FAKULTET ELEKTROTEHNIKE I RAČUNARSTVA

NUMERIČKA ANALIZA ELEKTROENERGETSKOG SUSTAVA

Predavanja

ZAGREB

DOM/

NUMERIČKA ANALIZA
ELEKTROENERGETSKOG SUSTAVA
prof. dr.sc. Z. Hebel

2004./2005.

LITERATURA:

Stefanin, Babić, U-Feuer: "Matične metode u analizi el. mreža"

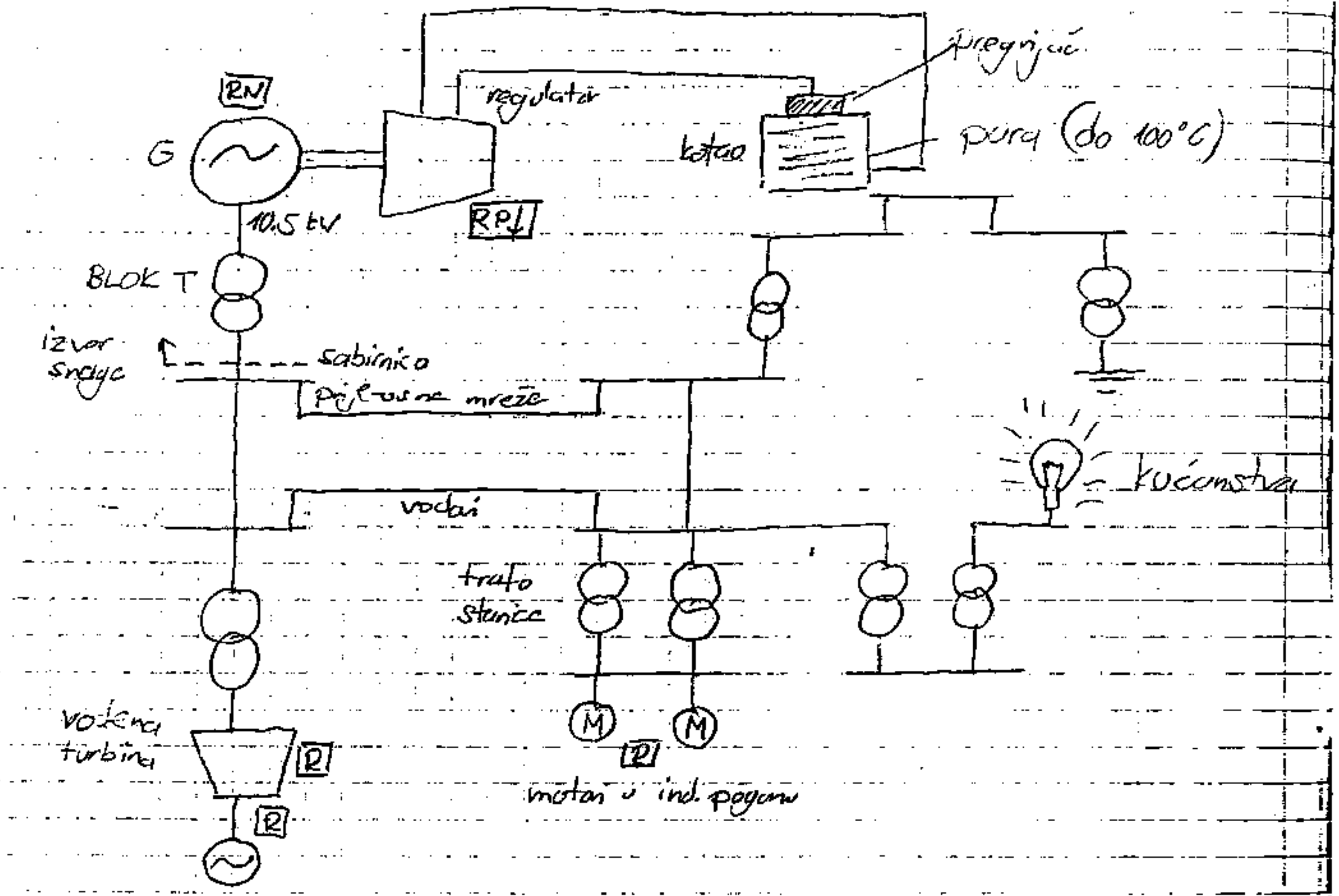
Ožegović: "Elektroenergetske mreže I–IV"

Stagg & El-Abiad: "Computer Methods in Power System Analysis
McGraw-Hill International Student Edition

I. MREŽA

-obrađivat ćemo kvazistacionarna stanja

-primjer EES-a sa osnovnim elementima



-konstrukcija generatora ovisi o pogonskom stroju (turbina – TG)

-najisplativiji su termički blokovi koji uz električnu energiju proizvode i tehnološku paru (Ina – tehnološka para je glavni produkt koji služi za frakcijsku destilaciju nafte. El. en. služi za pokrivanje vlastitih energetskih troškova)

- Toplana u Zagrebu i brdo poduzeća na Žitnjaku radi na tom principu

-parna turbina ima veći η od vodene turbine

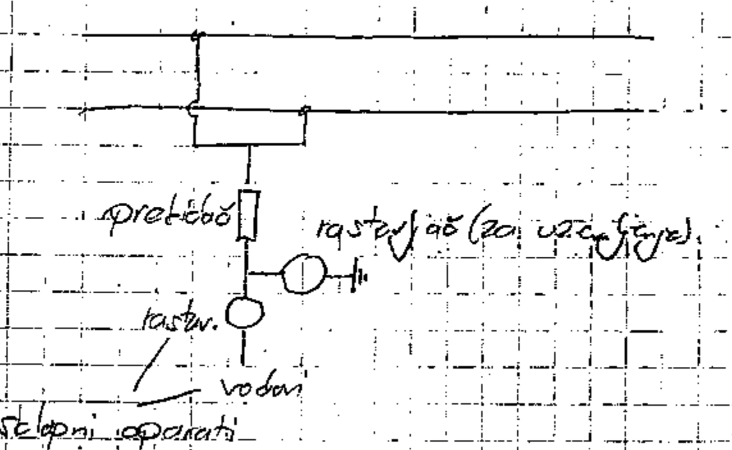
- koriste se još i plinske turbine

-ukoliko dođe do ispada ne smije doći do prekida tehnološkog procesa – zato postoje i dodatni izvori

-postoje tri važne veličine:

- → struja neovisna veličina
- → napon neovisna veličina
- → snaga zavisna veličina

STRUJA - povezana sa sklopnim aparatima



NEONOPOLNA SHEMA TRAFOSTANICE

- kad je prekidač uklopljen blokada omogućava konstantnost rada rastavljača (nemreš prekinut pogonsku)
- prekidač mora moći prekinuti struju kratkog spoja, radi se za nazivnu snagu
- struje KS su za 110, 220 i 400kV : 16kA, 25kA, 31,5kA, 40kA, 50kA i to su nazivne struje prekidača

NAPON - povezan sa potrošačima

SNAGA - važna kod mreža visokog napona

Ernestinovo:

- EES treba biti sinkroniziran (uvjeti koji su potrebni za sinkronizaciju generatora s mrežom)
- 110kV → prenosi oko 30MW prirodne snage (to je malo)
- Hrvatski EES za vrijeme rata je bio u kaosu i nije bio povezan jer je Ernestinovo bilo razrušeno → razbijen sinkroni rad izvora
- Istočna i Zapadna Europa nikad nisu bile povezane zbog različitih elektroenergetskih pravila Ernestinovo se rekonstruiralo i od jučer (10.10.2004.) je HEP imao čast pospojit 23 europske zemlje (Nordijske zemlje su same, V.B. isto)

- to je velika odgovornost, ali i veliki tehnološki dobitak u sigurnosti
- tu se može uvoziti, prodavati energija (kad ćemo imati više vode, bit ćemo tace). Sad imamo probni rok od 20 dana da se srede sekundarne regulacije po zemljama.

- u NAEES ćemo vijek računati STRUJU, NAPON, SNAGU i iz tih veličina računamo sve ostalo (presjek vodiča, izolaciju......)

11. 10. 2004.

PRORAČUN TOKOVA SNAGA

*Zadano → snage (djelatna i jalova) u potrošačkim čvorištima ↘ čvorište tereta

čvor ≠ čvorišta

čvor – matematski čvor

čvorišta – spoj nekoliko vodova

→ konfiguracija mreže

→ iznos napona i djelatne snage u generatorskim čvorištima (regulatori na prvoj slici)

→ napon u regulacijskim elektranama (kut 0) ↘ referentno čvorište $\longrightarrow U$

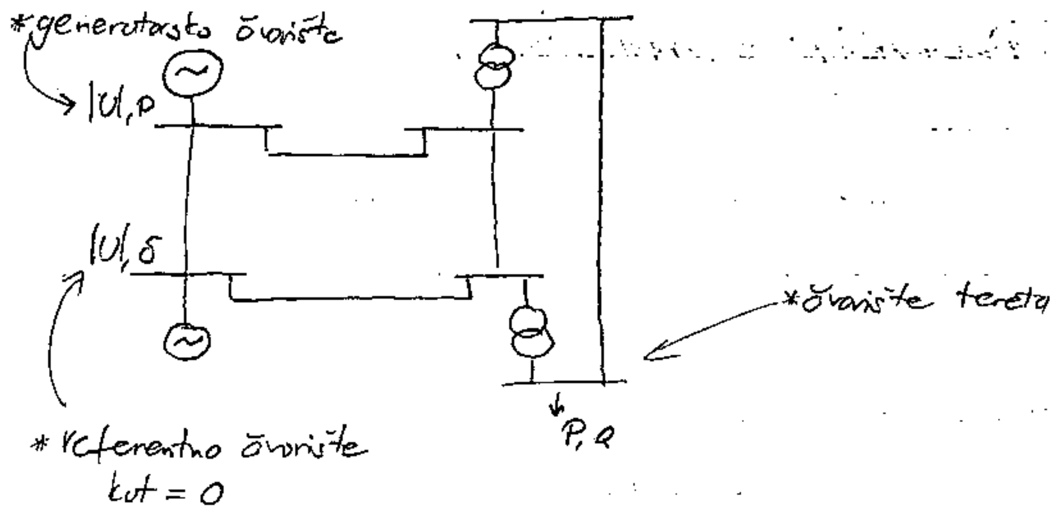
*Određuje se → napon u svim čvorištima (iznos i kut)

$$\begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix} \text{ - vektor stanja}$$

Iz toga se određuju:

- tokovi snaga u granama
- gubici djelatne i jalove snage u granama
- snaga regul. elektr.

Samo je jedan Mali Ivica!



- tu imamo 6 čvorišta
- mi računamo (u Hrvatskoj) oko 180 čvorišta
- za cijelu Europi bi nam trebalo 800 čvorišta (i to za samo 400kV i 220 kV)

PRORAČUN KRATKOG SPOJA

- početni – sa početnom reaktancijom. Postoji još i trajni i prelazni
- struja K.S. ima mehaničko i toplinsko djelovanje
- maximalni K.S. → zbog izbora opreme i zaštite

  računa se za nekoliko godina unaprijed (za odabir prekidača)

  oprema je standardizirana i baždari se na kA (16, 25, 40, 50, 60.)

  radije se ide na sekcioniranje mreže da se ne bi išlo na 50kA, jer je to opasno
- minimalni K.S. → zaštita mora reagirati i kad je struja K.S. minimalna jer i ta struja može izazvati prenapone i štetu, a zaštita je mora "primijetiti"

*Zadano → konfiguracija mreže, direktna, inverzna, nulta
→ uključeni generatori

struja: U_n

*Određuje se → tropolna rasklopna struja (snaga), struje jednopolnog kratkog spoja

PRORAČUN STABILNOSTI

- ima jako puno podvrsta
- računa se od nastanka kvara pa nekoliko minuta

→ dinamička – često se računa (kvar kada jedna jedinica elektrane od npr. 250MVA ispada)

- kapacitivno područje → nestabilni rad (P.H. ima veliko kap. opterećenje)

- proračunava se tako da se u pojedinim čvorištima računa K.S (u trajanju 0.2s, 1s, 2s) i mjeri vrijeme koje smije proći prije nego neki generator ispadne iz pogona

- kod proračuna je problem što imamo više naponskih nivoa (snaga je nepromjenjiva)
- treba nekako "izjednačit" sve naponske nivoe da nam proračun bude lakši (zbog invarijantnosti snage)

Metoda otpora – otpori se preračunavaju

→ $S = S'$ snaga u našem realnom 3f sustavu jednaka je snazi u našem relativnom sustavu

Preporuka by Hebel

→ $\boxed{U' = \frac{U_B}{U_N} \cdot U}$

(oko 1 0.9 ÷ 1.1)

U_B – BAZNI NAPON (uobičajeno 100kV)
– proizvoljni iznos

U_N – NAZIVNI NAPON

U – STVARNI NAPON (može biti ±10% u U_N)

U' – oko ±10% od U_B

$$\sqrt{3} \cdot U \cdot I = U' \cdot I'$$

$$\boxed{I' = \frac{\sqrt{3} \cdot U \cdot I}{U'} = \frac{\sqrt{3}}{U_B} \cdot U_N \cdot I}$$

Samo je jedan Mali Ivica!

$$\boxed{Z' = \frac{U'}{I'} = \frac{U_B}{U_N} \cdot U \cdot \frac{U_B}{\sqrt{3} \cdot U_N \cdot I} = \left(\frac{U_B}{U_N}\right)^2 \cdot \overbrace{\frac{U}{\sqrt{3} I}}^{Z} = \left(\frac{U_B}{U_N}\right)^2 \cdot Z}$$

$$\boxed{Y' = \left(\frac{U_N}{U_B}\right)^2 \cdot Y}$$

- sve otpore u sustavu pomoću njih pretvaramo u relativno područje pomoću lažnog otpora
- sve veličine ostaju sačuvane po jedinicama, mijenja se samo numerička vrijednost fizikalne jedinice

Metoda jediničnih vrijednosti [PU] - per unit

- ta metoda se sastoji u svođenju na jedinične vrijednosti (nema jedinice)
- pretpostavka:

→ S_B Bazna snaga (prirodna snaga)

→ $U_{B_1} = U_{N_1}$ Bazni naponi su jednaki nazivnima

$U_{B_2} = U_{N_2}$

⋮

$$\boxed{S_{pu} = \frac{S}{S_B}}$$

- Uobičajeno se uzima $P_B = 100\,MVA$
- bezdimenzionalna veličina

$$\boxed{U_{pu} = \frac{U}{U_B}}$$

- iznosi između $0.9 \div 1.1\ U_B$
- vrijedi za svaki naponski nivo

$$\boxed{I_{pu} = \frac{I}{I_B} = \frac{\frac{S}{\sqrt{3}U}}{\frac{S_B}{\sqrt{3}U_B}} = \frac{S}{S_B} \cdot \frac{U_B}{U} = \frac{S}{S_B} \cdot \frac{U_N}{U}}$$

$$\boxed{Z_{pu} = \frac{Z}{Z_B} = Z \cdot \frac{1}{\frac{U_B^2}{S_B}} = Z \cdot \frac{S_B}{U_B^2} = Z \cdot \frac{S_B}{U_N^2}}$$

BAZNI NAPONI su jednaki NAZIVNIMA!!!

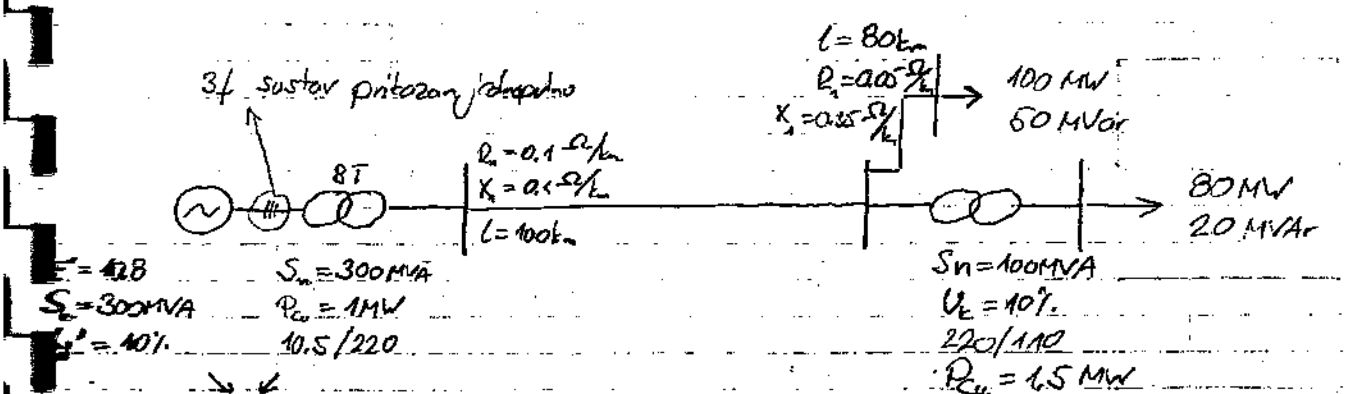
$$\boxed{Y_{pu} = \frac{Y}{Y_B} = Y \cdot \frac{U_N^2}{S_B}}$$

-sve pu veličine s jednodimenzionalne!!!

Metoda reduciranih admitancija

- proračun KS
- sve pretvaramo u admitancije

METODA OTPORA	METODA R.A.	METODA PU
$U' = \frac{U_B}{U_N} \cdot U$ $\longleftrightarrow$	$U_B = 1kV$ $\longrightarrow U_r' = \frac{U}{U_N}$ $S_B = 1 \longleftarrow$	$U_{pu} = \frac{U}{U_{Bi}} = \frac{U}{U_{ni}}$
$I' = \frac{\sqrt{3}}{U_B} \cdot U_N \cdot I$	$I_r' = \sqrt{3} U_N \cdot I$	$I_{pu} = \frac{S}{S_B} = \frac{\sqrt{3} \cdot U_N \cdot I}{S_B}$
$Z' = \left(\frac{U_B}{U_N}\right)^2 \cdot Z$	$Z_r = \frac{Z}{U_N^2}$	$Z_{pu} = Z \cdot \frac{S}{U_N^2}$
$Y' = \left(\frac{U_N}{U_B}\right)^2 \cdot Y$	$Y_r = U_n^2 \cdot Y$	$Y_{pu} = \frac{U_n^2}{S_B} \cdot Y$

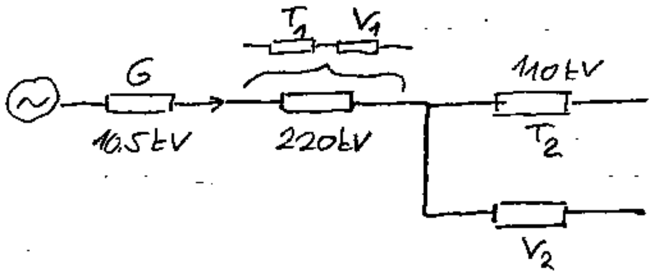


Trafoi sa blokom u gen. imaju istu snagu

Metoda otpora

$S = S'$

$U' = \frac{U_B}{U_n} \cdot U$



$U_B = 100 kV$

$X_g = 0.1 \cdot \frac{U_{gn}^2}{S_n} = 0.1 \cdot \frac{10.5^2}{300} = 0.03675\ \Omega$ (10.5 kV)

$* Z_{T_1} = \frac{U_{n(220)}^2}{S_n} \cdot \left[\frac{P_{Cu}}{S_n} + j\sqrt{U_k^2 - \left(\frac{P_{Cu}}{S_n}\right)^2} \right] [\Omega] = 0.5377 + j\,19.352$ (220kV)

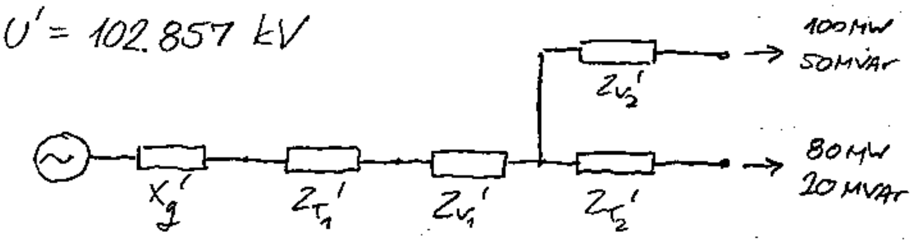
$\to$ ne u %

$* Z_{V_1} = Z_1 \cdot L = (0.1 + j\,0.4) \cdot 100$

$* Z_{T_1} = \frac{U_{n(110)}^2}{S_n} \left[\frac{P_{Cu_2}}{S_{n_2}} + j\sqrt{U_{k_2}^2 - \left(\frac{P_{Cu}}{S_n}\right)^2} \right] = \frac{12100}{10} \left[\frac{1.5}{100} + j\sqrt{0.1 - \left(\frac{1.5}{100}\right)^2} \right] =$

$= 1.815 + j\,11.963\ \Omega$ (110kV)

* $U' = 102.857 kV$



$X_g' = j\,X_g \left(\frac{100}{10.5}\right)^2 = j\,0.03675 \cdot \left(\frac{100}{10.5}\right)^2 = j\,3.3\dot{3}$

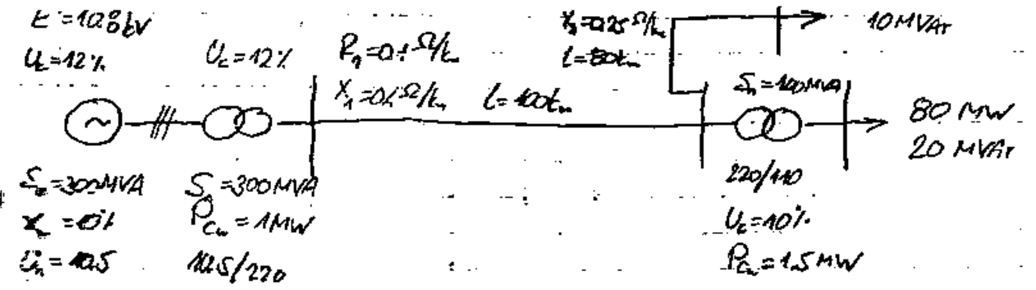
$Z_{T_1}' = Z_{T_1} \left(\frac{100}{220}\right)^2 = (0.5377 + j\,19.352)\left(\frac{100}{220}\right)^2 = 0.111 + j\,3.998\ \Omega$

$Z_{V_1}' = Z_{V_1} \cdot \left(\frac{100}{220}\right)^2 = 2.066 + j\,8.246\ \Omega$

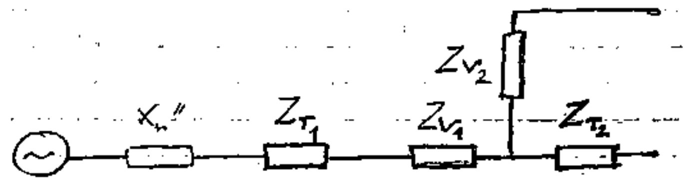
$Z_{V_2}' = Z_{V_2} \cdot \left(\frac{100}{220}\right)^2 = 0.82645 + j\,5.785\ \Omega$

$Z_{T_2}' = Z_{T_2} \cdot \left(\frac{100}{110}\right)^2 = 1.5 + j\,9.8868\ \Omega$

*cijeli EES je iste veličine kad ga svedemo na jedan naponski nivo

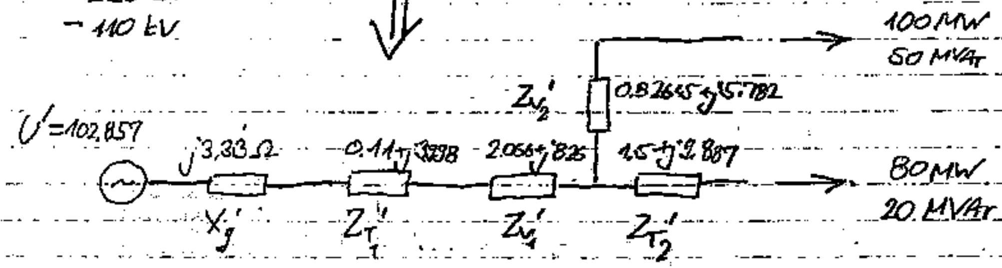


→ to smo već izračunali metodom otpora:



- 10.5 kV
- 220 kV
- 110 kV

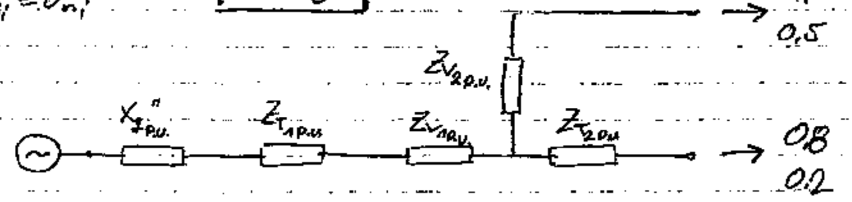
$$\Downarrow$$



→ još jedanput ⇒ metoda p.u.

$S_B = 100\,MVA$
$U_{B_i} = U_{n_i}$

$$\boxed{Z_B = \frac{U_B^2}{S_B}}$$



– djelatna (ali bez jedinice), jalova (podijeljeno s bazom)

$$E'' = \frac{10.8}{10.5} = 1.02857$$

$$X_{g_{p.u.}}'' = \frac{X_g}{Z_B} = \frac{X_g}{U_B^2} \cdot S_B = 0.03\dot{3}$$

$$Z_{T_{1p.u.}} = \frac{Z_{T_1}}{Z_B} = 0.5377 + j\,19.352 \cdot \frac{100}{220^2} = 0.0011 + j\,0.039983$$

$$Z_{V_{1p.u.}} = \frac{Z_{V_1}}{Z_B} = 0.02066 + j\,0.08264$$

$$Z_{V_{2p.u.}} = Z_{V_2} \cdot \frac{100}{220^2} = 0.008264 + j\,0.05785$$

$$Z_{T_{2p.u.}} = Z_{T_2} \cdot \frac{S_B}{110^2} = 0.015 + j\,0.09887$$

- te dva sustava su totalno ravnopravna

* Hebel preporučio da se služimo P.U. metodom.

Trafo npr. $^{10.5}/_{231}$ → $U_n = 10.5$ → $U_n = 220$!

- u PH će na sekundaru biti 231 (više zavoja)
- tu nam računica neće odgovarat pa se to mora uzet u obzir
  to su aktivne ems u trafou koje pokušavamo izbjeć →
  → izazivaju struje izjednačenja

TEOREMI ELEKTROTEHNIKE

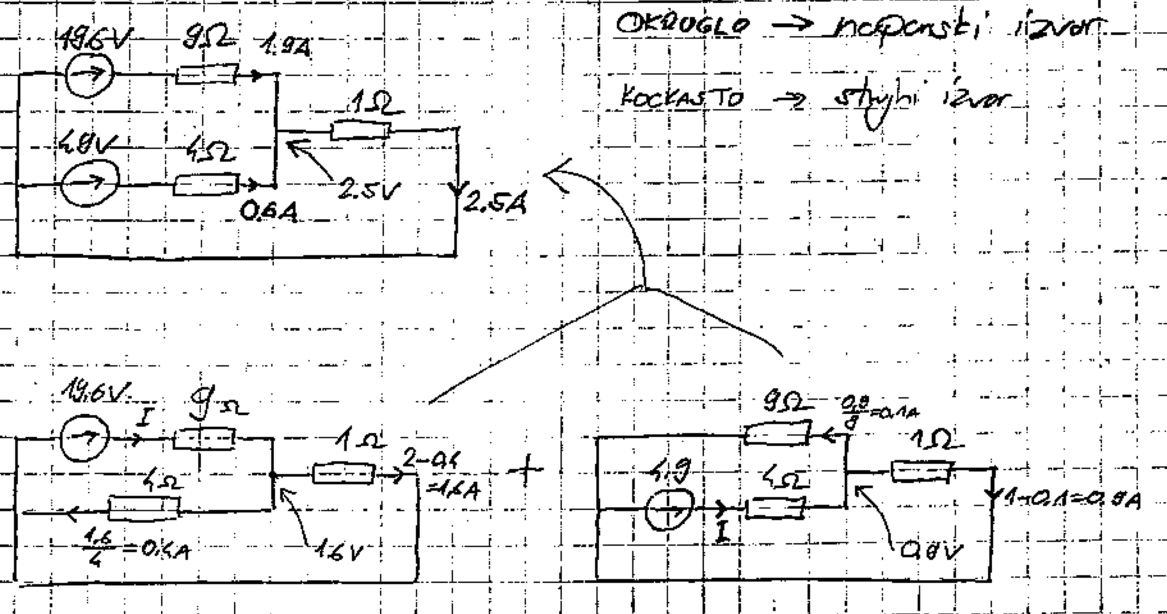
TEOREM SUPERPOZICIJE

→ istosmjerna, izmjenična, izmjenična s različitih frekvencija

Svaka ems proizvodi u linearnoj mreži struju neovisno o drugim ems.

OKRUGLO → naponski izvor

KOCKASTO → strujni izvor

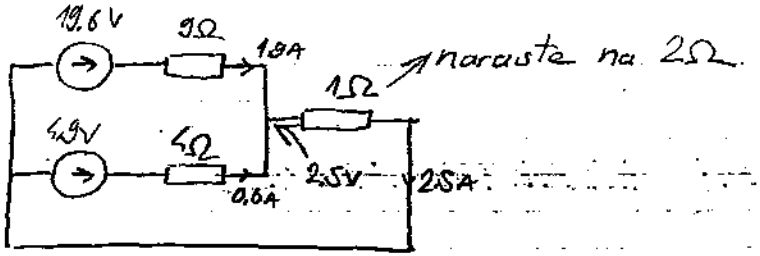


$$I = \frac{19.6}{9+\frac{4\cdot 1}{4+5}} = 2\,A \qquad I = \frac{4.9}{4+\frac{9\cdot 1}{9+1}} = 1A$$

TEOREM KOMPENZACIJE

→ radimo sa 200-tinjak čvorišta i jedno isključimo, onda sve poludi; da ne računamo sve ponovo koristimo ovaj teorem.

→ ako se u jednoj grani mreže poveća/smanji impedancija za ΔZ doći će do promjene struje u svim granama koju će proizvesti ems I·ΔZ spojena u seriju s granama



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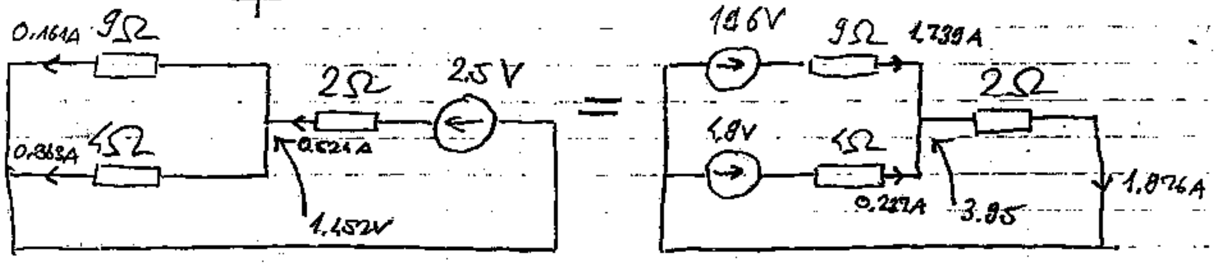
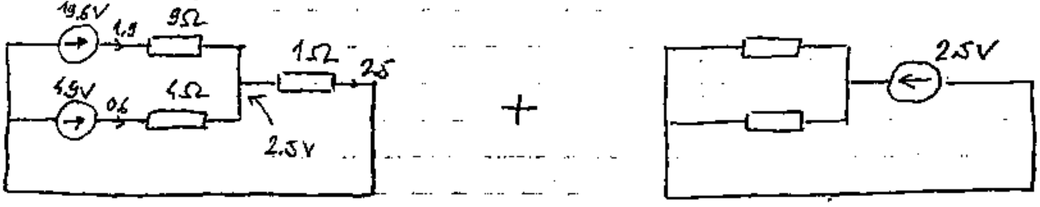


$$I = \frac{2,5}{2+\frac{4\cdot9}{13}} = \frac{2.5}{4,769} = 0,524\ A$$

$2\cdot0,524 = 1.048$ $2,5-1,048 = 1,452V$

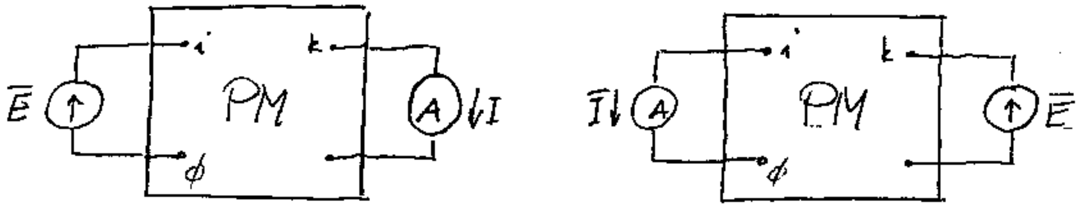
- Probaj sa smanjenjem impedancije od 1Ω na 0Ω:



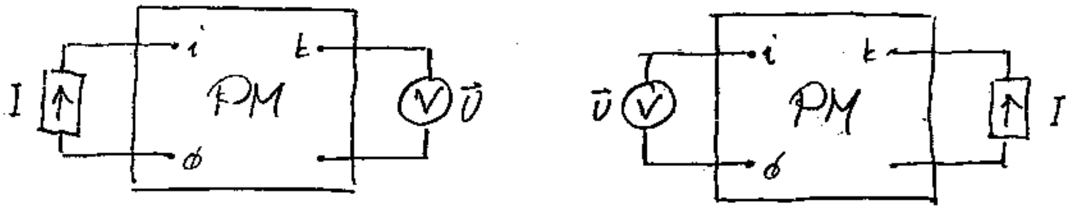
Samo je jedan Mali Ivica!

TEOREM RECIPROCITETA

→ za pasivne mreže (bez ems u granama)

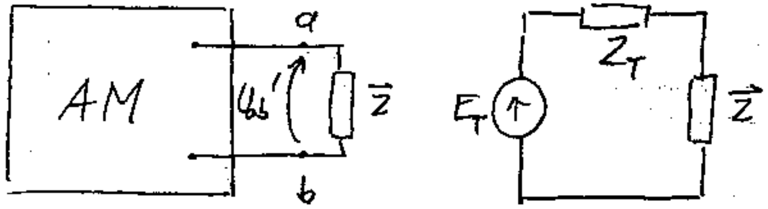


- ako zamijenimo strane, otklon će biti isti

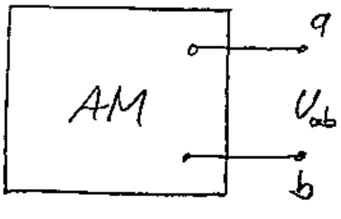


THEVENENOV TEOREM

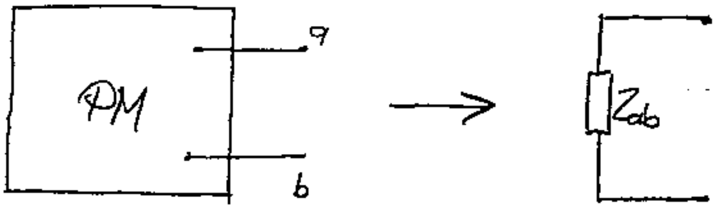
→ za aktivne mreže (imaju i pasivne elemente)



prvo odredjemo U_{ab}



kratko spajamo aktivne elemente i tražimo otpor između a i b



$E_T = U_{ab}$

$Z_T = Z_{ab}$

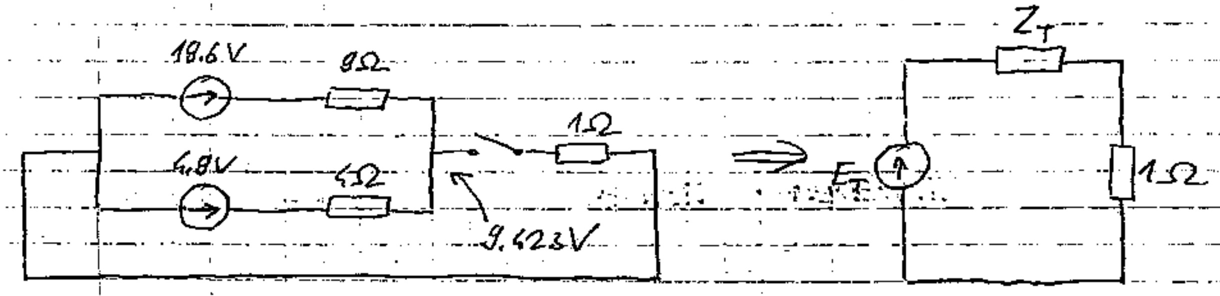
$$I = \frac{E_T}{Z_T + Z} = \frac{U_{ab}}{Z_{ab} + Z}$$

$$U_{ab}' = I \cdot Z = \frac{U_{ab}}{Z_{ab} + Z} \cdot Z$$

→ struja I proizvodi gubitke na Z_{ab} i oni nisu isti gubicima u stvarnoj mreži

→ Thevenenom računamo samo struje!

