SVEUČILIŠTE U ZAGREBU
FAKULTET ELEKTROTEHNIKE I RAČUNARSTVA

ANALIZA ELEKTROENERGETSKOG SUSTAVA

AUDITORNE VJEŽBE

2. DIO

ANALIZA ELEKTROENERGETSKOG SUSTAVA

Izdanje: Studeni 2011.

Ovaj "radni materijal" predstavlja kratki zapis dijela gradiva i zadataka koji se obrađuju u sklopu predmeta *Analiza elektroenergetskog sustava* po nastavnom programu FER II. Dio gradiva obuhvaćen je samo zadacima, a dio i malim objašnjenjem ili analizom.

PREPORUČENA LITERATURA:

[1.] Marija Ožegović, Karlo Ožegović: *Električne energetske mreže I-VI*, FESB Split, 1996-2006.

[2.] G. W. Stagg, A. H. El-Abiad: *Computer Methods in Power System Analysis*, McGraw-Hill, 1968.

[3.] B. Stefanini, S. Babić, M. Urbiha-Feuerbach: *Matrične metode u analizi električnih mreža*, Školska knjiga, Zagreb, 1975.

POPIS OZNAKA I KRATICA

B	susceptancija ($2\pi fC$) [S]
C	kapacitet [F]
G	vodljivost [S]
I	struja [A]
L	induktivitet [H]
P	djelatna snaga [W]
Q	jalova snaga [var]
R	otpor [Ω]
S	prividna snaga [VA]
U	linijski napon, napon [V]
V	fazni napon [V]
X	reaktancija ($2\pi fL$) [Ω]
Y	admitancija (G+jB) [S]
Z	impedancija (R+jX) [Ω]
z^*	konjugirano kompleksni broj

PRORAČUN TOKOVA SNAGA

1. U transformatorskoj stanici nalaze se 2 transformatora sa sljedećim podacima:

T1:	$S_n = 80MVA$	T2:	$S_n = 80MVA$
	$P_k = 2MW$		$P_k = 2MW$
	$u_k = 11\%$		$u_k = 11\%$
	$216/110$		$222/110$

Napon na primaru iznosi 215 kV, a snaga na sekundaru je 100+j30 MVA. Odredite napon na sekundaru, te tokove i gubitke snage na svakom transformatoru koristeći Gauss-Seidel metodu za Z matricom uz točnost $\varepsilon=10^{-4}$.

RJEŠENJE:

$$U_1 = 215kV$$

$$S_2 = -100 - j30MVA$$

$$\varepsilon = 10^{-4}$$

$$S_B = 100MVA$$

$$a_{1T1} = \frac{216}{220} = 0,982 \qquad a_{1T2} = \frac{222}{220} = 1,009$$

$$Y_T = \frac{1}{Z_T}$$

$$Z_T = \frac{U_n^{\ 2}}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{u_k^{\ 2} - \left(\frac{P_k}{S_n}\right)^2}\right][\Omega]$$

$$Z_T = \frac{S_B}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{u_k^{\ 2} - \left(\frac{P_k}{S_n}\right)^2}\right][p.u.]$$

$$Z_T = 0.03125 + j0.13390\ p.u.$$

$$Y_T\left[p.u.\right] = 1.65289 - j7.08241\ p.u.$$

$$Y_{1,2}' = \frac{Y_T\left[p.u.\right]}{a_{1T1}} = 1,68350 - j7,21356$$

$$Y_{10}' = Y_{12}'\left(\frac{1}{a_{1T1}} - 1\right) = 0,03118 - j0,13358$$

$$Y_{20}' = Y_{12}' a_{1T1}\left(1 - \frac{1}{a_{1T1}}\right) = -0,03061 + j0,13116$$

$$Y_{12}'' = \frac{Y_T\left[p.u.\right]}{a_{1T2}} = 1,63800 - j7,01860$$

$$Y_{10}'' = Y_{12}''\left(\frac{1}{a_{1T2}} - 1\right) = -0,01476 + j0,06323$$

$$Y_{20}'' = Y_{12}'' a_{1T2}\left(1 - \frac{1}{a_{1T2}}\right) = 0,01489 - j0,06381$$

Z_{22} – element Z-matrice, tj. invertirane Y-matrice (u ovom slučaju dimenzije 1x1)

$$Z_{22} = \frac{1}{Y_{12}' + Y_{12}''} = 0,01555 + j0,06663$$

$$Y_{20} = Y_{20}' + Y_{20}'' = -0,01572 + j0,06735$$

$$S_2^* = \frac{-100 + j30}{100} = -1 + j0,3$$

Pretpostavljamo: $U_2^{(0)} = 1 + j0$

$$I_2^{(0)} = \frac{S_2^*}{U_2^{(0)*}} - U_2^{(0)} Y_{20} = -0,98428 + j0,23265$$

$$U_2^{(1)} = U_1 + Z_{22} I_2^{(0)} = 0,94646 - j0,06197 \quad \rightarrow \quad \varepsilon^{(1)} = \left|U_2^{(1)} - U_2^{(0)}\right| = 0,08190$$

$$I_2^{(1)} = -1,02069 + j0,31978$$

$$U_2^{(2)} = 0,94009 - j0,06304 \quad \rightarrow \quad \varepsilon^{(2)} = \left|U_2^{(2)} - U_2^{(1)}\right| = 6,46133 \cdot 10^{-3}$$

$$I_2^{(2)} = -1,02713 + j0,32439$$

$$U_2^{(3)} = 0,93968 - j0,06340 \quad \rightarrow \quad \varepsilon^{(3)} = \left|U_2^{(3)} - U_2^{(2)}\right| = 5,42260 \cdot 10^{-4}$$

$$I_2^{(3)} = -1,02742 + j0,325$$

$$U_2^{(4)} = 0,93964 - j0,06341 \quad \rightarrow \quad \varepsilon^{(4)} = \left|U_2^{(4)} - U_2^{(3)}\right| = 4,58929 \cdot 10^{-5}$$

$$U_2 = U_2^{(4)} = 0,93964 - j0,06341\ p.u. = 103,6\underline{\left|-3,86°\right.}kV$$

Tokovi i gubitci snage:

$$S_{1-2}' = U_1\left[\left(U_1 - U_2\right)Y_{12}' + U_1 Y_{10}'\right]^* = U_1\left(U_1^* - U_2^*\right)Y_{12}'^* + \left|U_1\right|^2 \cdot Y_{10}'^*$$

$$S_{1-2}' = 0,539 + j0,289\ p.u.$$

$$S_{2-1}' = U_2\left(U_2^* - U_1^*\right)Y_{12}^* + \left|U_2\right|^2 \cdot Y_{20}^*$$

$$S_{2-1}' = -0,527 - j0,238\ p.u.$$

$$\Delta S' = S_{1-2}' + S_{2-1}' = 0,012 + j0,050 = 1,2 + j5,0 MVA$$

$$S_{1-2}'' = 0,481 + j0,096\ p.u.$$

$$S_{2-1}'' = -0,473 - j0,062\ p.u.$$

$$\Delta S'' = 0,008 + j0,034 = 0,8 + j3,4 MVA$$

$$\Delta S = \Delta S' + \Delta S'' = 2 + j8,4 MVA$$

2. U transformatorskoj stanici se nalaze 2 transformatora sa sljedećim podacima:

T1:	$S_n = 80MVA$	T2:	$S_n = 80MVA$
	$P_k = 2MW$		$P_k = 2MW$
	$u_k = 11\%$		$u_k = 11\%$
	216/110		222/110

Napon na primaru iznosi 215 kV, a snaga na sekundaru je 100+j30 MVA. Odredite napon na sekundaru, te tokove i gubitke snage na svakom transformatoru koristeći Gauss-Seidel metodu sa Y matricom uz točnost ε=10^{-4}.

RJEŠENJE:

$$U_1 = 215kV$$
$$S_2 = -100 - j30MVA$$
$$\varepsilon = 10^{-4}$$
$$S_B = 100MVA$$
$$a_{1T1} = \frac{216}{220} = 0,982 \qquad a_{1T2} = \frac{222}{220} = 1,009$$
$$U_1 = \frac{215}{220} = 0,97727 \ p.u.$$

$$Y_{12}' = 1,6835 - j7,21356 \qquad Y_{10}' = 0,03118 - j0,13358 \qquad Y_{20}' = -0,03061 + j0,13116$$
$$Y_{12}'' = 1,638 - j7,0186 \qquad Y_{10}'' = -0,01476 + j0,06323 \qquad Y_{20}'' = 0,01489 - j0,06381$$

$$Y = \begin{vmatrix} Y_{12}' + Y_{12}'' + Y_{10}' + Y_{10}'' & -Y_{12}' - Y_{12}'' \\ -Y_{12}' - Y_{12}'' & Y_{12}' + Y_{12}'' + Y_{20}' + Y_{20}'' \end{vmatrix}$$

$$Y = \begin{vmatrix} 3,33792 - j14,30251 & -3,3215 + j14,23216 \\ -3,3215 + j14,23216 & 3,30578 - j14,16481 \end{vmatrix}$$

Opći član:

$$U_i^{(k+1)} = \frac{KL_i}{\left(U_i^{(k)}\right)^*} - \sum_{j=1}^{i-1} YL_{i,j} \cdot U_j^{(k+1)} - \sum_{j=i+1}^{n} YL_{i,j} \cdot U_j^{(k)}$$
$$i = 1,\ldots,n$$
$$i \neq ref$$

KL_i članova ima (n-1) – u našem slučaju 2-1=1.

