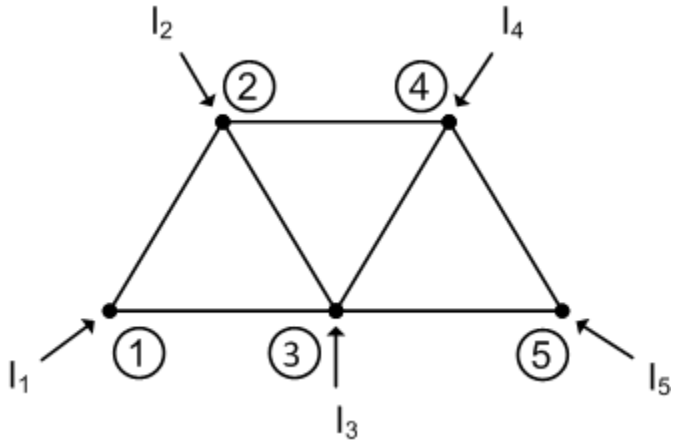
ANALIZA ELEKTROENERGETSKOG SUSTAVA

Predavanje br. 6.

- METODA ČVORIŠTA
  - Sve naponske izvore pretvorimo u strujne izvore



- Zadano:
  a) $y_{1-2}, y_{1-3}, \ldots . \; y_{i-j}$
  b) $I_1, I_2, I_3, I_4 \rightarrow I_5$ se računa
  c) + predznak struje koja ulazi u mrežu

- 1. Kirchoffov zakon za sva čvorišta

$$I_1 = (U_1 - U_2) \cdot y_{1-2} + (U_1 - U_3) \cdot y_{1-3} + \cdots + (U_1 - U_n) \cdot y_{1-n}$$
$$I_2 = (U_2 - U_1) \cdot y_{2-1} + (U_2 - U_3) \cdot y_{2-3} + \cdots + (U_2 - U_n) \cdot y_{2-n}$$
$$\vdots$$
$$I_{n-1} = (U_{n-1} - U_1) \cdot y_{(n-1)-1} + \cdots + (U_{n-1} - U_n) \cdot y_{(n-1)-n}$$

  - (***n-1***) jednadžbi s ***n*** nepoznanica
  - n-ta jednadžba je $\sum_{i=1}^{n} I_i = 0$
  - prema tome moramo znati još jedan napon U_n a ostalih (***n-1***) ćemo izračunati

- Ako sada uredimo sustav linearnih jednadžbi dobije se:

$$\begin{vmatrix} I_1 \\ I_2 \\ \vdots \\ I_{n-1} \end{vmatrix} = \begin{vmatrix} \sum_{i=2}^{n} y_{1-i} & -y_{1-2} & \cdots & -y_{1-(n-1)} \\ -y_{2-1} & \sum_{\substack{i=1 \\ i\neq 2}}^{n} y_{2-i} & \cdots & -y_{2-(n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ -y_{(n-1)-1} & -y_{(n-1)-2} & \cdots & \sum_{\substack{i=1 \\ i\neq n-1}}^{n} y_{(n-1)-i} \end{vmatrix} \cdot \begin{vmatrix} U_1 - U_n \\ U_2 - U_n \\ \vdots \\ U_{(n-1)} - U_n \end{vmatrix}$$

- Kraće: $\bar{I} = \bar{Y} \cdot \Delta\bar{U}$
- gdje je: $Y_{i,j} = -y_{i-j}$

$$Y_{i,i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} y_{i-j}$$

- $\bar{Y}$ - matrica admitancije čvorišta (simetrična)
- Elementi:
  - dijagonalni $Y_{i,i}$ (vlastita admitancija čvorišta)
  - vandijagonalni $Y_{i,j}$ (međusobna admitancija čvorišta)