19.10.2004.

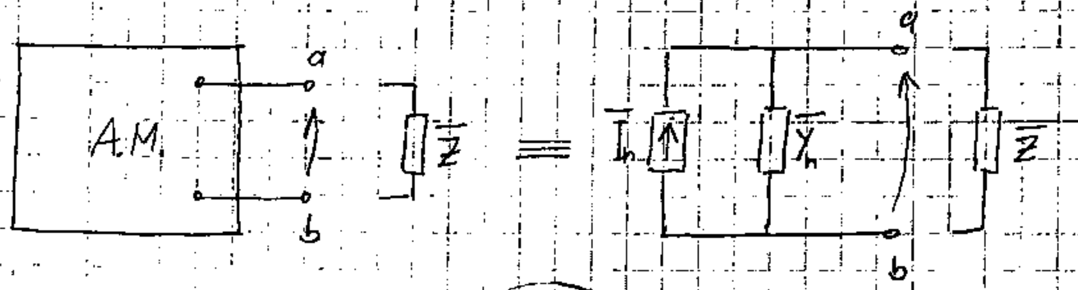


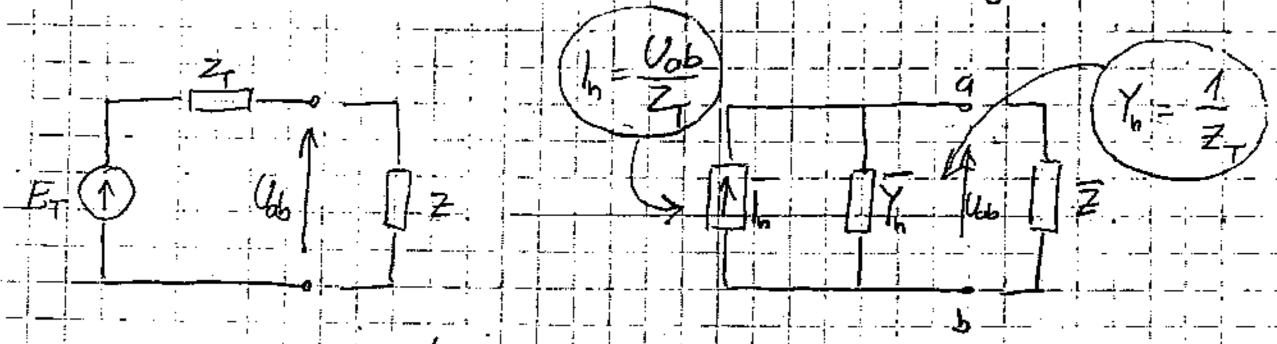
$$\frac{19.6 - 4.9}{13} = \frac{14.7}{13} = 1,1308\,A \qquad Z_T = \frac{9 \cdot 4}{9+4} = 2.77\,\Omega$$

$$19.6 - 9 \cdot 1,1308 = 9.423\,V \qquad I = \frac{9.423}{2.77 + 1} = 2.5\,A$$

→ u praksi su mreže aktivne i napućene sa strujnim izvorima. Zato ćemo koristiti NORTONOV EKVIVALENT.

NORTONOV TEOREM





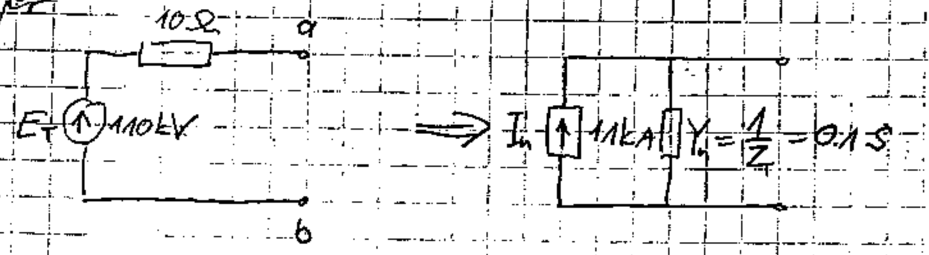
$E_T = U_{ab}$ $\quad E_T = \frac{I_n}{Y_n}$ $\qquad U_{ab} = \frac{I_n}{Y_n}$

Ⓣ Ⓝ

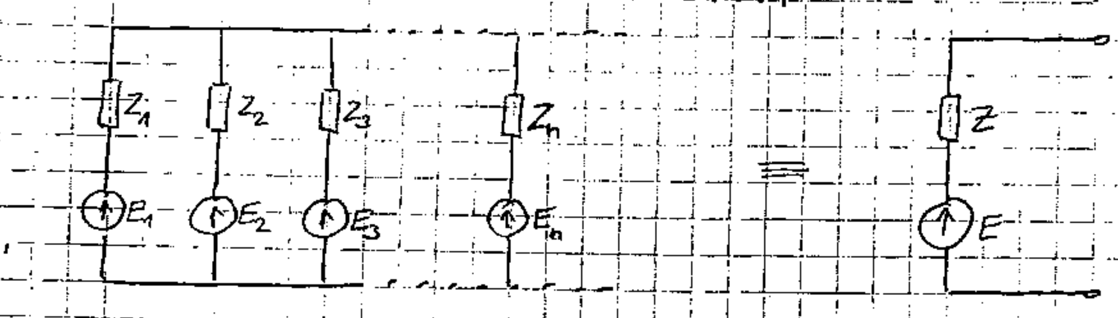
Od Thevenenove mreže se može dobiti Nortonova uz:

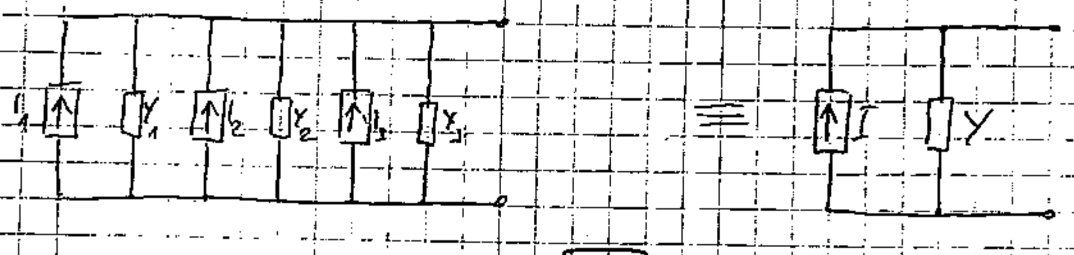
$E_T \rightarrow I_n \qquad Y_n = \frac{1}{Z_T}$

Primjer



MILLMANOV TEOREM





Samo je jedan Mali Ivica!

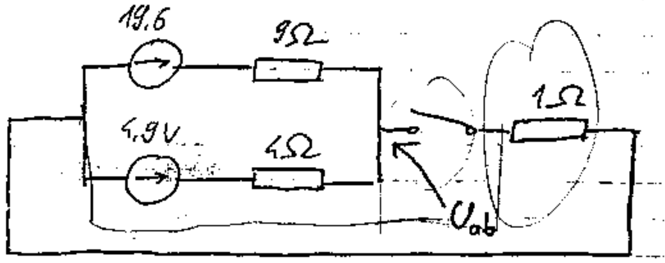
$I=\sum_1 I_i \qquad Y=\sum_1 Y_i$

$I_i=\frac{E_i}{Z_i} \qquad Y_i=\frac{1}{Z_i}$

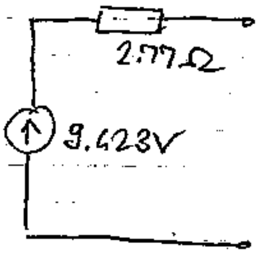
$\boxed{I=\sum_1^n \frac{E_i}{Z_i}} \qquad \boxed{Y=\sum_1^n \frac{1}{Z_i}} \qquad E=\frac{I}{Y} \qquad Z=\frac{1}{Y}$

$$\boxed{E=\frac{\sum_{i=1}^n \frac{E_i}{Z_i}}{\sum_{i=1}^n \frac{1}{Z_i}} \qquad Z=\frac{1}{\sum_{i=1}^n \frac{1}{Z_i}}}$$

~ opet naš svepristutni primjer ~



Već smo $\Longrightarrow$ izračunali pomoću T



→ odmah se uvrsti u Teorem

$$E_T=\frac{\sum_1^n \frac{E_i}{Z_i}}{\sum_1^n \frac{1}{Z_i}}=\frac{\frac{19.6}{9}+\frac{4.9}{4}}{\frac{1}{9}+\frac{1}{4}}=\frac{2.177+1.225}{0.3611}=9.423V$$

$$Z=\frac{1}{\sum_1^n \frac{1}{Z_i}}=\frac{1}{\frac{1}{9}+\frac{1}{4}}=2.77\Omega$$

Samo je jedan Mali Ivica!

TRANSFORMATORI

→ Trafo je pasivan element. Pri prijenosu diže napon, za potrošače ga spušta. Energija se prenosi pomoću magnetskog polja → fizikalno, to je aktivan element

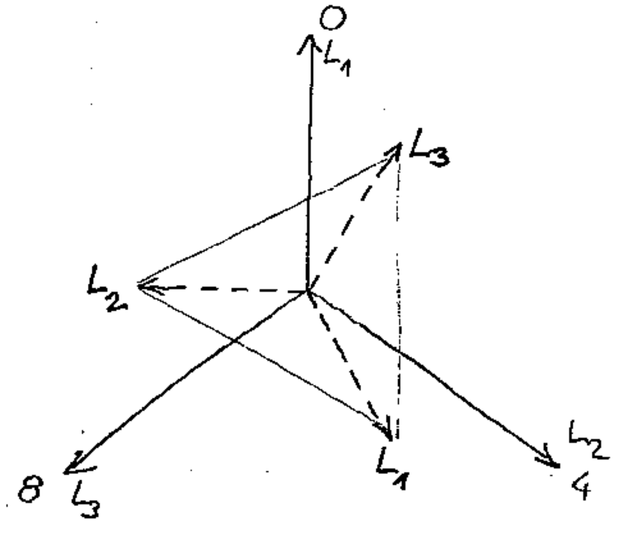
→ Najbitnija karakteristika: 2 naponska nivoa čiji omjer zovemo prijenosni omjer

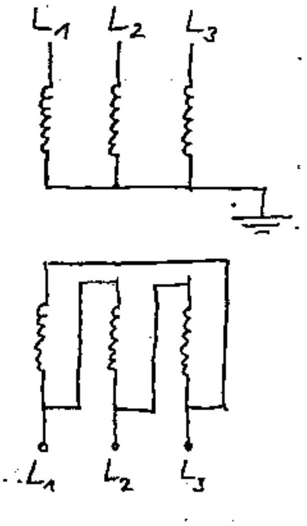
* Spojevi
  Y,y zvijezda
  D,d trokut
  Z,z cik cak

Velika slova → visokonaponska strana !
ne miješaj to sa primarnom stranom!

Primar i sekundar se određuju pomoću smjera energije
Spajanje je jako bitno...

Primer
Yd5 → zakret A i a faze





→ Uzemljenje zvjezdišta trafoa → bitno za izračun KS
Ne uzemljuju se 35kV i 20kV mreže. Ako mreža nije uzemljena onda se faza koja je spojena sa zemljom zove ZEMLJOSPOJ

→ Na 110kV mreži se vrši uzemljenje i to samo jednog trafoa od njih hrpu

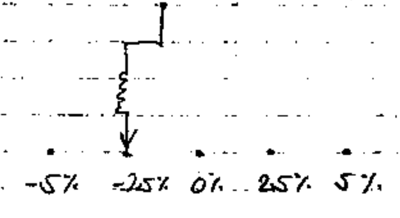
otpor na 300A
induktivitet Petersenta } koristimo za uzemljenje 35kV mreža

* Za napajanje X/0.4 koristimo Yz5 Dy5 za veće napone
Yy0 nije uzemljen

* Za 110/x
220/x Yy0 s tercijarom

→ Glavni problem je "ugađati" napon. Treba regulirati trafo.
Dvije vrste regulacije 1. Regulacija pod teretom (110/x)
2. Regulacija u P.H. (X/0.4)

Postoji ±4% ±5% / sa 3 ili 5 položaja



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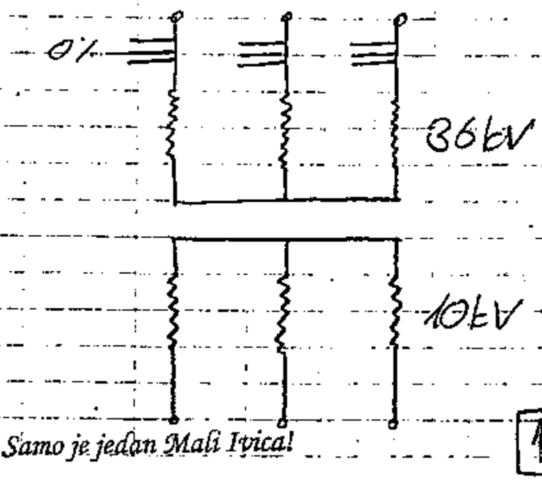


* ako na primaru poraste napon → porast će i na sekundaru
→ povećati broj zavoja na primaru

* obrnuto, pada li se napon na sekundaru, povećaš zavoje na primaru
nije poželjno mijenjat puno zavoja

→ Druga vrsta regulacije jest mjerenje položaja sklopke za vrijeme opterećenja, ta mora osigurati velik broj zamjene zavoja (od 12% do 15% sa korakom od 1.5%)

→ Ta sklopka je skupa ko cijeli trafo

Svi distributivni trafoi (110/35(20),(10)) imaju takvu sklopku

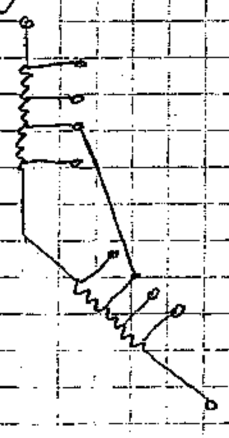
Mrežni trafoi 400/220 400/110 220/110

Ernestinovo

Ukratko: Na sekundaru uvijek imamo 36.5, 38 kV (to je više od 35 kV)

Kad se na mrežu spoji neki brutalno veliki potrošač on ruši napon. Manji napon na sekundaru → zahtijeva se promjena zavoja na primaru. To je zadaća regulacijskih sklopki

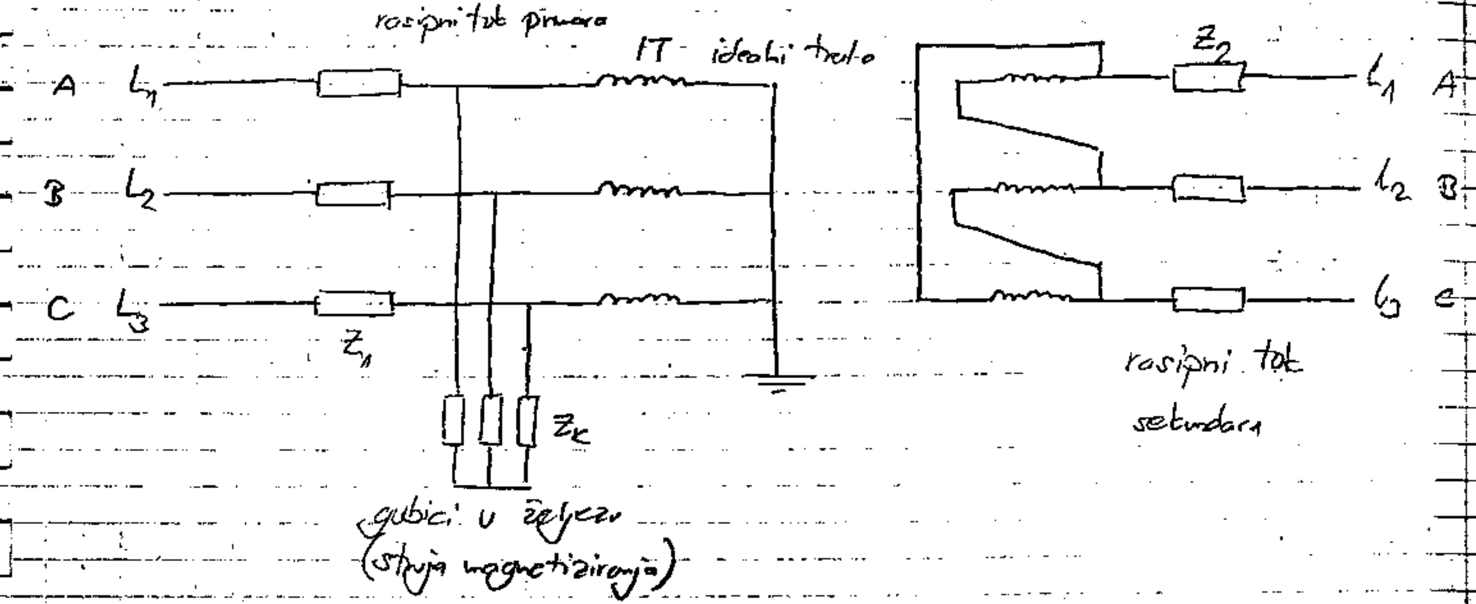
Poprečna (kosa) regulacija je uvedena da se promijene tokovi djelatne snage u mreži (400/220 ili 400/110)



– može biti uzdužna, poprečna, kosa

Bitno Ne možemo djelovati sa regulacijom na djelatnu snagu već samo na jalovu snagu

Matematički model



→ impedancija koju uzimamo u proračun je posljedica rasipnog magn. toka

→ strujni trafo mora biti kratko spojen da postoji struja sekundara koja će svojim djelovanjem (svojim AZ) oborit majku pomoćnoj struji tako da ona ne naraste previše. Inače sva struja primara postane struja magnetiziranja → jezgra ode u p.m.

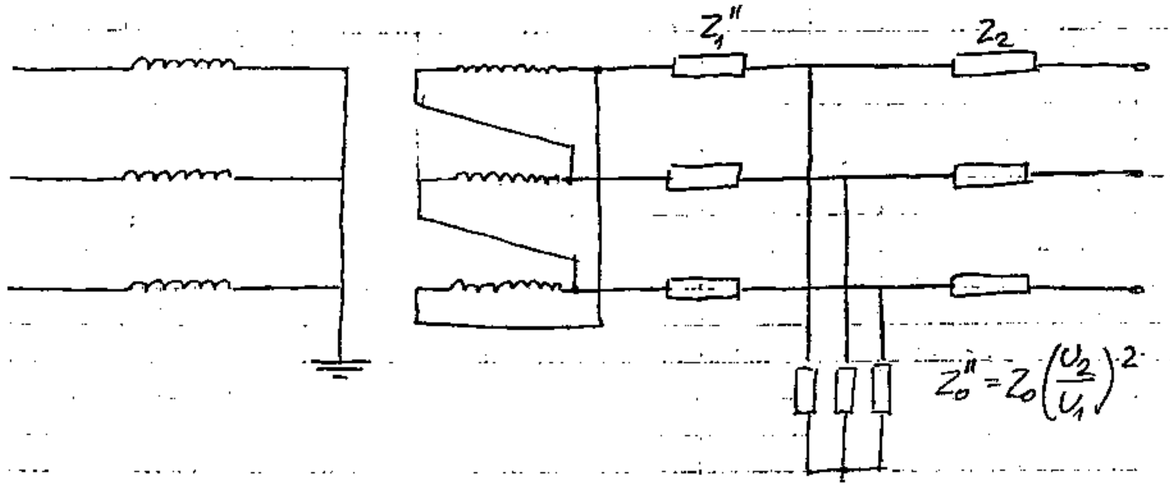
pri proračunu trafoa potrebno je primarne impedancije preračunati na sekundarnu stranu.

prim → sek
$$S = \frac{U_1^2}{Z_1} = \frac{U_2^2}{Z_1''} \qquad \boxed{Z_1'' = Z_1\left(\frac{U_2}{U_1}\right)^2}$$

snaga je ista i na prim. i sek.

sek → prim
$$Z_2' = Z_2\left(\frac{U_1}{U_2}\right)^2$$

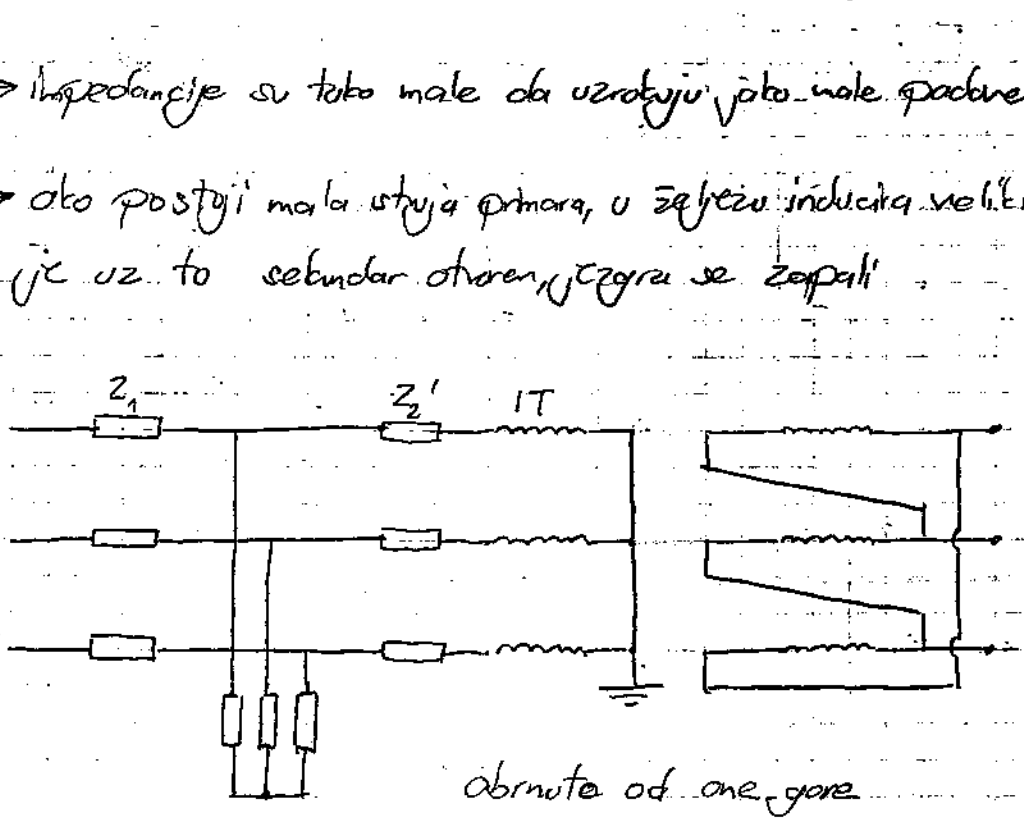
fizikalna slika idealnog trafoa



$Z_0'' = Z_0\left(\frac{U_2}{U_1}\right)^2$

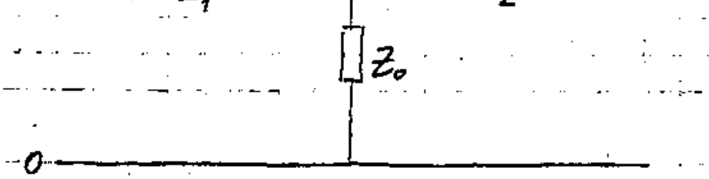
→ impedancije su tako male da uzrokuju jako male padove napona

→ ako postoji mala struja primara, u željezu inducira veliki napon, a ako je uz to sekundar otvoren, jezgra se zapali



obrnuta od one gore

nadomjesna shema



Z_1 - omski otpor primarnog (bakar)

Z_2 - omski otpor sekundarnog, rasipni (preračunat na primar)

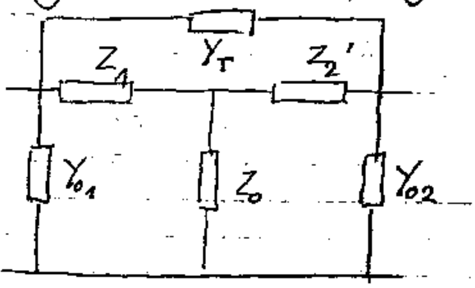
Z_0 - iz pokusa P.H.

*To je T shema i ima jedno fiktivno čvorište između primara i sekundara. To nije dobro jer struja magnetiziranja mora proći samo kroz primar.

Onda mi to pretvaramo u Π shemu

*Iz opće zvijezde se može pretvoriti u opći poligon, ali obrnuto nemremo! Jedino trokut u zvijezdu

→ Iz zvijezde u opći poligon prelazimo u admitancijama



$$*\,Y_T = \frac{\frac{1}{Z_1}\cdot\frac{1}{Z_2'}}{\frac{1}{Z_1}+\frac{1}{Z_2'}+\frac{1}{Z_0}} = \frac{Z_0}{Z_2' Z_0 + Z_1 Z_0 + Z_1\cdot Z_2'} =$$

$$= \frac{1}{Z_2' + Z_1 + \frac{Z_1\cdot Z_2'}{Z_0}}$$

$$*\,Z_T = Z_1 + Z_2' + \cancelto{0}{\frac{Z_1\cdot Z_2'}{Z_0}}$$

Z_0 je $\approx 100\cdot Z_1$ pa taj dio zanemarujemo

$$\boxed{Z_T = Z_1 + Z_2'}$$

Fizikalno gledajući Y_{02} ni ne postoji, ali jebiga...

$$Y_{01} = \frac{\frac{1}{Z_1}\cdot\frac{1}{Z_0}}{\frac{1}{Z_1}+\frac{1}{Z_2'}+\frac{1}{Z_0}} = \frac{Z_2'}{Z_2' Z_0 + Z_1 Z_0 + Z_1 Z_2'} = \frac{1}{Z_0 + Z_1 + \frac{Z_1 Z_0}{Z_2'}}$$

$$*\,Z_{01} = Z_0 + Z_1 + \frac{Z_0 Z_1}{Z_2'}$$

$\frac{Z_1}{Z_2'} \approx 1$ ako je preračunato na isti napon, pa je:

$$*\,Y_{01} = \frac{1}{Z_0(1+1+\cancelto{0}{\frac{Z_1}{Z_0}})} = \frac{1}{2Z_0} = \frac{Y_0}{2} \qquad \boxed{Z_{01} = 2\cdot Z_0}$$

Probaj sam riješit Y_{02} .....

→ Na ovom našem modelu više nemamo obtimu ems jer smo pretpostavili da su nam omjeri broja zavoja i nazivnih napona isti.....
Znači, lakše nam je raditi sa pasivnom mrežom

POKUS KRATKOG SPOJA

Sekundar kratko spojen, čekamo da poteče nazivna struja
$U_{K\%}$ u postocima

* Nazivna struja → iz nazivne snage S_n i nazivnog napona I_n
i nije loše još gurnut wattmetar i izmjerit P_K (gubici u bakru)
$I^2 \cdot R$ su gubici i na primaru i na sekundaru

* primarni AŽ potežu struju nešto nešto bla.....

$$U_{K\%} = \frac{I_n \cdot Z_T}{\frac{U_n}{\sqrt{3}}} \cdot 100 = \frac{\frac{S_n}{\cancel{\sqrt{3}}U_n} \cdot \cancel{\sqrt{3}} \cdot Z}{U_n} \cdot 100 = \frac{S}{U_n^2} Z \cdot 100$$

→ fazno

$$\boxed{Z_T = \frac{U_{K\%}}{100} \frac{U_n^2}{S_n} \left[\Omega\right]} \qquad \frac{kV^2}{MVA} = \frac{V}{A}$$

to je samo iznos, a to je impedancija koja ima realni i imaginarni
zato iskoristimo gubitke u bakru

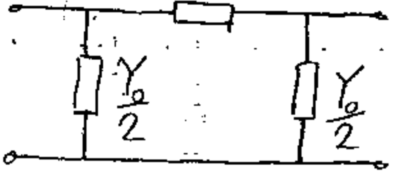
$$P_K = 3 \cdot I_n^2 \cdot R = 3 \cdot \left(\frac{S_n}{\sqrt{3}U_n}\right)^2 \cdot R = \frac{S_n^2}{U_n^2} \cdot R$$

$$\boxed{R = \frac{P_K U_n^2}{S_n^2}}$$

$$X = \sqrt{Z^2 - R^2} = \sqrt{U_K^2 \cdot \frac{U_n^4}{S_n^2} - P_K^2 \frac{U_n^4}{S_n^4}} = \frac{U_n^2}{S_n} \sqrt{U_K^2 - \frac{P_K^2}{S_n^2}}$$

$$\boxed{Z = \frac{U_n^2}{S_n} \left[\frac{P_K}{S_n} + j\sqrt{U_K^2 - \frac{P_K^2}{S_n^2}}\right] \quad \left[\Omega\right]}$$

$$Z_T = \frac{U_n^2}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{u_k^2 - \left(\frac{P_k}{S_n}\right)^2}\right]$$



Y_o računamo pomoću pokusa praznog hoda.

POKUS PRAZNOG HODA

→Trafo se spoji na U_n, sekundar otvoren, mjerimo snagu na ulazu, struju magnetiziranja.

$$*I_o = \frac{U_n}{\sqrt{3}} \cdot Y_o \qquad Y_o = \frac{\sqrt{3} I_o}{U_n}$$

$I_o = i_o \cdot I_n$ koliki dio nazivne struje je struja P.H. (magnetiziranja

$$*Y_o = \frac{\sqrt{3} \cdot i_o \cdot I_n}{U_n} = \frac{\cancel{\sqrt{3}} i_o}{U_n} \cdot \frac{S_n}{\cancel{\sqrt{3}} \cdot U_n} = i_o \cdot \frac{S_n}{U_n^2}$$

$$i_o = (0.01 \div 0.05)$$

$$P_o = 3\left(\frac{U_n}{\sqrt{3}}\right)^2 \cdot G_o$$

G_o je djelatni dio od Y_o

$$\boxed{P_o = U_n^2 \cdot G_o} \Longrightarrow G_o = \frac{P_o}{U_n^2}$$

$$B_o = \sqrt{Y_o^2 - G_o^2} = \sqrt{i_o^2 \cdot \frac{S_n^2}{U_n^4} - \frac{P_o^2}{U_n^4}} = \frac{S_n}{U_n^2}\sqrt{i_o^2 - \frac{P_o^2}{S_n^2}}$$

$$Y_o = \frac{P_o}{U_n^2} \ominus j\frac{S_n}{U_n^2}\sqrt{i_o^2 - \frac{P_o^2}{S_n^2}}$$

→ zato jer je induktivnog karaktera !
(tako je kod admitancija)

$$\boxed{Y_o = \frac{S_n}{U_n^2}\left[\frac{P_o}{S_n} - j\sqrt{i_o^2 - \frac{P_o^2}{S_n^2}}\right]} \quad [S]$$

$$\frac{MVA}{MV^2}$$

Metoda otpora

$$Z_T' = \left(\frac{U_B}{U_n}\right)^2 \cdot Z_T = \frac{U_B^2}{\cancel{U_n^2}} \cdot \frac{\cancel{U_n^2}}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{U_k^2 - \frac{P_k^2}{S_n^2}}\right] = \frac{U_B^2}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{u_k - \left(\frac{P_k}{S_n}\right)^2}\right] \; [\Omega]$$

$$Y_0' = \frac{U_n^2}{U_B^2} \cdot Y_0 = \frac{\cancel{U_n^2}}{U_B^2} \cdot \frac{S_n}{\cancel{U_n^2}}\left[\frac{P_0}{S_n} - j\sqrt{i_0^2 - \left(\frac{P_0}{S_n}\right)^2}\right] = \frac{S_n}{U_B^2}\left[\frac{P_0}{S_n} - j\sqrt{i_0^2 - \frac{P_0^2}{S_n^2}}\right] \; [S]$$

$$Z_{T p.u.} = \frac{Z_T}{Z_B} = \frac{S_B}{U_n^2} \cdot Z_T = \frac{S_B}{\cancel{U_n^2}} \frac{\cancel{U_n^2}}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{U_k^2 - \frac{P_k^2}{S_n^2}}\right] = \frac{S_B}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{U_k^2 - \frac{P_k^2}{S_n^2}}\right] \; p.u.$$

S_B obično 100 MVA

$$Y_{0 p.u.} = \frac{\cancel{U_n^2}}{S_B} \cdot \frac{S_n}{\cancel{U_n^2}}\left[\frac{P_0}{S_n} - j\sqrt{i_0^2 - \left(\frac{P_0}{S_n}\right)^2}\right] = \frac{S_n}{S_B}\left[\frac{P_0}{S_n} - j\sqrt{i_0^2 - \left(\frac{P_0}{S_n}\right)^2}\right] \; p.u.$$

→ to sve su numeričke vrijednosti naše Π sheme

110 kV   30 MVA
220 kV   160 MVA

kontrolne veličine prirodne snage

* termička snaga ovisi o vodičima

240 kvadrata termička granica je 120 MVA

400 kV Žerjavinec 1200 MVA (to je dosta za cijelu Hrvatsku)

Konkretne veličine

	mali T	veliki T	
$U_{k\%}$ =	4%	— 15%	od U_n
P_k =	2.5%	— 0.5%	od S_n
i_0 =	2.5%	— 0.5%	od I_n
P_0 =	15%	— 40%	od P_k

mali T   veliki T
20 kVA ÷ 400 MVA

→ za 110/x su gubici u željezu znatno veći od onih u bakru

49 GWh željezo   18 GWh bakar

$\cdot 10^6$ = [kWh]   svaki kWh gubitaka košta 40 lipa

→ u praksi nemamo prijenosni omjer jednak nazivnom omjeru zbog unutarnjih padova napona

→ u Ernestinovu se nalazi trafo 400/115 kV. Reg. sklopka se nalazi na primaru zbog manjih struja. Ta sklopka je $\pm 12 \times 1.25 \%$: ima 13 položaja tako da možeš mijenjati broj namota primara u slučaju višeg ili nižeg napona na sekundaru

→ ±12 položaja sa 1.25% korak
13. položaj je nazivni napon
(onaj u sredini)

→ zato što na primaru postoji više namota (4.54% zbog padova napona) imamo u modelu to znači da imamo aktivnu EMS koja smeta za proračun ⇒ toga se moramo riješiti

Primjer:

$S_n = 100 MVA$

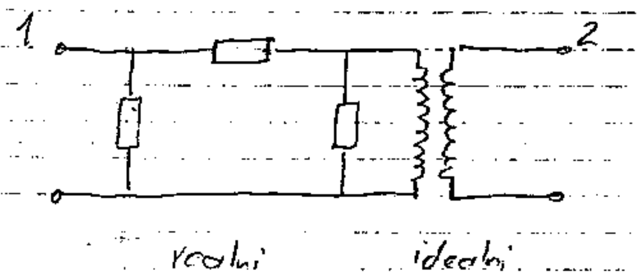
$u_{k\%} = 12\%$

$i_0 = 1\%$

$P_k = 1\%$ od S_n

$P_0 = 25\% \, P_k$

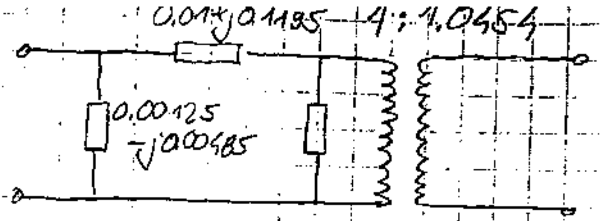
$A = 220/115 \; / \; 220/110$



$$Z_{T_{p.u.}} = \frac{S_B}{S_n}\left[\frac{P_k}{S_n} + j\sqrt{u_k^2 - \left(\frac{P_k}{S_n}\right)^2}\right] = 0.01 + j\,0.1195$$

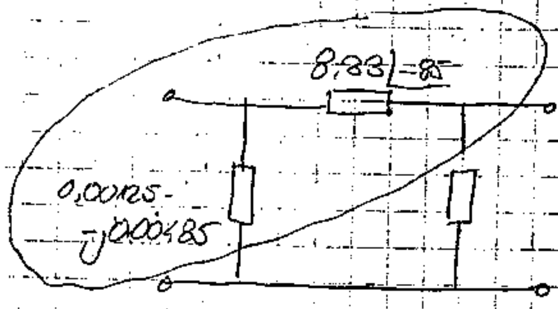
$* \; S_B = S_n = 100 MVA$

$$Y_{0_{p.u.}} = \frac{S_n}{S_B}\left[\frac{P_0}{S_n} - j\sqrt{i_0^2 - \left(\frac{P_0}{S_n}\right)^2}\right] = 0.0025 - j\,0.0097$$



