ANALIZA ELEKTROENERGETSKOG SUSTAVA

Predavanje br. 7.

- VRSTE ČVORIŠTA
  - Čvorište tereta (poznato P, Q)
  - Generatorsko čvorište (poznato |V|, P, Q_{min}, Q_{max})
  - Čvorište regulacijske elektrane (poznato |V|, $\angle\delta$)

- SNAGE U ČVORIŠTIMA (1)

$$\vec{S}_i = \vec{U}_i \cdot \vec{I}_i^{*} = P_i + jQ_i$$

$$\vec{U}_i = \left|\vec{U}_i\right| \cdot e^{j\delta_i}$$

$$\vec{I}_i = \sum_{j=1}^{n} \vec{Y}_{ij} \cdot \vec{U}_j \quad \Rightarrow \quad \vec{I}_i^{*} = \sum_{j=1}^{n} \vec{Y}_{ij}^{*} \cdot \vec{U}_j^{*}$$

- SNAGE U ČVORIŠTIMA (2)

$$\vec{U}_j^* = \left|\vec{U}_j\right| \cdot e^{-j\delta_j}$$

$$\vec{Y}_{ij} = \left|\vec{Y}_{ij}\right| \cdot e^{j\Theta_{ij}} \longrightarrow$$

Admitancije čvorišta
(elementi matrice admitancija čvorišta)

$$\vec{Y}_{ij}^* = \left|\vec{Y}_{ij}\right| \cdot e^{-j\Theta_{ij}}$$

$$\vec{S}_i = \vec{U}_i \cdot \vec{I}_i^* = \vec{U}_i \cdot \sum_{j=1}^{n} \vec{Y}_{ij}^* \cdot \vec{U}_j^*$$

$$\vec{S}_i = \left|\vec{U}_i\right| \cdot e^{j\delta_i} \cdot \sum_{j=1}^{n} \left|\vec{Y}_{ij}\right| \cdot e^{-j\Theta_{ij}} \cdot \left|\vec{U}_j\right| \cdot e^{-j\delta_j}$$

$$\vec{S}_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{Y}_{ij}\right| \cdot \left|\vec{U}_j\right| \cdot e^{j(\delta_i - \delta_j - \Theta_{ij})}$$

- SNAGE U ČVORIŠTIMA (3)

$$\vec{S}_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \left[\cos\left(\delta_i - \delta_j - \Theta_{ij}\right) + j\sin(\delta_i - \delta_j - \Theta_{ij})\right]$$

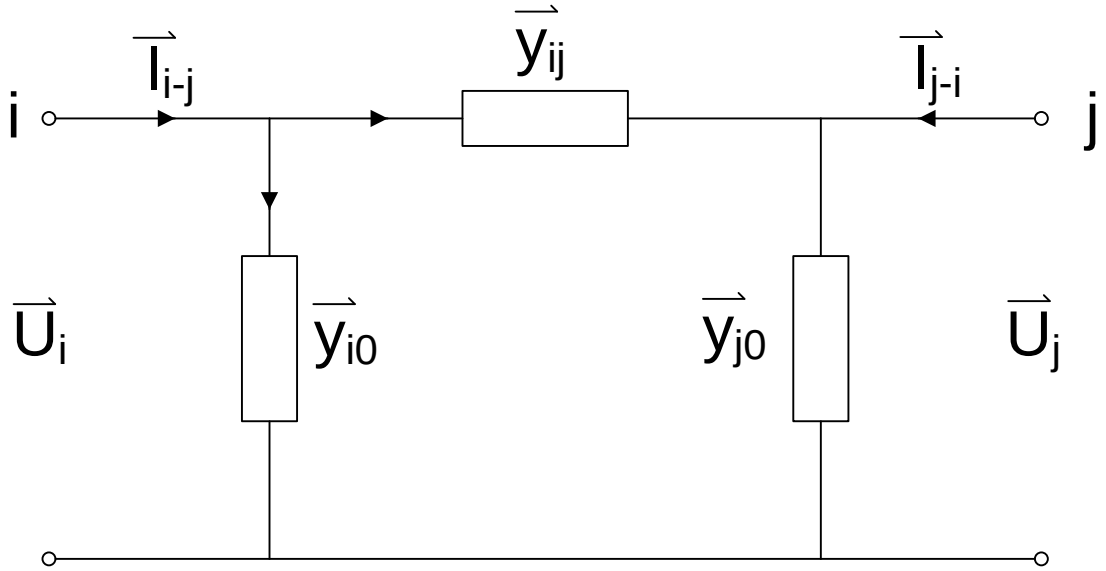
  - DJELATNA SNAGA U ČVORIŠTU i (P_i)