$YL_{i,j}$ članova ima koliko ima van dijagonalnih elemenata – elementi u referentnom retku.

$$KL_2 = \frac{S_2^*}{Y_{2,2}} = \frac{-1 + j0,3}{3,30578 - j14,16481} = 0,0717771\underline{|240,16°} = -0,03571 - j0,06226$$

$$YL_{2,1} = \frac{Y_{2,1}}{Y_{2,2}} = \frac{-3,3215 + j14,23216}{3,30578 - j14,16481} = 1,004755\underline{|180°} = -1,004755$$

$$U_2^{(0)} = 1 + j0$$

$$U_2^{(1)} = \frac{KL_2}{\left(U_2^{(0)}\right)^*} - YL_{2,1} \cdot U_1$$

$$U_2^{(1)} = 0,94621 - j0,06226$$

Izaberemo $\alpha = 1,2$ (faktor ubrzanja)

$$\Delta U_2^{(1)} = U_2^{(1)} - U_2^{(0)} = 0,94621 - j0,06226 - 1 = -0,05379 - j0,06226$$

$$\left|U_2^{(1)} - U_2^{(0)}\right| = 0,08228$$

$$U_{2ub}^{(1)} = U_{2ub}^{(0)} + \alpha \Delta U_2^{(1)} = 1 + j0 + 1,2\left(-0,05379 - j0,06226\right) =$$
$$= 0,93545 - j0,07471 = 0,93843\underline{|-4,566°}$$

$$U_2^{(2)} = \frac{KL_2}{\left(U_{2ub}^{(1)}\right)^*} - YL_{2,1} \cdot U_1 = \frac{0,07177\underline{|240,16°}}{0,93843\underline{|4,566°}} + 0,98192 = 0,93871 - j0,06310$$

$$\Delta U_2^{(2)} = U_2^{(2)} - U_{2ub}^{(1)} = 0,93871 - j0,06310 - 0,93545 + j0,07471 = 0,00326 + j0,01161$$

$$\left|U_2^{(2)} - U_{2ub}^{(1)}\right| = 0,01206$$

$$U_{2ub}^{(2)} = U_{2ub}^{(1)} + \alpha \Delta U_2^{(2)} = 0,93545 - j0,07471 + 1,2\left(0,00326 + j0,01161\right) =$$
$$= 0,93936 - j0,06078 = 0,94132\underline{|-3,70°}$$

$$U_2^{(3)} = \frac{0,07177\underline{|240,16°}}{0,94132\underline{|3,70°}} + 0,98192 = 0,93979 - j0,06355$$

$$\Delta U_2^{(3)} = U_2^{(3)} - U_{2ub}^{(2)} = 0,93979 - j0,06355 - 0,93936 + j0,06078 = 0,00043 - j0,00277$$

$$\left|U_2^{(3)} - U_{2ub}^{(2)}\right| = 0,00280$$

$$U_{2ub}^{(3)} = U_{2ub}^{(2)} + \alpha \Delta U_2^{(3)} = 0,93988 - j0,06410 = 0,94206\underline{|-3,90°}$$

$$U_2^{(4)} = \frac{0,07177\underline{|240,16°}}{0,94206\underline{|3,90°}} + 0,98192 = 0,93960 - j0,06335 = 0,94173\underline{|-3,86°}$$

$$\Delta U_2^{(4)} = U_2^{(4)} - U_{2ub}^{(3)} = -0,00027 + j0,00075$$

$$\left|U_2^{(4)} - U_{2ub}^{(3)}\right| = 0,00080$$

$$U_{2ub}^{(4)} = U_{2ub}^{(3)} + \alpha \Delta U_2^{(4)} = 0,93955 - j0,06320 = 0,94167\underline{|-3,85°}$$

$$U_2^{(5)} = \frac{0,07177\angle 240,16°}{0,94167\angle 3,85°} + 0,98192 = 0,93965 - j0,06342$$

$$\Delta U_2^{(5)} = U_2^{(5)} - U_{2ub}^{(4)} = 0,0001 - j0,00022$$

$$\left| U_2^{(5)} - U_{2ub}^{(4)} \right| = 0,00024$$

$$U_{2ub}^{(5)} = U_{2ub}^{(4)} + \alpha \Delta U_2^{(5)} = 0,93966 - j0,06346 = 0,94180\angle -3,86°$$

$$U_2^{(6)} = \frac{0,07177\angle 240,16°}{0,94180\angle 3,86°} + 0,98192 = 0,93963 - j0,06340$$

$$\Delta U_2^{(6)} = U_2^{(6)} - U_{2ub}^{(5)} = -0,00003 + j0,00006$$

$$\left| U_2^{(6)} - U_{2ub}^{(5)} \right| = 0,00007$$

$$U_2 = 0,93963 - j0,06340 = 0,94177\angle -3,86°$$

$$U_2 = 103,6\angle -3,86° \; kV$$

3. Zadani su podaci za transformatore u stanici:

T1: $S_n = 80MVA$, $P_k = 2MW$, $u_k = 11\%$, $216/110$

T2: $S_n = 80MVA$, $P_k = 2MW$, $u_k = 11\%$, $222/110$

Napon na primaru iznosi $U_1 = 215kV$, a snaga sekundara je $S_2 = -100 - j30MVA$. Odredite napon na sekundaru i tokove snage kroz transformatore koristeći Newton-Raphson metodu uz zadanu točnost $\varepsilon = 10^{-2}$.

RJEŠENJE:

$$S_B = 100MVA$$

$$a_{1T1} = \frac{216}{220} = 0,98182 \qquad a_{1T2} = \frac{222}{220} = 1,009$$

$$Z_T = 0,03125 + j0.13390 \ p.u.$$

$$Y_T = \frac{1}{Z_T}$$

$$Y_{1,2}' = \frac{Y_T}{a_{1T1}} = 1,6835 - j7,21356 \, p.u.$$

$$Y_{1,0}' = Y_{12}'\left(\frac{1}{a_{1T1}} - 1\right) = 0,03118 - j0,13358 \, p.u.$$

$$Y_{2,0}' = Y_{1,2}' \cdot a_{1T1}\left(1 - \frac{1}{a_{1T1}}\right) = -0,03061 + j0,13116 \, p.u.$$

$$Y_{1,2}'' = 1,638 - j7,0186 \, p.u.$$

$$Y_{1,0}'' = -0,01476 + j0,06323 \, p.u.$$

$$Y_{2,0}'' = 0,01489 - j0,06381 \, p.u.$$

$$[Y] = \begin{bmatrix} 3,33792 - j14,30231 & -3,3215 + j14,23216 \\ -3,3215 + j14,23216 & 3,30578 - j14,16481 \end{bmatrix} = \begin{bmatrix} 14,68685\angle -76.863^\circ & 14,61461\angle 103.137^\circ \\ 14,61461\angle 103.137^\circ & 14,54545\angle -76.863^\circ \end{bmatrix}$$

$$U_1 = \frac{215}{220} = 0,977273 + j0 \qquad (ref.čvorište)$$

$$U_2^{(0)} = 1 + j0 \qquad (pretpostavljeni\ napon)$$

Za sva nezavisna čvorišta vrijedi:

$$P_{i,izr}^{(k)} = U_i^{(k)} \sum_{j=1}^{n} Y_{i,j} \cdot U_j^{(k)} cos\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad ; i = 1, 2, 3, \dots, n-1 \quad (ako\ je\ n - to\ čvorište\ referentno)$$

$$Q_{i,izr}^{(k)} = U_i^{(k)} \sum_{j=1}^{n} Y_{i,j} \cdot U_j^{(k)} sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{ij}\right) \quad ; i = 1, 2, 3, \dots, n-1-g \quad (g - generatorska\ čvorišta)$$

Referentno čvorište 1:

$$i = 2$$

$$P_2^{(0)} = U_2^{(0)} \left[Y_{2,1} \cdot U_1 \cos\left(\delta_2^{(0)} - \delta_1 - \Theta_{2,1}\right) + Y_{2,2} \cdot U_2^{(0)} \cos\left(-\Theta_{2,2}\right) \right] =$$

$$= 1 \cdot 14,61461 \cdot 0,977273 \cdot \cos\left(-103,137\right) + 1 \cdot 14,54545 \cdot \cos\left(76,863\right) = 0,05977$$

$$Q_2^{(0)} = U_2^{(0)} \left[Y_{2,1} \cdot U_1 \sin\left(\delta_2^{(0)} - \delta_1 - \Theta_{2,1}\right) + Y_{2,2} \cdot U_2^{(0)} \sin\left(-\Theta_{2,2}\right) \right] =$$

$$= 1 \cdot 14,61461 \cdot 0,977273 \cdot \sin\left(-103,137\right) + 1 \cdot 1 \cdot 14,54545 \cdot \sin\left(76,863\right) = 0,25611$$

$$\Delta P_2^{(0)} = P_2^{ZAD} - P_2^{(0)} = -1 - 0,0598 = -1,05977$$

$$\Delta Q_2^{(0)} = Q_2^{ZAD} - Q_2^{(0)} = -0,3 - 0,25611 = -0,55611$$

J₁ za $i \neq ref.$**:**

$$\left(\frac{\partial P_i}{\partial \delta_i}\right)^{(k)} = -\sum_{\substack{j=1 \\ j \neq i}}^{n} U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{i,j} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{i,j}\right) \qquad i = 1, 2, \dots, n-1$$

$$\left(\frac{\partial P_i}{\partial \delta_j}\right)^{(k)} = U_i^{(k)} \cdot U_j^{(k)} \cdot Y_{i,j} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{i,j}\right) \qquad i = 1, 2, \dots, n-1;\ j = 1, 2, \dots, n-1$$