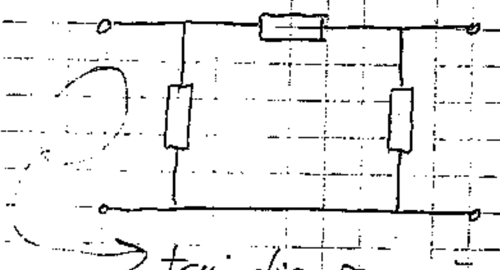
$$Y_T = \frac{1}{Z_T} = \frac{1}{0,01+j0,1195} = \frac{0,01-j0,1195}{0,01^2+0,1195^2} = 0.686 - j8.3$$

$$Y_T = 8,33 \angle{-85,22} \text{ p.u.}$$



obrati pažnju na (−)

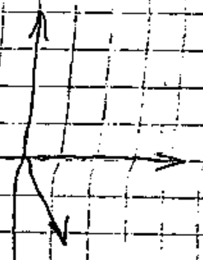
i jedna i druga ADMITANCIJA su induktivnog karaktera dok je kod VODOVA obično ▯ admitancija bila kapacitivna (sa +), a vodovima induktivnog karaktera



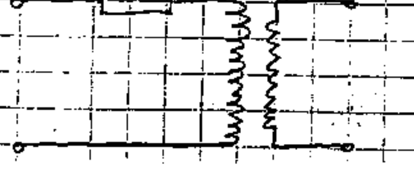
→ taj dio se može zanemariti

* kod trafoa su induktivni gubici puno puta veći od djelatnih

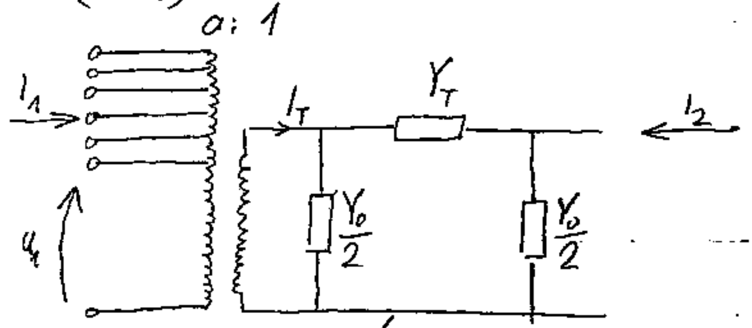
→ u kompleksnoj ravnini:



→ zato je:



* moramo naći mehanizam da trafo pretvorimo u pasivni element, a da pri tome ne zanemarimo prijenosni omjer (0.956)



$$220/115\,kV \;/\; 220/110\,kV = \frac{220}{220}\cdot\frac{110}{115} = 0.956:1$$

* na primar imaš više napona po jednom zavoju nego na sekundar zato smanjivanjem broja zavoja na primaru povećavaš iznos napona na sekundar

na primaru je $\frac{1}{0.956}$ napona više nego na sekundaru, a to je 4.54%

$$\boxed{a = 0.85 \div 1.15}$$

$$\frac{U_1}{U_T} = a = \frac{I_T}{I_1}$$

$$I_T = (U_T - U_2)\cdot Y_T + U_T\cdot\frac{Y}{2}$$

$$I_1 = \frac{I_T}{a} = (U_T - U_2)\cdot\frac{Y_T}{a} + \frac{U_T}{a}\cdot\frac{Y_0}{2}$$

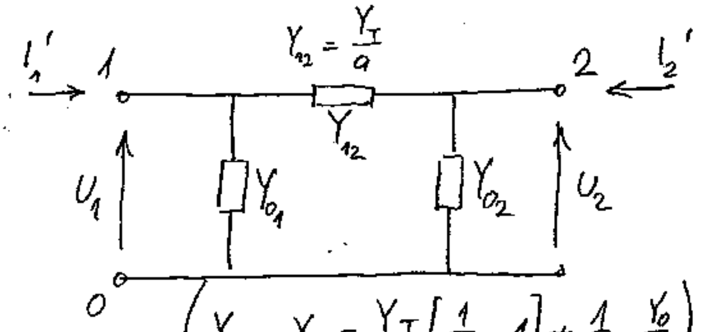
$$= \left(\frac{U_1}{a} - U_2\right)\frac{Y_T}{a} + \frac{U_1}{a^2}\cdot\frac{Y_0}{2} = (U_1 - U_2\cdot a)\frac{Y_T}{a^2} + \frac{U_1}{a^2}\cdot\frac{Y_0}{2}$$

$$I_2 = (U_2 - U_T)\cdot Y_T + U_2\cdot\frac{Y_0}{2} = \left(U_2 - \frac{U_1}{a}\right)Y_T + U_2\frac{Y_0}{2}$$

$$= (U_2\cdot a - U_1)\cdot\frac{Y_T}{a} + U_2\cdot\frac{Y_0}{2}$$

$$\boxed{\begin{aligned} I_1 &= (U_1 - aU_2)\frac{Y_T}{a^2} + \frac{U_1}{a^2}\cdot\frac{Y_0}{2} \\ I_2 &= (U_2\cdot a - U_1)\frac{Y_T}{a} + U_2\cdot\frac{Y_0}{2} \end{aligned}}$$

I_1 i I_2 kao funkcije od
U_1 i U_2



$$\left(Y_{01} = Y_{02} = \frac{Y_T}{a}\left[\frac{1}{a} - 1\right] + \frac{1}{a^2}\,\frac{Y_0}{2}\right)$$

Y-one moramo dobiti tako da vrijede ISTE naponske prilike kao i za prethodne sheme. ZA SVAKO STANJE!!! !

$$I_1' = (U_1 - U_2)Y_{12} + U_1\cdot Y_{01}$$

$$I_2' = (U_2 - U_1)Y_{12} + U_2\cdot Y_{02}$$

1. Kratko spajamo točku 2, a na ulaz stavljamo U_1

$U_2 = 0$

$U_1 = 1$

$$\left.\begin{aligned} I_1 &= 1\cdot\frac{Y_T}{a^2} + \frac{1}{a^2}\cdot\frac{Y_0}{2} \\ I_2 &= -1\cdot\frac{Y_T}{a} \end{aligned}\right\}$$ za približnu π-shemu (onu prije)

$$I_1' = Y_{12}\cdot 1 + 1\cdot Y_{01}$$

$$I_2' = -Y_{12}\cdot 1$$

$$\left.\begin{aligned}&\frac{Y_T}{a^2} + \frac{1}{a^2}\cdot\frac{Y_0}{2} = Y_{12} + Y_{01}\\ &Y_{12} = -\frac{Y_T}{a}\end{aligned}\right\} \Longrightarrow \begin{aligned}&Y_{01} = \frac{1}{a^2}\left[Y_T + \frac{Y_0}{2}\right] - \frac{Y_T}{a} \quad \circledast\\ &Y_{01} = \frac{Y_T}{a}\left[\frac{1}{a} - 1\right] + \frac{1}{a^2}\cdot\frac{Y_0}{2}\end{aligned}$$

2. Kratko spajamo točku 1, narinemo U_2 na 2

$$U_1 = 0$$
$$U_2 = 1$$

$$\left.\begin{aligned}&I_1 = -\frac{Y_T}{a}\\ &I_2 = Y_T + \frac{Y_0}{2}\end{aligned}\right\} \text{za približnu } \Pi \text{ shemu}$$

$$I_1' = -Y_{12}$$
$$I_2' = Y_{12} + Y_{02}$$

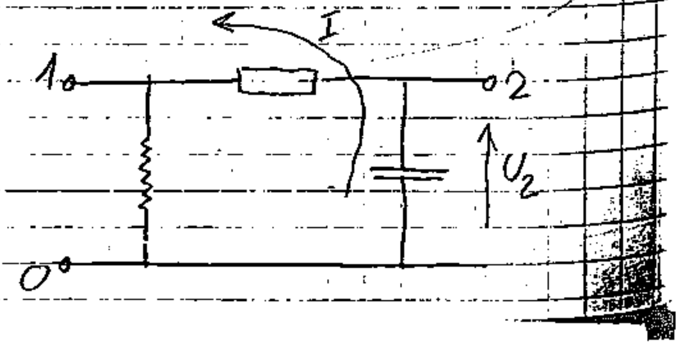
$$Y_T + \frac{Y_0}{2} = \frac{Y_T}{a} + Y_{02}$$
$$Y_{02} = Y_T\left(1 - \frac{1}{a}\right) + \frac{Y_0}{2} \quad \circledast$$

$$Y_{12} = \frac{Y_T}{a}$$
$$Y_{02} = \frac{Y_T}{a}\left(1 - \frac{1}{a}\right) + \frac{Y_0}{2}$$

→ Sa ovako dobivenim parametrima, više nam ne treba IT

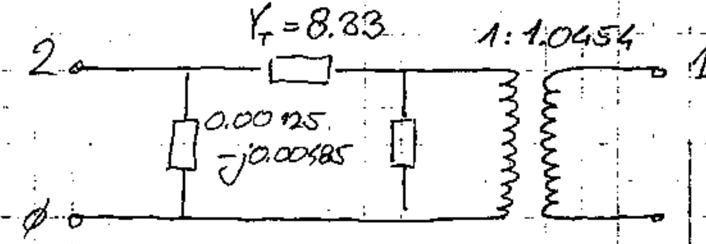
Za $a<1$ na izlazu dobivamo veći napon U_2 → formule

$$\circledast \quad \left.\begin{aligned}&Y_{01} = \underbrace{\frac{Y_T}{a}}_{>Y_T}\underbrace{\left[\frac{1}{a} - 1\right]}_{>0} + \cancel{\frac{1}{a^2}\,\frac{Y_0}{2}} \nearrow \emptyset\\ &Y_{02} = Y_T\underbrace{\left(1 - \frac{1}{a}\right)}_{\substack{<0\\ \text{kapacitet}}} + \cancel{\frac{Y_0}{2}} \nearrow \emptyset\end{aligned}\right\}$$



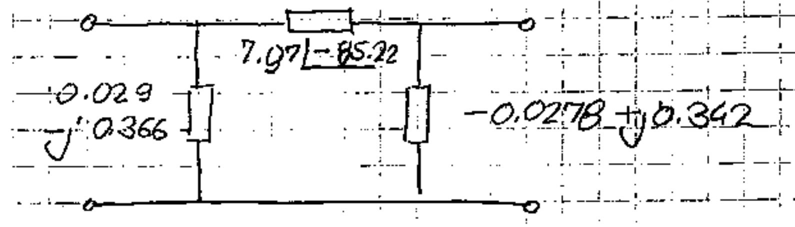
*Struja I pioi u drugom smjeru i napon se povećava

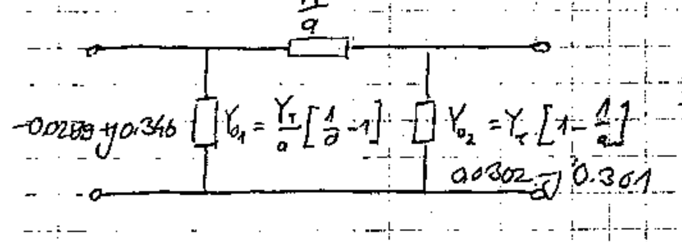
Prethodni primjer



$Y_{02} = Y_T\left(1 - \frac{1}{a}\right) = 8,33 \angle -85.22°$

$Y_{01} =$





$\Delta I_1 = U_1(-0.0289 + j\,0.352)$

$\downarrow$

$\Delta I_2 = U_2(0.0302 - j\,0.361)$

u P.H. ništa ne teče na prijem

$\Sigma I = 0$

$\Delta I_1 + \Delta I_2 = 0$

$-\Delta I_1 = \Delta I_2$

$-U_1(-0.0289 + j\,0.346) = U_2(0.0302 - j\,0.361)$

$$\frac{U_1}{U_2} = \frac{0.0802 - j\,0.361}{0.0289 - j\,0.346} = \frac{0.3628 \cancel{\angle -85.22}}{0.3474 \cancel{\angle -85.22}} = 1,04545$$

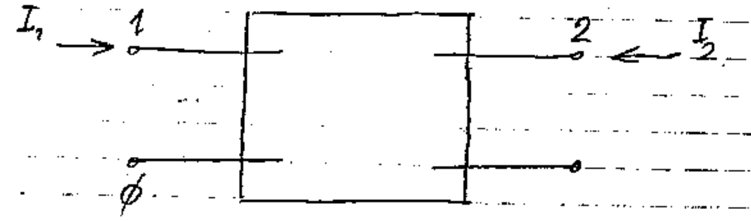
$$-U_1 \cdot \frac{\cancel{Y_T}}{a}\left[\frac{1}{a} - 1\right] = U_2 \cdot \cancel{Y_T}\left[1 - \frac{1}{a}\right]$$

$$\vdots$$

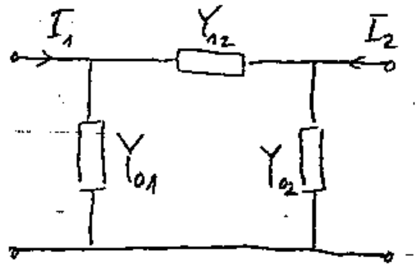
$$U_1\left[\frac{1}{a} - \frac{1}{a^2}\right] = U_2\left[1 - \frac{1}{a}\right]$$

$$\boxed{\frac{U_1}{U_2} = \frac{\cancel{1 - \frac{1}{a}}}{\frac{1}{a}\left[\cancel{\frac{1}{a} - 1}\right]} = a}$$

→ to vrijedi uvjet, ali kad uzmemo u obzir Y_0 onda nije baš stroz, al' jebiga



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$



$$I_1 = (U_1 - U_2)\,Y_{12} + U_1\,Y_{01} = U_1(Y_{12} + Y_{01}) + U_2(-Y_{12})$$

$$I_2 = (U_2 - U_1)\,Y_{12} + U_2\,Y_{02} = U_1(-Y_{12}) + U_2(Y_{12} + Y_{02})$$

$$\begin{bmatrix} y_{12} + Y_{01} & -y_{12} \\ -y_{12} & y_{12} + Y_{02} \end{bmatrix} = \begin{bmatrix} \frac{Y_T}{a} + \frac{Y_T}{a}\left[\frac{1}{a} - 1\right] & -\frac{y_T}{a} \\ -\frac{y_T}{a} & \frac{Y_T}{a} + Y_T\left[1 - \frac{1}{a}\right] \end{bmatrix} =$$

matrica admitancija čvorišta

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{Y_T}{a^2} & -\frac{y_T}{a} \\ -\frac{y_T}{a} & Y_T \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

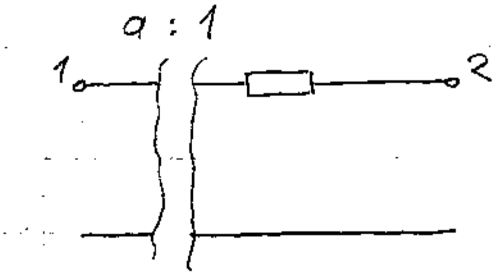
* ako je ta matrica ogromna.... samo 3 elementa promijenim i dobio sam istu matricu ?????

nekad se svi mijenjaju, ako promijenimo prijenosni omjer trafo a

→ zato treba naći neki trafo kojem ćemo mijenjat model

→ <u>Gauss Seidelova</u> metoda

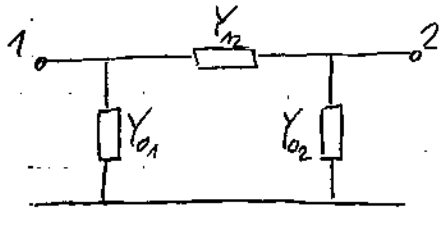
→ Sad zanemarimo Y_0



$$\frac{U_1}{U_T} = a = \frac{I_T}{I_1}$$

$$I_1 = (U_1 - aU_2)\frac{Y_T}{a^2}$$

$$I_2 = (U_2 a - U_1)\frac{Y_T}{a}$$



$$I_1' = (U_1 - U_2)Y_{12} + U_1 \cdot Y_{01}$$

$$I_2' = (U_2 - U_1)Y_{12} + U_2 \cdot Y_{02}$$

treba nam bit $I_1 = I_1'$

Promjenom prijenosnog omjera ne želimo da nam se Y matrica mijenja, pa neš malo probajmo

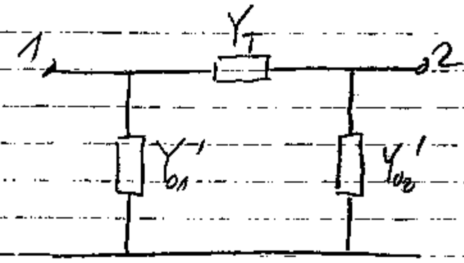
$$\left(U_1 - aU_2\right)\frac{Y_T}{a^2} = \left(U_1 - U_2\right) Y_T + U_1 \cdot Y_{01}$$

$$* \; Y_{01} = \frac{1}{U_1}\left[\left(U_1 - aU_2\right)\frac{Y_T}{a^2} - \left(U_1 - U_2\right)Y_T\right] = Y_T\left[\left(\frac{1}{a^2}-1\right) - \frac{U_2}{U_1}\left(\frac{1}{a}-1\right)\right] =$$

$$= Y_T\left(\frac{1}{a^2}-1\right) - \frac{U_2}{U_1} Y_T\left(\frac{1}{a}-1\right)$$

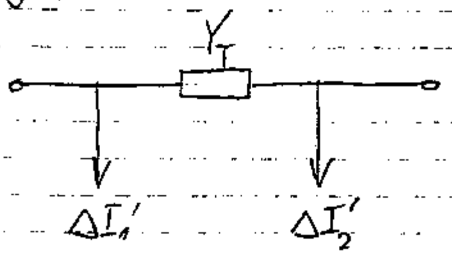
Y_{01} je f-ja napona, ali se zato uzdužna impedancija ne mijenja.

$$* \; Y_{02} = \ldots\ldots = \frac{U_1}{U_2} Y_T\left(1-\frac{1}{a}\right)$$



te poprečne admitancije množimo s naponom i dobijemo neke struje

→ Ova crtica na Y_{01}' nam označava da se uzdužna impedancija ne mijenja



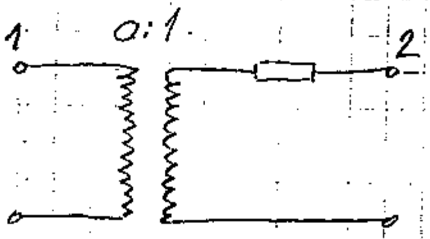
$$\Delta I_1' = U_1 \cdot Y_{01}' =$$
$$= U_1 \cdot Y_T\left(\frac{1}{a^2}-1\right) - U_2\left(\frac{1}{a}-1\right)$$

$$\Delta I_2' = U_2 \cdot Y_{02}' =$$
$$= U_1 \cdot Y_T\left(1-\frac{1}{a}\right)$$

U svakoj iteraciji računamo struje čvorišta

→ vidi se da ΔI_2 ovisi o naponu u ① čvorištu.

→ to su bila dva modela trafoa, sad ćemo učit treći koji je isti al drugačiji...

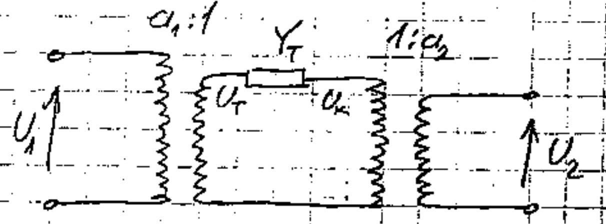


ovo je idealno

*al mi zbog nečega napravimo i na primaru nenazivni broj zavoja npr.

npr. 231/115

400/115   uz nazivni 380



$$\frac{U_1}{U_t} = a_1 = \frac{I_T}{I_1} \qquad \frac{U_2}{U_k} = a_2 = \frac{I_k}{I_2}$$

$$* \; I_t = (U_t - U_k) Y_T$$

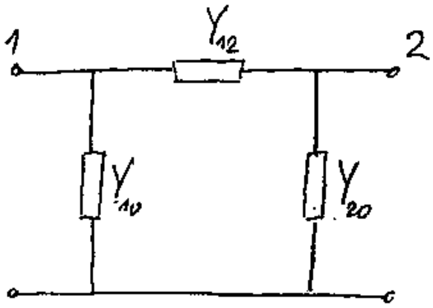
$$I_1 \cdot a_1 = \left(\frac{U_1}{a_1} - \frac{U_2}{a_2}\right) Y_t$$

$$\boxed{I_1 = (U_1 \cdot a_2 - U_2 \cdot a_1)\frac{Y_t}{a_1^2 a_2}} = \left(U_1 \frac{a_2}{a_1} - U_2\right)\frac{Y_t}{a_1 a_2}$$

$$* \; I_k = (U_k - U_t) Y_t$$

$$I_2 \cdot a_2 = \left(\frac{U_2}{a_2} - \frac{U_1}{a_1}\right) Y_t$$

$$\boxed{I_2 = U_2 \cdot \cdot \;\; ''} = \left(U_2 \frac{a_1}{a_2} - U_1\right)\frac{Y_t}{a_1 a_2}$$



$$I_1' = (U_1' - U_2')Y_{12} + U_1' Y_{10}$$

$$I_2' = (U_2' - U_1')\cdot Y_{12} + U_2' Y_{20}$$

bla bla → idemo na KS u čvorištu 2

Kratki spoj u čvorištu 2 a $U_1 = U_1' = 1$

$$I_1 = \frac{Y_t}{a_1^2} = \frac{a_2}{a_1}\cdot\frac{Y_t}{a_1 a_2} \qquad I_1' = Y_{12} + Y_{10} \qquad \Bigg| \qquad Y_{12} = +\frac{Y_t}{a_1 \cdot a_2}$$

$$I_2 = -\frac{Y_t}{a_1 a_2} \qquad I_2' = -Y_{12} \qquad \Bigg| \qquad Y_{10} = \frac{Y_t}{a_1 a_2}\left(\frac{a_2}{a_1} - 1\right)$$

Kratki spoj u čvorištu 1 a $U_2 = 1$

$$I_1 = -\frac{Y_t}{a_1 a_2} \qquad I_1' = -Y_{12} \qquad \Bigg| \qquad Y_{12} = \frac{Y_t}{a_1 \cdot a_2}$$

$$I_2 = \frac{a_1}{a_2}\cdot\frac{Y_t}{a_1 a_2} = \frac{Y_t}{a_2^2} \qquad I_2' = Y_{12} + Y_{20} \qquad \Bigg| \qquad Y_{20} = \frac{Y_t}{a_1 a_2}\left(\frac{a_1}{a_2} - 1\right)$$

*kad je $a_2 = 1$ (zašto? ne znam...) onda dobivamo: → naš model

$$Y_{10} = \frac{Y_t}{a_1}\left(\frac{1}{a_1} - 1\right) \qquad Y_{20} = \frac{Y_t}{a_1}\left(\frac{a_1}{1} - 1\right) = Y_t\left(1 - \frac{1}{a_1}\right)$$

$$\Rightarrow \quad Y = \begin{vmatrix} Y_{12} + Y_{10} & -Y_{12} \\ -Y_{12} & Y_{12} + Y_{20} \end{vmatrix}$$

matricu admitancija treba već sad znat

Elementi:

11 zbroj međuadmitancije i admitancije tog čvorišta

12 = 21 (simetrična) negativna vrijednost međ...

22 zbroj međ... i admitancije tog. čvorišta

$$Y = \dots = \begin{vmatrix} \frac{Y_t}{a_1a_2} + \frac{Y_t}{a_1a_2}\left(\frac{a_2}{a_1} - 1\right) & -\frac{Y_t}{a_1a_2} \\ -\frac{Y_t}{a_1a_2} & \frac{Y_t}{a_1a_2} + \frac{Y_t}{a_1a_2}\left(\frac{a_1}{a_2} - 1\right) \end{vmatrix} =$$

$$= \begin{vmatrix} \frac{Y_t}{a_1 \cdot a_2} \cdot \frac{a_2}{a_1} & -\frac{Y_t}{a_1a_2} \\ -\frac{Y_t}{a_1a_2} & \frac{Y_t}{a_1a_2} \cdot \frac{a_1}{a_2} \end{vmatrix} = \begin{vmatrix} \frac{Y_t}{a_1^2} & -\frac{Y_t}{a_1a_2} \\ -\frac{Y_t}{a_1a_2} & \frac{Y_t}{a_2^2} \end{vmatrix}$$

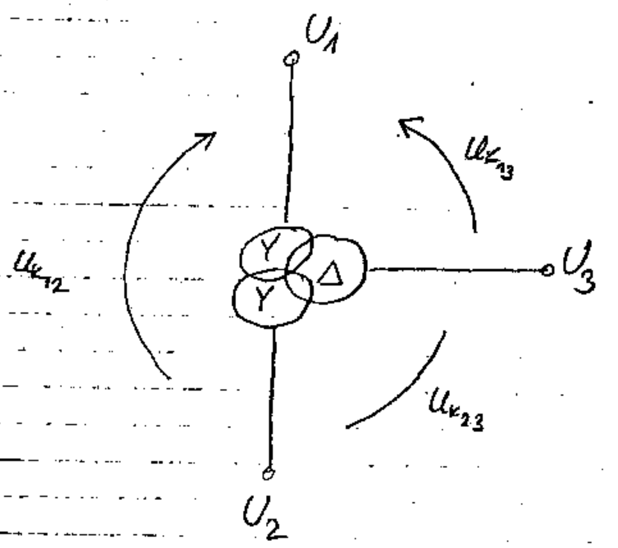
$$\begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = Y \cdot \begin{vmatrix} U_1 \\ U_2 \end{vmatrix}$$

→ to je najopćenitiji model trafoa, s tim da je a_1 promjenjiv, a a_2 je konstantan (samo na primaru imamo promjenjivi prijenosni omjer)

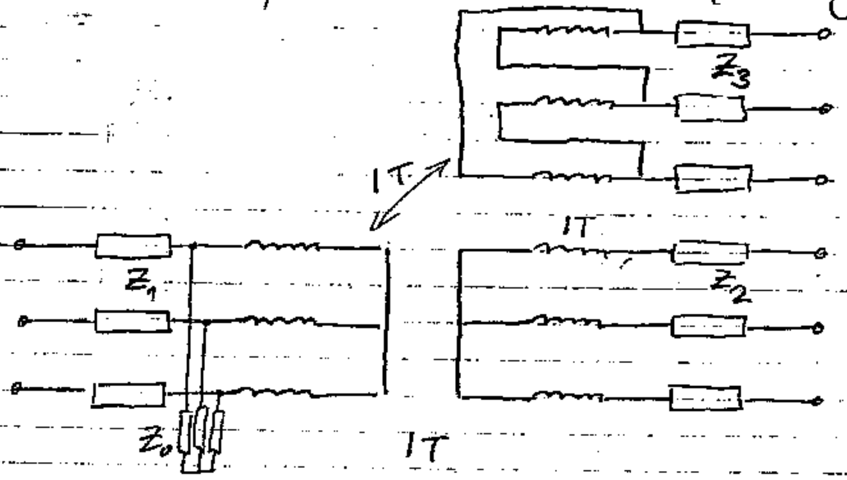
Ista grupa spoja
Isti prijenosni omjer
Snage se razlikuju do 1/3
} uvjeti za paralelu trafoa

U EES-u su najčešće tronamotni transformatori (tercijar, spajaju se u trokut ili seriju i tako poništavamo nulte komponente struje)
Takav tercijar nam ne čini tronamotni trafo. Ali imamo i takvih (u Tumbrima), na tercijar se priključi određeni teret (prigušnica)

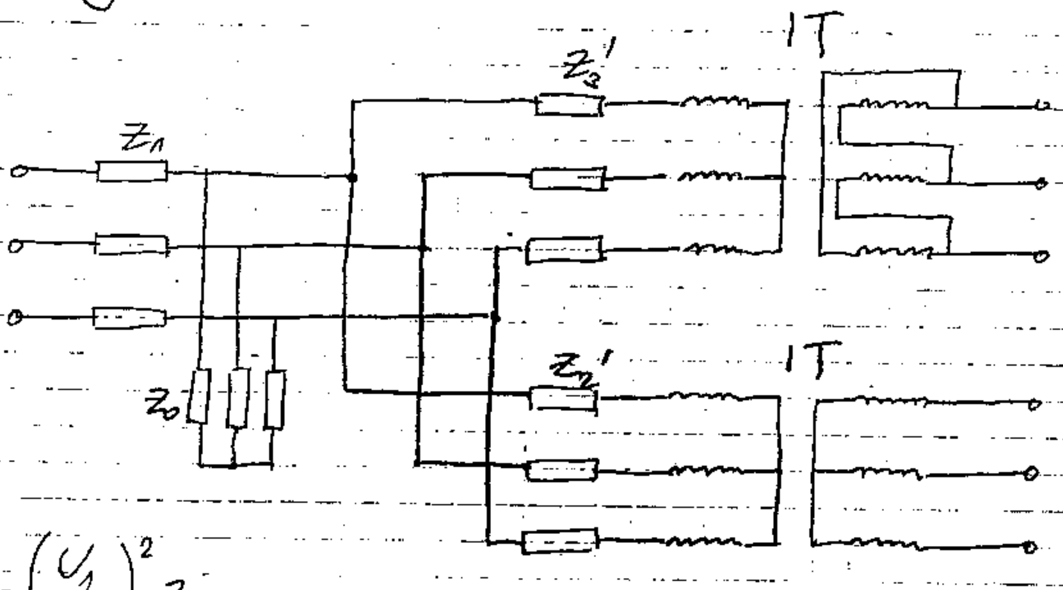
TRONAMOTNI TRAFO



tu se isto rade pokusi PH i KS (određujemo gubitke)



tercijar ne treba graditi za punu snagu, već od prilike 1/3 snage



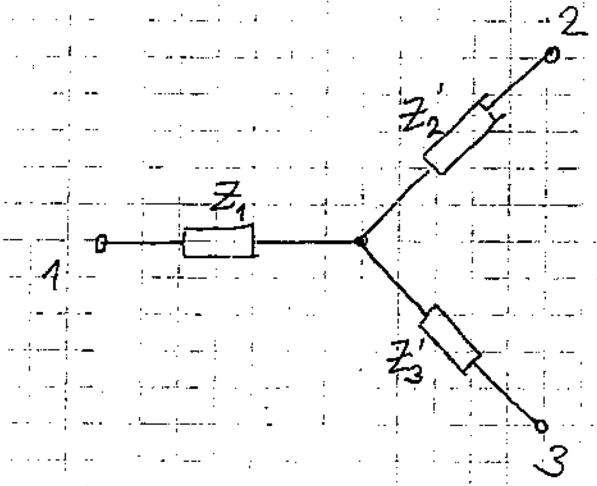
$$Z_3' = \left(\frac{U_1}{U_2}\right)^2 Z_3$$

$$Z_2' = \left(\frac{U_1}{U_2}\right)^2 Z_2$$

Z_{12}, Z_{13}, Z_{23}

$$\boxed{\begin{aligned}
Z_{12} &= \frac{U_{n1}^2}{S_{n12}}\left[\frac{P_{k12}}{S_{n12}} + j\sqrt{u_{k12}^2 - \left(\frac{P_{k12}}{S_{n12}}\right)^2}\right]\\
Z_{13} &= \frac{U_{n1}^2}{S_{n13}}\left[\frac{P_{k13}}{S_{n13}} + j\sqrt{u_{k13}^2 - \left(\frac{P_{k13}}{S_{n13}}\right)^2}\right]\\
Z_{23} &= \frac{U_{n2}^2}{S_{n23}}\left[\frac{P_{k23}}{S_{n23}} + j\sqrt{u_{k23}^2 - \left(\frac{P_{k23}}{S_{n23}}\right)^2}\right]
\end{aligned}}$$

iz pokusa KS



da bi bili veseli, moramo trafo matematički modelirati.

$$Z_{12} = Z_1 + Z_2 \longrightarrow Z_1 + Z_{23} - Z_3 = Z_1 + Z_{23} - Z_{13} + Z_1$$

$$Z_{13} = Z_1 + Z_3$$

$$Z_{23} = Z_2 + Z_3$$

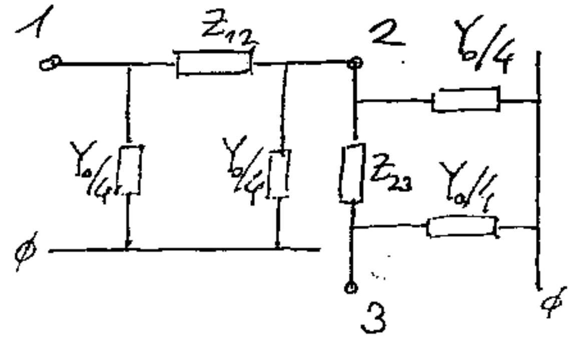
$$\Longrightarrow \boxed{\begin{aligned}
Z_1 &= \frac{1}{2}\left(Z_{12} - Z_{23} + Z_{13}\right)\\
Z_2 &= \frac{1}{2}\left(Z_{12} + Z_{23} - Z_{13}\right)\\
Z_3 &= \frac{1}{2}\left(Z_{13} + Z_{23} - Z_{12}\right)
\end{aligned}}$$

Z_{12}, Z_{12}, Z_{23} → iz pokusa KS

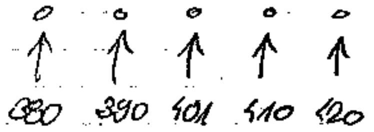
Z_1, Z_2, Z_3 → računamo iz toga ✓

Može se desiti da je Z_2 mali u odnosu na Z_1 i Z_3, a čak se može i desiti da bude NEGATIVAN.

Onda dobijemo L-shemu....

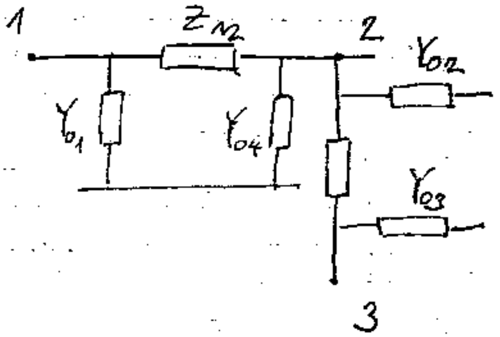


U Tumbrima nemremo mijenjat pod opterećenjem ali imamo nekakvu sklopku..



400/115/31,5

$\underline{1,045/1,05} = 0,996$



to je stvarna shema

Naša mreža proizvodi 400MVAr, a potrošimo ih oko 50. To je OK, jer nam podiže napon, ali samo do neke granice

Ernestinovo 100MVAr

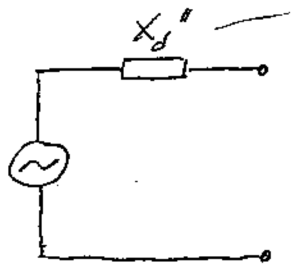
Tumbri 50MVAr

→ to su u stvari nekakve PRIGUŠNICE

GENERATORI

→ Oni nam ne trebaju za proračun tokova snage, već na proračun kratkog spoja

Kratki spoj je brz, a generator treba neko vrijeme da ga osjeti, pa nam je važan rasipni magnetski tok



* djelatni dio je zanemariv, a ovaj X je rasipno magnetsko polje
- zadaje se u Ω, ali je to teško usporedit sa veličinama u mreži, pa koristimo nazivnu snagu

$$\boxed{\frac{X_d''[\Omega]}{U_{ng}^2} \cdot S_n \cdot 100\% = X_d''[\%]}$$

$\frac{U_n^2}{S_n}$ - bazna impedancija

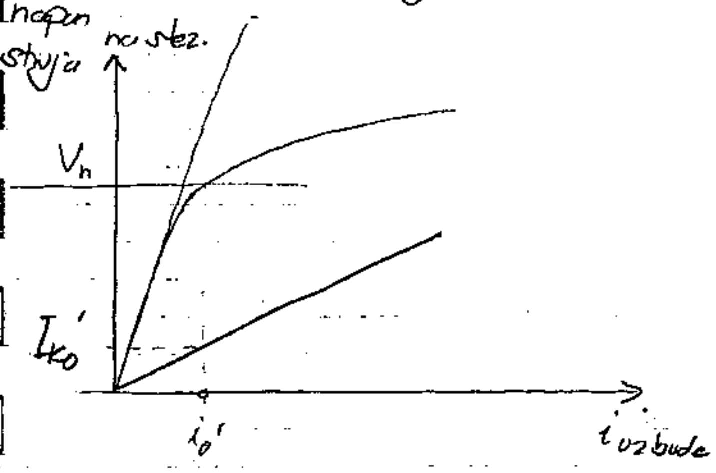
ovo je najjednostavniji model, a inače je dosta kompliciran (TG, HG):

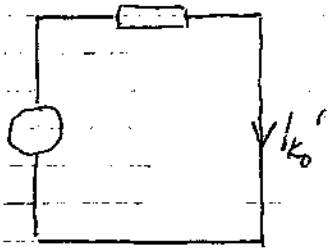
TG — 3000 rpm (pogonski stroj - parna turbina)
HG — sporohodne mašine (magnetska slika mu je full komplicirana)

* X_d' - prolazna reaktancija generatora
  X_d - sinkrona reaktancija generatora, rasipni magnetski tok rotora ⇒ struja trajnog KS = nazivnoj struji ($X_d = 100\,\%$)

* Prigušni namot se stavlja da priguše prodor magnetskog polja u rotor

Sinkrona reaktancija





nakon kratkog spajanja pričekamo neko vrijeme da armatura počne djelovat i da smanji to polje (stac. stanje)
nakon nekoliko minuta...

