}^0} \cdot I_m^i$$

$$^{Z}U_m^d + Z_{mm}^d \cdot I_m^d = Z_{mm}^i \cdot \left(-I_m^d - I_m^0 \right)$$

$$^{Z}U_m^d + Z_{mm}^d \cdot I_m^d + Z_{mm}^i \cdot I_m^d + \frac{Z_{mm}^i}{Z_{mm}^0} \cdot \left({}^{Z}U_m^d + Z_{mm}^d \cdot I_m^d \right) = 0$$

$$^{Z}U_m^d \cdot \left(1 + \frac{Z_{mm}^i}{Z_{mm}^0} \right) + \left(Z_{mm}^d + Z_{mm}^i + \frac{Z_{mm}^d \cdot Z_{mm}^i}{Z_{mm}^0} \right) \cdot I_m^d = 0$$

Dvofazni kratki spoj sa zemljom

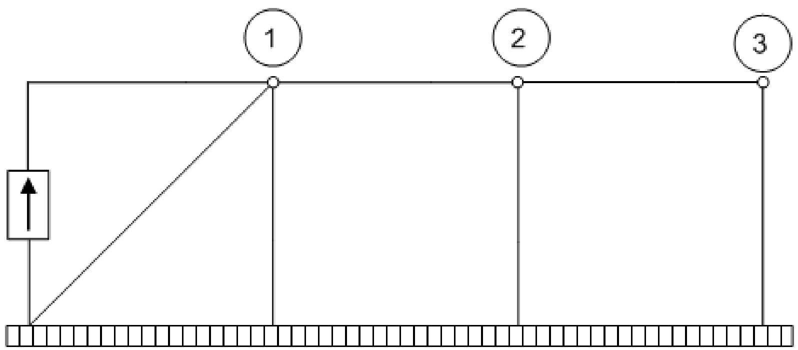
$$I_m^d = -{}^Z U_m^d \cdot \frac{Z_{mm}^0 + Z_{mm}^i}{Z_{mm}^d \cdot Z_{mm}^0 + Z_{mm}^i \cdot Z_{mm}^0 + Z_{mm}^d \cdot Z_{mm}^i}$$

$$\begin{aligned} I_m^i &= \frac{{}^Z U_m^d}{Z_{mm}^i} - {}^Z U_m^d \cdot \frac{Z_{mm}^d}{Z_{mm}^i} \cdot \frac{Z_{mm}^0 + Z_{mm}^i}{Z_{mm}^d \cdot Z_{mm}^0 + Z_{mm}^i \cdot Z_{mm}^0 + Z_{mm}^d \cdot Z_{mm}^i} \\ &= {}^Z U_m^d \cdot \left(\frac{Z_{mm}^d \cdot Z_{mm}^0 + Z_{mm}^i \cdot Z_{mm}^0 + Z_{mm}^d \cdot Z_{mm}^i - Z_{mm}^d \cdot Z_{mm}^0 - Z_{mm}^d \cdot Z_{mm}^i}{Z_{mm}^i \cdot \left(Z_{mm}^d \cdot Z_{mm}^0 + Z_{mm}^i \cdot Z_{mm}^0 + Z_{mm}^d \cdot Z_{mm}^i \right)} \right) \\ &= {}^Z U_m^d \cdot \left(\frac{Z_{mm}^0}{Z_{mm}^d \cdot Z_{mm}^0 + Z_{mm}^i \cdot Z_{mm}^0 + Z_{mm}^d \cdot Z_{mm}^i} \right) \end{aligned}$$

$$I_m^0 = {}^Z U_m^d \cdot \left(\frac{Z_{mm}^i}{Z_{mm}^d \cdot Z_{mm}^0 + Z_{mm}^i \cdot Z_{mm}^0 + Z_{mm}^d \cdot Z_{mm}^i} \right)$$

