BDA - Project

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Introduction

The motivation

Neonatal mortality is related to the number of deaths in the first 28 days of a child's life. This is a high-risk period and according to UNICEF data, the average number of deaths is estimated to be 17 per 1000 live births in 2019 [1]. Neonatal mortality is classified into an early neonatal mortality which is between the first 7 days of birth and a late neonatal mortality which occurs after 7 days of birth. On the other hand, perinatal mortality refers to the death of the child during the fetal period in conjunction to early neonatal mortality. It is clear that there are huge differences in perinatal and neonatal mortality rates between continents and between developed and emerging countries. This project will attempt to model the probability of perinatal and late neonatal mortality in 2020 in different regions in Colombia (a developing country in Latin America) and compare it with the US, which is among the richest countries in the world.

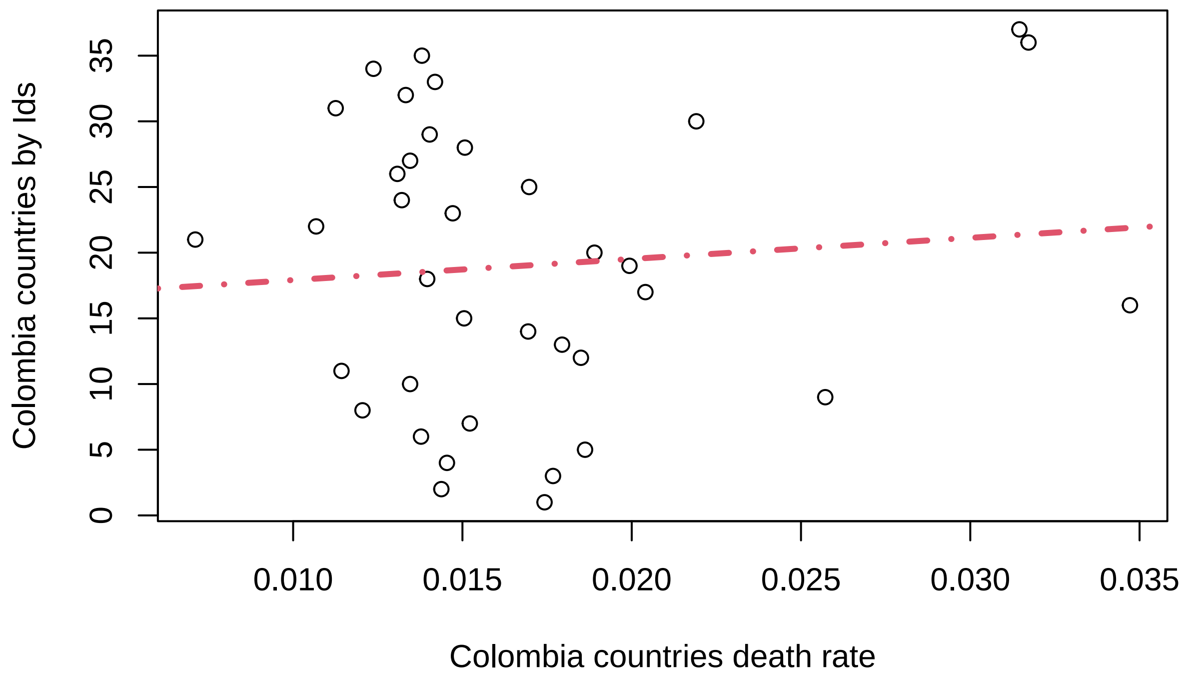
The main modeling idea

The general idea for modeling the probability of perinatal and late neonatal mortality is to treat the cases of death from this condition as "successes" among the total number of births in the same time period. With this in mind, we came up with a beta-binomial model that is natural in these cases.

Some illustrative figure

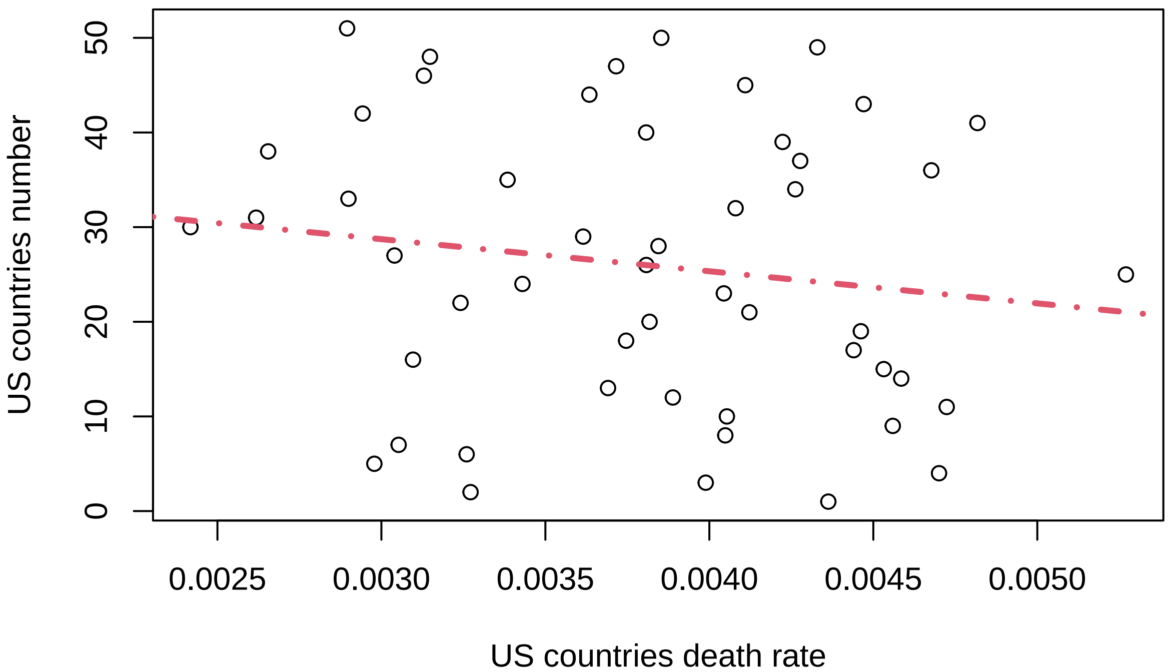
```r
# Read Excel file and convert it to DataFrame
library("readxl")
library("rstan")
exceldata = read_excel("Colombia2020.xlsx")
dfData= data.frame(exceldata)
deaths = dfData$Deaths
births = dfData$Lbirths+ dfData$Deaths
dfData$rate = (deaths/births)

dfData$id <- seq.int(nrow(dfData))
plot(dfData$id ~ dfData$rate, title="Perinatal and  neonatal mortality for Colombia",
     ylab="Colombia countries by Ids",
     xlab="Colombia countries death rate")
exceldata = read_excel("a.xlsx")
dfA= data.frame(exceldata)
exceldata = read_excel("b.xlsx")
```

```
dfB= data.frame(exceldata)
lm1 <- lm(id ~ rate, data = dfData)
abline(lm1, col = 2, lty = 4, lw = 3)
```



And here about the US data:

```
exceldata = read_excel("US2020.xlsx")
dfData= data.frame(exceldata)
deaths = dfData$Deaths
births = dfData$Births
dfData$rate = (deaths/births)
dfData$id <- seq.int(nrow(dfData))
plot(dfData$id ~ dfData$rate, title="The perinatal and neonatal mortality for US",
     xlab="US countries death rate", ylab="US countries number")
exceldata = read_excel("aUS.xlsx")
dfA= data.frame(exceldata)
exceldata = read_excel("bUS.xlsx")
dfB= data.frame(exceldata)
lm1 <- lm(id ~ rate, data = dfData)
abline(lm1, col = 2, lty = 4, lw = 3)
```



Data

For Colombia, data were found on the website of the National Institute of Health of the country, data were collected manually in an excel file from epidemiological reports from the year 2017 to 2020. The data collected were the number of births and the number of deaths in the perinatal and late neonatal period by year and in 38 regions. Data for the United States was collected from the CDC wonder website in the Infant Death Records. In the case of Colombia, there are no published models for this type of infant mortality and only predictive models for the whole country were found in terms of US data.

Model description

The following section describe the two models used in this problem, hierarchical and separate.

Separate model

The parameter of interest is θ which corresponds to the probability late neonatal and perinatal death among regions. The likelihood is computed as:

$$p(y_i|\theta_i) = \binom{n_i}{y_i}\theta_i^{y_i}\,(1-\theta_i)^{y_i}$$

Being y_i the number of deaths, n_i the number of births for each region.

Prior

Informative priors were chosen in both models since we also have data from previous years. The mean of a and b values for the previous years were used in the hierarchical model and separate models.

The prior in this case was obtained using beta distribution since parameter a resemble number of successes and $a + b$ the total number of trials.

So, the prior in this case was computed as:

$$\theta_i \sim Beta(\bar{a}_i, \bar{b}_i)$$

Being $\bar{a}_i, \bar{b}_i$ the mean of of deaths and live births between each region and computed from previous years.

Hierarchical model

The likelihood remains the same as in the previous model. But in this case we use a transformation to compute hyper-prior.

Parameter μ is equal to the probability of death, that is $a/(a + b)$ and η corresponds to the total population $(a + b)$.

Hyper-priors

$$\eta \sim exponential(x), \mu_i \sim Beta(a_i, b_i)$$

Prior

$$\theta_i \sim Beta(\alpha, \beta)$$

given that $\alpha = \mu\eta$ and $\beta = (1 - \mu)\eta$.

Stan code

Separate model

```
write("// Stan Separate model
//Beta-binomial Separate model

data {
  int<lower=0> N; // Number of states
  int<lower=0> y[N]; //Number of neonatal deaths
  int<lower=0> n[N]; //Number of births
  real aMean[N]; //Minimun a value
  real bMean[N]; //Minimun b Value


}

// The parameters accepted by the model
```

```
parameters {
  vector<lower=0, upper=1>[N] theta;

}

// The model to be estimated. We model the output
// 'y' to be normally distributed with mean 'mu'
// and standard deviation 'sigma'.
model {

  //Priors

    for (j in 1:N) {
      theta[j] ~ beta(aMean[j], bMean[j]);
    }

    //Likelihood
    for (k in 1:N){
      y[k] ~ binomial(n[k], theta[k]);
    }

}

generated quantities {

    //Log Likelihood ratios
    vector[N] log_lik;
    real ypred[N];

      for(j in 1:N){
        log_lik[j] = binomial_lpmf(y[j] | n[j], theta[j]);
      }


    for(j in 1:N){
        ypred[j]= binomial_rng(n[j],theta[j]);
        }
  }
// The posterior predictive distribution",
"separate_model.stan")
```

Hierarchical model

```
write("// Stan Beta-binomial Hierarchical model
data {

  int<lower=0> N; // Number of states
  int<lower=0> y[N]; //Number of neonatal deaths
  int<lower=0> n[N];
  real a[N]; //Mean a value
  real b[N]; //Mean b Value
```

```
  real<lower=0> e;

}

// The parameters accepted by the model
parameters {
  real<lower=0,upper=1> mu;
  real<lower=0> eta;
  real<lower=0,upper=1> theta[N];
}

transformed parameters {
  real<lower=0> alpha;
  real<lower=0> beta;
  alpha = eta* mu ;
  beta = eta*(1-mu);
}


model {
    //Hyper-priors
    eta ~ exponential(e);




    //Prior
    for (k in 1:N){

     mu ~ beta(a[k],b[k]);
     theta[k] ~ beta(alpha,beta);

    //Likelihood
      y[k] ~ binomial(n[k], theta[k]);
    }

  }
generated quantities {

  vector[N] log_lik;
  real ypred[N];

  for(i in 1:N){
        log_lik[i] = binomial_lpmf(y[i]|n[i], theta[i]);
      }


    for(j in 1:N){
        ypred[j]= binomial_rng(n[j],theta[j]);
        }
}// The posterior predictive distribution",
"hierachichal_model.stan")
```

Running the model

US data

Separate model

First we need to read the data and transform it into a data frame.

```
library("readxl")
library("rstan")
exceldata = read_excel("US2020.xlsx")
dfData= data.frame(exceldata)
```

Then we read the files that have the mean and of the number of live births and the number of deaths in previous years. In the case of US average was obtained from 2007-2018

```
exceldata = read_excel("aUS.xlsx")
dfA= data.frame(exceldata)
exceldata = read_excel("bUS.xlsx")
dfB= data.frame(exceldata)
```

Then we compute the model with the data

```
library(loo)

deaths = dfData$Deaths
births = dfData$Births

stan_data <- list(
  N = 51,
  y = deaths,
  n = births,
  aMean = dfA$Mean,
  bMean = dfB$Mean
)

sm <- rstan::stan_model(file = "separate_model.stan")
separate_modelUS <- rstan::sampling(sm, data = stan_data, refresh= 0)
```

Hierarchical model

```
sm <- rstan::stan_model(file = "hierarchical_model.stan")
stan_data_hierachichal <- list(
  N=51,
  y = deaths,
  n = births,
  a = dfA$Mean,
  b = dfB$Mean,
  e=1/60000
)
hierarchical_modelUS <- rstan::sampling(sm, data = stan_data_hierachichal,refresh=0)
```

Colombia data

First we need to read the data and transform it into a data frame.

```
exceldata = read_excel("Colombia2020.xlsx")
dfData= data.frame(exceldata)
```

Then we read the files that have the mean and of the number of live births and the number of deaths in previous years. In the case of Colombia these values were obtained using the mean from 2017 to 2019.

```
exceldata = read_excel("a.xlsx")
dfA= data.frame(exceldata)
exceldata = read_excel("b.xlsx")
dfB= data.frame(exceldata)
```

Separate model

```
library(bayesplot)
library(loo)

deaths = dfData$Deaths
births = dfData$Lbirths+ dfData$Deaths

stan_data <- list(
  N = 37,
  y = deaths,
  n = births,
  aMean = dfA$Mean,
  bMean = dfB$Mean
)

sm <- rstan::stan_model(file = "separate_model.stan")
separate_modelCol <- rstan::sampling(sm, data = stan_data, refresh= 0)
```

Hierarchical model

```
sm <- rstan::stan_model(file = "hierarchical_model.stan")

stan_data_hierachichal <- list(
  N=37,
  y = deaths,
  n = births,
  a = dfA$Mean,
  b = dfB$Mean,
  e=1/20000
)
hierarchical_modelCol <- rstan::sampling(sm, data = stan_data_hierachichal,refresh=0)
```

Convergence diagnostics ($\hat{R}$, ESS, divergences) and what was done if the convergence was not good with the first try.