$$P_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \cos\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

  - JALOVA SNAGA U ČVORIŠTU i (Q_i)

$$Q_i = \left|\vec{U}_i\right| \cdot \sum_{j=1}^{n} \left|\vec{U}_j\right| \cdot \left|\vec{Y}_{ij}\right| \cdot \sin\left(\delta_i - \delta_j - \Theta_{ij}\right)$$

- SNAGE U GRANAMA



$$\vec{I}_{i-j} = \left(\vec{U}_i - \vec{U}_j\right)\cdot \vec{y}_{i-j} + \vec{U}_i \cdot \vec{y}_{i0}$$

$$\vec{S}_{i-j} = \vec{U}_i \cdot \vec{I}^{*}_{i-j} = \vec{U}_i\left[\left(\vec{U}^{*}_i - \vec{U}^{*}_j\right)\cdot \vec{y}^{*}_{i-j} + \vec{U}^{*}_i \cdot \vec{y}^{*}_{i0}\right]$$

$$\vec{I}_{j-i} = \left(\vec{U}_j - \vec{U}_i\right)\cdot \vec{y}_{i-j} + \vec{U}_j \cdot \vec{y}_{j0}$$

$$\vec{S}_{j-i} = \vec{U}_j \cdot \vec{I}^{*}_{j-i} = \vec{U}_j\left[\left(\vec{U}^{*}_j - \vec{U}^{*}_i\right)\cdot \vec{y}^{*}_{i-j} + \vec{U}^{*}_j \cdot \vec{y}^{*}_{j0}\right]$$

- GUBICI SNAGE U GRANAMA

$$\Delta\vec{S} = \vec{S}_{i-j} + \vec{S}_{j-i}$$

$$\Delta\vec{S} = \left(\vec{U}_i^* - \vec{U}_j^*\right)\cdot \vec{y}_{i-j}^* \cdot \left(\vec{U}_i - \vec{U}_j\right) + \left|\vec{U}_i\right|^2 \cdot \vec{y}_{i0}^* + \left|\vec{U}_j\right|^2 \cdot \vec{y}_{j0}^*$$

– Napomena: $\vec{y}_{i-j}$ - uzdužna admitancija grane

$\vec{Y}_{ij}$ - međusobna admitancija čvorišta (između čvorišta **i** i **j**),element matrice admitancija čvorišta

$$\vec{Y}_{ij} \neq \vec{y}_{i-j}$$

- PRORAČUN TOKOVA SNAGA
  - Osnovne metode proračuna tokova snaga:
    - METODA GAUSS SEIDEL POMOĆU Z MATRICE
    - METODA GAUSS SEIDEL POMOĆU Y MATRICE
    - METODA NEWTON RAPHSON

- METODA GAUSS-SEIDEL POMOĆU Z MATRICE
  - Mreža od ***n*** čvorišta – jedno čvorište referentno