Nakon nekog vremena (nekoliko minuta) proteče struja I_{ko}' koja je od prilike jednaka nazivnoj struji

$$X_d = \frac{V_n}{I_{ko}'}$$

$$X_n = \frac{V_n}{I_n}$$

$$X_{d\%} = \frac{X_d}{X_n} \cdot 100\%$$

$$\boxed{X_{d\%} = \frac{\frac{V_n}{I_{ko}'}}{\frac{V_n}{I_n}} = \frac{I_n}{I_{ko}} \cdot 100\%} = 1$$

nama su poznati napon i snaga pa ćemo "probat" pomoću toga izrazit X:

$$\boxed{X_{d\%} = X_d \cdot \frac{I_n}{V_n} \cdot 100 = X_d \cdot \frac{3V_n \cdot I_n}{3V_n^2} = X_d \cdot \frac{S_n}{U_{ng}^2} \cdot 100\%}$$

$$\boxed{X_d\,[\Omega] = \frac{X_d\%}{100} \cdot \frac{U_{ng}^2}{S_n}\,[\Omega]}$$

X_d se može poopćit i računat u trenutku nastanka KS....

"...prodre u rotor i poništi svoj uzrok..."

u početku da $10 \cdot I_n \Longrightarrow X_d = 10\%$.

to se može sredit sa nekim vanjskim otporima i trafoima

Index d znači uzdužna struja (u fazi sa rotorom)

$$\boxed{X_d''\,[\Omega] = \frac{X_d''}{100} \cdot \frac{U_n^2}{S_n}}$$

a sad malo brijemo po p.u. veličinama

$$\text{p.u.} \Longrightarrow X_d\,[\text{p.u.}] = \frac{X_d''\%}{100} \cdot \frac{\cancel{U_{ng}^2}}{S_n} \cdot \frac{S_B}{\cancel{U_{ng}^2}}$$

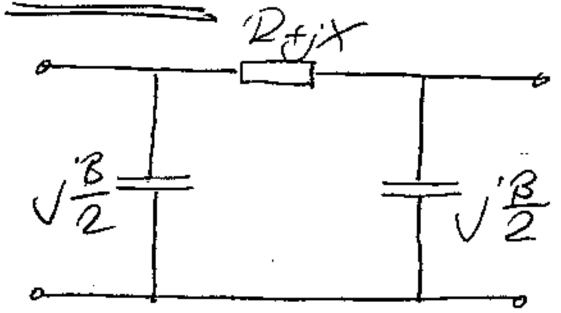
$$\boxed{X_d\ \text{p.u.} = \frac{X_d''\%}{100} \cdot \frac{S_B}{S_n}}$$

→ veći generator → manji X_d p.u.

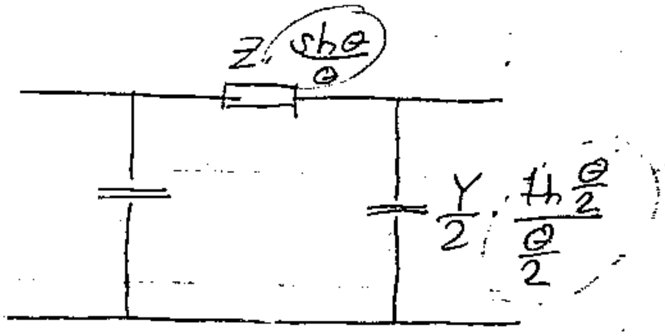
KS-em određujemo reaktancije

X_d – sinkrona reaktancija
X_d' – prolazna reaktancija
X_d'' – početna reaktancija

110kV



→ do 200km je jako točna
iznad toga nam treba točna π shema



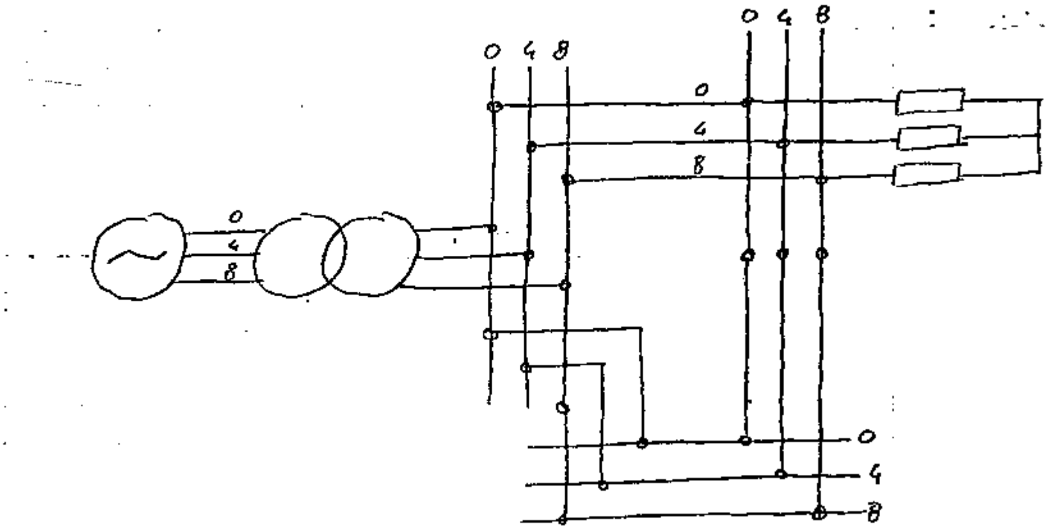
→ korekcioni faktori

to je prevelika komplikacija za premalo dobitka

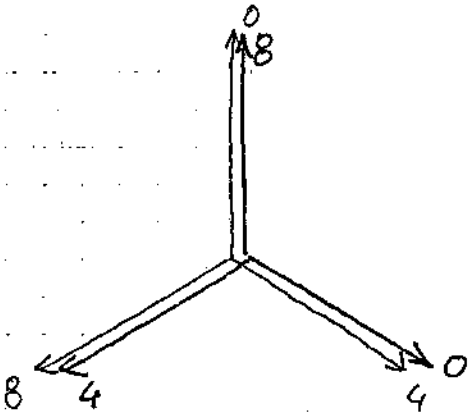
Alučel	R_1 [Ω/km]	X_1 [Ω/km]	CB_1 [mS/km]
150/25	0,19	0,42	0.00275
240/40	0,12	0.41	0.00278
360/60	0,08	0.40	0.00285
490/65	0,06	0.40	0,003
2×490/65	0,03	0.31	0.01380

Znamo vodove, trafoe, generatore, referentne elektrane, snage u čvorištima → i sad računamo

* Mreže se mogu crtati jednopolno ili tropolno i onda treba uzet taj čemu s čim pospojit.



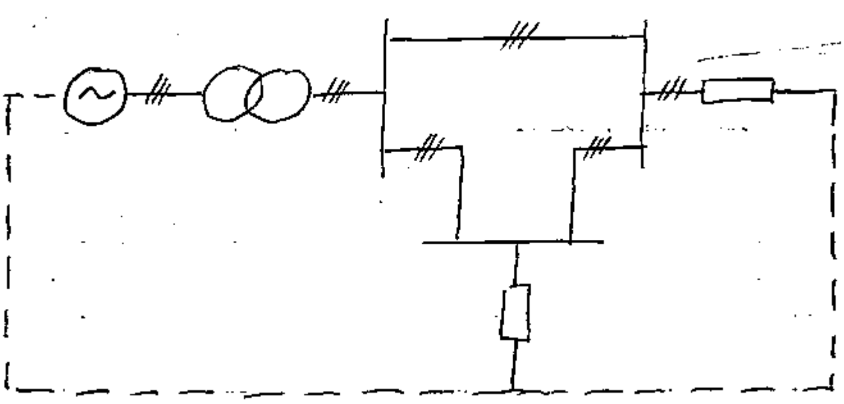
Prije rekonekcije u Europi, Hrvatska je prije spajanja sa Srbijom morala promijeniti redoslijed faza jer uvijek moraju biti spojene 0 i 0, 4 i 4, 8 i 8.....



Natjerali smo frekvenciju da se naš vektor zavrti brže i da stignemo europski vektor. To je trajalo oko 2 sata.... I onda je nastupila sinkronizacija

Sad već svi prodaju struju i dobili smo jedinstveno tržište, koje smo i htjeli...

Taj tropolni način prikazivanja smo skužili, ali za neke proračune nam je ipak prekompliciran jer smo jadni.



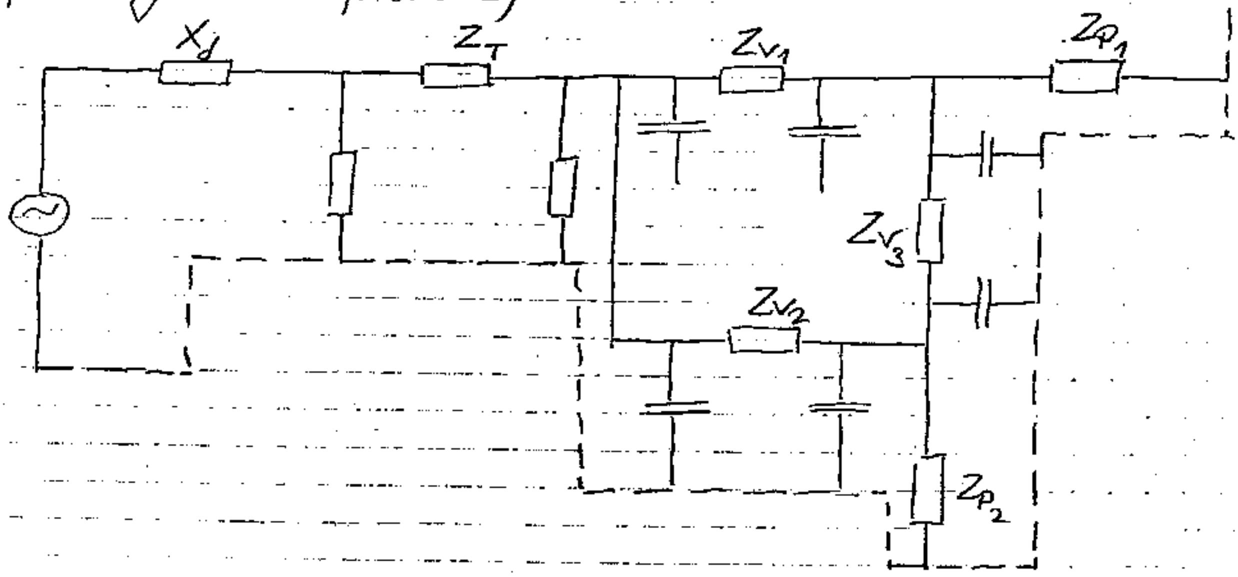
ak se zna da je trofazno

nulta sabirnica

~ jednopolni grafički model ~

Pretpostavke:

→ simetrično opterećenje (naponi pomaknuti za 120° i sve faze jednako opterećene)



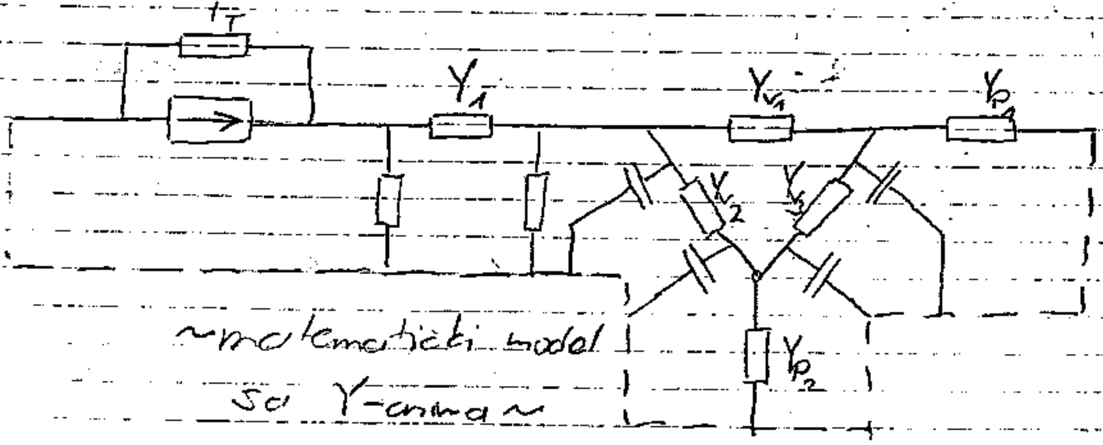
~matematički model~

V - vod

P - potrošač

T - trafo

To ćemo rješavat metodom čvorišta



~matematički model sa Y-ama~

Model potrošača:

1. Konstantni otpor

$S = f(U^2)$ žarulje, lučne peći

$S = \frac{U^2}{Z}$

2. Konstantna struja

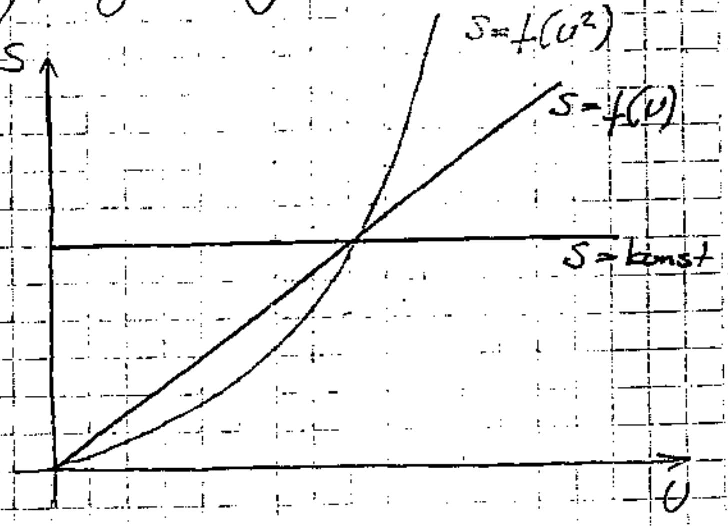
$S = f(U)$ proizvodnja aluminija, naftna industrija

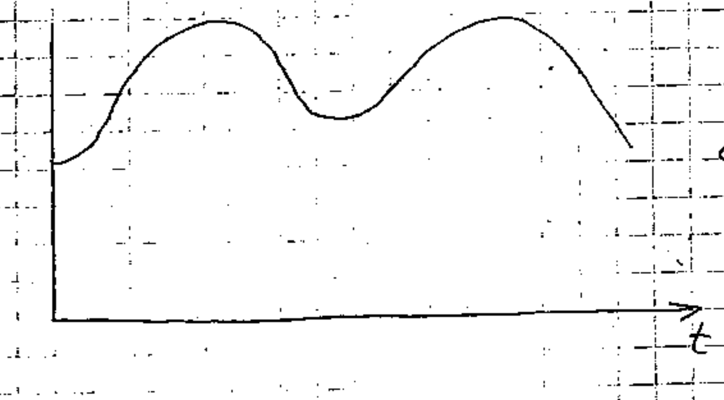
3. Konstantna snaga

S = konst.

- motori s konstantnim teretom, (pumpa za vodu)
- visokonaponska mreža (promjena napona na 110 kV neće imat utjecaja iza trafoa na neke potrošače jer ima miljon regulatora)

Najbolje rješenje je kombinacija ovih tri, ali nama je najtočniji ovaj 3. model potrošača..





distributivna stanica i njen dnevni dijagram opterećenja

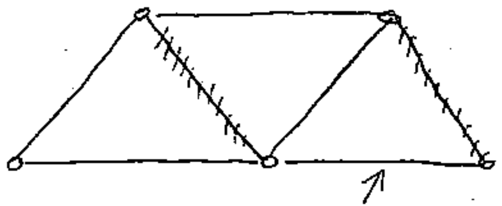
110/x su regulacijski trafoi!

20 x ±1.5% 20 preklopki...

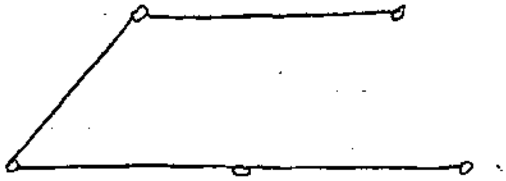
Sad nam treba i nekakva topološka relacija svega toga...

Mi jadnici ćemo koristiti samo neke elemente te TOPOLOGIJE. Tu imamo dosta pretpostavki, lema i ostalih pizdarija koje ćemo uzet u obzir.

Ako se u nekoj petlji suvisle mreže ukloni bilo koja grana mreže, mreža ostaje suvisla! Kod nam ostane jedan zatvoreni dio onda smijemo samo iz njega ukloniti granu.



ovu onda ne smijemo micat jer će to čvorište ostat u zraku



Ovo je <u>stablo</u> jer nemremo proć iz jednog čvorišta pa opet u njega, bez da neku prođemo dvaput

n – broj čvorišta

g_{min} – minimalni broj grana

g_{max} – maximalni broj grana

g – stvarni broj grana

p – broj petlji

$$g_{min} = n - 1$$

$$g_{max} = \frac{n(n-1)}{2}$$

broj nezavisnih grana = temeljni broj petlji

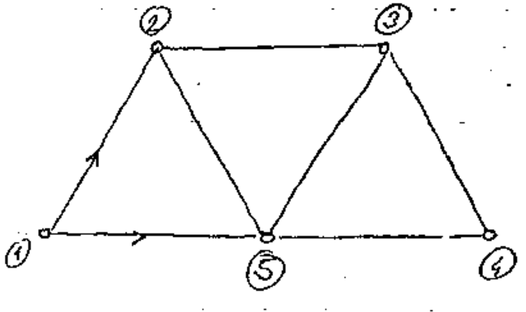
$$p = g - g_{min} = g - n + 1$$

Npr. $n = 5$ $g = 7$

$g_{min} = 4$ $p = 7 - 4 = 3$

$g_{max} = 10$

= nastavljamo sa IMIL-om



* Imamo zadanu el. veličinu svake grane i admitanciju grane između čvorišta i & j

Zadano još I_1, I_2, I_3, I_4 (nezavisne struje)
dogovor: ⊕ predznak ima struja koja ULAZI u mrežu
⊖ predznak ima struja koja IZLAZI iz mreže

→ I. KZ za čvorište ① (općeniti slučaj)

$$I_1 = (U_1 - U_2) \cdot Y_{1-2} + (U_1 - U_3) Y_{1-3} + (U_1 - U_4) Y_{1-4} + (U_1 - U_5) Y_{1-5}$$

$$I_2 = (U_2 - U_1) Y_{2-1} + (U_2 - U_3) Y_{2-3} + (U_2 - U_4) Y_{2-4} + (U_2 - U_5) Y_{2-5}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$I_{n-1} = (U_{n-1} - U_1) Y_{(n-1)-1} + (U_{n-1} - U_2) Y_{(n-1)-2} + \ldots + (U_{n-1} - U_n) Y_{(n-1)-n}$$

~ sustav od n-1 nezavisnih jednadžbi, n-ta jednadžba je linearno zavisna ~

$U_1, \ldots, U_n \Rightarrow$ nepoznanice
Napon u referentnom ili zavisnom čvorištu $\Rightarrow$ poznat

$$I_1 = U_1 \cdot \sum_{i=2}^{n} Y_{i-1} + U_2 (-Y_{1-2}) + \ldots + U_n (-Y_{1-n})$$

$$I_2 = U_1 (-Y_{2-1}) + U_2 \sum_{\substack{i=1 \\ i \neq 2}}^{n} Y_{2-i} + \ldots + U_n (-Y_{2-n})$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$I_{n-1} = U_1 (-Y_{(n-1)-1}) + U_2 (-Y_{(n-1)-2}) + \ldots + U_{n-1} \sum_{\substack{i=1 \\ i \neq n-1}}^{n} Y_{(n-1)-i} + U_n (-Y_{(n-1)-n})$$

Referentno čvorište - točka u kojoj MORAMO imat zadan napon (jedan od U_i, $i=1,\dots,n$) da bi mogli riješiti sustav od $n-1$ nepoznanica

$$1. \; -U_n\left[\sum_{i=2}^{n} Y_{1-i} - Y_{1-2} - Y_{1-3} - \dots - Y_{1-n}\right]$$

$$2. \; -U_2\left[\sum_{\substack{i=1\\ i\neq 2}}^{n} Y_{2-i} - Y_{2-1} - Y_{2-3} - \dots - Y_{2-n}\right]$$

$$\left.\begin{aligned}
I_1 &= (U_1-U_n)\sum_{i=2}^{n} Y_{1-i} + (U_2-U_n)(-Y_{1-2}) + \dots + (U_{n-1}-U_n)(-Y_{1-(n-1)})\\
I_2 &= (U_1-U_n)(-Y_{2-1}) + (U_2-U_n)\sum_{\substack{i=1\\ i\neq 2}}^{n} Y_{2-i} + \dots + (U_{n-1}-U_n)(-Y_{2-(n-1)})\\
&\vdots\\
I_{n-1} &= (U_1-U_n)(-Y_{(n-1)-1}) + (U_2-U_n)(-Y_{(n-1)-2}) + \dots + (U_{n-1}-U_n)\sum_{\substack{i=1\\ i\neq n-1}}^{n} Y_{(n-1)-i}
\end{aligned}\right\} n-1 \text{ jednadžba}$$

Napon u n-tom čvorištu nam je poznat

$$\begin{bmatrix} I_1\\ I_2\\ \vdots\\ I_{n-1}\end{bmatrix} = \underbrace{\begin{bmatrix} \sum\limits_{i=2}^{n} Y_{1-i} & -Y_{1-2} & -Y_{1-3} & \dots & -Y_{1-n}\\ -Y_{2-1} & \sum\limits_{\substack{i=1\\ i\neq 2}}^{n} Y_{2-i} & -Y_{2-3} & \dots & -Y_{2-n}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ -Y_{(n-1)-1} & -Y_{(n-1)-2} & -Y_{(n-1)-3} & \dots & \sum\limits_{\substack{i=1\\ i\neq n-1}}^{n} Y_{(n-1)-i}\end{bmatrix}} \cdot \begin{bmatrix} U_1-U_n\\ U_2-U_n\\ \vdots\\ U_{n-1}-U_n\end{bmatrix}$$

MATRICA ADMITANCIJA ČVORIŠTA

(FULL VAŽNA MATRICA) najvažnija!

Ova matrica opisuje karakter mreže:

→ dijagonalni elementi $\boxed{Y_{ii} = \sum\limits_{\substack{j=1\\ j\neq i}}^{n} y_{i-j}}$ suma admitancija grana koje ulaze u zadano čvorište

→ vandijagonalni elementi $\boxed{Y_{ij} = -y_{i-j}}$

Y_{ii} vlastita admitancija čvorišta i

Y_{ij} međusobna admitancija između čvorišta i & j

$$I_n = -\sum_{i=1}^{n-1} I_i$$

$$[I] = [Y]\cdot[\Delta U]'_{n-1}$$

$$[\Delta U] = [Y]^{-1}\cdot[I]$$

$\boxed{[Y]^{-1} = [Z]}$ matrica koja se rjeđe koristi, ali je nužna za proračun kratkog spoja

${}^{1}_{n}[I] = {}^{1}_{n}[Y]^{n}\cdot[U]^{1}_{n}$ → nerješiv, ali vrijedi za slučaj kada imamo zadano referentno čvorište između 1-n

Z_{ii} vlastita impedancija čvorišta

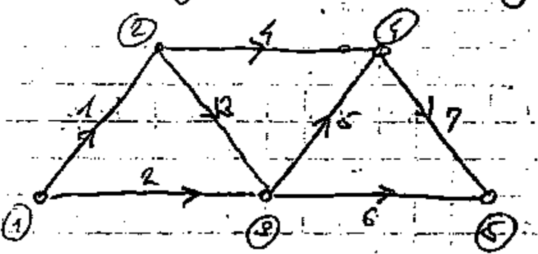
Z_{ij} međusobna impedancija čvorišta

Y matrica – sadrži brdo nula

Z matrica – sadrži elemente $\neq 0$

M - matrica incidencije

M = broj čvorova × broj grana



orijentacija grana:

niže čvorište → višem čvorištu

$$M = \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline ① & 1 & 1 & & & & & \\ ② & -1 & & 1 & & & & \\ ③ & & -1 & -1 & & 1 & 1 & \\ ④ & & & & & -1 & & 1 \\ ⑤ & & & & & & -1 & -1 \end{array}$$

referentni čvor

$$Y = M \cdot y \cdot M^T$$

Y – matrica admitancija čvorišta

M – spojna (incidentna) matrica

y – matrica admitancija grana

$$y = \begin{array}{|c|c|c|c|c|c|c|} \hline y_{1-2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline & y_{1-3} & & & & & \\ \hline & & y_{2-3} & & & & \\ \hline & & & y_{2-4} & & & \\ \hline & & & & y_{3-4} & & \\ \hline & & & & & y_{3-5} & \\ \hline & & & & & & y_{4-5} \\ \hline \end{array}$$

$$Y = \begin{array}{|c|c|c|c|c|c|c|} \hline 1 & 1 & & & & & \\ \hline -1 & & 1 & 1 & & & \\ \hline & -1 & -1 & & 1 & 1 & \\ \hline & & & -1 & -1 & & 1 \\ \hline & & & & & -1 & -1 \\ \hline \end{array} \times \begin{array}{|c|c|c|c|c|c|c|} \hline y_{1-2} & & & & & & \\ \hline & y_{1-3} & & & & & \\ \hline & & y_{2-3} & & & & \\ \hline & & & y_{2-4} & & & \\ \hline & & & & y_{3-4} & & \\ \hline & & & & & y_{3-5} & \\ \hline & & & & & & y_{4-5} \\ \hline \end{array} \times \begin{array}{|c|c|c|c|c|} \hline 1 & -1 & & & \\ \hline 1 & & -1 & & \\ \hline & 1 & -1 & & \\ \hline & 1 & & -1 & \\ \hline & & 1 & -1 & \\ \hline & & 1 & & -1 \\ \hline & & & 1 & -1 \\ \hline \end{array} =$$

$$M \cdot y = \begin{bmatrix} y_{1-2} & y_{1-3} & 0 & 0 & 0 & 0 & 0 \\ -y_{1-2} & 0 & y_{2-3} & y_{2-4} & 0 & 0 & 0 \\ 0 & -y_{1-3} & -y_{2-3} & 0 & y_{3-4} & y_{3-5} & 0 \\ 0 & 0 & 0 & -y_{2-4} & y_{3-4} & 0 & y_{4-5} \\ 0 & 0 & 0 & 0 & 0 & y_{3-5} & -y_{4-5} \end{bmatrix}$$

$$Y = \underline{M} \cdot y \cdot M^T = \begin{bmatrix} y_{1-2} & y_{1-3} & & & & & \\ -y_{1-2} & & y_{2-3} & y_{2-4} & & & \\ & -y_{1-3} & -y_{2-3} & & y_{3-4} & y_{3-5} & \\ & & & -y_{2-4} & -y_{3-4} & & y_{4-5} \\ & & & & & -y_{3-5} & -y_{4-5} \end{bmatrix} \times \begin{bmatrix} 1 & -1 & & & \\ 1 & & -1 & & \\ & 1 & -1 & & \\ & 1 & & -1 & \\ & & 1 & -1 & \\ & & 1 & & -1 \\ & & & 1 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} y_{1-2}+y_{1-3} & -y_{1-2} & -y_{1-3} & 0 & 0 \\ -y_{1-2} & y_{1-2}+y_{2-3}+y_{2-4} & -y_{2-3} & -y_{2-4} & 0 \\ -y_{1-3} & -y_{2-3} & y_{1-3}+y_{2-3}+y_{3-4}+y_{3-5} & -y_{3-4} & -y_{3-5} \\ 0 & -y_{2-4} & -y_{3-4} & y_{2-4}+y_{3-4}+y_{4-5} & -y_{4-5} \\ 0 & 0 & -y_{3-5} & -y_{4-5} & y_{3-5}+y_{4-5} \end{bmatrix}$$

$5 \times 5 = 25$ elemenata u matrici

$7 \Rightarrow 5 \neq 0$ dijagonalnih elemenata

7 grana ⇒ 7 vandijagonalnih elemenata

$7 \times 2 = 14$ elemenata $\neq 0$

$25 - 14 - 5 = 6$ elemenata $= 0$

JE MATRICE SE MORAJU ZNAT!

Z-matrica impedancije

Inverz matrice (npr. 100×100) pomoću determinanti ne bi stigli izračunat cijeli život. Zato su majstori zmislili nešto drugo...

$Y \rightarrow$ inverz običnog broja $\frac{1}{Y}$

$Y \rightarrow$ inverz matrice Y^{-1}

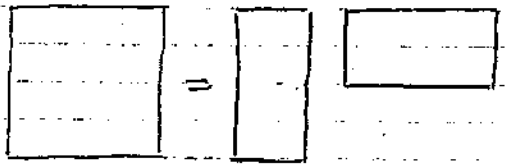
→ Sad ćemo izbjegat inverz matrice

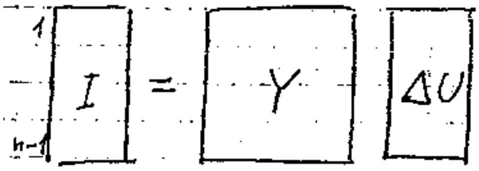
$A \cdot A^{-1} = E$

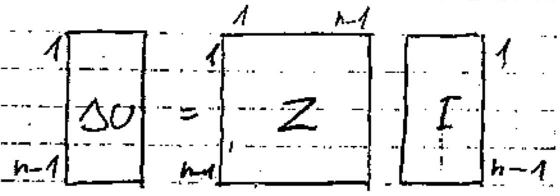
↳ jedinična matrica

$B = A \cdot E = A$

$\boxed{i \times j \cdot j \times k}$ uvjet da se smije množiti



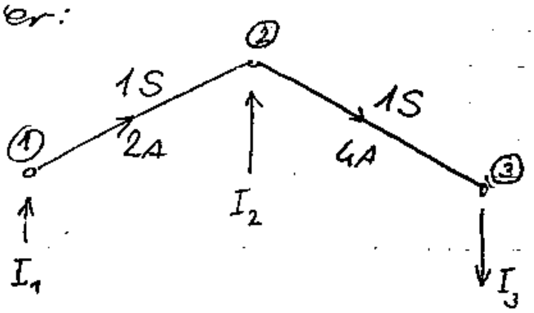




U_n = zadano

→ Idemo sad to vidjet na jednom primjeru

Primjer:



$I_1 = 2A$

$I_2 = 4A$

$$Y = \begin{array}{|c|c|c|} \hline 1 & -1 & 0 \\ \hline -1 & 2 & -1 \\ \hline 0 & -1 & 1 \\ \hline \end{array}$$

$$Y_{n-1} = \begin{bmatrix} \textcircled{1} & -1 \\ -1 & 2 \end{bmatrix}$$

I onda se pri stavi recipročna vrijednost, onaj koji nije u tom stupcu i tom retku (gdje je PIVOT) iznosi:

→ 2 – umnožak ovih ostalih (u njegovom redu i stupcu)

$$Y^{(1)} = \begin{vmatrix} \frac{1}{1} & -\frac{1}{1} \\ -\frac{-1}{1} & 2-1 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & \textcircled{1} \end{vmatrix}$$

$$\boxed{\begin{array}{ll} 11 = \frac{1}{pivot} & 12 = \frac{y_{12}}{pivot} \\ 21 = -\frac{y_{21}}{pivot} & 22 = y_{22} - \frac{y_{12}}{pivot} \end{array}}$$

I sad dalje neka (ovo kroz ovo plus ovaj puta pivotao dijete...)

$$Y^{(2)} = \begin{vmatrix} 1 - \frac{-1 \cdot 1}{1} & -\frac{-1}{1} \\ \frac{1}{1} & \frac{1}{1} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = Z = Y^{-1}$$

$$\boxed{\begin{array}{ll} 11 = y_{11}^{(1)} - \frac{y_{12}^{(1)} \cdot y_{21}^{(1)}}{pivot^{(1)}} & 12 = -\frac{y_{12}^{(1)}}{pivot^{(1)}} \\ 21 = \frac{y_{21}^{(1)}}{pivot^{(1)}} & 22 = \frac{1}{pivot^{(1)}} \end{array}}$$

Koraka koliko i stupaca....

$U_3 = \phi$ uzemljili smo ③ čvorište

$I_3 = -\sum_{i=1}^{2} I_i$

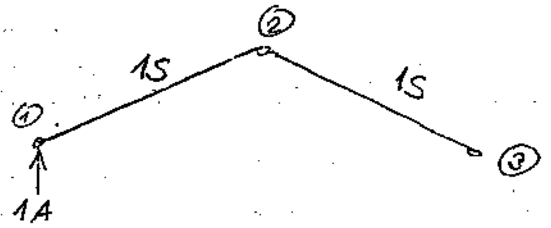
$I_3 = -6A$

$$\begin{vmatrix}\Delta U_1\\ \Delta U_2\end{vmatrix} = \begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix}\cdot\begin{vmatrix}2\\ 4\end{vmatrix} = \begin{vmatrix}8\\ 6\end{vmatrix}$$

$I_{1-2} = (U_1 - U_2)y_{1-2} = 2\cdot 1 = 2A$

$I_{2-3} = (U_2 - U_3)\cdot y_{2-3} = (6-0)\cdot 1 = 6A$

→ Idemo mi malo izanalizirat tu Z matricu



$Z = \begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix}$ kako smo struju promijenili:

$Z\cdot\begin{bmatrix}1\\ 0\end{bmatrix} = \begin{bmatrix}2\\ 1\end{bmatrix}\begin{matrix}U_1\\ U_2\end{matrix}$ ili $Z\cdot\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}1\\ 1\end{bmatrix}$

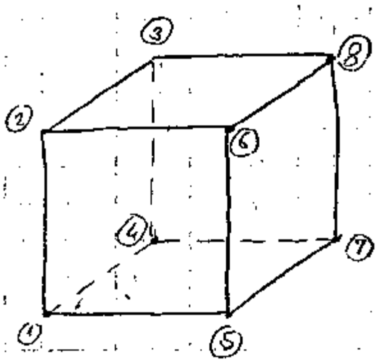
Pokazuje napone u promatranoj mreži u određenom stupcu ako narinemo 1A.

Z_{21} – napon u čvorištu ② ako je u ① ušlo 1A

Zato ju zovemo i matricom distribucije napona.

Problem kocke

To smo kakti iz OE radili...



Idemo sredit matricu admitancije.

$$Y=\begin{bmatrix} 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ -1 & \boxed{3} & \boxed{-1} & \boxed{0} & 0 & -1 & 0 \\ 0 & \boxed{-1} & \boxed{3} & \boxed{-1} & 0 & 0 & 0 \\ -1 & \boxed{0} & \boxed{-1} & \boxed{3} & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 3 & -1 & -1 \\ 0 & -1 & 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & -1 & -1 & -1 & 0 & 3 \end{bmatrix} \quad \begin{matrix} \Sigma=0 \\ \Sigma=0 \\ \Sigma=1 \\ \Sigma=0 \\ \Sigma=0 \\ \Sigma=1 \\ \Sigma=1 \end{matrix}$$

Moramo napisat 7 koraka, al ne moramo ić po redu...
Možemo ić npr. 3 stupac, pa 5 stupac itd....
Prvo idemo eliminirat stupce koji imaju najviše Ø jer oni u tim retku mijenjaju najmanje elemenata...