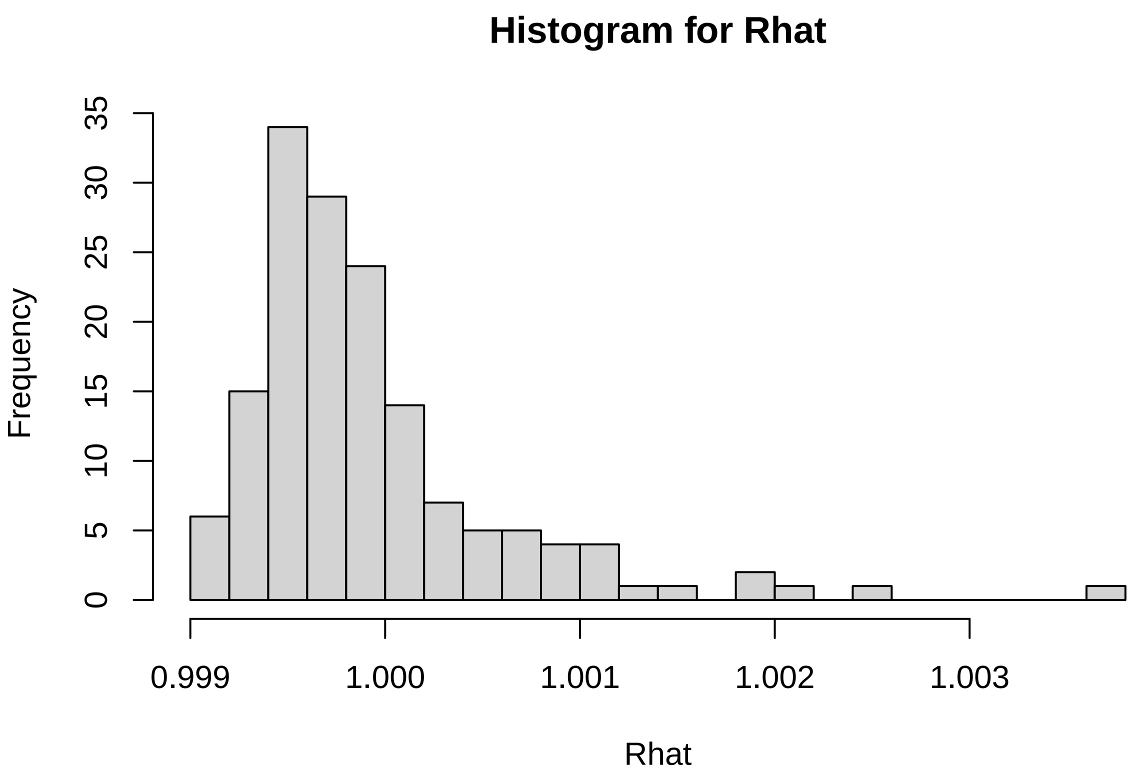
For convergence diagnostics Rhat and Bulk_ESS values were used. This statistic compares between chain and within chain drawns for each of the model parameters. If chains have not converged well, Rhat will be larger than 1.

US Data

Separate model

Below there are the histograms of Rhat for each one of the models:

```
s <- summary(separate_modelUS)$summary
Rhat <- s[,10]
hist(Rhat,
    main="Histogram for Rhat",
     xlab="Rhat",
     breaks=20)
```




```
check_divergences(separate_modelUS)
```

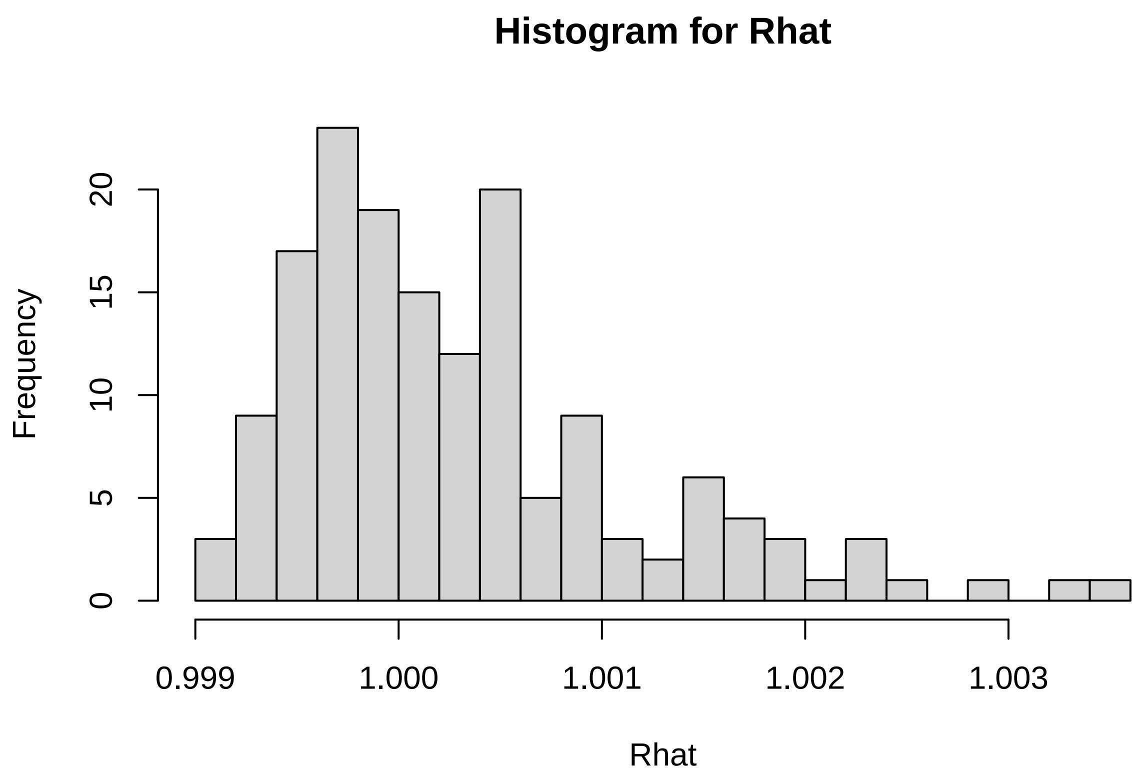
```
## 0 of 4000 iterations ended with a divergence.
```

```
check_treedepth(separate_modelUS)
```

```
## 0 of 4000 iterations saturated the maximum tree depth of 10.
```

Hierarchical model

```
s <- summary(hierarchical_modelUS)$summary
Rhat <- s[,10]
hist(Rhat,
    main="Histogram for Rhat",
     xlab="Rhat",
     breaks=20)
```




```
check_divergences(hierarchical_modelUS)
```

```
## 0 of 4000 iterations ended with a divergence.
```

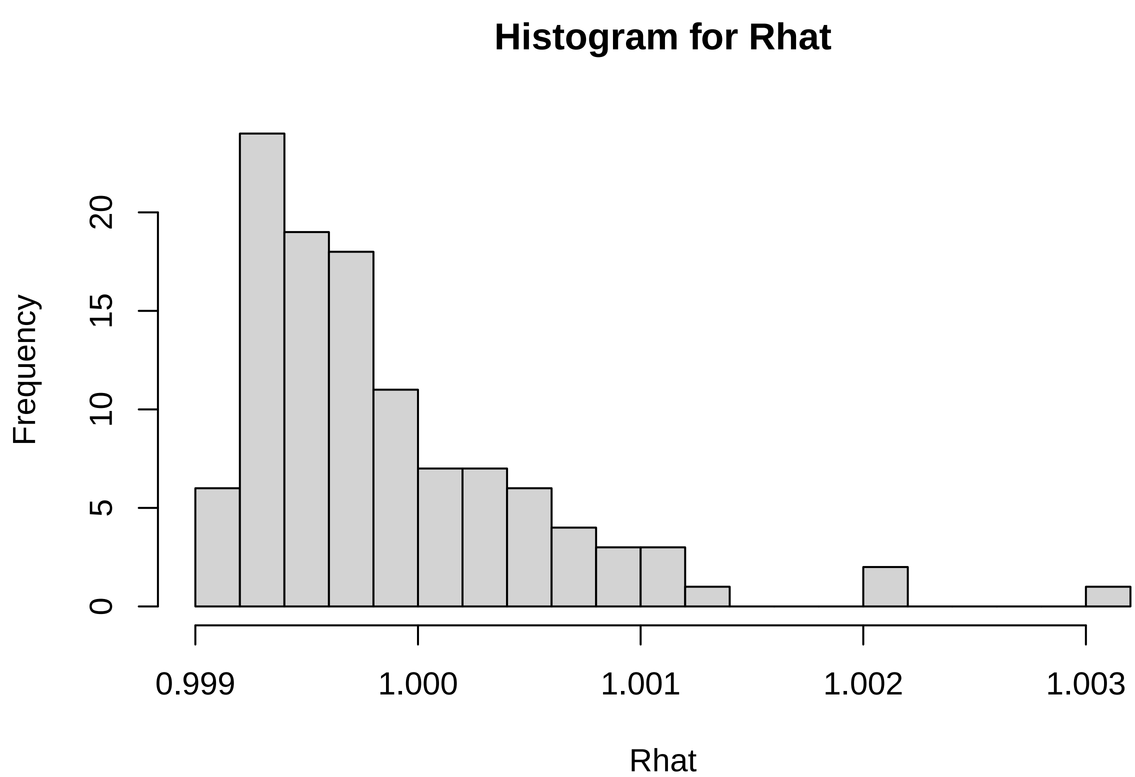
```
check_treedepth(hierarchical_modelUS)
```

```
## 0 of 4000 iterations saturated the maximum tree depth of 10.
```

Colombia data

Separate model

```
s <- summary(separate_modelCol)$summary
Rhat <- s[,10]
hist(Rhat,
    main="Histogram for Rhat",
     xlab="Rhat",
     breaks=20)
```




```
check_divergences(separate_modelCol)
```

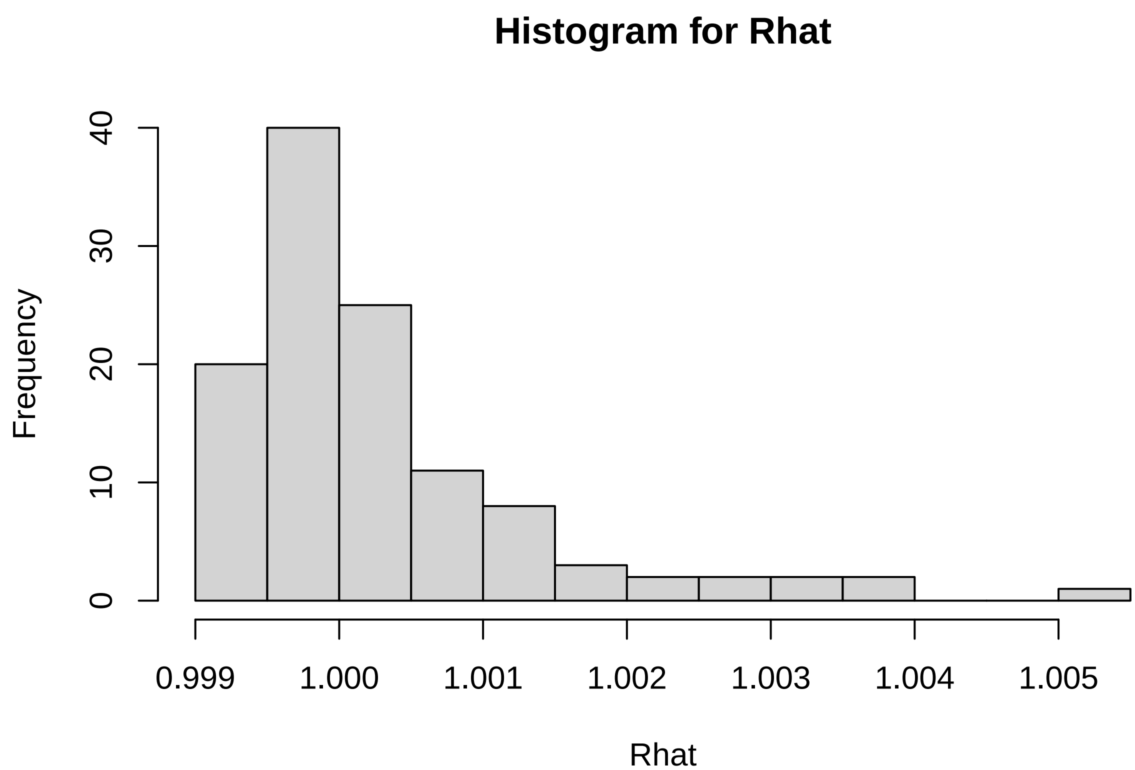
```
## 0 of 4000 iterations ended with a divergence.
```

```
check_treedepth(separate_modelCol)
```

```
## 0 of 4000 iterations saturated the maximum tree depth of 10.
```

Hierarchical model

```r
s <- summary(hierarchical_modelCol)$summary
Rhat <- s[,10]
hist(Rhat,
    main="Histogram for Rhat",
     xlab="Rhat",
     breaks=20)
```




```r
check_divergences(hierarchical_modelCol)
```

```
## 0 of 4000 iterations ended with a divergence.
```

```r
check_treedepth(hierarchical_modelCol)
```

```
## 0 of 4000 iterations saturated the maximum tree depth of 10.
```

In all of these histograms we see that Rhat values are really close to 1 and none of Rhat values are below 1.05 which is recommended in Stan official website.

