Numerička analiza elektroenergetskog sustava

- AUDITORNE VJEŽBE -

Numerička analiza elektoenergetskih sustava

Izdanje: Siječanj 2007.

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Vježbe: dr. sc. Marko Delimar

Ovaj "radni materijal" predstavlja kratki zapis dijela gradiva i zadataka koji se obrađuju u sklopu auditornih vježbi predmeta *Numerička analiza elektroenergetskih* sustava po nastavnom programu FER I. Dio gradiva obuhvaćen je samo zadacima, a dio i malim objašnjenjem ili analizom.

Preporučena literatura:

Marija Ožegović, Karlo Ožegović: *Električne energetske mreže I-VI*, FESB Split, 1996-2006.
G. W. Stagg, A. H. El-Abiad: *Computer Methods in Power System Analysis*, McGraw-Hill, 1968.
B. Stefanini, S. Babić, M. Urbiha-Feuerbach: *Matrične metode u analizi električnih mreža*, Školska knjiga, Zagreb, 1975.

Popis oznaka i kratica

φ	kut napona [°] [rad]	P	djelatna snaga [W] [pu]
B	susceptancija ($2\pi fC$) [S]	Q	jalova snaga [var] [pu]
C	kapacitet [F]	R	otpor [Ω]
G	vodljivost [S]	S	prividna snaga [VA] [pu]
I	struja [A] [pu]	U	linijski napon, napon [V] [pu]
L	induktivitet [H]	V	fazni napon [V]
		X	reaktancija ($2\pi fL$) [Ω] [pu]
		Y	admitancija (G+jB) [S] [pu]
		Z	impedancija (R+jX) [Ω] [pu]
		z^*	konjugirano kompleksni broj

Metoda za proračun mreža

Metoda otpora	Metoda reduciranih admitancija	Metoda p.u. (per unit)
$U' = \frac{U_B}{U_n} \cdot U$	$U_r = \frac{U}{U_n}$	$U_{[p.u.]} = \frac{U}{U_n}$
$I' = \frac{\sqrt{3} \cdot U_n}{U_B} \cdot I$	$I_r = \sqrt{3} \cdot U_n \cdot I$	$I_{[p.u.]} = \frac{S}{S_B} \cdot \frac{U_n}{U} = \frac{\sqrt{3} \cdot U_n}{S_B} \cdot I$
$Z' = \left(\frac{U_B}{U_n}\right)^2 \cdot Z$	$Z_r = \frac{Z}{U_n^2}$	$Z_{[p.u.]} = \frac{S_B}{U_n^2} \cdot Z$
$Y' = \left(\frac{U_n}{U_B}\right)^2 \cdot Y$	$Y_r = U_n^2 \cdot Y$	$Y_{[p.u.]} = \frac{U_n^2}{S_B} \cdot Y$

Teorem superpozicije (za aktivne mreže)

Svaka EMS proizvodi u linearnoj mreži struje nezavisno od struja proizvedenih nekim drugim EMS-om. Konačno stanje dobiva se sumiranjem pojedinačnih struja i analogno pojedinačnih padova napona.

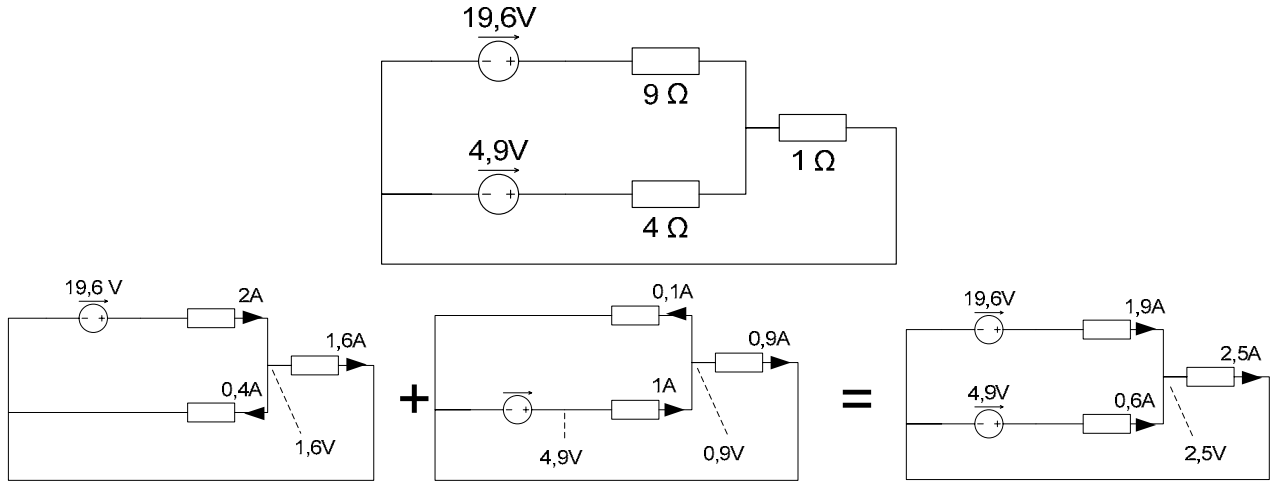
Primjena:

a) samo za linearne mreže
b) za heterogene struje (izmjenične i istosmjerne, simetrične i nesimetrične, različitih frekvencija)

Postupak:

1) uzimamo samo jedan izvor, a druge kratko spojimo
2) izračunamo struje po granama i napone u čvorištima
3) prve 2 točke ponovimo za sve ostale izvore
4) zbrojimo struje u granama od pojedinih izvora i napone po čvorištima

Primjer:



$$R = \frac{1 \cdot 4}{1+4} + 9 = \frac{49}{5}$$

$$19,6 = \frac{196}{100}$$

$$I = \frac{\frac{196}{100}}{\frac{49}{5}} = 2A$$

$$R = \frac{1 \cdot 9}{1+9} + 4 = \frac{49}{10}$$

$$I = \frac{\frac{49}{10}}{\frac{49}{10}} = 1A$$

Teorem kompenzacije (za aktivne mreže)

Ako se u jednoj grani mreže poveća impedancija za $\Delta\vec{Z}$, promjena struja u svim granama bit će jednaka struji, koju daje izvor $\Delta\vec{Z}\cdot\vec{I}$, spojen u seriju s promijenjenom granom kojom je prije tekla struja $\vec{I}$.

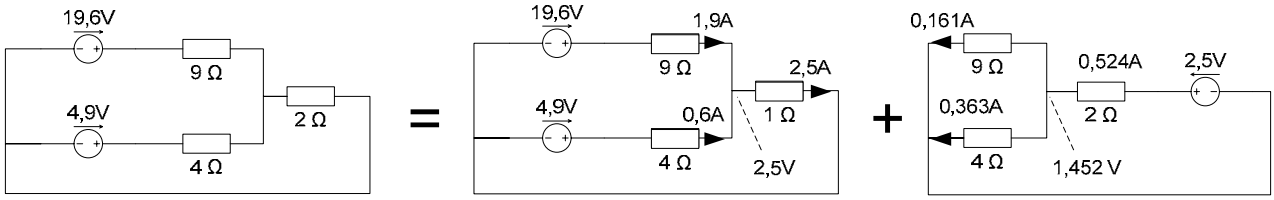
Primjena:

a) za mreže koje su već analizirane, samo nam se sada jedna impedancija promijenila
b) za aktivne i linearne mreže

Postupak:

1) postaviti stanje koje je prije izračunato nekom od metoda
2) tom stanju dodati struje u granama i napone u čvorištima izračunate samo za izvor $\Delta\vec{Z}\cdot\vec{I}$ (ostali su kratko spojeni)

Primjer:

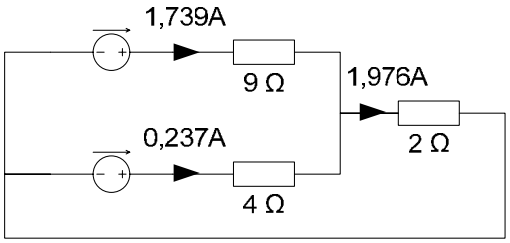


Nova EMS = $\Delta Z\cdot I=\left(Z_{poč}-Z_{kon}\right)\cdot I=\left(1-2\right)\cdot 2,5=-2,5V$

$$R=\frac{4\cdot 9}{4+9}+2=\frac{62}{13}=4,769\Omega$$

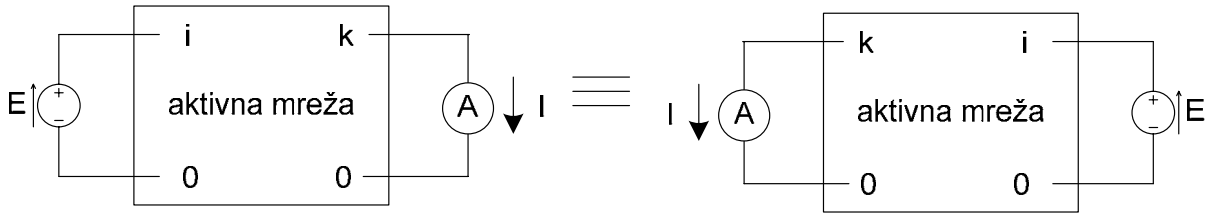
$$I=\frac{2,5}{4,796}=0,524A \quad ; \quad 0,524\cdot 2=1,048 \quad ; \quad 2,5-1,048=1,452V$$

$$I_1=\frac{1,452}{9}=0,161 \quad ; \quad I_2=\frac{1,452}{4}=0,363$$



Teorem reciprociteta

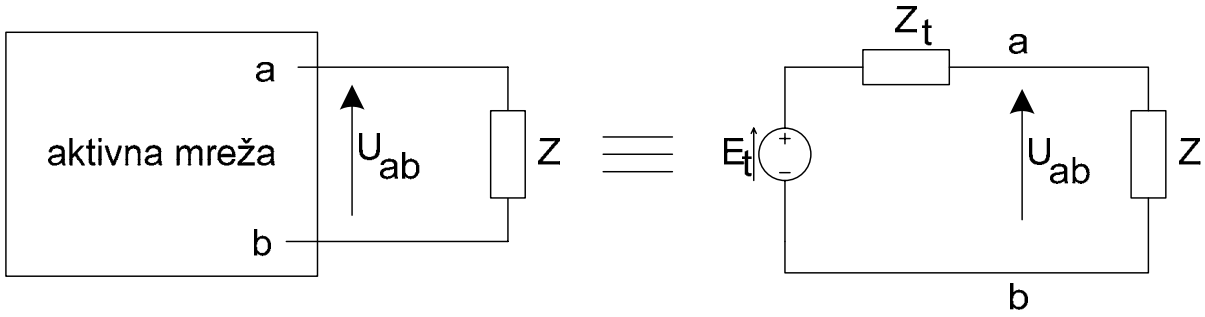
Neka u pasivnoj mreži djeluje samo jedna EMS na odabranom paru čvorova. Ampermetar priključen na neki drugi par čvorava pokazat će određeni otklon. Ako zamijenimo ampermetar i EMS otklon će biti isti.



Teorem vrijedi i ako umjesto naponskog izvora E uzmemo strujni izvor I, a na drugom kraju mjerimo napon V.

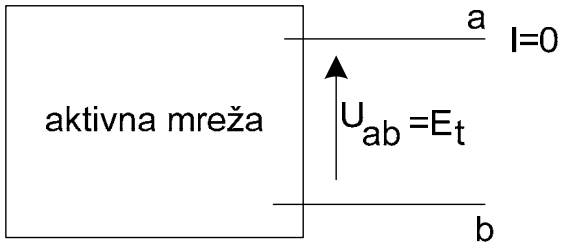
Theveninov teorem

Svaku stvarnu aktivnu mrežu promatranu iz 2 čvora možemo nadomjestiti fiktivnom mrežom, koja je serijski spoj impedancije $\overrightarrow{Z_T}$ i EMS-e $\overrightarrow{E_T}$, a naziva se Theveninov ekvivalent.

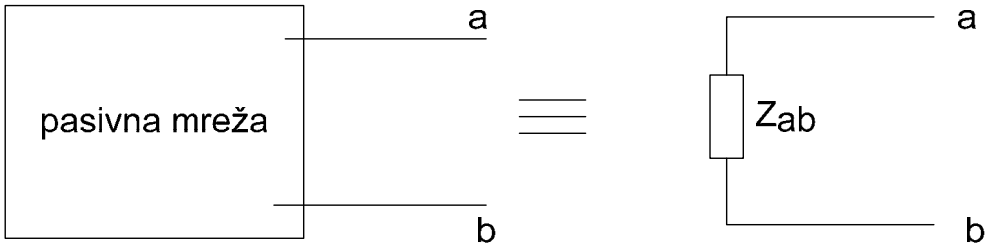


Postupak: Odrediti $\overrightarrow{E_T}$ i $\overrightarrow{Z_T}$

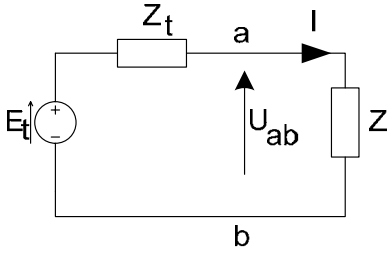
1) $\overrightarrow{E_T}$ određujemo tako da otvorimo mrežu na čvorištima a i b i izmjerimo taj napon U_{ab}



2) $\overrightarrow{Z_T}$ odredimo tako da aktivnoj mreži kratko spojimo sve EMS-ove i dobijemo pasivnu mrežu, a zatim izmjerimo impedanciju između čvorišta a i b



Theveninov ekvivalent:



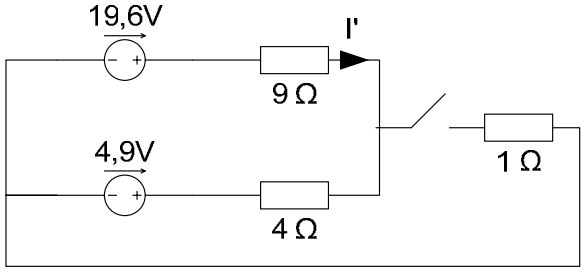
$$\vec{I} = \frac{\vec{E}_T}{\vec{Z}_T + \vec{Z}}$$

$$\vec{U}_{ab}' = \vec{E}_T - I\vec{Z}_T = \vec{E}_T - \frac{\vec{E}_T}{\vec{Z}_T + \vec{Z}} \cdot \vec{Z}_T = \vec{E}_T \left(1 - \frac{\vec{Z}_T}{\vec{Z}_T + \vec{Z}}\right)$$

$$U_{ab}' = \vec{E}_T \frac{\vec{Z}}{\vec{Z}_T + \vec{Z}}$$

Sa energetskog stajališta aktivna mreža i Theveninov ekvivalent ne smiju se poistovjetiti jer u slučaju praznog hoda tj. otvorenih stezaljki a i b u Theveninovu ekvivalentu ne teče nikakva struja, dok u aktivnoj mreži teče struja te postoje i gubici.