Dvofazni kratki spoj sa zemljom

- Primjer:



$$Z^d = Z^i = \begin{vmatrix} 0.385 & 0.154 & 0.077 \\ 0.154 & 0.461 & 0.230 \\ 0.077 & 0.230 & 0.615 \end{vmatrix} \qquad Z^0 = \begin{vmatrix} 0.770 & 0.308 & 0.154 \\ 0.308 & 0.922 & 0.460 \\ 0.154 & 0.460 & 1.230 \end{vmatrix}$$

Dvofazni kratki spoj sa zemljom

$$I_m^d = -0.385 \cdot \frac{0.77 + 0.385}{0.385 \cdot 0.77 + 0.385 \cdot 0.77 + 0.385 \cdot 0.385} = -0.60\ A$$

$$I_m^i = 0.385 \cdot \frac{0.77}{0.385 \cdot 1.925} = 0.4\ A$$

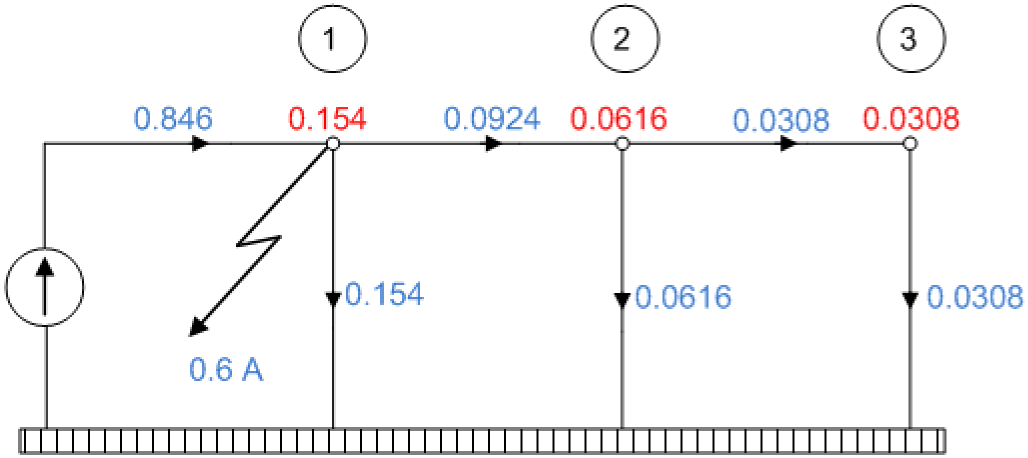
$$I_m^0 = 0.385 \cdot \frac{0.385}{0.385 \cdot 1.925} = 0.2\ A$$

$$\begin{vmatrix} U_1^d \\ U_2^d \\ U_3^d \end{vmatrix}^B = \begin{vmatrix} U_1^d \\ U_2^d \\ U_3^d \end{vmatrix}^Z + \left| Z^d \right| \cdot \begin{vmatrix} I_m^d \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0.385 \\ 0.154 \\ 0.077 \end{vmatrix} - \begin{vmatrix} 0.385 \cdot 0.6 \\ 0.154 \cdot 0.6 \\ 0.077 \cdot 0.6 \end{vmatrix} = \begin{vmatrix} 0.154 \\ 0.0616 \\ 0.0308 \end{vmatrix}$$

$$\begin{vmatrix} U_1^i \\ U_2^i \\ U_3^i \end{vmatrix}^B = \left| Z^i \right| \cdot \begin{vmatrix} I_m^i \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0.385 \cdot 0.4 \\ 0.154 \cdot 0.4 \\ 0.077 \cdot 0.4 \end{vmatrix} = \begin{vmatrix} 0.154 \\ 0.0616 \\ 0.0308 \end{vmatrix}$$