Additionally if you check the **Appendix** section and **Convergence monitoring** all values of Bulk Effective sample size are above 100 so the models are considered good.

Finally, there were not any warnings regarding iterations ending in divergence or saturating the maximum tree depth of 10 for any of the models.

Model improvements

Separate model

For the separate model, the prior was first tried to be computed using another hyper-prior information assuming that parameters a (number of deaths) and b (number of living births) will follow a normal distribution based on past data. The following lines contain the previous model priors:

```
model {


  //Hyper priors
  for (i in 1:N) {
      a[i] ~ normal(aMean[i], aStd[i]) T[L,]; //Number of successes parameter
      b[i] ~ normal(bMean[i], bStd[i]) T[L,]; //Number of no Success
    }

  //Priors

    for (j in 1:N) {
      theta[j] ~ beta(a[j], b[j]);
    }

    //Likelihood
    for (k in 1:N){
      y[k] ~ binomial(n[k], theta[k]);
    }

}
```

However, doing the k-pareto analysis, there were many values between 0.7 and 1 and even higher than 1, so this choice was discarded.

Hierarchical model

Many values were chosen for hyper-priors of parameters μ and η. We start using gamma function for parameter η and one single distribution for μ. Like this:

```
model {


model {
  //Hyper-priors
  eta ~ gamma(s,t);


  mu ~ beta(a,b);

    //Prior
    for (k in 1:N){
      theta[k] ~ beta(alpha,beta);

    //Likelihood
```

```
      y[k] ~ binomial(n[k], theta[k]);

  }

}
```

However this common parameter η distribution was changed to exponential since it is a weakly hyper-prior and μ different distributions were obtained depending on the region. This changes improved k-pareto diagnostics a little bit.

Model comparison

US Data

Separate model

```r
separate_extract_log_lik <- extract_log_lik(separate_modelUS,
                                             parameter_name = "log_lik",
                                             merge_chains = FALSE);
r_eff <- relative_eff(exp(separate_extract_log_lik), cores = 2)
separate_model_loo <- loo(separate_extract_log_lik, r_eff = r_eff, cores = 2)
print(separate_model_loo)
```

```
##
## Computed from 4000 by 51 log-likelihood matrix
##
##          Estimate   SE
## elpd_loo   -232.7  9.4
## p_loo        30.9  4.6
## looic       465.5 18.9
## ------
## Monte Carlo SE of elpd_loo is NA.
##
## Pareto k diagnostic values:
##                          Count Pct.    Min. n_eff
## (-Inf, 0.5]   (good)     18    35.3%   451
##  (0.5, 0.7]   (ok)       24    47.1%   98
##    (0.7, 1]   (bad)       9    17.6%   23
##    (1, Inf)   (very bad)  0     0.0%   <NA>
## See help('pareto-k-diagnostic') for details.
```

Hierarchical model

```r
hierarchical_extract_log_lik <- extract_log_lik(hierarchical_modelUS,
                                                 parameter_name = "log_lik",
                                                 merge_chains = FALSE);
r_eff <- relative_eff(exp(hierarchical_extract_log_lik), cores = 2)
hierarchical_model_loo <- loo(hierarchical_extract_log_lik, r_eff = r_eff, cores = 2)
print(hierarchical_model_loo)
```

```
##
## Computed from 4000 by 51 log-likelihood matrix
##
##          Estimate   SE
## elpd_loo   -229.3  5.7
## p_loo        34.0  2.7
## looic       458.5 11.5
## ------
## Monte Carlo SE of elpd_loo is NA.
##
## Pareto k diagnostic values:
##                          Count Pct.    Min. n_eff
## (-Inf, 0.5]   (good)      3     5.9%   864
##  (0.5, 0.7]   (ok)       17    33.3%   123
##    (0.7, 1]   (bad)      26    51.0%   20
##    (1, Inf)   (very bad)  5     9.8%   11
## See help('pareto-k-diagnostic') for details.
```

Colombia Data

Separate model

```r
separate_extract_log_lik <- extract_log_lik(separate_modelCol,
                                            parameter_name = "log_lik",
                                            merge_chains = FALSE);
r_eff <- relative_eff(exp(separate_extract_log_lik), cores = 2)
separate_model_loo <- loo(separate_extract_log_lik, r_eff = r_eff, cores = 2)
print(separate_model_loo)
```

```
##
## Computed from 4000 by 37 log-likelihood matrix
##
##          Estimate   SE
## elpd_loo   -165.0  6.7
## p_loo        21.8  3.9
## looic       330.1 13.5
## ------
## Monte Carlo SE of elpd_loo is NA.
##
## Pareto k diagnostic values:
##                          Count Pct.    Min. n_eff
## (-Inf, 0.5]   (good)     11    29.7%   739
##  (0.5, 0.7]   (ok)       18    48.6%   413
##    (0.7, 1]   (bad)       7    18.9%   81
##    (1, Inf)   (very bad)  1     2.7%   12
## See help('pareto-k-diagnostic') for details.
```

Hierarchical model

```r
hierarchical_extract_log_lik <- extract_log_lik(hierarchical_modelCol,
                                                parameter_name = "log_lik",
                                                merge_chains = FALSE);
r_eff <- relative_eff(exp(hierarchical_extract_log_lik), cores = 2)
hierarchical_model_loo <- loo(hierarchical_extract_log_lik, r_eff = r_eff, cores = 2)
print(hierarchical_model_loo)
```

```
##
## Computed from 4000 by 37 log-likelihood matrix
##
##          Estimate   SE
## elpd_loo   -174.1  5.8
## p_loo        32.6  4.2
## looic       348.1 11.7
## ------
## Monte Carlo SE of elpd_loo is NA.
##
## Pareto k diagnostic values:
##                          Count Pct.    Min. n_eff
## (-Inf, 0.5]   (good)      0     0.0%   <NA>
##  (0.5, 0.7]   (ok)        8    21.6%   118
##    (0.7, 1]   (bad)      23    62.2%   30
##    (1, Inf)   (very bad)  6    16.2%   3
## See help('pareto-k-diagnostic') for details.
```

Looking at k-pareto analysis since the majority of k values are below 0.7 in the separate model, this model was chosen as the best for both data sets. In colombian data the elpd_loo is greater in the separate model, that is another reason why this model was chosen.

Posterior checking and results

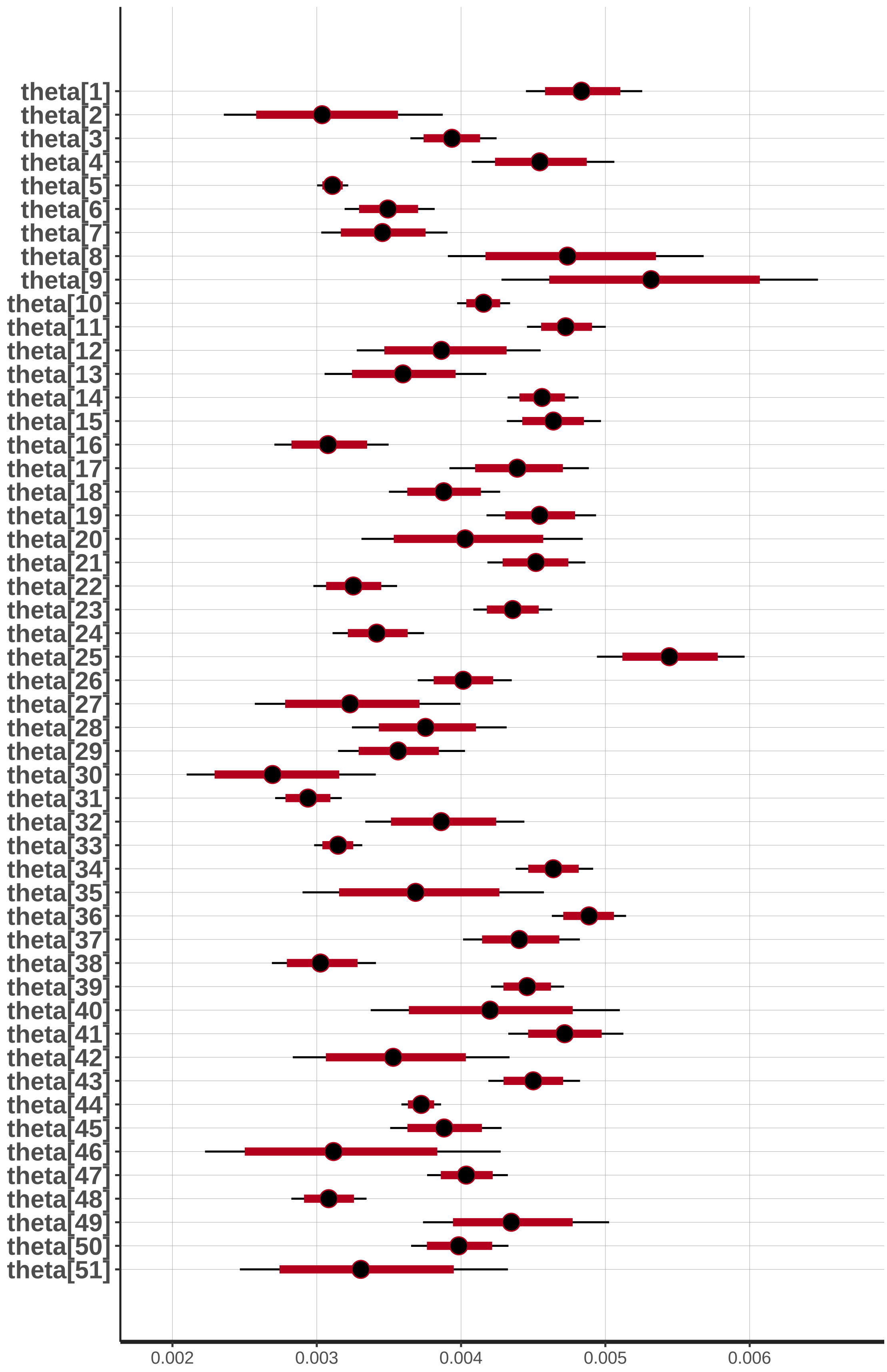
As the best model was the separate, we will use the posterior draws of this model to compare the theta parameter for the two countries.

The US:

```r
plot(separate_modelUS, pars=c('theta'))
```

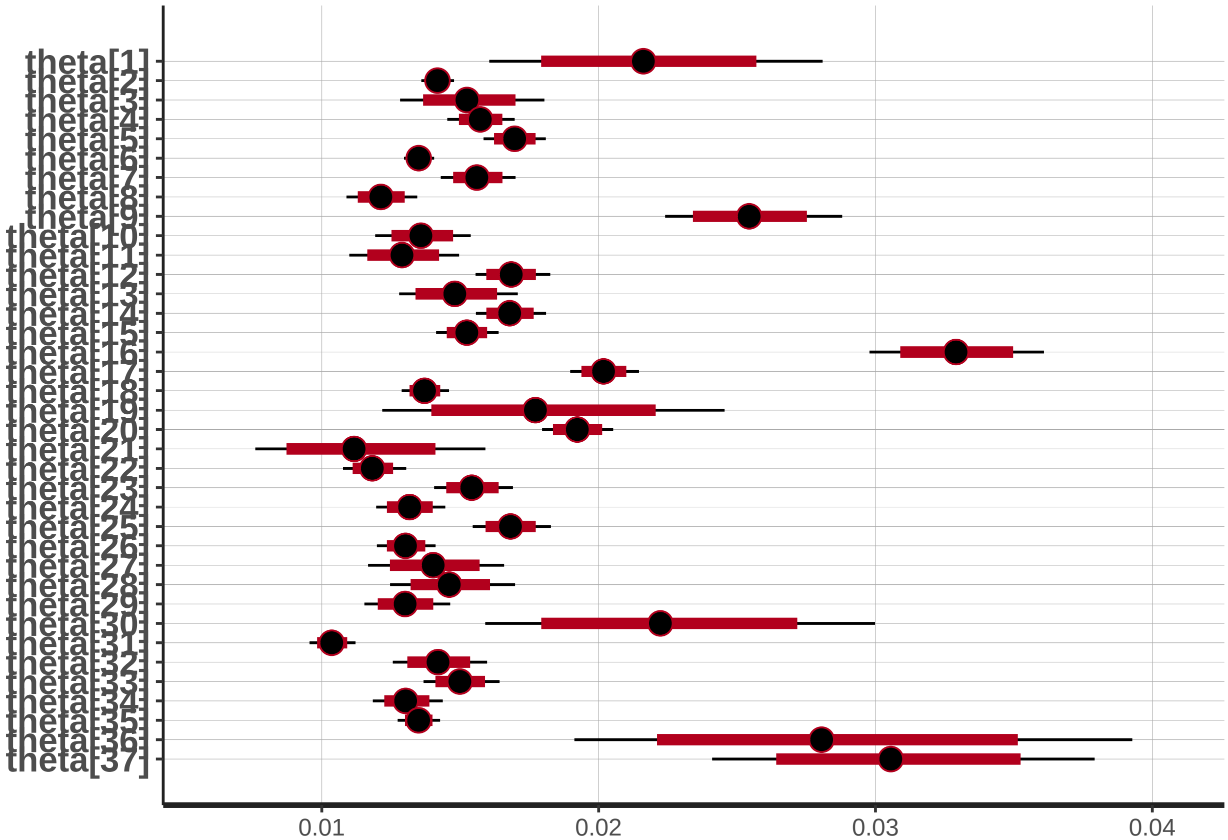
```
## ci_level: 0.8 (80% intervals)
```

```
## outer_level: 0.95 (95% intervals)
```


theta[1]
theta[2]
theta[3]
theta[4]
theta[5]
theta[6]
theta[7]
theta[8]
theta[9]
theta[10]
theta[11]
theta[12]
theta[13]
theta[14]
theta[15]
theta[16]
theta[17]
theta[18]
theta[19]
theta[20]
theta[21]
theta[22]
theta[23]
theta[24]
theta[25]
theta[26]
theta[27]
theta[28]
theta[29]
theta[30]
theta[31]
theta[32]
theta[33]
theta[34]
theta[35]
theta[36]
theta[37]
theta[38]
theta[39]
theta[40]
theta[41]
theta[42]
theta[43]
theta[44]
theta[45]
theta[46]
theta[47]
theta[48]
theta[49]
theta[50]
theta[51]
0.002
0.003
0.004
0.005
0.006

Colombia:

```
plot(separate_modelCol, pars=c('theta'))
```



As you can see from the figures, which represent the confidence interval and the mean value of the probability of death in the perinatal and late neonatal period, the probability parameter is drastically larger among all Colombian states compared to the ones in the US.

