

Exercice 20  $X(\Omega) \subset \mathbb{N}$   $\Delta$

$$F_x: \mathbb{R} \rightarrow [0, 1]$$

$$x \mapsto F_x(x) = P(X \leq x)$$

$$1) F_x(2,3) = P(X \leq 2,3)$$

$$= P(X=0) + P(X=1) + P(X=2) \quad (X \text{ prend des valeurs entières})$$

$$2) \lim_{x \rightarrow -\infty} F_x(x) = \lim_{x \rightarrow -\infty} P(X \leq x) = 0 \quad \text{car } X(\Omega) \subset \mathbb{N} \\ x < 0 \text{ vide}$$

$$\lim_{x \rightarrow +\infty} F_x(x) = \lim_{x \rightarrow +\infty} P(X \leq x) = P(\Omega) = 1$$

$$3) \{X=k\} = \{X \leq k\} \setminus \{X \leq k-1\} \quad \text{car } X(\Omega) \in \mathbb{N}$$

$$P(X=k) = P(X \leq k) - P(X \leq k-1) = F_x(k) - F_x(k-1)$$

$$4) \text{ Soit } (a,b) \in \mathbb{R}^2 \quad a < b$$

$$P(a < x \leq b)$$

$$\{a < x \leq b\} = \{a < x\} \cap \{x \leq b\}$$

$$\text{or } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

Ainsi

$$P(a < x \leq b) = P(a < x) + P(x \leq b) - P(a < x \text{ ou } x \leq b)$$

$$= 1 - P(X \leq a) + P(X \leq b) - P(X \in \mathbb{N})$$

$$= 1 - F_x(a) + F_x(b) - 1 = F_x(b) - F_x(a)$$