$m=1$, $k=3$

$$Y^{(1)}=\begin{array}{|c|c|c|c|c|c|c|} \hline 3 & -1 & 0 & -1 & -1 & 0 & 0 \\ \hline -1 & \boxed{8/3} & 1/3 & -1/3 & \boxed{0} & \boxed{-1} & 0 \\ \hline 0 & -1/3 & 1/3 & -1/3 & 0 & 0 & -1 \\ \hline -1 & -1/3 & 1/3 & 8/3 & 0 & 0 & -1 \\ \hline -1 & \boxed{0} & 0 & 0 & \boxed{3} & \boxed{-1} & -1 \\ \hline 0 & \boxed{-1} & 0 & 0 & \boxed{-1} & \boxed{3} & 0 \\ \hline 0 & 0 & 0 & -1 & -1 & 0 & 3 \\ \hline \end{array}$$

$m = 2 \qquad k = 6$

$$Y^{(2)} = \begin{array}{|c|c|c|c|c|c|c|}
\hline
3 & -1 & 0 & -1 & -1 & 0 & 0 \\
\hline
-1 & 7/3 & -1/3 & -1/3 & -1/3 & 1/3 & 0 \\
\hline
0 & -1/3 & 1/3 & -1/3 & 0 & 0 & 0 \\
\hline
-1 & -1/3 & 1/3 & \boxed{8/3} & \boxed{3} & 0 & \boxed{-1} \\
\hline
-1 & -1/3 & 0 & \boxed{0} & \boxed{8/3} & 1/3 & \boxed{-1} \\
\hline
0 & -1/3 & 0 & 0 & -1/3 & 1/3 & 0 \\
\hline
0 & 0 & 0 & \boxed{-1} & \boxed{-1} & 0 & \boxed{3} \\
\hline
\end{array}$$

$* \ \text{broj} - \dfrac{\text{suc.red} \cdot \text{sud.stp}}{\text{pivot}}$

$* \ -\dfrac{\text{broj}}{\text{pivot}}$

$* \ \dfrac{\text{broj}}{\text{pivot}}$

$m = 3 \qquad k = 7$

$$Y^{(3)} = \begin{array}{|c|c|c|c|c|c|c|}
\hline
\boxed{3} & \boxed{-1} & 0 & \boxed{-1} & \boxed{-1} & 0 & 0 \\
\hline
\boxed{-1} & \boxed{7/3} & 1/3 & \boxed{-1/3} & \boxed{-1/3} & 1/3 & 0 \\
\hline
0 & -1/3 & 1/3 & -1/3 & 0 & 0 & 0 \\
\hline
\boxed{-1} & \boxed{-1/3} & 1/3 & \boxed{7/3} & \boxed{-1/3} & 0 & 1/3 \\
\hline
\boxed{-1} & \boxed{-1/3} & 0 & \boxed{-1/3} & \boxed{7/3} & 1/3 & 1/3 \\
\hline
0 & -1/3 & 0 & 0 & -1/3 & 1/3 & 0 \\
\hline
0 & 0 & 0 & -1/3 & -1/3 & 0 & 1/3 \\
\hline
\end{array}$$

Mijenjaju se oni elementi koji su u ravnini s k-tim (označenim) retkom i stupcom a taj element u k-tom je različit od 0. Zvuči bolesno, al' pogle pa ćeš skužit.

*PIVOT uvijek pozitivan

*Taj redak ne mijenja predznak već se samo dijeli s pivotom.

$m=4 \qquad k=1$

$$Y^{(4)} = \begin{array}{c|c|ccccc}
\boxed{1/3} & \boxed{-1/3} & \boxed{0} & \boxed{-1/3} & \boxed{-1/3} & \boxed{0} & 0 \\ \hline
\boxed{1/3} & \boxed{5/3} & \boxed{1/3} & \boxed{-2/3} & \boxed{-2/3} & \boxed{1/3} & 0 \\ \hline
\boxed{0} & \boxed{-1/3} & \boxed{1/3} & \boxed{-1/3} & \boxed{0} & \boxed{0} & 0 \\
\boxed{1/3} & \boxed{-2/3} & \boxed{1/3} & \boxed{6/3} & \boxed{-2/3} & \boxed{0} & 1/3 \\
\boxed{1/3} & \boxed{-2/3} & \boxed{0} & \boxed{-2/3} & \boxed{6/3} & \boxed{1/3} & 1/3 \\
\boxed{0} & \boxed{-1/3} & \boxed{0} & \boxed{0} & \boxed{-1/3} & \boxed{1/3} & 0 \\
0 & 0 & 0 & -1/3 & -1/3 & 0 & 1/3
\end{array}$$

$m=5 \qquad k=2$

$$Y^{(5)} = \begin{array}{ccc|c|ccc}
7/18 & 1/6 & 1/18 & -4/9 & -4/9 & 1/18 & 0 \\
1/6 & 1/2 & 1/6 & -1/3 & -1/3 & 1/6 & 0 \\
1/18 & 1/6 & 7/18 & -4/9 & -1/9 & 1/18 & 0 \\ \hline
4/9 & 1/3 & 4/9 & 16/9 & -8/9 & 1/9 & 1/3 \\ \hline
4/9 & 1/3 & 1/9 & -8/9 & 16/9 & 4/9 & 1/3 \\
1/18 & 1/6 & 1/18 & -1/9 & -4/9 & 7/18 & 0 \\
0 & 0 & 0 & -1/2 & -1/3 & 0 & 1/3
\end{array}$$

u sljedećem koraku će se
svi elementi promijeniti (jer su
u označenom retku i stupcu
svi različiti od nule)

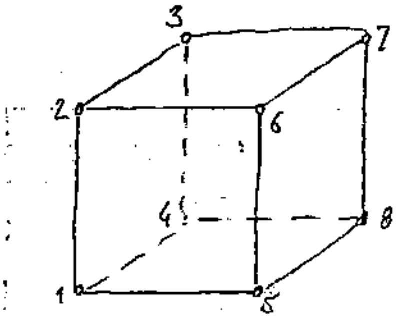
$m=6 \qquad k=4$

$$Y^{(6)} = \begin{array}{cccc|c|cc}
1/2 & 3/12 & 1/6 & 1/4 & -2/3 & 1/12 & 1/2 \\
1/4 & 9/16 & 1/4 & 3/16 & -1/2 & 3/16 & 1/16 \\
1/6 & 1/4 & 1/2 & 1/4 & -1/3 & 1/12 & 1/12 \\
1/4 & 3/16 & 1/4 & 9/16 & -1/2 & 1/16 & 3/16 \\ \hline
2/3 & 1/2 & 1/3 & 1/2 & 4/3 & 1/2 & 1/2 \\ \hline
1/12 & 3/16 & 1/12 & 1/16 & -1/2 & 57/144 & 1/48 \\
1/12 & 1/16 & 1/12 & 3/16 & -1/2 & 1/48 & 19/48
\end{array}$$

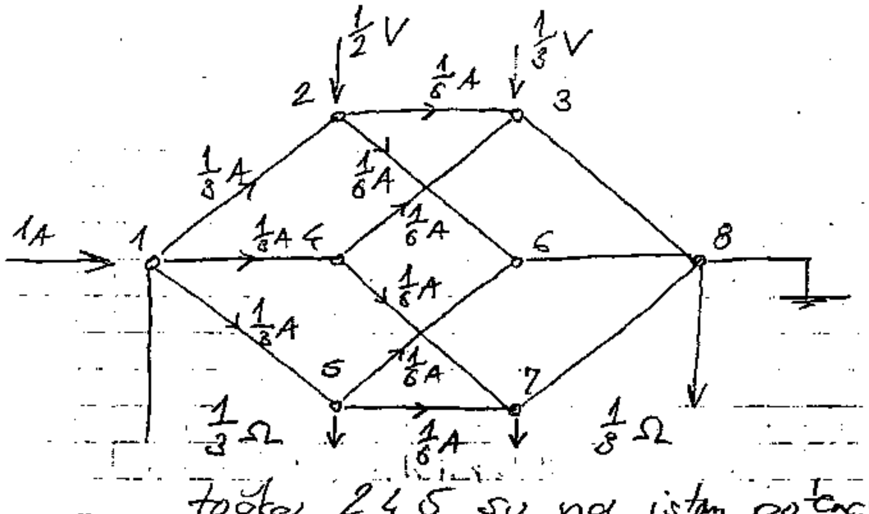
$$Y^{(7)} = \begin{bmatrix} 5/6 & 1/2 & 1/3 & 1/2 & 1/2 & 1/3 & 1/3 \\ 1/2 & 3/4 & 3/8 & 3/8 & 3/8 & 3/8 & 1/4 \\ 1/3 & 3/8 & 7/12 & 3/8 & 1/4 & 5/24 & 5/24 \\ 1/4 & 3/8 & 3/8 & 3/4 & 3/8 & 1/4 & 3/8 \\ 1/2 & 3/8 & 1/4 & 3/8 & 3/4 & 3/8 & 3/8 \\ 1/3 & 3/8 & 5/24 & 1/4 & 3/8 & 7/12 & 5/24 \\ 1/3 & 1/4 & 5/24 & 3/8 & 3/8 & 5/24 & 7/12 \end{bmatrix} = Z$$

Z_{11} - otpor mreže između čvorišta 1 i referentnog čvorišta

*



za npr. $Z_{18} = \frac{1}{3}$ ako narinemo struju od 1A na 1, a uzemljimo 8, dobit ćemo $\frac{1}{3}$V u čvorištu 1



točka 2, 4, 5 su na istom potencijalu

POTENCIJALI

$\frac{5}{6} - \frac{1}{3} = \frac{1}{2}$ $\quad Z_{12} = Z_{14} = Z_{15} = \frac{1}{2}$

$\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$ $\quad Z_{13} = Z_{16} = Z_{17} = \frac{1}{3}$

$Z_{22} = \frac{3}{4} \Omega$

$Z_{21} = Z_{12} = \frac{1}{2} \Omega$

$Z_{22} = \frac{3}{8} \Omega$

$Z_{24} = 3/8 \Omega$

$Z_{25} = Z_{26} = \frac{3}{8} \Omega$ imaju isti potencijal

U VN mreži ne raspolažemo sa strujama, već sa snagama

<u>3 vrste čvorišta:</u>

- čvorište tereta P, Q
- čvorište generatorsko $P, |U|, \overbrace{Q_{min} \text{ i } Q_{max}}^{\text{granice}}$
- čvorište regulacijsko $|U|, \delta \Rightarrow |U|, \delta = 0$

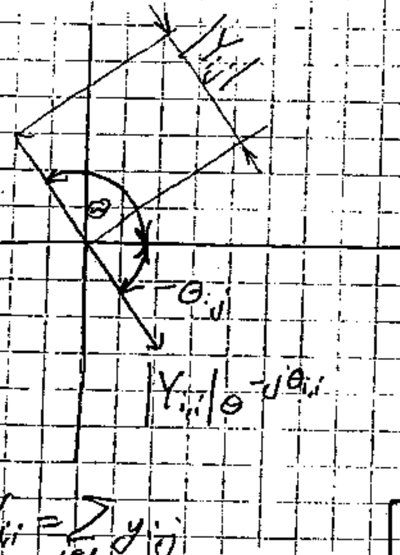
→ Snage su funkcije više varijabli → iterativne metode rješavanja

$S = U \cdot I^* = P + jQ$	$S = U^* I = P - jQ$
+ induktivno	− induktivno
− kapacitivno	+ kapacitivno

$S_i = U_i \cdot I_i^*$ $\quad U_i = |U_i| e^{j\delta_i}$

$U_j = |U_j| e^{j\delta_j}$

$Y_{ij} = -y_{ij} = -(G_{ij} - jB_{ij}) = |Y_{ij}| e^{j\theta_{ij}}$



dijagonalni 4 kvadrant

vandijagonalni 2 kvadrant

$Y_{ii} = \sum_{j \neq 1} y_{ij}$

$$I_i = \overset{1}{[}Y\overset{n}{]}\cdot[U]$$

$$* \quad S_i = U_i \cdot \sum_{i=1}^{n} Y_{ij}^{*} U_j^{*}$$

$$* \quad S_i = U_i \cdot e^{j\delta_i} \cdot \sum |U_j| e^{j\delta_j} \cdot |Y_{ij}| \cdot e^{j\theta_{ij}}$$

$$= U_i \cdot e^{j\delta_i} \cdot \sum |U_j|\,|Y_{ij}| \left[\cos(\delta_i - \delta_j - \theta_{ij}) + j\sin(\delta_i - \delta_j - \theta_{ij})\right]$$

$$* \quad P_i = |U_i| \sum_{j=1}^{n} |U_j||Y_{ij}| \cos(\delta_i - \delta_j - \theta_{ij}) \qquad i = 1, \ldots, n-1$$

$$* \quad Q_i = |U_i| \sum_{j=1}^{n} |U_j||Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij}) \qquad i = 1, \ldots, n-1$$

Tako smo dokazali da snage u i-tom čvorištu ovise o svim snagama svih čvorišta.

$$* \quad P_i = |U_i|^2 |Y_{ii}| \cdot \cos(\theta_{ii}) + |U_i| \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} |U_j||Y_{ij}| \cos(\delta_i - \delta_j - \theta_{ij})$$

$$* \quad Q_i = -|U_i|^2 \cdot |Y_{ii}| \sin(\theta_{ii}) + |U_i| \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} |U_j||Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij})$$

čvorište tereta → tražimo U, δ

čvorište generatorsko → tražimo δ_i, Q

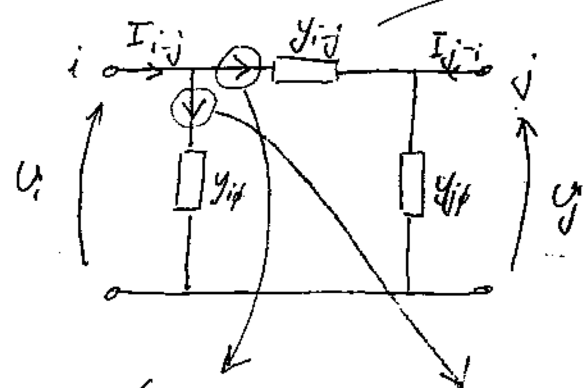
ako je $U_i = E_i + j F_i \quad Y_{ij} = G_{ij} + j B_{ij}$ & $S = U_i \sum_{j=1}^{n} U_j^{*} Y_{ij}^{*}$:

$$P_i = \sum_{j=1}^{n} \left[E_i \cdot (E_j G_{ij} - F_j B_{ij}) + F_i (F_j G_{ij} + E_j B_{ij})\right]$$

$$Q_i = \sum_{j=1}^{n} \left[F_i \cdot (E_j G_{ij} - F_j B_{ij}) + E_i (F_j G_{ij} + E_j B_{ij})\right]$$

$i = 1, \ldots, n \;,\; i \neq ref$

vektor stanja je onaj vektor koji sadrži sve napone po iznosu i kutu u mreži



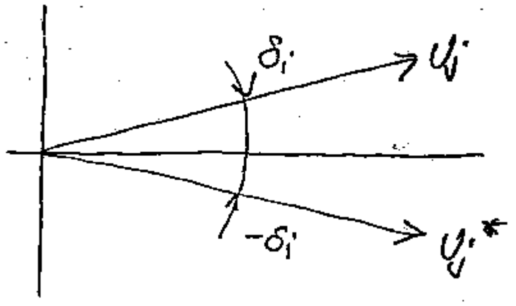
ovo nije element matrice već uzdužna admitancija

$$I_{i-j} = (U_i - U_j)\cdot y_{i-j} + U_i \cdot y_{i0}$$

$$S_{i-j} = U_i \cdot I_{i-j}^* = U_i\left[(U_i^* - U_j^*)y_{i-j}^* + U_i^* \cdot y_{i0}^*\right]$$

$$I_{j-i} = (U_j - U_i)y_{i-j} + U_j \cdot y_{j0}$$

$$S_{j-i} = U_j \cdot I_{j-i}^* = U_j \cdot (U_j^* - U_i^*)\cdot y_{i-j}^* + |U_j|^2 \cdot y_{j0}^*$$



* gubici $\rightarrow \Delta S = S_{i-j} + S_{j-i} = y_{i-j}\,(U_i^* - U_j^*)(U_i - U_j) + |U_i|^2 y_{i0}^* + |U_j|^2 y_{j0}$

$$\Delta S = \underbrace{(U_i^* - U_j^*)\cdot y_{i-j}^*(U_i - U_j)}_{\Delta P \text{ je uvijek } +} \ominus \underbrace{\left(|U_i|^2 - |U_j|^2\right)\cdot \frac{B_{i-j}}{2}}_{\Delta Q \text{ je } -}$$

dodatak jal. E zbog kap. vodova

zato je

$$y_{i0}^* = y_{j0}^* = -\frac{B_{i-j}}{2}$$

ΔS = kvadratna forma

U svakoj kvadratnoj formi izraz je pozitivan i uvjet vrijedi:

$\Delta P > 0$ — fizikalno: gubici energije koji se pretvaraju u toplinu

$\Delta Q > 0$ — fizikalno: gubici jalove snage

↘ ↙

kad se ova dva ponište, vod je opterećen PRIRODNOM SNAGOM

Idealna snaga – nema djelatnih gubitaka

Hibridne matrice – možemo imati i do 3 ref. čvorišta

– nije se održala u praksi

GAUSS-SEIDELOVA METODA

$$(U_i - U_{ref}) = \sum_{j=1}^{n-1} Z_{ij} \cdot I_j \qquad n = \text{ref. čv.}$$

ij → nezavisna čvorišta

Postoji jedno zavisno čvorište REFERENTNO

$U_i =$

$U_{ref} = |U_{ref}| \angle 0^\circ \rightarrow \delta = 0$

$Z = Y^{-1}$

$$I_j = \frac{S_j^*}{U_j^*}$$

$$\boxed{\begin{array}{l} U_1 - U_n = Z_{11} I_1 + Z_{12} I_2 + \ldots + Z_{1,n-1} I_{n-1} \\ \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ U_i - U_n = Z_{i1} I_1 + Z_{i2} I_2 + \ldots + Z_{i,n-1} I_{n-1} \\ \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ U_{n-1} - U_n = Z_{n-1,1} I_1 + Z_{n-1,2} I_2 + \ldots + Z_{n-1,n-1} I_{n-1} \end{array}}$$

0 - ta iteracija S_i je zadano

$$U_i^{(0)} = 1 + j\phi \qquad i=1,\dots,n-1$$

$$\boxed{I_i^{(0)} = \frac{S_i^*}{U_i^{*(0)}} - y_{i0} U_i^{(0)}} \qquad i=1,\dots,n-1$$

Matricu Z dobivamo uzimavši u obzir samo uzdužne parametre, ne uzimam u obzir kapacitete vodova i prijenosne omjere transformatora.

Poprečne grane ćemo mijenjati kao nadomjesne struje.

1. iteracija

$$U_1^{(1)} = Z_{11} I_1^{(0)} + Z_{12} I_2^{(0)} + \dots + Z_{1,n-1} I_{n-1}^{(0)} + U_n \Longrightarrow \boxed{I_1^{(1)} = \frac{S_1^*}{(U_1^{(1)})^*} - y_{10} U_1^{(1)}}$$

$$U_2^{(1)} = Z_{21} I_1^{(0)} + Z_{22} I_2^{(0)} + \dots + Z_{2,n-1} I_{n-1}^{(0)} + U_n \Longrightarrow \boxed{I_2^{(1)} = \frac{S_2^*}{(U_2^{(1)})^*} - y_{20} U_2^{(1)}}$$

$\vdots$

OPĆENITO: $\boxed{I_j^{(k+1)} = \frac{S_j^*}{U_1^{(k+1)*}} - y_{j0} U_j^{(k+1)}}$

JOŠ OPĆENITIJE:

$$U_i^{k+1} = U_n + \sum_{j=1}^{n-1} Z_{ij} \cdot I_j^{(k+1)} + \sum_{v=1}^{n-1} Z_{ij} \cdot I_j^{(k)}$$

$$I^{k+1} = \frac{S_j^*}{(U_j^{(k+1)})^*} - y_{j0} \cdot U_j^{k}$$

$$I_j^{k} = \frac{S_j^*}{(U_j^{k})^*} - y_{j0} \cdot U_j^{k}$$

$$\text{Max} \left| U_i^{(k+1)} - U_i^{(k)} \right| \leq \varepsilon$$

$$\left.\begin{array}{l} \text{Max Re} \left| U_i^{k+1} - U_i^{k} \right| \\ \text{Im} \left| U_i^{k+1} - U_i^{k} \right| \end{array}\right\} \leq \varepsilon$$

TOČNOST BY HEBEL
ε=0,0001

Samo je jedan Mali Ivica!

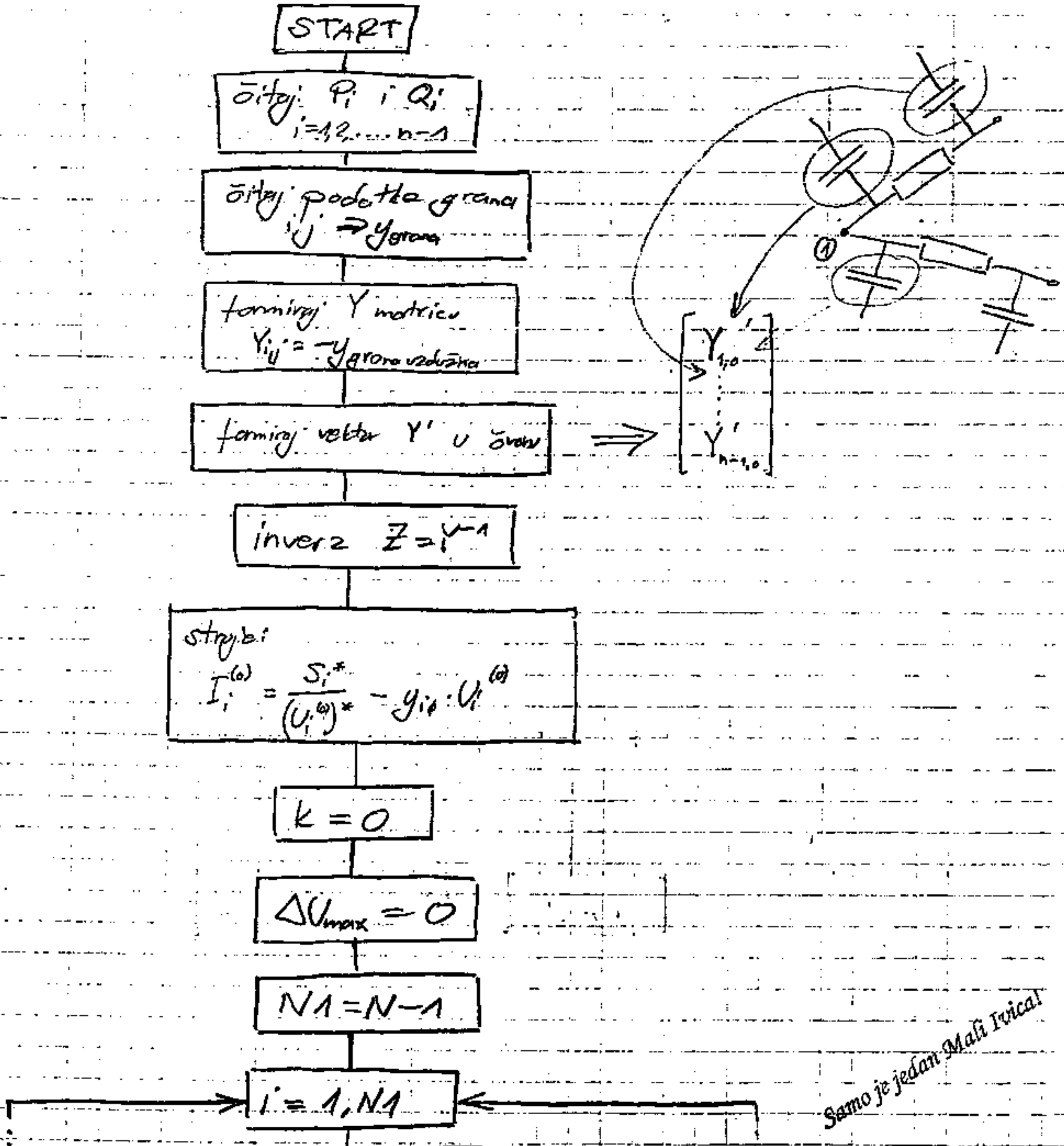
Metoda je povoljna za mali broj čvorišta (20), ali je zato konvergentna do jaja! SAMO 5 - 10 iteracija

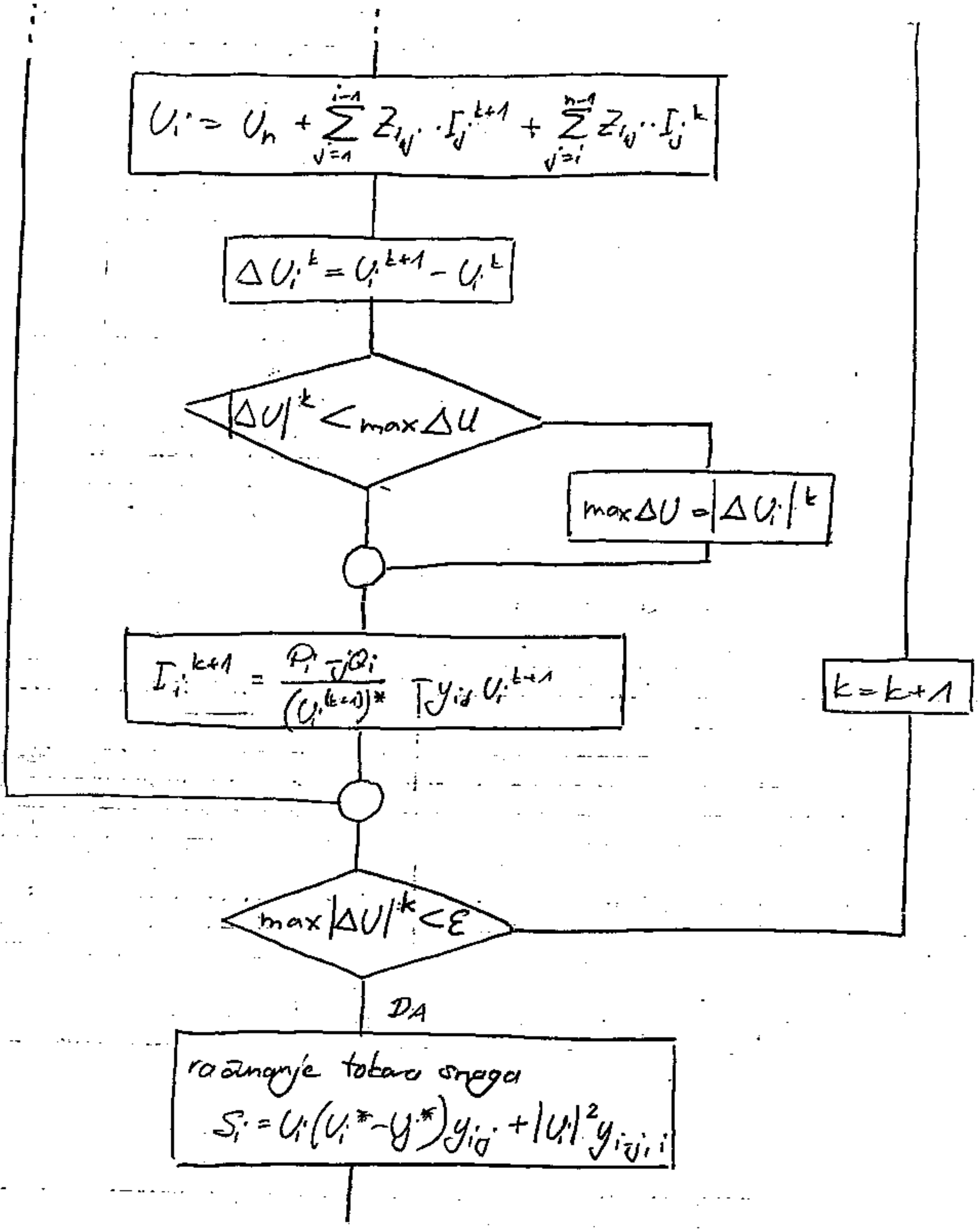
U tim mrežama imamo jedno referentno i teretna čvorišta

Nema generatorskih čvorišta zbog kompliciranosti, pa ga prebacimo u teretno čvorište

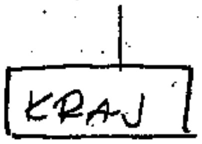
Ova je metoda OK za oko 100 čvorišta (više od toga baš i ne...)

Za 100 čvorišta matrica impedancija ima 100 × 100 = 10000 elemenata. Tu iteraciju rade kompjutori. Ovako bi izgledao algoritam programa računanja:



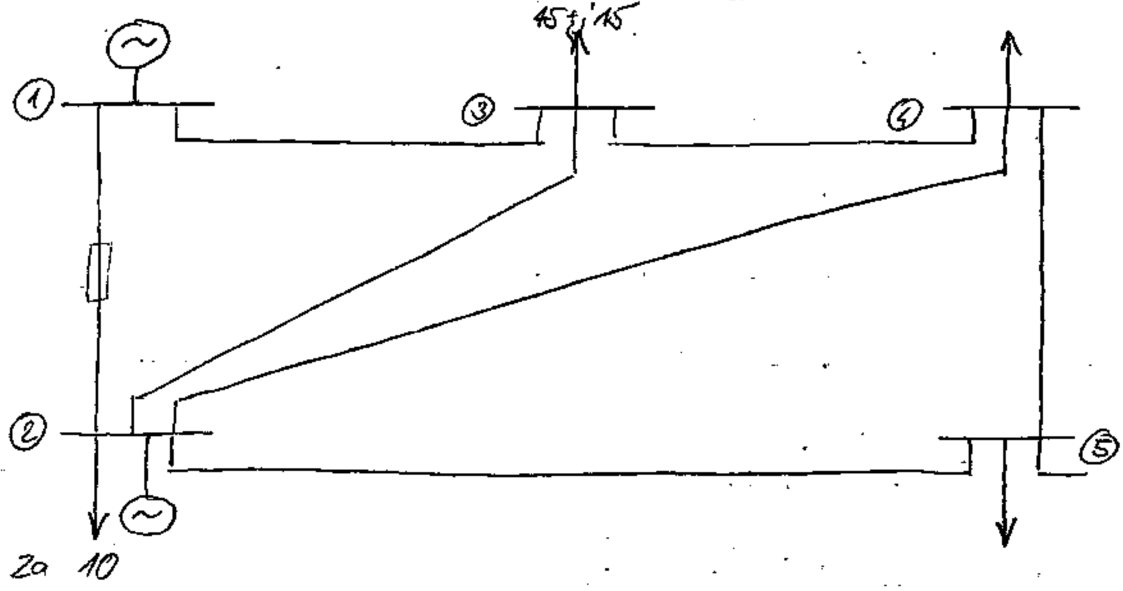


Poanta ovog sranja je vektor stanja. Njega treba dobit pod svaku cijenu



Thank God!

Primjer:



PAZI! To izbaci iz mreže pa se u račun uzimo $-45-j15$

Zadano:

ČVOR	GENERATOR			TERET		y_{io}
	U	MW	MVAr	MW	MVAr	
1.	$1{,}06+j0$	/	/	/	/	$j0.055$
2.		40	30	20	10	$j0.085$
3.		0	0	45	15	$j0.055$
4.				40	5	$j0.055$
5.				60	10	$j0.04$

Vodovi:

i	j	Z_{ij} (P.U.)	$Y_{ij}/2$ (P.U.)	y_{ij}
1	②	$0.02+j0.06$	$j0.03$	$5-j15$
1	3	$0.08+j0.24$	$j0.025$	$1.25-j3.75$
②	3	$0.06+j0.18$	$j0.02$	$1.66-j5$
②	4	$0.06+j0.18$	$j0.02$	$1.66-j5$
②	5	$0.04+j0.12$	$j0.015$	$2.5-j7.5$
3	4	$0.01+j0.03$	$j0.01$	$10-j30$
4	5	$0.08+j0.24$	$j0.025$	$1.25-j3.75$

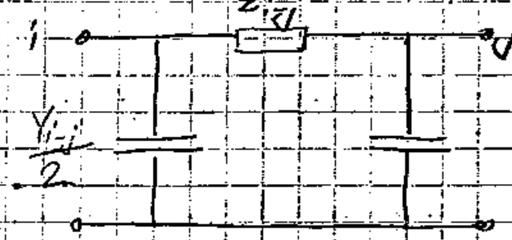
→ Kak se dobiju ovi y_{i-j} (u tablici 2)

$$y_{1-2} = \frac{1}{z_{i-j}} = \frac{1}{0.02+j0.06} = \frac{0.02-j0.06}{0.02-j0.06} = 5-j15$$

Npr. za ovaj stupac u tablici "Zadano":
Prvo se tablica 2 pogleda... $Y_{i-j}/2$ iz redova di su zaokruženi brojevi u prvim stupcima se pozbrajaju

Npr. za 2. čvorište:

$$j0.03 + j0.02 + j0.02 + j0.015 = j0.085$$



* Sad idemo napravit matricu admitancija čvorišta

Dijagonalni elementi: Zbroj svih admitancija koje dodiruju čvorište Npr. 1,1 $1.25-j3.75+5-j15$

3,3 1,3 + 2,3 + 3,4 iz tablice 2

Vandijagonalni elementi: Zapisujemo ih kakvi jesu, ali obrnutog predznaka

$Y_{⑤}$ singularna UVIJEK i ZAUVIJEK!

6.25 - j18.75	-5 + j15	-1.25 + j3.75	0	0
-5 + j15	10.83 - j32.5	-1.66 + j5	-1.66 + j5	-2.5 + j7.5
-1.25 + j3.75	-1,66 + j5	12.916 - j38.75	-10 + j30	0
0	-1,66 + j5	-10 + j30	12.916 - j38.75	-1.25 + j3.75
0	-2.5 + j7.5	0	-1.25 + j3.75	3.75 - j11.25

Čvorište 1 je referentno, pa odbacujemo 1. stupac i 1. red.

$Z = Y^{-1} =$

0,016857 + j0.050571	0,012571 + j0.03771	0,013428 + j0,0402857	0,0151743 + j0,057143
0,012571 + j0,03771	0,0297143 + j0,089143	0.0262857 + j0.0788571	0,017143 + j0,0514286
0,013428 + j0,0402857	0,0262857 + j0,0788571	0,0317143 + j0,08514	0,0185238 + j0,0585715
0,0151743 + j0,057143	0,017143 + j0,0514286	0,0185238 + j0,0585714	0,043654 + j0,1308521

→ Dijagonalni su u Z matrici uvijek najveći

Imaginarni dio = 3× realni dio

Svi su pozitivni → naponi će biti pozitivni

$\boxed{S_B = 100 \text{ MVA}}$

G − T
40 − 20 = 20

G − T
30 − 10 = 20

$\frac{20}{S_B} = 0.2$

tablica 1,

→ računamo struje:

$$I_2^{(0)} = \frac{S_2^*}{(U_2^{(0)})^*} - y_{20}^* \cdot U_2^{(0)} = \frac{\textcircled{0.2} - j\textcircled{0.2}}{1} - j0.085 \cdot 1 = 0.2 - j0.285$$

$$I_3^{(0)} = \frac{S_3^*}{(U_3^{(0)})^*} - y_{30}^* \cdot U_3^{(0)} = \frac{-0.45 + j0.15}{1 - j0} - j0.055 \cdot 1 = -0.45 + j0.095$$

$$I_4^{(0)} = \frac{-0.4 + j0.05}{1 - j0} - j0.055 \cdot 1 = -0.4 - j0.005$$

$$I_5^{(0)} = \frac{-0.6 + j0.1}{1} - j0.04 \cdot 1 = -0.6 + j0.06$$

$$U_2^{(1)} = U_1 + Z_{2,1} \cdot I_1^{(0)} + Z_{2,2} \cdot I_2^{(0)} + Z_{2,3} I_3^{(0)} + Z_{2,4} \cdot I_4^{(0)} + Z_{2,5} \cdot I_5^{(0)} = 1.06 + (0.016857 + j0.050571) \cdot$$
$$\cdot (0.2 - j0.285) + (0.012571 + j0.03771)(-0.45 + j0.095) + (0.013428 + j0.0402857)(-0.4 - j0.005) +$$

$$+(0.0151743+j0.047148)(-0.6+j0.06)=1.05112-j0.05399$$

$$\Delta U_2 = U_2^{(1)} - U_2^{(0)} = 0.00888 - j0.05391$$ "nismo ni blizu rješenja"

$$I_2^{(1)} = \frac{S_2^*}{(U_2^{(1)})^*} - y_{20} \cdot U_2^{(1)} = \frac{0.2-j0.2}{1.05112-j0.05399} - j0.085 \cdot (1.05112+j0.05399) =$$

$$= 0.17544 - j0.208887$$

$$U_3^{(1)} = U_1 + Z_{3,2} I_3^{(0)} + Z_{3,4} I_4^{(0)} + Z_{3,5} I_5^{(0)} = 1.02777 - j0.09581$$

$$I_3^{(1)} = \frac{-0.45+j0.15}{1.02777-0.09581} - j0.0055(1.02777-j0.09581) = -0.42585 - j0.12813$$

$$U_4^{(1)} = U_1 + Z_{4,2} I_2^{(1)} + Z_{4,3} I_3^{(1)} + Z_{4,4} I_4^{(1)} + Z_{4,5} I_5^{(1)} = 1.02521 - j0.0992$$

$$I_4^{(1)} = \frac{-0.4+j0.05}{1.02521+j0.0992} - j0.055(1.02521-j0.0992) = -0.38782+j0.02933$$

$$U_5^{(1)} = U_1 + Z_{5,2} I_2^{(1)} + Z_{5,3} I_3^{(1)} + Z_{5,4} \cdot I_4^{(1)} + Z_{5,5} I_5^{(1)} = 1.01913 - j0.11403$$

$$I_5^{(1)} = -0.57518 + j0.1212$$

Idemo na 2. iteraciju

$$U_2^{(2)} = U_1 + Z_{2,2} I_2^{(1)} + Z_{2,3} I_3^{(1)} + Z_{2,4} I_4^{(1)} + Z_{2,5} I_5^{(1)}$$

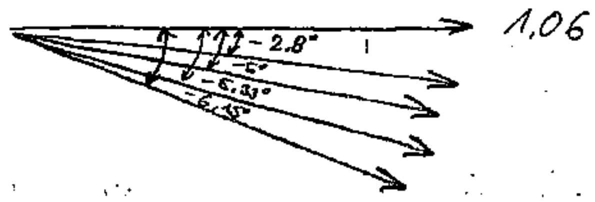
Iteracija	2	3	4	5
0	1+j0	1+j0	1+j0	1+j0
1	1,05112-j0,05399	1,02777-j0,09581	1,02529-j0,0992	1,01913-j0,11403
2	1,09622-j0,05286	1,02041-j0,08837	1,01824-j0,09455	1,0122-j0,10841
3	1,04622-j0,05129	1,02035-j0,09924	1,01818-j0,08502	1,01212-j0,10908
	$1{,}04748\angle -2.8^\circ$	$1{,}02425\angle -5^\circ$	$1{,}02361\angle -5.33^\circ$	$1{,}018\angle -6.45^\circ$

* Tek nam 3. iteracija ispunjava našu točnost 0.001 ÷ 0.0001

Da smo dobili pozitivan kut značilo bi da smo zjebali...