J₄ za $i \neq ref.$**:**

$$\left(\frac{\partial Q_i}{\partial U_i}\right)^{(k)} = 2 \cdot U_i^{(k)} \cdot Y_{i,i} \cdot \sin\left(-\Theta_{ii}\right) + \sum_{\substack{j=1 \\ j \neq i}}^{n} U_j^{(k)} \cdot Y_{i,j} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{i,j}\right) \qquad i = 1, 2, \dots, n-1-g$$

$$\left(\frac{\partial Q_i}{\partial U_j}\right)^{(k)} = U_i^{(k)} \cdot Y_{i,j} \cdot \sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \Theta_{i,j}\right) \qquad i = 1, 2, \dots, n-1-g;\ j = 1, 2, \dots, n-1-g$$

$$\left(\frac{\partial P_2}{\partial \delta_2}\right)^{(0)} = -U_2^{(0)} \cdot U_1 \cdot Y_{2,1} \cdot \sin\left(\delta_2^{(0)} - \delta_1^{(0)} - \Theta_{2,1}\right) =$$

$$= -1 \cdot 0,977273 \cdot 14,61461 \cdot \sin\left(-103,137\right) =$$

$$= 13,90871$$

$$\left(\frac{\partial Q_2}{\partial U_2}\right)^{(0)} = 2 \cdot U_2^{(0)} \cdot Y_{2,2} \cdot \sin\left(-\Theta_{2,2}\right) + U_1 \cdot Y_{2,1} \cdot \sin\left(\delta_2^{(0)} - \delta_1^{(0)} - \Theta_{2,1}\right) =$$

$$= 2 \cdot 1 \cdot 14,54545 \cdot \sin\left(76,863\right) + 0,977273 \cdot 14,61461 \cdot \sin(-103,137) =$$

$$= 14,4209$$

$$\Delta P_2^{(0)} = \left(\frac{\partial P_2}{\partial \delta_2}\right)^{(0)} \cdot \Delta\delta_2^{(0)} \qquad \Rightarrow \qquad -1,05977 = 13,90871 \cdot \Delta\delta_2^{(0)}$$

$$\Delta Q_2^{(0)} = \left(\frac{\partial Q_2}{\partial U_2}\right)^{(0)} \cdot \Delta U_2^{(0)} \qquad \Rightarrow \qquad -0,55611 = 14,4209 \cdot \Delta U_2^{(0)}$$

$$\Delta\delta_2^{(0)} = -\frac{1,05977}{13,90871} = -0,07619$$

$$\Delta U_2^{(0)} = -\frac{0,55611}{14,4209} = -0,03856 \qquad \longrightarrow \qquad \text{kraj 0. iteracijskog koraka}$$

$$U_2^{(1)} = U_2^{(0)} + \Delta U_2^{(0)} = 1 - 0,03856 = 0,96144$$

$$\delta_2^{(1)} = \delta_2^{(0)} + \Delta\delta_2^{(0)} = -0,07619 \ \ rad = -4,366°$$

$$P_2^{(1)} = 0,96144 \cdot 14,61461 \cdot 0,977273 \cdot \cos\left(-103,137 - 4,366\right) +$$

$$+ 0,96144 \cdot 14,54545 \cdot 0,96144 \cdot \cos(76,863) =$$

$$= -1,07396$$

$$Q_2^{(1)} = 0,961437 \cdot 14,61461 \cdot 0,977273 \cdot \sin(-103,137 - 4,366) +$$

$$+ 0,961437 \cdot 14,54545 \cdot 0,961437 \cdot \sin(76,863) =$$

$$= -0,00258$$

$$\Delta P_2^{(1)} = P_2^{ZAD} - P_2^{(1)} = -1 + 1,07396 = 0,07396$$

$$\Delta Q_2^{(1)} = Q_2^{ZAD} - Q_2^{(1)} = -0,3 + 0,00258 = -0,29742$$

$$\left(\frac{\partial P_2}{\partial \delta_2}\right)^{(1)} = -0,96144 \cdot 0,97728 \cdot 14,61461 \cdot \sin(-103,137 - 4,366) =$$

$$= 13,09599$$

$$\left(\frac{\partial Q_2}{\partial U_2}\right)^{(1)} = 2 \cdot 0,96144 \cdot 14,54545 \cdot \sin(76,869) + 0,97728 \cdot 14,61451 \cdot \sin(-103,137 - 4,36564) =$$

$$= 13,61591$$

$$\Delta P_2^{(1)} = \left(\frac{\partial P_2}{\partial \delta_2} \right)^{(1)} \cdot \Delta \delta_2^{(1)} \qquad \Rightarrow \qquad 0,07396 = 13,09599 \cdot \Delta \delta_2^{(1)}$$

$$\Delta Q_2^{(1)} = \left(\frac{\partial Q_2}{\partial U_2} \right)^{(1)} \cdot \Delta U_2^{(1)} \qquad \Rightarrow \qquad -0,29742 = 13,61591 \cdot \Delta U_2^{(1)}$$

$$\Delta \delta_2^{(1)} = \frac{0,07396}{13,09599} = 0,00565$$

$$\Delta U_2^{(1)} = -\frac{0,29742}{13,61591} = -0,02184$$

$$U_2^{(2)} = U_2^{(1)} + \Delta U_2^{(1)} = 0,96144 - 0,021844 = 0,93959$$

$$\delta_2^{(2)} = \delta_2^{(1)} + \Delta \delta_2^{(1)} = -0,07055 = -4,0421°$$

$$P_2^{(2)} = -1,04507$$

$$Q_2^{(2)} = -0,31583$$

$$\Delta P_2^{(2)} = 0,04507$$

$$\Delta Q_2^{(2)} = 0,01583$$

$$\vdots$$

$$\Delta P_2^{(3)} = -0,00252$$

$$\Delta Q_2^{(3)} = 0,01385$$

$$\vdots$$

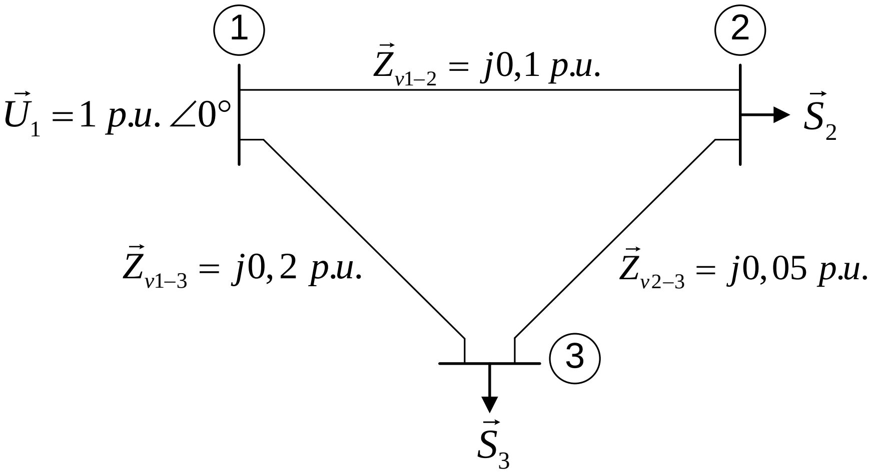
$$U_2^{(4)} = 0,94188$$

$$\delta_2^{(4)} = -0,06723 = -3,85190°$$

$$\Delta P_2^{(4)} = -0,00219$$

$$\Delta Q_2^{(4)} = 0,00079$$

4. Odredite numeričke vrijednosti Jakobijeve matrice u 0-toj iteraciji za proračun tokova snaga Newton-Raphsonovom metodom za mrežu na slici.



