Exercice 1 : A)

1) $(462)_7 = 4 \times 7^2 + 6 \times 7 + 2 = 196 + 42 + 2 = 240_{10}$

2) $(4BA)_{12} = 4 \times 12^2 + 11 \times 12 + 10 = 576 + 132 + 10 = 718_{10}$

3) $(11101101)_2 = 128 + 64 + 32 + 8 + 4 + 1 = 237_{10}$

4) $(1022)_3 = 27 + 6 + 2 = 35$

5) $(377)_8 = 3 \times 8^2 + 7 \times 8 + 7 = 255$

6) $(BAC)_{16} = 11 \times 16^2 + 10 \times 16 + 12 = 2988$

7) $(12AD)_{16} = 1 \times 16^3 + 2 \times 16^2 + 10 \times 16 + 13 = 4781$

B) 1) 275 / 2 = 137 r = 1
137 / 2 = 68 r = 1
68 / 2 = 34 r = 0
34 / 2 = 17 r = 0
17 / 2 = 8 r = 1
8 / 2 = 4 r = 0
4 / 2 = 2 r = 0
2 / 2 = 1 r = 0
1 / 2 = 0 r = 1

$275_{10} = 100010011_2$

2) 564 / 2 = 282 r = 0
282 / 2 = 141 r = 0
141 / 2 = 70 r = 1
70 / 2 = 35 r = 0
35 / 2 = 17 r = 1
17 / 2 = 8 r = 1
8 / 2 = 4 r = 0
4 / 2 = 2 r = 0
2 / 2 = 1 r = 0
1 / 2 = 0 r = 1

$564_{10} = 1000110100_2$

4) 3201 /2 = 1600 r = 1
1600 /2 = 800 r = 0
800 /2 = 400 r = 0
400 /2 = 200 r = 0
200 /2 = 100 r = 0
100 /2 = 50 r = 0
50 /2 = 25 r = 0
25 /2 = 12 r = 1
12 /2 = 6 r = 0
6 /2 = 3 r = 0
3 /2 = 1 r = 0
1 /2 = 0 r = 1

$(3201)_{10} = 1000\,1000\,0001_2$
$= C81_{16}$

8) 687 /16 = 42 r = 15
42 /16 = 2 r = 10
2 /16 = 0 r = 2

$687_{10} = 2AF_{16}$

5) 4321 /8 = 540 r = 1
540 /8 = 67 r = 4
67 /8 = 8 r = 3
8 /8 = 1 r = 0
1 /8 = 0 r = 1

$(687)_{10} = 1034_8$

Exercice 2

$(AC7E)_{16} = 1010\ 1100\ 0101\ 1110_2$
$(BCD)_{16} = 1011\ 1100\ 1101_2$
$(1234)_{16} = 0001\ 0010\ 0011\ 0100_2$
$(5567)_8 = 101\ 101\ 110\ 111_2$
$(1111\ 1001\ 1011\ 0010)_2 = F9B2_{16}$
$(001\ 111\ 100\ 110\ 110\ 010)_2 = 174\,662_8$

Exercice 2 (Suite).

$(ABDC)_{16} = 1010\ 1011\ 1101\ 1100_2 = 125734_8$

$(2074)_{16} = 010\ 000\ 111\ 100 = 43C_{16}$

Exercice 3:

1) $b^2 + 3b + 2 = 3 \times 10^1$

$b^2 + 3b - 28 = 0$

$\Delta = 121$

$S = \dfrac{-3 + 11}{2} = 4$

donc $(132)_4 = (30)_{10}$

2) $2 \times 16 + 10 = 3b + 6$

$42 = 3b + 6$

$3b - 36 = 0 \iff b = 12$

donc $(2A)_{10} = (36)_{12}$

3) $(2b^1 + 2b^0)(2b^1 + b^0) = 5b^2 + 2b^0$

$(2b + 2)(2b + 1) = 5b^2 + 2$

Exercice 3:

1. $(101)_a = (401)_b$

$a^2+1 = 4b^2+1$

$a^2-(2b)^2 = 0$

$(a-2b)(a+2b)=0$

Soit $a-2b=0$ ou $a+2b=0$

ce qui est impossible car une base ≥ 2

2. $(501)_a = (5001)_b$

$5a^2+1 = 5b^4+1$

$5a^2-5b^4 = 0$ de plus $b \geq 6$ et $a \geq 6$

On prend $b=6$

Ainsi $a=36$ et $b=6$

3. $(12)_a = (1002)_b$

$a+2 = b^3+2$ avec $b \geq 3$ et $a \geq 3$

On prend $b=3$

$a-b^3 = 0 \Rightarrow b=3 \quad a=27$

Exercice 4:

$N \equiv a\ [b] \iff \exists k \in \mathbb{Z} \quad N = b \times k + a$

N pair → $N \equiv 0\ [2]$

N impaire → $N \equiv 1\ [2]$

$N = (a_n \times a_{n-1} \times \dots \times a_0) = \sum_{i=0}^{n} a_i b^i$

- 1er cas: b est pair → $b \equiv 0\ [2]$

$b^i \equiv 0\ [2] \quad \forall i \in [1; n]$

$a_i b^i \equiv 0\ [2] \quad \forall i \in [1, n]$

$\sum_{i=1}^{n} a_i b^i \equiv 0\ [2]$

$N = \sum a_i b^i + a_0 b^0 \equiv a\ [2]$

Ainsi si b est pair, le Nb sera pair en base 10 si son symbole de poids faible est pair.

- 2e cas:

$b^i \equiv 1\ [2]$

$a_i b^i \equiv a_i\ [2]$

$\sum_{i=0}^{n} a_i b^i = \sum_{i=0}^{n} a_i\ [2]$

Ainsi si b est impaire le Nb sera pair en base 10 si la somme de ses symbole est pai

Exercice 5:

A)

1. $(1101,011)_2 = 2^3 + 2^2 + 1 + 2^{-2} + 2^{-3}$

$= 8 + 4 + 1 + 0,25 + 0,125$

$= 13,375_{10}$

2. $(123,42)_8 = 8^2 + 2\times 8 + 3 + 4\times 8^{-1} + 2\times 8^{-2}$

$= 2^6 + 2^4 + 3 + 2^2 \times 2^{-3} + 2\times 2^{-6}$

$= 64 + 16 + 3 + 0,5 + 0,03125$

$= 83,53125_{10}$

3. $(BAC,028)_{16} = 11\times 16^2 + 10\times 16 + 12 + 2\times 16^{-2} + 8\times 16^{-3}$

$= 2988 + 2\times 2^{-8} + 2^3 \times 2^{-12}$

$= 2988 + 2^{-7} + 2^{-9}$

$= 2988 + 0,0078125 + 0,001953125$

$= 2988,009765625$

B)

1. $(164,76)_{10}$

$164 / 8 = 20 \quad r = 4$

$20 / 8 = 2 \quad r = 4$

$2 / 8 = 0 \quad r = 2$

$0,76 \times 8 = 6,08$

$0,08 \times 8 = 0,64$

$0,64 \times 8 = 5,12$

$(164,76)_{10} = (244,605)_8$

2. $(24,42)_{10}$

$24/2 = 12 \quad r = 0$

$12/2 = 6 \quad r = 0$

$6/2 = 3 \quad r = 0$

$3/2 = 1 \quad r = 1$

$1/2 = 0 \quad r = 1$

$0,42 \times 2 = 0,84$

$0,84 \times 2 = 1,68$

$0,68 \times 2 = 1,36$

$0,36 \times 2 = 0,72$

$0,72 \times 2 = 1,44$

$0,44 \times 2 = 0,88$

$0,88 \times 2 = 1,96$

$(24,42)_{10} = (11000, 0110101)_2$

3. $(69,23)_{10}$

$69/16 = 4 \quad r = 5$

$4/16 = 0 \quad r = 4$

$0,23 \times 16 = 3,68$

$0,68 \times 16 = 10,88$

$0,88 \times 16 = 14,08$

$(69,23)_{10} = (45, 3AE)_{16}$

4. $(111010001111, 0101110111)_2$

0111 0100 0111 , 0101 1101 1000

$(7\ 4\ 7, 5\ D\ 8)_{16}$

5. $(101110100, 100110111)_2$

101 110 100 , 100 110 110

$(5\ 6\ 4, 4\ 6\ 6)_8$