- Rješenje problema je određivanje vektora $\Delta\bar{U}$

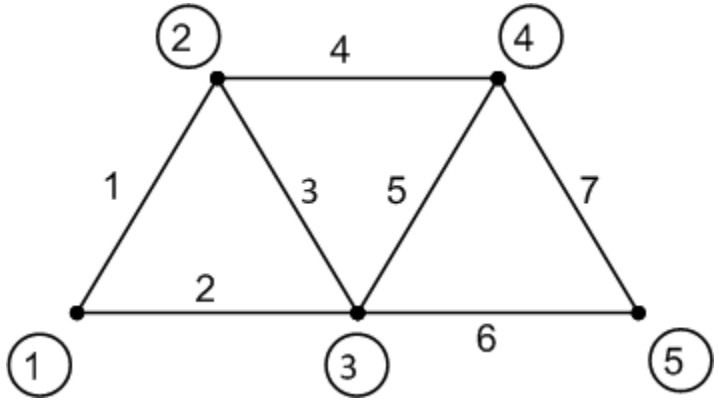
$$\Delta\bar{U} = \bar{Z} \cdot \bar{I} \qquad \text{gdje je} \qquad Z = Y^{-1}$$

  - Z - matrica impedancija čvorišta
  - $z_{i,i}$ - vlastita impedancija čvorišta
  - $z_{i,j}$ - međusobna impedancija čvorišta

$$Y = M \cdot y \cdot M^T$$

  - M - spojna matrica
  - M je (***n***x***g***) matrica koja daje vezu između čvorišta i grana

- Primjer:



$M =$

$n \downarrow$ \ $g \rightarrow$	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1)	1	1	0	0	0	0	0
(2)	-1	0	1	1	0	0	0
(3)	0	-1	-1	0	1	1	0
(4)	0	0	0	-1	-1	0	1
(5)	0	0	0	0	0	-1	-1