Primjer:



1) Određivanje $\overrightarrow{E_T}$

$$\vec{I}' = \frac{19,6-4,9}{13} = \frac{14,7}{13} = 1,1307 \quad ; \quad \vec{I} \cdot 9 = 10,1736$$

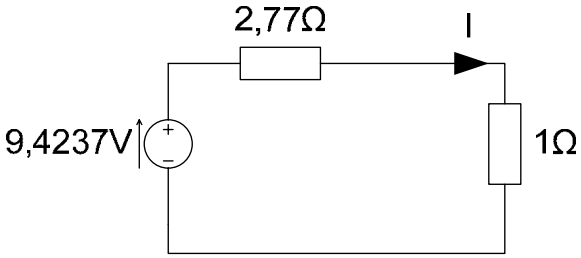
$$\vec{E}_T = 19,6 - 10,1763 = 9,4237V$$

2) Određivanje $\overrightarrow{Z_T}$

$$9 \,||\, 4 = \vec{Z}_T = \frac{4 \cdot 9}{4+9} = \frac{36}{13} = 2,77\Omega$$

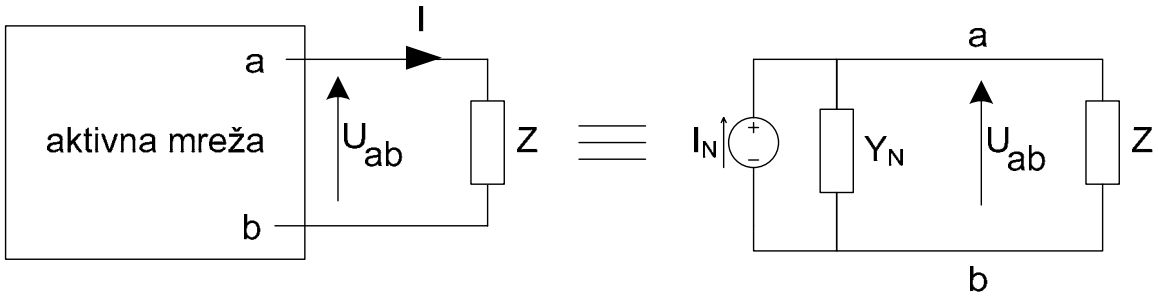
$$\vec{I} = \frac{\vec{E}_T}{\vec{Z}_T + \vec{Z}} = \frac{9,4237}{2,77+1} = 2,5A$$

$$\vec{U}_{ab}' = \frac{\vec{E}_T}{\vec{Z}_T + \vec{Z}} \cdot \vec{Z} = 2,5 \cdot 1 = 2,5V$$



Nortonov teorem (za aktivne mreže)

Svaku stvarnu aktivnu mrežu promatranu iz 2 čvora možemo nadomjestiti fiktivnom mrežom, koja je paralelni spoj admitancije $\vec{Y}_N$ i strujnog izvora $\vec{I}_N$



Postupak:

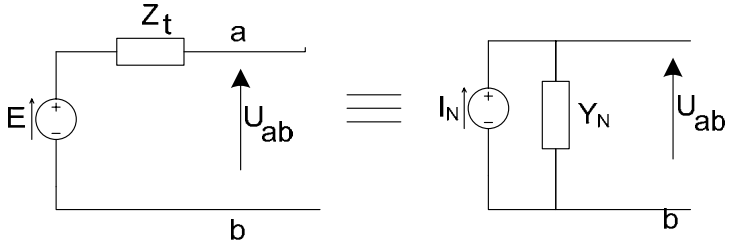
Odrediti $\vec{I}_N$ i $\vec{Y}_N$:

1) $\vec{I}_N$ odrediom tako da u aktivnoj mreži kratko spojimo stezaljke a i b te izmjerimo struju $\vec{I}_N$
2) $\vec{Y}_N$ odredimo tako da sve strujne izvore otspojimo, a naponske kratko spojimo, te izmjerimo admitanciju takve pasivne mreže

2. način:

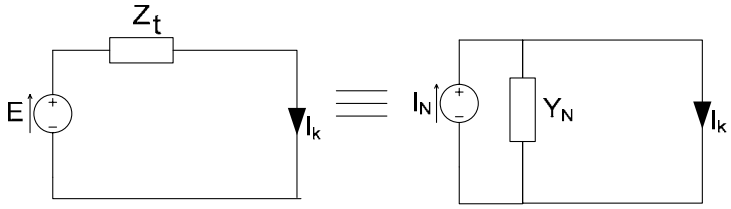
Odredimo Theveninov ekvivalent $\overrightarrow{E_T}$ i $\overrightarrow{Z_T}$ i zatim uz pomoć pokusa P.H. i K.S. odredimo Nortonov ekvivalent

P.H.:



$$\vec{U}_{ab} = \vec{E}_T$$
$$\vec{I}_N = \vec{U}_{ab} \cdot \vec{Y}_N = \vec{E}_T \cdot \vec{Y}_N \quad (1)$$

K.S.:



$$\vec{I}_K = \vec{I}_N$$
$$\vec{E}_T = \vec{Z}_T \vec{I}_K = \vec{Z}_T \vec{I}_N$$
$$\vec{I}_N = \frac{\vec{E}_T}{\vec{Z}_T} \quad (2)$$

Iz izraza (1) i (2) slijedi: $\vec{I}_N = \vec{E}_T \cdot \vec{Y}_N$

$$\vec{I}_N = \frac{\vec{E}_T}{\vec{Z}_T}$$

$$\vec{Z}_T = \frac{1}{\vec{Y}_N}$$

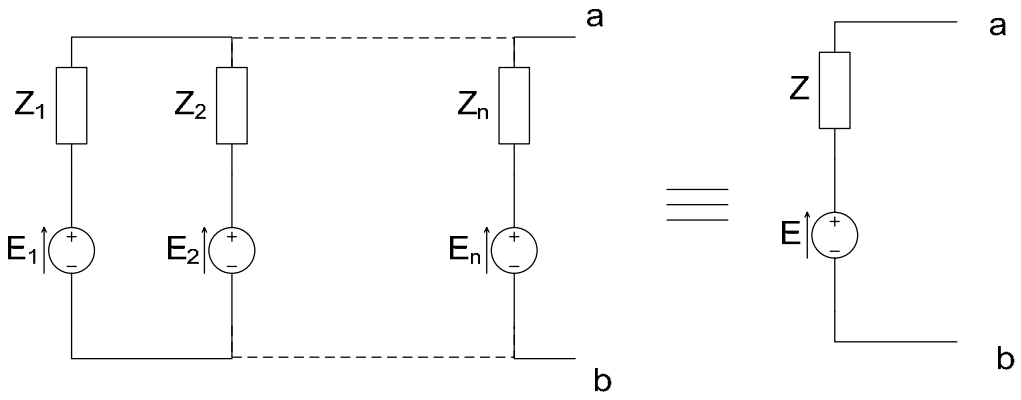
Dakle dobili smo:

$$\boxed{\vec{I}_N = \frac{\vec{E}_T}{\vec{Z}_T} \quad ; \quad \vec{Y}_N = \frac{1}{\vec{Z}_T}}$$

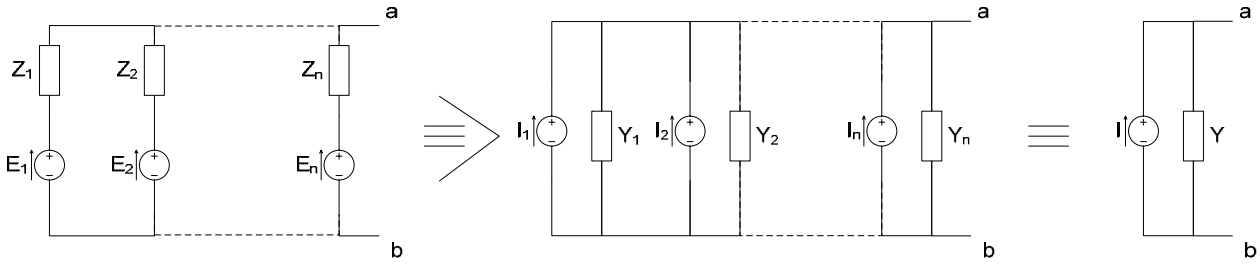
Millmanov teorem (za aktivne mreže)

Kompletnu aktivu mrežu ekvivalentiramo uz pomoć Nortonovog i Theveninovog teorema u impedantni ili admitantni oblik.

1. način:



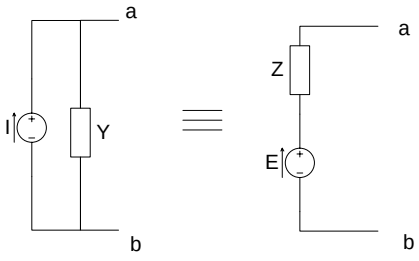
2. način (bolji):



$$\vec{I}_i = \frac{\vec{E}_i}{\vec{Z}_i} \qquad \vec{I} = \sum_{i=1}^{n} \vec{I}_i = \sum_{i=1}^{n} \frac{\vec{E}_i}{\vec{Z}_i}$$

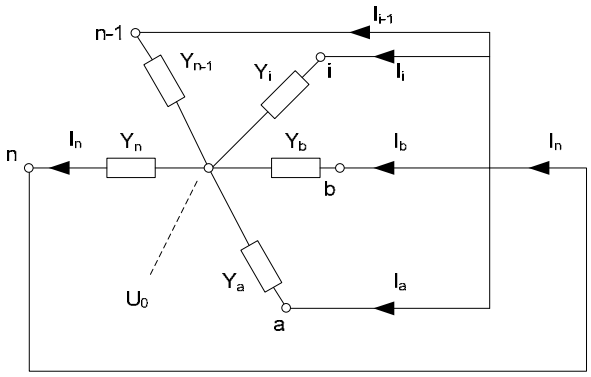
$$\vec{Y}_i = \frac{1}{\vec{Z}_i} \qquad \vec{Y} = \sum_{i=1}^{n} \vec{Y}_i$$

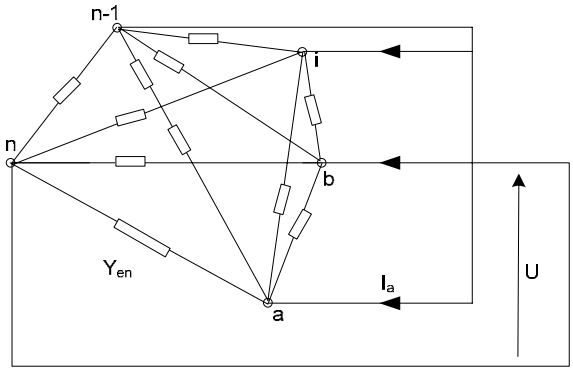
Zatim se ponovo možemo vratiti na impedantni oblik



$$\boxed{\vec{E} = \frac{\vec{I}}{\vec{Y}} = \frac{\sum_{i=1}^{n} \frac{\vec{E}_i}{\vec{Z}_i}}{\sum_{i=1}^{n} \frac{1}{\vec{Z}_i}}} \qquad \boxed{Z = \frac{1}{\vec{Y}} = \frac{1}{\sum_{i=1}^{n} \frac{1}{\vec{Y}_i}}}$$

Izrazi za pretvorbu opće zvijezde u opći poligon





$$I_n = \frac{\left(Y_a + Y_b + ... + Y_i + Y_{n-1}\right) \cdot Y_n}{\left(Y_a + Y_b + ... + Y_i + Y_{n-1}\right) + Y_n} \cdot U = \frac{\left(Y_a + Y_b + ... + Y_i + Y_{n-1}\right) \cdot Y_n}{\sum_{i=a}^{n} Y_i} \cdot U$$

$$I_a = \left(U - U_0\right) Y_a \qquad ; \qquad I_n = \left(U - U_0\right) \cdot \left(Y_a + Y_b + ... + Y_i + ... + Y_{n-1}\right)$$

$$\frac{I_a}{I_n} = \frac{\left(U - U_0\right) Y_a}{\left(U - U_0\right) \cdot \left(Y_a + Y_b + ... + Y_i + ... + Y_{n-1}\right)} = \frac{Y_a}{\left(Y_a + Y_b + ... + Y_i + ... + Y_{n-1}\right)}$$

$$I_a = I_n \frac{Y_a}{\left(Y_a + Y_b + ... + Y_i + ... + Y_{n-1}\right)} = \frac{\left(Y_a + Y_b + ... + Y_i + ... + Y_{n-1}\right) Y_n \cdot Y_a}{\sum_{i=1}^{n} Y_a \cdot \left(Y_a + Y_b + ... + Y_i + ... + Y_{n-1}\right)} \cdot U$$

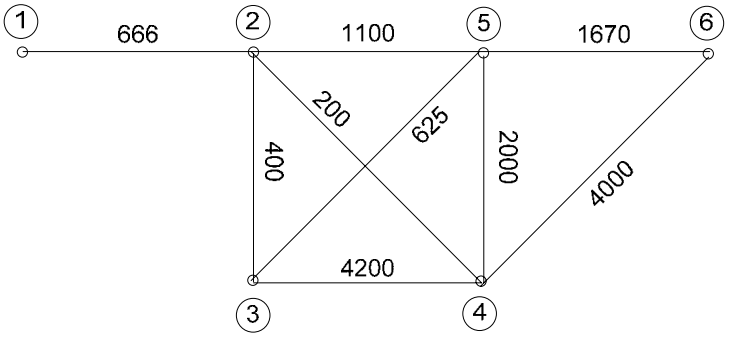
$$I_a = U \cdot \frac{Y_n \cdot Y_a}{\sum_{i=a}^{n} Y_i} \Rightarrow$$

$$\Rightarrow Y_{an} = \frac{Y_n \cdot Y_a}{\sum_{i=a}^{n} Y_i}$$

$$I_a = U \cdot Y_{en} \qquad \Rightarrow$$

$$\boxed{Y_{jk} = \frac{Y_j \cdot Y_k}{\sum_{i=a}^{n} Y_i}}$$

Primjer za transfiguraciju mreže



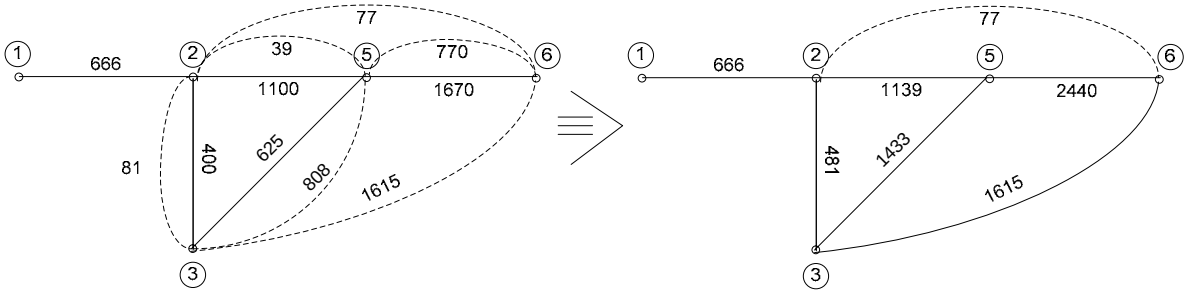
(zadržati čvorišta 1, 3 i 6)

Čvorište 4:

$$\sum Y = 4200 + 2000 + 4000 + 200 = 10400$$

$$Y_{23} = \frac{4200 \cdot 200}{10400} = 81 \quad ; \qquad Y_{25} = \frac{200 \cdot 2000}{10400} = 39 \quad ; \qquad Y_{26} = \frac{200 \cdot 4000}{10400} = 77$$

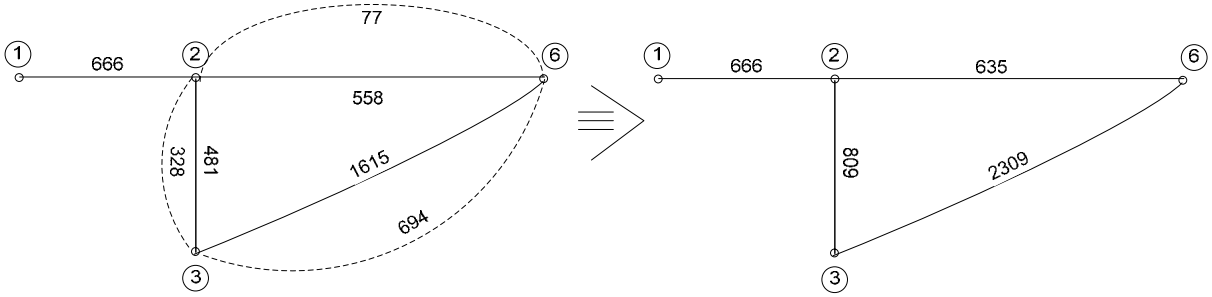
$$Y_{35} = \frac{4200 \cdot 2000}{10400} = 808; \qquad Y_{36} = \frac{4200 \cdot 4000}{10400} = 1615; \qquad Y_{56} = \frac{2000 \cdot 4000}{10400} = 770$$



Čvorište 5:

$$\sum Y = 1139 + 2440 + 1433 = 5012$$

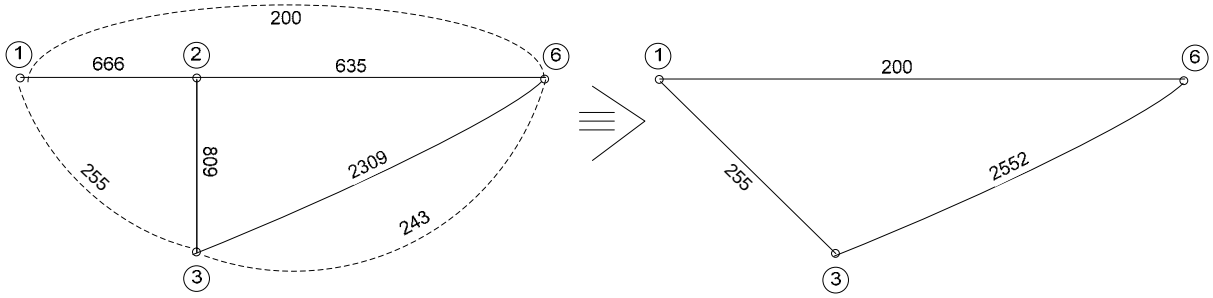
$$Y_{23} = \frac{1139 \cdot 1439}{5012} = 328 \qquad ; \qquad Y_{26} = \frac{1139 \cdot 2440}{5012} = 538; \qquad Y_{36} = \frac{1499 \cdot 2440}{5012} = 694$$



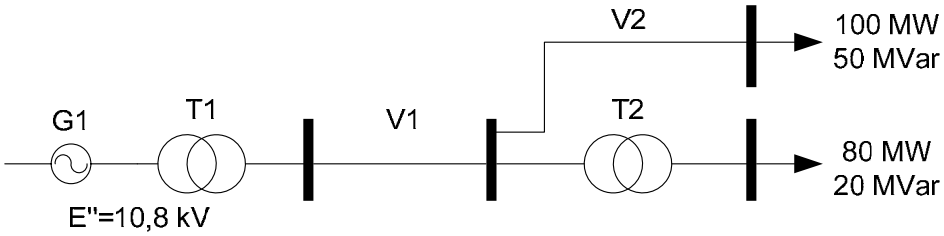
Čvorište 2:

$$\sum Y = 666 + 635 + 809 = 2100$$

$$Y_{13} = \frac{666 \cdot 809}{2100} = 255 \quad ; \qquad Y_{16} = \frac{666 \cdot 635}{2100} = 200; \; Y_{36} = \frac{809 \cdot 635}{2100} = 243$$



ZADATAK 1 Za prijenosni sustav prikazan na slici zadani su podaci za pojedine elemente.



