

ANALIZA ELEKTROENERGETSKOG SUSTAVA

Predavanje br. 9.

- ISTOSMJERNI MODEL TOKOVA SNAGA

$$P_i = \sum_{j=1}^n U_i \cdot U_j \cdot Y_{ij} \cdot \cos(-\Theta_{ij} + \delta_i - \delta_j)$$

$$Q_i = \sum_{j=1}^n U_i \cdot U_j \cdot Y_{ij} \cdot \sin(-\Theta_{ij} + \delta_i - \delta_j)$$

– Pretpostavimo: $U_i \approx U_j \approx 1.0$

$$Y_{ij} = B_{ij} e^{j90^\circ}$$

$$Y_{ii} = B_{ii} e^{-j90^\circ} \quad (\text{Zanemarimo } G_{ij} \text{ i } G_{ii})$$

$$\cos(\alpha - 90^\circ) = \cos \alpha \cdot \cos 90^\circ + \sin \alpha \cdot \sin 90^\circ = \sin \alpha$$

$$\sin(\alpha - 90^\circ) = \sin \alpha \cdot \cos 90^\circ - \cos \alpha \cdot \sin 90^\circ = -\cos \alpha$$

$$P_i = B_{ii} \cdot \cos(-90^\circ) + \sum_{\substack{j=1 \\ j \neq i}}^n B_{ij} \cdot \sin(\delta_i - \delta_j)$$

$$Q_i = B_{ii} \cdot \sin(90^\circ) - \sum_{\substack{j=1 \\ j \neq i}}^n B_{ij} \cdot \cos(\delta_i - \delta_j)$$

- Zanemarimo Q_i ($Q_i = 0$). Tada slijedi da je kut $(\delta_i - \delta_j)$ jako mali pa je:

$$P_i = \sum_{\substack{j=1 \\ j \neq i}}^n B_{ij} \cdot (\delta_i - \delta_j) \quad (\text{za jako male kuteve vrijedi: } \sin(*) \approx *)$$

- Izmnožimo i sredimo:

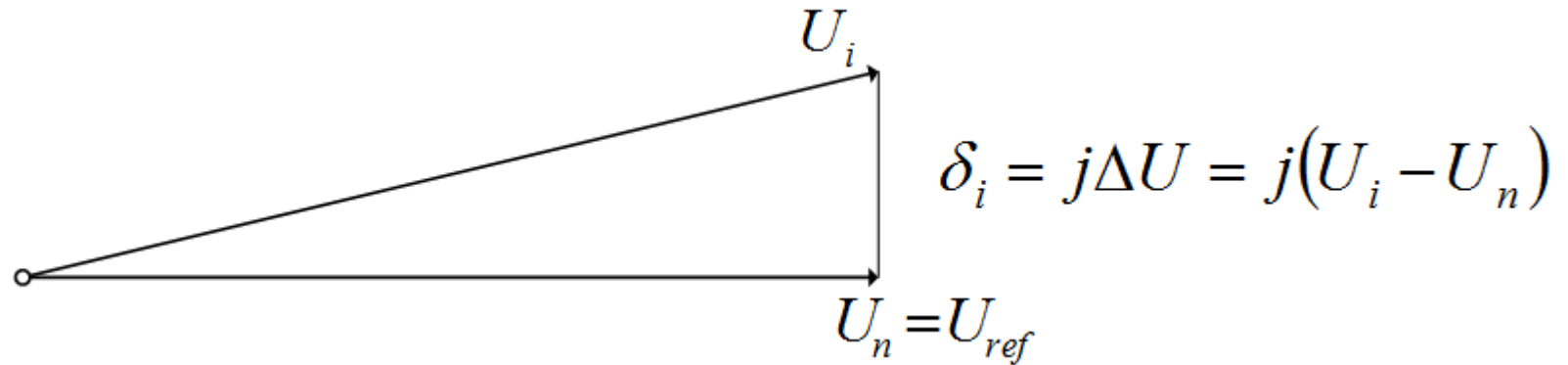
$$P_i = \sum_{j=1}^{n-1} B_{ij} \cdot \delta_j \quad \delta_n = \delta_{ref} = 0^\circ$$

$$|P| = |B| \cdot |\delta|$$

$|B| [(n-1) \times (n-1)]$ - matrica susceptancije, realna matrica

– Isto je ako imamo:

$$\begin{vmatrix} P_1 \\ \vdots \\ P_i \\ \vdots \\ P_{n-1} \end{vmatrix} = j|B| \cdot \begin{vmatrix} U_1 - U_{ref} \\ \vdots \\ U_i - U_{ref} \\ \vdots \\ U_{n-1} - U_{ref} \end{vmatrix}$$