RJEŠENJE:

$$U_1 = 1 + j0\ p.u.$$
$$U_2^{(0)} = 1 + j0\ p.u.$$
$$U_3^{(0)} = 1 + j0\ p.u.$$

$$Y_{1-2} = -j10\ p.u.$$
$$Y_{1-3} = -j5\ p.u.$$
$$Y_{2-3} = -j20\ p.u.$$

$$Y = -j\begin{vmatrix} 15 & -10 & -5 \\ -10 & 30 & -20 \\ -5 & -20 & 25 \end{vmatrix} p.u. = \begin{vmatrix} 15\angle -90° & 10\angle 90° & 5\angle 90° \\ 10\angle 90° & 30\angle -90° & 20\angle 90° \\ 5\angle 90° & 20\angle 90° & 25\angle -90° \end{vmatrix} p.u.$$

J1:

$$\frac{\partial P_2}{\partial \delta_2} = -U_2^{(0)} \cdot U_1 \cdot Y_{21} \cdot sin\left(\delta_2{}^{(0)} - \delta_1 - \Theta_{21}\right) — U_2^{(0)} \cdot U_3^{(0)} \cdot Y_{23} \cdot sin\left(\delta_2{}^{(0)} - \delta_3{}^{(0)} - \Theta_{23}\right) = 30$$

$$\frac{\partial P_2}{\partial \delta_3} = U_2^{(0)} \cdot U_3^{(0)} \cdot Y_{23} \cdot sin\left(\delta_2{}^{(0)} - \delta_3{}^{(0)} - \Theta_{23}\right) = -20$$

$$\frac{\partial P_3}{\partial \delta_2} = U_3^{(0)} \cdot U_2^{(0)} \cdot Y_{32} \cdot sin\left(\delta_3{}^{(0)} - \delta_2{}^{(0)} - \Theta_{32}\right) = -20$$

$$\frac{\partial P_3}{\partial \delta_3} = -U_3^{(0)} \cdot U_1 \cdot Y_{31} \cdot sin\left(\delta_3{}^{(0)} - \delta_1 - \Theta_{31}\right) — U_3^{(0)} \cdot U_2^{(0)} \cdot Y_{32} \cdot sin\left(\delta_3{}^{(0)} - \delta_2{}^{(0)} - \Theta_{32}\right) = 25$$

J2:

$$\frac{\partial P_2}{\partial U_2} = 2 \cdot U_2^{(0)} \cdot Y_{22} \cdot cos(-\Theta_{22}) + U_1 \cdot Y_{21} \cdot cos\left(\delta_2{}^{(0)} - \delta_1 - \Theta_{21}\right)$$
$$+U_3^{(0)} \cdot Y_{23} \cdot cos\left(\delta_2{}^{(0)} - \delta_3{}^{(0)} - \Theta_{23}\right) = 0$$

$$\frac{\partial P_2}{\partial U_3} = U_2^{(0)} \cdot Y_{23} \cdot cos\left(\delta_2{}^{(0)} - \delta_3{}^{(0)} - \Theta_{23}\right) = 0$$
$$\frac{\partial P_3}{\partial U_2} = U_3^{(0)} \cdot Y_{32} \cdot cos\left(\delta_3{}^{(0)} - \delta_2{}^{(0)} - \Theta_{32}\right) = 0$$
$$\frac{\partial P_3}{\partial U_3} = 2 \cdot U_3^{(0)} \cdot Y_{33} \cdot cos(-\Theta_{33}) + U_1 \cdot Y_{31} \cdot cos\left(\delta_3{}^{(0)} - \delta_1 - \Theta_{31}\right)$$
$$+U_2^{(0)} \cdot Y_{32} \cdot cos\left(\delta_3{}^{(0)} - \delta_2{}^{(0)} - \Theta_{32}\right) = 0$$

J3:

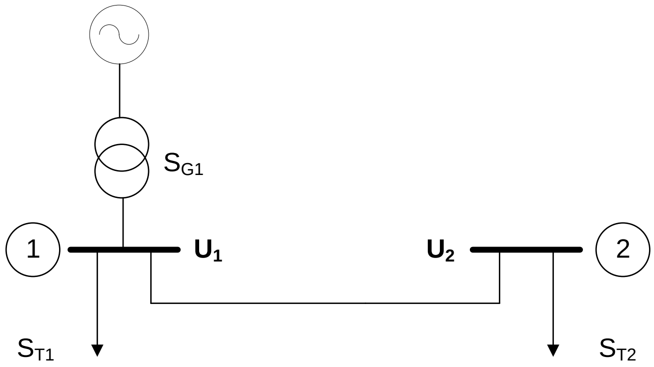
$$\frac{\partial Q_2}{\partial \delta_2} = U_2^{(0)} \cdot U_1 \cdot Y_{21} \cdot cos\left(\delta_2{}^{(0)} - \delta_1 - \Theta_{21}\right) + U_2^{(0)} \cdot U_3^{(0)} \cdot Y_{23} \cdot cos\left(\delta_2{}^{(0)} - \delta_3{}^{(0)} - \Theta_{23}\right)$$
$$= 0$$
$$\frac{\partial Q_2}{\partial \delta_3} = -U_2^{(0)} \cdot U_3^{(0)} \cdot Y_{23} \cdot cos\left(\delta_2{}^{(0)} - \delta_3{}^{(0)} - \Theta_{23}\right) = 0$$
$$\frac{\partial Q_3}{\partial \delta_2} = -U_3^{(0)} \cdot U_2^{(0)} \cdot Y_{32} \cdot cos\left(\delta_3{}^{(0)} - \delta_2{}^{(0)} - \Theta_{32}\right) = 0$$
$$\frac{\partial Q_3}{\partial \delta_3} = U_3^{(0)} \cdot U_1 \cdot Y_{31} \cdot cos\left(\delta_3{}^{(0)} - \delta_1 - \Theta_{31}\right) + U_3^{(0)} \cdot U_2^{(0)} \cdot Y_{32} \cdot cos\left(\delta_3{}^{(0)} - \delta_2{}^{(0)} - \Theta_{32}\right)$$
$$= 0$$

J4:

$$\frac{\partial Q_2}{\partial U_2} = 2 \cdot U_2^{(0)} \cdot Y_{22} \cdot sin(-\Theta_{22}) + U_1 \cdot Y_{21} \cdot sin\left(\delta_2{}^{(0)} - \delta_1 - \Theta_{21}\right)$$
$$+U_3^{(0)} \cdot Y_{23} \cdot sin\left(\delta_2{}^{(0)} - \delta_3{}^{(0)} - \Theta_{23}\right) = 30$$
$$\frac{\partial Q_2}{\partial U_3} = U_2^{(0)} \cdot Y_{23} \cdot sin\left(\delta_2{}^{(0)} - \delta_3{}^{(0)} - \Theta_{23}\right) = -20$$
$$\frac{\partial Q_3}{\partial U_2} = U_3^{(0)} \cdot Y_{32} \cdot sin\left(\delta_3{}^{(0)} - \delta_2{}^{(0)} - \Theta_{32}\right) = -20$$
$$\frac{\partial P_3}{\partial U_3} = 2 \cdot U_3^{(0)} \cdot Y_{33} \cdot sin(-\Theta_{33}) + U_1 \cdot Y_{31} \cdot sin\left(\delta_3{}^{(0)} - \delta_1 - \Theta_{31}\right)$$
$$+U_2^{(0)} \cdot Y_{32} \cdot sin\left(\delta_3{}^{(0)} - \delta_2{}^{(0)} - \Theta_{32}\right) = 25$$

$$J^{(0)} = \begin{vmatrix} 30 & -20 & 0 & 0 \\ -20 & 25 & 0 & 0 \\ 0 & 0 & 30 & -20 \\ 0 & 0 & -20 & 25 \end{vmatrix}$$

5. Za mali sustav na slici, napravite proračun tokova snaga Gauss-Seidelovom metodom pomoću Y-matrice.



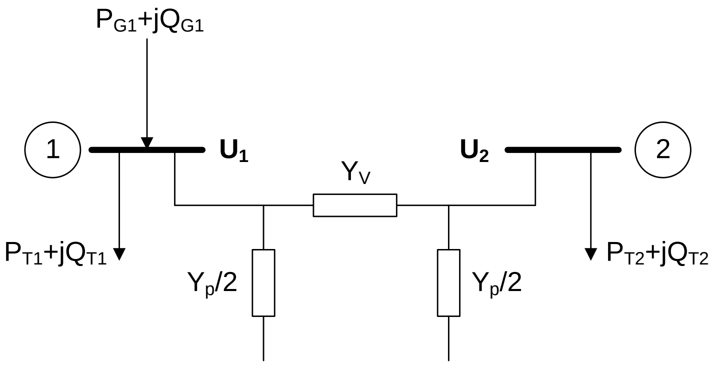
Zadano je:

$U_1 = 1,05\underline{|0°}[p.u.]$ $\qquad$ $Y_V = 0,3044 - j1,87996[p.u.]$

$S_{T1} = 1,15 + j0,31[p.u.]$ $\qquad$ $\frac{Y_p}{2} = j0,06366[p.u.]$

$S_{T2} = 0,45 + j0,20[p.u.]$ $\qquad$ $|U_2^{(0)}| = 0,95$; $\delta_2^{(0)} = -13,5°$; $\varepsilon = 10^{-4}$

