Exercice 4 :

1) $C_n^p = \frac{n}{p} C_{n-1}^{p-1}$

$$\frac{n}{p} C_{n-1}^{p-1} = \frac{n}{p} \frac{(n-1)!}{(p-1)!(n-p)!} = \frac{n(n-1)!}{p(p-1)!(n-p)!} = \frac{n!}{p!(n-p)} = C_n^p$$

2) $\sum_{p=1}^{n} p C_n^p = p \sum_{p=1}^{n} C_n^p = p \sum_{p=1}^{n} \frac{n}{p} C_{n-1}^{p-1} = n \sum_{p=1}^{n} C_{n-1}^{p-1}$

on pose $k = p-1$

Ce qui donne

$$n \sum_{k=0}^{n-1} C_{n-1}^{k} = n(1+1)^{n-1} = n2^{n-1}$$

Exercice 5 :

$C_n^k C_{n-k}^{p-k} = C_p^k C_n^p$?

$$C_n^k C_{n-k}^{p-k} = \frac{n!}{k!(n-k)!} \times \frac{(n-k)!}{(p-k)!(n-k-p+k)} = \frac{n!}{k!(n-k)!} \times \frac{(n-k)!}{(p-k)!(n-p)!}$$

$$= \frac{n!}{k!(p-k)!(n-p)!} \times 1 = \frac{n!}{k!(p-k)!(n-p)!} \times \frac{p!}{p!}$$

$$= \frac{p!\,n!}{k!\,p!\,(p-k)!\,(n-p)!}$$

$$= \frac{p!}{k!(p-k)!} \times \frac{n!}{p!(n-p)!} = C_p^k C_n^p$$