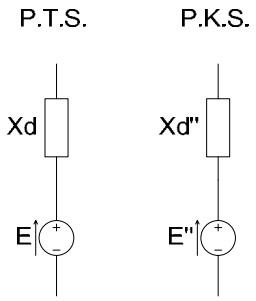
G1	$S_n = 300MVA$	**T1**	$S_n = 300MVA$	**T2**	$S_n = 100MVA$
	$X_d'' = 10\%$		$U_{n1}/U_{n2} = 220kV/10,5kV$		$U_{n1}/U_{n2} = 220kV/110kV$
	$U_n = 10,5kV$		$P_{Cu} = 1MW$		$P_{Cu} = 1,5MW$
	$E'' = 10,8kV$		$u_k = 12\%$		$u_k = 10\%$
V1	$l = 100km$	**V2**	$l = 80km$		
	$X_1 = 0,4\Omega/km$		$X_1 = 0,35\Omega/km$		
	$R_1 = 0,1\Omega/km$		$R_1 = 0,05\Omega/km$		

a) Metodom otpora preračunaj sve elemente zadanog prijenosnog sustava na bazni napon 100 kV
b) Uz pomoć metode p.u. preračunaj na baznu snagu $S_B = 100MVA$ elemente zadanog prijenosnog sustava

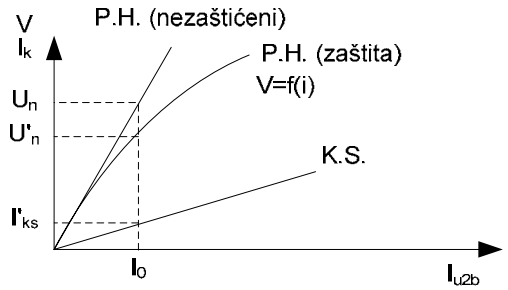
Rješenje:

Zanemarenja:

1. **G1 – turbogenerator**
   - za potrebe proračuna mreža:



   - R generatora je zanemaren
   - definicija sinhrone reaktacije:



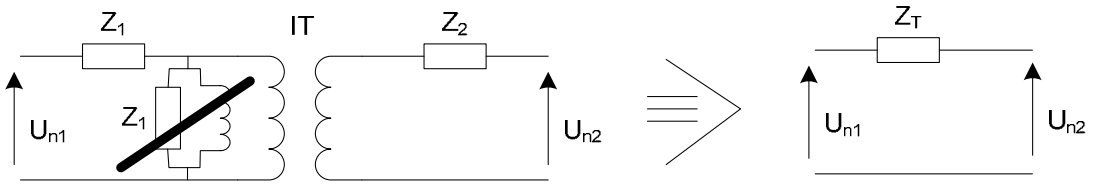
$$X_n = \frac{U_n}{I_n}, \qquad X_d[\%] = \frac{X_d[\Omega]}{X_n} \cdot 100 = \frac{\frac{U_n}{I_{ks}'}}{\frac{U_n}{I_n}} \cdot 100 = \frac{I_n}{I_{ks}'} \cdot 100$$

$$X_d[\%] = \frac{I_n}{I_{ks}'} \cdot 100 \cdot \frac{3U_n}{3U_n} = \frac{V_n}{I_{ks}'} \cdot \frac{3I_n}{3U_n} \cdot 100 = X_d[\%] \cdot \frac{3I_n}{3U_n} \cdot 100$$

$$X_d[\%] = X_d[\Omega] \cdot \frac{3U_n V_n}{3U_n^{\,2}} \cdot 100$$

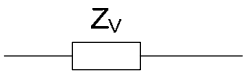
$$\boxed{\begin{aligned} X_d[\%] &= X_d[\Omega] \cdot \frac{S_n}{U_n^{\,2}} \cdot 100 \\ X_d[\Omega] &= \frac{X_d[\%]}{100} \cdot \frac{U_n^{\,2}}{S_n} \end{aligned}}$$

2. **T1, T2**
   - transformator nazivnog prijenosnog omjera
   - zanemareni su gubici praznog hoda (struja magnetiziranja i gubici u Fe)
   - nema poprečnih grana u modelu transformatora



$$Z_T = \frac{U_{n1}^{\,2}}{S_n}\left[\frac{P_{Cu}}{S_n} + j\sqrt{u_k^{\,2} - \left(\frac{P_{Cu}}{S_n}\right)^2}\right]$$

3. V1, V2
   - zanemarene su poprečne grane (admitancije)
   - model voda:



$$Z_V = Z_{V1} \cdot l$$

$$X_g[\Omega] = \frac{X_d''[\%]}{100} \cdot \frac{U_n^2}{S_n} = \frac{10}{100} \cdot \frac{10,5^2}{300} \left[\frac{(kV)^2}{MVA} = \frac{MV^2}{MVA} = \Omega \right]$$

$$X_g[\Omega] = 0,03675\Omega$$

$$Z_{T1}[\Omega] = \frac{U_{n2}^2}{S_n} \left[\frac{P_{Cu}}{S_n} + j\sqrt{u_k^2 + \left(\frac{P_{Cu}}{S_n}\right)^2} \right] = \frac{220^2}{300} \left[\frac{1}{300} + j\sqrt{0,12^2 - \left(\frac{1}{300}\right)^2} \right]$$

$$Z_{T1}[\Omega] = 0,537 + j19,352[\Omega]$$

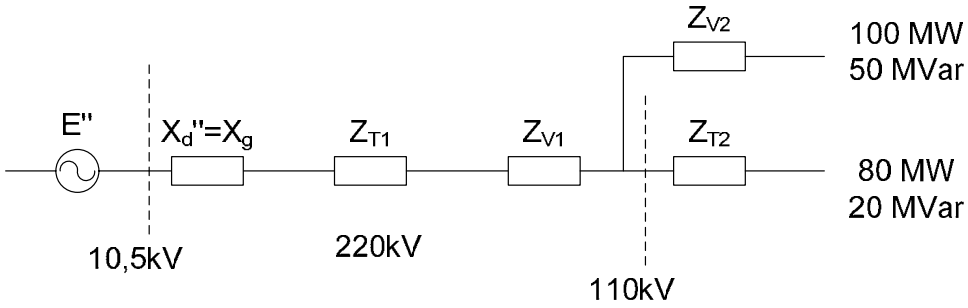
$$Z_{V1} = Z_1 \cdot l_1 = (0,1 + 0,4) \cdot 100 = 10 + j40[\Omega]$$

$$Z_{V2} = Z_2 \cdot l_2 = (0,05 + 0,35) \cdot 180 = 4 + j28[\Omega]$$

$$Z_{T2} = \frac{U_{n2}^2}{S_n} \left[\frac{P_{Cu}}{S_n} + j\sqrt{u_k^2 + \left(\frac{P_{Cu}}{S_n}\right)^2} \right] = \frac{110^2}{100} \left[\frac{1,5}{100} + j\sqrt{0,1^2 - \left(\frac{1,5}{100}\right)^2} \right]$$

$$Z_{T2} = 1,815 + j11,963$$

Model:



a)

$$U_B = 100kV$$

$$U_g' = \frac{U_B}{U_n} \cdot E'' = \frac{100}{10,5} \cdot 10,8 = 102,857kV$$

$$X_g' = \left(\frac{U_B}{U_n}\right)^2 \cdot X_g = \left(\frac{100}{10,5}\right)^2 \cdot 0,03675 = j3,333\Omega$$

$$Z_{T1}' = \left(\frac{U_B}{U_n}\right)^2 \cdot Z_{T1} = \left(\frac{100}{220}\right)^2 \cdot (0,537 + j19,352) = 0,1109 + j3,998\Omega$$

$$Z_{V1}' = \left(\frac{U_B}{U_n}\right)^2 \cdot Z_{V1} = \left(\frac{100}{220}\right)^2 \cdot (10 + j40) = 2,066 + j8,2645\Omega$$

$$Z_{V2}' = \left(\frac{U_B}{U_n}\right)^2 \cdot Z_{V2} = \left(\frac{100}{220}\right)^2 \cdot (4 + j28) = 0,8264 + j5,785\Omega$$

$$Z_{T2}' = \left(\frac{U_B}{U_n}\right)^2 \cdot Z_{T2} = \left(\frac{100}{110}\right)^2 \cdot (1,815 + j11,963) = 1,5 + j9,8868\Omega$$

b)

$$S_B = 100MVA$$

$$U_g\left[p.u.\right] = \frac{E''}{U_B} = \frac{E''}{U_n} = \frac{10,8}{10,5} = 1,02857\ \ p.u.$$

$$X_g\left[p.u.\right] = X_g\left[\Omega\right] \cdot \frac{S_B}{U_n^{\ 2}} = 0,03675 \cdot \frac{100}{10,5^2} = \ j0,0333 p.u.$$

$$Z_{T1}\left[p.u.\right] = Z_{T1}\left[\Omega\right] \cdot \frac{S_B}{U_{n2}^{\ 2}} = \left(0,537 + \ j19,352\right) \cdot \frac{100}{220^2} = 0,001109 + \ j0,03998 p.u.$$

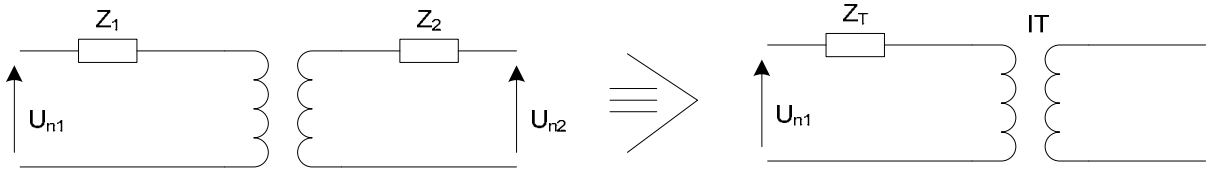
$$Z_{V1}\left[p.u.\right] = Z_{V1}\left[\Omega\right] \cdot \frac{S_B}{U_n^{\ 2}} = \left(10 + \ j40\right) \cdot \frac{100}{220^2} = 0,02066 + \ j0,082645 p.u.$$

$$Z_{V2}\left[p.u.\right] = Z_{V2}\left[\Omega\right] \cdot \frac{S_B}{U_n^{\ 2}} = \left(4 + \ j28\right) \cdot \frac{100}{220^2} = 0,008264 + \ j0,05785 p.u.$$

$$Z_{T2}\left[p.u.\right] = Z_{T2}\left[\Omega\right] \cdot \frac{S_B}{U_n^{\ 2}} = \left(1,815 + \ j11,963\right) \cdot \frac{100}{110^2} = 0,015 + \ j0,09887 p.u.$$

DODATAK:

Transformator



$Z_T = Z_1 + Z_2'$ $\qquad Z_2' = \left(\frac{U_1}{U_2}\right)^2 Z_2$ -> vrijedi za nominalni prijenosni omjer

Definicija: (iz pokusa k.s.)

$$u_k[\%] = \frac{I_n \cdot Z_T}{U_n/\sqrt{3}} \cdot 100$$

U_n – napon ove strane transformatora na kraju svoditi obje impedancije (u našem slučaju U_{n1})

U_k – napon na primaru da u slučaju kad je sekundar kratko spojen poteče kroz trafo I_n (U_k – fazni napon)

$$u_k[\%] = \frac{I_n \cdot Z_T}{U_n/\sqrt{3}} \cdot 100$$

$$Z_T = \frac{u_k[\%] \cdot U_n}{\sqrt{3} \cdot I_n \cdot 100} = \frac{u_k[\%] \cdot U_n^2}{S_n \cdot 100} = \frac{u_k[\%]}{100} \cdot \frac{U_n^2}{S_n} \qquad [\Omega]$$

Koliki je kut impedancije? Odredit ćemo ga pomoću mjerenja djelatne snage pri pokusu k.s.

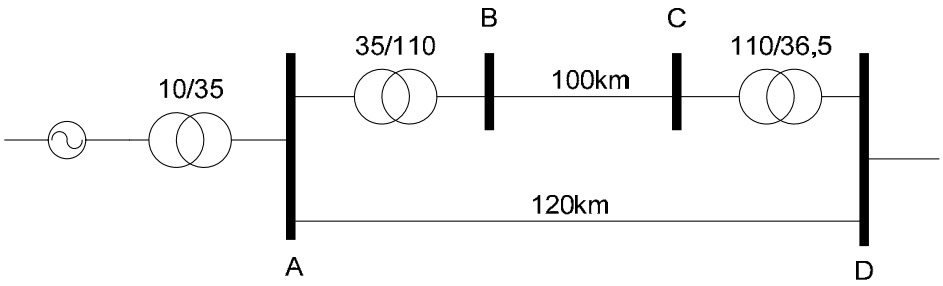
$$P_k = 3I_n^2 \cdot R = 3\left[\frac{S_n}{\sqrt{3} \cdot U_n}\right] \cdot R = \frac{S_n^2}{U_n^2} \cdot R$$

$$R = \frac{P_k}{S_n^2} \cdot U_n^2 \quad [\Omega] \qquad X = \sqrt{Z^2 - R^2} = \frac{U_n^2}{S_n}\sqrt{(u_k)^2 - \left(\frac{P_k}{S_n}\right)^2}$$

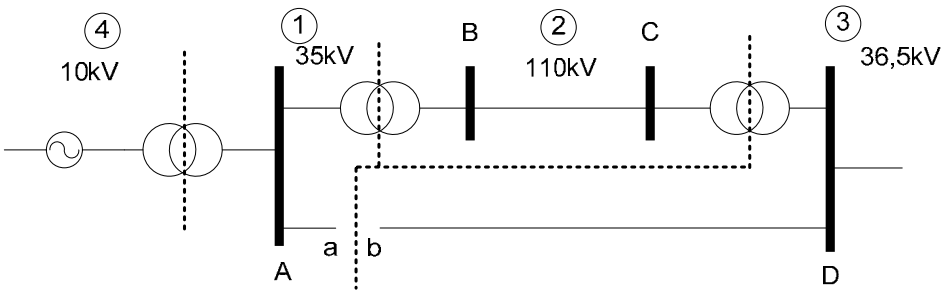
$$Z_T = \frac{P_k}{S_n^2} \cdot U_n^2 + j \cdot \frac{U_n^2}{S_n}\sqrt{(u_k)^2 - \left(\frac{P_k}{S_n}\right)^2}$$

$$\boxed{Z_T = \frac{U_n^2}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{(u_k)^2 - \left(\frac{P_k}{S_n}\right)^2}\right] \qquad [\Omega]}$$

ZADATAK 2 U mreži na slici odredi prilike u praznom hodu za $U_A = 34kV$. Za sve transformatore $u_k = 10\%$, a nazivna snaga je 20 MVA. (Stvarni prijenosni omjer transformatora jednak je nazivnom.)
Podaci generatora: $U_n = 10,5kV;\ S_n = 20MVA;\ X_d = 115\%$.
Reaktancija voda je $X_V = 0,4\Omega / km$.