RJEŠENJE:



$$U_1 = |U_1|\underline{|0°}$$
$$U_2 = |U_2|\underline{|\delta_2}$$

$$|Y| = \begin{vmatrix} 0,3044 - j1,8163 & -0,3044 + 1,87996 \\ -0,3044 + 1,87996 & 0,3044 - j1,8163 \end{vmatrix}$$

$$S_2 = -S_{T2} = -0,45 - j0,20[p.u.]$$

$$KL_2 = \frac{S_2^*}{Y_{2,2}} = 0,267395\underline{|-123,47647°} = -0,14749 - j0,22304$$

$$YL_{2,1} = \frac{Y_{2,1}}{Y_{2,2}} = 1,034108\underline{|179,68343°} = -1,03409 + j0,00571$$

$$U_2^{(1)} = \frac{KL_2}{(U_2^{(0)})^*} - YL_{2,1} \cdot U_1 = 0,88002 - j0,19804$$

$$\Delta U_2^{(1)} = U_2^{(1)} - U_2^{(0)} = -0,04373 + j0,02373$$

$$\left|\Delta U_2^{(1)}\right| = 0,04975$$

$$U_2^{(2)} = \frac{KL_2}{(U_2^{(1)})^*} - YL_{2,1} \cdot U_1 = 0.87199 - j0,21133$$

$$\Delta U_2^{(2)} = U_2^{(2)} - U_2^{(1)} = -0,00804 - j0,01328$$

$$\left|\Delta U_2^{(2)}\right| = 0,001552$$

$$\vdots$$

$$U_2^{(7)} = \frac{KL_2}{(U_2^{(6)})^*} - YL_{2,1} \cdot U_1 = 0.86591 - j0,21018$$

$$\Delta U_2^{(7)} = U_2^{(7)} - U_2^{(6)} = -5,72523 \cdot 10^{-5} + j3,12535 \cdot 10^{-5}$$

$$\left|\Delta U_2^{(7)}\right| = 6,52274 \cdot 10^{-5}$$

$$U_2 = U_2^{(7)} = 0.86591 - 0,21018 \ p.u. = 0,89105\angle -13,64347° \ p.u.$$

$$S_{1-2} = U_1\left(U_1^* - U_2^*\right)Y_{1,2}^* + \left|U_1\right|^2 \cdot \frac{Y_p^*}{2}$$

$$S_{1-2} = 0,47373 + j0,22602$$

$$S_{2-1} = U_2\left(U_2^* - U_1^*\right)Y_{1,2}^* + \left|U_2\right|^2 \cdot \frac{Y_p^*}{2} =$$

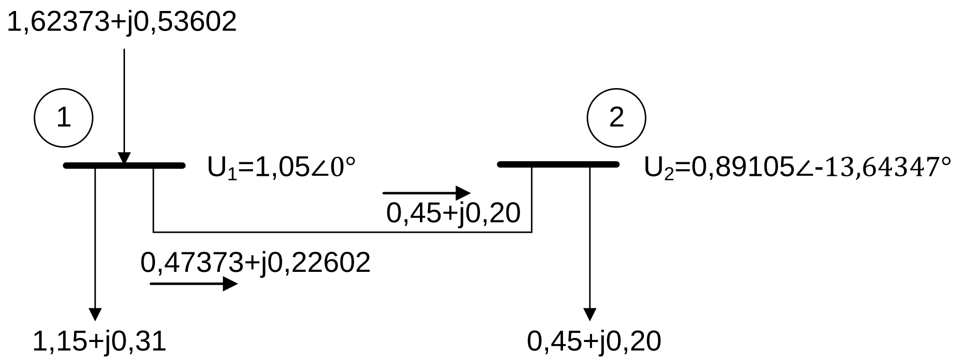
$$S_{2-1} = -0,44996 - j0,19999$$

$$\Delta S = S_{1-2} + S_{2-1}$$

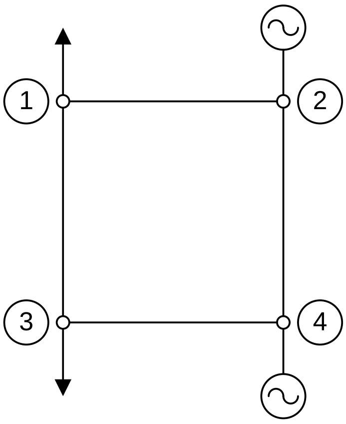
$$\Delta S = 0,02376 + j0,02603$$

$$P_{1-2} = 0,47373 p.u. \qquad P_{G1} = P_{1-2} + P_{T1} = 0,47373 + 1,15 = 1,62373\left[p.u.\right]$$

$$Q_{1-2} = 0,22602 p.u. \qquad Q_{G1} = Q_{1-2} + Q_{T1} = 0,22602 + 0,31 = 0,53602\left[p.u.\right]$$


1,62373+j0,53602
1
2
U1=1,05∠0°
U2=0,89105∠-13,64347°
0,45+j0,20
0,47373+j0,22602
1,15+j0,31
0,45+j0,20

6. Zadana je mreža na slici. Prema istosmjernom modelu, odredite tokove snaga u granama. (S_B=100MVA).



4 – ref.

$$X_{2-4} = j0{,}1\,[p.u.]$$
$$X_{3-4} = j0{,}15\,[p.u.]$$
$$X_{1-2} = j0{,}2\,[p.u.]$$
$$X_{1-3} = j0{,}1[p.u.]$$
$$P_3 = -50MW \quad \rightarrow \quad -0{,}5[p.u.]$$
$$P_1 = -40MW \quad \rightarrow \quad -0{,}4[p.u.]$$
$$P_2 = 30MW \quad \rightarrow \quad 0{,}3[p.u.]$$

$$P_{1-2} = ?; \quad P_{1-3} = ? \;\; ; \;\; P_{4-3} = ? \;\; ; \;\; P_{2-4} = ?$$

RJEŠENJE:

$$Y_{2-4} = -j10[p.u.]$$
$$Y_{3-4} = -j6{,}667[p.u.]$$
$$Y_{1-2} = -j5[p.u.]$$
$$Y_{1-3} = -j10[p.u.]$$

$$Z = j\begin{vmatrix} 0{,}13636 & 0{,}04545 & 0{,}08181 \\ 0{,}04545 & 0{,}08181 & 0{,}02727 \\ 0{,}08181 & 0{,}02727 & 0{,}10909 \end{vmatrix}$$

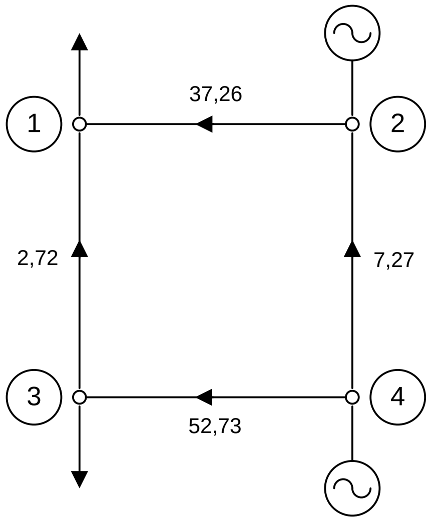
$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = [Z]\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$\delta_1 = j0{,}13636 \cdot (-0{,}4) + j0{,}04545 \cdot 0{,}3 + j0{,}08181 \cdot (-0{,}5) = -j0{,}08181$$
$$\delta_2 = j0{,}04545 \cdot (-0{,}4) + j0{,}08181 \cdot 0{,}3 + j0{,}02727 \cdot (-0{,}5) = -j0{,}007272$$
$$\delta_3 = j0{,}08181 \cdot (-0{,}4) + j0{,}02727 \cdot 0{,}3 + j0{,}10909 \cdot (-0{,}5) = -j0{,}079088$$

$$P_{1-2} = \frac{\delta_1 - \delta_2}{X_{1-2}} = \frac{-j0{,}08181 + j0{,}007272}{j0{,}1} = -0{,}3727\,[p.u.]$$
$$P_{1-3} = \frac{\delta_1 - \delta_3}{X_{1-3}} = \frac{-j0{,}08181 + 0{,}079088}{j0{,}1} = -0{,}02722\,[p.u.]$$
$$P_{2-4} = \frac{\delta_2 - \delta_4}{X_{2-4}} = \frac{-j0{,}007272 + 0}{j0{,}1} = -0{,}07272\,[p.u.]$$
$$P_{3-4} = \frac{\delta_3 - \delta_4}{X_{3-4}} = \frac{-j0{,}079088 + 0}{j0{,}15} = -0{,}52725\,[p.u.]$$

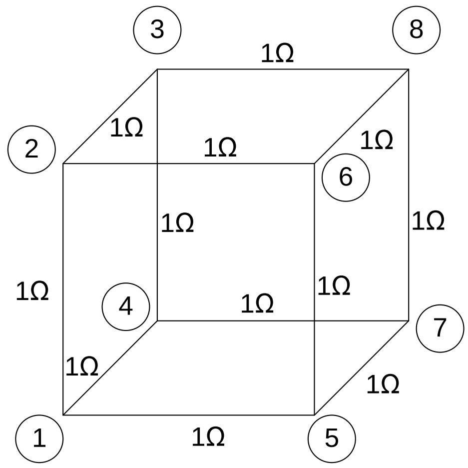
$P_{1-2} = -37,26MW$ $\qquad P_{1-3} = -2,72MW$

$P_{2-4} = -7,27MW$ $\qquad P_{3-4} = -52,73MW$



DODATAK A: INVERZIJA MATRICE

PRIMJER Zadana je "konfiguracija otpora" prema slici. Odredi nadomjesni otpor između točaka 1 i 8.



(referentno čvorište 8)

RJEŠENJE:

Nadomjesni otpor će se izračunati tako da se prvo odredi matrica admitancija čvorišta Y, a zatim njenom inverzijom matrica impedancija čvorišta Z.

Matrica Y:

$$\vec{Y} = \begin{vmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 3 & -1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 3 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 3 & -1 & -1 \\ 0 & -1 & 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{vmatrix}$$

Za proračun matrice Z se koristi metoda inverzije matrice Y pomoću Gauss-Jordanove eliminacije.

