Exercice 25 :

Modèle : On lance une piece truquée

p = proba d'avoir "Pile"

X = rang d'apparition du 1^{er} "Pile"

$X(\Omega) = [\![1 ; +\infty[\![= \mathbb{N}^*$

$\{X=k\}$ = Au $k-1$ premiers lancers on a eu "Face" et au kieme on a "Pile"

$P(X=k) = q^{k-1}p = p(1-p)^{k-1} \quad q \in]0;1[$

Explications de $\sum_{1}^{+\infty}$

$$\sum_{k=0}^{+\infty} q^k = \lim_{n \to +\infty} \sum_{k=0}^{n} q^k \qquad \lim_{n\to+\infty} q^n = \begin{cases} 0 & -1<q<1 \\ +\infty & q>1 \\ \varnothing & q \leq -1 \end{cases}$$

$$= \lim_{n\to+\infty} \frac{1-q^{n+1}}{1-q} \qquad K = k-1$$

$$= q\frac{1}{1-q}$$

ex:

$$\sum_{k=1}^{+\infty} P(X=k) = \sum_{k=1}^{+\infty} pq^{k-1} = p\sum_{k=1}^{+\infty} q^{k-1} = p\sum_{k=0}^{+\infty} q^K = p \times \frac{1}{1-q} = \frac{p}{p} = 1$$

⊛ $\sum_{k=1}^{+\infty} k q^{k-1}$

$$\sum_{k=0}^{n} q^k = \frac{1-q^{n+1}}{1-q}$$

$$\sum_{k=1}^{n} k q^{k-1} = \frac{-(n+1)q^n (1-q) + (1-q^{n+1})}{(1-q)^2}$$

$$\Rightarrow \lim_{n \to +\infty} \sum_{k=1}^{n} k q^{k-1} = \frac{1}{(1-q)^2}$$

$$P(X=k) = p q^{k-1}$$

⊛ Esperance :

$$E(X) = \sum_{k=1}^{+\infty} k\, P(X=k)$$
$$= \sum_{k=1}^{+\infty} k\, p q^{k-1} = p \sum_{k=1}^{+\infty} k q^{k-1}$$
$$= \frac{p}{(1-q)^2} = \frac{p}{p^2} = \frac{1}{p}$$

⊛ Variance

$$V(X) = E(X(X-1)) + E(X) - (E(X))^2$$

$$E(X(X-1)) = \sum_{k=2}^{+\infty} k(k-1)\, P(X=k)$$
$$= \sum_{k=2}^{+\infty} k(k-1)\, p\, q^{k-1} = \frac{p}{q} \sum_{k=2}^{+\infty} k(k-1)\, q^{k-2}$$
$$= \frac{2pq}{(1-q)^3} = \frac{2q}{p^2}$$

Exercice 25 :

$$V(x) = \frac{2q}{p^2} + \frac{1}{p} - \frac{1}{p^2} = \frac{2q + p - 1}{p^2}$$

$$= \frac{2(1-p) + p - 1}{p^2} = \frac{1-p}{q^2}$$

2) X = nb d'apparition du 1er Pile

$$P(X=4) = \left(\frac{2}{3}\right)^3 \times \frac{1}{3}$$

$$= \frac{8}{81}$$