$$Y =$$

1	1	0	0	0	0	0
-1	0	1	1	0	0	0
0	-1	-1	0	1	1	0
0	0	0	-1	-1	0	1
0	0	0	0	0	-1	-1

$\cdot$

y1-2						
	y1-3					
		y2-3				
			y2-4			
				y3-4		
					y3-5	
						y4-5

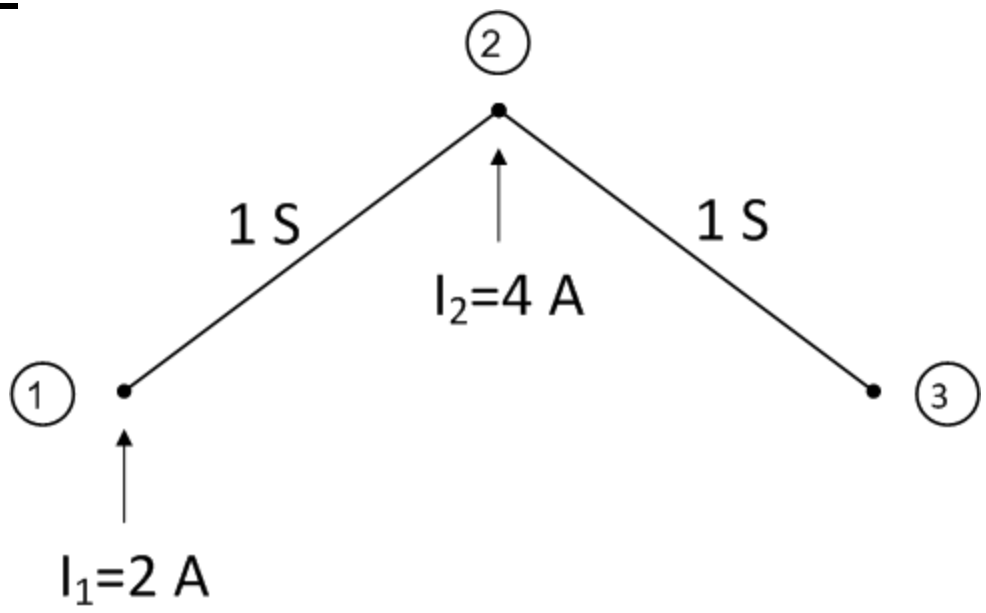
$\cdot$

1	-1	0	0	0
1	0	-1	0	0
0	1	-1	0	0
0	1	0	-1	0
0	0	1	-1	0
0	0	1	0	-1
0	0	0	1	-1

$$Y = \begin{vmatrix} y_{1-2}+y_{1-3} & -y_{1-2} & -y_{1-3} & 0 & 0 \\ -y_{1-2} & y_{1-2}+y_{2-3}+y_{2-4} & -y_{2-3} & y_{2-4} & 0 \\ -y_{1-3} & -y_{2-3} & y_{1-3}+y_{2-3}+y_{3-4}+y_{3-5} & -y_{3-4} & -y_{3-5} \\ 0 & y_{2-4} & -y_{3-4} & y_{2-4}+y_{3-4}+y_{3-5} & -y_{4-5} \\ 0 & 0 & -y_{3-5} & -y_{4-5} & y_{3-4}+y_{4-5} \end{vmatrix}$$

- Z-matrica se ne može tako dobiti
  - Inverzija pomoću determinante – veliki broj operacija
  - Numerički postupak pomoću Gaussovog postupka (Frobeniusovih matrica)

- Primjer:



$$Y_S = \left[\begin{array}{cc|c} 1 & -1 & 0 \\ -1 & 2 & -1 \\ \hline 0 & -1 & 1 \end{array}\right]$$

Y= singularna matrica

Čvorište (3) je referentno

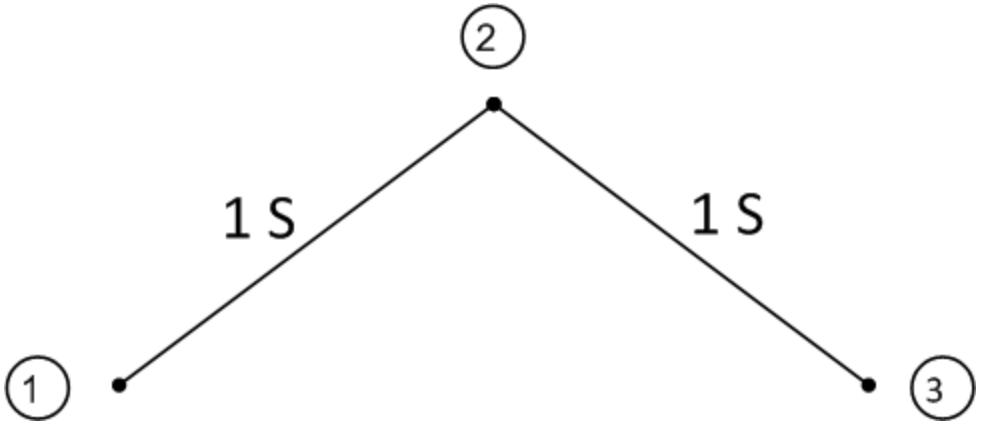
$$Y = \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}$$

$$Y^{(1)} = \begin{vmatrix} \frac{1}{1} & \frac{-1}{1} \\ -\frac{-1}{1} & 2 - \frac{1}{1} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$Y^{(2)} = \begin{vmatrix} 1 + \frac{1}{1} & -\frac{-1}{1} \\ \frac{1}{1} & \frac{1}{1} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = Y^{-1} = Z$$