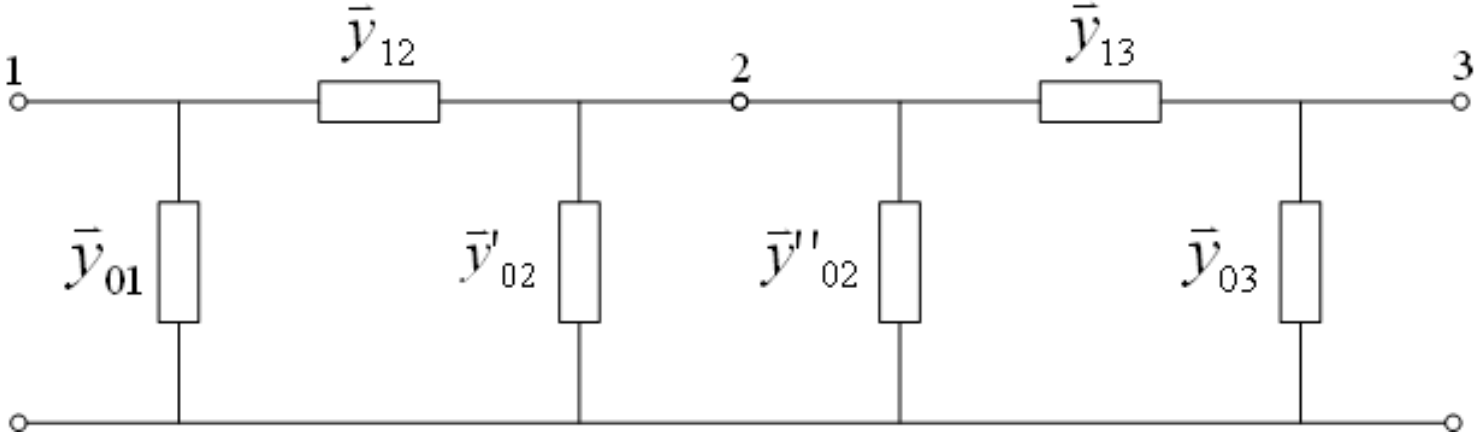
$$\vec{U}_i - \vec{U}_{ref.} = \sum_{\substack{j=1 \\ j \neq ref}}^{n} \vec{Z}_{ij} \cdot \vec{I}_j$$

$$\left|\Delta\vec{U}\right| = \left|\vec{Z}\right| \cdot \left|\vec{I}\right|$$

$$\left|\vec{Z}\right| = \left|\vec{Y}\right|^{-1}$$

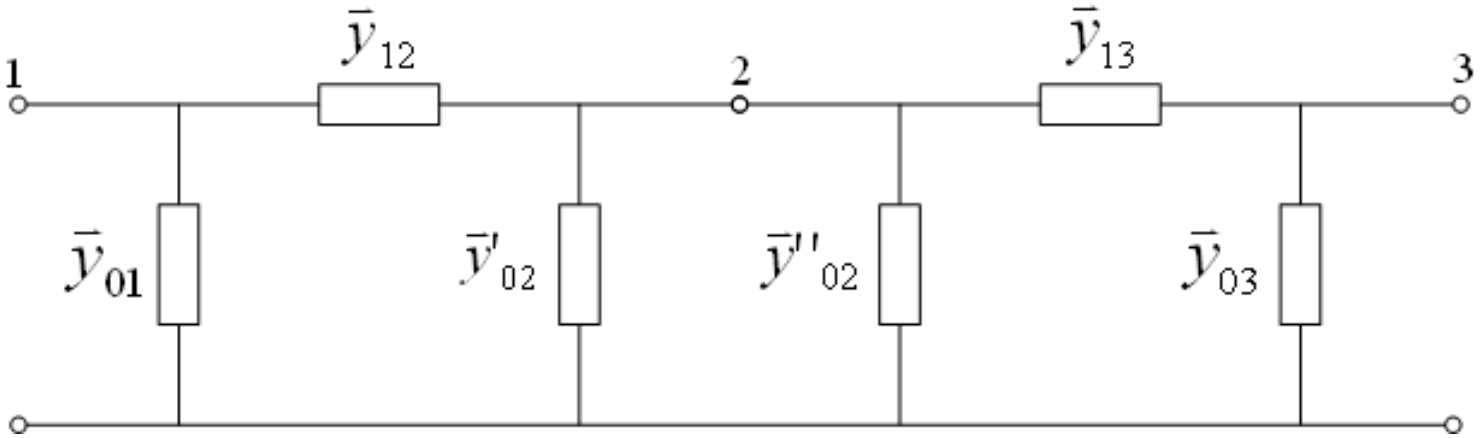
  - Matrica **Y** se dobije uzimajući u obzir samo uzdužne parametre grana
  - Poprečne admitancije grana sačinjavaju novu matricu **Y´**

