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Workload balance in Put-to-light systems

1. Problem definition

A put-to-light classification system processes orders which form a batch that is denoted by the set $P = \{P_1,\ P_2, \dots, P_{\mathrm{i}}\ \}$. Each order is composed of a variety of SKUs. There may be repetition which means that some orders share the same SKU although in different quantities.

The SKUs are defined in the set $R = \{R_1,\ R_2, \dots, R_{\mathrm{m}}\}$. The SKUs in an order cannot be divided into different batches, it is considered that 100% of the SKUs must be classified before the batch ends, unless there is no availability in the inventory, only in this case is the classification of less than the total considered. For this study, inventory inaccuracies are not considered, and the availability of all units is assumed.

These orders will be classified in a position $S = \{S_1,\ S_2, \dots, S_{\mathrm{k}}\}$ that is part of a zone $Z = \{Z_1,\ Z_2, \dots, Z_{\mathrm{j}}\}$. Enough capacity is assumed in all positions, allowing to classify all the units for each SKU that makes up an order.

Order classification begins with the authentication by workers $T = \{T_1,\ T_2, \dots, T_{\mathrm{n}}\}$. At the beginning of the process, they scan an identification number that was assigned to them so that the system can detect who is going to introduce SKUs in each position of the zone. Only one worker can be authenticated in a zone. It is considered that this worker moves through the positions of their zone at a constant speed v to deposit the SKUs. In addition, due to their experience, they present different results in terms of productivity depending on their stage in the learning curve of the process.

Table 1 summarizes the notation of the proposed nonlinear programming model. The proposed model assigns orders to the PTL system, considering the processing times of each order that are affected by the number of SKUs that compose it and the distances to the PTL positions.

Sets	
P	Orders contained in the batch, index $\boldsymbol{i}$
Z	Zones that make up the put-to-light system, index $\boldsymbol{j}$

S	Positions that make up the zones of the put-to-light system, index $\boldsymbol{k}$
R	SKUs of each order that is part of the batch, index $\boldsymbol{m}$
T	Workers who are assigned to each zone of the put-to-light system, index $\boldsymbol{n}$
Parameters	
ns_j	Number of positions that are part of each zone $\boldsymbol{j}$ of the put-to-light system
s_{jk}	Binary parameter, takes the value of 1 if the position $\boldsymbol{k} \in \boldsymbol{S}$ is part of the zone $\boldsymbol{j} \in \boldsymbol{Z}$ of the put-to-light system, 0 otherwise
d_{jk}	Distance in meters that must be traveled from the scan located in the zone $\boldsymbol{j} \in \boldsymbol{Z}$ to deposit SKUs at position $\boldsymbol{k} \in \boldsymbol{S}$
rp_{im}	Binary parameter, takes the value of 1 if the SKU $\boldsymbol{m} \in \boldsymbol{R}$ is part of the order $\boldsymbol{i} \in \boldsymbol{P}$, 0 otherwise
tr_{im}	Time to classify a SKU $\boldsymbol{m} \in \boldsymbol{R}$ with all its units in an order $\boldsymbol{i} \in \boldsymbol{P}$
v	Average speed traveled of the workers of the put-to-light system

Table 1. Notation

3.1 Decision variables

To assign orders to achieve a balance in the workloads across zones, we define two binaries and four real variables. The binary variable X_{ik} takes the value of one if order i is assigned to be classified at position k. It should be noted that an order cannot be split into multiple positions, therefore, it can only be assigned to one zone. ~~Additionally, a worker can only be assigned to one zone, so the variable~~ ~~L_{nj} represents the zone in which the worker should be assigned considering his/her expertise~~.
The variable Y_i determines the time it takes to classify an order with all its units considering the distance to the position which it is assigned. Finally, the variable W_j denotes the total classification time per zone. Additionally, the variables W_{max} and W_{min} represent the maximum and minimum classification times in the zones.

3.2 Objective functions and constraints

We focused on four alternative objective functions that were independently implemented with the goal of balancing the classification times in the zone that make up the PTL system. All these objective functions are evaluated in test and realistic instances to determine which ones yield feasible and optimal solutions. This implies that all orders for sorting into the PTL system must be assigned, each order must be assigned to a single position, and workers must be assigned one per zone.

In the first function, we minimize the average classification time in the zones. The objective is defined as:

$$Minimize\ Average\_Time = Sum(W_j)/|Z| \tag{1}$$

In the second, we minimize the difference between the maximum and minimum time obtained from the classification in the zones. The objective function is denoted as follows:

$$Minimize\ Difference\_Time = W_{max} - W_{min} \tag{2}$$

In the third function we seek to minimize the maximum classification time with the following objective function:

$$Minimize\ Maximum\_Time = W_{max} \tag{3}$$

Applying the notation in Table 1, three models consisting of one the objective functions mentioned above (1) to (3) and constraints (4) to (13) are formulated.

$$\sum_{k \epsilon S} X_{ik} = 1\ \forall i \in P \tag{4}$$

$$\sum_{i \epsilon P} X_{ik} \leq 1\ \forall k \in S \tag{5}$$

$$\sum_{k \epsilon S} \sum_{i \epsilon P} X_{ik} * s_{jk} \leq ns_j\ \forall j \in Z \tag{6}$$

$$Y_i = \sum_{k \epsilon S} \sum_{m \in R} \sum_{j \in Z} \sum_{n \in T} ((tr_{im} + 2(d_{jk}/v))) * rp_{im} * X_{ik} * s_{jk}\ \forall i \in P \tag{7}$$

$$W_j = \sum_{i \epsilon P} \sum_{k \in S} Y_i * X_{ik} * s_{jk}\ \forall j \in Z \tag{8}$$

$$W_{min} \leq W_j \leq W_{max} \tag{9}$$

$$X_{ik} \in \{0,1\} \ \forall \iota \in P, \forall k \in S \tag{10}$$

$$Y_i \geq 0 \ \forall \iota \in P \tag{11}$$

$$W_j \geq 0 \ \forall j \in Z \tag{12}$$

$$W_{min}, W_{max} \geq 0 \tag{13}$$

Constraint (4) guarantees that an order will be assigned to only one position of PTL, ensuring that orders are not divided into different positions. Furthermore, constraint (5) ensures that each position has a single order assigned for the batch to be classified or that does not have any assignment. Expression (6) controls that the orders assigned to a zone do not exceed the total positions of the zone. Constraint (7) refers to the execution time of an order that considers all times involved in the process: product scanning, counting, transfer and deposit of units, confirmation at each position and return time to the scanner to continue with the classification. These mentioned times are included in the parameter tr_{im}. Expression (8) considers that the execution times in each zone is the sum of the classification times of the orders assigned to a zone. This constraint is nonlinear (quadratic), so it considers the product of a binary variable X_{ik} and a real variable Y_i. Additionally, constraint (9) guarantees that the execution time of the zone is between the maximum and minimum classification time of the zones that compose the put-to-light system. Finally, expressions (10), (11), (12), and (13) define the domain of the decision variables.