Rješenje:



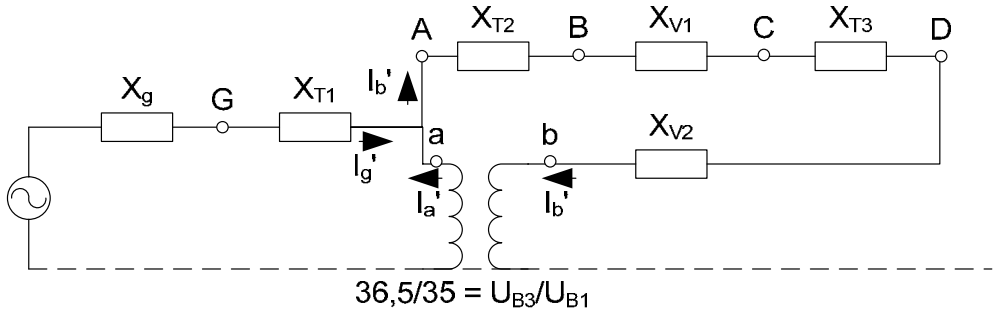
- prije otvaranja zamke vrijedi $\vec{U}_a = \vec{U}_b$
- nakon otvaranja zamke:

$\vec{U}_a = \vec{U}_a \cdot \frac{U_{B1}}{U_{B1}}$; $\vec{U}_b' = \vec{U}_b \frac{U_{B1}}{U_{B3}}$ (po metodi otpora, mrežu svodimo na U_{B1})

- kad je zamka zatvorena

$U_a = U_b \Rightarrow U_a' = U_b' \cdot \frac{U_{B3}}{U_{B1}} \Rightarrow \frac{U_a'}{U_b'} = \frac{U_{B3}}{U_{B1}} \neq 1$

- dakle, u reduciranoj mreži ($U_a' \neq U_b'$) moramo dodati na tom mjestu EMS, tj. idealni transformator sa prijenosnim omjerom U_{b3}/U_{b1}



Omjeri baznih napona:

$$\frac{U_{B1}}{U_{B2}} = \frac{35}{110} = 0,318 \qquad \frac{U_{B1}}{U_{B3}} = \frac{35}{36,5} = 0,959 \qquad \frac{U_{B1}}{U_{B4}} = \frac{35}{10} = 3,5$$

Stvarne reaktancije:

$$x_g = \frac{x_{d\%}}{100} \cdot \frac{U_n^{\ 2}}{S_n} = \frac{115}{100} \cdot \frac{10,5^2}{20} = 6,339\Omega$$

$$x_{T1} = x_{T2} = \frac{u_{k\%}}{100} \cdot \frac{U_n^{\ 2}}{S_n} = \frac{10}{100} \cdot \frac{35^2}{20} = 6,125\Omega$$

$$x_{V1} = 0,4 \cdot 100 = 40\Omega$$

$$x_{T3} = \frac{10}{100} \cdot \frac{110^2}{20} = 60,5\Omega$$

$$x_{V2} = 0,4 \cdot 120 = 48\Omega$$

Reducirane reaktancije

$$x_g' = x_g \cdot \left(\frac{U_{B1}}{U_{B4}}\right)^2 = 6,339 \cdot 3,5^2 = 77,653\Omega$$

$$x_{T1}' = x_{T2}' = 6,125 \cdot \left(\frac{U_{B1}}{U_{B1}}\right)^2 = 6,125\Omega$$

$$x_{V1}' = 40 \cdot \left(\frac{U_{B1}}{U_{B2}}\right)^2 = 40 \cdot 0,318^2 = 4,045\Omega$$

$$x_{T3}' = 60,5 \cdot 0,318^2 = 6,118\Omega$$

$$x_{V2}' = 48 \cdot 0,959^2 = 44,145\Omega$$

Proračun u reduciranoj mreži

$$U_A = U_A' = 34kV$$

$$U_B = 34 \cdot \frac{35}{36,5} = 32,6kV$$

$$U_A' - U_B' = 34 - 32,6 = 1,4kV$$

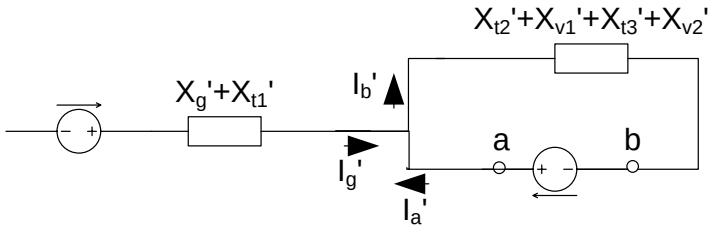
$$\sum X' = j60,433\Omega$$

$$I_b' = \frac{1,4}{j60,433} = -j23,2A$$

$$I_a' = I_b' \cdot \frac{35}{36,5} = -j22,25A$$

$$I_g' = I_b' - I_a'$$

$$I_g' = j23,2 + j22,25 = -j0,95A$$



$$U_a' = 34 + j0kV$$

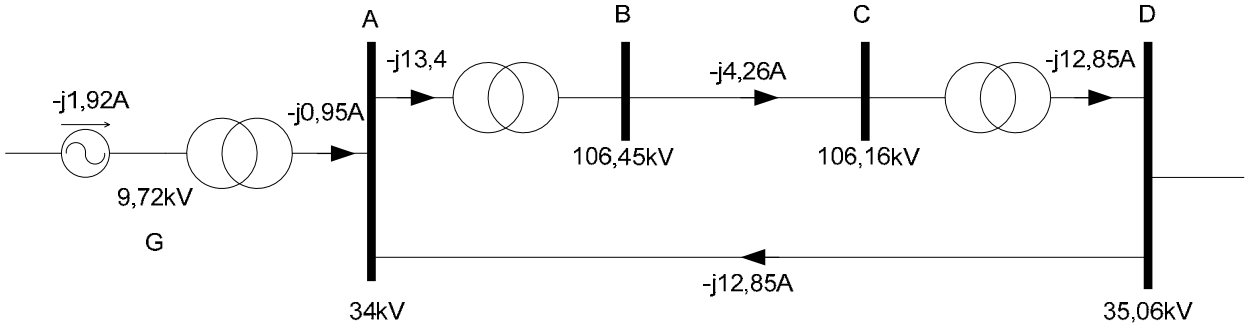
$$U_D' = 32,6 + (-j23,2)\cdot j44,145\cdot 10^{-3} = 33,62kV$$

$$U_C' = 33,62 + (-j23,2)\cdot j6,118\cdot 10^{-3} = 33,76kV$$

$$U_B' = 33,76 + (-j23,2)\cdot j4,045\cdot 10^{-3} = 33,85kV$$

$$U_A' = 33,85 + (-j23,2)\cdot j6,125\cdot 10^{-3} = 34kV$$

$$U_G' = 34 + (-j0,95)\cdot j6,125\cdot 10^{-3} = 34,006kV$$



Stvarne vrijednosti:

$$U_A = 34kV \qquad I_a = \frac{I_a'}{\sqrt{3}}\cdot\left(\frac{U_{B1}}{U_{B1}}\right) = -j12,85A$$

$$I_b = \frac{I_b'}{\sqrt{3}}\cdot\left(\frac{U_{B1}}{U_{B3}}\right) = \frac{-j23,2}{\sqrt{3}}\cdot 0,959 = -j12,85A$$

$$I_g = \frac{-j0,95}{\sqrt{3}}\cdot\left(\frac{U_{B1}}{U_{B4}}\right) = -j1,92A \qquad I_{V1} = -j4,26A$$

$$U_B = U_B'\cdot\frac{1}{0,318} = 106,45kV \qquad U_C = U_C'\cdot\frac{1}{0,318} = 106,16kV$$

$$U_D = U_D'\cdot\frac{1}{0,959} = 35,06kV \qquad U_G = U_G'\cdot\frac{1}{3,5} = 9,72kV$$

Napomena:

Kod ove metode otpora koju koriste neki autori (npr. Ožegović) imamo jedan bazni napon na koji preračunavamo cijelu mrežu (to je u ovom zadatku U_{b1}=35 kV) i ostale bazne napone koji su određeni stvarnim prijenosnim omjerima transformatora (u ovom zadatku U_{b2} =110 kV, U_{b3} = 36,5 kV, U_{b4} = 10 kV).

Za razliku od metode otpora koju smo mi radili ovdje se reducirane reaktancije dobivaju od stvarnih po formuli:

$$X_i' = X_i\cdot\left(\frac{U_{B1}}{U_{Bi}}\right)^2$$

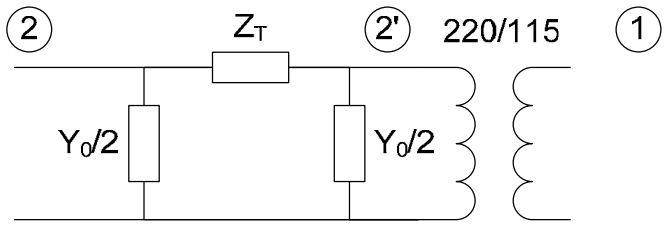
pri čemu je X_i (stvarna reaktancija u području napona „i“) računata na naponski nivo U_{Bi} koji ne mora biti jednak nazivnom naponu.

Jedino kod generatora se U_{Bi} ne uzima od transformatora već je određen podacima o generatoru.

ZADATAK 3 Zadan je transformator sa podacima prema listi.

a) Odredi parametre π-sheme tog transformatora koristeći model s idealnim transformatorom
b) Odredi parametre π-sheme tog transformatora koristeći model bez idealnog transformatora.

$S_n = 100MVA$
$u_k = 12\%$
$i_0 = 1\%$
$P_k = 1\% \;\; S_n = 0,01S_n$
$P_0 = 25\% \;\; P_k = 0,0025S_n$



Rješenje:

a)

Preračunava se na onu stranu na kojoj nije regulaciona sklopka tj. gdje je nazivni napon – obično je to VN strana (ovdje nije).

$$Z_T = \frac{U_n^{\,2}}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{u_k^{\,2} - \left(\frac{P_k}{S_n}\right)^2}\right] = \frac{220}{100}\left[\frac{0,01S_n}{S_n} + j\sqrt{0,12^2 - (0,01)^2}\right]$$

$$Z_T = 4,84 + j57,88[\Omega]$$

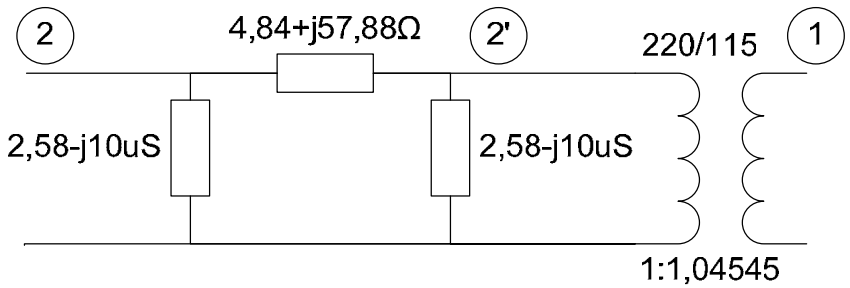
$$Y_T = \frac{1}{Z_T} = \frac{1}{58,082\underline{|85,22°}} = 0,0172\underline{|-85,22°} \;\; [S]$$

$$Y_0 = \frac{S_n}{U_n^{\,2}}\left[\frac{P_0}{S_n} - j\sqrt{i_0^{\,2} - \left(\frac{P_0}{S_n}\right)^2}\right] = \frac{110}{220^2}\left[\frac{0,0025S_n}{S_n} - j\sqrt{(0,01)^2 - 0,0025}\right]$$

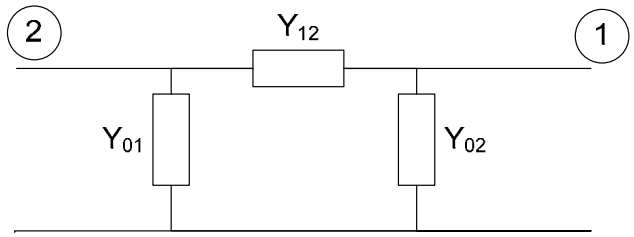
$$Y_0 = (5,16 - j20)\cdot 10^{-6} \;\; [S] = 5,16 - j20 \;\; [\mu S]$$

$$\frac{Y_0}{2} = 2,58 - j10[\mu S]$$

$$a = \frac{\frac{115}{220}}{\frac{110}{220}} = \frac{115}{110} = 1,04545$$



b)



$$Y_{12} = \frac{Y_T}{a}$$

$$Y_{01} = \frac{Y_T}{a}\left(\frac{1}{a} - 1\right) + \frac{1}{a^2}\frac{Y_0}{2}$$

$$Y_{02} = Y_T\left(\frac{1}{a} - 1\right) + \frac{1}{a^2}\frac{Y_0}{2}$$

$$Y_{12} = \frac{0,0172\underline{|-85,22°}}{1,04545} = 0,01645\underline{|-85,22°}\,[S] = 0,00137 - j0,01639[S]$$

$$Y_{01} = 0,01645\underline{|-85,22°}\left(\frac{1}{1,04545} - 1\right) + \frac{1}{1,04545^2}\cdot 10,327\underline{|-75,53°}\cdot 10^{-6} =$$

$$= 7,1515\cdot 10^{-4}\underline{|94,78°} + 9,445\cdot 10^{-6}\underline{|-75,53°} =$$

$$= (-5,96 + j71,27)\cdot 10^{-5} + (2,36 - j9,145)\cdot 10^{-6} =$$

$$= (-59,6 + j712,7 + 2,36 - j9,145)\cdot 10^{-6} =$$

$$= -57,25 + j703,555\mu S$$

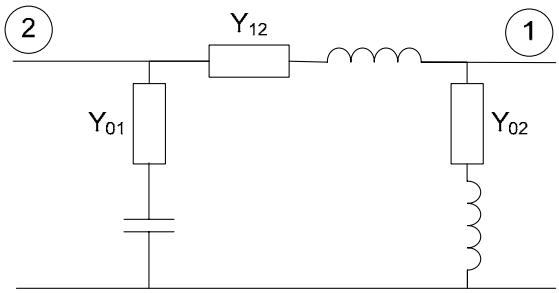
$$Y_{02} = 0,0172\underline{|-85,22°}\left(1 - \frac{1}{1,04545}\right) + (2,58 - j10)\cdot 10^{-6} =$$

$$= (6,23 - j74,515)\cdot 10^{-5} + (2,36 - j9,145)\cdot 10^{-6} =$$

$$= (62,3 - j74,515 + 2,58 - j10)\cdot 10^{-6} =$$

$$= (64,88 - j84,515)\cdot 10^{-6} =$$

$$= 64,88 - j84,515\mu S$$



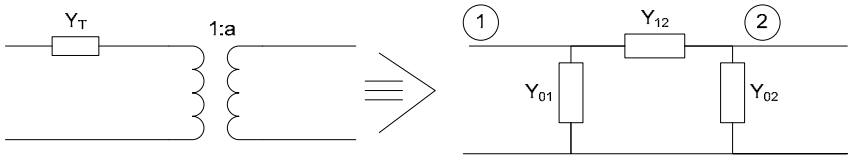
$$Y_{12} = 0,00137 - j0,01639[S]$$

$$Y_{01} = -57,24 + j703,555[\mu S]$$

$$Y_{02} = 64,88 - j84,515[\mu S]$$

Zanemarenja:

1) $\frac{Y_0}{2}$ **tj. i_0 i P_0**

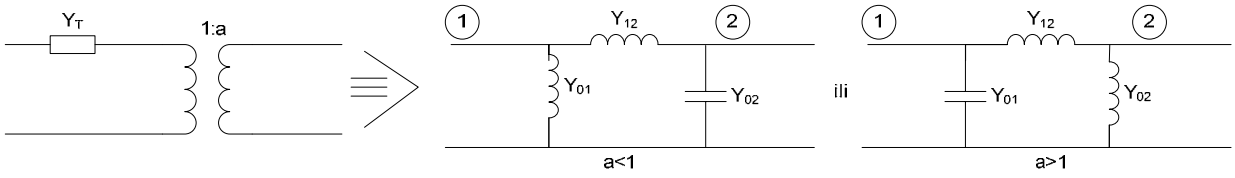


$$Y_{12} = \frac{Y_T}{a}$$

$$Y_{01} = \frac{Y_T}{a}\left(\frac{1}{a} - 1\right) + \frac{1}{a^2}\frac{Y_0}{2}$$

$$Y_{01} = Y_T\left(1 - \frac{1}{a}\right) + \frac{Y_0}{2}$$

2) Često zanemarimo i Pk



$$Z_T = j\frac{U_n^{\,2}}{S_n}u_k = jX_T$$

$$Y_T = -j\frac{1}{X_T} = -jB_T$$

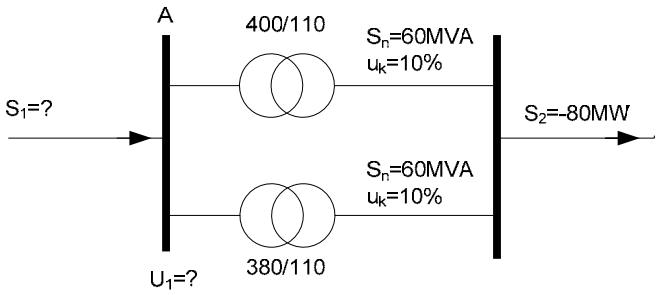
$$y_{01} = -j\ldots \qquad y_{02} = +j\ldots$$

$$y_{01} = +j\ldots. \qquad y_{02} = -j\ldots$$

ZADATAK 4 U transformatorskoj stanici paralelno su spojena 2 transformatora nazivne snage 60 MVA i napona kratkog spoja u_k=10%. Preklopka regulacione sklopke jednog od njih nalazi se na prijenosnom odnosu 400/110, a drugog 380/110. Odredi snagu i napon na primaru ako je napon na sekundaru jednak U_2=115kV i ako je transformatorska stanica opterećena sa S_2 = -80MW.

