Exercice 2 Cours

Soit $(a, b) \in \mathbb{R}^2$

$$\forall n \in \mathbb{N} \quad (a+b)^n = \sum_{k=0}^{n} C_n \quad a^k \, b^{n-k}$$

Preuve : recurrence

- $n = 0$

$$(a+b)^0 = 1$$

$$\sum_{k=0}^{0} C_n^k \, a^k \, b^{n-k} = C_0^0 \, a^0 \, b^0 = 1$$

donc vrai au rang 1.

- Ⓗ $(a+b)^n = \sum_{k=0}^{n} C_n^k \, a^n \, b^{n-k}$

AD : $(a+b)^{n+1} = \sum_{k=0}^{n+1} C_{n+1}^k \, a^k \, b^{n-1-k}$

$$\begin{aligned}
(a+b)^{n+1} &= (a+b)^n \, (a+b) \\
&= \sum_{k=0}^{n} C_n^k \, a^n \, b^{n-k} \\
&= a \sum_{k=0}^{n} C_n^k \, a^n \, b^{n-k} + b \sum C_n^k \, a^n \, b^{n-k} \\
&= \sum_{k=0}^{n} C_n^k \, a^{n+1} \, b^{n-k} + \sum_{k=0}^{n} C_n^k \, a^n \, b^{n-k+1}
\end{aligned}$$

On pose $K = k+1$ et $K = k$

On a donc :

$$(a+b)^{n+1} = \sum_{k=1}^{n+1} C_n^{k-1} a^k b^{n+1-k} + \sum_{K=0}^{n} C_n^k a^k b^{n+1-k}$$

$$= \sum_{k=1}^{n} C_n^{k-1} a^k b^{n+1-k} + \sum_{k=1}^{n} C_n^k a^k b^{n+1-k} + C_0^0 a^{n+1} b^0 + C_0^0 a^0 b^{n+1}$$

$$= \sum_{k=1}^{n} \left(C_n^{k-1} + C_n^k \right) a^k b^{n+1-k} + a^{n+1} + b^{n+1}$$

$$= \sum_{k=1}^{n} C_{n+1}^k a^k b^{n+1-k} + C_{n+1}^{n+1} a^{n+1-(n-1)} + C_{n+1}^0 a^0 b^{n+1-0}$$

$$= \sum_{k=1}^{n+1} C_{n+1}^k a^k b^{n+1-k}$$