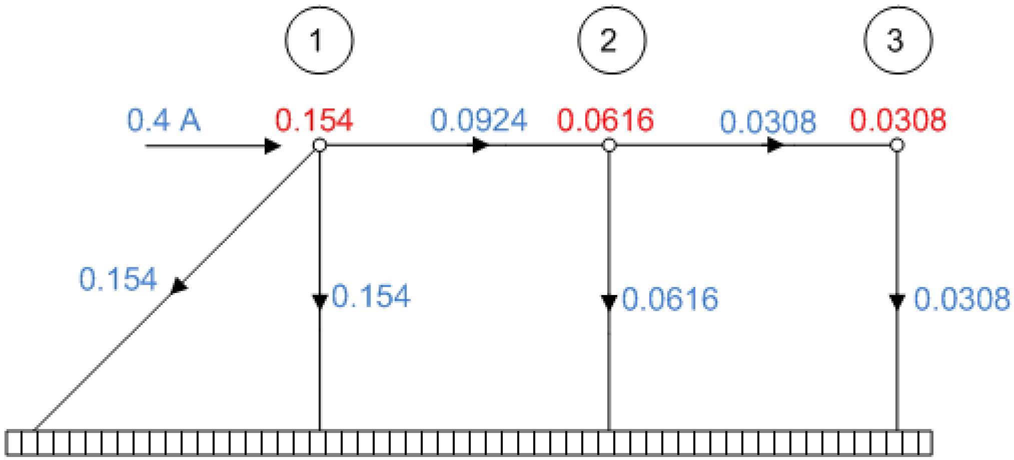
Dvofazni kratki spoj sa zemljom

$$\begin{vmatrix} U_1^0 \\ U_2^0 \\ U_3^0 \end{vmatrix}^B = \left|Z^0\right| \cdot \begin{vmatrix} I_m^0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0.77 \cdot 0.2 \\ 0.308 \cdot 0.2 \\ 0.154 \cdot 0.2 \end{vmatrix} = \begin{vmatrix} 0.154 \\ 0.0616 \\ 0.0308 \end{vmatrix}$$

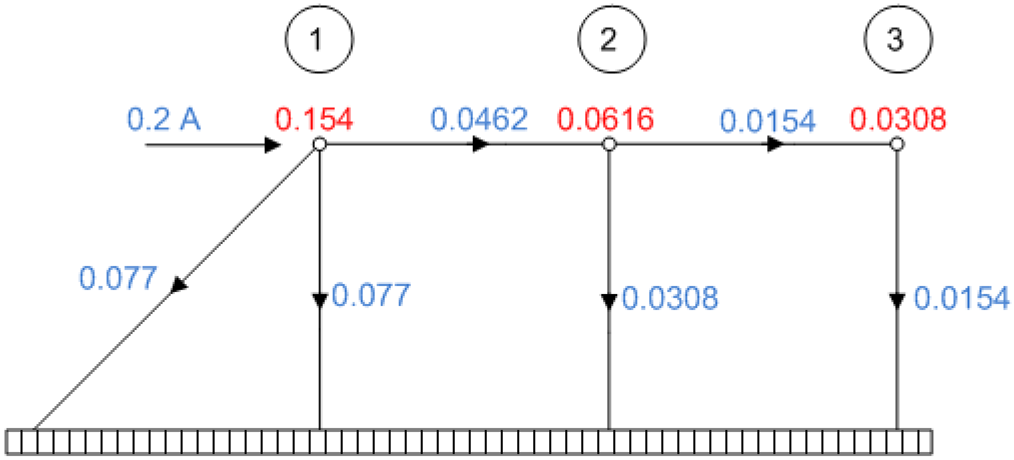


Direktna mreža

Dvofazni kratki spoj sa zemljom



Inverzna mreža



Nulta mreža

Dvofazni kratki spoj sa zemljom

$$\begin{vmatrix} {}^{R}U_1 \\ {}^{S}U_1 \\ {}^{T}U_1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} U_1^0 \\ U_1^d \\ U_1^i \end{vmatrix} =$$