1) Pivot $\rightarrow \vec{Y}(3,3)$

$$\vec{Y} = \begin{vmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 3 & -1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 3 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 3 & -1 & -1 \\ 0 & -1 & 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{vmatrix}$$

2) Pivot $\rightarrow \vec{Y}(6,6)$

$$\vec{Y}^{(1)} = \begin{vmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & \frac{8}{3} & \frac{1}{3} & -\frac{1}{3} & 0 & -1 & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 \\ -1 & -\frac{1}{3} & \frac{1}{3} & \frac{8}{3} & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 3 & -1 & -1 \\ 0 & -1 & 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{vmatrix}$$

3) Pivot $\rightarrow \vec{Y}(7,7)$

$$\vec{Y}^{(2)} = \begin{vmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & \frac{7}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 \\ -1 & -\frac{1}{3} & \frac{1}{3} & \frac{8}{3} & 0 & 0 & -1 \\ -1 & -\frac{1}{3} & 0 & 0 & \frac{8}{3} & \frac{1}{3} & -1 \\ 0 & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{vmatrix}$$

Vrijede sljedeći izrazi za m+1 iteraciju:

Pivot:

$$Y_{i,i}^{(m+1)} = \frac{1}{Y_{i,i}^{(m)}}$$

Elementi precrtanog retka:

$$Y_{i,j}^{(m+1)} = \frac{Y_{i,j}^{(m)}}{Y_{i,i}^{(m)}} \quad ; j = 1,\dots,N; \quad j \neq i$$

Elementi precrtanog stupca:

$$Y_{j,i}^{(m+1)} = -\frac{Y_{j,i}^{(m)}}{Y_{i,i}^{(m)}} \quad ; j = 1,\dots,N; \quad j \neq i$$

Vanjski elementi:

$$Y_{k,l}^{(m+1)} = Y_{k,l}^{(m)} - \frac{Y_{k,i}^{(m)} \cdot Y_{i,l}^{(m)}}{Y_{i,i}^{(m)}}$$

$; k = 1,\dots,N; \; k \neq i$

$; l =,\dots,N; \; l \neq i$

4) Pivot $\rightarrow \vec{Y}(1,1)$

$$\vec{Y}^{(3)} = \begin{vmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & \frac{7}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 \\ -1 & -\frac{1}{3} & \frac{1}{3} & \frac{7}{3} & -\frac{1}{3} & 0 & \frac{1}{3} \\ -1 & -\frac{1}{3} & 0 & -\frac{1}{3} & \frac{7}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & \frac{1}{3} \end{vmatrix}$$

5) Pivot $\rightarrow \vec{Y}(2,2)$

$$\vec{Y}^{(4)} = \begin{vmatrix} \frac{1}{3} & -\frac{1}{3} & 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & 0 \\ \frac{1}{3} & \frac{6}{3} & \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} & \frac{4}{3} & -\frac{2}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & 0 & -\frac{2}{3} & \frac{6}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & \frac{1}{3} \end{vmatrix}$$

6) Pivot $\rightarrow \vec{Y}(4,4)$

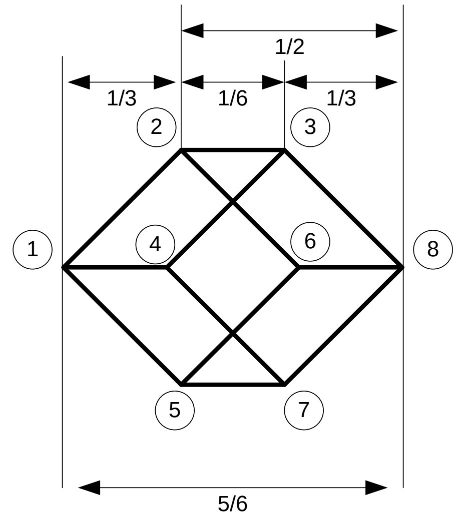
$$\vec{Y}^{(5)} = \begin{vmatrix} \frac{7}{18} & \frac{1}{6} & \frac{1}{18} & -\frac{4}{9} & -\frac{4}{9} & \frac{1}{18} & 0 \\ \frac{1}{6} & \frac{3}{6} & \frac{1}{6} & -\frac{2}{6} & -\frac{2}{6} & \frac{1}{6} & 0 \\ \frac{1}{18} & \frac{1}{6} & \frac{7}{18} & -\frac{4}{9} & -\frac{1}{9} & \frac{1}{18} & 0 \\ \frac{4}{9} & \frac{2}{6} & \frac{4}{9} & \frac{16}{9} & -\frac{8}{9} & \frac{1}{9} & \frac{1}{3} \\ \frac{4}{9} & \frac{2}{6} & \frac{1}{9} & -\frac{8}{9} & \frac{16}{9} & \frac{4}{9} & \frac{1}{3} \\ \frac{1}{18} & \frac{1}{6} & \frac{1}{18} & -\frac{1}{9} & -\frac{4}{9} & \frac{7}{18} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & \frac{1}{3} \end{vmatrix}$$

7) Pivot $\rightarrow \vec{Y}(5,5)$

$$\vec{Y}^{(6)} = \begin{vmatrix} \frac{1}{2} & \frac{3}{12} & \frac{1}{6} & \frac{1}{4} & -\frac{2}{3} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{4} & \frac{9}{16} & \frac{1}{4} & \frac{3}{16} & -\frac{1}{2} & \frac{3}{16} & \frac{1}{16} \\ \frac{1}{6} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & -\frac{1}{3} & \frac{1}{12} & \frac{1}{18} \\ \frac{1}{4} & \frac{3}{6} & \frac{1}{4} & \frac{9}{16} & -\frac{1}{2} & \frac{1}{16} & \frac{3}{16} \\ \frac{2}{3} & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & \frac{4}{3} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{12} & \frac{3}{16} & \frac{1}{12} & \frac{1}{16} & -\frac{1}{2} & \frac{57}{141} & \frac{1}{48} \\ \frac{1}{12} & \frac{1}{16} & \frac{1}{12} & \frac{3}{16} & -\frac{1}{2} & \frac{1}{18} & \frac{19}{48} \end{vmatrix}$$

8) Inverz matrice $\vec{Y}$

$$\vec{Y}^{(7)} = \begin{vmatrix} \frac{5}{6} & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & \frac{1}{2} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{3}{4} & \frac{3}{8} & \frac{6}{16} & \frac{3}{8} & \frac{3}{8} & \frac{1}{4} \\ \frac{1}{3} & \frac{3}{8} & \frac{7}{12} & \frac{3}{8} & \frac{1}{4} & \frac{5}{24} & \frac{5}{24} \\ \frac{1}{2} & \frac{3}{8} & \frac{3}{8} & \frac{3}{4} & \frac{3}{8} & \frac{1}{4} & \frac{3}{8} \\ \frac{1}{2} & \frac{3}{8} & \frac{1}{4} & \frac{3}{8} & \frac{3}{4} & \frac{3}{8} & \frac{3}{8} \\ \frac{1}{3} & \frac{3}{8} & \frac{5}{24} & \frac{1}{4} & \frac{3}{8} & \frac{7}{12} & \frac{5}{24} \\ \frac{1}{3} & \frac{1}{4} & \frac{5}{24} & \frac{3}{8} & \frac{3}{8} & \frac{5}{24} & \frac{7}{12} \end{vmatrix}$$



$I_1 = 1A$

$U_2 = 0$

$$I_{1,n} = I_{1,2} = I_{1,5} = {}^{1}\!/_{3}\,A \;\Rightarrow\; U_2 = U_4 = U_5 = \frac{5}{6} - \frac{1}{3} = \frac{1}{2}V$$

$$\Rightarrow\; U_3 = U_6 = U_7 = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}V$$

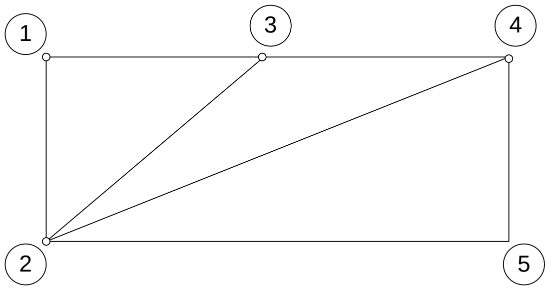
$Z_{i,j}$ - napon u čvorištu „*j*“ ako u „*i*-to“ čvorište narinemo 1A

DODATAK B: REDUKCIJA PASIVNE MREŽE

Često je, zbog potrebe pojednostavljivanja proračuna, potrebno analiziranu mrežu svesti na prihvatljivu veličinu. Jedan od načina kojim je to moguće ostvariti je redukcija pasivnog dijela mreže. Prije negoli se postupak redukcije provede potrebno je odrediti koji dio mreže je poželjno i moguće reducirati. Dio mreže koji se „uklanja“ će nakon redukcije zamijeniti nadomjesne grane između čvorišta koja su povezivala reducirani i nereducirani (promatrani) dio mreže.

Postupak redukcije pasivne mreže je moguće provesti na više načina.

PRIMJER Za mrežu na slici, reduciraj mrežu na čvorišta 1, 2 i 3. Y matricu formiraj kao za proračun tokova snaga pomoću Gauss-Seidel metode sa Z matricom (u ovom slučaju u Y matricu ne ulaze poprečne grane).