* Iznos napona je mjerilo za Q, a kut je mjerilo za P

Glavnu naponsku kvalitetu diktira napon u referentnom čvorištu



PRORAČUN TOKOVA SNAGA UZ POMOĆ Y MATRICE

$$\boxed{[\bar{I}] = [\bar{Y}] \cdot [\bar{U}]}$$ → sadrži sve napone

→ singularna matrica

$I_1 = Y_{11} \cdot U_1 + Y_{12} U_2 + \dots + Y_{1n} U_n$

$\vdots$

$I_{n-1} = Y_{n-1,1} \cdot U_1 + Y_{n-1,2} \cdot U_2 + \dots + Y_{n-1,n} U_n$

prebacimo U-ove

$$U_1 = \frac{1}{Y_{11}} \left[I_1 - Y_{12} U_2 - Y_{13} U_3 - \dots - Y_{1n} U_n \right]$$

$$U_2 = \frac{1}{Y_{22}} \left[I_2 - Y_{21} U_1 - Y_{23} U_3 - \dots - Y_{2n} U_n \right]$$

$$\vdots$$

$$U_n = \frac{1}{Y_{nn}} \left[I_n - Y_{n1} U_1 - \dots \dots Y_{n,n-1} U_{n-1} \right]$$

$m = n-1$

r – referentno čvorište

Hebel je odabro da je referentno čvorište n ???

U drugoj jednadžbi fali referentna točka. To smo šatro htjeli

$$\boxed{U_p = \frac{1}{Y_{pp}} \left[I_p - \sum_{\substack{i=1 \\ i \neq p}}^{n} Y_{pi} \cdot U_i \right]}$$ $p = 1, \dots, n \quad p \neq ref$

$$I_p = \frac{S_p^*}{U_p^*} = \frac{P_p - jQ_p}{U_p^*}$$

$$\boxed{U_p = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{U_p^*} - \sum_{\substack{i=1 \\ i \neq p}}^{n} Y_{pi} U_i \right]}$$ $p = 1, \dots n \quad p \neq ref$

→ napon iz neke druge iteracije

$$\boxed{\frac{P_p - jQ_p}{Y_{pp}} = KL_p}$$ konstantno za vrijeme iteracija $$\boxed{\frac{Y_{pi}}{Y_{pp}} = YL_{pi}}$$

$$\boxed{U_p = KL_p \cdot \frac{1}{U_p^*} - \sum_{\substack{i=1 \\ i\neq p}}^{n} \frac{Y_{pi}}{Y_{pp}} \cdot U_i}$$

$$U_1^{k+1} = \frac{KL_1}{(U_1^k)^*} - YL_{12} \cdot U_2^k - YL_{13} U_3^k - \ldots\ldots - YL_{1n} U_n^k$$

$$U_2^{k+1} = \frac{KL_2}{(U_2^k)^*} - YL_{21} \cdot U_1^k - YL_{23} U_3^k - \ldots\ldots - YL_{2,n} U_n^k$$

$$\vdots$$

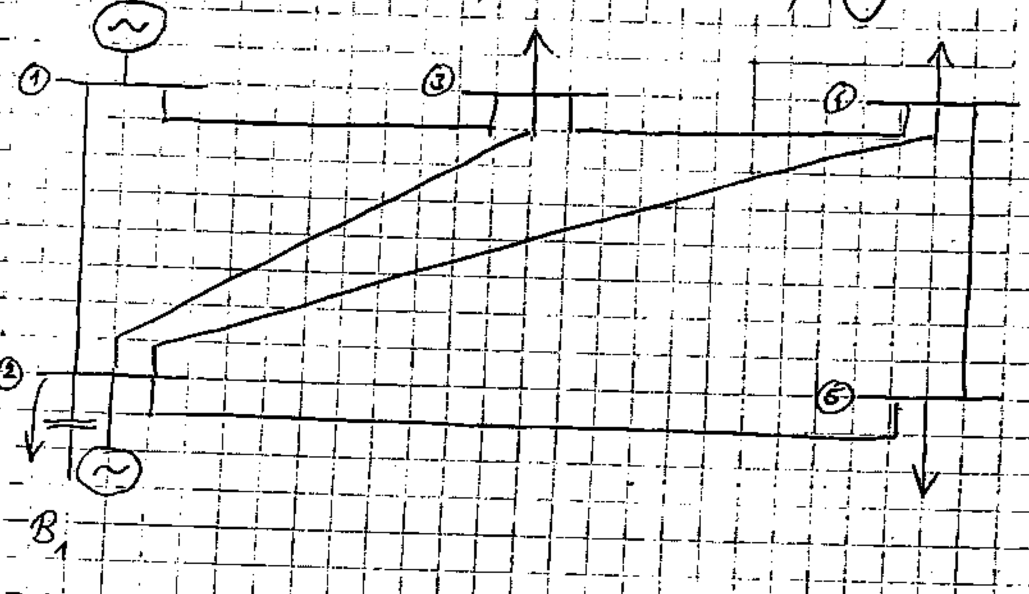
$$U_p^{k+1} = \frac{KL_p}{(U_p^k)^*} - YL_{p1} U_1^{k+1} - \sum_{i=1}^{p-1} Y_{pi} U_i^{k+1} - \ldots\ldots - \sum_{i=p+1}^{n} Y_{pi} U_i^k$$

$$U_n^{k+1} = \frac{KL_n}{(U_n^k)^*} - \sum_{i=1}^{n-1} Y_{ni} U_i^{k+1}$$

→ prednost ove metode je u tome što Y matrica ima, neovisno o broju čvorišta, relativno malo pribrojnika, a sve ostalo su nule. Nasuprot tome, Z matrica ima milijun * milijun elemenata

→ Nedostatak je sedammilijunačetiristošesdesettisućatristadvadesetčetiri iteracije

Po 100. put idemo opet na naš primjer:



Samo je jedan Mali Ivica!

B1

$j0.055$

$\boxed{j0.085}$

$j0.055$

$j0.055$

$j0.04$

$$Y=\begin{array}{c|c|c|c|c} & 2 & 3 & 4 & 5 \\ \hline 2 & 10.883-j32.415 & -1.66+j5 & -1.66+j5 & -2.5+j7.5 \\ 3 & -1,66+j5 & 12,916-j38.695 & 10+j30 & 0 \\ 4 & -1.66+j5 & 10.+j30 & 12,916-j38.69 & -1.25+j3.75 \\ 5 & -2.5+j7.5 & 0 & -1.25+j3.75 & 3.75-j11.21 \end{array}$$

$$\begin{array}{r} 15 \\ +\ 5 \\ +\ 5 \\ +\ 7.5 \\ \hline -j32.5 \\ +j0.085 \\ \hline -j32.415 \end{array}$$

$$\boxed{\begin{array}{l} Y_{ij} = -y_{ij} \\ Y_{ii} = \sum_{j=1}^{n} y_{ij} + B_i \end{array}}$$

sa odgovarajućim predznacima

$$KL_2 = \frac{P_2 - jQ_2}{Y_{22}} = \frac{0.2 - j0.2}{10.83 - j32.415} \rightarrow \text{PU vrijednosti} = 1.0074 + j0.0037$$

$$KL_3 = \frac{P_3 - jQ_3}{Y_{33}} = \frac{-0.45 + j0.15}{12.916 - j38.695} = -0.00698 - j0.0093$$

$$KL_4 = \frac{P_4 - jQ_4}{Y_{44}} = \frac{-0.4 + j0.05}{12.916 - j38.695} = -0.00427 - j0.00891$$

$$KL_5 = \frac{P_5 - jQ_5}{Y_{55}} = \frac{-0.6 + j0.1}{3.75 - j11.21} = -0.02413 - j0.03545$$

* tablica iz koje se čupaju vrijednosti za P i Q (to je već zadano prije)

	P (MW)	Q (MVAr)
2	40-20	30-10
3	-45	-15
4	-40	-5
5	-60	-10

$$YL_{2,1} = \frac{Y_{21}}{Y_{22}} = \frac{-5+j15}{10,8\dot{3}-j32,415} = -0,4623+j0.00036$$

$$YL_{23} = \frac{Y_{23}}{Y_{22}} = \frac{-1,66+j15}{10,8\dot{3}-j32,415} = -0,15421+j0.00012$$

$$YL_{24} = \frac{Y_{24}}{Y_{22}} = \frac{-1,66+j15}{10,83-j32,415} = -0,15421+j0.00012$$

$$YL_{25} = \frac{Y_{25}}{Y_{22}} = \frac{-2,5+j7,5}{10,8\dot{3}-j32,415} = -0,23131+j0.00018$$

$\searrow$ Podaci u proširenoj tablici:

-5+j15	$10,83\dot{3}-j32,415$	-1,66+j5	
-1,125+j3.75	-1,66+j5		
0			
0		0	

$$YL_{3,1} = \frac{Y_{3,1}}{Y_{3,3}} = \frac{-1.25+j\,3.75}{12.816-j\,38.685} = -0.0969+j\,0.00004$$

$$YL_{3,2} = \frac{Y_{3,2}}{Y_{3,3}} = -0.12920+j\,0.00006$$

$$YL_{3,4} = \frac{Y_{3,4}}{Y_{3,3}} = -0.77518+j\,0.00033$$

$$YL_{4,2} = \frac{Y_{4,2}}{Y_{4,4}} = -0.1292+j\,0.00006$$

$$YL_{4,3} = \frac{Y_{4,3}}{Y_{4,4}} = -0.77518+j\,0.0003$$

$$YL_{4,5} = \frac{Y_{4,5}}{Y_{44}} = -0.0969+j\,0.00004$$

$$YL_{5,2} = \frac{Y_{5,2}}{Y_{55}} = -0.66881+j\,0.00072$$

$$YL_{5,4} = \frac{Y_{5,4}}{Y_{55}} = -0.3344+j\,0.00036$$

Glavna pretpostavka

$$\boxed{\begin{array}{ll} U_i^0 = 1+j\,0 & i=2,\dots,5 \\ U_1 = 1.06+j\,0 & \text{UVIJEK} \end{array}}$$

$$\boxed{\frac{KL_2}{(U_2)^*} = \frac{P_2-j\,Q_2}{Y_{2,2}\cdot U_2^*}}$$

$$U_2^1 = \frac{KL_2}{(U_2^0)^*} - YL_{2,1}\cdot U_1 - YL_{2,3}\,U_3^0 - YL_{2,4}\cdot U_4^0 - YL_{2,5}\cdot U_5^0$$

$$= \frac{0.0074+j\,0.0037}{1-j\,0} - (-0.46263+j\,0.00036)\cdot 1.06 - (-0.1514+j\,0.00012)(1+j\,0) -$$

$$-(-0.15421+j\,0.00012)(1+j\,0) - (-0.23131+j\,0.00018)(1+j\,0) =$$

$$= 1.03752+j\,0.00290$$

→ Već ga imamo, pa zašto ne... U_1 je referentno i isto za sve iteracije

$$U_3^1 = \frac{KL_3}{(U_3^0)^*} - YL_{3,1}\cdot U_1 - YL_{3,2}\cdot U_2^1 - YL_{3,4}\cdot U_4^0 =$$

$$= 1.0069-j\,0.00921$$

Ovo sve skupa mi je slabo konvergentno, Hebel je nestrpljiv, pa ćemo gurnut nekakav FAKTOR UBRZANJA

$$\Delta U_2 = U_2^{(1)} - U_2^{(0)} = 1.03752 + j0.0029 - (1 + j0) = 0.03752 + j0.0029$$

$$* \ \Delta U_{2ubr}^{(1)} = \Delta U_2 \cdot \alpha$$

α može bit različit za realni i imaginarni dio...

$(1.2 \div 2)$ mi uzmemo 1.4

$$\Delta U_{2ubr} = 0.0375 \cdot 1.4 + j0.0029 \cdot 1.4 = 0.05253 + j0.00406$$

$$U_{2ubr}^{1} = U_2^{0} + \Delta U_{2ubr} = 1.05253 + j0.00406$$

To sad metnemo u formulu za U_3^1 (Hebel je to, naravno, samo dopisao tam)

$$* \ \Delta U_3^{(1)} = 0.0069 - j0.00921$$

$$U_{3ubr}^{(1)} = 1.4(0.0069 - j0.00921) + U_3^0 = 1.00966 - j0.01289$$

Razlika između susjednih iteracija bi trebala bit ispod 0.001 + j0.001 i to pokušavamo dobit.....

$$* \ U_4^{(1)} = \frac{KL_4}{(U_4^0)^*} - YL_{4,2} \cdot U_{2ubr}^{(1)} - YL_{4,3} \cdot U_{3ubr}^{(1)} - YL_{4,5} \cdot U_5^0 = 1.01128 - j0.01881$$

$$\Delta U_4^{(1)} = 0.01128 - j0.01881 \Longrightarrow U_{4ubr}^{(1)} = 1.01579 - j0.02635$$

$$* \ U_5^{(1)} = \frac{KL_5}{(U_5^0)^*} - YL_{5,2} \cdot U_{2ubr}^{(1)} - YL_{5,4} \cdot U_{4ubr}^{(1)} = 1.01949 - j0.05267$$

$$U_{5ubr}^{(1)} = 1.4(0.01949 - j0.05267) + (1 + j0) = 1.02728 - j0.07374$$

2. iteracija:

$$U_2^{(2)} = \frac{KL_2}{\left(U_2^{(1)}\right)^*} - YL_{2,1} \cdot U_1 - YL_{2,3} \cdot U_{3ubr}^{(1)} - YL_{2,4} \cdot U_{4ubrz}^{(1)} - YL_{2,5} \cdot U_{5ubr}^{(1)}$$

$\Delta U_2^{(2)} = U_2^{(2)} - U_2^{(1)} =$ pa provjeravamo točnost i riješimo α

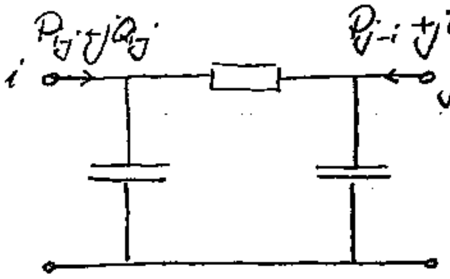
$$U_{2ubr}^{(2)} = \Delta U_2^{(2)} \cdot \alpha + U_2^{(1)}$$

$$U_3^{(2)} = \frac{KL_3}{\left(U_3^{(1)}\right)^*} - YL_{3,1} U_1 - YL_{3,2} U_{2ubr}^{(2)} - YL_{3,4} \cdot U_4^{(1)}$$

	ČVORIŠTA			
ITERACIJE	2	3	4	5
1	0,05253 +j'0,00406 1,05253 +j'0,00406	1,00866 -j0,01289	1,01579 -j'0,02635	1,02727 -j'0,07374
2	-0,00724 -j' 1,04528 -j'0,03015	1,02454 -j'0,04227	1,02451 -j'0,06353	1,01025 -j0,08032
3	0,00204 -j 0,00603 1,04732 -j'0,03618.			
4	1,04964 -j'0,0473			
5	1,04749 -j'0,05016			
6	1,04708 -j0,05057			
7	1,04676 -j0,05127			
⋮	⋮			
10	-0,00007 -j'0,00003 1,04623 -j0,05126	0,00012 -j 0,00001 1,02036 -j'0,08917	-0,00007 -j'0,00002 1,0192 -j'0,09504	-0,00008 -j'0,00000 1,01211 -j0,10904

Usporedimo sa tablicom u Gauss-Seidelu i zadovoljavamo točnost (ali! tu je 10 iteracija, a tamo je bilo 3) mogli smo uzeti drugi α

Sad smo gotovi s tih sedamstomilijardi iteracija, i pišemo dalje, al Habel se zapričo, pa nije reko naslov...



$$P_{ij}+jQ_{ij}=U_i\left(U_i^*-U_j^*\right)\cdot y_{ij}^* + \underbrace{U_i\cdot U_i^*}_{|U_i|^2}\cdot\frac{y_{i-j}^*}{2}$$

$$\rightarrow P_{1-2}+jQ_{1-2}=U_1\left(U_1^*-U_2^*\right)y_{1-2}^*+U_1\cdot U_1^*\cdot\frac{y_{1-2}^*}{2}=$$

$$=1.06(1.06-1.04623-j0.05126)(5+j15)+1.06^2\cdot(-j0.03)=$$

$$=0,888-j0.086\longrightarrow 88.8-j8,6\text{ MVA}$$

jer je $S_B=100\text{MVA}$

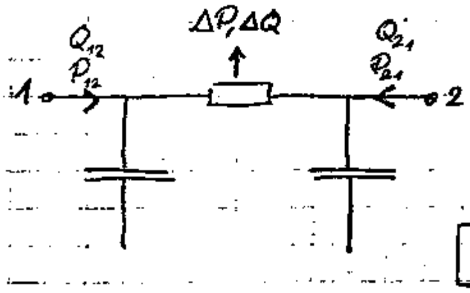
$$\rightarrow P_{2-1}+jQ_{2-1}=U_2\left(U_2^*-U_1^*\right)y_{1-2}^*+|U_2|^2\cdot\frac{y_{1-2}^*}{2}=$$

$$=(1.04623-j0.05126)(1.04623+j0.05126-1.06)(5+j15)+1.0478^2(-j0.03)$$

$$=-0,874+j0,062\longrightarrow -87,4+j6,2\text{ MVA}$$

$$\Delta P_{1-2}+j\Delta Q_{1-2}=P_{12}+P_{21}+j\left(Q_{12}+Q_{21}\right)=1,4\text{MW}-2,4\text{ MVAr}$$

$\boxed{\Delta P\text{ mora bit pozitivan}}$

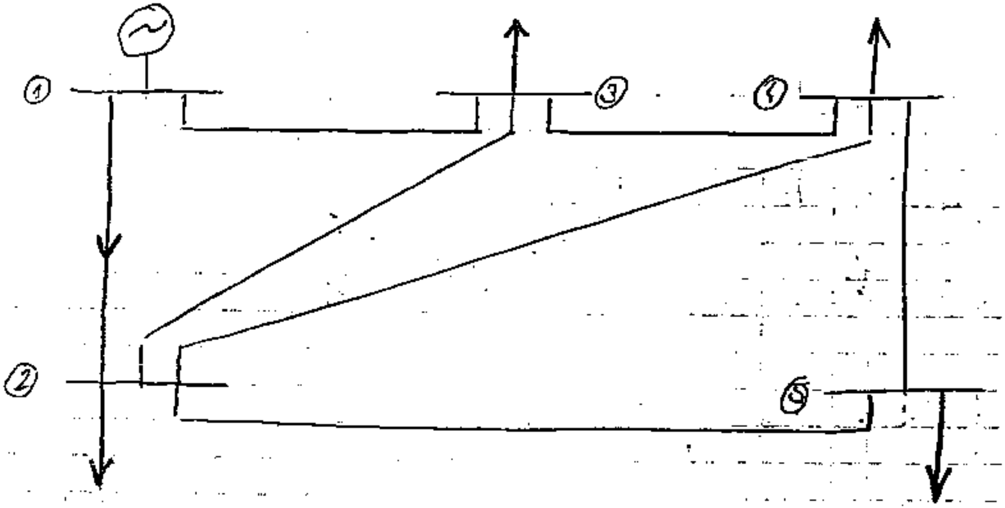


← Zato se zbrajaju...

$$\boxed{P_{i-j} + jQ_{i-j} = U_i\left(U_i^{*} - U_j^{*}\right) y_{ij}^{*} + U_i^{2}\, y_{i0}^{*}}$$

$$\boxed{P_{j-i} + jQ_{j-i} = U_j\left(U_j^{*} - U_i^{*}\right) y_{j-i}^{*} + U_j^{2}\, y_{j0}^{*}}$$

Zbrojeno daje $\Delta P + j\Delta Q$



Onda izračunamo sve tokove snage (2-3, 3-2, 3-4, 4-3, 4-5, 5-4, 2-5, 5-2 1-3, 3-1)

VOD	P [MW]	Q [MVAr]	ΔP [MW]	ΔQ [MVAr]
1-2	88,8	-8,6	1,4	-2,4
2-1	-87,4	6,2		
1-3	40,7	1,1	1,2	-1,9
3-1	-38,5	-3		
2-3	24,7	3,5	0,4	-3,3
3-2	-24,3	-6,8		
2-4	27,9	30	0,4	-2,9
4-2	-27,5	5,9		
2-5	54,8	7,4	1,1	0,2
5-2	-53,7	-7,2		
3-4	18,9	-5,1	~ 0	-1,9
4-3	-18,9	3,2		
4-5	6,3	-2,3	~0	-5,1
5-4	-6,3	-2,8		

PROIZVODI JALOVU SNAGU (OPTEREĆEN ISPOD PRIRODNE SNAGE)

KOD DJELATNE SNAGE POZITIVNA MORA BIT VEĆA OD NEGATIVNE (GUBICI)

→ Sad još moramo izračunati snagu u našoj referentnoj elektrani (kod nas generator u ① čvoru)
Mora pokriti razliku snaga između elektrane i potrošača, te gubitke u vodovima:

$$\boxed{P_1 + jQ_1 = -\sum_{i=2}^{5} P_i + j - \sum_{i=2}^{5} Q_i + \Delta P + \Delta Q} =$$

$$= -40 + 20 + \underset{\textcircled{3}}{45} + \underset{\textcircled{1}}{40} + \underset{\textcircled{5}}{60} + j(-30 + 10 + 15 + 5 + 10) =$$

jer je generator, a s ove strane jednadžbe su oni negativni

$$= \underbrace{125 + j10} + \underbrace{4.5 - j17.3}$$

iz tablice s vodovima

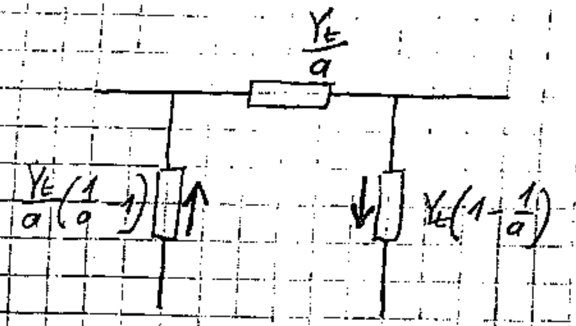
→ to smo sve izvukli iz tablice "Zadano"

$$= (129.5 - j7.3) \text{ MVA}$$

*Ako oduzmemo djelatne i jalove iz prethodne tablice (prva dva reda)....

$$\begin{array}{r} 88.8 \\ +40.7 \\ \hline 129.5 \end{array} \qquad \begin{array}{r} -8.6 \\ +1.1 \\ \hline -7.5 \end{array}$$

Malo krivo, al jebiga...

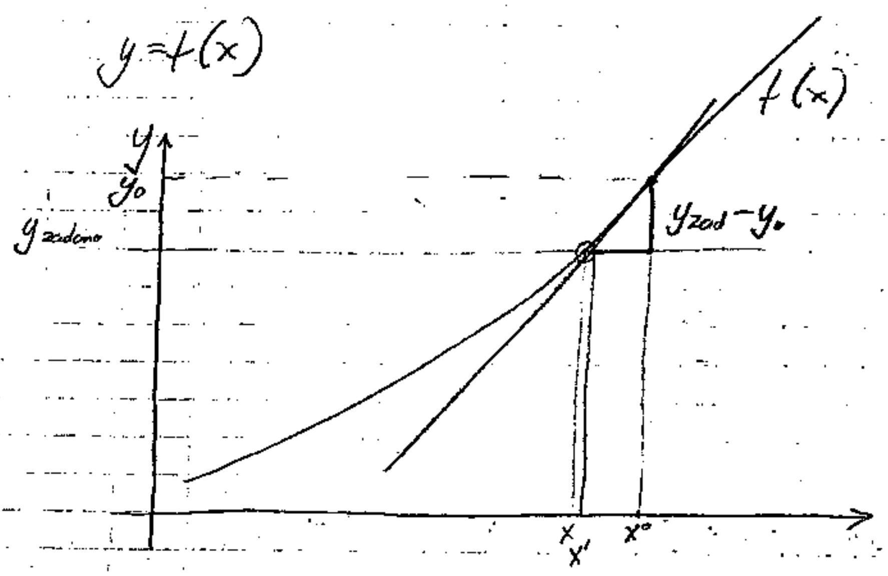


Samo vodovi mogu imati gubitke Q manje od nule ("proizvodi induktivnu snagu")
Kada je Q veći od nule onda je to kapacitivno ("Troši induktivnu snagu")

NEWTON-RAPHSON-ova METODA

Jecabena metoda

$y=f(x)$



I teško nam je odredit taj pripadajući x, pa pljusnemo neki približni x_0

$\left.\frac{\partial f}{\partial x}\right|_{x_0}$ tangenta (tanga)

$$\left(x'-x^{\circ}\right)\left.\frac{\partial F}{\partial x}\right|_{x_0}=y^{\circ}-y_{zad}$$

$$x'-x^{\circ}=\left(\left.\frac{\partial f}{\partial x}\right|_{x_0}\right)^{-1}\left(y^{\circ}-y_{zad}\right)$$

$$\boxed{x'=x^{\circ}+\left(\left.\frac{\partial f}{\partial x}\right|_{x_0}\right)^{-1}\left(y^{\circ}-y_{zad}\right)}$$

→ I tak se mi tangentama približavamo pravom rješenju i to je dosta konvergentno (uf, kak volim tu riječ).

Mi ćemo samo imat funkcije više varijabli, al to nije bed...

Jednadžbe snaga:

$$\boxed{\begin{aligned} P_i &= U_i \cdot \sum_{j=1}^{n} U_j Y_{ij} \cos(\delta_i - \delta_j - \Theta_{ij}) \\ Q_i &= U_i \cdot \sum_{j=1}^{n} U_j \cdot Y_{ij} \sin(\delta_i - \delta_j - \Theta_{ij}) \end{aligned}} \qquad i = 1, \dots, n-1$$

- ovo su sve iznosi napona i admitancija $(|U_i|, |Y_{ij}|)$

$$P_i = \dots = U_i^2 \cdot Y_{ii} \cdot \cos(\Theta_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^{n} U_i \cdot U_j \cdot Y_{ij} \cos(\delta_i - \delta_j - \Theta_{ij})$$

$$Q_i = \dots = -U_i^2 \cdot Y_{ii} \sin(\Theta_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^{n} U_i \cdot U_j \cdot Y_{ij} \sin(\delta_i - \delta_j - \Theta_{ij})$$

- Za referentno čvorište nema ovih formula...

* Broj snaga = $n-1$ (bez referentnog)

* Broj jalovih snaga = $n-g-1$ (bez referentnog i generatorskih

Mi tražimo $U_i \angle \delta_i$ $\quad U_i \rightarrow 1 \dots (n-g-1)$

$\delta_i \rightarrow 1 \dots (n-1)$

- jer u generatorskim čvorištima imamo poznat napon
- dakle imamo $\boxed{n-1+n-g-1 = 2n-2-g}$ čvorišta

Kak ćemo započet?

Izmislimo početni X_0.... Isto ko i u Gauss-Seidelu

Postupak:

→ ① Zadamo X^0 $\quad U_i^0 \angle \delta_i^0 = 1 \angle 0 \quad i = 1 \dots n-g-1$

$|U_i| = |U_g|$ za generatorska čvorišta

$\delta_g = 0$

I sad idemo s formulama za P_i i Q_i u 0. iteraciji

- $P_i^0 = f(U_i^0; \delta_i^0)$ * f je cos $\quad i = 1 \dots n-1$
- $Q_i^0 = f(U_i^0; \delta_i^0)$ * f je sin $\quad i = 1 \dots n-g-1$

$P_{i,zad} - P_i^0 = \Delta P^0$

$$
\begin{array}{c|c|cccc|cccc|c}
 & & 1 & \longrightarrow & & n-1 & 1 & \longrightarrow & & n-1-g & \\
\hline
\Delta P_1 & 1 & \frac{\partial P_1}{\partial \delta_1} & \frac{\partial P_1}{\partial \delta_2} & \cdots & \frac{\partial P_1}{\partial \delta_{n-1}} & \frac{\partial P_1}{\partial U_1} & \frac{\partial P_1}{\partial U_2} & \cdots & \frac{\partial P_1}{\partial U_{n-1}} & \Delta\delta_1 \\
\Delta P_2 & \downarrow & \frac{\partial P_2}{\partial \delta_1} & \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_{n-1}} & \frac{\partial P_2}{\partial U_1} & \frac{\partial P_2}{\partial U_2} & \cdots & \frac{\partial P_2}{\partial U_{n-1}} & \Delta\delta_2 \\
\vdots & & \vdots & & & \vdots & \vdots & & & \vdots & \\
\Delta P_{n-1} & n-1 & \frac{\partial P_{n-1}}{\partial \delta_1} & \frac{\partial P_{n-1}}{\partial \delta_2} & \cdots & \frac{\partial P_{n-1}}{\partial \delta_{n-1}} & \frac{\partial P_{n-1}}{\partial U_1} & \frac{\partial P_{n-1}}{\partial U_2} & \cdots & \frac{\partial P_{n-1}}{\partial U_{n-1}} & \Delta\delta_{n-1} \\
\hline
\Delta Q_1 & = \; 1 & \frac{\partial Q_1}{\partial \delta_1} & \frac{\partial Q_1}{\partial \delta_2} & \cdots & \frac{\partial Q_1}{\partial \delta_{n-1}} & \frac{\partial Q_1}{\partial U_1} & \frac{\partial Q_1}{\partial U_2} & \cdots & \frac{\partial Q_2}{\partial U_{n-1-g}} & \Delta U_1 \\
\Delta Q_2 & \downarrow & \vdots & & & \vdots & \vdots & & & \vdots & \Delta U_2 \\
\vdots & & & & & & & & & & \\
\Delta Q_{n-1-g} & n-1-g & \frac{\partial Q_{n-1-g}}{\partial \delta_1} & & \cdots & \frac{\partial Q_{n-1-g}}{\partial \delta_{n-1-g}} & \frac{\partial Q_{n-1-g}}{\partial U_1} & & \cdots & \frac{\partial Q_{n-1-g}}{\partial U_{n-1-g}} & \Delta U_{n-1-g}
\end{array}
$$

$$\Delta X \cdot \frac{\partial f(x)}{\partial x}\bigg|_{x^0} = \Delta P$$

Ova derivacija je, u stari, Jacobijeva matrica

$$\left(\frac{\partial f(x)}{\partial x}\right) \cdot \Delta x = \Delta y$$

→ ② Izračunaj P_i^0, Q_i^0

→ ③ Izračunaj ΔP_i^0 i ΔQ_i^0

→ ④ Izračunaj Jacobijevu matricu

$$\underset{(n-1)\times(n-1)}{\frac{\partial P}{\partial \delta}}, \quad \underset{(n-1)\times(n-1-g)}{\frac{\partial P}{\partial U}}, \quad \underset{(n-1-g)\times(n-1)}{\frac{\partial Q}{\partial \delta}}, \quad \underset{(n-1-g)\times(n-1-g)}{\frac{\partial Q}{\Delta U}} \Leftarrow \text{Matrice}$$

$(n-1)\times(n-1)$	$(n-1)$ $\times(n-1-g)$
$(n-1-g)$ $\times(n-1)$	$(n-1-g)$ $\times(n-1-g)$

Kvadratna matrica

!!!

$$\left|\Delta y\right| = \left|M\right| \cdot \left|\Delta x\right|$$

Poznato Poznato Traži se

→ Mi smo u stvari, imaginarne matrice pretvorili u redke (snage podijelimo na P i Q, napon podijelimo na iznos i kut).

Jacobijeva (prazna) → jer su neke admitancije 0

* Dakle, idemo opet malo ponoviti

1. Pretpostavimo $U_i = 1 \quad i = 1 \ldots n-1-g$

$U_k = U_{k\,zad} \quad k = 1 \ldots g$

$\delta_i = 0$ za $i = 1 \ldots n-1$

2. $P_{i_{izr}}^{0} = \sum_{j=1}^{n} U_i^0 U_j^0 Y_{ij} \cos(\delta_i^0 - \delta_j^0 - \Theta_{ij}) \quad i = 1 \ldots n-1$

$Q_{i_{izr}}^{0} = \sum_{j=1}^{n} U_i U_j Y_{ij} \sin(\delta_i - \delta_j - \Theta_{ij}) \quad i = 1 \ldots n-1-g$

U gen. čvorištima nemamo Q, već imamo zadano U i P.

3. Vektor $\Delta P_i = P_{i,zad}^{0} - P_{i,izr}^{0} \quad i = 1 \ldots n-1$

$\Delta Q_i = Q_{i,zad}^{0} - Q_{i,izr}^{0} \quad i = 1 \ldots n-1-g$

4. Postavljamo Jacobijevu matricu (ima ih 4 komada)

$\frac{\partial P}{\partial \delta}, \frac{\partial P}{\partial U}, \frac{\partial Q}{\partial \delta}, \frac{\partial Q}{\partial U}$ 4 sub matrice

Derivacije je brzina promjene (snage o kutu i naponu).

$J_1 = \frac{\partial P}{\partial \delta} \longrightarrow \frac{\partial P_i}{\partial \delta_i}$ dijagonalni elementi te sub-matrice (tu su promjene najveće i najvažnija

$$P_i = U_i^2 Y_{ii} \cos(-\theta_{ij}) + \sum_{\substack{j=1 \\ j\neq i}}^{n} U_i U_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$

↳ konstanta, tak da nam taj dio nije funkcija δ_i pa je to NULA

$$\frac{\partial P_i}{\partial \delta_i} = -\sum_{\substack{j=1 \\ j\neq i}}^{n} U_i U_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$

dijagonalni elementi → suma svih vandijagonalnih

$$\frac{\partial P_i}{\partial \delta_j} = U_i U_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$

nama je funkcija samo našeg j, ostali u sumi onda daju 0 (derivacija konstante)

${}_2 = \frac{\partial P}{\partial U} \longrightarrow \frac{\partial P_i}{\partial U_i}$

$$\frac{\partial P_i}{\partial U_i} = 2U_i Y_{ii} \cos(-\theta_{ii}) + \sum_{\substack{j=1 \\ j\neq i}}^{n} U_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$

$$\frac{\partial P_i}{\partial U_j} = U_i Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$

${}_3 = \frac{\partial Q}{\partial \delta} \longrightarrow \frac{\partial Q_i}{\partial \delta_i}$

$$Q_i = U_i^2 Y_{ii} \sin(-\theta_{ii}) + \sum_{\substack{j=n \\ j\neq i}}^{n} U_i U_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j=1}^{n-1} U_i U_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \qquad i=1 \ldots n-1-g$$

$$\frac{\partial Q_i}{\partial \delta_j} = -U_i U_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \qquad i=1, \ldots n-1-g$$

$j=1, \ldots n-1$

- $\frac{\partial Q}{\partial U} \longrightarrow \frac{\partial Q_i}{\partial U_i}$.

$$\frac{\partial Q_i}{\partial U_i} = 2U_i Y_{i,i} \sin(-\Theta_{i,i}) + \sum_{j=1}^{n-1} U_j Y_{ij} \sin(\delta_i - \delta_j - \Theta_{ij}) \qquad i=1 \dots n-1-g$$

$$\frac{\partial Q_i}{\partial U_j} = U_i Y_{ij} \sin(\delta_i - \delta_j - \Theta_{ij})$$

$J_1 = \dots \sin$

$J_2 = \dots \cos$

→ gdje god je cos, Θ_{ij} je element matrice admitancije (oko 90°), a δ_i i δ_j su mali i približno su isti, pa im je razlika skoro 0. Zato, $\cos(70°) \approx$ mali, a $\sin(70°) \approx$ veći

Zato, da bi ubrzali proračun: $\cos\Theta_{ij}$ i $\sin\Theta_{ij}$

pa dobijeno:

$$\begin{array}{|c|c|} \hline J_1 & \emptyset \\ \hline \emptyset & J_4 \\ \hline \end{array} \qquad \boxed{\begin{array}{l} \delta_i - \delta_j \cong \emptyset \\ \delta_i - \delta_j - \Theta_{ij} \cong -\Theta_{ij} \end{array}}$$

$$\begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix} = \begin{array}{|c|c|} \hline J_1 & / \\ \hline / & J_4 \\ \hline \end{array} \cdot \begin{vmatrix} \Delta\delta_1 \\ \vdots \\ \Delta\delta_{n-1} \\ \Delta U_1 \\ \vdots \\ \Delta U_{n-1-g} \end{vmatrix}$$

(J nije stvja već matrica)

$$\Delta P = J_1 \cdot \Delta\delta \longrightarrow \Delta\delta^1 = J_1^{-1} \Delta P^0$$

$$\Delta Q = J_4 \cdot \Delta U \longrightarrow \Delta U^1 = J_4^{-1} \cdot \Delta Q^0$$

6 Sredit napone $U_i^{(1)} = U_i^0 + \Delta U$

$\delta_i^{(1)} = \delta_1^0 + \Delta\delta$

7. Računamo

$$P_{i\,izr}' = f(U_i', \delta_i')$$

$$Q_{i\,izr}' = f(U_i', \delta')$$

$$\Delta P' = P_{zad} - P_{izr}$$

$$\Delta Q' = Q_{zad} - Q_{izr}$$

$\boxed{\text{IF}}$ $\Delta P < \varepsilon$

$\Delta Q < \varepsilon$

$\swarrow$ $\boxed{\text{NO}}$ $\searrow$ $\boxed{\text{YES}}$

Ponavljamo proračun s točkom 4

Završili smo postupak i proračun

→ Habel je nacrto nekaj kaj niko nije skužio, ali uglavnom: da ne računamo na-stop Jacobijevu matricu, zadržimo prvu tangentu i translatiramo ju.....