Rješenje:

Zadano:



Metoda *per unit*: $S_B = 60MVA$

Tr1:

$$a_1 = \frac{\frac{400}{110}}{\frac{400}{110}} = 1$$

$$y_{12}' = y_T = \frac{S_n}{S_B \cdot ju_k} = -j\frac{1}{u_k} = -j10\,p.u.$$

$$y_{01}' = 0$$

$$y_{02}' = 0$$

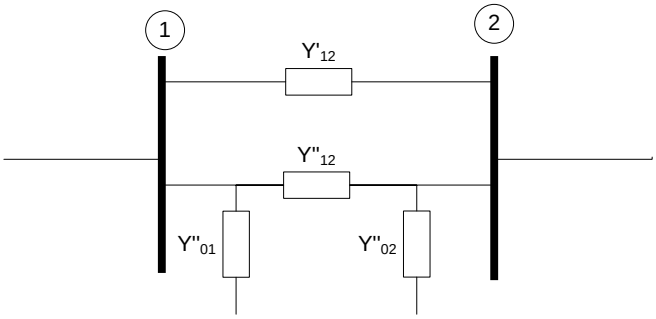
Tr2:

$$a_2 = \frac{\frac{380}{110}}{\frac{400}{110}} = \frac{380}{400} = 0,95$$

$$y_{12}'' = \frac{y_T''}{a_2} = \frac{S_n}{a_2 \cdot S_B \cdot ju_k} = -j10,526\,p.u.$$

$$y_{01}'' = \frac{y_T''}{a_2}\left(\frac{1}{a_2} - 1\right) = -j0,554\,p.u.$$

$$y_{02}'' = y_T''\left(1 - \frac{1}{a_2}\right) = j0,5264\,p.u.$$



$$Y = \begin{vmatrix} Y_{12}' + Y_{12}'' + Y_{01}'' & -j\left(Y_{12}' + Y_{12}''\right) \\ -\left(Y_{12}' + Y_{12}''\right) & Y_{12}' + Y_{12}'' + Y_{02}'' \end{vmatrix} = \begin{vmatrix} -j21,0803 & j20,5263 \\ j20,5263 & -j20 \end{vmatrix}$$

$$\begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = |Y| \cdot \begin{vmatrix} U_1 \\ U_2 \end{vmatrix}$$

$$U_2\left[p.u.\right]=\frac{U_2\left[kV\right]}{U_{2n}}=\frac{115}{110}=1,04545\,p.u.$$

$$S_2\left[p.u.\right]=\frac{S_2\left[MVA\right]}{S_B}=-\frac{80}{60}=-1,333\,p.u.$$

$$I_2=\left(\frac{S_2}{U_2}\right)^*=\frac{S_2}{U_2}=\frac{-1,333}{1,04545}=-1,2754\,p.u.$$

$$I_2=Y_{21}\cdot U_1+Y_{22}\cdot U_2\ \Rightarrow\qquad U_1=\frac{I_2-Y_{22}\cdot U_2}{Y_{21}}=\frac{-1,2754+\ j20\cdot 1,04545}{j20,5263}$$

$$U_1=1,01865+\ j0,0621\,p.u.$$

$$I_1=Y_{11}\cdot U_1+Y_{12}\cdot U_2=-j21,0803\left(1,01865+\ j0,0621\right)+\ j20,5263\cdot 1,04545=$$

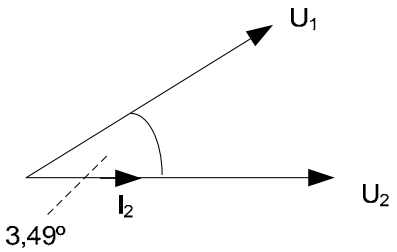
$$=1,3091-\ j0,014135\,p.u.$$

$$S_1=U_1\cdot I_1^*=1,02054\underline{|3,49^\circ}\cdot 1,30918\underline{|0,62^\circ}=1,336\underline{|4,11^\circ}=1,3326+\ j0,0958\,p.u.$$

$$S_1=79,956+\ j5,75MVA=80+\ j5,75MVA$$

$$U_1=407,5+\ j24,84kV$$

ANALIZA:



$U_1=408,256\underline{|3,49^\circ}$

U_1 prethodi U_2

Tok snage: od (1) prema (2)

$$S_2=-80MW$$

$$S_1=80+\ j5,75MVA$$

$$\Delta S=S_2+S_1=\ j5,75MVar$$

ZADATAK 5 Odredi napon na sekundaru i snagu na primaru dvaju paralelnih transformatora koji su u praznom hodu, ako je na primaru narinut nazivni napon 220kV. Ostali podaci:

T1 $u_k = 10\%$ $S_n = 100MVA$ $a_p = 220/110$

T2 $u_k = 12\%$ $S_n = 100MVA$ $a_p = 209/110$

Rješenje:

$S_B = S_n = 100MVA$

$Z_T[p.u.] = ju_k \qquad \rightarrow \qquad Y_T[p.u.] = -j\frac{1}{u_k}$

T1:

$$a_1 = \frac{\frac{220}{110}}{\frac{220}{110}} = 1$$

$$Y_{T1}[p.u.] = -j\frac{1}{0,1} = -j10\,p.u.$$

$$Y_{12}' = -j10\,p.u.$$

$$Y_{01}' = 0 \qquad Y_{02}' = 0$$

T2:

$$a_2 = \frac{\frac{209}{110}}{\frac{220}{110}} = 0,95$$

$$Y_{T1}[p.u.] = -j\frac{1}{0,12} = -j8,333\,p.u.$$

$$Y_{12}' = \frac{-j8,333}{a_2} = -j8,772\,p.u.$$

$$Y_{01}' = \frac{Y_{T2}}{a_2}\left(\frac{1}{a_2} - 1\right) = \frac{-j8,33}{0,95}\left(\frac{1}{0,95} - 1\right) = -j0,462\,p.u.$$

$$Y_{02}' = Y_{T2}\left(1 - \frac{1}{a_2}\right) = -j8,33\left(1 - \frac{1}{0,95}\right) = j0,439\,p.u.$$

$$\vec{Y} = \begin{vmatrix} -j19,234 & j18,772 \\ j18,772 & -j18,333 \end{vmatrix}$$

$$\begin{vmatrix} \vec{I}_1 \\ \vec{I}_2 \end{vmatrix} = \left|\vec{Y}\right| \cdot \begin{vmatrix} \vec{U}_1 \\ \vec{U}_2 \end{vmatrix}$$

$$\vec{U}_1[p.u.] = \frac{\vec{U}_1}{\vec{U}_n} = \frac{220}{220} = 1p.u.$$

$$\vec{I}_2[p.u.] = 0$$

$$\vec{I}_1[p.u.] = -j19,234\vec{U}_1 + j18,772\vec{U}_2$$

$$0 = j18,772\vec{U}_1 - j18,333\vec{U}_1 \qquad \left\{\vec{U}_1 = 1\right\}$$

$$\vec{U}_2 = \frac{j18,772}{j18,333} = 1,0239 \text{ p.u.}$$

$$\vec{U}_2[p.u.] = \frac{\vec{U}_2}{\vec{U}_{2n}} \qquad \Rightarrow \qquad U_2 = 110 \cdot 1,0239 = 112,63 \text{ kV}$$

$$\vec{I}_1 = -j19,234 + j18,772 \cdot 1,0239 = -j0,01335 \text{ p.u.}$$

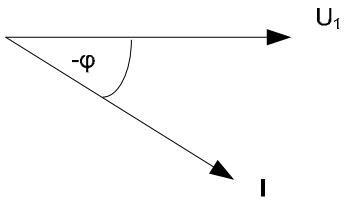
$$\vec{I}_1^{\,*}[p.u.] = j0,01335 \text{ p.u.}$$

$$\vec{S}_1[p.u.] = \vec{U}_1 \cdot \vec{I}_1^{\,*} = 1 \cdot j0,01335 = j0,01335 \text{ p.u.}$$

$$\vec{S}_1 = \vec{S}_1[p.u.] \cdot S_B = j1,335 \text{ Mvar}$$

NAPOMENA:

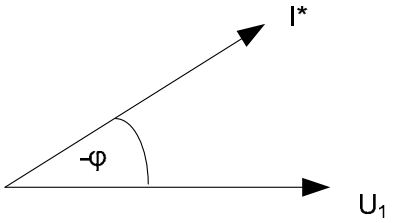
$\vec{S} = \vec{U} \cdot \vec{I}^{\,*}[p.u.]$ zbog dogovora da je *U* u referentnoj osi i da *P*+j*Q* označava induktivni karakter snage (pri čemu su *P* i *Q* veći od 0)



-> ovo je karakteristika induktivnog potrošača (da bi snaga bila pozitivna treba uzeti struju konjugirano kompleksnu)

$$\vec{S} = \vec{U}\vec{I} = S\angle -\varphi$$

(ovo nije naš dogovor)



$$\vec{S} = S\angle +\varphi$$

$$\vec{S} = \vec{U}\vec{I}^{\,*}$$

(ovo je naš dogovor)

$$Z_T = \frac{U_n^{\,2}}{S_n}\left[\frac{P_n}{S_n} + j\sqrt{\left(\frac{u_k^{\,2}}{100}\right) - \left(\frac{P_n}{S_n}\right)^2}\right] \quad [\Omega]$$

$$Z_T[p.u.] = Z_T[\Omega] \cdot \frac{S_B}{U_n^{\,2}} = \frac{U_n^{\,2}}{S_n} \cdot j\frac{u_k}{100} \cdot \frac{S_B}{U_n^{\,2}} \qquad (\text{za } S_B = S_n)$$

$$Z_T[p.u.] = j\frac{u_k}{100}$$

$$Y_T[p.u.] = -j\frac{1}{u_k}$$

ZADATAK 6 U transformatorskoj stanici nalaze se dva transformatora sa slijedećim podacima:

T1 $S_n = 150MVA$ T2 $S_n = 150MVA$

$u_k = 10\%$ $u_k = 10\%$

$a = \frac{220}{110}$

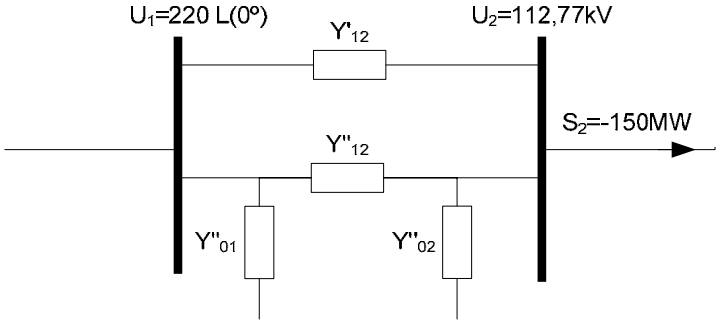
Koliki prijenosni odnos mora imati 2. transformator da bi napon na sekundaru bio 112,77 kV, uz zadanu snagu S_2 = -150 MW i nazivni napon na primaru?

Rješenje:

$U_1 = 220kV$

$S_2 = -150MW$

$a = ?$



$S_B = 150MVA$

$Y_T = -j10\,p.u.$

$Y_{12}' = -j10\,p.u.$ $\quad Y_{12}'' = -\frac{j10}{a_2}$ $\quad Y_{01}'' = \frac{Y_T}{a_2}\left(\frac{1}{a_2} - 1\right)$

$Y_{02}'' = Y_T\left(1 - \frac{1}{a_2}\right)$

$$\begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \left|\vec{Y}\right| \begin{vmatrix} U_1 \\ U_2 \end{vmatrix}$$

$$\left|\vec{Y}\right| = \begin{vmatrix} Y_T + \frac{Y_T}{a_2} + \frac{Y_T}{a_2}\left(\frac{1}{a_2} - 1\right) & -Y_T\left(1 + \frac{1}{a_2}\right) \\ -Y_T\left(1 + \frac{1}{a_2}\right) & Y_T + \frac{Y_T}{a_2} + Y_T\left(1 - \frac{1}{a_2}\right) \end{vmatrix} = \begin{vmatrix} Y_T\left(1 + \frac{1}{a_2^2}\right) & -Y_T\left(1 + \frac{1}{a_2}\right) \\ -Y_T\left(1 + \frac{1}{a_2}\right) & 2Y_T \end{vmatrix}$$

$$\vec{I}_1 = Y_T\left(1 + \frac{1}{a_2^2}\right)\vec{U}_1 - Y_T\left(1 + \frac{1}{a_2}\right)\vec{U}_2$$

$$\vec{I}_2 = -Y_T\left(1 + \frac{1}{a_2}\right)\vec{U}_1 + 2Y_T\vec{U}_2 \qquad \vec{I}_2 = \frac{\vec{S}_2^{\,*}}{\vec{U}_2^{\,*}}$$

$$\frac{\vec{S}_2^{\,*}}{\vec{U}_2^{\,*}} = -Y_T\left(1+\frac{1}{a_2}\right)\vec{U}_1 + 2Y_T\vec{U}_2 \quad / \cdot U_2^{\,*}$$

$$\vec{S}_2^{\,*} = -Y_T\left(1+\frac{1}{a_2}\right)\vec{U}_1\vec{U}_2^{\,*} + 2Y_T\left|\vec{U}_2\right|^2$$

$$\vec{U}_2^{\,*}\left(1+\frac{1}{a_2}\right) = \frac{2Y_T\left|\vec{U}_2\right|^2}{Y_T\vec{U}_1} - \frac{S_2^{\,*}}{Y_T\vec{U}_1} = \frac{2\left|\vec{U}_2\right|^2}{\vec{U}_1} - \frac{\vec{S}_2^{\,*}}{Y_T\vec{U}_1}$$

$$\vec{S}_2\left[p.u.\right] = -1p.u. \qquad \vec{U}_1\left[p.u.\right] = 1 + j0 \qquad Y_T\left[p.u.\right] = -j10\,p.u.$$

$$\left|\vec{U}_2\right|\left[p.u.\right] = \frac{112,77}{110} = 1,02518\,p.u.$$

$$\frac{2\cdot 1,02518^2}{1} - \frac{-1}{-j10\cdot 1} = 2,102 + j0,1p.u. = 2,104\underline{|2,724°}$$

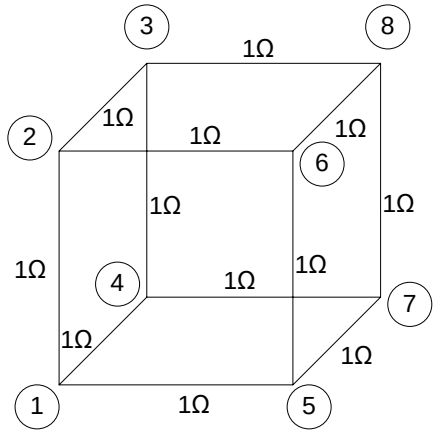
$$\vec{U}_2^{\,*}\left(1+\frac{1}{a_2}\right) = \left|\vec{U}_2\right|\left(1+\frac{1}{a_2}\right)\underline{|-\varphi} = 2,104\underline{|2,724°}$$

$$\varphi = -2,724°$$

$$1+\frac{1}{a_2} = \frac{2,104}{1,02518} = 2,0523$$

$$a_2 = \frac{1}{1,0523} = 0,95$$

ZADATAK 7 Zadana je "konfiguracija otpora" prema slici. Odredi nadomjesni otpor između točaka 1 i 8.