	$Z_{i-j}\,[p.u.]$	$\frac{Y_{i-j}{}'}{2}\,[p.u.]$	$Y_{i-j}\,[p.u.]$
1-2	0,02+j0,06	j0,003	5-j15
1-3	0,03+j0,24	j0,025	1,25-j3,75
2-3	0,06+j0,18	j0,02	1,66-j5
2-4	0,06+j0,18	j0,02	1,66-j5
2-5	0,04+j0,12	j0,015	2-j7,5
3-4	0,01+j0,03	j0,01	10-j30
4-5	0,08+j0,24	j0,025	1,25-j3,75

RJEŠENJE:

$$\vec{Y}=\begin{vmatrix} 6,25-j18,75 & -5+j15 & -1,25+j3,75 & 0 & 0 \\ -5+j15 & 10,833-j32,5 & -1,66+j5 & -1,66+j5 & -2,5+j7,5 \\ -1,25+j3,75 & -1,66+j5 & 12,916-j38,75 & -10+j30 & 0 \\ 0 & -1,66+j5 & -10+j30 & 12,916-j38,75 & -1,25+j3,75 \\ 0 & -2,5+j7,5 & 0 & -1,25+j3,75 & 3,75-j11,25 \end{vmatrix}$$

$$\vec{Y}=\begin{vmatrix} 19,76\underline{|-71,6^\circ} & 15,81\underline{|108,4^\circ} & 3,95\underline{|108,4^\circ} & 0 & 0 \\ 15,81\underline{|108,4^\circ} & 34,26\underline{|-71,6^\circ} & 5,27\underline{|108,4^\circ} & 5,27\underline{|108,4^\circ} & 7,91\underline{|108,4^\circ} \\ 3,95\underline{|108,4^\circ} & 5,27\underline{|108,4^\circ} & 40,85\underline{|-71,6^\circ} & 31,62\underline{|108,4^\circ} & 0 \\ 0 & 5,27\underline{|108,4^\circ} & 31,62\underline{|108,4^\circ} & 40,85\underline{|-71,6^\circ} & 3,95\underline{|108,4^\circ} \\ 0 & 7,91\underline{|108,4^\circ} & 0 & 3,95\underline{|108,4^\circ} & 11,86\underline{|-71,6^\circ} \end{vmatrix}$$

$$\vec{Y}=\left|\begin{array}{cccc|c} 19,76 & -15,81 & -3,95 & 0 & 0 \\ -15,81 & 34,26 & -5,27 & -5,27 & -7,91 \\ -3,95 & -5,27 & 40,85 & -31,62 & 0 \\ 0 & -5,27 & -31,622 & 40,85 & -3,95 \\ \hline 0 & -7,91 & 0 & -3,95 & 11,86 \end{array}\right|$$

$$\vec{Y}^{1234}=\begin{vmatrix} 19,76 & -15,81 & -3,95 & 0 \\ -15,81 & 28,98 & -5,27 & -7,90 \\ -3,95 & -5,27 & 40,85 & -31,62 \\ 0 & -7,90 & -31,62 & 39,53 \end{vmatrix} \rightarrow \begin{vmatrix} 19,76 & -15,81 & -3,95 \\ -15,81 & 27,40 & -11,59 \\ -3,95 & -11,59 & 15,56 \end{vmatrix}$$

$$\vec{Y}_{23}^{-1}=\begin{vmatrix} 27,40 & -11,59 \\ -11,59 & 15,56 \end{vmatrix}=\begin{vmatrix} 18,77 & 0,745 \\ -0,745 & 0,0643 \end{vmatrix}=\begin{vmatrix} 0,0533\underline{|71,6^\circ} & 0,0397\underline{|71,6^\circ} \\ 0,0397\underline{|71,6^\circ} & 0,0939\underline{|71,6^\circ} \end{vmatrix}=$$

$$=\begin{vmatrix} 0,0168+j0,0506 & 0,0125+j0,0377 \\ 0,0125+j0,0377 & 0,0296+j0,0891 \end{vmatrix}=|Z_{23}|$$

DODATAK C: LU DEKOMPOZICIJA

Proračun tokova snaga se svodi na rješavanje sustava ***nelinearnih*** jednadžbi (snaga-napon) korištenjem nekih od iterativnih postupaka. U takvim postupcima se do rješenja dolazi kroz niz iterativnih koraka, unutar kojih je uobičajeno nužno rješavati sustav ***linearnih*** jednadžbi oblika:

$$[A] \cdot [x] = [B] \tag{1}$$

Pri tome su matrice $[A]$ (dimenzija nxn) i $[B]$ (dimenzija nx1) matrice poznatih vrijednosti, a $[x]$ (dimenzija nx1) vektor traženih vrijednosti.

Kao što je poznato,ovakve sustave linearnih jednadži je moguće rješavati na više načina. Međutim, matrica $[A]$ je kod tokova snaga često velikih dimenzija i s velikim brojem članova kojima je vrijednost jednaka nuli (rijetka matrica). Kako bi se uštedilo na vremenu računanja, kao i na memoriji skladištenja podataka, pokazalo se kako je za rješavanje takvog sustava linearnih jednadžbi poželjno korištenje metode LU dekompozicije, ili neke od njenih varijanti.

LU dekompozicija podrazumijeva rastav regularne (nesingularne) matrice $[A]$ u umnožak donje trokutaste matrice ($[L]$ ili $[D]$) i gornje trokutaste matrice ($[U]$ ili $[G]$):

$$[A] = [D] \cdot [G] \tag{2}$$

Uvrštavanjem u jednadžbu (1) rastava (2) može se pisati:

$$[D] \cdot [G] \cdot [x] = [B] \tag{3}$$

Ukoliko se uvede zamjena:

$$[G] \cdot [x] = [z] \tag{4}$$

, odnosno:

$$[D] \cdot [z] = [B] \tag{5}$$

, moguće je rješavanje sustava jednadžbi (1) podijeliti u dva koraka:

1. Korištenje tzv. ***supstitucije naprijed,*** odnosno rješavanje izraza (5) kako bi se izračunao vektor $[z]$.
2. Rješavanje sustava jednadžbi (4), odnosno korištenje tzv. ***supstitucije natrag*** kako bi se odredio traženi vektor $[x]$.

Prednost korištenja ovakve metode u rješavanju sustava *n* linearnih jednadžbi je u tome što je dekompoziciju matrice $[A]$ potrebno obaviti samo jednom, supstituciju naprijed *n* puta, te supstituciju natrag *n* puta, čime se za veliki broj *n* značajno smanjuje ukupno vrijeme računanja.

Dakle, za sustav od *n* linearnih jednadžbi je matrica $[A]$ dimenzija nxn te se može pisati:

$$[A] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \tag{6}$$

LU dekompozicijom je potrebno dobiti sljedeći rezultat:

$$[A] = [D] \cdot [G] = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ d_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ d_{n1} & d_{n2} & \cdots & 1 \end{bmatrix} \cdot \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ 0 & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & g_{nn} \end{bmatrix} \tag{7}$$

LU dekompozicija se može provesti korištenjem Gaussove metode eliminacije pomoću sljedećeg algoritma:

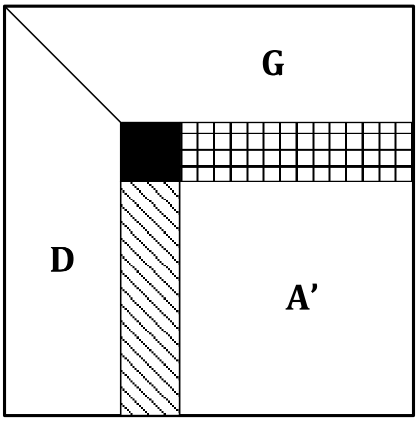
Za (*k=1, 2, ... , n-1*)
{

Za (*i=k+1, ... , n*)
{
$d_{ik} = a_{ik}/a_{kk}$;
}

Za (*i=k, ... , n*)
{
$g_{ki} = a_{ki}$;
} (8)

Za (j=*k+1, ... , n*)
{
Za (*i=k+1, ... , n*)
{
$a_{ij} = a_{ij} - d_{ik} \cdot a_{kj}$;
}
}
}

Postupak dekompozicije se može ilustrativno prikazati priloženom slikom.



Pri tome kvadrat predstavlja matricu $[A]$, crnom bojom je označen element a_{kk} (pivot u koraku k), zatamnjeni horizontalni pravokutnik redak gornje trokutaste matrice, a zatamnjeni vertikalni pravokutnik stupac donje trokutaste matrice. Slovom **D** su označeni već proračunati elementi matrice $[D]$, slovom **G** proračunati elementi matrice $[G]$, a s **A'** nepreračunati elementi matrice $[A]$.

Naravno, provedena dekompozicija predstavlja tek uvodni korak u rješavanju sustava linearnih jednadžbi. Kako bi se izračunao vektor traženih vrijednosti potrebno je ponajprije provesti supstituciju naprijed, odnosno izračunati vektor $[z]$ iz izraza (5) korištenjem matrice $[D]$ i vektora poznatih vrijednosti $[B]$.