- Matrica **Y**
  - Kod ove metode u matricu **Y** ulaze samo uzdužne admitancije
  - Poprečne admitancije se koriste za formiranje matrice **Y´**
  - Primjer:



$$\vec{Y} = \begin{bmatrix} \vec{y}_{12} & -y_{12} & 0 \\ -y_{12} & \vec{y}_{12} + \vec{y}_{13} & -\vec{y}_{13} \\ 0 & -\vec{y}_{13} & \vec{y}_{13} \end{bmatrix}$$

- Matrica **Y´**
  - Primjer:



$$\vec{Y}' = \begin{bmatrix} \vec{y}_{01} \\ \vec{y}'_{02} + \vec{y}''_{02} \\ \vec{y}_{03} \end{bmatrix}$$

- Uz $\vec{U}_n = \vec{U}_{ref}$ vrijedi:

$$\vec{U}_1 - \vec{U}_n = \vec{Z}_{11} \cdot \vec{I}_1 + \vec{Z}_{12} \cdot \vec{I}_2 + \ldots + \vec{Z}_{1(n-1)} \cdot \vec{I}_{n-1}$$

$$\vec{U}_i - \vec{U}_n = \vec{Z}_{i1} \cdot \vec{I}_1 + \vec{Z}_{i2} \cdot \vec{I}_2 + \ldots + \vec{Z}_{i(n-1)} \cdot \vec{I}_{n-1}$$

$$\vec{U}_{n-1} - \vec{U}_n = \vec{Z}_{(n-1)1} \cdot \vec{I}_1 + \vec{Z}_{(n-1)2} \cdot \vec{I}_2 + \ldots + \vec{Z}_{(n-1)(n-1)} \cdot \vec{I}_{n-1}$$

- Struje u čvorištima je potrebno odrediti pomoću injekcija snaga u čvorištima

- POSTUPAK PRORAČUNA
  1. korak
     - Učitavanje podataka o mreži (konfiguracija, admitancije grana)
     - Učitavanje podataka o injekcijama snage u čvorištima
  2. korak
     - Formiranje matrice **Y´** (samo poprečne admitancije grana)
     - Formiranje matrice **Y** (samo uzdužne admitancije grana)
  3. Korak
     - Računanje matrice **Z** ($\left|\vec{Z}\right| = \left|\vec{Y}\right|^{-1}$)
  4. Korak
     - Početne vrijednosti napona čvorišta: $\vec{U}_i^{(0)} = 1 + j0 \text{ p.u.} = 1\angle 0° \text{ p.u.}$
  5. korak
     - Računanje struja u čvorištima (nulta iteracija, k=0):

$$\vec{I}_i^{(0)} = \frac{\vec{S}_i^*}{\vec{U}_i^{*(0)}} - Y_i^{'} \cdot \vec{U}_i^{(0)} \qquad i = 1,2,\ldots,n-1$$

6. korak

– Računanje napona $\vec{\mathrm{U}}_{\mathrm{i}}^{(1)}$ i struja $\vec{\mathrm{I}}_{\mathrm{i}}^{(1)}$ u čvorištima (k=1):

$$\vec{U}_1^{(1)} = \vec{U}_{ref} + \vec{Z}_{11} \cdot \vec{I}_1^{(0)} + \vec{Z}_{12} \cdot \vec{I}_2^{(0)} + \ldots + \vec{Z}_{1(n-1)} \cdot \vec{I}_{(n-1)}^{(0)}$$