(referentni čvor 8)

Rješenje:

$$\vec{Y} = \left|\begin{array}{ccc|cccc} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 3 & -1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 3 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 3 & -1 & -1 \\ 0 & -1 & 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{array}\right|$$

Precrtani stupac:

$$Y_{i,j}^{(n)} = -\frac{Y_{i,j}^{(n-1)}}{Y_{j,j}^{(n-1)}} \quad ; i = 1,...,N;\ i \neq j$$

Precrtani redak

$$Y_{j,i}^{(n)} = -\frac{Y_{j,i}^{(n-1)}}{Y_{j,j}^{(n-1)}} \quad ; i = 1,...,N;\ i \neq j$$

$$\vec{Y}^{(1)} = \left|\begin{array}{cccccc|c} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & 8/3 & 1/3 & -1/3 & 0 & -1 & 0 \\ 0 & -1/3 & 1/3 & -1/3 & 0 & 0 & 0 \\ -1 & -1/3 & 1/3 & 8/3 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 3 & -1 & -1 \\ 0 & -1 & 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{array}\right|$$

Vanjski elementi

$$Y_{k,l}^{(n)} = Y_{k,l}^{(n-1)} - \frac{Y_{k,j}^{(n-1)} \cdot Y_{j,l}^{(n-1)}}{Y_{j,j}^{(n-1)}}$$

$$; k = 1,...,N;\ k \neq j$$

$$; l =,...,N;\ l \neq j$$

$$\vec{Y}^{(2)} = \left|\begin{array}{ccccccc} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & 7/3 & 1/3 & -1/3 & -1/3 & 1/3 & 0 \\ 0 & -1/3 & 1/3 & -1/3 & 0 & 0 & 0 \\ -1 & -1/3 & 1/3 & 8/3 & 0 & 0 & -1 \\ -1 & -1/3 & 0 & 0 & 8/3 & 1/3 & -1 \\ 0 & -1/3 & 0 & 0 & -1/3 & 1/3 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{array}\right|$$

Pivot

$$Y_{j,j}^{(n)} = \frac{1}{Y_{j,j}^{(n-1)}}$$

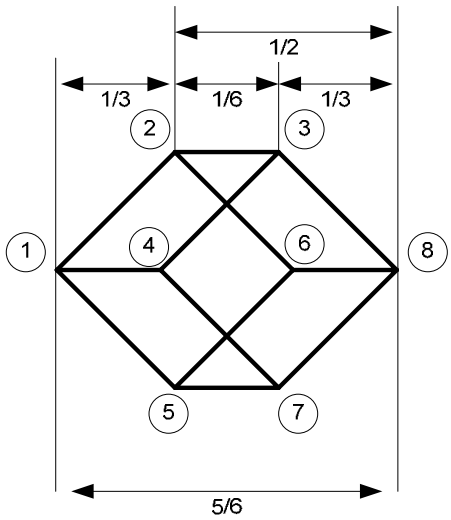
$$\vec{Y}^{(3)} = \begin{vmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & 7/3 & -1/3 & -1/3 & -1/3 & 1/3 & 0 \\ 0 & -1/3 & 1/3 & -1/3 & 0 & 0 & 0 \\ -1 & -1/3 & 1/3 & 7/3 & -1/3 & 0 & 1/3 \\ -1 & -1/3 & 0 & -1/3 & 7/3 & 1/3 & 1/3 \\ 0 & -1/3 & 0 & 0 & -1/3 & 1/3 & 0 \\ 0 & 0 & 0 & -1/3 & -1/3 & 0 & 1/3 \end{vmatrix}$$

$$\vec{Y}^{(6)} = \begin{vmatrix} 1/2 & 3/12 & 1/6 & 1/4 & -2/3 & 1/12 & 1/12 \\ 1/4 & 9/16 & 1/4 & 3/16 & -1/2 & 3/16 & 1/16 \\ 1/6 & 1/4 & 1/2 & 1/4 & -1/3 & 1/12 & 1/18 \\ 1/4 & 3/6 & 1/4 & 9/16 & -1/2 & 1/16 & 3/16 \\ 2/3 & 1/2 & 1/3 & 1/2 & 4/3 & 1/2 & 1/2 \\ 1/12 & 3/16 & 1/12 & 1/16 & -1/2 & 57/141 & 1/48 \\ 1/12 & 1/16 & 1/12 & 3/16 & -1/2 & 1/18 & 19/48 \end{vmatrix}$$

$$\vec{Y}^{(4)} = \begin{vmatrix} 1/3 & -1/3 & 0 & -1/3 & -1/3 & 0 & 0 \\ 1/3 & 6/3 & 1/3 & 2/3 & -2/3 & 1/3 & 0 \\ 0 & -1/3 & 1/3 & -1/3 & 0 & 0 & 0 \\ 1/3 & -2/3 & 1/3 & 4/3 & -2/3 & 0 & 1/3 \\ 1/3 & -2/3 & 0 & -2/3 & 6/3 & 1/3 & 1/3 \\ 0 & -1/3 & 0 & 0 & -1/3 & 1/3 & 0 \\ 0 & 0 & 0 & -1/3 & -1/3 & 0 & 1/3 \end{vmatrix}$$

$$\vec{Y}^{(7)} = \begin{vmatrix} 5/6 & 1/2 & 1/3 & 1/2 & 1/2 & 1/3 & 1/3 \\ 1/2 & 3/4 & 3/8 & 6/16 & 3/8 & 3/8 & 1/4 \\ 1/3 & 3/8 & 7/12 & 3/8 & 1/4 & 5/24 & 5/24 \\ 1/2 & 3/8 & 3/8 & 3/4 & 3/8 & 1/4 & 3/8 \\ 1/2 & 3/8 & 1/4 & 3/8 & 3/4 & 3/8 & 3/8 \\ 1/3 & 3/8 & 5/24 & 1/4 & 3/8 & 7/12 & 5/24 \\ 1/3 & 1/4 & 5/24 & 3/8 & 3/8 & 5/24 & 7/12 \end{vmatrix}$$

$$\vec{Y}^{(5)} = \begin{vmatrix} 7/18 & 1/6 & 1/18 & -4/9 & -4/9 & 1/18 & 0 \\ 1/6 & 3/6 & 1/6 & -2/6 & -2/6 & 1/6 & 0 \\ 1/18 & 1/6 & 7/18 & -4/9 & -1/9 & 1/18 & 0 \\ 4/9 & 2/6 & 4/9 & 16/9 & -8/9 & 1/9 & 1/3 \\ 4/9 & 2/6 & 1/9 & -8/9 & 16/9 & 4/9 & 1/3 \\ 1/18 & 1/6 & 1/18 & -1/9 & -4/9 & 7/18 & 0 \\ 0 & 0 & 0 & -1/3 & -1/3 & 0 & 1/3 \end{vmatrix}$$



$I_1 = 1A$

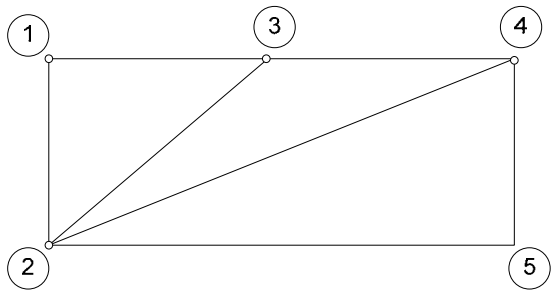
$U_2 = 0$

$$I_{1,n} = I_{1,2} = I_{1,5} = 1/3\,A \;\Rightarrow\; U_2 = U_4 = U_5 = \frac{5}{6} - \frac{1}{3} = \frac{1}{2}V$$

$$\Rightarrow\; U_3 = U_6 = U_7 = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}V$$

$Z_{i,j}$ - napon u čvorištu „*j*“ ako u „*i*-to“ čvorište narinemo 1A

ZADATAK 8 Za mrežu na slici, reduciraj mrežu na čvorišta 1, 2 i 3 (redukcija pasivne mreže). Y matricu formiraj kao za proračun tokova snaga pomoću Gauss-Seidel metode sa Z matricom (pri čemu poprečne grane ne ulaze u Y matricu).



Rješenje:

	$Z_{i-j}[p.u.]$	$\frac{Y_{i-j}'}{2}[p.u.]$	$Y_{i-j}[p.u.]$
1-2	0,02+j0,06	j0,003	5-j15
1-3	0,03+j0,24	j0,025	1,25-j3,75
2-3	0,06+j0,18	j0,02	1,66-j5
2-4	0,06+j0,18	j0,02	1,66-j5
2-5	0,04+j0,12	j0,015	2-j7,5
3-4	0,01+j0,03	j0,01	10-j30
4-5	0,08+j0,24	j0,025	1,25-j3,75

$$\vec{Y} = \begin{vmatrix} 6,25-j18,75 & -5+j15 & -1,25+j3,75 & 0 & 0 \\ -5+j15 & 10,833-j32,5 & -1,66+j5 & -1,66+j5 & -2,5+j7,5 \\ -1,25+j3,75 & -1,66+j5 & 12,916-j38,75 & -10+j30 & 0 \\ 0 & -1,66+j5 & -10+j30 & 12,916-j38,75 & -1,25+j3,75 \\ 0 & -2,5+j7,5 & 0 & -1,25+j3,75 & 3,75-j11,25 \end{vmatrix}$$

$$\vec{Y} = \begin{vmatrix} 19,76\underline{|-71,6°} & 15,81\underline{|108,4°} & 3,95\underline{|108,4°} & 0 & 0 \\ 15,81\underline{|108,4°} & 34,26\underline{|-71,6°} & 5,27\underline{|108,4°} & 5,27\underline{|108,4°} & 7,91\underline{|108,4°} \\ 3,95\underline{|108,4°} & 5,27\underline{|108,4°} & 40,85\underline{|-71,6°} & 31,62\underline{|108,4°} & 0 \\ 0 & 5,27\underline{|108,4°} & 31,62\underline{|108,4°} & 40,85\underline{|-71,6°} & 3,95\underline{|108,4°} \\ 0 & 7,91\underline{|108,4°} & 0 & 3,95\underline{|108,4°} & 11,86\underline{|-71,6°} \end{vmatrix}$$

$$\vec{Y} = \left|\begin{array}{cccc|c} 19,76 & -15,81 & -3,95 & 0 & 0 \\ -15,81 & 34,26 & -5,27 & -5,27 & -7,91 \\ -3,95 & -5,27 & 40,85 & -31,62 & 0 \\ 0 & -5,27 & -31,622 & 40,85 & -3,95 \\ \hline 0 & -7,91 & 0 & -3,95 & 11,86 \end{array}\right|$$

$$\vec{Y}^{1234} = \begin{vmatrix} 19,76 & -15,81 & -3,95 & 0 \\ -15,81 & 28,98 & -5,27 & -7,90 \\ -3,95 & -5,27 & 40,85 & -31,62 \\ 0 & -7,90 & -31,62 & 39,53 \end{vmatrix} \rightarrow \begin{vmatrix} 19,76 & -15,81 & -3,95 \\ -15,81 & 27,40 & -11,59 \\ -3,95 & -11,59 & 15,56 \end{vmatrix}$$

$$\vec{Y}_{23}^{-1} = \begin{vmatrix} 27,40 & -11,59 \\ -11,59 & 15,56 \end{vmatrix} = \begin{vmatrix} 18,77 & 0,745 \\ -0,745 & 0,0643 \end{vmatrix} = \begin{vmatrix} 0,0533\underline{|71,6°} & 0,0397\underline{|71,6°} \\ 0,0397\underline{|71,6°} & 0,0939\underline{|71,6°} \end{vmatrix} =$$

$$= \begin{vmatrix} 0,0168 + j0,0506 & 0,0125 + j0,0377 \\ 0,0125 + j0,0377 & 0,0296 + j0,0891 \end{vmatrix} = |Z_{23}|$$

ZADATAK 9 U transformatorskoj stanici nalaze se 2 transformatora sa slijedećim podacima:

T1 $S_n = 80MVA$ T2 $S_n = 80MVA$

$P_k = 2MW$ $P_k = 2MW$

$u_k = 11\%$ $u_k = 11\%$

$a_1 = {}^{216}\!/_{110}$ $a_1 = {}^{222}\!/_{110}$

Napon na primaru iznosi 215 kV, a snaga na sekundaru je 100+j30 MVA. Odredi napon na sekundaru, te tokove i gubitke snage na svakom transformatoru koristeći Gauss-Seidel metodu za Z matricom.

$U_1 = 215kV$

$S_2 = -100 - j30MVA \quad \rightarrow \quad U_2,\ \Delta S',\ \Delta S'',\ S_1$

$\varepsilon = 10^{-4}$

Rješenje:

$S_B = 100MVA$

$a_1 = \frac{216}{220} = 0,982 \qquad a_2 = \frac{222}{220} = 1,009$

$$Y_T = \frac{1}{Z_T} = \frac{1}{\frac{U_n^{\,2}}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{u_k^{\,2} - \left(\frac{P_k}{S_n}\right)^2}\right]} \cdot \frac{\frac{U_n^{\,2}}{S_n}\left[\frac{P_k}{S_n} - j\sqrt{u_k^{\,2} - \left(\frac{P_k}{S_n}\right)^2}\right]}{\frac{U_n^{\,2}}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{u_k^{\,2} - \left(\frac{P_k}{S_n}\right)^2}\right]} = \frac{\frac{U_n^{\,2}}{S_n}\left[\frac{P_k}{S_n} - j\sqrt{u_k^{\,2} - \left(\frac{P_k}{S_n}\right)^2}\right]}{\left(\frac{U_n^{\,2}P_k^{\,2}}{S_n^{\,4}}\right) + \left[u_k^{\,2} - \left(\frac{P_k}{S_n}\right)^2\right]\frac{U_n^{\,4}}{S_n^{\,2}}} =$$

$$= \frac{\frac{U_n^{\,2}}{S_n}\left[\frac{P_k}{S_n} - j\sqrt{u_k^{\,2} - \left(\frac{P_k}{S_n}\right)^2}\right]}{u_k^{\,2}\frac{U_n^{\,2}}{S_n}} = \frac{S_n}{U_n^{\,2}} \cdot \frac{1}{u_k^{\,2}}\left[\frac{P_k}{S_n} - j\sqrt{u_k^{\,2} - \left(\frac{P_k}{S_n}\right)^2}\right]$$

$$Y_T[p.u.] = Y_T \cdot \frac{U_n^{\,2}}{S_B} = \frac{S_n}{S_B \cdot u_k^{\,2}}\left[\frac{P_k}{S_n} - j\sqrt{u_k^{\,2} - \left(\frac{P_k}{S_n}\right)^2}\right] \quad [p.u.]$$

$$Y_{1,2}{}' = \frac{Y_T[p.u.]}{a_1} = 1,6835 - j7,21356$$

$$Y_{10}{}' = Y_{12}{}'\left(\frac{1}{a_1} - 1\right) = 0,03118 - j0,13358$$

$$Y_{20}{}' = Y_{12}{}' a_1'\left(1 - \frac{1}{a_1}\right) = -0,03061 + j0,13116$$

$$Y_{12}'' = 1,638 - j7,0186$$

$$Y_{10}'' = -0,01476 + j0,06323$$

$$Y_{20}'' = 0,01489 - j0,06381$$

Z_{22} – element *Z*-matrice, tj. invertirane *Y*-matrice (u ovom slučaju dimenzije 1x1)

$$Z_{22} = \frac{1}{Y_{12}' + Y_{12}''} = 0,01555 + j0,06663$$

$$Y_{20} = Y_{20}' + Y_{20}'' = -0,01572 + j0,06735$$

$$S_2^* = \frac{-100 + j30}{100} = -1 + j0,3$$

pretpostavljamo $U_2^{(0)} = 1 + j0$

$$I_2^{(0)} = \frac{S_2^*}{U_2^{(0)*}} - U_2^{(0)} Y_{20} = -0,98428 + j0,23265$$

$$U_2^{(1)} = U_1 + Z_{22} I_2^{(0)} = 0,94646 - j0,06197$$

$$I_2^{(1)} = -1,02069 + j0,31978$$

$$U_2^{(2)} = 0,94009 - j0,06304$$

$$I_2^{(2)} = -1,02713 + j0,92439$$

$$U_2^{(3)} = 0,93968 - j0,06340$$

$$I_2^{(3)} = -1,02742 + j0,325$$

$$U_2^{(4)} = 0,93964 - j0,06341 = 103,6\underline{|-3,86°}kV$$

$$S_{1-2}' = U_1\left[\left(U_1 - U_2\right)Y_{12}' + U_1 Y_{10}'\right]^* = U_1\left(U_1^* - U_2^*\right)Y_{12}^* + \left|U_1\right|^2 \cdot Y_{10}^*$$

$$S_{2-1}' = U_2\left(U_1^* - U_2^*\right)Y_{12}^* + \left|U_2\right|^2 \cdot Y_{20}^*$$

$$S_{1-2}' = 0,539 + j0,289 \qquad S_{2-1}' = -0,527 - j0,238$$

$$\Delta S' = S_{1-2}' + S_{2-1}' = 0,012 + j0,051 = 1,2 + j5,1MVA$$

$$S_{1-2}'' = 0,481 + j0,096 \qquad S_{2-1}'' = -0,473 - j0,062$$

$$\Delta S'' = 0,008 + j0,034 = 0,8 + j3,4MVA$$

$$\Delta S = \Delta S' + \Delta S'' = 2 + j8,5MVA$$

ZADATAK 10 U transformatorskoj stanici nalaze se 2 transformatora sa slijedećim podacima:

T1 $S_n = 80MVA$ $P_k = 2MW$ $u_k = 11\%$ $a_1 = {}^{216}\!/_{110}$

T2 $S_n = 80MVA$ $P_k = 2MW$ $u_k = 11\%$ $a_1 = {}^{222}\!/_{110}$

Napon na primaru iznosi 215 kV, a snaga na sekundaru je 100+j30 MVA. Odredi napon na sekundaru, te tokove i gubitke snage na svakom transformatoru koristeći Gauss-Seidel metodu za Y matricom.