Regarding within-country comparison and using these posterior draws summary graph we can rank the top 5 regions of both countries with worst perinatal and late neonatal mortality rate.

In Colombia the top 5 worst states are: 5. Cordoba ($\bar{\theta}$) of 0.0202, 4. Buenaventura ($\bar{\theta}$) of 0.0284 3. Vaupes ($\bar{\theta}$) of 0.0284, 2. Vichada ($\bar{\theta}$) of 0.0306 and 1. Choco with a ($\bar{\theta}$) of 0.0330.

In the US, the top 5 worst states are 5. South Carolina ($\bar{\theta}$) of 0.00472, 4. Alabama ($\bar{\theta}$) of 0.00484 3. Ohio ($\bar{\theta}$) of 0.00488, 2. Columbia ($\bar{\theta}$) of 0.00534 and 1. Mississippi with a ($\bar{\theta}$) of 0.00545.

It is interesting that Choco, Vichada and Vaupes are among the top 5 poorest states in Colombia [2] while South Carolina, Columbia, Mississipi and Alabama are between the top 10 poorest states in the US [3].

With this in mind, there is reason to suspect that the state wealth variable is related to the rate of perinatal and late neonatal mortality.

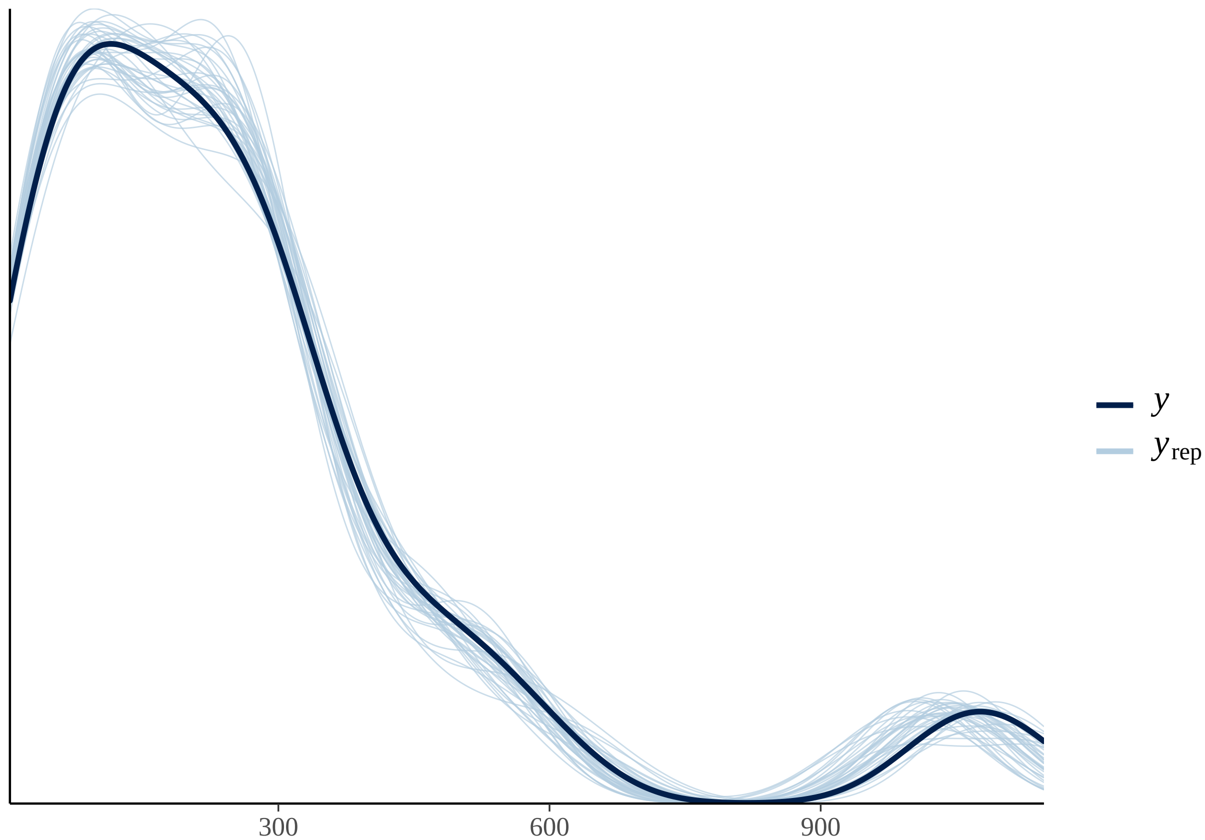
Posterior predictive checking

The posterior predictive checking was done using the best model which was the separate one in Colombian data.

```r
y_pred <- as.matrix(separate_modelCol, pars = "ypred")
dim(y_pred)
```

```
## [1] 4000   37
```

```r
ppc_dens_overlay(deaths, y_pred[1:37, ])
```



As you can see from the figure, the predictions are quite similar to the real value of y (number of deaths in neonatal and perinatal period). The predictive performance is quite good.

Sensitivity analysis with respect to prior choices (i.e. checking whether the result changes a lot if prior is changed)

Separate model

As mentioned in model improvements, some other priors were chosen before, specifically hyper-priors

```
  //Hyper priors
  for (i in 1:N) {
      a[i] ~ normal(aMean[i], aStd[i]) T[L,]; //Number of successes parameter
      b[i] ~ normal(bMean[i], bStd[i]) T[L,]; //Number of no Success
    }
```

The results of the posterior draws were almost the same but when performing k-pareto diagnostics, results were worse (more k values above 0.7).

Hierarchical

As mentioned in model improvements, some other priors were chosen before, specifically hyper-priors

```
//Hyper-priors
eta ~ gamma(s,t);
```

Many values were manually selected from $s.t$ to resemble te distribution of the total number of births, but the k-pareto diagnostics gave worse results (more k values above 0.7).

Discussion of issues and potential improvements.

The hierarchical model did not perform well in k-pareto diagnostics, many k values were above 0.7. Maybe to improve our model we can work with the data points of the previous years, first to improve k-values and second to make a predictive model by state so that the probability of perinatal and neonatal mortality by state can be forecasted in future years. Probably using models that fit data series such as ARIMA or ARIMAx.

Conclusion what was learned from the data analysis.

From our point of view, we learned and reinforced how to model using hierarchical and non-hierarchical models in case of binomial trials. Also we learn how to work on the model (choosing priors is not easy!) and also how to do all kinds of calculations to get information about convergence and how well the model fits the data. We also learned about model comparison and basically we have the tools to do Bayesian modeling in any area.

Self-reflection of what the group learned while making the project.

We as a group learned how we can work together first in finding the data then doing some research to find the appropriate model for the problem we want to solve. We learned how we can improve models by sharing our knowledge and experience and finally doing our best together to improve the proposed model and solution to the related problem

References

[1] "Neonatal Mortality." UNICEF DATA, 20 July 2021, https://data.unicef.org/topic/child-survival/neonatal-mortality/. [2] Semana Magazine. "Pobreza En Colombia: Estos Son Los Departamentos Más Afectados." Semana.com Últimas Noticias De Colombia y El Mundo, 21 Dec. 2020, https://www.semana.com/economia/articulo/pobreza-en-colombia-estos-son-los-departamentos-mas-afectados/202026/. [3] November 10, 2021. "Top 10 Poorest States in the U.S." Friends Committee On National Legislation, https://www.fcnl.org/updates/2021-11/top-10-poorest-states-us. Vehtari, Aki, and Markus Paasiniemi. "BDA3 Demos Comparison of 2 Groups with Binomial." Aalto University. Niemi, Jarad. "Hierarchical Models." PPT file. Iowa State University, 2019. Hu, Jim Albert and

Jingchen. "Probability and Bayesian Modeling." Chapter 10 Bayesian Hierarchical Modeling, 30 July 2020, https://bayesball.github.io/BOOK/bayesian-hierarchical-modeling.html. Brody-Moore, Peter, "Bayesian Hierarchical Meta-Analysis of Asymptomatic Ebola Seroprevalence" (2019). CMC Senior Theses. 2228.https://scholarship.claremont.edu/cmc_theses/2228