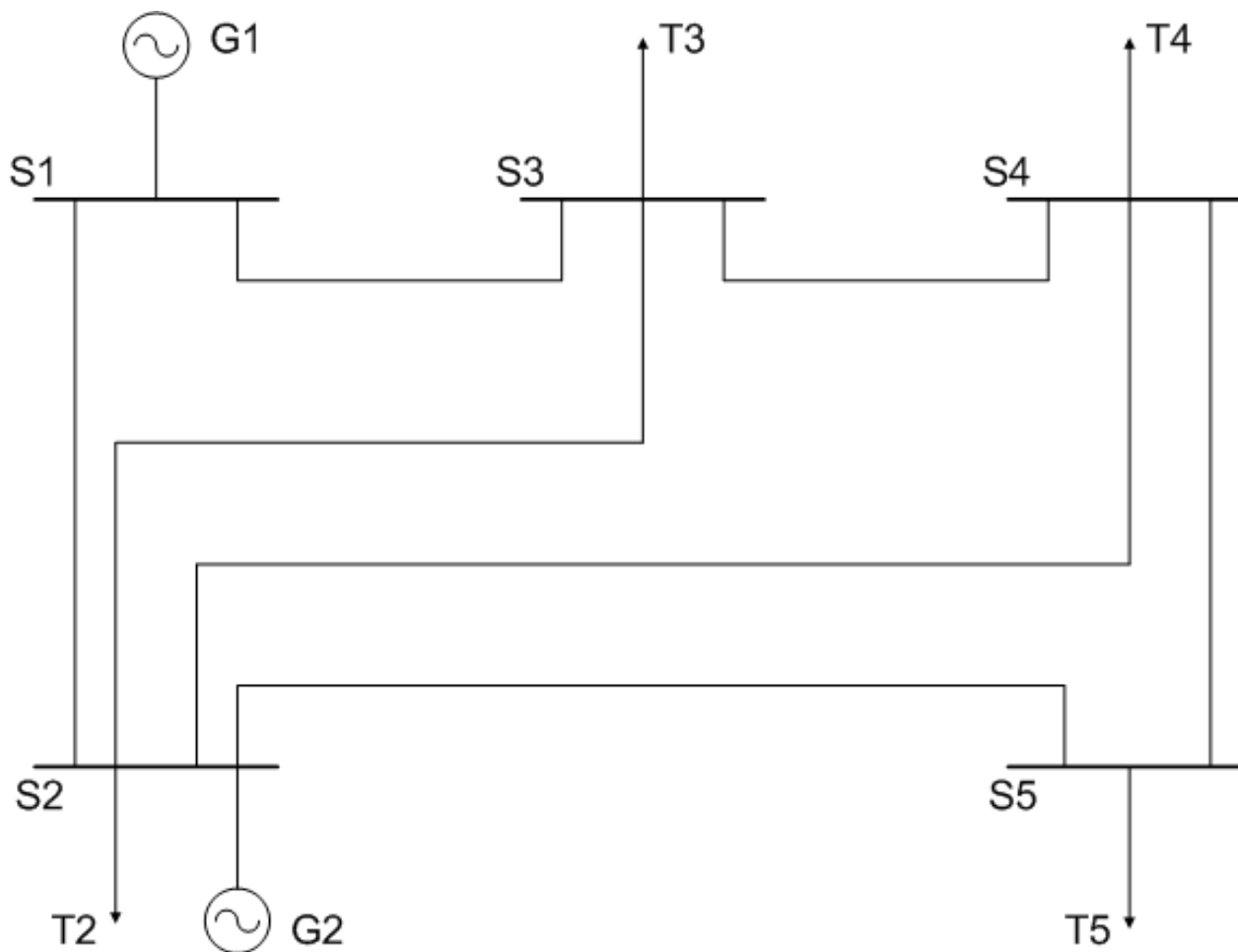
- Kod matrice B dijagonale su (-), a vandijagonalni elementi (+) radi pravila stvaranja matrice admitancija

$$|\Delta U| = j|B|^{-1} \cdot |P|$$

$$P_{i-j} = \frac{j(\Delta U_i - \Delta U_j)}{jX_{i-j}}$$

$$P_{i-j} = -P_{j-i} \quad \text{jer je } R=0$$

- Primjer



- Zadano:

Čv.	Pi (MW)	Pi (p.u.)
2.	20	0.2
3.	-45	-0.45
4.	-40	-0.4
5.	-60	-0.6

i	j	X	B
1	2	j0.06	-j16.667
1	3	j0.24	-j4.167
2	3	j0.18	-j5.556
2	4	j0.18	-j5.556
2	5	j0.12	-j8.333
3	4	j0.03	-j33.333
4	5	j0.24	-j4.167

$$P_1 = -\sum_{i=2}^5 P_i = -20 + 45 + 40 + 60 = 125 \text{ MW}$$

$$B_5 = -j \begin{vmatrix} 20.833 & -16.667 & -4.167 & 0 & 0 \\ -16.667 & 36.111 & -5.556 & -5.556 & -8.333 \\ -4.167 & -5.556 & 43.055 & -33.333 & 0 \\ 0 & -5.556 & -33.333 & 43.055 & -4.167 \\ 0 & -8.333 & 0 & -4.167 & 12.5 \end{vmatrix}$$

