Día: Mes: Año:

Fracciones Parciales:

$$X(s) = \frac{2s^3+8s^2+4s+8}{s(s+1)(s^2+4s+8)} = \frac{K1}{s} + \frac{K2}{s+1} + \frac{A}{s+2+j2} + \frac{A^*}{s+2-j2}$$

$\hookrightarrow K1 = sX(s)\Big|_{s=0} = \dfrac{s(2s^3+8s^2+4s+8)}{s(s+1)(s^2+4s+8)} = \dfrac{8}{8} = 1$

$\hookrightarrow K2 = (s+1)X(s)\Big|_{s=-1} = \dfrac{\cancel{(s+1)}(2s^3+8s^2+4s+8)}{s\cancel{(s+1)}(s^2+4s+8)} = -2$

$\hookrightarrow A = (s+2+j2)X(s)\Big|_{s=-2-j2} = \cancel{(s+2+j2)}\left\{\dfrac{2s^3+8s^2+4s+8}{s(s+1)\cancel{(s+2+j2)}(s+2-j2)}\right\} = \dfrac{3}{2} + j\dfrac{1}{2}$

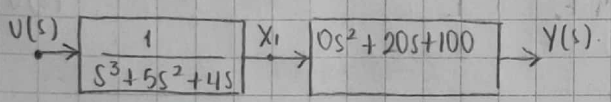
$\hookrightarrow A^* = \dfrac{3}{2} - j\dfrac{1}{2}$

Obteniendo así que $X(s) = \dfrac{1}{s} - \dfrac{2}{s+1} + \dfrac{3/2 + j\,1/2}{s+2+j2} + \dfrac{3/2 - j\,1/2}{s+2-j2}$

Ejemplo: 12.1 del Libro de Norman Nise: Diseño de sistema control.

Diseñar un sistema de control con planta $G(s) = \dfrac{20(s+5)}{s(s+1)(s+4)}$, con $OS = 9{,}5\%$ y $ts = 0{,}74$ seg.

*



- $\dfrac{X_1(s)}{U(s)} = \dfrac{1}{s^3+5s^2+4s}$

  $[s^3+5s^2+4s]\,X_1(s) = U(s)$

  $\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u$

- $\dfrac{Y(s)}{X_1(s)} = 20s+100$

  $Y(s) = X_1(s)\,[20s+100]$

  $y = 20\dot{x}_1 + 100x_1$

- Variables de estado

  $q_1 = x_1$

  $q_2 = \dot{q}_1 = \dot{x}_1$

  $q_3 = \dot{q}_2 = \ddot{x}_1$

  $\dot{q}_3 = \dddot{x}_1$

- $\dot{q}_3 = u - 5q_3 - 4q_2$

  $y = 20q_2 + 100q_1$

tuffy

Día: Mes: Año:

- Espacio de estados:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$

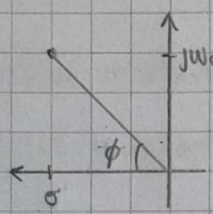
- $OS\% = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100 = 9{,}5.$

$$\frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = \ln(0{,}095)$$

$$\frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = -2{,}3539.$$

$\hookrightarrow \zeta \cong 0{,}5996.$

- $s = \sigma + j\omega_d$, donde $\sigma = \zeta\omega_n$.



- $\zeta = \cos\phi$

$\hookrightarrow \phi = \cos^{-1}(0{,}5996) \cong 53{,}16°.$

- $ts = \dfrac{4}{\sigma} = 0{,}74.$

$\hookrightarrow \sigma = \dfrac{4}{0{,}74} \cong 5{,}41.$

- $\sigma = \zeta\omega_n$

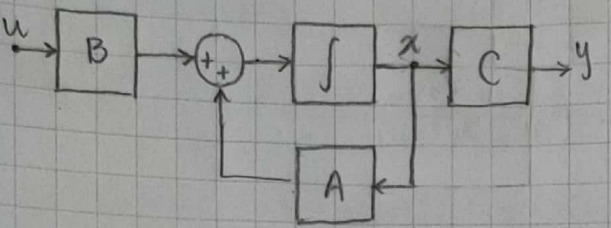
$\hookrightarrow \omega_n = \dfrac{\sigma}{\zeta} = \dfrac{5{,}41}{0{,}5996} = 9{,}0227 \ rad/s.$

- $\omega_d = \omega_n\sqrt{1-\zeta^2} = 9{,}0227\sqrt{1-(0{,}5996)^2} \cong 7{,}2207 \ rad/s.$