$$\vec{I}_1^{(1)} = \frac{\vec{S}_1^{*}}{\vec{U}_1^{*(1)}} - \vec{U}_1^{(1)} \cdot Y_1^{'}$$

$$\vec{U}_2^{(1)} = \vec{U}_{ref} + \vec{Z}_{21} \cdot \vec{I}_1^{(1)} + \vec{Z}_{22} \cdot \vec{I}_2^{(0)} + \ldots + \vec{Z}_{2(n-1)} \cdot \vec{I}_{(n-1)}^{(0)}$$

$$\vec{I}_2^{(1)} = \frac{\vec{S}_2^{*}}{\vec{U}_2^{*(1)}} - \vec{U}_2^{(1)} \cdot Y_2^{'}$$

$$\vdots$$

$$\vec{U}_i^{(1)} = \vec{U}_{ref} + \sum_{j=1}^{i-1} \vec{Z}_{ij} \cdot \vec{I}_j^{(1)} + \sum_{j=i}^{n-1} \vec{Z}_{ij} \cdot \vec{I}_j^{(0)}$$

$$\vec{I}_i^{(1)} = \frac{\vec{S}_i^{*}}{\vec{U}_i^{*(1)}} - \vec{U}_i^{(1)} \cdot Y_i^{'} \qquad za\ i = 1,2,\ldots,n-1$$

7. Korak
   - Provjera da li izračunate vrijednosti napona $\vec{U}_i^{(1)}$ zadovoljavaju unaprijed postavljeni uvjet točnosti :

$$\left|(\vec{U}_i^{(1)} - \vec{U}_i^{(0)})\right| < \varepsilon$$

$$\varepsilon = 0.001 \div 0.0001 \quad \text{(najčešće)}$$

   - Ako je postavljeni uvjet zadovoljen za svaki **i** – KRAJ PRORAČUNA, konačno rješenje je vektor stanja $\vec{U}_i^{(1)}$ (i=1,2,…, n-1)
   - U suprotnom prelazak u sljedeću iteraciju (k=2) korištenjem izračunatog vektora struja u čvorištima $\vec{I}_i^{(1)}$, te ponovnim izvršavanjem koraka 6 (dakle računaju se $\vec{U}_i^{(2)}$ i $\vec{I}_i^{(2)}$) i koraka 7
   - Ako postavljeni uvjet nije zadovoljen ni u drugoj iteraciji prelazi se u treću (k=3) korištenjem rezultata iz druge itd. dok se ne ostvari tražena točnost

- Općenito za neku iteraciju **k+1** vrijede sljedeći izrazi:

$$\vec{U}_i^{(k+1)} = \sum_{j=1}^{i-1} \vec{Z}_{ij} \cdot \vec{I}_j^{(k+1)} + \sum_{j=i}^{n-1} \vec{Z}_{ij} \cdot \vec{I}_j^{(k)} + \vec{U}_{ref}$$

$$\vec{I}_i^{(k+1)} = \frac{\vec{S}_i^*}{\vec{U}_i^{*(k+1)}} - \vec{U}_i^{(k+1)} \cdot Y_i^{'} \qquad za\ i = 1,2,...,n-1$$