- Inverzna matrica

$$B_5^{-1} = j \begin{vmatrix} 0.05057 & 0.03771 & 0.04029 & 0.04714 \\ 0.03771 & 0.08914 & 0.07886 & 0.05143 \\ 0.04029 & 0.07886 & 0.09514 & 0.05857 \\ 0.04714 & 0.05143 & 0.05857 & 0.13095 \end{vmatrix}$$

$$j \begin{vmatrix} \Delta U_2 \\ \Delta U_3 \\ \Delta U_4 \\ \Delta U_5 \end{vmatrix} = |B|^{-1} \cdot \begin{vmatrix} 0.2 \\ -0.45 \\ -0.4 \\ -0.6 \end{vmatrix} = j \begin{vmatrix} -0.05126 \\ -0.09497 \\ -0.10063 \\ -0.11571 \end{vmatrix}$$

$$P_{1-2} = \frac{j(\Delta U_1 - \Delta U_2)}{jX_{12}} = \frac{j[0 - (-0.05126)]}{j0.06} = 0.85433 \rightarrow 85.4 \text{ MW}$$

$$P_{1-3} = \frac{j[0 - (-0.09497)]}{j0.24} = 0.396 \rightarrow 39.6 \text{ MW}$$

$$P_{2-3} = \frac{j[-0.05126 - (-0.09497)]}{j0.18} = 0.243 \rightarrow 24.3 \text{ MW}$$

$$P_{2-4} = 27.4 \text{ MW}$$

$$P_{2-5} = 53.7 \text{ MW}$$

$$P_{3-4} = 18.9 \text{ MW}$$

$$P_{4-5} = 6.3 \text{ MW}$$

- Primjer izračunat s Y_{ij}

Čv.	P_i (MW)	P_i (p.u.)
2.	20	0.2
3.	-45	-0.45
4.	-40	-0.4
5.	-60	-0.6

i	j	Z	Y	Y
1	2	$0.02+j0.06$	$5-j15$	15.81
1	3	$0.08+j0.24$	$1.25-j3.75$	3.95
2	3	$0.06+j0.18$	$1.66-j5$	5.27
2	4	$0.06+j0.18$	$1.66-j5$	5.27
2	5	$0.04+j0.12$	$2.5-j4.5$	7.9
3	4	$0.01+j0.03$	$10-j30$	31.62
4	5	$0.08+j0.24$	$1.25-j3.75$	3.95

$$Y = \begin{vmatrix} 19.76 & -15.81 & -3.95 & 0 & 0 \\ -15.81 & 34.25 & -5.27 & -5.27 & -7.9 \\ -3.95 & -5.27 & 40.84 & -31.62 & 0 \\ 0 & -5.27 & -31.62 & 40.84 & -3.95 \\ 0 & -7.9 & 0 & -3.95 & 11.85 \end{vmatrix}$$

$$Y^{-1} = \begin{vmatrix} 0.05332 & 0.03977 & 0.04248 & 0.04970 \\ 0.03977 & 0.0940 & 0.08315 & 0.05423 \\ 0.04248 & 0.08315 & 0.10032 & 0.06176 \\ 0.04970 & 0.5423 & 0.06176 & 0.13811 \end{vmatrix}$$

$$\begin{bmatrix} \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \end{bmatrix} = |Y|^{-1} \cdot \begin{bmatrix} 0.2 \\ -0.45 \\ -0.4 \\ -0.6 \end{bmatrix} = \begin{bmatrix} -0.054 \\ -0.10014 \\ -0.1061 \\ -0.122 \end{bmatrix}$$

$$P_{1-2} = (\delta_1 - \delta_2) \cdot Y_{1-2} = 0.054 \cdot 15.81 = 0.85374 \rightarrow 85.4 \text{ MW} \quad (88.8 \text{ MW})$$

$$P_{1-3} = (\delta_1 - \delta_3) \cdot Y_{1-3} = 0.10014 \cdot 3.95 = 0.395553 \rightarrow 39.6 \text{ MW} \quad (40.7 \text{ MW})$$

$$P_{2-3} = (\delta_2 - \delta_3) \cdot Y_{2-3} = (-0.054 + 0.10014) \cdot 5.27 = 0.243 \rightarrow 24.3 \text{ MW} \quad (24.7 \text{ MW})$$

$$P_{2-4} = (\delta_2 - \delta_4) \cdot Y_{2-4} = (-0.054 + 0.10061) \cdot 5.27 = 0.275 \rightarrow 27.5 \text{ MW} \quad (27.9 \text{ MW})$$

$$P_{2-5} = 53.7 \text{ MW} \quad (54.8 \text{ MW})$$

$$P_{3-4} = 18.9 \text{ MW} \quad (18.9 \text{ MW})$$

$$P_{4-5} = 6.3 \text{ MW} \quad (8.3 \text{ MW})$$

Napomena : crvenom bojom su označeni rezultati dobiveni metodom Gauss-Seidel pomoću Y matrice