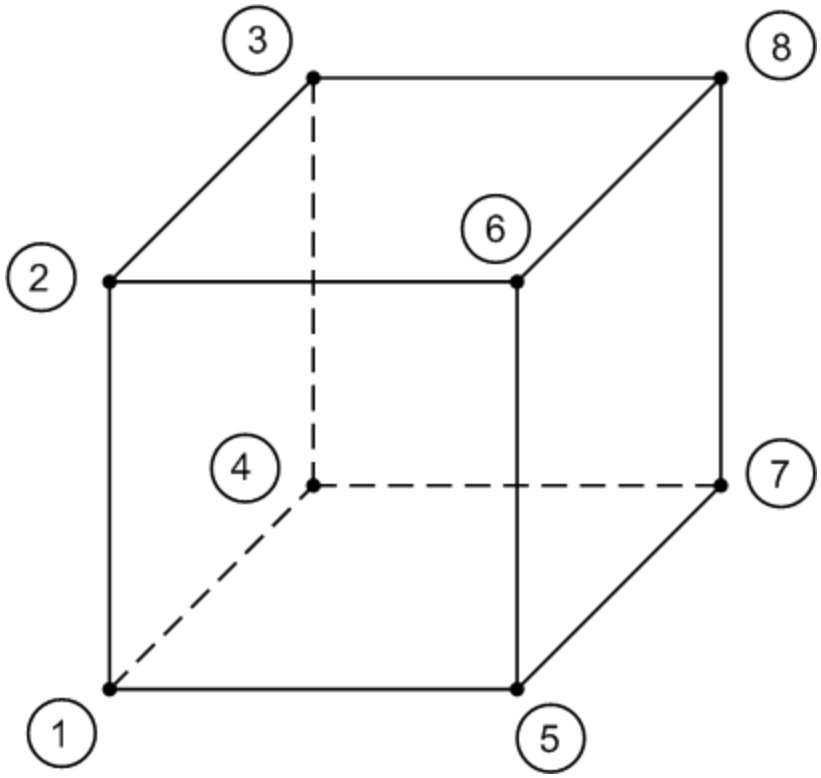
$$\begin{vmatrix}\Delta U_1\\ \Delta U_2\end{vmatrix} = \begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix} \cdot \begin{vmatrix}2\\ 4\end{vmatrix} = \begin{vmatrix}8\\ 6\end{vmatrix}\ V$$

$$(M \cdot y)^T \cdot \Delta U = \begin{vmatrix}1 & -1 & 0\\ 0 & 1 & -1\end{vmatrix} \cdot \begin{vmatrix}8\\ 6\\ 0\end{vmatrix} = \begin{vmatrix}2\\ 6\end{vmatrix}\ A$$



$$Z = \begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix}$$

- Problem kocke:



- m=1 , k=3

$$Y = \left|\begin{array}{ccc|c|ccc} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & \boxed{3} & \boxed{-1} & \boxed{0} & 0 & -1 & 0 \\ \hline 0 & \boxed{-1} & \boxed{3} & \boxed{-1} & 0 & 0 & 0 \\ \hline -1 & \boxed{0} & \boxed{-1} & \boxed{3} & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 3 & -1 & -1 \\ 0 & -1 & 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{array}\right|$$

- m=2 , k=6

$$Y^{(1)} = \left|\begin{array}{ccccc|c|c} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & \boxed{\frac{8}{3}} & \frac{1}{3} & -\frac{1}{3} & \boxed{0} & \boxed{-1} & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 \\ -1 & -\frac{1}{3} & \frac{1}{3} & \frac{8}{3} & 0 & 0 & -1 \\ -1 & \boxed{0} & 0 & 0 & \boxed{3} & \boxed{-1} & -1 \\ \hline 0 & \boxed{-1} & 0 & 0 & \boxed{-1} & \boxed{3} & 0 \\ \hline 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{array}\right|$$

- Ukoliko je pivot Y_{ii} :
  - Pivot
$$Y_{ii}^{(m+1)} = \frac{1}{Y_{ii}^{(m)}}$$
  - Elementi Y_{ij} (u istom retku kao i pivot, $j = 1,2,...n;\ j \neq i$)
$$Y_{ij}^{(m+1)} = \frac{Y_{ij}^{(m)}}{Y_{ii}^{(m)}}$$
  - Elementi Y_{ji} (u istom stupcu kao i pivot, $j = 1,2,...n;\ j \neq i$)
$$Y_{ji}^{(m+1)} = -\frac{Y_{ji}^{(m)}}{Y_{ii}^{(m)}}$$
  - Ostali elementi Y_{kl} ($k = 1,2,...n;\ k \neq i,\ l = 1,2,...n;\ l \neq i$)
$$Y_{kl}^{(m+1)} = Y_{kl}^{(m)} - \frac{Y_{ki}^{(m)} \cdot Y_{il}^{(m)}}{Y_{ii}^{(m)}}$$