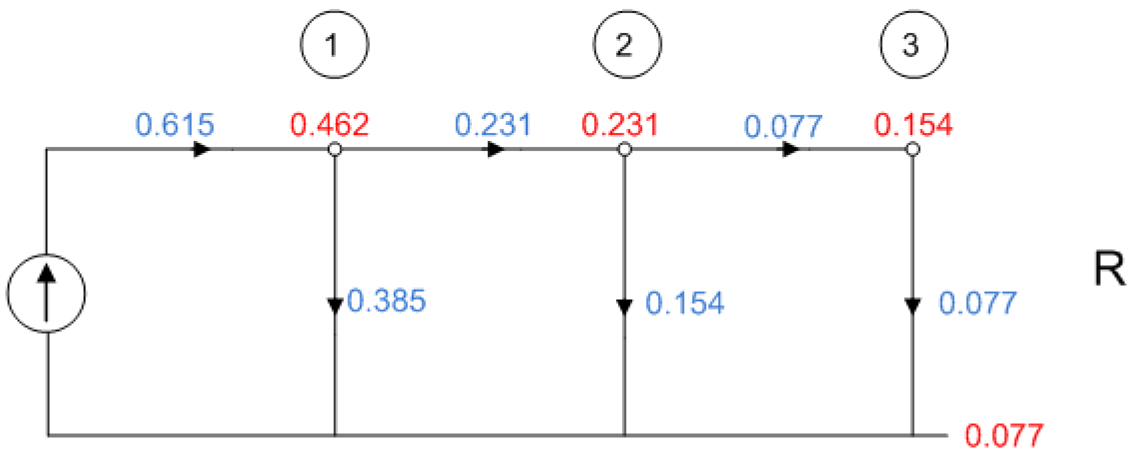
$$= \begin{vmatrix} 0.154+0.154+0.154 \\ 0.154+\left(-0.77-j0.133\right)+\left(-0.77+j0.133\right) \\ 0.154+\left(-0.77+j0.133\right)+\left(-0.77-j0.133\right) \end{vmatrix} = \begin{vmatrix} 0.462 \\ 0 \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} {}^{R}I_1 \\ {}^{S}I_1 \\ {}^{T}I_1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.2 \\ -0.6 \\ 0.4 \end{vmatrix}$$

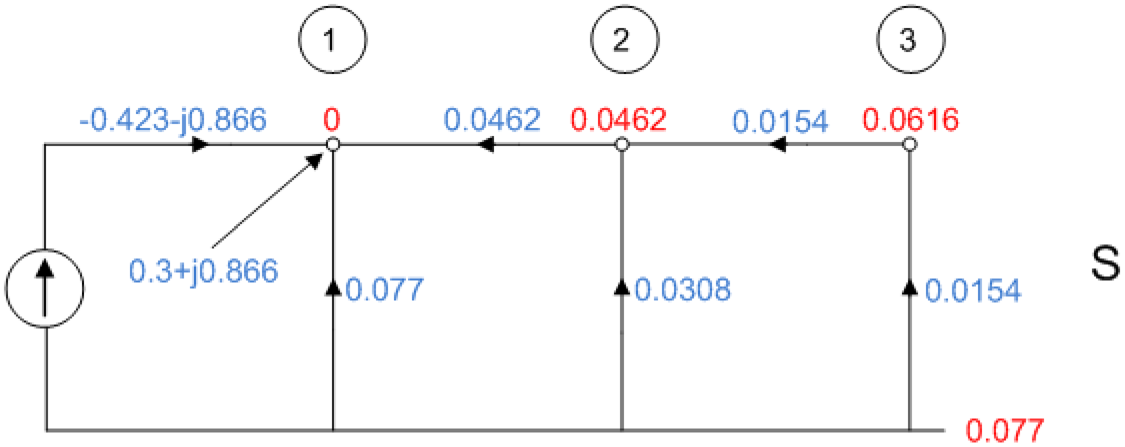
$$= \begin{vmatrix} 0 \\ 0.2+0.3+j0.5196-0.2+j0.3464 \\ 0.2+0.3-j0.3464-0.2-j0.5196 \end{vmatrix} = \begin{vmatrix} 0 \\ 0.3+j0.866 \\ 0.3-j0.866 \end{vmatrix}$$

