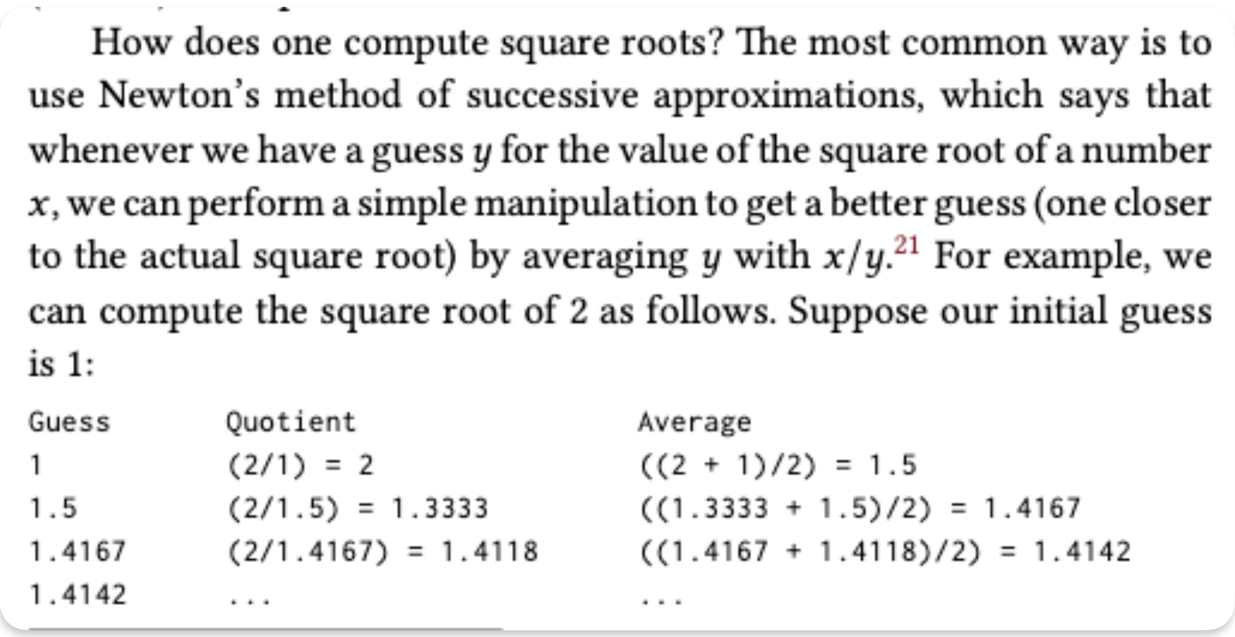
Newton's Successive Approximation



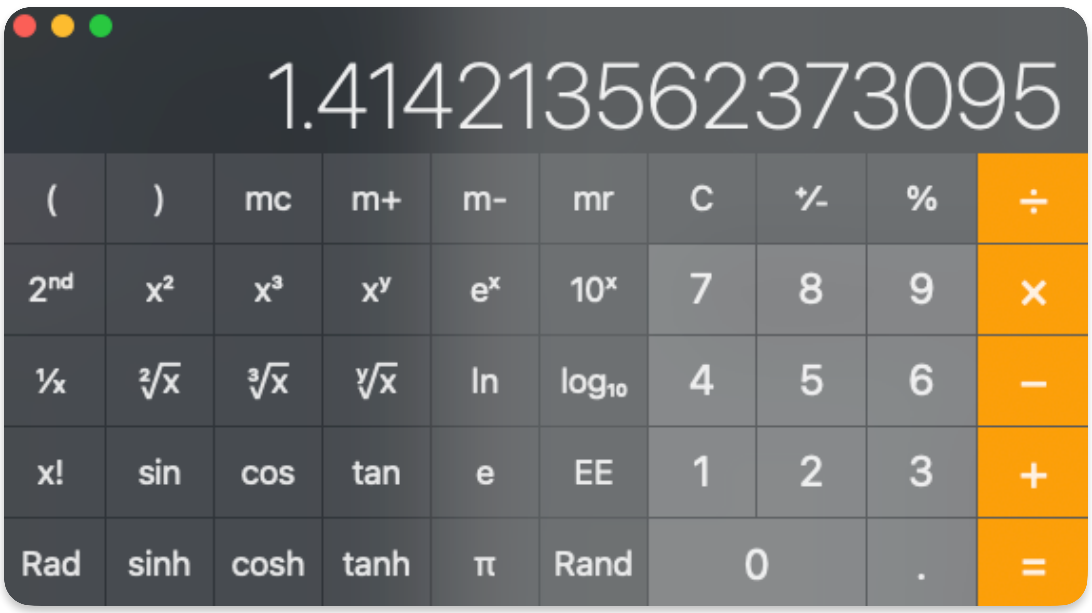
How does one compute square roots? The most common way is to use Newton's method of successive approximations, which says that whenever we have a guess y for the value of the square root of a number x, we can perform a simple manipulation to get a better guess (one closer to the actual square root) by averaging y with x/y.[21] For example, we can compute the square root of 2 as follows. Suppose our initial guess is 1:

Guess	Quotient	Average
1	(2/1) = 2	((2 + 1)/2) = 1.5
1.5	(2/1.5) = 1.3333	((1.3333 + 1.5)/2) = 1.4167
1.4167	(2/1.4167) = 1.4118	((1.4167 + 1.4118)/2) = 1.4142
1.4142	...	...

y	$x/y = q$	$(q + y) / 2 = y_2$
y_2	$x/y_2 = q_2$	$(q_2 + y_2) / 2 = y_3$

Example: *Square Root of **2***

1	2/1 = 2	(2 + 1) / 2 = *1.5*
1.5	2/1.5 = 1.3333	(1.3333 + 1.5) / 2 = *1.4167*
1.4167	2/1.4167 = 1.4118	(1.4167 + 1.4118) / 2 = *1.4142*



1.414213562373095

( ) mc m+ m- mr C ⁺∕₋ % ÷
2nd x² x³ xʸ eˣ 10ˣ 7 8 9 ×
¹⁄x ²√x ³√x ʸ√x ln log₁₀ 4 5 6 –
x! sin cos tan e EE 1 2 3 +
Rad sinh cosh tanh π Rand 0 . =

This could keep going, so you generally need to provide some threshold test in order to stop the recursion.

For example:

```
(define (good-enough? guess x)
  (< (abs (- (square guess) x)) 0.001))
```