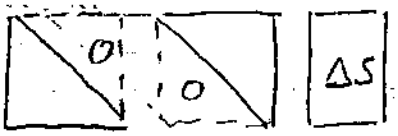
→ Sad dio koji nisam kurca skužio:

$$J \cdot \Delta x = \Delta S$$

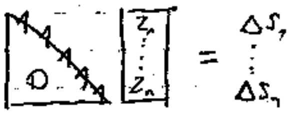
$$D \cdot G \cdot \Delta x = \Delta S$$

D - donja

G - gornja



$$Z_1 = \Delta S_1$$



$$d_{21} \cdot Z_1 + 1 \cdot Z_2 = \Delta S_2 \longrightarrow Z_2 = \Delta S_2 - d_{21} \cdot Z_1$$

$$d_{31} \cdot Z_1 + d_{32} \cdot Z_2 + 1 \cdot Z_3 \longrightarrow Z_3 = \Delta S_3 - d_{32} \cdot Z_2 - d_{31} Z_1$$

$$\boxed{Z_i = \Delta S_i - \sum_{j=1}^{i-1} d_{ij} \cdot Z_j}$$

Supstitucija unaprijed ????

Supstitucija natrag

$$g_{nn} \cdot \Delta x_n = z_n \qquad z_n = \frac{g_{nn}}{\Delta x_n}$$

$$\begin{vmatrix} g_{11} & \cdots & g_{1n} \\ & \ddots & \vdots \\ & & g_{nn} \end{vmatrix} \begin{vmatrix} \Delta x_1 \\ \vdots \\ \Delta x_n \end{vmatrix} = \begin{vmatrix} z_1 \\ \vdots \\ z_n \end{vmatrix}$$

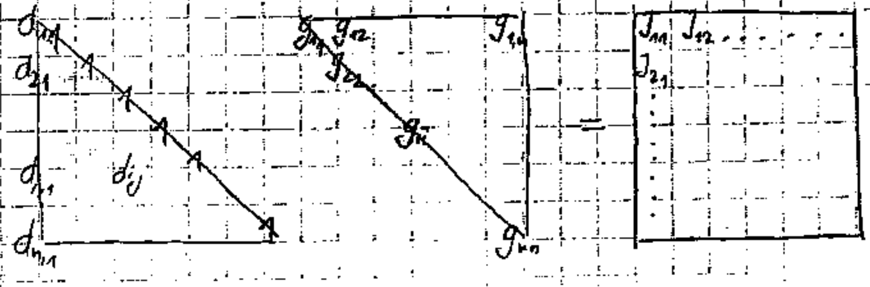
$$G \cdot \Delta x = z$$

$$g_{n-1,n-1} + g_{n-1,n} \cdot \Delta x_n = z_{n-1} \qquad \Delta x_{n-1} = \frac{z_{n-1} - g_{n-1,n} \cdot \Delta x_n}{g_{n-1,n-1}}$$

$$\boxed{\Delta x_i = \frac{z_i - \sum_{j=n}^{i+1} g_{ij} \cdot \Delta x_j}{g_{i,i}}}$$

E, al kak odredit one dvije trokut matrice

$$D \cdot G = J$$



1. $g_{11} = J_{11}$
   $g_{12} = J_{12}$
   $g_{13} = J_{13}$
   $\vdots$
   $g_{1n} = J_{1n}$

2. $J_{21} = d_{21} \cdot g_{11} \rightarrow d_{21} = \frac{J_{21}}{g_{11}}$
   $J_{31} = d_{31} \cdot g_{11} \rightarrow d_{31} = \frac{J_{31}}{g_{11}}$
   $d_{41} = \frac{J_{41}}{g_{11}}$
   $\vdots$
   $d_{n1} = \frac{J_{n1}}{g_{11}}$

3.

20.12.2004

Rekli smo da je ukupno

$$\boxed{n-1+(n-1-g)=2n-2-g}$$

sad računamo element j_{22} koji se dobije množenjem 2. retka donje i 2. stupca gornje:

$$j_{22}=\underbrace{g_{12}\cdot d_{21}}_{\text{to je poznato od maloprije}}+\underbrace{g_{22}}_{\text{ovo se traži}}$$

$$g_{22}=I_{22}-g_{12}\cdot d_{21}$$

$$j_{23}=g_{13}\cdot d_{21}+g_{23}\Longrightarrow\boxed{g_{23}=j_{23}-g_{13}\cdot d_{21}}$$

$$\vdots$$

$$\boxed{g_{2i}=j_{2i}-g_{1i}\cdot d_{21}}$$

Tak smo dobili i drugi redak gornje matrice:

$$\boxed{g_{2n}=j_{2n}-g_{1n}\cdot d_{21}}$$

* Sad tražimo drugi stupac donje matrice:

$$I_{32}=g_{12}\cdot d_{31}+d_{32}\cdot g_{22}\Longrightarrow d_{32}=\frac{I_{32}-g_{12}\cdot d_{31}}{g_{22}}$$

$$d_{42}=\frac{I_{42}-g_{12}\cdot d_{41}}{g_{22}}$$

A općenito za drugi stupac:

$$d_{n,2}=\frac{I_{n,2}-g_{12}\cdot d_{n,1}}{g_{22}}$$

* A sad, 3. redak g matrice

$$j_{33}=g_{13}\cdot d_{31}+g_{23}\cdot d_{32}+g_{33}$$

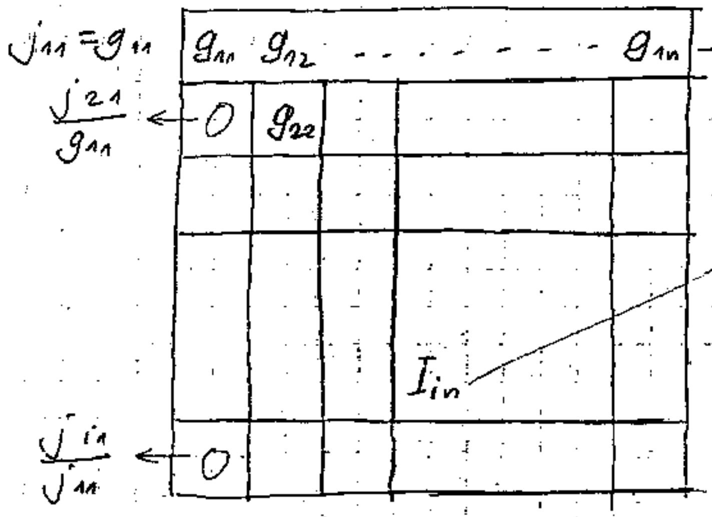
$$\Longrightarrow g_{33}=j_{33}-\sum_{i=1}^{2}g_{i3}\cdot d_{3i}$$

$$g_{34}=j_{34}-\sum_{i=1}^{2}g_{i4}\cdot d_{3i}$$

$$\boxed{g_{3n} = j_{3n} - \sum_{i=1}^{2} g_{in} \cdot d_{3i}}$$

U praksi se ova suma odvija na slijedeći način:

$$g_{3n} = J_{3n} - g_{1n} \cdot \underbrace{d_{31}}_{} - g_{2n} \cdot \underbrace{d_{32}}_{} \longrightarrow \text{već je izračunato}$$



→ 1. redak je isti i samo ga se prepiše.

$$\overset{(1)}{I_{in}} = \overset{(0)}{I_{ik}} - g_{1k} \cdot d_{ki}$$

$i = 2 \dots n$

$k = 2 \dots n$

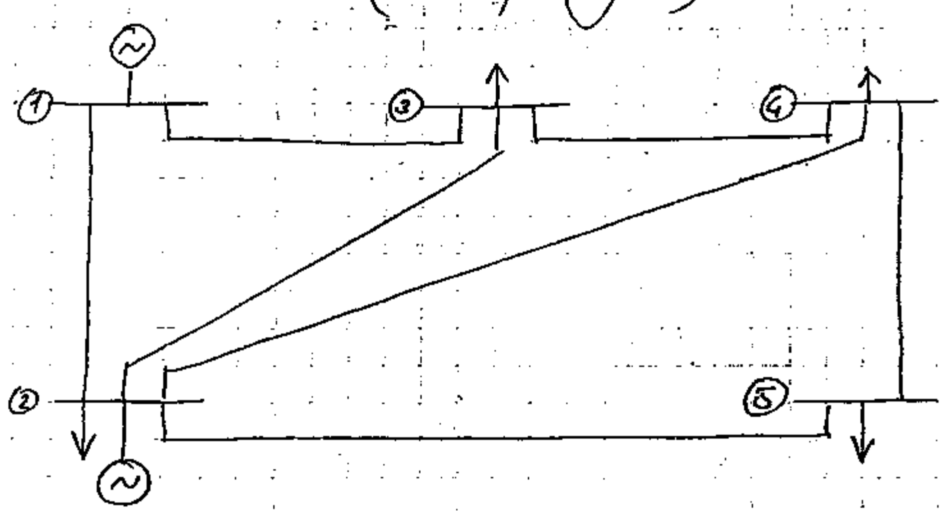
tak se svi preračunaju

To preračunamo pa idemo na drugi redak

→ drugi stupac se analogno prvom samo podijeli sa g_{22}

$$\overset{(2)}{I_{ik}} = \overset{(1)}{I_{ik}} - g_{2k} \cdot d_{k2}$$

N-R metoda (isti primjer):



Podatke imamo od prije, a sad nam treba Y matrica

$$Y = \begin{vmatrix} 6.25-j18.695 & -5+j15 & -1.25+j3.75 & 0 & 0 \\ -5+j15 & 10.83-j32.415 & -1.66+j5 & -1.66+j5 & 2.3+j7.5 \\ -1.25+j3.75 & -1.66+j5 & 12.916-j38.695 & -10+j30 & 0 \\ 0 & -1.66+j5 & -10+j30 & 12.916-j38.695 & -1.25+j3.7 \\ 0 & -2.5+j7.5 & 0 & -1.25+j3.75 & 3.75-j11.21 \end{vmatrix}$$

dodani su kapaciteti u dijagonalnim elementima
(zato se malo promijeni kut)

*To sve je pametno pretvorit u polarni oblik jer se u izrazima za P_i i Q_i traže iznosi i kutevi.

$$Y = \begin{vmatrix} 19.712\angle{-71.5145} & 15.81\angle{108.435} & 3.95\angle{108.435} & 0 & 0 \\ 15.81\angle{108.435} & 34.18\angle{-71.52} & 5.27\angle{108.435} & 5.27\angle{108.435} & 0 \\ 3.95\angle{108.435} & 5.27\angle{108.435} & 40.79\angle{108.435} & 31.62\angle{108.435} & 0 \\ 0 & 5.27\angle{108.435} & 31.62\angle{108.435} & 40.79\angle{-71.54} & 3.95\angle{108.435} \\ 0 & 7.9\angle{108.435} & 0 & 3.95\angle{108.435} & 11.82\angle{-71.52} \end{vmatrix}$$

1.06 i uvijek isto!

0 u 0. iteraciji

$$\begin{aligned} P_{2,izr}^{(0)} &= U_2^{(0)} \cdot U_1 \cdot Y_{21} \cdot \cos(\delta_2 - \delta_2 - \Theta_{21}) + \\ &+ U_2 \cdot U_2 \cdot Y_{22} \cdot \cos(-\Theta_{22}) + U_2 \cdot U_3 \cdot Y_{32} \cdot \cos(\delta_2^0 - \delta_3^0 - \Theta_{32}) + \\ &+ U_2^0 \cdot U_4^0 \cdot Y_{24} \cdot \cos(\delta_2^0 - \delta_4^0 - \Theta_{24}) + U_2^0 \cdot U_5^0 \cdot Y_{25} \cos(\delta_2 - \delta_5^0 - \Theta_{25}) \\ &= 1 \cdot 1{,}06 \cdot 15{,}81 \cdot \cos(-108{,}435) + 1 \cdot 1 \cdot 34{,}18 \cdot \cos(71{,}52) + \\ &+ 1 \cdot 1 \cdot 5{,}27 \cdot \cos(-108{,}435) + 1 \cdot 1 \cdot 5{,}27 \cos(-108{,}435) + \\ &+ 1 \cdot 1 \cdot 7{,}9 \cos(-108{,}435) = \\ &= -5{,}3 + 10{,}833 - 1{,}66 - 1{,}66 - 2{,}5 = -0{,}3 \end{aligned}$$

(ovo su sve djelatni dijelovi Y-matrice)

Analogno se dobije i za jalovu snagu

$$Q_{2,izr}^{(0)} = -15{,}9 + 32{,}417 - 5 - 5 - 7{,}49 = -0{,}985$$

(imaginarni dijelovi 2. stupca)

(P.U.)

Sad imamo snagu u čvorištu 2 u nultoj iteraciji (al to je sve netočno, jer još nemamo točne napone)

21.12.2006.

$P_{2izr}^{0} = -0.3$ $\qquad$ $P_{4izr}^{0} = 0$

$Q_{2izr}^{0} = -0.985$ $\qquad$ $Q_{4izr}^{0} = -0.055$

$P_{3izr}^{0} = -0.075$ $\qquad$ $P_{5izr}^{0} = 0$

$Q_{3izr}^{0} = -0.28$ $\qquad$ $P_{5izr}^{0} = -0.04$

④ i ⑤ čvorišta nisu vezana za referentno pa su im djelatne snage 0

$* \ \Delta P_2^0 = P_{2zad} - P_{2izr}^0 = 0.2 - (-0.3) = 0.5$ (prevelika razlika, nama treba $\varepsilon = 0.0001$)

$\Delta P_3^0 = P_{3zad} - P_{3izr}^0 = -0.45 + 0.075 = -0.875$

↓ − jer je to teret

$\Delta P_4^0 = P_{4zad} - P_{4izr}^0 = -0.4 + 0 = -0.4$

$\Delta P_5^0 = -0.6 + 0 = -0.6$

$* \ \Delta Q_2^0 = 0.2 + 0.985 = 1.185$

$\Delta Q_3 = -0.15 + 0.28 = 0.13$

$\Delta Q_4 = -0.05 + 0.055 = 0.005$

$\Delta Q_5 = -0.10 + 0.04 = -0.06$

$$\boxed{\begin{aligned} \left(\frac{\partial P_i}{\partial \delta_j}\right)^0 &= U_i \cdot U_j \cdot Y_{ij} \cdot \sin(-\Theta_{ij}) \\ \left(\frac{\partial P_i}{\partial \delta_i}\right) &= -U_i^0 \sum_{\substack{j=1 \\ j\neq 1}}^{n} U_j^0 \cdot Y_{ij} \cdot \sin(\delta_i^0 - \delta_j^0 - \Theta_{ij}) \end{aligned}}$$

$$\begin{vmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \\ \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_4 \\ \Delta Q_5 \end{vmatrix} = \begin{vmatrix} 0.5 \\ -0.375 \\ -0.4 \\ -0.6 \\ 1.185 \\ 0.13 \\ 0.005 \\ -0.06 \end{vmatrix} = \left|\begin{array}{cccc|cccc} 33.4 & -5 & -5 & -7.5 & 10.533 & -1.66 & -1.66 & -2.5 \\ -5 & 38.975 & -30 & 0 & -1.66 & 12.84 & -10 & 0 \\ -5 & -30 & 38.75 & -3.75 & -1.66 & -10 & 12.916 & -1.25 \\ -7.5 & 0 & -3.75 & 11.25 & -2.5 & -0 & -1.25 & 3.75 \\ \hline -11.338 & 1.66 & 1.66 & 2.5 & 21.43 & -5 & -5 & -7.5 \\ 1.66 & -12.991 & 10 & 0 & -5 & 38.45 & -30 & 0 \\ 1.66 & 10 & -12.816 & 1.25 & -5 & -30 & 38.64 & -3.75 \\ 2.5 & 0 & 1.25 & -3.75 & -7.5 & 0 & -3.75 & 11.17 \end{array}\right|$$

U lijevom dijelu matrice dijagonalni elementi su negativni, a vandijagonalni pozitivni.... vandijagonalni su 3 puta manji nego u četverokutu matrice iznad ili desno.

$$\begin{vmatrix} \Delta P_2 \\ \vdots \\ \vdots \\ \Delta Q_5 \end{vmatrix} = \begin{vmatrix} \text{ko odozgora} \end{vmatrix} \cdot \begin{vmatrix} \Delta \delta_2' \\ \vdots \\ \Delta \delta_5' \\ \Delta U_2' \\ \Delta U_5' \end{vmatrix}$$

$$\frac{\partial P}{\partial \delta} = J_1 \quad \frac{\partial P}{\partial U} = J_2 \quad \frac{\partial Q}{\partial \delta} = J_3 \quad \frac{\partial Q}{\partial U} = J_4$$

$$\begin{vmatrix} \Delta \delta' \\ \Delta U' \end{vmatrix} = \begin{vmatrix} -0.05068 \\ -0.0911 \\ -0.09733 \\ -0.11268 \\ 0.05494 \\ 0.03134 \\ 0.03091 \\ 0.026 \end{vmatrix}$$

$* \delta_2^1 = \delta_2^0 \pm \Delta\delta^1 = -0.05068 \quad (-2.905°)$

$\delta_3^1 = \delta_3^0 + \Delta\delta^1 = -0.0911 \quad (-3.22°)$

$\delta_4^1 = \delta_4^0 + \Delta\varepsilon^1 = -0.09733 \quad (-5.577°)$

$\delta_5^1 = \delta_5^0 + \Delta\delta^1 = -0.11288 \quad (-6.455°)$

$* U_2^1 = U_2^0 + \Delta U_2^1 = 1,05449$

$U_3^1 = U_2^0 + \Delta U_3^1 = 1,03134$

$U_4^1 = U_4^0 + \Delta U_4^1 = 1,03091$

$U_5^1 = U_5^0 + \Delta U_5^1 = 1,026$

Sad idemo u drugu iteraciju jer nam ovi ΔU-ovi ne zadovoljavaju $\varepsilon = 0.0001$.

$$P_{2izr}^{1} = U_2^{1} \cdot \sum_{j=1}^{5} U_j Y_{2j} \cdot \cos\left(\delta_2^{1} - \delta_j^{1} - \theta_{2j}\right) =$$

$$= 1.05449 \cdot 1.06 \cdot Y_{21} \cos\left(108.435 - 2.904 - 0\right) +$$

$$+ 1.05449 \cdot 1.05449 \cdot 34.18 \cdot \cos\left(71.52\right) + \ldots = 0.2738$$

$$\Delta P^{1} = P_{2zad} - P_{2izr}^{1} = 0.2 - 0.2738 = -0.0738$$ već je bolje (3. iteracijski zadovoljili naše zahtjeve za ε)

I onda pičimo:

$$\begin{vmatrix} \Delta P \\ \Delta Q \end{vmatrix} = \begin{vmatrix} J_1 & 0 \\ 0 & J_4 \end{vmatrix} \begin{vmatrix} \Delta \delta \\ \Delta U \end{vmatrix} \longrightarrow \begin{matrix} \Delta P = J_1 \cdot \Delta \delta \\ \Delta Q = J_4 \cdot \Delta U \end{matrix}$$

Tu ubacimo $\emptyset$ da si pojednostavimo račun (P ovisi o δ, a Q o U). To nas sjebe za par iteracija više al je ipak lakše računati....

$$\begin{vmatrix} \Delta \delta_2 \\ \Delta \delta_3^{1} \\ \Delta \delta_4^{1} \\ \Delta \delta_5^{1} \\ \Delta U_2^{1} \\ \Delta U_3^{1} \\ \Delta U_4^{1} \\ \Delta U_5^{1} \end{vmatrix} = \begin{vmatrix} -0.03443 \\ -0.08239 \\ -0.0889 \\ -0.10589 \\ 0.07518 \\ 0.0607 \\ 0.0674 \\ 0.06785 \end{vmatrix}$$

*ovo su "veće" vrijednosti za U od one prethodne matrice (vektora), ali zato u 5 jednostavnijih iteracija elegantnije dođeš do rješenja...

Jer, u Jacobijevom algoritmu su nam trebale 3 iteracije, a ovom pojednostavljenom smo ju trebali zavrtiti 5 puta, ali jebiga → lakše je

ISTOSMJERNI MODEL

$$P_i = \sum_{j=1}^{n} \underbrace{U_i U_j}_{U_i = U_j \approx 1} Y_{ij} \cos(-\theta_{ij} + \delta_i - \delta_j)$$ → u NAEES se približava $-90°$

$\cos(\alpha - 90) = \sin\alpha$

$$Q_i = \sum U_i U_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$ $\sin(\alpha - 90°) = -\cos\alpha$

* $U_i = U_j = 1$ $\quad Y_{ij} = B_{ij} \cdot e^{j90}$

$Y_{ii} = B_{ii} e^{-j90}$

Pretpostavimo da nema djelatne komponente (=struja je nema)

$$P_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} (\delta_i - \delta_j) \qquad \boxed{P_i = \sum_{j=1}^{n-1} B_{ij} \cdot \delta_j}$$ $\delta_n = 0°$ referentno čvor.

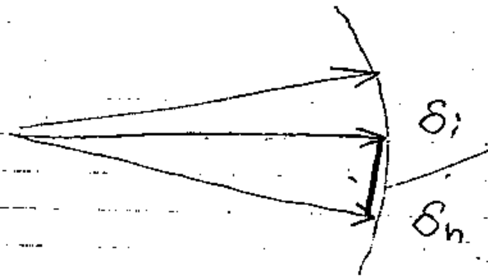
U matričnom obliku:

$$|P|_{n-1}^{1} = |B|^{n-1} \cdot |\delta|$$ → matrica susceptancija (nema mag. dijela)

B_{ij} — element Y matrice i to njen imaginarni dio

Ovaj istosmjerni model je jako sličan izmjeničnom i daje slične rezultate. Ali, ovdje ne dobijemo naponske prilike!

10.1.2005.



$\rightarrow U_i - U_n = j\Delta U$

$$|P_i| = j \cdot |B| \cdot |U_i - U_{ref}|$$

Proces proračuna

Napravimo matricu B

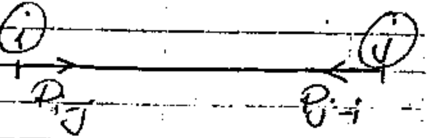
$$\begin{vmatrix} - & + & + & + & + & + \\ + & - & + & + & + & + \\ + & + & - & + & + & + \\ + & + & + & - & + & + \\ + & + & + & + & - & + \\ + & + & + & + & + & - \end{vmatrix} \longrightarrow -j \begin{vmatrix} + & - & - & - & - & - \\ - & + & - & - & - & - \\ - & - & + & - & - & - \\ - & - & - & + & - & - \\ - & - & - & - & + & - \\ - & - & - & - & - & + \end{vmatrix}$$

$$|P_i| = -j \begin{vmatrix} & \end{vmatrix} \cdot \begin{vmatrix} \Delta U \\ \Delta U \end{vmatrix}$$

$$\Rightarrow j \cdot \begin{vmatrix} \Delta U \\ \vdots \\ \Delta U \end{vmatrix} = \begin{vmatrix} X \end{vmatrix} \cdot \begin{vmatrix} P_i \end{vmatrix}$$

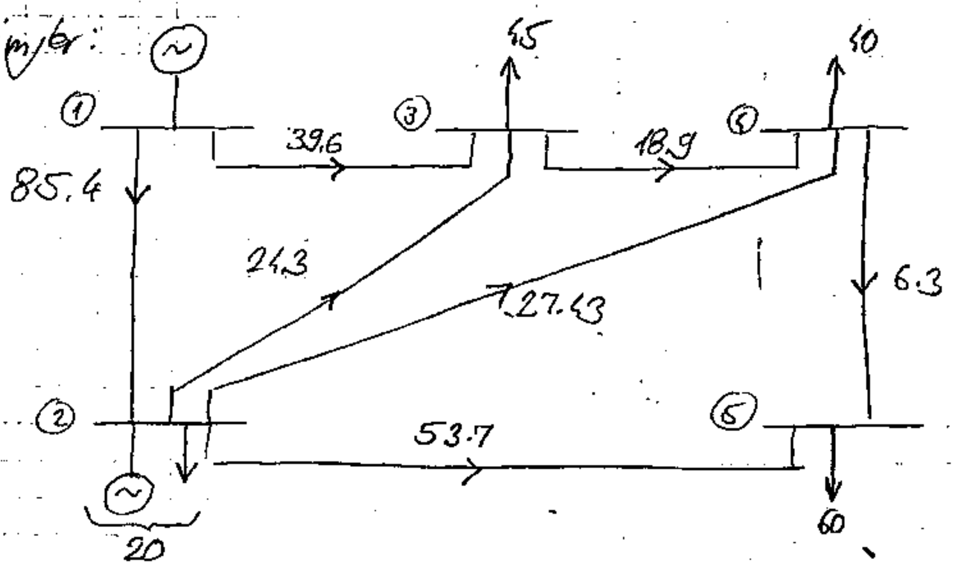
$$P_{i-j} = \frac{\cancel{j}\Delta U_i - \cancel{j}\Delta U_j}{\cancel{j}X_{i-j}} \Rightarrow P_{i-j} \text{ je realni broj!}$$

$$P_{j-i} = \frac{\cancel{j}\Delta U_j - \cancel{j}\Delta U_i}{\cancel{j}X_{i-j}} = -P_{i-j}$$



Dakle, nema gubitaka jer smo zanemarili djelatni otpor R.

Primjer:



1.

2. 20 MW 0,2

3. -45MW -0.45

4. -40MW -0.40

5. -60 MW -0.60

$\Big\}$ 1,25 → 125

	X	B
1-2	$j0.06$	$-j16.\dot{6}$
1-3	$j0.24$	$-j4.1\dot{6}$
2-3	$j0.18$	$-j5.5\dot{5}$
2-4	$j0.18$	$-j5.55$
2-5	$j0.12$	$-j8.33$
3-4	$j0.03$	$-j33.\dot{3}$
4-5	$j0.24$	$-j4.1\dot{6}$

$$B_{⑤} = j\begin{bmatrix} 20.833 & -16.\dot{6} & 4.16 & 0 & 0 \\ -16.6 & 36.111 & -5.55 & -5.55 & -8.3\dot{3} \\ -4.16 & -5.5\dot{5} & 43.055 & -33.3\dot{3} & 0 \\ 0 & -5.5\dot{5} & -3.3\dot{3} & 43.0\dot{5} & -4.16\dot{6} \\ 0 & -8.3\dot{3} & 0 & 4.16\dot{6} & 12.5 \end{bmatrix}$$

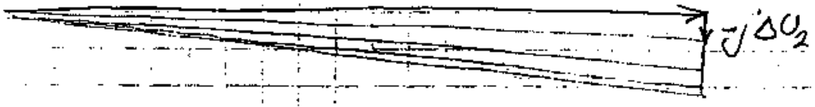
$$\begin{vmatrix} 0.2 \\ -0.45 \\ -0.40 \\ -0.60 \end{vmatrix} = \begin{vmatrix} & B_{\odot} & \end{vmatrix} , \begin{vmatrix} \Delta U_2 \\ \Delta U_3 \\ \Delta U_4 \\ \Delta U_5 \end{vmatrix} \qquad \delta_1 = \Delta U_1 = 0$$

$$j \begin{vmatrix} \Delta U_2 \\ \Delta U_3 \\ \Delta U_4 \\ \Delta U_5 \end{vmatrix} = j \begin{vmatrix} & B^{-1} & \end{vmatrix} \begin{vmatrix} 0.2 \\ -0.45 \\ -0.4 \\ -0.6 \end{vmatrix}$$

$$B^{-1} = \begin{vmatrix} 0.0507 & 0.03771 & 0.04029 & 0.3714 \\ 0. & 0.08914 & 0.07886 & 0.05143 \\ & & 0.09514 & 0.05857 \\ & & & 0.1309 \end{vmatrix} \quad \text{simetrična}$$

$$\Delta U = \begin{vmatrix} -j0.05126 \\ -j0.09497 \\ -j0.10063 \\ -j0.11571 \end{vmatrix}$$

vidimo da su to imaginarni dijelovi (jako manji od jedinice - dijelovi)



Ovaj j možemo ispustit jer znamo da je on tamo. Pa možemo napisat:

$$P = B \cdot \Delta U$$

$$\Delta U = \begin{vmatrix} -\cancel{j}0.05126 \\ \cancel{j}0.09497 \\ -\cancel{j}0.10063 \\ -\cancel{j}0.11571 \end{vmatrix}$$

$$P_{1-2} = \frac{\Delta U_1 - \Delta U_2}{X_{1-2}} = \frac{0-(-0.05126)}{0.06} = 0.854 \rightarrow 85.4 \text{ MW}$$

$$P_{1-3} = \frac{\Delta U_1 - \Delta U_3}{X_{1-3}} = \frac{0-(-0.09487)}{0.24} = 0.3957 \rightarrow 39.6 \text{ MW}$$

$$P_{2-3} = \frac{\Delta U_2 - \Delta U_3}{X_{2-3}} = \frac{-0.05126+0.09487}{0.18} = 0.2428 \rightarrow 24.3 \text{ MW}$$

$$P_{2-4} = \frac{\Delta U_2 - \Delta U_4}{X_{2-4}} = \frac{-0.05126+0.10063}{0.18} = 0.2743 \rightarrow 27.43 \text{ MW}$$

$$P_{2-5} = \frac{\Delta U_2 - \Delta U_5}{X_{2-5}} = \frac{-0.05126+0.11571}{0.12} = 0.537 \rightarrow 53.7 \text{ MW}$$

$$P_{3-4} = \frac{\Delta U_3 - \Delta U_4}{X_{3-4}} = \frac{-0.09487+0.10063}{0.03} = 0.1887 \rightarrow 18.87 \text{ MW}$$

$$P_{4-5} = \frac{\Delta U_4 - \Delta U_5}{X_{4-5}} = \frac{-0.10063+0.11571}{0.24} = 0.063 \rightarrow 6.3 \text{ MW}$$

* Ucrtamo to u shemu i provjerimo po Kichhofu dal pašu snage (pozbrajamo i pooduzimamo) sa zadanima.

Važno je da usporedimo sa izmjerenim tokovima snage (jer smo zanemarili R) → zadatak prije desetak stranica (tablica)

Zaključujemo da smo blizu (najveća je razlika kod referentnog čvorišta)

S ovim istosmjernim modelom nemremo realno procijenit naponske prilike, ili kad određujemo gubitke

	Z	Y	|Y|
1-2	$0.02+j0.06$	$5-j15$	15,81
1-3	$0.08+j0.24$	$1,25-j3.7$	3.95
2-3	$0.06+j0.18$	$1.66-j5$	5.27
2-4	$0.06+j0.18$	$1.66-j5$	5.27
2-5	$0.04+j0.12$	$2.5-j7.5$	7.9
3-4	$0.01+j0.03$	$10-j30$	31,62
4-5	$0.08+j0.24$	$1,25-j3.75$	3.95

$$Y=\begin{vmatrix} 34.25 & -5.27 & -5.27 & -7.9 \\ -5.27 & 40.84 & -31.62 & 0 \\ -5.27 & -31.62 & 40.84 & -3.95 \\ -7.9 & 0 & -3.95 & 11.85 \end{vmatrix}$$

dijagonale poz.
vandijagonalne neg.

$$Z=Y^{-1}=\begin{vmatrix} 0,05332 & 0,03977 & 0,04248 & 0,04970 \\ 0,03977 & 0,094 & 0,08315 & 0,05423 \\ 0,04248 & 0,08315 & 0,10032 & 0,06176 \\ 0,4970 & 0,05423 & 0,6176 & 0,13811 \end{vmatrix}$$

$$\begin{vmatrix} \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \end{vmatrix} = \begin{vmatrix} Y^{-1} \end{vmatrix} \begin{vmatrix} 0.2 \\ -0.45 \\ -0.4 \\ -0.6 \end{vmatrix} = \begin{vmatrix} -0.054 \\ -0.1014 \\ -0.061 \\ -0.122 \end{vmatrix}$$

Onda napravimo faktor $\frac{0.054}{0.05126}=1,0534$

$\frac{0.1014}{0.08407}=1.054$ uzmemo $\boxed{1.05}$

$\frac{16.6}{15.81}=1.0499$

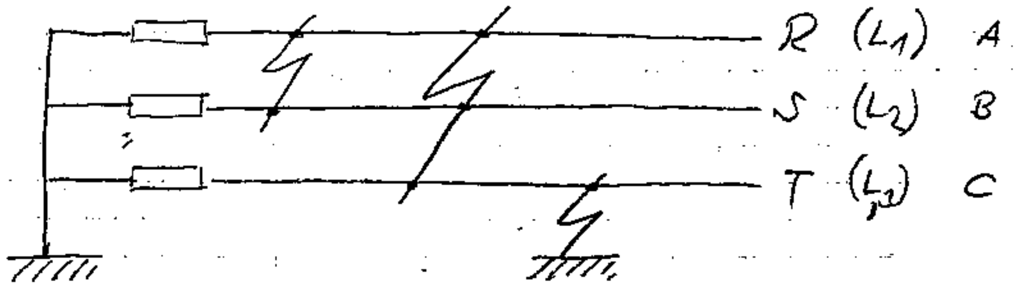
$$|P|=|Y||\delta|$$

Mreža Hrvatske ima 200 čvorišta i u Y matrici je u sigurno 90 tamada $\emptyset$.

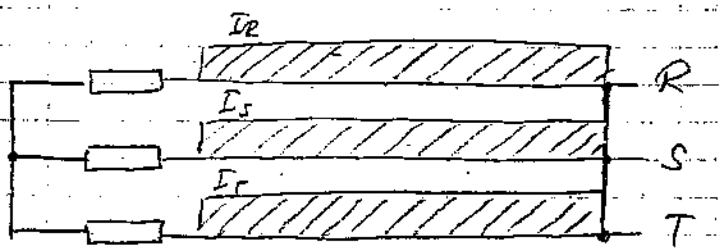
Tak smo završili tokove snaga...

11. 1. 2005.