Appendix

Convergence monitoring

US Data

Separate model

```
sUS<- monitor(separate_modelUS)$Bulk_ESS
```

```
## Inference for the input samples (4 chains: each with iter = 2000; warmup = 0):
##
##                    Q5       Q50       Q95      Mean   SD  Rhat Bulk_ESS
## theta[1]          0.0       0.0       0.0       0.0  0.0     1     7149
## theta[2]          0.0       0.0       0.0       0.0  0.0     1     6492
## theta[3]          0.0       0.0       0.0       0.0  0.0     1     7188
## theta[4]          0.0       0.0       0.0       0.0  0.0     1     6583
## theta[5]          0.0       0.0       0.0       0.0  0.0     1     5984
## theta[6]          0.0       0.0       0.0       0.0  0.0     1     6756
## theta[7]          0.0       0.0       0.0       0.0  0.0     1     6728
## theta[8]          0.0       0.0       0.0       0.0  0.0     1     5760
## theta[9]          0.0       0.0       0.0       0.0  0.0     1     6410
## theta[10]         0.0       0.0       0.0       0.0  0.0     1     6668
## theta[11]         0.0       0.0       0.0       0.0  0.0     1     7088
## theta[12]         0.0       0.0       0.0       0.0  0.0     1     6024
## theta[13]         0.0       0.0       0.0       0.0  0.0     1     6442
## theta[14]         0.0       0.0       0.0       0.0  0.0     1     6359
## theta[15]         0.0       0.0       0.0       0.0  0.0     1     5993
## theta[16]         0.0       0.0       0.0       0.0  0.0     1     6336
## theta[17]         0.0       0.0       0.0       0.0  0.0     1     6823
## theta[18]         0.0       0.0       0.0       0.0  0.0     1     5161
## theta[19]         0.0       0.0       0.0       0.0  0.0     1     5801
## theta[20]         0.0       0.0       0.0       0.0  0.0     1     6581
## theta[21]         0.0       0.0       0.0       0.0  0.0     1     6998
## theta[22]         0.0       0.0       0.0       0.0  0.0     1     6372
## theta[23]         0.0       0.0       0.0       0.0  0.0     1     6945
## theta[24]         0.0       0.0       0.0       0.0  0.0     1     6642
## theta[25]         0.0       0.0       0.0       0.0  0.0     1     5609
## theta[26]         0.0       0.0       0.0       0.0  0.0     1     6632
## theta[27]         0.0       0.0       0.0       0.0  0.0     1     7208
## theta[28]         0.0       0.0       0.0       0.0  0.0     1     6532
## theta[29]         0.0       0.0       0.0       0.0  0.0     1     6744
## theta[30]         0.0       0.0       0.0       0.0  0.0     1     5865
## theta[31]         0.0       0.0       0.0       0.0  0.0     1     7939
## theta[32]         0.0       0.0       0.0       0.0  0.0     1     7022
```

```
## theta[33]          0.0       0.0       0.0       0.0  0.0     1     6424
## theta[34]          0.0       0.0       0.0       0.0  0.0     1     5808
## theta[35]          0.0       0.0       0.0       0.0  0.0     1     6944
## theta[36]          0.0       0.0       0.0       0.0  0.0     1     6922
## theta[37]          0.0       0.0       0.0       0.0  0.0     1     6922
## theta[38]          0.0       0.0       0.0       0.0  0.0     1     5154
## theta[39]          0.0       0.0       0.0       0.0  0.0     1     7185
## theta[40]          0.0       0.0       0.0       0.0  0.0     1     5864
## theta[41]          0.0       0.0       0.0       0.0  0.0     1     6218
## theta[42]          0.0       0.0       0.0       0.0  0.0     1     6573
## theta[43]          0.0       0.0       0.0       0.0  0.0     1     7306
## theta[44]          0.0       0.0       0.0       0.0  0.0     1     7162
## theta[45]          0.0       0.0       0.0       0.0  0.0     1     7523
## theta[46]          0.0       0.0       0.0       0.0  0.0     1     6585
## theta[47]          0.0       0.0       0.0       0.0  0.0     1     7408
## theta[48]          0.0       0.0       0.0       0.0  0.0     1     6210
## theta[49]          0.0       0.0       0.0       0.0  0.0     1     6987
## theta[50]          0.0       0.0       0.0       0.0  0.0     1     5848
## theta[51]          0.0       0.0       0.0       0.0  0.0     1     6334
## log_lik[1]        -7.6      -5.1      -3.8      -5.3  1.2     1     7083
## log_lik[2]        -3.9      -2.8      -2.7      -3.0  0.5     1     2601
## log_lik[3]        -4.8      -3.9      -3.8      -4.1  0.4     1     1821
## log_lik[4]        -4.8      -3.7      -3.5      -3.8  0.5     1     2746
## log_lik[5]        -8.1      -5.8      -4.7      -6.0  1.1     1     6023
## log_lik[6]        -5.8      -4.1      -3.6      -4.3  0.7     1     5872
## log_lik[7]        -6.3      -4.1      -3.3      -4.4  1.0     1     6635
## log_lik[8]        -5.1      -3.4      -2.8      -3.6  0.8     1     5596
## log_lik[9]        -5.2      -3.3      -2.8      -3.6  0.8     1     5341
## log_lik[10]       -6.0      -4.6      -4.3      -4.8  0.6     1     4891
## log_lik[11]       -5.1      -4.2      -4.1      -4.4  0.4     1     1694
## log_lik[12]       -3.9      -3.1      -3.0      -3.3  0.3     1     1912
## log_lik[13]       -4.2      -3.2      -3.1      -3.4  0.4     1     2236
## log_lik[14]       -5.2      -4.3      -4.2      -4.4  0.4     1     2258
## log_lik[15]       -5.2      -4.0      -3.9      -4.2  0.5     1     2988
## log_lik[16]       -4.3      -3.4      -3.3      -3.6  0.4     1     1717
## log_lik[17]       -4.5      -3.6      -3.5      -3.7  0.4     1     1798
## log_lik[18]       -5.0      -3.8      -3.6      -4.0  0.5     1     3194
## log_lik[19]       -4.8      -3.8      -3.7      -4.0  0.4     1     2348
## log_lik[20]       -4.0      -3.0      -2.8      -3.1  0.4     1     2646
## log_lik[21]       -7.4      -5.0      -3.9      -5.2  1.1     1     7130
## log_lik[22]       -4.5      -3.7      -3.6      -3.9  0.3     1     1728
## log_lik[23]       -7.7      -5.2      -4.1      -5.5  1.1     1     6652
## log_lik[24]       -4.6      -3.8      -3.6      -3.9  0.4     1     1883
## log_lik[25]       -4.8      -3.7      -3.6      -3.9  0.5     1     3069
## log_lik[26]       -5.8      -4.1      -3.7      -4.4  0.7     1     5671
## log_lik[27]       -3.8      -2.8      -2.7      -3.0  0.4     1     2930
## log_lik[28]       -4.3      -3.3      -3.2      -3.5  0.4     1     2300
## log_lik[29]       -4.4      -3.5      -3.3      -3.6  0.4     1     2195
## log_lik[30]       -4.1      -2.8      -2.6      -3.0  0.5     1     4003
## log_lik[31]       -8.3      -5.6      -4.0      -5.8  1.4     1     7872
## log_lik[32]       -4.6      -3.4      -3.2      -3.6  0.5     1     3006
## log_lik[33]       -9.6      -6.4      -4.6      -6.7  1.5     1     6411
## log_lik[34]       -8.7      -5.9      -4.3      -6.1  1.4     1     5835
## log_lik[35]       -4.2      -2.9      -2.7      -3.1  0.5     1     3517
```

```
## log_lik[36]      -6.7      -4.8      -4.2      -5.0  0.8     1     6293
## log_lik[37]      -4.9      -3.8      -3.6      -3.9  0.4     1     3088
## log_lik[38]      -6.6      -4.3      -3.3      -4.5  1.0     1     5217
## log_lik[39]      -7.1      -4.9      -4.1      -5.2  1.0     1     7330
## log_lik[40]      -4.3      -3.0      -2.8      -3.2  0.5     1     4493
## log_lik[41]      -4.9      -3.9      -3.7      -4.0  0.4     1     2700
## log_lik[42]      -5.2      -3.3      -2.7      -3.5  0.8     1     6586
## log_lik[43]      -4.8      -4.0      -3.9      -4.1  0.4     1     1775
## log_lik[44]      -6.6      -4.9      -4.5      -5.2  0.7     1     5395
## log_lik[45]      -5.4      -3.9      -3.6      -4.1  0.6     1     4809
## log_lik[46]      -3.2      -2.4      -2.3      -2.6  0.3     1     1943
## log_lik[47]      -7.7      -5.2      -4.0      -5.4  1.2     1     7300
## log_lik[48]      -4.9      -3.9      -3.7      -4.0  0.4     1     2545
## log_lik[49]      -4.0      -3.2      -3.1      -3.3  0.3     1     2001
## log_lik[50]      -5.1      -3.9      -3.7      -4.1  0.5     1     3583
## log_lik[51]      -3.8      -2.6      -2.4      -2.8  0.5     1     4518
## ypred[1]        246.0     279.0     315.0     279.9 20.8     1     4617
## ypred[2]         20.0      31.0      43.0      31.0  6.9     1     4542
## ypred[3]        282.0     317.0     354.0     317.3 21.6     1     4910
## ypred[4]        143.0     168.0     196.0     168.4 16.2     1     4515
## ypred[5]       1341.0    1415.5    1490.0    1415.0 45.0     1     4437
## ypred[6]        191.0     220.0     249.0     219.8 17.8     1     4689
## ypred[7]         99.0     120.0     142.0     120.3 13.3     1     4903
## ypred[8]         37.0      50.0      65.0      50.6  8.5     1     4339
## ypred[9]         35.0      49.0      64.0      49.1  8.7     1     4648
## ypred[10]       860.0     920.0     982.0     920.2 37.2     1     3957
## ypred[11]       547.0     596.0     645.0     596.5 30.0     1     4477
## ypred[12]        50.0      66.0      83.0      65.9  9.9     1     4256
## ypred[13]        60.0      77.0      95.0      77.2 10.7     1     4854
## ypred[14]       610.0     660.0     713.0     660.8 31.5     1     4478
## ypred[15]       342.0     379.0     419.0     379.0 23.7     1     4152
## ypred[16]        96.0     116.0     139.0     116.8 13.3     1     4235
## ypred[17]       134.0     159.0     186.0     159.2 15.6     1     4408
## ypred[18]       180.0     209.0     239.0     209.1 18.1     1     4659
## ypred[19]       239.0     271.0     304.0     271.0 19.8     1     4486
## ypred[20]        36.0      50.0      64.0      49.7  8.6     1     4357
## ypred[21]       286.0     320.0     357.0     320.7 21.5     1     4720
## ypred[22]       197.0     225.0     256.0     225.1 18.0     1     4319
## ypred[23]       436.0     480.0     524.0     479.9 26.4     1     4691
## ypred[24]       200.0     230.0     262.0     230.4 18.6     1     4888
## ypred[25]       174.0     201.0     230.0     201.3 17.3     1     4551
## ypred[26]       262.0     294.0     329.0     294.6 20.5     1     5292
## ypred[27]        26.0      37.0      50.0      37.5  7.4     1     4676
## ypred[28]        77.0      96.0     116.0      95.9 11.8     1     5013
## ypred[29]       105.0     127.0     150.0     127.3 13.7     1     4621
## ypred[30]        22.0      32.0      45.0      32.5  6.9     1     4052
## ypred[31]       264.0     297.0     333.0     297.7 21.0     1     4909
## ypred[32]        71.0      89.0     109.0      89.3 11.7     1     4655
## ypred[33]       660.0     712.0     764.0     712.0 32.0     1     4629
## ypred[34]       507.0     552.0     601.0     552.3 28.5     1     4619
## ypred[35]        28.0      39.0      53.0      39.5  7.7     1     4802
## ypred[36]       609.0     660.0     710.0     660.3 31.2     1     4438
## ypred[37]       191.0     219.0     250.0     219.6 18.2     1     4495
## ypred[38]       106.0     128.0     151.0     128.2 13.8     1     4083
```

```
## ypred[39]       557.0     605.0     655.0     605.3 29.9     1     4442
## ypred[40]        32.0      44.0      58.0      44.5  8.1     1     4739
## ypred[41]       235.0     267.0     300.0     267.6 19.8     1     4313
## ypred[42]        30.0      42.0      56.0      42.0  7.9     1     4592
## ypred[43]       325.0     362.0     401.0     362.8 23.0     1     4474
## ypred[44]      1335.0    1410.0    1487.0    1410.5 46.0     1     5001
## ypred[45]       158.0     183.0     212.0     183.8 16.5     1     4835
## ypred[46]         9.0      17.0      26.0      17.1  5.0     1     4499
## ypred[47]       364.0     403.0     445.0     403.0 24.6     1     4812
## ypred[48]       233.0     265.0     298.0     265.1 19.9     1     4921
## ypred[49]        63.0      79.0      98.0      79.6 10.8     1     5057
## ypred[50]       224.0     255.0     289.0     255.6 19.8     1     4104
## ypred[51]        13.0      22.0      31.0      21.8  5.6     1     4806
## lp__         -199370.7 -199361.3 -199354.0 -199361.7  5.0     1     1589
##              Tail_ESS
## theta[1]        2951
## theta[2]        2838
## theta[3]        2755
## theta[4]        3085
## theta[5]        3017
## theta[6]        2522
## theta[7]        3133
## theta[8]        3016
## theta[9]        2921
## theta[10]       3333
## theta[11]       2852
## theta[12]       3210
## theta[13]       3012
## theta[14]       2975
## theta[15]       2748
## theta[16]       2479
## theta[17]       2311
## theta[18]       2779
## theta[19]       2823
## theta[20]       2854
## theta[21]       3204
## theta[22]       3213
## theta[23]       2967
## theta[24]       3029
## theta[25]       2859
## theta[26]       2659
## theta[27]       2970
## theta[28]       2766
## theta[29]       2852
## theta[30]       2996
## theta[31]       2857
## theta[32]       2637
## theta[33]       2824
## theta[34]       2962
## theta[35]       2875
## theta[36]       2982
## theta[37]       3035
## theta[38]       3243
## theta[39]       2916
```

```
## theta[40]        2762
## theta[41]        2828
## theta[42]        2696
## theta[43]        2909
## theta[44]        2991
## theta[45]        2923
## theta[46]        2601
## theta[47]        3251
## theta[48]        3139
## theta[49]        2725
## theta[50]        2922
## theta[51]        2774
## log_lik[1]       2951
## log_lik[2]       2666
## log_lik[3]       2034
## log_lik[4]       2836
## log_lik[5]       3068
## log_lik[6]       2522
## log_lik[7]       3271
## log_lik[8]       3016
## log_lik[9]       3410
## log_lik[10]      3651
## log_lik[11]      2048
## log_lik[12]      2133
## log_lik[13]      2657
## log_lik[14]      2411
## log_lik[15]      3050
## log_lik[16]      1725
## log_lik[17]      1671
## log_lik[18]      2992
## log_lik[19]      2540
## log_lik[20]      2668
## log_lik[21]      3210
## log_lik[22]      2753
## log_lik[23]      2967
## log_lik[24]      1975
## log_lik[25]      2678
## log_lik[26]      3129
## log_lik[27]      2426
## log_lik[28]      2615
## log_lik[29]      2414
## log_lik[30]      3197
## log_lik[31]      2857
## log_lik[32]      2707
## log_lik[33]      2824
## log_lik[34]      2962
## log_lik[35]      3211
## log_lik[36]      3039
## log_lik[37]      3001
## log_lik[38]      3251
## log_lik[39]      3453
## log_lik[40]      3354
## log_lik[41]      2562
## log_lik[42]      2988
```

```
## log_lik[43]      2217
## log_lik[44]      2991
## log_lik[45]      2923
## log_lik[46]      2380
## log_lik[47]      3251
## log_lik[48]      2999
## log_lik[49]      1842
## log_lik[50]      2874
## log_lik[51]      3100
## ypred[1]         3941
## ypred[2]         3915
## ypred[3]         3941
## ypred[4]         3923
## ypred[5]         3871
## ypred[6]         3770
## ypred[7]         3565
## ypred[8]         3779
## ypred[9]         4146
## ypred[10]        3819
## ypred[11]        3671
## ypred[12]        3600
## ypred[13]        3972
## ypred[14]        4016
## ypred[15]        3899
## ypred[16]        3956
## ypred[17]        3032
## ypred[18]        3715
## ypred[19]        3864
## ypred[20]        3703
## ypred[21]        3984
## ypred[22]        3931
## ypred[23]        3868
## ypred[24]        3969
## ypred[25]        4078
## ypred[26]        3899
## ypred[27]        3974
## ypred[28]        3866
## ypred[29]        3839
## ypred[30]        3706
## ypred[31]        4221
## ypred[32]        3856
## ypred[33]        4092
## ypred[34]        3944
## ypred[35]        3748
## ypred[36]        3738
## ypred[37]        3846
## ypred[38]        3633
## ypred[39]        4062
## ypred[40]        3785
## ypred[41]        3966
## ypred[42]        3417
## ypred[43]        3926
## ypred[44]        4050
## ypred[45]        3845
```

```
## ypred[46]       3562
## ypred[47]       3777
## ypred[48]       4071
## ypred[49]       3959
## ypred[50]       3495
## ypred[51]       3727
## lp__            2097
##
## For each parameter, Bulk_ESS and Tail_ESS are crude measures of
## effective sample size for bulk and tail quantities respectively (an ESS > 100
## per chain is considered good), and Rhat is the potential scale reduction
## factor on rank normalized split chains (at convergence, Rhat <= 1.05).
```