- m=3 , k=7

$$Y^{(2)} = \left[\begin{array}{cccccc|c} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & \frac{7}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 \\ -1 & -\frac{1}{3} & \frac{1}{3} & \frac{8}{3} & 0 & 0 & -1 \\ -1 & -\frac{1}{3} & 0 & 0 & \frac{8}{3} & \frac{1}{3} & -1 \\ 0 & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 \\ \hline 0 & 0 & 0 & -1 & -1 & 0 & 3 \end{array}\right]$$

- m=4 , k=1

$$Y^{(3)} = \left|\begin{array}{c|ccccccc} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ \hline -1 & \frac{7}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & 0 \\ -1 & -\frac{1}{3} & \frac{1}{3} & \frac{7}{3} & -\frac{1}{3} & 0 & \frac{1}{3} \\ -1 & -\frac{1}{3} & 0 & -\frac{1}{3} & \frac{7}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & \frac{1}{3} \end{array}\right|$$

- m=5 , k=2

$$Y^{(4)} = \left|\begin{array}{ccc|ccc|c} 1/3 & -1/3 & 0 & -1/3 & -1/3 & 0 & 0 \\ \hline -1 & 6/3 & 1/3 & -2/3 & -2/3 & 1/3 & 0 \\ \hline 0 & -1/3 & 1/3 & -1/3 & 0 & 0 & 0 \\ 1/3 & -2/3 & 1/3 & 6/3 & -2/3 & 0 & 1/3 \\ 1/3 & -2/3 & 0 & -2/3 & 6/3 & 1/3 & 1/3 \\ 0 & -1/3 & 0 & 0 & -1/3 & 1/3 & 0 \\ 0 & 0 & 0 & -1/3 & -1/3 & 0 & 1/3 \end{array}\right|$$

- m=6 , k=4

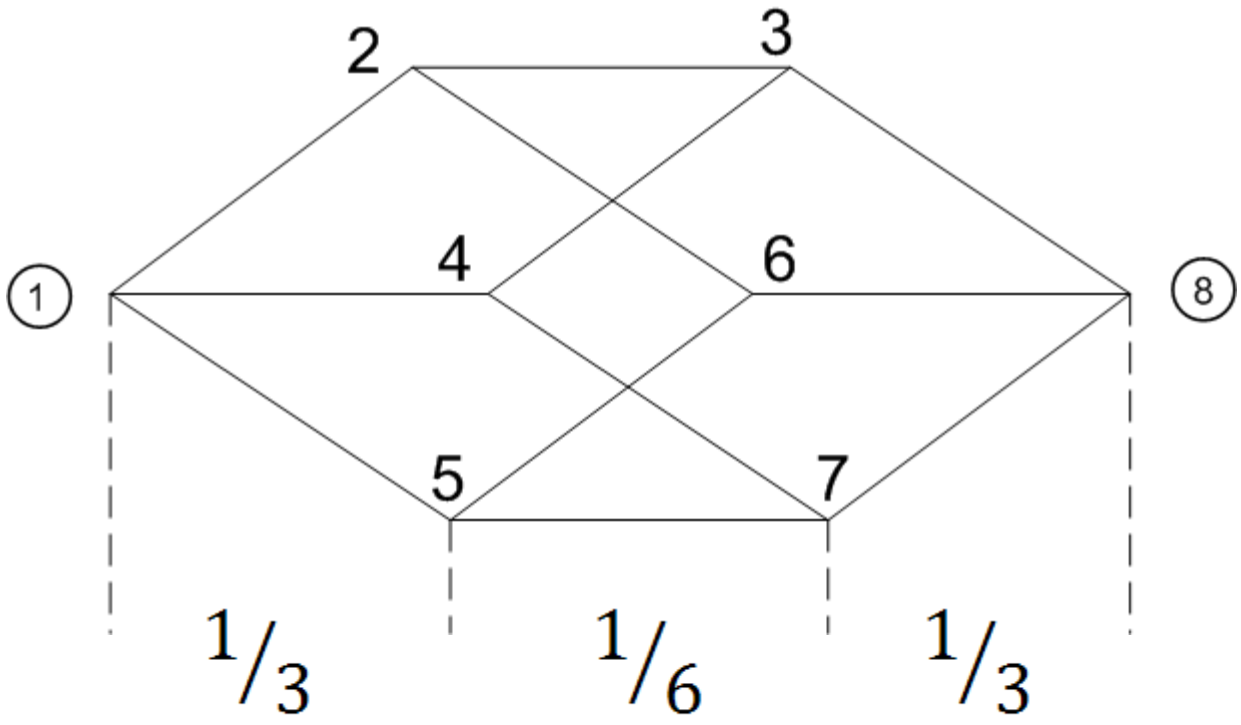
$$Y^{(5)} = \left|\begin{array}{ccc|c|cc|c} 7/18 & 1/6 & 1/18 & -4/9 & -4/9 & 1/18 & 0 \\ 1/6 & 1/2 & 1/6 & -1/3 & -1/3 & 1/6 & 0 \\ 1/18 & 1/6 & 7/18 & -4/9 & -1/9 & 1/18 & 0 \\ \hline 4/9 & 1/3 & 4/9 & 16/9 & -8/9 & 1/9 & 1/3 \\ \hline 4/9 & 1/3 & 1/9 & -8/9 & 16/9 & 4/9 & 1/3 \\ 1/18 & 1/6 & 1/18 & -1/9 & -4/9 & 7/18 & 0 \\ 0 & 0 & 0 & -1/3 & -1/3 & 0 & 1/3 \end{array}\right|$$