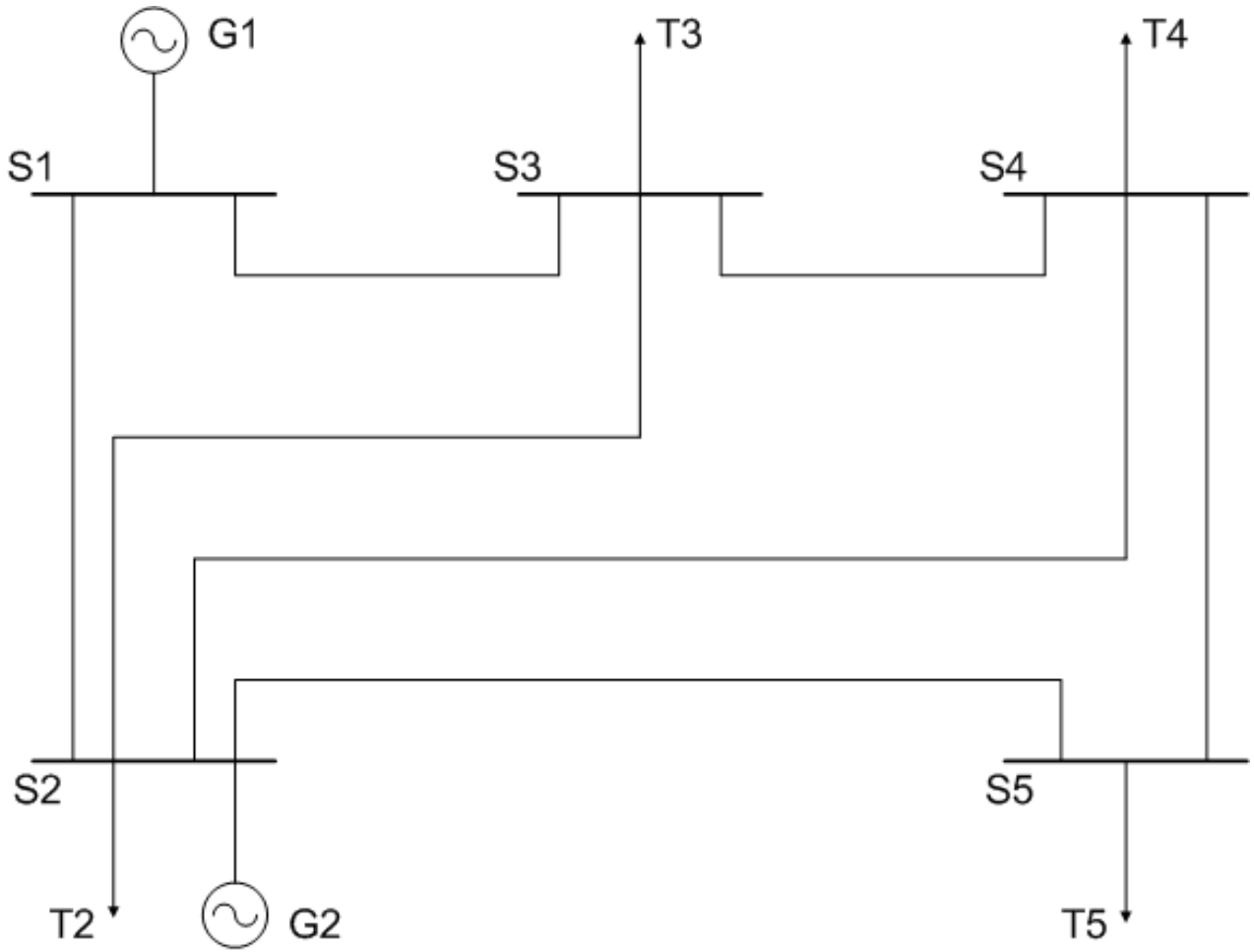
- Proračun se zaustavlja kada je ispunjeno:

$$\left| \left(\vec{U}_i^{(k+1)} - \vec{U}_i^{(k)} \right) \right| < \varepsilon \qquad za\ i = 1,2,...,n-1$$

- Traženo rješenje (vektor stanja) je:

$$\vec{U}_i^{(k+1)} = \left| \vec{U}_i^{(k+1)} \right| \angle \delta_i^{k+1}$$

- PRIMJER



- Zadano:

i	j	$\vec{Z}_{i-j}\ (p.u.)$	$y'_{i-j}/2\ (p.u.)$	$\vec{y}_{i-j}$
1	2	0.02+j0.06	j0.03	5-j15
1	3	0.08+j0.24	j0.025	1.25-j3.75
2	3	0.06+j0.18	j0.02	1.66-j5
2	4	0.06+j0.18	j0.02	1.66-j5
2	5	0.04+j0.12	j0.015	2.5-j7.5
3	4	0.01+j0.03	j0.01	10-j30
4	5	0.08+j0.24	j0.025	1.25-j3.75