$U_1 = 215kV$

$S_2 = -100 - j30MVA \quad \rightarrow \quad U_2,\ \Delta S',\ \Delta S'',\ S_1$

$\varepsilon = 10^{-4}$

Rješenje:

$S_B = 100MVA \qquad a_1 = \frac{216}{220} = 0,982 \qquad a_2 = \frac{222}{220} = 1,009 \qquad U_1 = \frac{215}{220} = 0,97727$

$Y_{12}' = 1,6835 - j7,21356 \qquad Y_{10}' = 0,03118 - j0,13358 \qquad Y_{20}' = -0,03061 + j0,13116$

$Y_{12}'' = 1,638 - j7,0186 \qquad Y_{10}'' = -0,01476 + j0,06323 \qquad Y_{20}'' = 0,01489 - j0,06381$

$$Y = \begin{vmatrix} Y_{12}' + Y_{12}'' + Y_{10}' + Y_{10}'' & -Y_{12}' - Y_{12}'' \\ -Y_{12}' - Y_{12}'' & Y_{12}' + Y_{12}'' + Y_{20}' + Y_{20}'' \end{vmatrix}$$

$$Y = \begin{vmatrix} 3,33792 - j14,30251 & -3,3215 + j14,23216 \\ -3,3215 + j14,23216 & 3,30578 - j14,16481 \end{vmatrix}$$

Opći član:

$$U_i^{(k+1)} = \frac{KL_i}{\left(U_i^{(k)}\right)^*} - \sum_{j=1}^{i-1} YL_{i,j} \cdot U_j^{(k+1)} - \sum_{j=i+1}^{n} YL_{i,j} \cdot U_j^{(k)}$$

$i = 1,...,n$

$i \neq ref$

KL_i članova ima (n-1) – u našem slučaju 2-1=1.

$YL_{i,j}$ članova ima koliko ima van dijagonalnih elemenata – elementi u referentnom retku.

$$KL_2 = \frac{S_2^*}{Y_{2,2}} = \frac{-1 + j0,3}{3,30578 - j14,16481} = 0,0717771\underline{|240,16°} = -0,03571 - j0,06226$$

$$YL_{2,1} = \frac{Y_{2,1}}{Y_{2,2}} = \frac{-3,3215 + j14,23216}{3,30578 - j14,16481} = 1,004755\underline{|180°} = -1,004755$$

$$U_2^{(0)} = 1 + j0$$

$$U_2^{(1)} = \frac{KL_2}{\left(U_2^{(0)}\right)^*} - YL_{2,1} \cdot U_1$$

$$YL_{2,1} \cdot U_1 = -1,004755 \cdot 0,97727 = -0,98192$$

$$U_2^{(1)} = \frac{0,07177\angle 240,16°}{1} + 0,98192 = -0,03571 - j0,06226 + 0,98192$$

$$U_2^{(1)} = 0,94621 - j0,06226$$

Izaberemo $\alpha = 1,2$ (faktor ubrzanja)

$$\Delta U_2^{(1)} = U_2^{(1)} - U_{2ub}^{(0)} = 0,94621 - j0,06226 - 1 = -0,05379 - j0,06226$$

$$U_{2ub}^{(1)} = U_{2ub}^{(0)} + \alpha \Delta U_2^{(1)} = 1 + j0 + 1,2\left(-0,05379 - j0,06226\right) =$$

$$= 0,93545 - j0,07471 = 0,93843\angle -4,566°$$

$$U_2^{(2)} = \frac{KL_2}{\left(U_{2ub}^{(1)}\right)^*} - YL_{2,1} \cdot U_1 = \frac{0,07177\angle 240,16°}{0,93843\angle 4,566°} + 0,98192 = 0,93871 - j0,06310$$

$$\Delta U_2^{(2)} = U_2^{(2)} - U_{2ub}^{(1)} = 0,93871 - j0,06310 - 0,93545 + j0,07471 = 0,00326 + j0,01161$$

$$U_{2ub}^{(2)} = U_{2ub}^{(1)} + \alpha \Delta U_2^{(2)} = 0,93545 - j0,07471 + 1,2\left(0,00326 + j0,01161\right) =$$

$$= 0,93936 - j0,06078 = 0,94132\angle -3,70°$$

$$U_2^{(3)} = \frac{0,07177\angle 240,16°}{0,94132\angle 3,70°} + 0,98192 = 0,93979 - j0,06355$$

$$\Delta U_2^{(3)} = U_2^{(3)} - U_{2ub}^{(2)} = 0,93979 - j0,06355 - 0,93936 + j0,06078 = 0,00043 - j0,00277$$

$$U_{2ub}^{(3)} = U_{2ub}^{(2)} + \alpha \Delta U_2^{(3)} = 0,93988 - j0,06410 = 0,94206\angle -3,90°$$

$$U_2^{(4)} = \frac{0,07177\angle 240,16°}{0,94206\angle 3,90°} + 0,98192 = 0,93960 - j0,06335 = 0,94173\angle -3,86°$$

$$\Delta U_2^{(4)} = U_2^{(4)} - U_{2ub}^{(3)} = -0,00027 + j0,00075$$

$$U_{2ub}^{(4)} = U_{2ub}^{(3)} + \alpha \Delta U_2^{(4)} = 0,93955 - j0,06320 = 0,94167\angle -3,85°$$

$$U_2^{(5)} = \frac{0,07177\angle 240,16°}{0,94167\angle 3,85°} + 0,98192 = 0,93965 - j0,06342$$

$$\Delta U_2^{(5)} = U_2^{(5)} - U_{2ub}^{(4)} = 0,0001 - j0,00022$$

$$U_{2ub}^{(5)} = U_{2ub}^{(4)} + \alpha \Delta U_2^{(5)} = 0,93966 - j0,06346 = 0,94180\angle -3,86°$$

$$U_2^{(6)} = \frac{0,07177\angle 240,16°}{0,94180\angle 3,86°} + 0,98192 = 0,93963 - j0,06340$$

$$\Delta U_2^{(6)} = U_2^{(6)} - U_{2ub}^{(5)} = -0,00003 + j0,00006$$

$$\boxed{\begin{aligned} U_2 &= 0,93963 - j0,06340 = 0,94177\angle -3,86° \\ U_2 &= 103,6\angle -3,86°\ kV \end{aligned}}$$

ZADATAK 11 Zadani su podaci za transformatore u stanici.

T1 $S_n = 80MVA$ $P_k = 2MW$ $u_k = 11\%$ $a_1 = \frac{216}{110}$

T2 $S_n = 80MVA$ $P_k = 2MW$ $u_k = 11\%$ $a_1 = \frac{222}{110}$

Napon na primaru iznosi $U_1 = 215kV$, a snaga sekundara je $S_2 = -100 - j30MVA$. Odrediti napon na sekundaru i tokove snage kroz transformatore koristeći Newton-Rapshon metodu. $\varepsilon = 10^{-2}$.

Rješenje:

$S_B = 100MVA$

$$a_1 = \frac{\frac{216}{110}}{\frac{220}{110}} = 0,98182 \qquad a_2 = \frac{\frac{222}{110}}{\frac{220}{110}} = 1,009$$

$$Y_{1,2}{}' = \frac{1}{a_1}\left[\frac{S_n}{S_B u_k{}^2}\cdot\left(\frac{P_n}{S_n} - j\sqrt{u_k{}^2 - \left(\frac{P_n}{S_n}\right)^2}\right)\right] = 1,6835 - j7,21356\,p.u.$$

$$Y_{1,0}{}' = Y_{12}{}'\left(\frac{1}{a_1} - 1\right) = 0,03118 - j0,13358\,p.u.$$

$$Y_{2,0}{}' = Y_{1,2}{}'\cdot a_1\left(1 - \frac{1}{a_1}\right) = -0,03061 + j0,13116\,p.u.$$

$$Y_{1,2}{}'' = 1,638 - j7,0186\,p.u.$$
$$Y_{1,0}{}'' = -0,01476 + j0,06323\,p.u.$$
$$Y_{2,0}{}'' = 0,01489 - j0,06381\,p.u.$$

$$[Y] = \begin{bmatrix} 3,33792 - j14,30231 & -3,3215 + j14,23216 \\ -3,3215 + j14,23216 & 3,30578 - j14,16481 \end{bmatrix} = \begin{bmatrix} 14,68685\underline{|-76.863°} & 14,61461\underline{|103.137°} \\ 14,61461\underline{|103.137°} & 14,54545\underline{|-76.863°} \end{bmatrix}$$

$$U_1 = \frac{215}{220} = 0,977273 + j0 \quad (ref.čvorište)$$

$$U_2{}^{(0)} = 1 + j0 \quad (pretpostavljeni\ napon)$$

Za sva nezavisna čvorišta vrijedi:

$$P_{i,izr}{}^{(k)} = U_i{}^{(k)}\sum_{j=1}^{n} Y_{i.j}\cdot U_j{}^{(k)}\cos\left(-\Theta_{i,j} + \delta_i{}^{(k)} - \delta_j{}^{(k)}\right)\ ;\ i = 1,...,n\ ;\ i \neq j$$

$$Q_{i,izr}{}^{(k)} = U_i{}^{(k)}\sum_{j=1}^{n} Y_{i.j}\cdot U_j{}^{(k)}\cos\left(-\Theta_{i,j} + \delta_i{}^{(k)} - \delta_j{}^{(k)}\right)\ ;\ i = 1,...,n\ ;\ i \neq j\ ;\ i \neq gen.$$

Referentno čvorište 1:

$i = 2$

$$P_2^{(0)} = U_2^{(0)}\left[Y_{2,1}\cdot U_1\cos\left(-\Theta_{2,1}+\delta_2^{(0)}-\delta_1\right)+Y_{2,2}\cdot U_2^{(2)}\cos\left(-\Theta_{2,2}\right)\right]=$$
$$=1\cdot 14,61461\cdot 0,977273\cdot\cos\left(-103,137\right)+1\cdot 14,54545\cdot\cos\left(76,863\right)=$$
$$=-3,24612+3,3058=0,0598$$

$$Q_2^{(0)} = U_2^{(0)}\left[Y_{2,1}\cdot U_1\sin\left(-\Theta_{2,1}+\delta_2^{(0)}-\delta_1\right)+Y_{2,2}\cdot U_2^{(2)}\sin\left(-\Theta_{2,2}\right)\right]=$$
$$=1\cdot 14,61461\cdot 0,977273\cdot\sin\left(-103,137\right)+1\cdot 1\cdot 14,54545\cdot\sin\left(76,863\right)=$$
$$=13,9087+14,16481=0,25611$$

$$\Delta P_2^{(0)} = P_2^{ZAD}-P_2^{(0)}=-1-0,0598=-1,0598$$

$$\Delta Q_2^{(0)} = Q_2^{ZAD}-Q_2^{(0)}=-0,3-0,25611=-0,55611$$

$\mathbf{I_1}$ za $i, j \neq ref.$**:**

$$\left(\frac{\partial P_i}{\partial \delta_j}\right)^{(k)} = U_i^{(k)}\cdot U_j^{(k)}\cdot Y_{i,j}\cdot\sin\left(-\Theta_{i,j}+\delta_i^{(k)}-\delta_j^{(k)}\right)\quad i\neq j$$

$$\left(\frac{\partial P_i}{\partial \delta_i}\right)^{(k)} = -\sum_{\substack{j=1\\ j\neq i}}^{n} U_i^{(k)}\cdot U_j^{(k)}\cdot Y_{i,j}\cdot\sin\left(-\Theta_{i,j}+\delta_i^{(k)}-\delta_j^{(k)}\right)$$

$\mathbf{I_n}$ za $i, j \neq ref.$**:**

$$\left(\frac{\partial Q_i}{\partial U_j}\cdot U_j\right)^{(k)} = U_i^{(k)}\cdot U_j^{(k)}\cdot Y_{i,j}\cdot\sin\left(-\Theta_{i,j}+\delta_i^{(k)}-\delta_j^{(k)}\right)\qquad i\neq j$$

$$\left(\frac{\partial Q_i}{\partial U_i}\cdot U_j\right)^{(k)} = 2U_i^{(k)}\cdot U_i^{(k)}\cdot Y_{i,j}\cdot\sin\left(\Theta_{ii}\right)+\sum_{\substack{j=1\\ j\neq i}}^{n} U_i^{(k)}\cdot U_j^{(k)}\cdot Y_{i,j}\cdot\sin\left(-\Theta_{i,j}+\delta_i^{(k)}-\delta_j^{(k)}\right)$$

$$\left(\frac{\partial P_2}{\partial \delta_2}\right)^{(0)} = -U_2^{(0)}\cdot U_1^{(0)}\cdot Y_{2,1}\cdot\sin\left(-\Theta_{i,j}+\delta_2^{(0)}-\delta_1^{(0)}\right)=$$
$$=-1\cdot 0,977273\cdot 14,61461\cdot\sin\left(-103,137\right)=13,9087$$

$$\left(\frac{\partial Q_2}{\partial U_2}\cdot U_2\right)^{(0)} = 2U_2^{(0)}\cdot U_2^{(0)}\cdot Y_{2,2}\cdot\sin\left(-\Theta_{2,2}\right)+U_2^{(0)}\cdot U_1\cdot Y_{2,2}\cdot\sin\left(-\Theta_{2,2}+\delta_2^{(0)}-\delta_1^{(0)}\right)=$$
$$=2\cdot 1\cdot 1\cdot 14,54545\cdot\sin\left(76,863\right)+1\cdot 0,977273\cdot 14,61461\cdot\sin(-103,137)=14,4209$$

$$\Delta P_2^{(0)} = \left(\frac{\partial P_2}{\partial \delta_2}\right)^{(0)}\Delta\delta_2^{(0)} \qquad \Rightarrow \qquad -1,0598=13,9087\cdot\Delta\delta_2^{(0)}$$

$$\Delta Q_2^{(0)} = \left(\frac{\partial Q_2}{\partial U_2}\cdot U_2\right)^{(0)}\left(\frac{\Delta U_2}{U_2}\right)^{(0)} \qquad \Rightarrow \qquad -0,55611=14,4209\cdot\left(\frac{\Delta U_2}{U_2}\right)^{(0)}$$

$$\Delta\delta_2^{(0)} = -\frac{1,0598}{13,9087} = -0,076197$$

$$\left(\frac{\Delta U_2}{U_2}\right)^{(0)} = -\frac{0,55611}{14,4209} = -0,038563 \qquad \longrightarrow \qquad \text{kraj 0. iteracijskog koraka}$$

$$U_2^{(1)} = U_2^{(0)} + \left(\frac{\Delta U_2}{U_2}\right)^{(0)} \cdot U_2^{(0)} = 1 - 0,038569 = 0,961437$$

$$\delta_2^{(1)} = \delta_2^{(0)} + \Delta\delta_2^{(0)} = -0,076197 \;\; rad = -4,366°$$

$$\begin{aligned} P_2^{(1)} &= 0,961437 \cdot 14,61461 \cdot 0,977273 \cdot \cos\left(-103,137 - 4,366\right) + \\ &\quad + 0,961437 \cdot 14,54545 \cdot 0,961437 \cdot \cos(76,863) = \\ &= -4,12988 + 3,05984 = -1,074 \end{aligned}$$

$$\begin{aligned} Q_2^{(1)} &= 0,961437 \cdot 14,61461 \cdot 0,977273 \cdot \sin(-103,137 - 4,366) + \\ &\quad + 0,961437 \cdot 14,54545 \cdot 0,961437 \cdot \sin(76,863) = \\ &= -13,09599 + 13,09338 = -0,00255 \end{aligned}$$

$$\Delta P_2^{(1)} = P_2^{ZAD} - P_2^{(1)} = -1 + 1,074 = 0,074$$

$$\Delta Q_2^{(1)} = Q_2^{ZAD} - Q_2^{(1)} = -0,3 + 0,00255 = -0,29745$$

$$\left(\frac{\partial P_2}{\partial \delta_2}\right)^{(1)} = -0,961437 \cdot 0,977279 \cdot 14,61461 \cdot \sin(-103,137 - 4,366) = 13,09983$$

$$\begin{aligned} \left(\frac{\partial Q_2}{\partial U_2} \cdot U_2\right)^{(1)} &= 2 \cdot 0,961437^2 \cdot 14,54545 \cdot \sin(76,869) + \\ &\quad + 0,961437 \cdot 0,977273 \cdot 14,61451 \cdot \sin(-107,303) = \\ &= 26,186736 - 13,09393 = 13,090826 \end{aligned}$$

$$\Delta P_2^{(1)} = \left(\frac{\partial P_2}{\partial \delta_2}\right)^{(1)} \cdot \Delta\delta_2^{(1)} \qquad \Rightarrow \qquad 0,075 = 13,09393 \cdot \Delta\delta_2^{(1)}$$

$$\Delta Q_2^{(1)} = \left(\frac{\partial Q_2}{\partial U_2} \cdot U_2\right)^{(1)} \cdot \left(\frac{\Delta U_2}{U_2}\right)^{(1)} \qquad \Rightarrow \qquad -0,29745 = 13,090825 \cdot \left(\frac{\Delta U_2}{U_2}\right)^{(1)}$$

$$\Delta\delta_2^{(1)} = \frac{0,074}{13,09593} = 0,00363 = 0,3237°$$

$$\left(\frac{\Delta U_2}{U_2}\right)^{(1)} = -\frac{0,29745}{13,09393} = -0,02272$$

$$U_2^{(2)} = U_2^{(1)} + \left(\frac{\Delta U_2}{U_2}\right)^{(1)} \cdot U_2^{(1)} = 0,961437 - 0,02272 \cdot 0,961437 = 0,93959$$

$$\Delta\delta_2^{(2)} = \delta_2^{(1)} + \Delta\delta_2^{(1)} = -4,366 + 0,3237 = -4,0423$$

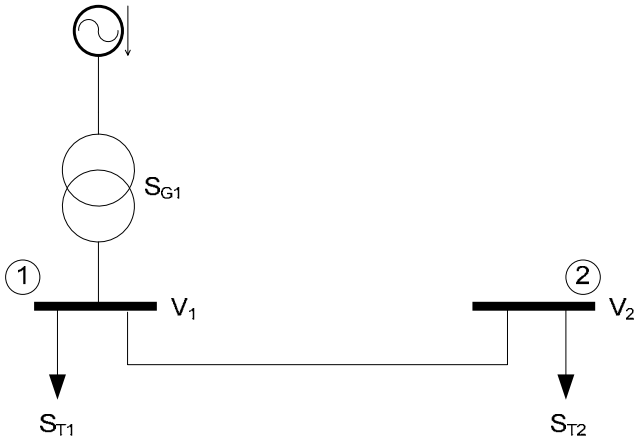
$$\begin{aligned} P_2^{(2)} &= 0,93939 \cdot 14,61461 \cdot 0,977273 \cdot \cos(-103,137 - 4,0423) + \\ &\quad + 0,93939 \cdot 14,54545 \cdot \cos(76,863) = -3,96367 + 2,91863 = -1,045 \end{aligned}$$

$$Q_2^{(2)} = \ldots$$

$$\Delta P_2^{(2)} = 0,045$$

$$\Delta Q_2^{(2)} = \ldots$$

ZADATAK 12 Za mali sustav na slici, napravi proračun tokova snaga Gauss-Seidelovom metodom pomoću Y-matrice.