- m=7 , k=5

$$Y^{(6)} = \left|\begin{array}{cccc|c|cc} 1/2 & 3/12 & 1/6 & 1/4 & -2/3 & 1/12 & 1/12 \\ 1/4 & 9/16 & 1/4 & 3/16 & -1/2 & 3/16 & 1/16 \\ 1/6 & 1/4 & 1/2 & 1/4 & -1/3 & 1/12 & 1/12 \\ 1/4 & 3/16 & 1/4 & 9/16 & -1/2 & 1/16 & 3/16 \\ \hline 2/3 & 1/2 & 1/3 & 1/2 & 4/3 & 1/2 & 1/2 \\ \hline 1/12 & 3/16 & 1/12 & 1/16 & -1/2 & 57/144 & 1/48 \\ 1/12 & 1/16 & 1/12 & 3/16 & -1/2 & 1/48 & 19/48 \end{array}\right|$$

$$Z = Y^{-1} = Y^{(7)} = \begin{vmatrix} 5/6 & 1/2 & 1/3 & 1/2 & 1/2 & 1/3 & 1/3 \\ 1/2 & 3/4 & 3/8 & 3/8 & 3/8 & 3/8 & 1/4 \\ 1/3 & 3/8 & 7/12 & 3/8 & 1/4 & 5/24 & 5/24 \\ 1/2 & 3/8 & 3/8 & 3/4 & 3/8 & 1/4 & 3/8 \\ 1/2 & 3/8 & 1/4 & 3/8 & 3/4 & 3/8 & 3/8 \\ 1/3 & 3/8 & 5/24 & 1/4 & 3/8 & 7/12 & 5/24 \\ 1/3 & 1/4 & 5/24 & 3/8 & 3/8 & 5/24 & 7/12 \end{vmatrix}$$

- z_{11} - je nadomjesna impedancija između čvorišta 1-8
- $z_{1,i}$ - je napon u čvorištu “i” ako je struja u čvorištu 1. = 1A
- z_{22} - je nadomjesna impedancija između čvorišta 2-8


2
3
4
6
1
8
5
7
1/3
1/6
1/3