Hierarchical model

```
sUS<- monitor(hierarchical_modelUS)$Bulk_ESS
```

```
## Inference for the input samples (4 chains: each with iter = 2000; warmup = 0):
##
##                   Q5       Q50       Q95      Mean     SD  Rhat Bulk_ESS
## mu               0.0       0.0       0.0       0.0    0.0     1     7396
## eta           6811.1   10357.3   15427.2   10635.3 2650.7     1     2768
## theta[1]         0.0       0.0       0.0       0.0    0.0     1     5594
## theta[2]         0.0       0.0       0.0       0.0    0.0     1     5917
## theta[3]         0.0       0.0       0.0       0.0    0.0     1     6486
## theta[4]         0.0       0.0       0.0       0.0    0.0     1     5672
## theta[5]         0.0       0.0       0.0       0.0    0.0     1     6006
## theta[6]         0.0       0.0       0.0       0.0    0.0     1     5547
## theta[7]         0.0       0.0       0.0       0.0    0.0     1     5063
## theta[8]         0.0       0.0       0.0       0.0    0.0     1     6676
## theta[9]         0.0       0.0       0.0       0.0    0.0     1     5765
## theta[10]        0.0       0.0       0.0       0.0    0.0     1     5398
## theta[11]        0.0       0.0       0.0       0.0    0.0     1     7061
## theta[12]        0.0       0.0       0.0       0.0    0.0     1     5546
## theta[13]        0.0       0.0       0.0       0.0    0.0     1     5313
## theta[14]        0.0       0.0       0.0       0.0    0.0     1     5234
## theta[15]        0.0       0.0       0.0       0.0    0.0     1     4905
## theta[16]        0.0       0.0       0.0       0.0    0.0     1     5177
## theta[17]        0.0       0.0       0.0       0.0    0.0     1     5311
## theta[18]        0.0       0.0       0.0       0.0    0.0     1     5252
## theta[19]        0.0       0.0       0.0       0.0    0.0     1     5580
## theta[20]        0.0       0.0       0.0       0.0    0.0     1     5319
## theta[21]        0.0       0.0       0.0       0.0    0.0     1     6688
## theta[22]        0.0       0.0       0.0       0.0    0.0     1     5460
## theta[23]        0.0       0.0       0.0       0.0    0.0     1     7293
## theta[24]        0.0       0.0       0.0       0.0    0.0     1     5837
## theta[25]        0.0       0.0       0.0       0.0    0.0     1     5047
## theta[26]        0.0       0.0       0.0       0.0    0.0     1     5020
## theta[27]        0.0       0.0       0.0       0.0    0.0     1     4602
## theta[28]        0.0       0.0       0.0       0.0    0.0     1     6364
## theta[29]        0.0       0.0       0.0       0.0    0.0     1     5792
## theta[30]        0.0       0.0       0.0       0.0    0.0     1     4550
```

```
## theta[31]          0.0       0.0       0.0       0.0    0.0     1      5375
## theta[32]          0.0       0.0       0.0       0.0    0.0     1      5449
## theta[33]          0.0       0.0       0.0       0.0    0.0     1      6056
## theta[34]          0.0       0.0       0.0       0.0    0.0     1      7405
## theta[35]          0.0       0.0       0.0       0.0    0.0     1      6171
## theta[36]          0.0       0.0       0.0       0.0    0.0     1      5622
## theta[37]          0.0       0.0       0.0       0.0    0.0     1      6129
## theta[38]          0.0       0.0       0.0       0.0    0.0     1      5001
## theta[39]          0.0       0.0       0.0       0.0    0.0     1      5531
## theta[40]          0.0       0.0       0.0       0.0    0.0     1      6514
## theta[41]          0.0       0.0       0.0       0.0    0.0     1      6037
## theta[42]          0.0       0.0       0.0       0.0    0.0     1      4420
## theta[43]          0.0       0.0       0.0       0.0    0.0     1      5902
## theta[44]          0.0       0.0       0.0       0.0    0.0     1      4845
## theta[45]          0.0       0.0       0.0       0.0    0.0     1      6515
## theta[46]          0.0       0.0       0.0       0.0    0.0     1      5331
## theta[47]          0.0       0.0       0.0       0.0    0.0     1      6597
## theta[48]          0.0       0.0       0.0       0.0    0.0     1      6028
## theta[49]          0.0       0.0       0.0       0.0    0.0     1      5608
## theta[50]          0.0       0.0       0.0       0.0    0.0     1      5471
## theta[51]          0.0       0.0       0.0       0.0    0.0     1      6244
## alpha             27.3      41.7      61.9      42.7   10.6     1      2770
## beta            6784.0   10315.7   15365.0   10592.6 2640.1     1      2769
## log_lik[1]        -5.4      -3.9      -3.7      -4.1    0.6     1      2260
## log_lik[2]        -4.3      -2.9      -2.7      -3.1    0.6     1      4172
## log_lik[3]        -5.5      -4.0      -3.8      -4.2    0.6     1      1837
## log_lik[4]        -5.4      -3.7      -3.5      -4.0    0.7     1      2494
## log_lik[5]        -6.6      -4.8      -4.5      -5.1    0.8     1      2177
## log_lik[6]        -5.6      -3.8      -3.6      -4.1    0.7     1      2884
## log_lik[7]        -5.6      -3.6      -3.3      -3.9    0.8     1      3537
## log_lik[8]        -3.8      -2.9      -2.8      -3.1    0.4     1      1834
## log_lik[9]        -4.0      -3.0      -2.8      -3.1    0.5     1      3489
## log_lik[10]       -6.2      -4.5      -4.3      -4.8    0.7     1      1703
## log_lik[11]       -6.2      -4.4      -4.1      -4.6    0.8     1      1795
## log_lik[12]       -4.2      -3.2      -3.0      -3.3    0.4     1      1973
## log_lik[13]       -4.5      -3.3      -3.1      -3.5    0.5     1      2278
## log_lik[14]       -6.1      -4.4      -4.2      -4.7    0.7     1      2252
## log_lik[15]       -5.8      -4.1      -3.9      -4.3    0.7     1      2097
## log_lik[16]       -5.5      -3.6      -3.3      -3.9    0.8     1      3955
## log_lik[17]       -5.1      -3.7      -3.5      -3.9    0.6     1      2390
## log_lik[18]       -5.2      -3.8      -3.6      -4.0    0.6     1      2265
## log_lik[19]       -5.5      -3.9      -3.7      -4.2    0.6     1      2132
## log_lik[20]       -4.0      -3.0      -2.8      -3.1    0.4     1      2010
## log_lik[21]       -5.5      -4.0      -3.8      -4.2    0.6     1      1478
## log_lik[22]       -5.7      -3.9      -3.6      -4.2    0.7     1      2534
## log_lik[23]       -5.8      -4.2      -4.0      -4.4    0.7     1      1855
## log_lik[24]       -5.5      -3.9      -3.6      -4.1    0.7     1      2660
## log_lik[25]       -6.2      -4.0      -3.6      -4.3    0.9     1      3501
## log_lik[26]       -5.4      -3.9      -3.7      -4.2    0.6     1      2210
## log_lik[27]       -4.7      -3.1      -2.7      -3.3    0.7     1      5183
## log_lik[28]       -4.6      -3.4      -3.2      -3.6    0.5     1      2236
## log_lik[29]       -5.0      -3.5      -3.3      -3.8    0.6     1      2449
## log_lik[30]       -6.1      -3.7      -2.7      -4.0    1.1     1      4651
## log_lik[31]       -6.5      -4.1      -3.7      -4.5    1.0     1      3846
```

```
## log_lik[32]      -4.5     -3.3     -3.2     -3.5   0.5     1    1999
## log_lik[33]      -6.3     -4.4     -4.2     -4.7   0.8     1    2639
## log_lik[34]      -5.9     -4.2     -4.0     -4.5   0.7     1    1667
## log_lik[35]      -4.0     -2.9     -2.7     -3.1   0.5     1    4020
## log_lik[36]      -6.0     -4.4     -4.1     -4.6   0.7     1    2255
## log_lik[37]      -5.2     -3.8     -3.6     -4.0   0.6     1    2098
## log_lik[38]      -6.5     -3.9     -3.3     -4.2   1.1     1    4687
## log_lik[39]      -5.8     -4.3     -4.1     -4.5   0.6     1    2036
## log_lik[40]      -3.8     -2.9     -2.8     -3.0   0.4     1    2367
## log_lik[41]      -5.7     -4.0     -3.7     -4.2   0.7     1    2675
## log_lik[42]      -5.0     -3.1     -2.7     -3.4   0.8     1    4975
## log_lik[43]      -5.7     -4.1     -3.9     -4.3   0.7     1    2291
## log_lik[44]      -6.5     -4.8     -4.5     -5.0   0.7     1    1833
## log_lik[45]      -5.1     -3.8     -3.6     -4.0   0.6     1    2169
## log_lik[46]      -3.7     -2.6     -2.3     -2.7   0.5     1    4983
## log_lik[47]      -5.6     -4.1     -3.9     -4.3   0.6     1    2097
## log_lik[48]      -5.9     -4.0     -3.7     -4.3   0.8     1    2635
## log_lik[49]      -4.4     -3.3     -3.1     -3.5   0.5     1    2284
## log_lik[50]      -5.3     -3.9     -3.7     -4.1   0.6     1    1991
## log_lik[51]      -4.3     -2.8     -2.4     -3.0   0.6     1    6377
## ypred[1]        215.0    249.0    285.0    248.8  21.5     1    4631
## ypred[2]         26.0     37.0     50.0     37.0   7.4     1    4505
## ypred[3]        282.0    322.0    363.0    322.5  24.8     1    4636
## ypred[4]        141.0    168.0    198.0    168.5  17.3     1    4929
## ypred[5]       1281.0   1363.0   1451.0   1364.6  51.7     1    4645
## ypred[6]        181.0    212.0    244.0    212.1  19.3     1    4557
## ypred[7]         91.0    113.0    139.0    113.8  14.3     1    4197
## ypred[8]         30.0     43.0     57.0     42.9   8.1     1    4557
## ypred[9]         28.0     39.0     52.0     39.3   7.5     1    4735
## ypred[10]       827.0    897.0    968.0    896.9  42.8     1    4653
## ypred[11]       535.0    589.0    645.0    589.3  33.7     1    4612
## ypred[12]        51.0     67.0     85.0     67.0  10.4     1    4476
## ypred[13]        63.0     81.0    101.0     81.1  11.7     1    4022
## ypred[14]       602.0    658.0    719.0    658.6  35.9     1    4570
## ypred[15]       323.0    365.0    408.0    365.9  26.0     1    4275
## ypred[16]       101.0    124.0    150.0    124.7  14.8     1    4468
## ypred[17]       131.0    157.0    185.0    157.5  16.6     1    4620
## ypred[18]       173.0    203.0    236.0    204.2  19.2     1    4837
## ypred[19]       226.0    261.0    299.0    261.5  22.0     1    4805
## ypred[20]        35.0     48.0     63.0     48.0   8.6     1    4404
## ypred[21]       253.0    291.0    331.0    291.2  23.5     1    5032
## ypred[22]       197.0    231.0    267.0    231.0  21.1     1    4971
## ypred[23]       397.0    443.0    493.0    443.5  29.2     1    4937
## ypred[24]       203.0    236.0    272.0    236.3  21.2     1    4794
## ypred[25]       156.0    185.0    215.0    184.9  18.3     1    4383
## ypred[26]       243.0    281.0    318.0    280.7  22.6     1    4520
## ypred[27]        28.0     40.0     53.0     40.2   7.7     1    4526
## ypred[28]        79.0     99.0    121.0     99.2  12.8     1    4826
## ypred[29]       107.0    132.0    158.0    131.9  15.3     1    4502
## ypred[30]        26.0     37.0     51.0     37.7   7.7     1    4061
## ypred[31]       241.0    277.0    318.0    278.2  23.4     1    4766
## ypred[32]        74.0     93.0    114.0     93.4  12.2     1    4696
## ypred[33]       608.0    666.0    725.0    666.6  36.0     1    5181
## ypred[34]       454.0    506.0    557.0    505.7  30.9     1    4737
```

```
## ypred[35]        27.0      39.0      52.0      39.2   7.6    1     4294
## ypred[36]       571.0     626.0     684.0     626.2  34.3    1     4742
## ypred[37]       180.0     211.0     244.0     211.0  19.4    1     4988
## ypred[38]        99.0     123.0     149.0     123.6  15.2    1     4458
## ypred[39]       518.0     571.0     627.0     571.1  33.2    1     4576
## ypred[40]        29.0      41.0      54.0      41.0   7.9    1     4766
## ypred[41]       229.0     266.0     302.0     265.6  21.9    1     4685
## ypred[42]        28.0      41.0      55.0      41.1   8.1    1     4027
## ypred[43]       317.0     357.0     401.0     357.0  25.9    1     4763
## ypred[44]      1295.0    1379.0    1468.0    1380.2  52.0    1     4804
## ypred[45]       163.0     193.0     225.0     193.1  18.8    1     4303
## ypred[46]        12.0      20.0      29.0      20.2   5.2    1     4315
## ypred[47]       332.0     373.0     419.0     374.2  26.0    1     4639
## ypred[48]       241.0     278.0     316.0     278.5  23.0    1     5136
## ypred[49]        59.0      76.0      96.0      76.8  11.1    1     4608
## ypred[50]       214.0     248.0     283.0     248.1  21.2    1     4599
## ypred[51]        15.0      23.0      33.0      23.4   5.7    1     4663
## lp__        -199242.3 -199232.2 -199224.1 -199232.6   5.5    1     1469
##             Tail_ESS
## mu              3023
## eta             2795
## theta[1]        2923
## theta[2]        3189
## theta[3]        2888
## theta[4]        2797
## theta[5]        3039
## theta[6]        3061
## theta[7]        2962
## theta[8]        2943
## theta[9]        3002
## theta[10]       2717
## theta[11]       2586
## theta[12]       2604
## theta[13]       3099
## theta[14]       3123
## theta[15]       2872
## theta[16]       3082
## theta[17]       3058
## theta[18]       3074
## theta[19]       2911
## theta[20]       3009
## theta[21]       3001
## theta[22]       2565
## theta[23]       2776
## theta[24]       3242
## theta[25]       2805
## theta[26]       2705
## theta[27]       2475
## theta[28]       3042
## theta[29]       2735
## theta[30]       2915
## theta[31]       3233
## theta[32]       2740
## theta[33]       3014
```

```
## theta[34]        3007
## theta[35]        2624
## theta[36]        3035
## theta[37]        2973
## theta[38]        3162
## theta[39]        2674
## theta[40]        3155
## theta[41]        2824
## theta[42]        2730
## theta[43]        2943
## theta[44]        2918
## theta[45]        3111
## theta[46]        3041
## theta[47]        3135
## theta[48]        3250
## theta[49]        3240
## theta[50]        3082
## theta[51]        2825
## alpha            2850
## beta             2795
## log_lik[1]       2673
## log_lik[2]       3244
## log_lik[3]       2369
## log_lik[4]       2596
## log_lik[5]       2620
## log_lik[6]       3192
## log_lik[7]       2962
## log_lik[8]       1996
## log_lik[9]       2981
## log_lik[10]      1901
## log_lik[11]      1982
## log_lik[12]      2526
## log_lik[13]      2505
## log_lik[14]      2631
## log_lik[15]      2499
## log_lik[16]      3284
## log_lik[17]      2621
## log_lik[18]      2506
## log_lik[19]      2743
## log_lik[20]      2115
## log_lik[21]      1921
## log_lik[22]      2876
## log_lik[23]      1722
## log_lik[24]      2850
## log_lik[25]      2724
## log_lik[26]      2416
## log_lik[27]      3152
## log_lik[28]      2329
## log_lik[29]      2562
## log_lik[30]      3339
## log_lik[31]      3165
## log_lik[32]      2514
## log_lik[33]      2878
## log_lik[34]      1763
```

```
## log_lik[35]     3438
## log_lik[36]     2911
## log_lik[37]     2284
## log_lik[38]     3573
## log_lik[39]     1779
## log_lik[40]     2439
## log_lik[41]     2627
## log_lik[42]     3637
## log_lik[43]     2443
## log_lik[44]     2332
## log_lik[45]     2355
## log_lik[46]     3041
## log_lik[47]     2055
## log_lik[48]     3112
## log_lik[49]     2898
## log_lik[50]     2397
## log_lik[51]     3160
## ypred[1]        3783
## ypred[2]        3790
## ypred[3]        3505
## ypred[4]        3870
## ypred[5]        3167
## ypred[6]        3641
## ypred[7]        3552
## ypred[8]        3892
## ypred[9]        3843
## ypred[10]       3273
## ypred[11]       3551
## ypred[12]       4055
## ypred[13]       3451
## ypred[14]       3706
## ypred[15]       3071
## ypred[16]       3777
## ypred[17]       3792
## ypred[18]       3552
## ypred[19]       3645
## ypred[20]       3512
## ypred[21]       3448
## ypred[22]       3671
## ypred[23]       3676
## ypred[24]       4064
## ypred[25]       3301
## ypred[26]       3884
## ypred[27]       4121
## ypred[28]       3666
## ypred[29]       3762
## ypred[30]       3671
## ypred[31]       3656
## ypred[32]       3667
## ypred[33]       3359
## ypred[34]       3760
## ypred[35]       3758
## ypred[36]       3516
## ypred[37]       3698
```

```
## ypred[38]       3890
## ypred[39]       3682
## ypred[40]       3889
## ypred[41]       3709
## ypred[42]       3834
## ypred[43]       3790
## ypred[44]       3513
## ypred[45]       3877
## ypred[46]       3588
## ypred[47]       3374
## ypred[48]       3818
## ypred[49]       3831
## ypred[50]       3472
## ypred[51]       4095
## lp__            2100
##
## For each parameter, Bulk_ESS and Tail_ESS are crude measures of
## effective sample size for bulk and tail quantities respectively (an ESS > 100
## per chain is considered good), and Rhat is the potential scale reduction
## factor on rank normalized split chains (at convergence, Rhat <= 1.05).
```