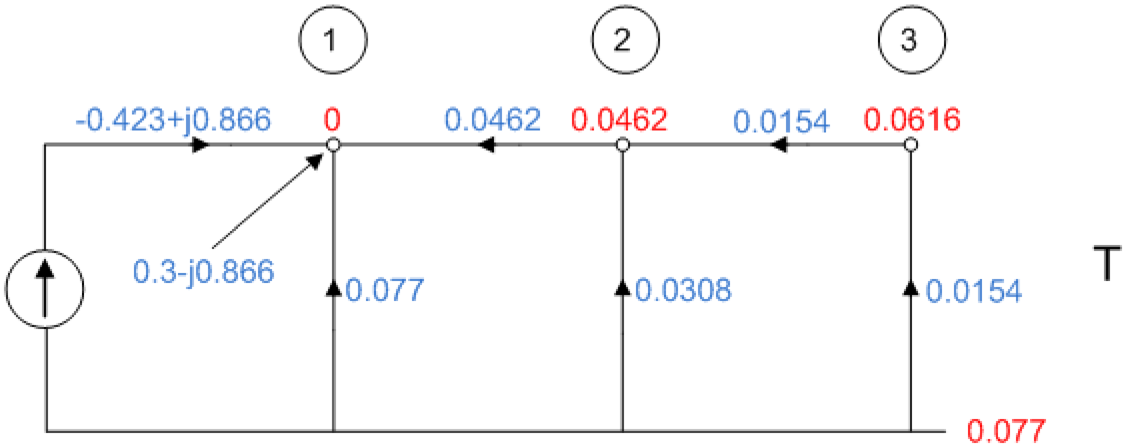
Dvofazni kratki spoj sa zemljom

$$\begin{vmatrix} {}^{R}I_{IZV} \\ {}^{S}I_{IZV} \\ {}^{T}I_{IZV} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} -0.077 \\ 0.846 \\ -0.154 \end{vmatrix} = \begin{vmatrix} 0.615 \\ -0.423 - j0.846 \\ -0.423 + j0.866 \end{vmatrix}$$



Dvofazni kratki spoj sa zemljom





Dvofazni kratki spoj sa zemljom

$$\begin{vmatrix} {}^{R}I_{1-2} \\ {}^{S}I_{1-2} \\ {}^{T}I_{1-2} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.0462 \\ 0.0924 \\ 0.0924 \end{vmatrix} = \begin{vmatrix} 0.231 \\ -0.0462 \\ -0.0462 \end{vmatrix}$$

$$\begin{vmatrix} {}^{R}I_{1-0} \\ {}^{S}I_{1-0} \\ {}^{T}I_{1-0} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.077 \\ 0.154 \\ 0.154 \end{vmatrix} = \begin{vmatrix} 0.385 \\ -0.077 \\ -0.077 \end{vmatrix}$$

$$\begin{vmatrix} {}^{R}I_{2-0} \\ {}^{S}I_{2-0} \\ {}^{T}I_{2-0} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.0308 \\ 0.0616 \\ 0.0616 \end{vmatrix} = \begin{vmatrix} 0.154 \\ -0.0308 \\ -0.0308 \end{vmatrix}$$

Dvofazni kratki spoj sa zemljom

$$\begin{vmatrix} {}^{R}I_{2-3} \\ {}^{S}I_{2-3} \\ {}^{T}I_{2-3} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.0154 \\ 0.0308 \\ 0.0308 \end{vmatrix} = \begin{vmatrix} 0.077 \\ -0.0154 \\ -0.0154 \end{vmatrix}$$

$$\begin{vmatrix} {}^{R}I_{3-0} \\ {}^{S}I_{3-0} \\ {}^{T}I_{3-0} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} 0.0154 \\ 0.0308 \\ 0.0308 \end{vmatrix} = \begin{vmatrix} 0.077 \\ -0.0154 \\ -0.0154 \end{vmatrix}$$

Pitanja i komentari