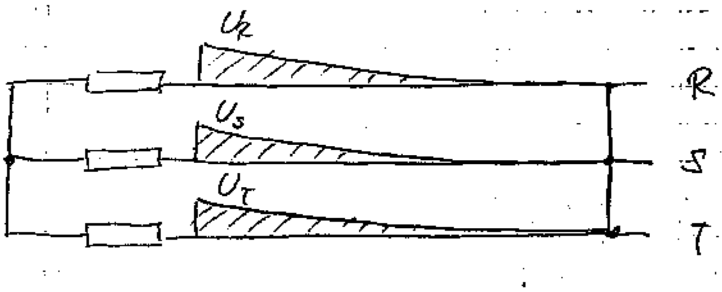
KRATKI SPOJ

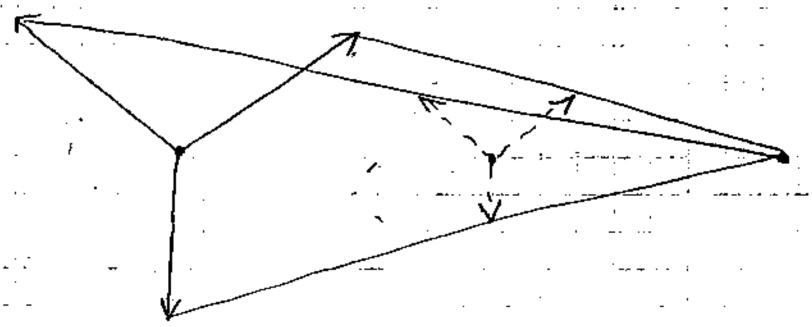


dvopolni tropolni jednopolni



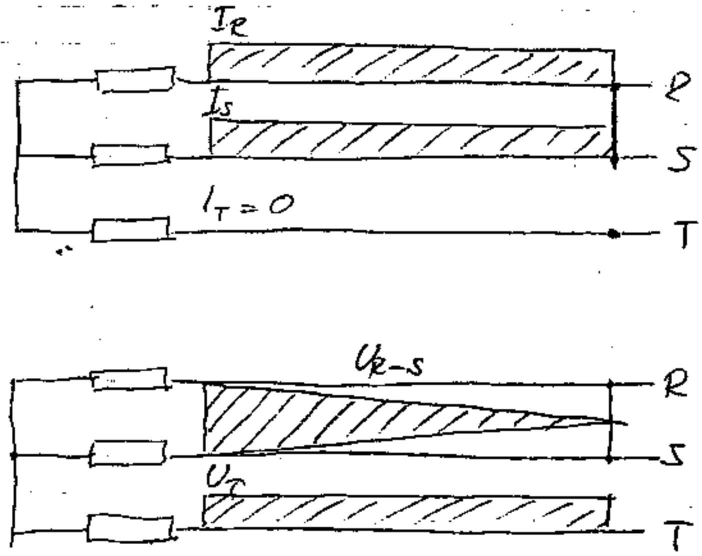
TROPOLNI

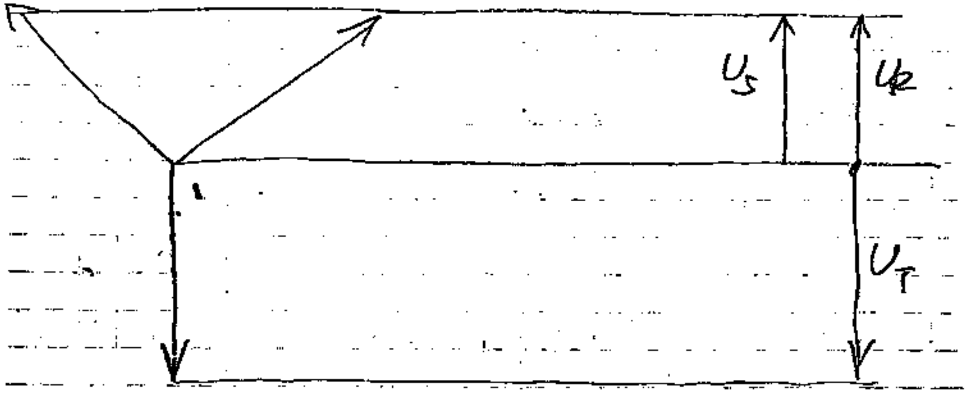




kut među naponima se zadržava, a naponi padaju prema $\emptyset$

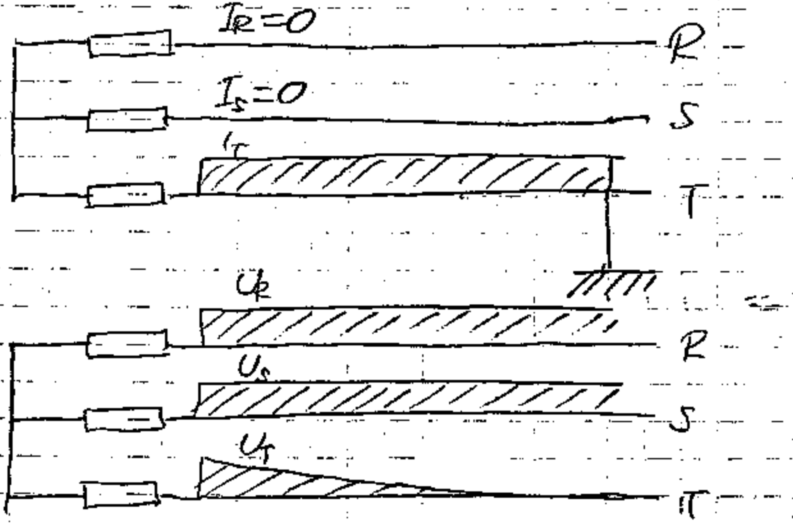
DVOPOLNI

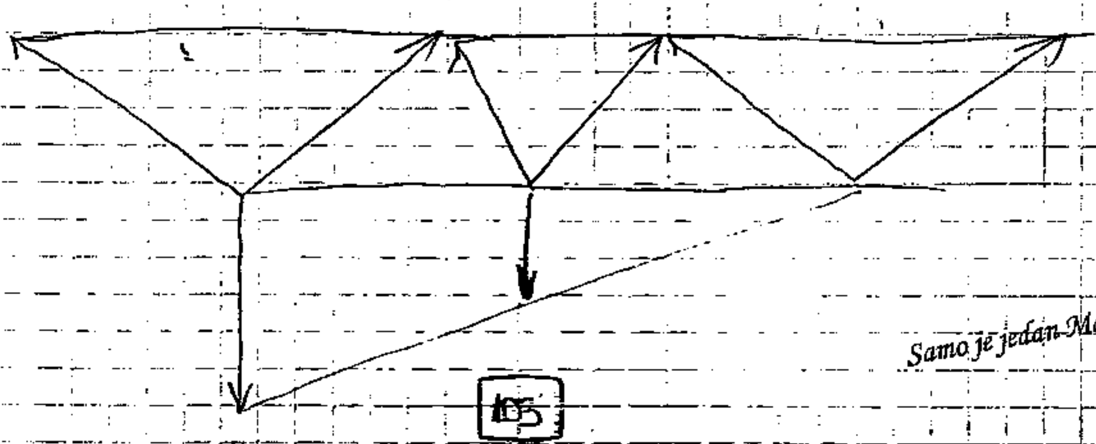


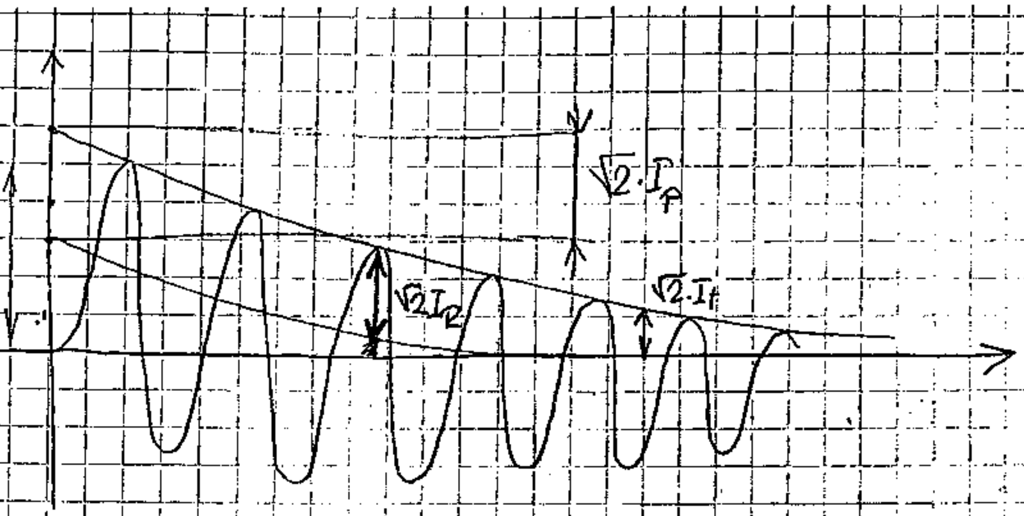


U_R i U_S su isti

JEDNOPOLNI







I_t – trajna struja efektivna

I_u – udarna struja tjemena

I_p – početna struja efektivna

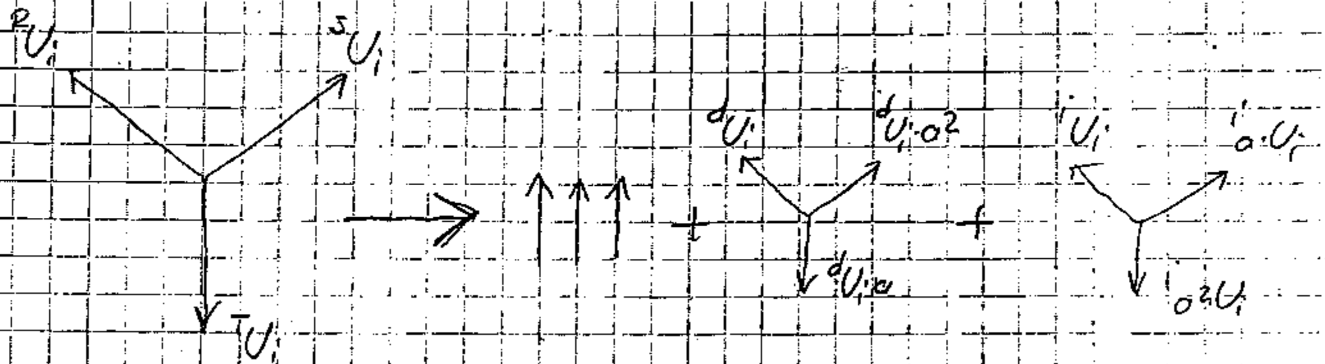
I_R – rasklopna struja efektivna

Svaki 3f sustav može se rastaviti na 3 3f sustava!

${}^{R}U_i = U_i^{0} + U_i^{d} + U_i^{i}$ i – čvor

${}^{S}U_i = U_i^{0} + a^2 U_i^{d} + a U_i^{i}$ R – faza

${}^{T}U_i = U_i^{0} + a U_i^{d} + a^2 U_i^{i}$



$$\boxed{\begin{aligned} a &= -0.5 + j\,\frac{\sqrt{3}}{2} \\ a^2 &= -0.5 - j\,\frac{\sqrt{3}}{2} \end{aligned}}$$

$$\begin{vmatrix} {}^{R}U_i \\ {}^{S}U_i \\ {}^{T}U_i \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} {}^{0}U_i \\ {}^{d}U_i \\ {}^{i}U_i \end{vmatrix}$$

$$\begin{vmatrix} {}^{0}U_i \\ {}^{d}U_i \\ {}^{i}U_i \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \cdot \begin{vmatrix} {}^{R}U_i \\ {}^{S}U_i \\ {}^{T}U_i \end{vmatrix}$$

17. 1. 2005.

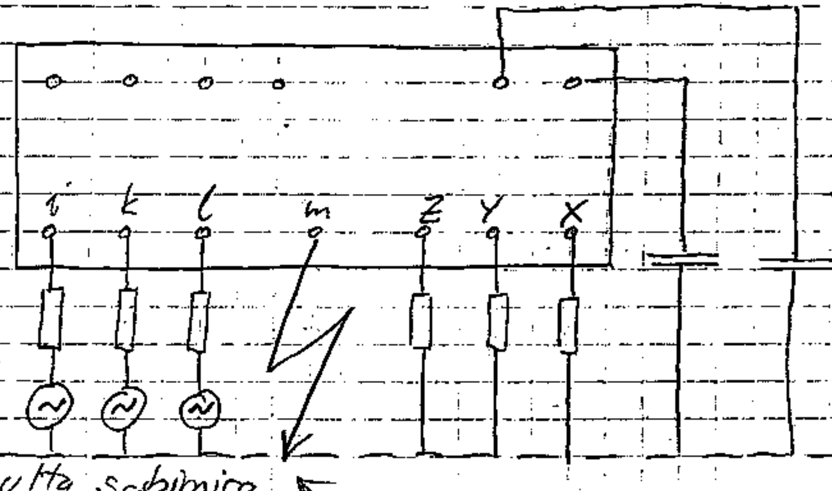
nastavak kratkog spoja

∅ sve struje u istoj vremenskoj osi

d kao kod 3f sustav

i obrnut smjer faza

Model mreže (fizikalna slika KS)



generatori u mreži

nulta sabirnica

tu se događi KS

svaki vod ili tak neki element mreže ima kapac. prema nultoj sabirnici

* E sad, kolka je struja?

kolki je napon?

kolike struje teku među čvorama?

kolki su naponi na čvorama?

$[I] = [Y]\cdot[U]$ nije singularna

$[U] = [Z]\cdot[I]$

$$[U] = [Z]\cdot\left\{\begin{vmatrix}0\\0\\I_i\\I_k\\I_l\end{vmatrix} + \begin{vmatrix}0\\0\\0\\I_m\\0\end{vmatrix}\right\}$$

$$[U] = \underbrace{|Z|\cdot|I_g|} + Z\cdot\begin{vmatrix}0\\I_m\\0\end{vmatrix}$$

to su naponi u čvorištima prije kratkog spoja

$$|U| = |\overset{\text{napon "zdrave" mreže}}{{}^zU}| + |Z|\begin{vmatrix}0\\0\\I_m\\0\end{vmatrix}$$

u m-čvorištu napon je nula jer je tamo nastao KS.

$$U_m = {}^zU_m + Z_{mm}\cdot I_m = 0 \Rightarrow \boxed{I_m = -\frac{{}^zU_m}{Z_{mm}}}$$ struja KS

$$\boxed{I_{KV} = -I_m = \frac{{}^zU_m}{Z_{mm}}}$$ struja kvara

Moramo naći Z_{mm} i napone u čvorištima u trenutku prije nastanka KS (metoda tokova snaga).

Ta metoda se koristi za:

1. IZBOR OPREME
2. ZA UDEŠENJE ZAŠTITE

Za <u>IZBOR OPREME</u> mi uzimamo ${}^zU_m$ malo veći $(1.1\cdot U_m)$, a za izračun Z_{mm} uzimaš X_{mm} (za koji vrijedi $X_{mm} < Z_{mm}$) tako da dobiješ veći I_m

Za <u>UDEŠENJE ZAŠTITE</u> uzimamo što točnije vrijednosti ${}^{z}U_m$ i Z_{mm} tako da zaštita može djelovati što točnije i selektivnije

VRSTE KRATKIH SPOJEVA

TROPOLNI KRATKI SPOJ

U slučaju da imaš 3p KS onda je ona slika za model mreže sa prethodne stranice, direktni sustav i naša matrična jednadžba dobiva malo slovo ⓓ

$$\boxed{{}^{d}I_{kv} = -{}^{d}I_m = \frac{{}^{z}U_m^{d}}{{}^{d}Z_{mm}}}$$

(dijagonalni elementi određuju struju KS u pojedinom čvorištu)

JEDNOPOLNI KRATKI SPOJ

Znamo da je

$$\left|{}^{d}U\right| = \left|{}^{d}U^{z}\right| + \left|{}^{d}Z\right| \cdot \begin{vmatrix} 0 \\ I_m \\ 0 \end{vmatrix}$$ direktni

$$\left|{}^{i}U\right| = \underset{\phi}{\underset{\parallel}{\left|{}^{i}U^{\phi}\right|}} + \left|{}^{i}Z\right| \cdot \begin{vmatrix} 0 \\ I_m \\ 0 \end{vmatrix} = \left|{}^{i}Z\right| \begin{vmatrix} 0 \\ {}^{i}I_m \\ 0 \end{vmatrix}$$ inverzni

$$\left|{}^{\phi}U\right| = \left|{}^{\phi}U^{z}\right| + \left|{}^{0}Z\right| \cdot \left|{}^{0}I_m\right| = \left|{}^{0}Z\right| \cdot \left|{}^{0}I_m\right|$$ nulti

Na mjestu KS:

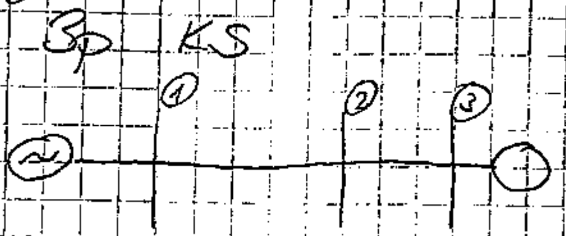
$$\left.\begin{aligned} {}^{d}U_m &= 0 \\ {}^{i}I_m &= 0 \\ {}^{\phi}I_m &= 0 \end{aligned}\right\}$$

$${}^{d}U_m = \phi = {}^{d}U_m^{z} + {}^{d}Z \cdot {}^{d}I_m$$

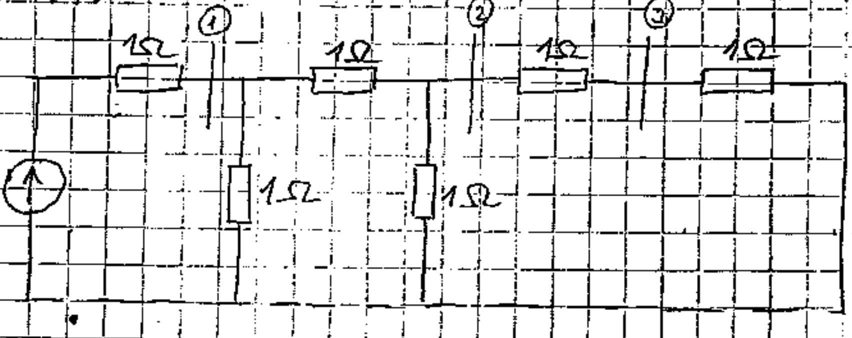
$$\boxed{{}^{d}I_m = -\frac{{}^{d}U_m^{z}}{{}^{d}Z}}$$

Primjer:

3p KS

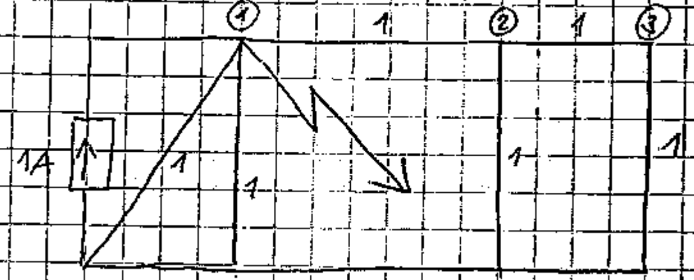


..ili...



Vod ima kapacitete

..ili..



Iz ove sheme se slaže matrica admitancije

$$Y = \begin{vmatrix} 3 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{vmatrix}$$

→ između 1 i 3 ide 0 jer nema direktne veze između njih

→ u čvorište 3 ulaze dvije 1

Invertiramo je i dobijamo Z:

$$Z = \begin{vmatrix} 0.385 & 0.154 & 0.077 \\ 0.154 & 0.461 & 0.23 \\ 0.077 & 0.23 & 0.615 \end{vmatrix}$$

$$\begin{vmatrix} U_1 \\ U_2 \\ U_3 \end{vmatrix} = |Z| \cdot \left\{ \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} + \begin{vmatrix} {}^{d}I_1 \\ \\ \end{vmatrix} \right\}$$

Ako je KS u ① onda je $U_1=$

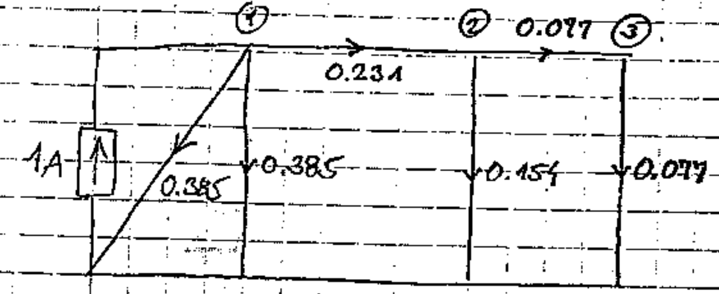
a struje ne teku kroz grane 1-2 i 2-3

$0 = 1 \cdot Z_{11} + {}^{d}I_1 \cdot Z_{11} \Rightarrow {}^{d}I_1 = -1$ tako da je

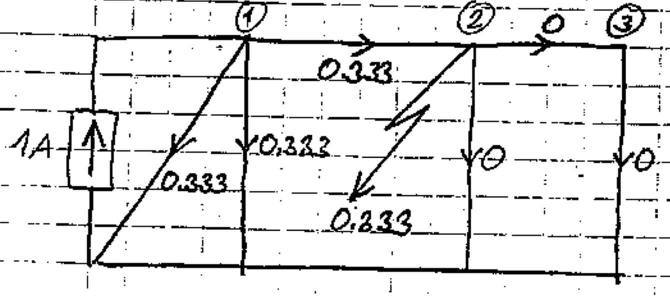
$$\begin{vmatrix} U_1 \\ U_2 \\ U_3 \end{vmatrix} = Z \cdot \begin{vmatrix} 1-1 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

* Ako KS dođe u čvorište ① onda je napon čvorišta nula, odnosno svi su naponi povezani tim čvorištem nula!

$$\begin{vmatrix} {}^dU_1^z \\ {}^dU_2^z \\ {}^dU_3^z \end{vmatrix} = Z \begin{vmatrix} 1 \\ \\ \end{vmatrix} = \begin{vmatrix} 0.385 \\ 0.154 \\ 0.077 \end{vmatrix}$$



* A sad, ako je KS u čvorištu ②:



$$\begin{vmatrix} {}^dU_1 \\ {}^dU_2 \\ {}^dU_3 \end{vmatrix} = \begin{vmatrix} 0.385 \\ 0.154 \\ 0.077 \end{vmatrix} + Z \cdot \begin{vmatrix} 0 \\ {}^dI_2 \\ 0 \end{vmatrix}$$

$$0 = 0.461 \cdot {}^dI_2 + 0.154$$

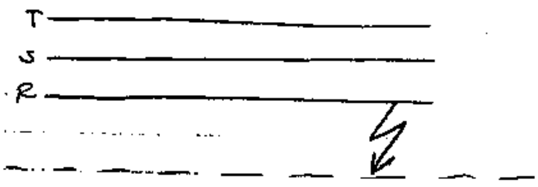
$${}^dI_2 = -\frac{0.154}{0.461} = -0.333$$

$$\begin{vmatrix} {}^dU_1 \\ {}^dU_2 \\ {}^dU_3 \end{vmatrix} = \begin{vmatrix} 0.385 \\ 0.154 \\ 0.077 \end{vmatrix} - \begin{vmatrix} 0.052 \\ 0.154 \\ 0.077 \end{vmatrix} = \begin{vmatrix} 0.333 \\ 0 \\ 0 \end{vmatrix}$$

Sad se vraćaš u shemu da pogledaš struje!

Jednopolni KS nastaje u 80% svih KS (to su prestaci i slične pizdarije), dok svi ostali nastupaju u 20% slučajeva

Npr. imaš 1p KS



onda je $\quad {}^{R}U_i = 0$

$\qquad\qquad {}^{S}I_i = {}^{T}I_1 = 0$

$$\begin{vmatrix} {}^{R}U_i \\ {}^{S}U_i \\ {}^{T}U_i \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \cdot \begin{vmatrix} {}^{0}U_i \\ {}^{d}U_i \\ {}^{i}U_i \end{vmatrix}$$

Ovo je rastavljanje na simetrične komponente

$${}^{0}U_i + {}^{d}U_i + {}^{i}U_i = 0$$
$${}^{S}I_i = I_i + a^2\,{}^{d}I_i + a\,{}^{i}I_i = 0$$
$${}^{T}I_i = {}^{0}I_i + a\,{}^{d}I_i + a^2\,{}^{i}I_i = 0$$

$$-{}^{0}I_i = {}^{d}I_i = {}^{i}I_i$$

$$|{}^{d}U| = |{}^{d}U^{z}| + |{}^{d}Z| \cdot \begin{vmatrix} 0 \\ {}^{d}I_i \\ 0 \end{vmatrix}$$

Napone imaš samo u direktnoj mreži

$$|{}^{i}U| = |{}^{i}Z| \cdot \begin{vmatrix} 0 \\ {}^{i}I_i \\ 0 \end{vmatrix}$$

$$|{}^{0}U| = |{}^{0}Z| \cdot \begin{vmatrix} 0 \\ I_i \\ 0 \end{vmatrix}$$

$${}^{\phi}U_i + {}^{d}U_i + {}^{i}U_i = \phi = {}^{0}Z_{ii}\,{}^{0}I_i + {}^{d}U_i^{z'} + {}^{d}Z_{ii}\,{}^{d}I_i + {}^{i}Z_{ii} \cdot {}^{i}I_i = \phi$$
$$\phi = {}^{z}U_i + ({}^{0}Z_{ii} + {}^{d}Z_{ii} + {}^{i}Z_{ii}) \cdot {}^{a}I_i$$

$$\boxed{{}^{d}I_i = -\frac{{}^{z}U_i}{{}^{0}Z_{ii} + {}^{d}Z_{ii} + {}^{i}Z_{ii}} = {}^{d}I_i = {}^{i}I_i}$$

Zato moramo imat matricu impedancije i ⓪ i ⓘ sustava

Samo je jedan Mali Ivica!

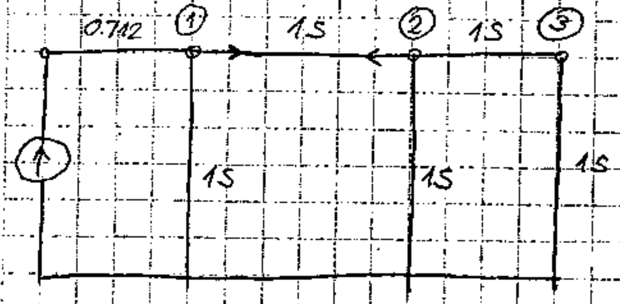
${}^{d}Z = {}^{i}Z$, ali ${}^{0}Z$ se razlikuje

18.1.2005.

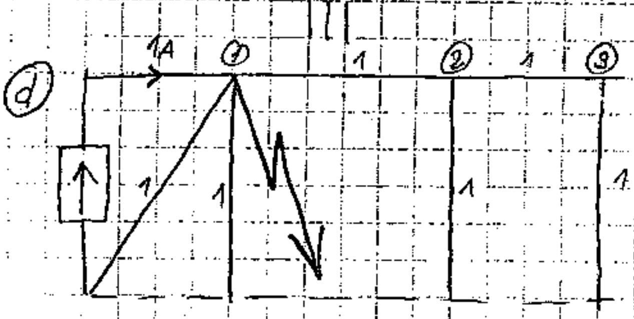
... nastavljamo s jednopolnim KS

$$\begin{vmatrix} {}^{R}I \\ {}^{S}I \\ {}^{T}I \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \begin{vmatrix} {}^{0}I_i \\ {}^{d}I_i \\ {}^{i}I_i \end{vmatrix} = \begin{vmatrix} {}^{R}I = 3{}^{0}I_i \\ {}^{S}I = 0 \\ {}^{T}I = 0 \end{vmatrix}$$

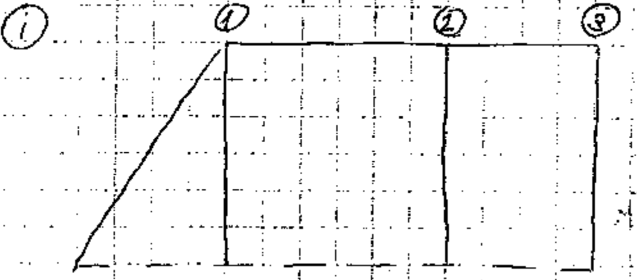
Jednadžbe koje smo već napisali vrijede pa crtamo mrežu

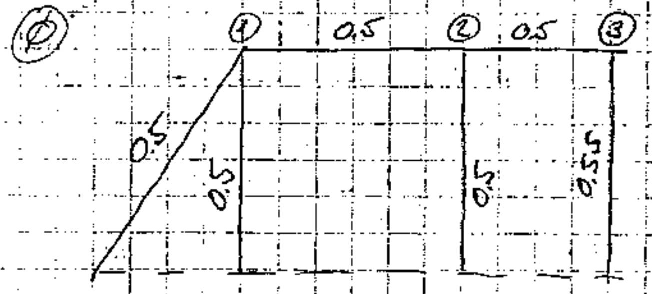


Imamo kvar u čvorištu ①



Ovo je direktna, a inverzna nema izvora (ems)





Samo je jedan Mali Ivica!

Kod nultog sustava su otpori veći (jer su struje u istom smjeru pa se magn polje zbraja), pa imamo vodljivost
0.5S → 2Ω

$$^{0}I_{①} = -\frac{0.385}{^{d}Z_{11} + {}^{i}Z_{11} + {}^{0}Z_{11}}$$

$$\begin{vmatrix} ^{z}U_1 \\ ^{z}U_2 \\ ^{z}U_3 \end{vmatrix} = \underbrace{\begin{vmatrix} 0.385 & 0.154 & 0.077 \\ 0.154 & 0.461 & 0.23 \\ 0.077 & 0.23 & 0.615 \end{vmatrix}}_{Z_d} \cdot \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$$

$^{z}U_1 = 0.385$ V

Idemo si nacrtat Y matricu ∅ sustava

$$⓪ \begin{vmatrix} 1.5 & -0.5 & 0 \\ -0.5 & 1.5 & -0.5 \\ 0 & -0.5 & 1 \end{vmatrix}$$

$$ⓓ \begin{vmatrix} 3 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{vmatrix}$$

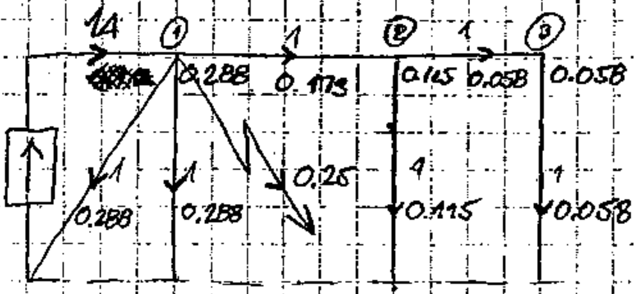
Dakle, ^{0}Z dobijemo tak da * pomnožimo s 2

$$\boxed{^{0}I_{①} = -\frac{0.385}{^{d}Z_{11} + {}^{i}Z_{11} + {}^{0}Z_{11}} = -\frac{0.385}{0.385 + 0.385 + 0.77} = -0.25 = {}^{i}I_{①} = I_{①}}$$

*

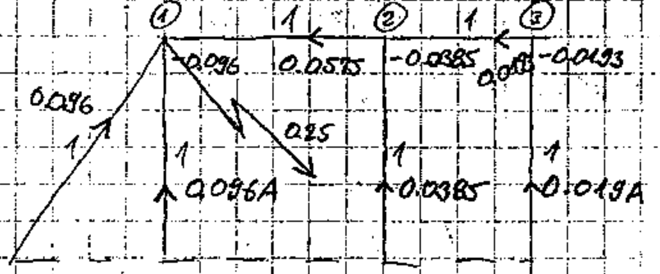
$$\begin{vmatrix} ^{d}U_1 \\ ^{z}U_2 \\ ^{z}U_3 \end{vmatrix} = \begin{vmatrix} 0.385 & 0.154 & 0.077 \\ 0.154 & 0.461 & 0.23 \\ 0.077 & 0.23 & 0.615 \end{vmatrix} \cdot \left\{ \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} + \begin{vmatrix} 0.25 \\ 0 \\ 0 \end{vmatrix} \right\} = \begin{vmatrix} 0.288 \\ 0.115 \\ 0.058 \end{vmatrix}$$

Pa smo dobili za ⓓ sustav



* A sad idemo rješavat ⓘ mrežu

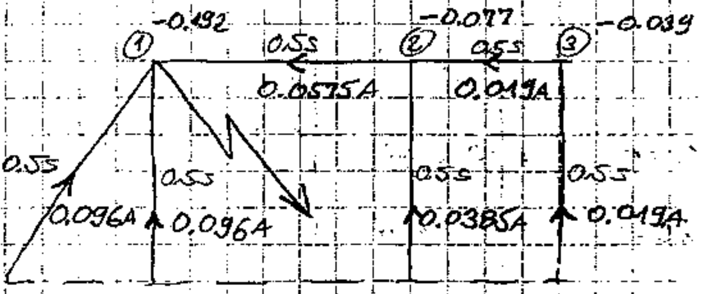
$$\begin{vmatrix} {}^{i}U_1 \\ {}^{i}U_2 \\ {}^{i}U_3 \end{vmatrix} = Z \begin{vmatrix} -0.25 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} -0.096 \\ -0.0385 \\ -0.0193 \end{vmatrix}$$



U ① moramo dobit onih 0.25

* I sad smo došli do 0 mreže

$$\begin{vmatrix} {}^{0}U_1 \\ {}^{0}U_2 \\ {}^{0}U_3 \end{vmatrix} = \begin{vmatrix} 0.77 & 0.308 & 0.154 \\ 0.308 & 0.922 & 0.46 \\ 0.154 & 0.46 & 1.23 \end{vmatrix} \cdot \begin{vmatrix} -0.25 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} -0.192 \\ -0.077 \\ -0.039 \end{vmatrix}$$



To nam sve treba jer hoćemo izračunati struje u R, S, T fazama pomoću matrice transformacije

$$\begin{vmatrix} {}^{R}I_1 \\ {}^{S}I_1 \\ {}^{T}I_1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \begin{vmatrix} \overset{{}^{0}I \searrow}{-0.25} \\ \overset{{}^{d}I \searrow}{-0.25} \\ \overset{{}^{i}I \searrow}{-0.25} \end{vmatrix} \Rightarrow \begin{vmatrix} -0.75 \\ 0 \\ 0 \end{vmatrix}$$

tu se misli na one poprečne struje koje izlaze, a ne na one struje grane!

pazi na − !

$$\begin{vmatrix} {}^{R}I_{izvora} \\ {}^{S}I_{izvora} \\ {}^{T}I_{izvora} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \begin{vmatrix} 0.096 \\ 0.712 \\ 0.096 \end{vmatrix} = \begin{vmatrix} 0.864 \\ -0.308-j0.533 \\ -0.308+j0.533 \end{vmatrix}$$

iz pne sheme (di je naponski izvor)

I sad gledamo napone u čvorištu

$$\begin{vmatrix} {}^{R}U_1 \\ {}^{S}U_1 \\ {}^{T}U_1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \begin{vmatrix} -0.192\,{}^{0}U_1 \\ 0.288\,{}^{d}U_1 \\ -0.096\,{}^{i}U_1 \end{vmatrix} = \begin{vmatrix} 0 \\ -0.288-j0.3325 \\ -0.288+j0.3825 \end{vmatrix}$$

i po računu, i logično...

Na mjestu kvara $U_R=0$, $I_S=0$, $I_T=0$. Dakle, impedancije ⓓ i ⓘ mreže su iste, a ⓪ mreže ovise o uzemljenjima i ostalim kriterijima iz RASKLOPNIH POSTROJENJA.

Ovisno gdje je KS, nekad ćemo morat računat napone više čvorišta i struje više grana...

*Al idemo mi sad vidjet napone "bolesne" mreže:

$$\begin{vmatrix} {}^{R}U_2 \\ {}^{S}U_2 \\ {}^{T}U_2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \begin{vmatrix} -0.077 \\ 0.115 \\ -0.0385 \end{vmatrix} = \begin{vmatrix} 0 \\ -0.115 -j \\ -0.1115 +j \end{vmatrix}$$

logično, al u S i T ne smije biti

$-0.075 + 0.115(-0.5 - j0.866) + 0.0385 \cdot (-0.5 + j0.866)$

$Re = -0.075 - 0.0575 + 0.01925 = -0.1115$

$I_m =$

$$\begin{vmatrix} {}^{R}I_{Trošilo} \\ {}^{S}I_{Trošilo} \\ {}^{T}I_{Trošilo} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \begin{vmatrix} -0.019 \\ 0.058 \\ -0.0193 \end{vmatrix} = \begin{vmatrix} 0.02 \\ \\ \\ \end{vmatrix}$$

Ipak ćemo imati nekakvu struju kroz trošilo.

Dakle, rezime...

Definiramo Y matricu metodom čvorišta. Invertiramo tu matricu i dobijemo Z, a možemo i onak da razdijelimo Y na gornju i donju.
Tada računamo napone zdrave mreže. Nakon toga možemo računat za bilo koju vrstu kratkog spoja (do sad 3p i 1
Kod 3p je [illegible], dok kod 1p imamo 3 mreže ⓓ, ⓘ i
i rješavamo 3 mreže. Računamo struje u granama i napone koji nam trebaju.
I onda se matricom transformacije vraćamo na R,S,T mreže.

* Kod DZ... Y matrica nije ista za proračun tokova snage i u proračunu kratkog spoja (mijenjaju se dijagonalni elementi)