Colombia Data

Separate model

```
sUS<- monitor(separate_modelCol)$Bulk_ESS
```

```
## Inference for the input samples (4 chains: each with iter = 2000; warmup = 0):
##
##                  Q5      Q50      Q95     Mean   SD  Rhat Bulk_ESS Tail_ESS
## theta[1]         0.0      0.0      0.0      0.0  0.0     1     8690     2599
## theta[2]         0.0      0.0      0.0      0.0  0.0     1     8780     2971
## theta[3]         0.0      0.0      0.0      0.0  0.0     1     8299     3168
## theta[4]         0.0      0.0      0.0      0.0  0.0     1     9520     2681
## theta[5]         0.0      0.0      0.0      0.0  0.0     1     8388     3466
## theta[6]         0.0      0.0      0.0      0.0  0.0     1    10700     2842
## theta[7]         0.0      0.0      0.0      0.0  0.0     1     9434     2489
## theta[8]         0.0      0.0      0.0      0.0  0.0     1     9531     2475
## theta[9]         0.0      0.0      0.0      0.0  0.0     1     7660     2946
## theta[10]        0.0      0.0      0.0      0.0  0.0     1     8582     3033
## theta[11]        0.0      0.0      0.0      0.0  0.0     1     9065     2749
## theta[12]        0.0      0.0      0.0      0.0  0.0     1     8937     3133
## theta[13]        0.0      0.0      0.0      0.0  0.0     1     7721     2783
## theta[14]        0.0      0.0      0.0      0.0  0.0     1     7402     3238
## theta[15]        0.0      0.0      0.0      0.0  0.0     1     8973     2833
## theta[16]        0.0      0.0      0.0      0.0  0.0     1     7727     2860
## theta[17]        0.0      0.0      0.0      0.0  0.0     1     8363     2601
## theta[18]        0.0      0.0      0.0      0.0  0.0     1     7806     2798
## theta[19]        0.0      0.0      0.0      0.0  0.0     1     7714     2678
## theta[20]        0.0      0.0      0.0      0.0  0.0     1     8363     2955
## theta[21]        0.0      0.0      0.0      0.0  0.0     1     8345     3505
## theta[22]        0.0      0.0      0.0      0.0  0.0     1     9161     2837
```

```
## theta[23]         0.0       0.0       0.0       0.0  0.0      1      8808      2983
## theta[24]         0.0       0.0       0.0       0.0  0.0      1      8080      2800
## theta[25]         0.0       0.0       0.0       0.0  0.0      1      9208      2825
## theta[26]         0.0       0.0       0.0       0.0  0.0      1      9539      2931
## theta[27]         0.0       0.0       0.0       0.0  0.0      1      8166      2699
## theta[28]         0.0       0.0       0.0       0.0  0.0      1      9051      2950
## theta[29]         0.0       0.0       0.0       0.0  0.0      1      8388      2971
## theta[30]         0.0       0.0       0.0       0.0  0.0      1      9231      2829
## theta[31]         0.0       0.0       0.0       0.0  0.0      1      8577      3178
## theta[32]         0.0       0.0       0.0       0.0  0.0      1      7655      2766
## theta[33]         0.0       0.0       0.0       0.0  0.0      1      7799      2648
## theta[34]         0.0       0.0       0.0       0.0  0.0      1      8068      3195
## theta[35]         0.0       0.0       0.0       0.0  0.0      1      8977      2888
## theta[36]         0.0       0.0       0.0       0.0  0.0      1      9355      3208
## theta[37]         0.0       0.0       0.0       0.0  0.0      1      7103      2823
## log_lik[1]       -4.4      -2.8      -2.4      -3.0  0.7      1      7248      3194
## log_lik[2]       -5.6      -4.5      -4.4      -4.7  0.5      1      3554      2936
## log_lik[3]       -5.9      -3.8      -3.1      -4.1  0.9      1      7899      3168
## log_lik[4]       -6.6      -4.6      -3.8      -4.8  0.9      1      7980      2824
## log_lik[5]       -8.4      -5.6      -4.2      -5.9  1.3      1      8469      3466
## log_lik[6]       -6.1      -4.7      -4.4      -4.9  0.6      1      5253      2948
## log_lik[7]       -5.0      -3.8      -3.7      -4.0  0.5      1      2704      2381
## log_lik[8]       -4.4      -3.6      -3.5      -3.7  0.3      1      1431      1936
## log_lik[9]       -4.2      -3.4      -3.3      -3.5  0.3      1      1628      2219
## log_lik[10]      -4.2      -3.4      -3.3      -3.5  0.3      1      1838      2107
## log_lik[11]      -5.4      -3.6      -3.0      -3.8  0.8      1      7232      2749
## log_lik[12]      -7.8      -5.2      -3.9      -5.4  1.2      1      8586      3182
## log_lik[13]      -8.3      -5.2      -3.5      -5.4  1.5      1      7730      2783
## log_lik[14]      -4.7      -3.9      -3.8      -4.0  0.3      1      2134      2462
## log_lik[15]      -4.9      -4.0      -3.8      -4.1  0.4      1      2017      2218
## log_lik[16]      -5.6      -3.9      -3.6      -4.2  0.7      1      5023      2860
## log_lik[17]      -5.1      -4.2      -4.0      -4.3  0.4      1      2161      2211
## log_lik[18]      -5.2      -4.2      -4.0      -4.3  0.4      1      2848      2717
## log_lik[19]      -3.9      -2.5      -2.4      -2.8  0.5      1      2986      2613
## log_lik[20]      -5.1      -4.1      -3.9      -4.2  0.4      1      2881      2737
## log_lik[21]      -5.3      -3.1      -2.1      -3.3  1.0      1      7917      3505
## log_lik[22]      -6.7      -4.5      -3.6      -4.7  1.0      1      8694      2837
## log_lik[23]      -5.4      -3.9      -3.6      -4.1  0.6      1      4897      2959
## log_lik[24]      -4.5      -3.7      -3.6      -3.8  0.4      1      1852      1882
## log_lik[25]      -4.7      -3.8      -3.7      -4.0  0.4      1      1775      2070
## log_lik[26]      -4.7      -3.9      -3.7      -4.0  0.4      1      1704      1723
## log_lik[27]      -4.2      -3.1      -3.0      -3.3  0.4      1      2433      2349
## log_lik[28]      -4.2      -3.3      -3.1      -3.4  0.4      1      2357      2512
## log_lik[29]      -5.5      -3.8      -3.4      -4.0  0.7      1      6728      2971
## log_lik[30]      -3.2      -2.4      -2.3      -2.6  0.3      1      1728      2006
## log_lik[31]      -7.0      -4.8      -3.8      -5.0  1.0      1      8258      3178
## log_lik[32]      -5.1      -3.6      -3.3      -3.8  0.6      1      5215      2797
## log_lik[33]      -5.4      -3.9      -3.6      -4.1  0.6      1      5904      3104
## log_lik[34]      -5.4      -3.8      -3.6      -4.1  0.7      1      5398      3195
## log_lik[35]      -5.6      -4.3      -4.1      -4.5  0.5      1      3753      3086
## log_lik[36]      -4.0      -2.6      -2.4      -2.8  0.6      1      3470      3224
## log_lik[37]      -4.0      -2.9      -2.8      -3.1  0.4      1      1961      2062
## ypred[1]         14.0      22.0      32.0      22.4  5.7      1      4684      3983
## ypred[2]        967.0    1030.0    1094.0    1029.9 38.5      1      5109      4110
```

```
## ypred[3]        48.0     63.0     79.0     62.9  9.5     1     4991     3905
## ypred[4]       263.0    296.0    330.0    296.2 20.7     1     4720     3724
## ypred[5]       333.0    369.0    406.0    369.3 22.6     1     4846     3949
## ypred[6]      1021.0   1086.0   1152.0   1086.4 40.2     1     5087     4029
## ypred[7]       230.0    261.0    295.0    261.4 19.5     1     5410     4116
## ypred[8]       140.0    164.0    190.0    164.3 15.5     1     4683     4077
## ypred[9]        92.0    113.0    134.0    113.0 12.8     1     4943     3809
## ypred[10]       89.0    109.0    131.0    109.3 12.8     1     4787     4020
## ypred[11]       62.0     79.0     98.0     79.5 10.8     1     4775     4031
## ypred[12]      255.0    287.0    321.0    287.7 20.3     1     4977     4168
## ypred[13]       71.0     90.0    110.0     90.1 11.7     1     4955     3978
## ypred[14]      268.0    301.0    335.0    301.1 20.4     1     4299     3864
## ypred[15]      309.0    347.0    385.0    346.7 23.1     1     4611     4025
## ypred[16]      176.0    203.0    233.0    203.7 17.0     1     4829     3640
## ypred[17]      459.0    503.0    549.0    503.4 27.2     1     5249     3430
## ypred[18]      442.0    484.0    529.0    484.2 26.5     1     4616     3242
## ypred[19]        9.0     16.0     25.0     16.1  4.9     1     5209     3689
## ypred[20]      380.0    419.0    463.0    420.1 25.3     1     4491     3691
## ypred[21]        7.0     14.0     23.0     14.4  4.7     1     4795     3866
## ypred[22]      173.0    200.0    229.0    200.7 17.2     1     4914     3879
## ypred[23]      199.0    228.0    259.0    228.3 18.5     1     5675     3720
## ypred[24]      174.0    202.0    231.0    201.9 17.1     1     5241     3877
## ypred[25]      243.0    275.0    309.0    275.6 19.8     1     4735     3866
## ypred[26]      256.0    287.0    323.0    288.1 20.8     1     5248     4093
## ypred[27]       49.0     64.0     82.0     64.8  9.9     1     5234     3768
## ypred[28]       63.0     80.0     98.0     80.6 10.9     1     4760     4048
## ypred[29]      105.0    127.0    150.0    127.2 13.7     1     4824     3864
## ypred[30]       10.0     17.0     26.0     17.3  5.0     1     4914     3572
## ypred[31]      242.0    273.0    308.0    273.5 20.0     1     4440     3853
## ypred[32]      110.0    133.0    157.0    133.3 14.4     1     4591     3800
## ypred[33]      186.0    213.0    242.0    213.1 17.2     1     5173     4213
## ypred[34]      181.0    209.0    238.0    209.3 17.3     1     5081     3732
## ypred[35]      532.0    578.0    629.0    579.1 29.4     1     5113     3628
## ypred[36]        9.0     17.0     26.0     17.0  5.1     1     4833     3506
## ypred[37]       30.0     43.0     57.0     42.9  8.0     1     5206     3866
## lp__      -97359.8 -97351.8 -97345.8 -97352.2  4.3     1     1520     2263
##
## For each parameter, Bulk_ESS and Tail_ESS are crude measures of
## effective sample size for bulk and tail quantities respectively (an ESS > 100
## per chain is considered good), and Rhat is the potential scale reduction
## factor on rank normalized split chains (at convergence, Rhat <= 1.05).
```