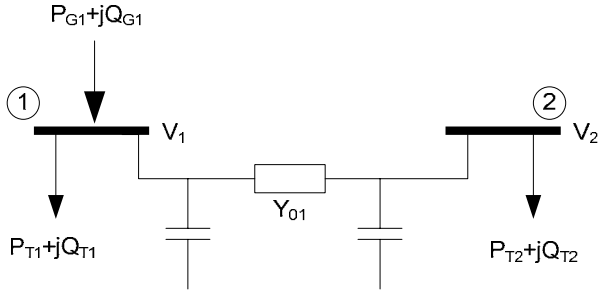
Zadano:

$V_1 = 1,05\underline{|0°}\,[p.u.]$ $\qquad Y_V = 0,3044 - j1,87996\,[p.u.]$

$S_{T1} = 1,15 + j0,31\,[p.u.]$ $\qquad \frac{Y_p}{2} = j0,06366\,[p.u.]$

$S_{T2} = 0,45 + j0,20\,[p.u.]$ $\qquad |V_2^{(0)}| = 0,95 \quad ; \qquad \delta_2^{(0)} = -13,5° \; ; \qquad \varepsilon = 10^{-4}$

Rješenje:



$V_1 = |V_1|\underline{|0°}$

$V_2 = |U_1|\underline{|\delta_2}$

$$|Y| = \begin{vmatrix} 0,3044 - j1,8163 & -0,3044 + 1,87996 \\ -0,3044 + 1,87996 & 0,3044 - j1,8163 \end{vmatrix} = \begin{vmatrix} 1,8416\underline{|-80,48561°} & 1,904443\underline{|99,197819°} \\ 1,904443\underline{|99,197819°} & 1,8416\underline{|-80,48561°} \end{vmatrix}$$

$|V_2^{(0)}| = 0,95\underline{|\delta_2^{(0)}} \qquad \delta_2^{(0)} = ?$

$P_2 = \frac{|V_1| \cdot |V_2|}{K} \sin\delta_2 \quad \Rightarrow \quad 0,45 = 1,05 \cdot 0,95(-1,87996) \cdot \sin\delta_2 \quad \Rightarrow \quad \delta_2 \approx -13,5°$

$V_2^{(0)} = 0,95\underline{|-13,5°}$

TERET = „ - ''

$$V_i^{(k+1)} = \frac{P_i - jQ_i}{Y_{i,i}} \cdot \frac{1}{\left(V_i^{(k)}\right)^*} - \sum_{j=1}^{i-1} \frac{Y_{i,j}}{Y_{i,i}} \cdot V_j^{(k+1)} - \sum_{j=i+1}^{n} \frac{Y_{i,j}}{Y_{i,i}} \cdot V_j^{(k)} \qquad , i \neq reg.$$

$$= KL_i \cdot \frac{1}{\left(V_i^{(k)}\right)^*} - \sum_{j=1}^{i-1} YL_{i,j} \cdot V_j^{(k+1)} - \sum_{j=i+1}^{n} YL_{i,j} \cdot V_j^{(k)}$$

$reg = 1 \qquad i \neq 1 \qquad j = 1,2$

$i = 2$

$$KL_2 = \frac{-P_2 + jQ_2}{Y_{2,2}} = \frac{0,492443\underline{|156,0375^\circ}}{1,841631\underline{|-80,48561^\circ}} = 0,267395\underline{|236,52311^\circ}$$

$$YL_{2,1} = \frac{Y_{2,1}}{Y_{2,2}} = \frac{1,90443\underline{|99,197819^\circ}}{1,841631\underline{|-80,48561^\circ}} = 1,0340997\underline{|179,68343^\circ}$$

$$V_2^{(1)} = \frac{KL_2}{(V_2^{(0)})^*} - YL_{2,1} \cdot V_1 = \frac{0,267395\underline{|236,52311^\circ}}{0,95\underline{|19,5^\circ}} - 1,0340997\underline{|179,68343^\circ} \cdot 1,05\underline{|0^\circ} =$$

$$= 0,2814684\underline{|223,02311} - 1,0838047\underline{|179,68343^\circ} =$$

$$= -0,2057755 - j0,192044 + 1,085788 - j0,0060182 =$$

$$= 0,8800125 - j0,198622 = 0,9020258\underline{|-12,684^\circ} p.u.$$

$$V_2^{(2)} = \frac{KL_2}{(V_2^{(1)})^*} - YL_{2,1} \cdot V_1 =$$

$\varepsilon = 10^{-4} \qquad V_2^{(7)} = 0,89103\underline{|-13,6448^\circ} \ [p.u.]$

$$S_{1-2} = V_1 \left(V_1^* - V_2^*\right) Y_{1,2}{}^* + |V_1|^2 \cdot \frac{Y_p{}^*}{2}$$

$$S_{1-2} = 1,05\underline{|0^\circ} \cdot \left(1,05 - 0,89103\underline{|13,6448^\circ}\right) \cdot \left(0,3044 + j1,87996\right) + 1,05^2 \cdot \left(-j0,06366\right) =$$

$$= 0,47377 + j0,22067 \ [p.u.]$$

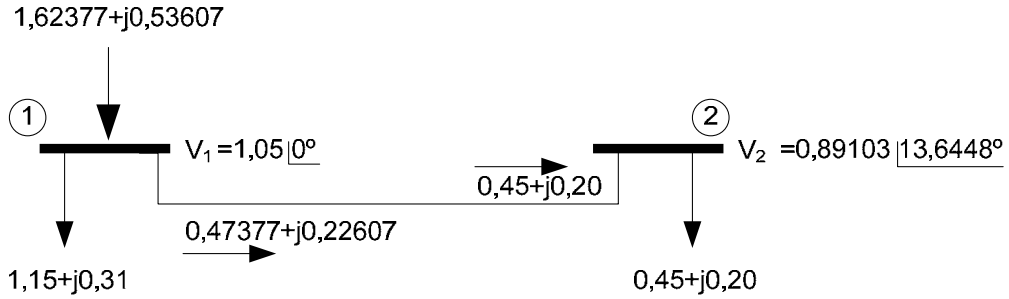
$$S_{2-1} = V_2 \left(V_1^* - V_2^*\right) Y_{1,2}{}^* + |V_2|^2 \cdot \frac{Y_p{}^*}{2} =$$

$$= 0,89103\underline{|-13,6448^\circ} \cdot \left(0,89103\underline{|13,6448^\circ} - 1,05\right) \cdot \left(0,3044 + j1,87996\right) + 0,89103^2 \left(-j0,06336\right)$$

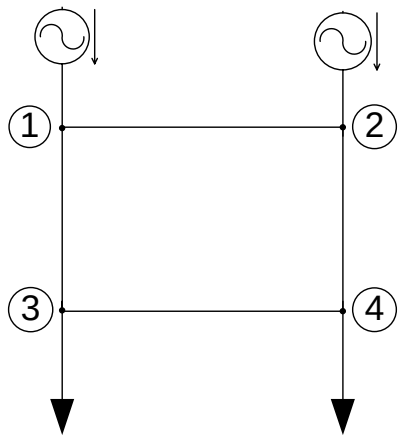
$$= -0,45000 - j0,20000 \ [p.u.]$$

$P_{1-2} = 0,47377\,p.u.$ $\quad P_{G1} = P_{1-2} + P_{T1} = 0,47377 + 1,15 = 1,62972\left[p.u.\right]$

$Q_{1-2} = 0,22067\,p.u.$ $\quad Q_{G1} = Q_{1-2} + Q_{T1} = 0,22607 + 0,31 = 0,53607\left[p.u.\right]$



ZADATAK 13 Zadana je mreža na slici. Prema istosmjernom modelu, odredi tokove snaga u granama. (S_B=100MVA).



$$X_{2-4} = j0,1\,[p.u.]$$
$$X_{3-4} = j0,15\,[p.u.]$$
$$X_{1-2} = j0,2\,[p.u.]$$
$$X_{1-3} = j0,1[p.u.]$$
$$P_3 = -50MW \rightarrow -0,5[p.u.]$$
$$P_1 = -40MW \rightarrow -0,4[p.u.]$$
$$P_2 = 30MW \rightarrow 0,3[p.u.]$$

$$P_{1-2} = ?; \quad P_{1-3} = ? \;;\; P_{4-3} = ? \;;\; P_{2-4} = ?$$

4 – ref.

Rješenje:

$$Y_{2-4} = -j10[p.u.]$$
$$Y_{3-4} = -j6,667[p.u.]$$
$$Y_{1-2} = -j5[p.u.]$$
$$Y_{1-3} = -j10[p.u.]$$

$$Z = j\begin{vmatrix} 0,13636 & 0,04545 & 0,08181 \\ 0,04545 & 0,08181 & 0,02727 \\ 0,08181 & 0,02727 & 0,10909 \end{vmatrix}$$

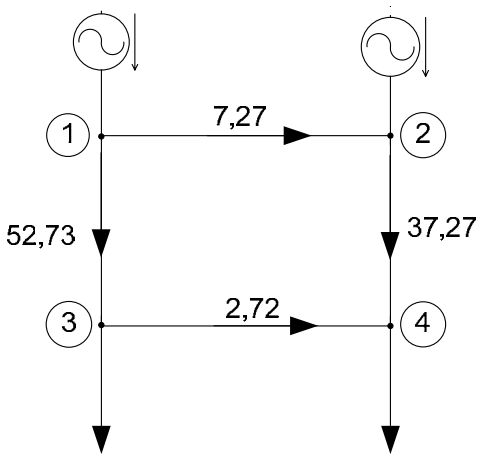
$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = [Z]\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$\delta_1 = j0,13636\cdot(-0,4) + j0,04545\cdot 0,3 + j0,08181\cdot(-0,5) = -j0,08181$$
$$\delta_2 = j0,04545\cdot(-0,4) + j0,08181\cdot 0,3 + j0,02727\cdot(-0,5) = -j0,007272$$
$$\delta_3 = j0,08181\cdot(-0,4) + j0,02727\cdot 0,3 + j0,10909\cdot(-0,5) = -j0,079088$$

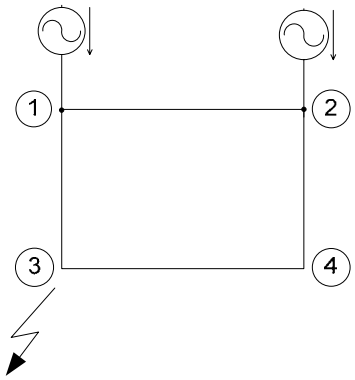
$$P_{1-2} = \frac{\delta_1 - \delta_2}{X_{1-2}} = \frac{-j0,08181 + j0,007272}{j0,1} = -0,3727\,[p.u.]$$
$$P_{1-3} = \frac{\delta_1 - \delta_3}{X_{1-3}} = \frac{-j0,08181 + 0,079088}{j0,1} = -0,02722\,[p.u.]$$
$$P_{2-4} = \frac{\delta_2 - \delta_4}{X_{2-4}} = \frac{-j0,007272 + 0}{j0,1} = -0,07272\,[p.u.]$$
$$P_{3-4} = \frac{\delta_3 - \delta_4}{X_{3-4}} = \frac{-j0,079088 + 0}{j0,15} = -0,52725\,[p.u.]$$

$P_{1-2} = -37,26MW$ $P_{1-3} = -2,72MW$

$P_{2-4} = -7,27MW$ $P_{3-4} = -52,73MW$



ZADATAK 14 Odredi struju kratkog spoja u vodu 1-3 u mreži na slici ako su zadane reaktancije generatora $X_{d4}" = X_{d2}" = j0{,}1$. Napon mreže je 110kV (radi određivanja struje u A). Simetrični kratki spoj je nastao u čvorištu 3 u trenutku kada je mreža bila neopterećena.



$$X_{2-4} = j0{,}1\,[p.u.] \qquad Y_{2-4} = -j10\,[p.u.]$$
$$X_{3-4} = j0{,}15\,[p.u.] \qquad Y_{3-4} = -j6{,}667\,[p.u.]$$
$$X_{1-2} = j0{,}2\,[p.u.] \qquad Y_{1-2} = -j5\,[p.u.]$$
$$X_{1-3} = j0{,}1[p.u.] \qquad Y_{1-3} = -j10[p.u.]$$
$$I_{1-3} = ?$$

Rješenje:

$$Y = -j\begin{vmatrix} 15 & -5 & -10 & 0 \\ -5 & 25 & 0 & -10 \\ -10 & 0 & 16{,}667 & -6{,}667 \\ 0 & -10 & -6{,}667 & 26{,}667 \end{vmatrix}$$

$$Z = Y^{-1} = j\begin{vmatrix} 0{,}1613 & 0{,}0516 & 0{,}1161 & 0{,}0484 \\ 0{,}0516 & 0{,}0643 & 0{,}0452 & 0{,}0355 \\ 0{,}1161 & 0{,}0452 & 0{,}1516 & 0{,}0548 \\ 0{,}0484 & 0{,}0355 & 0{,}0548 & 0{,}0645 \end{vmatrix}$$

$$\begin{bmatrix} U_1^k \\ U_2^k \\ 0 \\ U_4^k \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + j[Z]\begin{bmatrix} 0 \\ 0 \\ I_m \\ 0 \end{bmatrix}$$

$$0 = 1 + j0{,}1516 \cdot I_m \qquad \Rightarrow \qquad I_m = j\frac{1}{0{,}1516} = j6{,}596\,p.u.$$

$$U_1^k = 1 + j0{,}1161 \cdot I_m = 1 - 0{,}7658 = 0{,}2342\,p.u.$$

$$I_{1-3} = \frac{U_1^k - U_3^k}{X_{1-3}} = \frac{0{,}2342}{j0{,}1} = -j2{,}342\,p.u.$$

$$I_{1-3} = -j2{,}342 \cdot \frac{100}{\sqrt{3} \cdot 110} = -j1{,}229kA$$

$$U_2^k = 1 + j0,0452 \cdot I_m = 1 - 0,2981 = 0,7019\, p.u.$$

$$I_{2-1} = \frac{U_2^k - U_1^k}{X_{2-1}} = \frac{0,7019 - 0,2342}{j0,2} = -j2,339\, p.u.$$

$$U_4^k = 1 + j0,0548 \cdot I_m = 1 - 0,3615 = 0,6385 p.u.$$

$$I_{2-4} = \frac{U_2^k - U_4^k}{X_{2-4}} = \frac{0,7019 - 0,6385}{j0,1} = -j0,634\, p.u.$$

$$I_{4-3} = \frac{U_4^k - U_3^k}{X_{4-3}} = \frac{0,6385 - 0}{j0,15} = -j4,256\, p.u.$$