- Vrijedi da je: $\vec{y}_{i-j} = \frac{1}{\vec{Z}_{i-j}}$ (uzdužna admitancija grane)
- Čvorište 1 je referentno

- Zadano:

Čv.	Generator			Teret		$\vec{Y}_i'$
	U	MW	Mvar			
1.	1.06+j0	/	/	/	/	j0.055
2.		40	30	20	10	j0.085
3.		0	0	45	15	j0.055
4.				40	5	j0.055
5.				60	10	j0.04

- Bazna snaga: $S_B = 100\ MVA$
- Tražena točnost: $\varepsilon = 0.001$

- Matrica **Y**:

$\vec{Y}_{(5)} =$

6.25-j18.75	-5+j15	-1.25+j3.75	0	0
-5+j15	10.833-j32.5	-1.66+j5	-1.66+j5	-2.5+j7.5
-1.25+j3.75	-1.66+j5	12.916-j38.75	-10+j30	0
0	-1.66+j5	-10+j30	12.916-j38.75	-1.25+j3.75
0	-2.5+j7.5	0	-1.25+j3.75	3.75-j11.25

- Čvorište 1 je referentno pa odbacujemo 1. red i 1. stupac

$\vec{Y} =$

10.833-j32.5	-1.66+j5	-1.66+j5	-2.5+j7.5
-1.66+j5	12.916-j38.75	-10+j30	0
-1.66+j5	-10+j30	12.916-j38.75	-1.25+j3.75
-2.5+j7.5	0	-1.25+j3.75	3.75-j11.25

- Matrica impedancija čvorišta:

$$\vec{Z} = \vec{Y}^{-1} =$$

0.016857+ j0.050571	0.012571+ j0.03771	0.013428+ j0.0402857	0.0151743+ j0.047143
0.012571+ j0.03771	0.0297143+ j0.089143	0.0262857+ j0.0788571	0.017143+ j0.0514286
0.013428+ j0.0402857	0.0262857+ j0.0788571	0.0317143+ j0.09514	0.0185238+ j0.0585714
0.0151743+ j0.047143	0.017143+ j0.0514286	0.0185238+ j0.0585714	0.043651+ j0.1309524

$$\vec{I}_2^{(0)} = \frac{\vec{S}_2^*}{\vec{U}_2^{*(0)}} - Y_2^{'} \cdot \vec{U}_2^{(0)} = \frac{0.2 - j0.2}{1 - j0} - j0.085 \cdot 1 = 0.2 - j0.285$$

$$\vec{I}_3^{(0)} = \frac{\vec{S}_3^*}{\vec{U}_3^{*(0)}} - Y_3^{'} \cdot \vec{U}_3^{(0)} = \frac{-0.45 + j0.15}{1 - j0} - j0.055 \cdot 1 = -0.045 + j0.095$$

$$\vec{I}_4^{(0)} = \frac{\vec{S}_4^*}{\vec{U}_4^{*(0)}} - Y_4^{'} \cdot \vec{U}_4^{(0)} = \frac{-0.4 + j0.05}{1 - j0} - j0.055 \cdot 1 = -0.4 - j0.005$$

$$\vec{I}_5^{(0)} = \frac{\vec{S}_5^*}{\vec{U}_5^{*(0)}} - Y_5^{'} \cdot \vec{U}_5^{(0)} = \frac{-0.6 + j0.1}{1 - j0} - j0.04 \cdot 1 = -0.6 + j0.06$$

