Exercice 20 $X(\Omega) \subset \mathbb{N}$ ⚠

$F_X : \mathbb{R} \longrightarrow [0,1]$

$x \longmapsto F_X(x) = P(X \leq x)$

1) $F_X(2,3) = P(X \leq 2,3)$

$= P(X=0) + P(X=1) + P(X=2)$ (X prend des valeurs entières)

2) $\lim\limits_{x \to -\infty} F_X(x) = \lim\limits_{x \to -\infty} P(X \leq 0) = 0$ car $X(\Omega) \subset \mathbb{N}$

$x < 0$ vide

$\lim\limits_{x \to +\infty} F_X(x) = \lim\limits_{x \to +\infty} P(X \leq x) = P(\Omega) = 1$

3) $\{X = k\} = \{X \leq k\} \setminus \{X \leq k-1\}$ car $X(\Omega) \in \mathbb{N}$

$P(X=k) = P(X \leq k) - P(X \leq k-1) = F_X(k) - F_X(k-1)$

4) Soit $(a,b) \in \mathbb{R}^2$ $a<b$

$P(a < x \leq b)$

$\{a < x \leq b\} = \{a < x\} \cap \{x \leq b\}$

or $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$

Ainsi

$P(a < x \leq b) = P(a < x) + P(x \leq b) - P(a < x \text{ ou } x \leq b)$

$= 1 - P(X \leq a) + P(X \leq b) - P(X \in \mathbb{N})$

$= 1 - F_X(a) + F_X(b) - 1 = F_X(b) - F_X(a)$