Hierarchical model

```r
sUS<- monitor(hierarchical_modelCol)$Bulk_ESS
```

```
## Inference for the input samples (4 chains: each with iter = 2000; warmup = 0):
##
##                  Q5      Q50      Q95     Mean    SD  Rhat Bulk_ESS Tail_ESS
## mu              0.0      0.0      0.0      0.0   0.0     1     6991     3157
## eta           593.5    943.4   1497.7    979.0 281.3     1     3377     3313
## theta[1]        0.0      0.0      0.0      0.0   0.0     1     6301     3132
```

```
## theta[2]         0.0     0.0     0.0     0.0   0.0     1     6684     2862
## theta[3]         0.0     0.0     0.0     0.0   0.0     1     6279     3027
## theta[4]         0.0     0.0     0.0     0.0   0.0     1     7548     3142
## theta[5]         0.0     0.0     0.0     0.0   0.0     1     5908     2880
## theta[6]         0.0     0.0     0.0     0.0   0.0     1     6869     3285
## theta[7]         0.0     0.0     0.0     0.0   0.0     1     6106     2897
## theta[8]         0.0     0.0     0.0     0.0   0.0     1     6687     3149
## theta[9]         0.0     0.0     0.0     0.0   0.0     1     5875     3212
## theta[10]        0.0     0.0     0.0     0.0   0.0     1     6571     2933
## theta[11]        0.0     0.0     0.0     0.0   0.0     1     5589     3255
## theta[12]        0.0     0.0     0.0     0.0   0.0     1     5247     2641
## theta[13]        0.0     0.0     0.0     0.0   0.0     1     5922     2861
## theta[14]        0.0     0.0     0.0     0.0   0.0     1     5646     2845
## theta[15]        0.0     0.0     0.0     0.0   0.0     1     6196     2736
## theta[16]        0.0     0.0     0.0     0.0   0.0     1     4677     3071
## theta[17]        0.0     0.0     0.0     0.0   0.0     1     6131     2796
## theta[18]        0.0     0.0     0.0     0.0   0.0     1     6678     3032
## theta[19]        0.0     0.0     0.0     0.0   0.0     1     5904     2815
## theta[20]        0.0     0.0     0.0     0.0   0.0     1     6451     3116
## theta[21]        0.0     0.0     0.0     0.0   0.0     1     5200     2926
## theta[22]        0.0     0.0     0.0     0.0   0.0     1     6139     3231
## theta[23]        0.0     0.0     0.0     0.0   0.0     1     5719     2860
## theta[24]        0.0     0.0     0.0     0.0   0.0     1     6802     3279
## theta[25]        0.0     0.0     0.0     0.0   0.0     1     5989     2986
## theta[26]        0.0     0.0     0.0     0.0   0.0     1     6371     3326
## theta[27]        0.0     0.0     0.0     0.0   0.0     1     6444     3174
## theta[28]        0.0     0.0     0.0     0.0   0.0     1     6364     2919
## theta[29]        0.0     0.0     0.0     0.0   0.0     1     5376     2528
## theta[30]        0.0     0.0     0.0     0.0   0.0     1     5889     2674
## theta[31]        0.0     0.0     0.0     0.0   0.0     1     5735     3354
## theta[32]        0.0     0.0     0.0     0.0   0.0     1     5465     2648
## theta[33]        0.0     0.0     0.0     0.0   0.0     1     6982     3060
## theta[34]        0.0     0.0     0.0     0.0   0.0     1     6443     3212
## theta[35]        0.0     0.0     0.0     0.0   0.0     1     5556     2792
## theta[36]        0.0     0.0     0.0     0.0   0.0     1     4787     2961
## theta[37]        0.0     0.0     0.0     0.0   0.0     1     4838     2725
## alpha            8.6    13.9    22.0    14.4   4.1     1     3382     3284
## beta           584.7   929.5  1475.8   964.6 277.1     1     3377     3313
## log_lik[1]      -3.6    -2.5    -2.4    -2.7   0.5     1     2806     2537
## log_lik[2]      -6.3    -4.6    -4.4    -4.9   0.7     1     1815     2200
## log_lik[3]      -4.9    -3.3    -3.1    -3.5   0.6     1     2301     2338
## log_lik[4]      -5.6    -3.9    -3.7    -4.2   0.7     1     1504     2101
## log_lik[5]      -5.7    -4.1    -3.9    -4.4   0.6     1     2291     2659
## log_lik[6]      -6.2    -4.6    -4.4    -4.9   0.7     1     2014     2263
## log_lik[7]      -5.5    -3.9    -3.7    -4.2   0.7     1     1756     2159
## log_lik[8]      -5.2    -3.7    -3.5    -3.9   0.6     1     2138     2460
## log_lik[9]      -6.2    -3.7    -3.3    -4.1   1.0     1     4163     3563
## log_lik[10]     -5.0    -3.5    -3.3    -3.7   0.6     1     1778     2034
## log_lik[11]     -4.9    -3.2    -3.0    -3.5   0.7     1     2476     2770
## log_lik[12]     -5.9    -4.0    -3.8    -4.3   0.8     1     1862     1942
## log_lik[13]     -5.1    -3.5    -3.3    -3.7   0.7     1     2208     2762
## log_lik[14]     -5.5    -4.0    -3.8    -4.2   0.7     1     2128     2482
## log_lik[15]     -5.8    -4.0    -3.8    -4.3   0.7     1     1911     1770
## log_lik[16]     -7.6    -4.4    -3.6    -4.8   1.4     1     4210     3071
```

```
## log_lik[17]     -6.1     -4.3     -4.0     -4.6   0.8      1      2145     2210
## log_lik[18]     -5.9     -4.2     -4.0     -4.5   0.7      1      1835     2187
## log_lik[19]     -4.0     -2.6     -2.4     -2.8   0.6      1      3661     2784
## log_lik[20]     -5.9     -4.2     -3.9     -4.5   0.7      1      2154     2328
## log_lik[21]     -4.9     -2.7     -2.0     -3.0   0.9      1      5308     3195
## log_lik[22]     -5.3     -3.7     -3.5     -4.0   0.7      1      2227     2536
## log_lik[23]     -5.3     -3.8     -3.6     -4.1   0.6      1      1670     2384
## log_lik[24]     -5.4     -3.8     -3.6     -4.0   0.7      1      1713     2246
## log_lik[25]     -5.7     -4.0     -3.7     -4.2   0.7      1      1725     2235
## log_lik[26]     -5.7     -4.0     -3.7     -4.2   0.7      1      1989     2533
## log_lik[27]     -4.5     -3.2     -3.0     -3.4   0.6      1      2083     2617
## log_lik[28]     -4.7     -3.3     -3.1     -3.6   0.6      1      2124     2532
## log_lik[29]     -5.1     -3.6     -3.4     -3.8   0.6      1      1827     2284
## log_lik[30]     -4.3     -2.7     -2.3     -2.9   0.7      1      4274     2674
## log_lik[31]     -5.7     -4.0     -3.8     -4.3   0.7      1      2182     2563
## log_lik[32]     -5.0     -3.5     -3.3     -3.8   0.6      1      2077     2128
## log_lik[33]     -5.3     -3.8     -3.6     -4.0   0.6      1      2029     2316
## log_lik[34]     -5.4     -3.8     -3.6     -4.0   0.7      1      2067     2359
## log_lik[35]     -5.9     -4.3     -4.1     -4.6   0.7      1      1900     2465
## log_lik[36]     -6.3     -3.8     -2.5     -4.1   1.2      1      4866     2961
## log_lik[37]     -7.0     -4.1     -2.8     -4.4   1.4      1      4472     2359
## ypred[1]         9.0     16.0     25.0     16.6   5.0      1      4322     3810
## ypred[2]       972.0   1045.0   1120.0   1045.7  45.5      1      5102     3760
## ypred[3]        53.0     70.0     90.0     70.6  11.3      1      5055     3723
## ypred[4]       237.0    274.0    313.0    274.4  23.0      1      5346     3760
## ypred[5]       358.0    401.0    448.0    401.7  27.2      1      4951     3924
## ypred[6]      1035.0   1109.0   1188.0   1110.2  46.5      1      5314     4037
## ypred[7]       218.0    254.0    290.0    254.2  21.8      1      4937     3524
## ypred[8]       138.0    165.0    196.0    165.7  17.9      1      5772     3666
## ypred[9]        83.0    104.0    129.0    105.0  13.8      1      4248     3295
## ypred[10]       86.0    109.0    134.0    109.3  14.4      1      4953     3937
## ypred[11]       55.0     72.0     92.0     72.5  11.5      1      4289     3736
## ypred[12]      273.0    312.0    354.0    311.9  24.3      1      4178     3431
## ypred[13]       85.0    106.0    130.0    106.3  13.8      1      4716     3798
## ypred[14]      264.0    301.0    342.0    301.6  23.9      1      4621     3295
## ypred[15]      299.0    341.0    385.0    341.7  26.1      1      4863     3810
## ypred[16]      167.0    198.0    230.0    198.1  19.3      1      4810     3725
## ypred[17]      453.0    502.0    556.0    502.5  31.6      1      5004     3756
## ypred[18]      444.0    493.0    546.0    493.4  30.9      1      5079     3719
## ypred[19]        9.0     15.0     24.0     15.7   4.8      1      4900     4047
## ypred[20]      363.0    409.0    457.0    409.0  28.7      1      5625     3534
## ypred[21]        6.0     13.0     21.0     13.1   4.6      1      4094     3644
## ypred[22]      154.0    185.0    216.0    184.9  19.0      1      4801     3224
## ypred[23]      186.0    218.0    252.0    218.6  20.1      1      4688     3714
## ypred[24]      172.0    203.0    236.0    203.2  19.7      1      4894     3721
## ypred[25]      239.0    275.0    315.0    275.7  23.2      1      5048     3784
## ypred[26]      252.0    290.0    331.0    290.3  23.9      1      5298     3762
## ypred[27]       46.0     62.0     82.0     63.0  10.7      1      4674     3860
## ypred[28]       63.0     82.0    103.0     82.7  12.3      1      4598     3536
## ypred[29]      112.0    137.0    164.0    137.7  15.8      1      4804     3607
## ypred[30]        7.0     14.0     22.0     14.0   4.4      1      4638     3904
## ypred[31]      261.0    299.0    340.0    299.9  24.0      1      5124     4026
## ypred[32]      102.0    126.0    152.0    126.3  15.4      1      4501     3389
## ypred[33]      172.0    203.0    235.0    203.0  19.4      1      5005     3666
```

```
## ypred[34]      169.0    199.0    233.0    200.0  19.4     1     4791     3236
## ypred[35]      535.0    592.0    649.0    592.3  34.3     1     4289     3788
## ypred[36]        7.0     13.0     20.0     12.8   4.2     1     3903     3682
## ypred[37]       23.0     34.0     48.0     34.6   7.6     1     4409     4001
## lp__        -97360.6 -97352.2 -97345.9 -97352.6   4.5     1     1476     2319
##
## For each parameter, Bulk_ESS and Tail_ESS are crude measures of
## effective sample size for bulk and tail quantities respectively (an ESS > 100
## per chain is considered good), and Rhat is the potential scale reduction
## factor on rank normalized split chains (at convergence, Rhat <= 1.05).
```