- k=1:

$$\vec{U}_2^{(1)} = \vec{U}_1 + \vec{Z}_{22}\cdot\vec{I}_2^{(0)} + \vec{Z}_{23}\cdot\vec{I}_3^{(0)} + \vec{Z}_{24}\cdot\vec{I}_4^{(0)} + \vec{Z}_{25}\cdot\vec{I}_5^{(0)} = 1.05122 - j0.05399$$

$$\vec{I}_2^{(1)} = \frac{\vec{S}_2^{*}}{\vec{U}_2^{*(1)}} - Y_2^{'}\cdot\vec{U}_2^{(1)} = 0.17544 - j0.208887$$

$$\vec{U}_3^{(1)} = \vec{U}_1 + \vec{Z}_{32}\cdot\vec{I}_2^{(1)} + \vec{Z}_{33}\cdot\vec{I}_3^{(0)} + \vec{Z}_{34}\cdot\vec{I}_4^{(0)} + \vec{Z}_{35}\cdot\vec{I}_5^{(0)} = 1.02777 - j0.09581$$

$$\vec{I}_3^{(1)} = \frac{\vec{S}_3^{*}}{\vec{U}_3^{*(1)}} - Y_3^{'}\cdot\vec{U}_3^{(1)} = -0.42585 - j0.12813$$

$$\vec{U}_4^{(1)} = \vec{U}_1 + \vec{Z}_{42}\cdot\vec{I}_2^{(1)} + \vec{Z}_{43}\cdot\vec{I}_3^{(1)} + \vec{Z}_{44}\cdot\vec{I}_4^{(0)} + \vec{Z}_{45}\cdot\vec{I}_5^{(0)} = 1.02521 - j0.0992$$

$$\vec{I}_4^{(1)} = \frac{\vec{S}_4^{*}}{\vec{U}_4^{*(1)}} - Y_4^{'}\cdot\vec{U}_4^{(1)} = -0.38782 + j0.02933$$

$$\vec{U}_5^{(1)} = \vec{U}_1 + \vec{Z}_{52}\cdot\vec{I}_2^{(1)} + \vec{Z}_{53}\cdot\vec{I}_3^{(1)} + \vec{Z}_{54}\cdot\vec{I}_4^{(1)} + \vec{Z}_{55}\cdot\vec{I}_5^{(0)} = 1.01913 - j0.11403$$

$$\vec{I}_5^{(1)} = \frac{\vec{S}_5^{*}}{\vec{U}_5^{*(1)}} - Y_5^{'}\cdot\vec{U}_5^{(1)} = -0.57518 + j0.1212$$

- Provjera

$$\left|\vec{U}_2^{(1)} - \vec{U}_2^{(0)}\right| = \left|\left(1.05112 - j0.05399\right) - \left(1 + j0\right)\right|$$

$$= \left|0.05112 - j0.05399\right|$$

$$\left|0.05112 - j0.05399\right| > 0.001$$

- Uvjet je neispunjen (ostale napone nije ni potrebno dalje provjeravati)-prelazak u sljedeću iteraciju
- k=2

$$\vec{U}_2^{(2)} = \vec{U}_1 + \vec{Z}_{22} \cdot \vec{I}_2^{(1)} + \vec{Z}_{23} \cdot \vec{I}_3^{(1)} + \vec{Z}_{24} \cdot \vec{I}_4^{(1)} + \vec{Z}_{25} \cdot \vec{I}_5^{(1)}$$

$$\vec{I}_2^{(2)} = \frac{\vec{S}_2^*}{\vec{U}_2^{*(2)}} - Y_2^{'} \cdot \vec{U}_2^{(2)}$$

$$\vdots$$

Iteracija	Čv. 2	Čv. 3	Čv. 4	Čv. 5
0.	1+j0	1+j0	1+j0	1+j0
1.	1.05112-j0.05399	1.02777-j0.09581	1.02529-j0.0992	1.01913-j0.11403
2.	1.04622-j0.05286	1.02041-j0.08837	1.01924-j0.09454	1.0122-j0.10841
3.	1.04622-j0.05129	1.02035-j0.08924	1.01918-j0.09502	1.01212-j0.10908
	1.04748 ∠-28°	1.02425 ∠-5°	1.02361 ∠-5.33°	1.018 ∠-6.15°

- U trećoj iteraciji je zadovoljen postavljeni kriterij točnosti