1. SUPSTITUCIJA NAPRIJED

Vrijedi da je:

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ d_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ d_{n1} & d_{n2} & \cdots & 1 \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{9}$$

Do riješenja za vektor $[z]$ je moguće doći vrlo lako. Za element z_1 se raspisivanjem dobije:

$$z_1 = b_1 \tag{10}$$

Za element z_2 vrijedi:

$$\begin{gathered} d_{21} \cdot z_1 + z_2 = b_2 \\ z_2 = b_2 - d_{21} \cdot z_1 \end{gathered} \tag{11}$$

Za z_3:

$$\begin{gathered} d_{31} \cdot z_1 + d_{32} \cdot z_2 + z_3 = b_3 \\ z_3 = b_3 - d_{31} \cdot z_1 - d_{32} \cdot z_2 \end{gathered} \tag{12}$$

Općenito za neki element z_i vrijedi:

$$z_i = b_i - \sum_{j=1}^{i-1} d_{ij} \cdot z_j \tag{13}$$

2. SUPSTITUCIJA NATRAG

Supstitucijom natrag se određuje traženo rješenje, odnosno vektor $[x]$ korištenjem prethodno izračunatog vektora $[z]$ i gornje trokutaste matrice $[G]$:

$$\begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ 0 & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & g_{nn} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} \tag{14}$$

Iz izraza (14) se lako dobije za x_n:

$$\begin{gathered} g_{nn} \cdot x_n = z_n \\ x_n = \frac{z_n}{g_{nn}} \end{gathered} \tag{15}$$

Za x_{n-1}:

$$g_{n-1n-1} \cdot x_{n-1} + g_{n-1n} \cdot x_n = z_{n-1}$$
$$x_{n-1} = \frac{z_{n-1} - g_{n-1n} \cdot x_n}{g_{n-1n-1}} \tag{16}$$

Općenito vrijedi da je:

$$x_i = \frac{1}{g_{ii}} \cdot \left(z_i - \sum_{j=i+1}^{n} g_{ij} \cdot x_j \right)_n \tag{17}$$

PRIMJER: LU dekompozicijom odrediti gornju i donju trokutastu matricu zadane matrice A.

$$A = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 3 & -1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 3 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 3 & -1 & -1 \\ 0 & -1 & 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{bmatrix}$$

RJEŠENJE:

1. k=1

Za (*i=2, ... , 7*)
{
$d_{i1} = a_{i1}/a_{11}$;
}

Za (*i=1, ... , 7*)
{
$g_{1i} = a_{1i}$;
}

Za (j=*2, ... , 7*)
{
Za (*i=2, ... , 7*)
{
$a_{ij} = a_{ij} - d_{i1} \cdot a_{1j}$;
}
}
}

$$A^1 = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -\frac{1}{3} & \frac{8}{3} & -1 & -\frac{1}{3} & -\frac{1}{3} & -1 & 0 \\ 0 & -1 & 3 & -1 & 0 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{3} & -1 & \frac{8}{3} & -\frac{1}{3} & 0 & -1 \\ -\frac{1}{3} & -\frac{1}{3} & 0 & -\frac{1}{3} & \frac{8}{3} & -1 & -1 \\ 0 & -1 & 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{bmatrix}$$

2. k=2

Za (*i=3, ... , 7*)

{

$d_{i2} = a_{i2}/a_{22}$;

}

Za (*i=2, ... , 7*)

{

$g_{2i} = a_{2i}$;

}

Za (j=*3, ... , 7*)

{

Za (*i=3, ... , 7*)

{

$a_{ij} = a_{ij} - d_{i2} \cdot a_{2j}$;

}

}

}

$$A^2 = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -\frac{1}{3} & \frac{8}{3} & -1 & -\frac{1}{3} & -\frac{1}{3} & -1 & 0 \\ 0 & -\frac{3}{8} & \frac{21}{8} & -\frac{9}{8} & -\frac{1}{8} & -\frac{3}{8} & 0 \\ -\frac{1}{3} & -\frac{1}{8} & -\frac{9}{8} & \frac{21}{8} & -\frac{3}{8} & -\frac{1}{8} & -1 \\ -\frac{1}{3} & -\frac{1}{8} & -\frac{1}{8} & -\frac{3}{8} & \frac{21}{8} & -\frac{9}{8} & -1 \\ 0 & -\frac{3}{8} & -\frac{3}{8} & -\frac{1}{8} & -\frac{9}{8} & \frac{21}{8} & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{bmatrix}$$

3. k=3

$$A^3 = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -\frac{1}{3} & \frac{8}{3} & -1 & -\frac{1}{3} & -\frac{1}{3} & -1 & 0 \\ 0 & -\frac{3}{8} & \frac{21}{8} & -\frac{9}{8} & -\frac{1}{8} & -\frac{3}{8} & 0 \\ -\frac{1}{3} & -\frac{1}{8} & -\frac{3}{7} & \frac{15}{7} & -\frac{3}{7} & -\frac{2}{7} & -1 \\ -\frac{1}{3} & -\frac{1}{8} & -\frac{1}{21} & -\frac{3}{7} & \frac{55}{21} & -\frac{8}{7} & -1 \\ 0 & -\frac{3}{8} & -\frac{1}{7} & -\frac{2}{7} & -\frac{8}{7} & \frac{18}{7} & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{bmatrix}$$

4. k=4

$$A^4 = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -\frac{1}{3} & \frac{8}{3} & -1 & -\frac{1}{3} & -\frac{1}{3} & -1 & 0 \\ 0 & -\frac{3}{8} & \frac{21}{8} & -\frac{9}{8} & -\frac{1}{8} & -\frac{3}{8} & 0 \\ -\frac{1}{3} & -\frac{1}{8} & -\frac{3}{7} & \frac{15}{7} & -\frac{3}{7} & -\frac{2}{7} & -1 \\ -\frac{1}{3} & -\frac{1}{8} & -\frac{1}{21} & -\frac{1}{5} & \frac{38}{15} & -\frac{6}{5} & -\frac{6}{5} \\ 0 & -\frac{3}{8} & -\frac{1}{7} & -\frac{2}{15} & -\frac{6}{5} & \frac{38}{15} & -\frac{2}{15} \\ 0 & 0 & 0 & -\frac{7}{15} & -\frac{6}{5} & -\frac{2}{15} & \frac{38}{15} \end{bmatrix}$$

5. k=5

$$A^5 = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -\frac{1}{3} & \frac{8}{3} & -1 & -\frac{1}{3} & -\frac{1}{3} & -1 & 0 \\ 0 & -\frac{3}{8} & \frac{21}{8} & -\frac{9}{8} & -\frac{1}{8} & -\frac{3}{8} & 0 \\ -\frac{1}{3} & -\frac{1}{8} & -\frac{3}{7} & \frac{15}{7} & -\frac{3}{7} & -\frac{2}{7} & -1 \\ -\frac{1}{3} & -\frac{1}{8} & -\frac{1}{21} & -\frac{1}{5} & \frac{38}{15} & -\frac{6}{5} & -\frac{6}{5} \\ 0 & -\frac{3}{8} & -\frac{1}{7} & -\frac{2}{15} & -\frac{9}{19} & \frac{112}{57} & -\frac{40}{57} \\ 0 & 0 & 0 & -\frac{7}{15} & -\frac{9}{19} & -\frac{40}{57} & \frac{112}{57} \end{bmatrix}$$

6. k=6

$$A^6 = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -\frac{1}{3} & \frac{8}{3} & -1 & -\frac{1}{3} & -\frac{1}{3} & -1 & 0 \\ 0 & -\frac{3}{8} & \frac{21}{8} & -\frac{9}{8} & -\frac{1}{8} & -\frac{3}{8} & 0 \\ -\frac{1}{3} & -\frac{1}{8} & -\frac{3}{7} & \frac{15}{7} & -\frac{3}{7} & -\frac{2}{7} & -1 \\ -\frac{1}{3} & -\frac{1}{8} & -\frac{1}{21} & -\frac{1}{5} & \frac{38}{15} & -\frac{6}{5} & -\frac{6}{5} \\ 0 & -\frac{3}{8} & -\frac{1}{7} & -\frac{2}{15} & -\frac{9}{19} & \frac{112}{57} & -\frac{40}{57} \\ 0 & 0 & 0 & -\frac{7}{15} & -\frac{9}{19} & -\frac{5}{14} & \frac{12}{7} \end{bmatrix}$$

Dakle, iz navedenog je vidljivo da su matrice $[D]$ i $[G]$ jednake:

$$[D] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{3}{8} & 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{8} & -\frac{3}{7} & 1 & 0 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{8} & -\frac{1}{21} & -\frac{1}{5} & 1 & 0 & 0 \\ 0 & -\frac{3}{8} & -\frac{1}{7} & -\frac{2}{15} & -\frac{9}{19} & 1 & 0 \\ 0 & 0 & 0 & -\frac{7}{15} & -\frac{9}{19} & -\frac{5}{14} & 1 \end{bmatrix}$$

$$[G] = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ 0 & \frac{8}{3} & -1 & -\frac{1}{3} & -\frac{1}{3} & -1 & 0 \\ 0 & 0 & \frac{21}{8} & -\frac{9}{8} & -\frac{1}{8} & -\frac{3}{8} & 0 \\ 0 & 0 & 0 & \frac{15}{7} & -\frac{3}{7} & -\frac{2}{7} & -1 \\ 0 & 0 & 0 & 0 & \frac{38}{15} & -\frac{6}{5} & -\frac{6}{5} \\ 0 & 0 & 0 & 0 & 0 & \frac{112}{57} & -\frac{40}{57} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{12}{7} \end{bmatrix}$$

Dobiveni rezultat se može vrlo lako provjeriti ponovnim množenjem matrica $[D]$ i $[G]$, pri čemu dobiveni rezultat mora biti jednak matrici $[A]$.