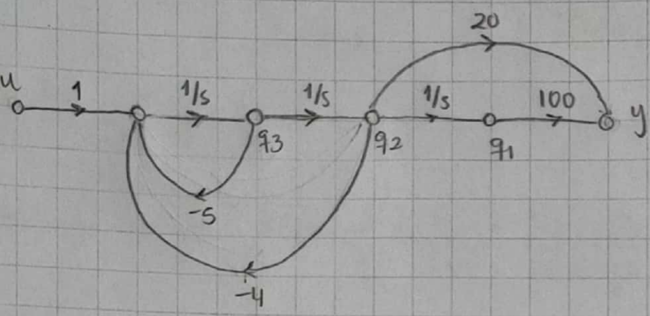
- $\omega_d = \sigma \tan(\phi) = 5{,}41 \tan(53{,}16) \cong 7{,}2212$

Día:____ Mes:____ Año:____

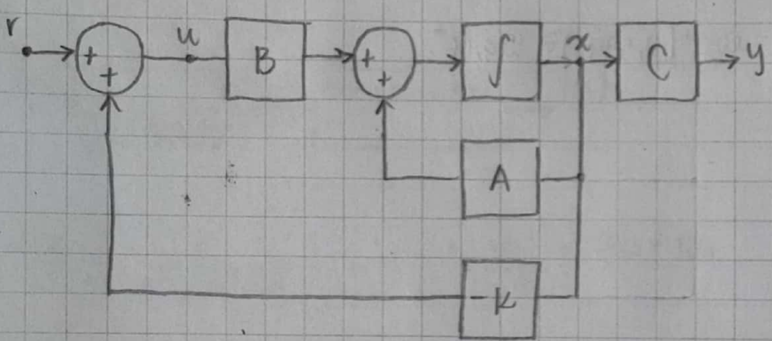
$\dot{x} = Ax + Bu$
$y = Cx$



Retomando el gemplo 12.1: Diagrama de flujo de señal.



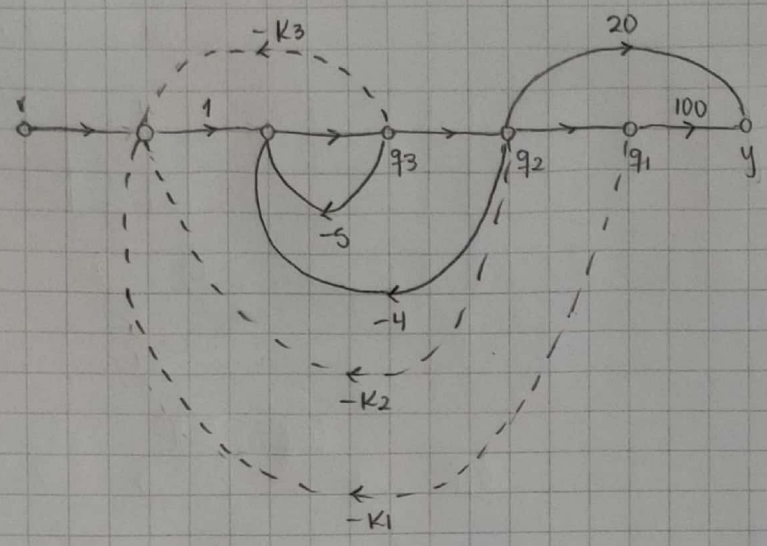
Redibujando el diagrama de bloques, agregando un lazo de realimentación



$\hookrightarrow \dot{x} = Ax + Bu$

$\dot{x} = Ax + B(r - Kx)$

$\dot{x} = (A - BK)x + Br.$



A partir del nuevo diagrama de flujo de señal, se debe reescribir el espacio de estados:

- $\dot{q}_3 = -5q_3 - 4q_2 + u$

  $\dot{q}_3 = -5q_3 - 4q_2 + (-K_3q_3 - K_2q_2 - K_1q_1) + r$

- $$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ -K_1 & -4-K_2 & -5-K_3 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

* Ecuación característica del sistema:

$$\text{Det}[sI-(A-BK)] = s^3 + (5+K_3)s^2 + (4+K_2)s^2 + K_1 s = 0.$$

- Polo dominante en lazo cerrado en $s = -5{,}4 + j7{,}21$.

- Cero de la planta en $s = 5$; con $s = 5{,}1$

  $(s+5{,}4-j7{,}2)(s+5{,}4+j7{,}2)(s+5{,}1)$, Ecuación característica

  $s^3 + 15{,}9s^2 + 136{,}22s + 413{,}83 = 0$

- Se compara la EC con la ultima obtenida:

  $K_3 = 15{,}9 - 5 = 10{,}9$

  $K_2 = 136{,}22 - 4 = 132{,}22$

  $K_1 = 413{,}83$